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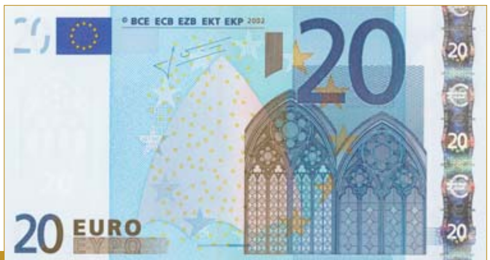

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1 The views expressed in this article do not necessarily reflect the views of the European Central Bank.

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Abstract

This paper compares the welfare implications of two widely used pricing assumptions in the New-Keynesian literature: Calvo-pricing vs. Rotemberg-pricing. We show that despite the strong similarities between the two assumptions to a first order of approximation, in general they might entail different welfare costs at higher order of approximation. In the special case of non-distorted steady state, the two pricing assumptions imply identical welfare losses to a second order of approximation.

Keywords: Calvo price adjustment; Rotemberg price adjustment; welfare; inflation; second-order approximation.

JEL classification: E3,E5.

Non technical summary

Most current dynamic stochastic general equilibrium (DSGE) models assume that prices (and/or wages) are sticky in nominal terms: i.e. they are not readjusted to the efficient level in each period. Under sticky prices, and in the face of exogenous shocks, there will generally be a gap between the efficient allocation of resources and the actual equilibrium allocation: e.g. too many or too few goods will be consumed and too many or too few hours will be worked. In general, it is not clear to which extent the welfare cost of price stickiness depends on the particular mechanism governing the price adjustments. In particular, the current macroeconomic literature has focused on a number of pricing mechanisms, of which the most common are Calvo price adjustments (Calvo, 1983) and Rotemberg price adjustments (Rotemberg, 1982).

This paper examines the welfare implications of these assumptions in the case when the steady state is inefficient. The main result is that the two assumptions give different results, with the tendency for Calvo to produce larger losses. As has been shown before, we confirm that in the special case when the steady state is efficient, the two assumptions produce identical results.

The mechanism proposed by Calvo assumes that each firm can re-set the price of its produce only at random intervals of time. This will make demand shift from one good to the other in a way that does not fully reflect the relative cost of production of the two goods. In other words, shocks will bring about a distribution of prices of goods that does not reflect the underlying marginal costs. Inflation, in this case, will be socially costly because it will be accompanied by an inefficient price dispersion.

The other pricing mechanism, proposed by Rotemberg, assumes that changing prices entails a real cost (e.g. in terms of goods or hours worked). In this case firms that produce at the same cost will set the same price, although this will not typically coincide with the efficient price. Shocks, therefore, will not produce an inefficient dispersion of prices. Instead, shocks will produce an additional consumption of scarce resources, which in turn will reduce social welfare.

One interesting feature of these two pricing mechanisms is that, to a first order of approximation, they yield the same set of dynamic equations: i.e. an identical Phillips curve. Most of the DSGE models that rely on these assumptions are estimated only up to the first order of approximation so that the two assumptions would be observationally equivalent.

Nevertheless, when the estimated DSGE models are used for welfare analysis (e.g. to assessing the welfare cost of alternative policies), the two pricing assumption might produce different results. So far, the literature has not shown exactly to which extent, and in which cases, the two pricing assumptions result in different welfare costs. Nisticò (2007) has shown that the two pricing assumptions imply identical welfare costs in a model with an efficient steady state (i.e. when efficiency is reached through subsidies to monopolistic firms). In this paper we show that this result holds only in that special case. In general, if the steady state is inefficient, the two pricing assumptions can yield different social costs of inflation. Whether the differences are quantitatively important will very likely depend on the other features of the model and in particular on the sources of shocks. Therefore, the results of our paper suggest that the particular pricing mechanism adopted in macroeconomic models could affect the derived policy prescriptions in important ways.

1 Introduction

Recently, a growing literature has dealt with the derivation of optimal policies in New-Keynesian models[1]. Most of this literature assumes that prices adjust at random intervals of time in a staggered fashion, following the pricing mechanism introduced by Calvo (1983) and Yun (1996). One of the alternative pricing assumptions widely discussed in the literature postulates that prices are adjusted only slowly to their optimal level in an identical way by all firms. Under this assumption, adjusting prices entails convex costs, as described by Rotemberg (1982).[2]

To a first order of approximation the two pricing assumptions are equivalent. Furthermore, in a recent paper, Nisticò (2007) shows that the two pricing assumptions (henceforth Calvo-pricing and Rotemberg-pricing) entail the same welfare losses when the steady state is efficient. Our paper shows that when the economy is allowed to fluctuate around an inefficient steady state, Calvo-pricing entails larger costs, for a given identical first order representation. Calvo-pricing implies a different degree of curvature of the economy than Rotemberg-pricing. By the Jensen-inequality, this implies that the expected value of the endogenous variables in the two models might differ. When the steady state of the economy is efficient, the linear term of the approximated welfare function drops out reducing the welfare measure to a function of quadratic terms only. On the contrary, when the steady state is inefficient, the linear term remains in the welfare function so that the different non-linearities are reflected in different expected levels of welfare.

The rest of the paper is organized as follows. First we describe the model used in the paper. In Section 3 we derive the first order representation of the economy under the two pricing assumptions. In Section 4 we derive the welfare measures and compare the two models. A final Section concludes the paper.

2 The Model

We use a very simple closed-economy dynamic stochastic general equilibrium (DSGE) model. This consists of a representative household purchasing a basket of differentiated goods and supplying homogeneous labor services. The differentiated goods are supplied by monopolistically competitive firms and produced using only labor services. Uncertainty is introduced via random fluctuations in labor productivity and in the subsidies paid to the firms. We introduce the latter shock as a source of inefficient fluctuations. The model is closed by assumptions regarding the decisions of the monetary authority concerning the short-run nominal interest rate.

We briefly discuss these assumptions in turn.

[1]See Woodford (2003) and the references cited therein.

[2]See for example Schmitt-Grohé and Uribe (2004).

2.1 Households

There is a continuum of identical households of unit mass. The preferences of the household are modelled as time-separable CRRA functions of the consumption basket (C) and of hours (l). Money (M) is introduced only for completeness, although it does not play any role in our discussion.

The household problem can be formalized as follows:

$$\max_{C_t, M_{t+1}, B_t, l_t} E_t \sum_{t=s}^{\infty} \beta^{t-s} \left\{ \frac{(C_t)^{1-\gamma}}{1-\gamma} + \frac{1}{1-\phi} \left(\frac{M_{t+1}}{P_t} \right)^{1-\phi} - \frac{l_t^{1+\varsigma}}{1+\varsigma} \right\} \tag{1a}$$

subject to the constraints

$$\begin{aligned} & \frac{B_t}{P_t} + C_t + \frac{M_{t+1}}{P_t} \\ & = \frac{M_t}{P_t} + \frac{R_{t-1} B_{t-1}}{P_t} + \frac{W_t}{P_t} l_t + \frac{\Pi_t}{P_t} + T_t \end{aligned} \tag{1b}$$

where B_t is a nominal bond in zero net supply, P_t is the aggregate price level, R_t is the short-run nominal interest rate paid on bonds, W_t is the nominal wage rate, Π_t is the share of profits rebated by the firms to the households and T_t is a transfer such that $M_{t+1} - M_t = T_t$. The first order conditions with respect to C_t,B_t and l_t are

$$C_t^{-\gamma} = \lambda_t \tag{2}$$

$$\lambda_t = R_t \beta E_t \lambda_{t+1} \frac{P_t}{P_{t+1}} \tag{3}$$

$$W_t = \zeta_t \frac{l_t^{\varsigma}}{\lambda_t} \tag{4}$$

where λ_t is the Lagrange's multiplier in the household maximization problem.

The consumption basket is defined as

$$C_t = \left(\int_0^1 c(i)^{\frac{\theta-1}{\theta}} \mathrm{di} \right)^{\frac{\theta-1}{\theta}}$$

where $c(i)$ denotes the specific i-th good. The associated price index is then given by

$$P_t = \left(\int_0^1 p(i)^{1-\theta} \mathrm{di} \right)^{\frac{1}{1-\theta}} \tag{5}$$

where $p(i)$ is the price of the i-th good.[3]

2.2 Firms

There is a continuum of monopolistically competitive firms of unit mass. Each firm produces a differentiated good. Each firm sets prices according to either the Calvo-pricing assumption or the Rotemberg-pricing assumption. We discuss the two pricing assumptions in the following subsections.

[3] From now on we drop the i-th index except for integrations.

2.2.1 Firms and price setting under Calvo pricing

Each firm chooses the optimal price by solving the following problem

$$\max_{p_t} E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{\lambda_{s+1}}{\lambda_s} \frac{\Pi_s}{P_s} = \max_{p_t} E_t \sum_{s=t}^{\infty} (\xi)^{s-t} \bar{R}_{s,t} \left[\frac{\omega_t \, p_t}{P_s} y_s - \frac{TC_s}{P_s} \right]$$

subject to

$$y_t = (\varepsilon_t l)$$

and to

$$y_t = \left(\frac{p_t}{P_t} \right)^{-\theta} C_t.$$

where TC denotes total costs of production, $\bar{R}_{s,t} = \beta^{s-t} \frac{\lambda_{s+1}}{\lambda_s}$ is the household nominal discount factor, ε_t is an AR(1) productivity shock and ω_t is a stochastic subsidy to firms.

The optimal price reduces to

$$\frac{p_t}{P_t} \equiv \bar{p}_t = \mu_f \frac{\bar{\mathbb{Q}}_{1,t}}{\bar{\mathbb{Q}}_{2,t}}$$

where

$$\overline{\mathbb{Q}}_{1,t}^{t} = mc_t C_s + \xi E_t \overline{R}_{t+1,t} \left(\pi_{t+1}\right)^{\theta} \overline{\mathbb{Q}}_{1,t+1}^{t}$$

and

$$\overline{\mathbb{Q}}_{2,t}^{t} = \omega_t C_t + \xi E_t \bar{R}_{t+1,t} \left(\pi_{t+1}\right)^{(\theta-1)} \overline{\mathbb{Q}}_{2,t+1}^{t}$$

where mc_t is the real marginal cost and $\pi_t \equiv \frac{P_t}{P_{t-1}}$.

2.2.2 Market clearing

Total demand for goods and labour must be equal to total supply of goods and labour, respectively. Bonds are in zero net supply.

Aggregate output.

$$\int_0^1 y(z)_t dz = C_t \int_0^1 \left(\bar{p}(z)_t\right)^{-\theta} dz = P_t^{\star} C_t$$

where

$$P_t^{\star} = (1-\xi)\, \bar{p}_t^{-\theta} + \xi \left(\pi_t\right)^{\theta} P_{t-1}^{\star} \tag{6}$$

is the price dispersion measure

Aggregate labor.

$$l_t = \int_0^1 l(z)_t \mathrm{dz} = \frac{mc_t}{W_t} \int_0^1 y(z)_t dz \tag{7}$$

Therefore the equilibrium wage can be written as

$$W_t = \frac{\zeta_t l_t^{\varsigma}}{\lambda_t^{c}}$$

Inflation. Noticing that in period t a fraction $(1-\xi)$ of firms sets the price p_t while a fraction ξ on average sets the price P_{t-1}, equation (5) gives

$$1 = \left[(1-\xi) (\bar{p}_t)^{1-\theta} + \xi \left(\frac{1}{\pi_t} \right)^{1-\theta} \right] \tag{8}$$

2.2.3 The Firms' problem under Rotemberg price setting

The firms' problem can now be formalized as follows:

$$\max_{p_t} E_t \sum_{s=t}^{\infty} (\beta)^{s-t} \bar{R}_{s,t} \left(\frac{\omega_t \, p_t}{P_t} y_t - TC_t - \frac{\phi}{2} \left(\frac{p_t}{p_{t-1}} - 1 \right)^2 C_t \right)$$

subject to

$$y_t = \varepsilon_t l.$$

and to

$$y_t = \left(\frac{p_t}{P_t} \right)^{-\theta} C_t.$$

After noting that all firms set the same price, so that $p_t = P_t$ the first order condition is

$$\begin{aligned} 0 &= (1-\theta) \, \omega_t y + \theta m c_t y - \phi \left(\pi_t - 1 \right) C_t \pi_t + \\ &+ E_t \beta \frac{\lambda_{t+1}^c}{\lambda_t^c} \phi \left(\pi_{t+1} - 1 \right) \left(\pi_{t+1} \right) C_{t+1} \end{aligned}$$

symmetry also implies that

$$y = Y_t = C_t + \frac{\phi}{2} \left(\pi_t - 1 \right)^2 C_t \tag{9}$$

Finally we can rewrite

$$\phi \left(\pi_t - 1 \right) Y_t \pi_t + (\theta - 1) \, \omega_t Y_t = \theta m c_t Y_t + E_t \beta \frac{\lambda_{t+1}^c}{\lambda_t^c} \phi \left(\pi_{t+1} - 1 \right) \left(\pi_{t+1} \right) Y_{t+1}$$

2.3 Monetary policy rule

The model can be closed by assumptions concerning the the monetary policy. In the numerical example we will consider the following policy rule:

$$R_t = \lambda_\pi \left(\frac{\pi_t}{\pi} - 1 \right)$$

3 First order approximation Phillips curves

To a first order of approximation, Calvo-pricing yields the following Phillips curve:

$$\widetilde{\pi}_t = \frac{(1-\xi\beta)(1-\xi)}{\xi}\widetilde{mc}_t - \frac{(1-\xi\beta)(1-\xi)}{\xi}\widetilde{\omega}_t + E_t\beta\widetilde{\pi}_{t+1} \quad (10)$$

Under Rotemberg-pricing to a first order of approximation the Phillips curve reduces to

$$\widetilde{\pi}_t = \frac{\omega(\theta-1)}{\phi}\widetilde{mc}_t + \frac{\omega(\theta-1)}{\phi}\widetilde{\omega}_t + E_t\beta\widetilde{\pi}_{t+1} \quad (11)$$

where $\widetilde{x} = \frac{x_t - x}{x}$ and where a variable without subscript denotes its steady-state value.

Therefor the two models are identical up to the coefficient on the marginal cost and on the cost-push shock.

Imposing that $\phi = \frac{\omega(\theta-1)\xi}{(1-\xi)(1-\xi\beta)}$ yields the same first-order dynamics in both models.

4 Welfare

Although the two models can be reduced to the same first-order representation, the welfare implications might differ. The following section derives the welfare function under the two specifications. Notice that from here on we denote $\widehat{x}_t = x_t - x$.

4.1 Welfare in the Calvo model

Under Calvo pricing the aggregate welfare function, conditional on information held by the policy maker at time t_0, takes the following form

$$V_{t_0} = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\mathcal{W}_t \quad (12)$$

where

$$\mathcal{W}_t \equiv \left\{\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{l_t^{1+\varsigma}}{1+\varsigma}\right\} \quad (13)$$

A second order Taylor approximation of the welfare function yields

$$\widehat{\mathcal{W}}_t = C^{-\gamma}\widehat{C}_t - l^{\varsigma}\widehat{l}_t - C^{-\gamma-1}\frac{1}{2}\gamma\widehat{C}_t^2 - l^{\varsigma-1}\frac{1}{2}\varsigma\widehat{l}_t^2 + t.i.p. + \mathcal{O}\left(\|\nu_t\|^3\right) \quad (14)$$

where ν_t is a vector containing the exogenous stochastic variables and where $t.i.p.$ collects the terms independent of policy (here first and second moments of the preference shock).

Noting that aggregate labor, adjusted for productivity, is identical to aggregate output we have that (henceforth omitting the error term for simplicity)

$$\widehat{l}_t + l\widehat{\varepsilon}_t + \widehat{l}_t\widehat{\varepsilon}_t = \frac{\omega(\theta-1)}{\theta}\left(C\widehat{P}_t^{\star} + \widehat{C}_t\right) \quad (15)$$

where we have taken into account the fact (shown in Appendix) that $P_t^\star$ is constant at its steady-state value when computed to a first order of approximation, so that the cross-product term does not appear in the expansion.

Combining equation (14) with equation (15) we obtain

$$\widehat{\mathcal{W}}_t^{\text{Calvo}} = \left(C^{-\gamma} - l^{\varsigma}\frac{\omega(\theta-1)}{\theta}\right)\widehat{C}_t + \\ -\frac{1}{2}C^{-\gamma-1}\gamma\widehat{C}_t^2 - l^{\varsigma-1}\frac{1}{2}\varsigma\widehat{l}_t^2 + l^{\varsigma}\widehat{\varepsilon}_t\widehat{l}_t - C\,l^{\varsigma}\frac{\omega(\theta-1)}{\theta}\widehat{P}_t^\star + t.i.p. \tag{16}$$

4.2 Welfare in the Rotemberg model

Under Rotemberg-pricing, equation (14) remain unchanged. What varies is equation (15). A second order expansion of equation (9) yields

$$\widehat{l}_t + l\widehat{\varepsilon}_t + \widehat{l}_t\widehat{\varepsilon}_t = \frac{\omega(\theta-1)}{\theta}\left(\widehat{C}_t + C\frac{\phi}{2}\widehat{\pi}_t^2\right) \tag{17}$$

By replacing this equation into equation (14) we obtain

$$\widehat{\mathcal{W}}_t^{\text{Rotemberg}} = \left(C^{-\gamma} - l^{\varsigma}\frac{\omega(\theta-1)}{\theta}\right)\widehat{C}_t + \\ -\frac{1}{2}C^{-\gamma-1}\gamma\widehat{C}_t^2 - l^{\varsigma-1}\frac{1}{2}\varsigma\widehat{l}_t^2 + l^{\varsigma}\widehat{\varepsilon}_t\widehat{l}_t - C\,l^{\varsigma}\frac{\phi}{2}\frac{\omega(\theta-1)}{\theta}\widehat{\pi}_t^2 + t.i.p. \tag{18}$$

4.3 Comparison under an efficient steady-state

We have seen that the two models are identical to a first order of approximation. Under the assumption $\frac{\omega(\theta-1)}{\theta} = 1$ we have that $C = l = 1$ so that the difference between the two measure of welfare just derived reduces to

$$\Delta^W \equiv \widehat{\mathcal{W}}_t^{\text{Calvo}} - \widehat{\mathcal{W}}_t^{\text{Rotemberg}} = \frac{\phi}{2}\widehat{\pi}_t^2 - \widehat{P}_t^\star \tag{19}$$

where, as shown in the Appendix

$$\widehat{P}_t^\star = \xi\widehat{P}_{t-1}^\star + 1/2\,\frac{\theta\,\xi}{1-\xi}\widehat{\pi}_t^2 \tag{20}$$

From a conditional welfare perspective have that

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\widehat{P}_t^\star = \frac{\xi}{1-\xi\beta}\widehat{P}_{t_0-1}^\star + E_{t_0}\sum_{t=t_0}^{\infty}\beta^t 1/2\,\frac{\theta\,\xi}{(1-\xi)(1-\beta\xi)}\widehat{\pi}_t^2 \tag{21}$$

so that

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta^W = \frac{\xi}{1-\xi\beta}\widehat{P}^{\star}_{t_0-1}+$$
$$+E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{\phi}{2}\left(1-\frac{\theta}{\omega(\theta-1)}\right)\widehat{\pi}_t^2 \tag{22}$$

where we have used the fact that, under the current assumptions, $\phi = \frac{\omega(\theta-1)\xi}{(1-\xi)(1-\xi\beta)}$. Since we have assumed that $\omega = \frac{\theta}{\theta-1}$ we can conclude that, the two pricing assumption yield identical welfare levels, conditional on $\widehat{P}^{\star}_{t_0-1} = 0$.

Notice that, from an unconditional-welfare perspective the last term in parenthesis in equation (22) would be

$$\left(1-\frac{(1-\xi\beta)}{(1-\xi)}\right) < 0. \tag{23}$$

Therefore, using an unconditional perspective, Calvo-pricing would result in larger welfare losses than Rotemberg-pricing.

4.4 Comparison under an inefficient steady-state

In the absence of subsidies to firms, i.e. if $\omega = 1$, equation (22) becomes

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta^W = \frac{C\, l^{\varsigma}\theta\xi}{(\theta-1)(1-\xi\beta)}\widehat{P}^{\star}_{t_0-1}+ \tag{24}$$
$$+\, l^{\varsigma}\left(1-\frac{(\theta-1)}{\theta}\right)E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\widehat{C}_t^{\text{Calvo}}-\widehat{C}_t^{\text{Rotemberg}}\right)$$
$$-\, C\frac{\phi}{2}l^{\varsigma}\left(1-\frac{(\theta-1)}{\theta}\right)E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\widehat{\pi}_t^2$$

This expression shows that the two pricing assumptions would produce the same level of welfare if and only if

$$\frac{\left(\widehat{C}_t^{\text{Calvo}}-\widehat{C}_t^{\text{Rotemberg}}\right)}{C} = \frac{\phi}{2}\pi_t^2 \tag{25}$$

In the next section we solve the model numerically to show that condition (25) does not hold in general.

4.4.1 Numerical example

Solving for the welfare gap analytically would be too cumbersome. In this subsection we provide instead a numerical example of the welfare gap when the steady state is inefficient.

For this purpose we provide two measures. The first is the conditional welfare gap while the second is the unconditional welfare gap. For this exercise we assumed $\beta = 0.99$, $\theta = 6$, $\gamma = 2$, $\varsigma = 2$, $\xi = 0.5$. As for the policy rule, we assume that the policy rate reacts only to current inflation with a coefficient of 1.5. We consider two shocks separately: a productivity shock and a shock to the subsidy to firms. These two shocks are intended to represent efficient shocks (no inflation-output trade off is produced) and inefficient shocks (an inflation-output trade-off is produced), respectively. Both shocks are assumed to be AR(1) with an auto-correlation coefficient of 0.9 and a standard deviation of 1. The results are presented in Table 1.[4] The values are expressed in welfare units.

Table 1: Welfare Gap

	Productivity	Subsidy
$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Delta^W =$	-20.86	-5.22
$E \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Delta^W =$	-22.51	-5.66

Looking more in details at the sources of these gaps we can see that expected consumption under Calvo-pricing is lower than expected consumption under Rotemberg-pricing. This consumption gap adds to the larger inefficiency wedge produced by inflation.

5 Conclusion

This paper has shown that two widespread assumptions in the current New-Keynesian literature concerning price adjustments can entail different welfare losses when the deterministic steady state of the economy is inefficient. The Calvo-pricing assumption implies a different curvature of the economy than the Rotemberg-pricing assumption. By Jensen-inequality, this implies that the expected value of the endogenous variables would, in general, differ across the two pricing mechanisms. Consequently, welfare is different across the two pricing assumptions.

[4]The results are obtained with DYNARE (version 4). The conditional mean is obtained iterating on the first- and second-order accurate state-space solution produced by DYNARE.

Appendix

6 Proof that $\widehat{P}^{\star} = 0 + \mathcal{O}\left(\|\nu\|^2\right)$

A second order expansion of (6) yields

$$\begin{aligned}\widehat{P}_t^{\star} &= \xi \widehat{P}_{t-1}^{\star} + \theta\left(\xi\widehat{\pi}_t - (1-\xi)\left(\widehat{\widetilde{p}}_t\right)\right) \\ &+ (1-\xi)\left(\frac{1}{2}\theta(\theta+1)\widehat{\widetilde{p}}_t^2\right) + \xi\left(\frac{1}{2}\theta(\theta-1)\widehat{\pi}_t^2 + \theta\widehat{\pi}_t\widehat{P}_{t-1}^{\star}\right)\end{aligned} \tag{26}$$

while a second order approximation of (8) produces

$$0 = \left[(1-\xi)\left((1-\theta)\widehat{\widetilde{p}}_t - \frac{1}{2}(1-\theta)\theta\widehat{\widetilde{p}}_t^2\right) + \xi\left((\theta-1)\widehat{\pi}_t + (\theta-1)(\theta-2)\widehat{\pi}_t^2\right)\right].$$

or

$$\xi\left(\widehat{\pi}_t\right) - (1-\xi)\left(\widehat{\widetilde{p}}_t\right) = \frac{1}{2}\left((1-\xi)\theta\left(-\widehat{\widetilde{p}}_t^2\right) - \xi(\theta-2)\left(\widehat{\pi}_t^2\right)\right). \tag{27}$$

By combining (26) with (27) we see that to a second order of approximation $\widehat{P}_t^{\star}$ depends only on quadratic terms. Therefore, to a first order of approximation $\widehat{P}_t^{\star} = 0 + \mathcal{O}\left(\|\nu\|^2\right)$. In particular by replacing the last expression into (26) and noting that

$$\left(\widehat{\widetilde{p}}_t\right)^2 = \left(\frac{\xi}{(1-\xi)}\right)^2\left(\widehat{\pi}_t\right)^2$$

we obtain

$$\begin{aligned}\widehat{P}_t^{\star} &= \xi\widehat{P}_{t-1}^{\star} - \theta\frac{1}{2}\left((1-\xi)\theta\left(\frac{\xi}{(1-\xi)}\right)^2\left(\widehat{\pi}_t\right)^2 + \xi(\theta-2)\left(\widehat{\pi}_t^2\right)\right) \\ &+ (1-\xi)\left(\frac{1}{2}\theta(\theta+1)\left(\frac{\xi}{(1-\xi)}\right)^2\left(\widehat{\pi}_t\right)^2\right) + \xi\left(\frac{1}{2}\theta(\theta-1)\widehat{\pi}_t^2 + \theta\widehat{\pi}_t\widehat{P}_{t-1}^{\star}\right)\end{aligned} \tag{28}$$

which reduces to

$$\widehat{P}_t^{\star} = \xi\widehat{P}_{t-1}^{\star} + 1/2\,\frac{\theta\,\xi}{1-\xi}\widehat{\pi}_t^2 \tag{29}$$

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