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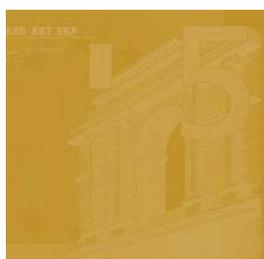
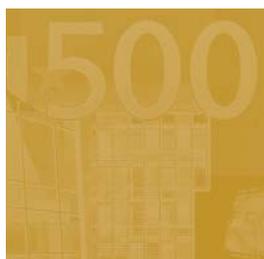
**RAMSEY MONETARY  
POLICY WITH LABOUR  
MARKET FRICTIONS**

by Ester Faia



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## WORKING PAPER SERIES

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# RAMSEY MONETARY POLICY WITH LABOUR MARKET FRICTIONS <sup>1</sup>

by Ester Faia <sup>2</sup>



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## Abstract

This paper studies the design of optimal monetary policy (in terms of unconstrained Ramsey allocation) in a framework with sticky prices and matching frictions. Furthermore I consider the role of real wage rigidities. Optimal policy features significant deviations from price stability in response to various shocks. This is so since search externalities generate an unemployment/inflation trade-off. In response to productivity shocks optimal policy is pro-cyclical when the worker's bargaining power is higher than the share of unemployed people in the matching technology and viceversa. This is so since when the workers' share of surplus is high there are many searching workers and few vacancies hence the monetary authority has an incentive to increase vacancy profitability by reducing the interest rate and increasing inflation. The opposite is true when the workers' share of surplus is high. This implies that optimal inflation volatility is U-shaped with respect to workers' bargaining power.

*JEL Codes:* E52, E24

*Keywords:* optimal monetary policy, matching frictions, wage rigidity.

## Non-Technical Summary

This paper derives optimal monetary policy in a model economy characterized by monopolistic competition and adjustment costs on pricing and matching frictions in the labour market. Furthermore I will also consider the role of real wage rigidity. Several papers in the recent literature study the effects of introducing matching frictions in a New Keynesian framework but very little has been done on the normative side. The assumption of monopolistic competition and adjustment cost on pricing a' la Rotemberg (1982) is needed to obtain non-neutral effects of monetary policy and to make a meaningful comparison across different monetary policy regimes. Introducing matching frictions a' la Mortensen and Pissarides (1999) in the labor market allows to consider frictional unemployment in the steady state and provides a rich dynamics for the formation and dissolution of employment relations. Finally the reason for considering the effects on the optimal policy design of real wage rigidity is twofold. First, several authors have argued that real wage rigidity helps to recover the typical unemployment-inflation trade-off commonly faced by central banks. Such trade-off, absent in standard new-keynesian models, is an essential feature to determine whether optimal monetary policy should deviate from full price stabilization. Secondly, some authors have shown that the introduction of real wage rigidity helps to resolve some inconsistencies between the standard matching friction model and the empirical evidence. The design of optimal policy in this paper follows the Ramsey approach according to which the monetary authority sets the optimal path of all variables in the economy by maximizing agents' welfare subject to the relations describing the competitive economy.

The economy described is characterized by three sources of inefficiency, both in the long and in the short run. The first is monopolistic competition which induces an inefficiently low level of output thereby calling for mild deviations from strict price stability<sup>1</sup>. The second type of distortion stems from the cost of adjusting prices which reduces output resources thereby calling for closing the "inflation gap". Finally the search theoretic framework is characterized by a congestion externality that tends to tighten the labour market. The chance that workers and firms have to match depends on the number of unemployed people or vacant firms in the market. Whether there is excessive vacancy creation or excessive unemployment depends on the bargaining power of workers: when the latter (hence the workers' share in the matching surplus) is too small there will be excessive vacancy creation due to the high profitability of a match for the firm and viceversa. In general Hosios (1990) has shown that the distance between current and efficient employment increases when the distance between workers' bargaining power and the fraction of searchers in the matching technology increases.

Optimal policy features significant deviations from price stability in response to various shocks. This is so since search externalities generate an unemployment/inflation trade-off which induces the monetary authority to strike a balance between reducing the cost of adjusting prices and increasing an inefficiently low employment. In response to productivity shocks optimal policy is pro-cyclical when the worker's bargaining power is higher than the share of unemployed people in the matching technology and viceversa. This is so since when the workers' share of surplus is high (firms' share is low) there are many searching workers and few vacancies hence the monetary authority has an incentive to increase vacancy profitability by reducing the interest rate and increasing inflation. The opposite is true when the workers' share of surplus is high. This also implies that optimal inflation volatility is U-shaped with respect to workers' bargaining power.

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<sup>1</sup>See Schmitt-Grohe and Uribe (2004) and Faia (2005) among others.

# 1 Introduction

Most central banks face an inflation-unemployment trade-off which implies that price stabilization can be achieved at the cost of higher unemployment. Moreover the ratio between the cost of inflation and the cost of unemployment is typically an increasing function of the degree of labor market rigidity. The trade-offs described suggest that the optimal monetary policy design should strike a balance between stabilizing inflation and fighting low employment. However most of the recent literature on the design of monetary policy concludes that optimality features either zero inflation or small deviations from price stability. This result has been established under different settings and characterizes both the closed and the open economy context<sup>1</sup>. One shortcoming of those studies is the lack of labor market rigidities which are a key ingredient in generating steady state unemployment and in characterizing employment fluctuations.

This paper derives optimal monetary policy in a model economy characterized by monopolistic competition and adjustment costs on pricing and matching frictions in the labour market. Furthermore I will also consider the role of real wage rigidity. Several papers in the recent literature study the effects of introducing matching frictions in a New Keynesian framework but very little has been done on the normative side. The assumption of monopolistic competition and adjustment cost on pricing a' la Rotemberg (1982) is needed to obtain non-neutral effects of monetary policy and to make a meaningful comparison across different monetary policy regimes. Introducing matching frictions a' la Mortensen and Pissarides (1999) in the labor market allows to consider frictional unemployment in the steady state and provides a rich dynamics for the formation and dissolution of employment relations<sup>2</sup>. Finally the reason for considering the effects on the optimal policy design of real wage rigidity is twofold. First, several authors have argued that real wage rigidity helps to recover the typical unemployment-inflation trade-off commonly faced by central banks<sup>3</sup>. Such trade-off, absent in standard new-keynesian models, is an essential feature to determine whether optimal monetary policy should deviate from full price stabilization. Secondly, some authors have

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<sup>1</sup>Zero inflation is the core result in the analysis of Woodford (2003), Clarida, Gali and Gertler (2000) who consider a monopolistic competitive framework with sticky prices a' la Calvo (1983). Lately Khan, King and Wolman (2003) and Schmitt-Grohe and Uribe (2004) have shown, using the Ramsey approach, that in presence of sticky prices and money distortions optimal policy implies small deviations from price stability and departure from the Friedman rule. The same is true in presence of capital accumulation, see Faia (2005). Finally Adao, Correia and Teles (2003) have shown by using a model with prices set one period in advance, that zero inflation is the optimal policy under a certain class of preferences.

<sup>2</sup>The introduction of matching frictions into a new Keynesian model has become common in the recent literature since it allows to replicate some empirical features. The laboratory economy that I use is very close to the one proposed in Krause and Lubik (2005). Several other authors, ranging from Walsh (2003) to Blanchard and Gali' (2005a,b), have recently introduced matching frictions and real wage rigidity into new Keynesian models.

<sup>3</sup>See Erceg, Henderson and Levin (2000), Canzoneri, Cumby and Diba (2004), and Blanchard and Gali' (2005) among others.

shown that the introduction of real wage rigidity helps to resolve some inconsistencies between the standard matching friction model and the empirical evidence<sup>4</sup>. The design of optimal policy in this paper follows the Ramsey approach according to which the monetary authority sets the optimal path of all variables in the economy by maximizing agents' welfare subject to the relations describing the competitive economy.

The economy described is characterized by three sources of inefficiency, both in the long and in the short run. The first is monopolistic competition which induces an inefficiently low level of output thereby calling for mild deviations from strict price stability<sup>5</sup>. The second type of distortion stems from the cost of adjusting prices which reduces output resources thereby calling for closing the "inflation gap". Finally the search theoretic framework is characterized by a congestion externality that tends to tighten the labour market. The chance that workers and firms have to match depends on the number of unemployed people or vacant firms in the market. Whether there is excessive vacancy creation or excessive unemployment depends on the bargaining power of workers: when the latter (hence the workers' share in the matching surplus) is too small there will be excessive vacancy creation due to the high profitability of a match for the firm and viceversa. In general Hosios (1990) has shown that the distance between current and efficient employment increases when the distance between workers' bargaining power and the fraction of searchers in the matching technology increases.

I find that in general optimal policy should deviate from price stability in response to different types of shocks. Contrary to previous studies deviations from the flexible price allocation are quantitatively significant. This is so search externalities generate an unemployment/inflation trade-off which induces the monetary authority to strike a balance between reducing the cost of adjusting prices and increasing an inefficiently low employment.

In response to productivity shocks optimal policy is pro-cyclical when the worker's bargaining power is higher than the share of unemployed people in the matching technology and countercyclical in the opposite case. The reason for this is as follows. When the workers' share of surplus is high and firms' share is low there are many searching workers and few vacancies. In this case the monetary authority has an incentive to increase vacancy profitability by reducing the interest rate and increasing demand. This obviously comes at the cost of higher inflation. The opposite is true when the workers' share of surplus is low. In general I find that the optimal inflation volatility is U-shaped with respect to workers' bargaining power and for given share of unemployed workers in the matching technology. In other words the monetary authority has an incentive to intervene and

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<sup>4</sup>See Hall (2003), Shimer (2003) and Krause and Lubik (2005).

<sup>5</sup>See Schmitt-Grohe and Uribe (2004) and Faia (2005) among others.

reduce the search externality by manipulating inflation only when the distance between current and efficient employment increases (e.g. when the distance between workers' bargaining power and the fraction of searchers in the matching technology increases).

The comparison between the dynamic under the Ramsey policy and the ones under other type of operational rules shows that the first induces higher variability of all variables. Under the types of rules considered, Taylor rule and the unemployment targeting, the monetary authority is only concerned with stabilization. On the contrary the Ramsey planner has the incentive to take full advantage of the productivity increase thereby exploiting all the benefits of the expansionary phase.

The findings in this paper are consistent with those in Cooley and Quadrini (2004). They study Ramsey monetary policy, both under commitment and discretion, in an economy with matching frictions and limited participation in financial market. Despite the fact that they have a different transmission mechanism they also reach the conclusion that the optimal policy should be procyclical in response to productivity shocks when the worker's bargaining power is higher than the share of unemployed people in the matching technology.

Blanchard and Gali' (2005) build a closed economy model with matching frictions and wage rigidity and conduct some normative analysis. They find that the output/inflation trade-off induces the monetary authority to deviate from strict inflation targeting. However their optimal policy analysis is based on a microfounded loss function and a loglinear approximation of the competitive equilibrium relations around a steady state where all distortions have been eliminated through an appropriate choice of the parameter space. This implies that the emergence of the output/inflation trade-off in their case is not directly related to the search externality, rather to the presence of sticky wages as in Erceg, Henderson and Levin (2000).

The paper proceeds as follow. Section 2 presents the model. Section 3 analyzes the optimal policy plan. Section 4 shows results for the long run optimal policy. Section 5 shows results for the optimal policy along the dynamics. Section 6 concludes.

## 2 The Model Economy

There is a continuum of agents whose total measure is normalized to one. The economy is populated by households who consume different varieties of goods, save and work. Households save in both non-state contingent securities and in an insurance fund that allows them to smooth income fluctuations associated with periods of unemployment. Each agent can indeed be either employed or unemployed. In the first case he receives a wage that is determined according to a Nash bargaining, in the second case he receives an unemployment benefit. The labor market is characterized

by matching frictions and exogenous job separation. The production sector acts as a monopolistic competitive sector which produces a differentiated good using labor as input and faces adjustment costs a' la Rotemberg (1982).

## 2.1 Households

Let  $c_t \equiv \int_0^1 [(c_t^i)^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}}$  be a Dixit-Stiglitz aggregator of different varieties of goods. The optimal allocation of expenditure on each variety yields is given by  $c_t = \left(\frac{p_t^i}{p_t}\right)^{-\epsilon} c_t$ , where  $p_t \equiv \int_0^1 [(p_t^i)^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}}$  is the price index. There is continuum of agents who maximize the expected lifetime utility<sup>6</sup>.

$$E_t \left\{ \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right\} \quad (1)$$

where  $c$  denotes aggregate consumption in final goods. Households supply labor hours inelastically  $h$  (which is normalized to 1). Total real labor income is given by  $w_t$  and is specified below. Unemployed households members,  $u_t$ , receive an unemployment benefit,  $b$ . The contract signed between the worker and the firm specifies the wage and is obtained through a Nash bargaining process. In order to finance consumption at time  $t$  each agent also invests in non-state contingent nominal bonds  $b_t$  which pay a gross nominal interest rate  $(1+r_t^n)$  one period later. As in Andolfatto (1996) and Merz (1995) it is assumed that workers can insure themselves against earning uncertainty and unemployment. For this reason the wage earnings have to be interpreted as net of insurance costs. Finally agents receive profits from the monopolistic sector which they own,  $\Theta_t$ , and pay lump sum taxes,  $\tau_t$ . The sequence of real budget constraints reads as follows:

$$c_t + \frac{b_t}{p_t} \leq w_t + bu_t + \frac{\Theta_t}{p_t} - \frac{\tau_t}{p_t} + (1+r_{t-1}^n) \frac{b_{t-1}}{p_t} \quad (2)$$

Households choose the set of processes  $\{c_t, b_t\}_{t=0}^{\infty}$  taking as given the set of processes  $\{p_t, w_t, r_t^n\}_{t=0}^{\infty}$  and the initial wealth  $b_0$ , so as to maximize (1) subject to (2). The following optimality conditions must hold:

$$\lambda_t = c_t^{-\sigma} \quad (3)$$

$$c_t^{-\sigma} = \beta(1+r_t^n) E_t \left\{ c_{t+1}^{-\sigma} \frac{p_t}{p_{t+1}} \right\} \quad (4)$$

<sup>6</sup>Let  $s^t = \{s_0, \dots, s_t\}$  denote the history of events up to date  $t$ , where  $s_t$  denotes the event realization at date  $t$ . The date 0 probability of observing history  $s^t$  is given by  $\rho_t$ . The initial state  $s^0$  is given so that  $\rho_0 = 1$ . Henceforth, and for the sake of simplifying the notation, let's define the operator  $E_t\{\cdot\} \equiv \sum_{s_{t+1}} \rho(s^{t+1}|s^t)$  as the mathematical expectations over all possible states of nature conditional on history  $s^t$ .

Equation (3) is the marginal utility of consumption and equation (4) is the Euler condition with respect to bonds. Optimality requires that No-Ponzi condition on wealth is also satisfied.

## 2.2 The Production Sector

Firms in the production sector sell their output in a monopolistic competitive market and meet workers on a matching market. The labor relations are determined according to a standard Mortensen and Pissarides (1999) framework. Workers must be hired from the unemployment pool and searching for a worker involves a fixed cost. Workers wages are determined through a Nash decentralized bargaining process which takes place on an individual basis.

### 2.2.1 Search and Matching in the Labor Market

The search for a worker involves a fixed cost  $\kappa$  and the probability of finding a worker depends on a constant return to scale matching technology which converts unemployed workers  $u$  and vacancies  $v$  into matches,  $m$ :

$$m(u_t, v_t) = mu_t^\xi v_t^{1-\xi} \quad (5)$$

where  $v_t = \int_0^1 v_{i,t} di$ . Defining labor market tightness as  $\theta_t \equiv \frac{v_t}{u_t}$ , the firm meets unemployed workers at rate  $q(\theta) = \frac{m(u_t, v_t)}{v_t} = m\theta_t^{-\xi}$ , while the unemployed workers meet vacancies at rate  $\theta_t q(\theta_t) = m\theta_t^{1-\xi}$ . If the search process is successful, the firm in the monopolistic good sector operates the following technology:

$$y_{i,t} = z_t n_{i,t} \quad (6)$$

where  $z_t$  is the aggregate productivity shock which follows a first order autoregressive process,  $e^{z_t} = e^{\rho z_{t-1} + \varepsilon_{z,t}}$ , and  $n_{i,t}$  is the number of workers hired by each firm. Matches are destroyed at an exogenous rate  $\rho^7$ . We are now in the position to determine the law of motion for the workers employed and the ones seeking for a job. Labor force is normalized to unity. The number of employed people at time  $t$  in each firm  $i$  is given by the number of employed people at time  $t-1$  plus the flow of new matches concluded in period  $t-1$  who did not discontinue the match:

$$n_{i,t} = (1 - \rho)(n_{i,t-1} + v_{i,t-1} q(\theta_{i,t-1})) \quad (7)$$

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<sup>7</sup>The alternative assumption of endogenous job destruction would induce, consistently with empirical observations, additional persistence to the model as shown in denHaan, Ramsey and Watson (2000). However due to the normative focus of this paper I choose the more simple assumption of exogenous job destruction. This greatly reduces the complexity of the numerical solution to the optimal policy problem without altering the results compared to the alternative assumption of endogenous job destruction. Indeed the main policy trade-offs do not change under the two alternative assumptions.

Unemployment is given by total labor force minus the number of employed workers:

$$u_t = 1 - n_t \quad (8)$$

Finally job creation rate is given by:

$$jc_t = \frac{(1 - \rho)v_{t-1}q(\theta_{t-1})}{n_{t-1}} \quad (9)$$

### 2.2.2 Monopolistic Firms

Firms in the monopolistic sector use labor to produce different varieties of consumption good and face a quadratic cost of adjusting prices. Wages are determined through the bargaining problem analyzed in the next section. Here I develop the dynamic optimization decision of firms choosing prices,  $p_{h,t}^i$ , number of employees,  $n_{i,t}$ , number of vacancies,  $v_{i,t}$ , to maximize the discounted value of future profits and taking as given the wage schedule. The representative firm chooses  $\{p_t^i, n_{i,t}, v_{i,t}\}$  to solve the following maximization problem (in real terms):

$$Max \Pi_{i,t} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left\{ \frac{p_t^i}{p_t} y_t^i - w_{i,t} n_{i,t} - \kappa v_{i,t} - \frac{\psi}{2} \left( \frac{p_t^i}{p_{t-1}^i} - 1 \right)^2 y_t^i \right\} \quad (10)$$

subject to

$$\text{s.to: } y_t^i = \left( \frac{p_t^i}{p_t} \right)^{-\epsilon} y_t = z_t n_{i,t} \quad (11)$$

$$\text{and: } n_{i,t} = (1 - \rho)(n_{i,t-1} + v_{i,t-1}q(\theta_{i,t-1})) \quad (12)$$

where  $\frac{\psi}{2} \left( \frac{p_t^i}{p_{t-1}^i} - 1 \right)^2 y_t^i$  represent the cost of adjusting prices,  $\psi$  can be thought as the sluggishness in the price adjustment process,  $\kappa$  as the cost of posting vacancies and  $w_t$  denotes the fact that the bargained wage might depend on time varying factors. Let's define  $mc_t$ , the lagrange multiplier on constraint (11), as the marginal cost of firms and  $\mu_t$ , the lagrange multiplier on constraint (12), as the marginal value of one worker. Since all firms will chose in equilibrium the same price and allocation we can now assume symmetry and drop the index  $i$ . First order conditions for the above problem read as follows:

- $n_t$  :

$$\mu_t = mc_t z_t - w_t + \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) ((1 - \rho) \mu_{t+1}) \quad (13)$$

- $v_t$  :

$$\frac{\kappa}{q(\theta_t)} = \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) ((1 - \rho) \mu_{t+1}) \quad (14)$$



•  $p_t$  :

$$1 - \psi(\pi_t - 1)\pi_t + \beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)[\psi(\pi_{t+1} - 1)\pi_{t+1}\frac{y_{t+1}}{y_t}] = (1 - mc_t)\varepsilon \quad (15)$$

Merging equations (13) and (14) rearranging we obtain the marginal cost of firms,  $mc_t$ :

$$mc_t = \frac{\mu_t - \frac{\kappa}{q(\theta_t)}}{z_t} + \frac{w_t}{z_t} \quad (16)$$

As already noticed in Krause and Lubik (2005) in a matching model the marginal cost of firms is not only given by the marginal productivity of each single employee,  $\frac{w_t}{z_t}$ , as it is in a standard walrasian model but contains an extra component,  $\frac{\mu_t - \frac{\kappa}{q(\theta_t)}}{z_t}$ , which depends on the future value of each employee. Posting vacancy is costly hence a successful match today is valuable also since it reduces future search costs.

### 2.2.3 Bellman Equations, Wage Setting and Nash Bargaining

The wage schedule is obtained through the solution to an individual Nash bargaining process. To solve for it we need first to derive the marginal values of a match for both, firms and workers. Those values will indeed enter the sharing rule of the bargaining process. Let's denote by  $V_t^J$  the marginal discounted value of a match for a firm:

$$V_t^J = mc_t z_t - w_t + E_t\left\{\left(\beta \frac{\lambda_{t+1}}{\lambda_t}\right)[(1 - \rho)V_{t+1}^J]\right\} \quad (17)$$

The marginal value of a match depends on real revenues minus the real wage plus the discounted continuation value. With probability  $(1 - \rho)$  the job remains filled and earns the expected value and with probability,  $\rho$ , the job is destroyed and has zero value. Using the equation (16) we can rewrite equation (17) as:

$$V_t^J = \frac{-\kappa}{q(\theta_t)} + E_t\left\{\left(\beta \frac{\lambda_{t+1}}{\lambda_t}\right)[(1 - \rho)V_{t+1}^J]\right\} \quad (18)$$

Since the net value of a match for the firm must be zero in equilibrium the following zero profit condition must be satisfied:

$$\frac{\kappa}{q(\theta_t)} = E_t\left\{\left(\beta \frac{\lambda_{t+1}}{\lambda_t}\right)[(1 - \rho)V_{t+1}^J]\right\} \quad (19)$$

Equation (19) is an arbitrage condition for the posting of new vacancies. It implies that in equilibrium the cost of posting a vacancy must equate the discounted expected return from posting the vacancy. For each worker, the values of being employed and unemployed are given by  $V_t^E$  and  $V_t^U$ :

$$V_t^E = [w_t + E_t\left\{\left(\beta \frac{\lambda_{t+1}}{\lambda_t}\right)[(1 - \rho)V_{t+1}^E + \rho V_{t+1}^U]\right\}] \quad (20)$$

$$V_t^U = [b + E_t\{(\beta \frac{\lambda_{t+1}}{\lambda_t})[\theta_t q(\theta_t)(1 - \rho)V_{t+1}^E + (1 - \theta_t q(\theta_t)(1 - \rho))V_{t+1}^U]\}] \quad (21)$$

where  $b$  denotes real unemployment benefits.

Workers and firms are engaged in a Nash bargaining process to determine wages. The optimal sharing rule of the standard Nash bargaining is given:

$$(V_t^E - V_t^U) = \frac{\varsigma}{1 - \varsigma} V_t^J \quad (22)$$

After substituting the previously defined value functions it is possible derive the following wage schedule:

$$w_t = \varsigma(m c_t z_t + \theta_t \kappa) + (1 - \varsigma)b \quad (23)$$

### 2.3 Equilibrium Conditions

Aggregate output is obtained by aggregating production of individual firms. I assume that there is exogenous government expenditure financed through lump sum taxation. Hence the resource constraint reads as follows:

$$y_t = n_t z_t = c_t + g_t + \kappa v_t + \frac{\psi}{2} (\pi_t - 1)^2 y_t^i \quad (24)$$

Furthermore I assume zero total net supply of bonds.

**Definition 1.** *A distorted competitive equilibrium for this economy is a sequence of allocation and prices  $\{c_t, u_t, n_t, v_t, \theta_t, \pi_t, y_t, w_t, r_t^n, m c_t\}_{t=0}^{\infty}$  which, for given initial  $B_0$  satisfies equations*

*(4), (7), (8), (13), (14), (15), (23),*

*(24) and  $\theta_t \equiv \frac{v_t}{u_t}$ .*

## 3 The Optimal Policy Problem

The optimal policy is determined by a monetary authority that maximizes the discounted sum of utilities of all agents given the constraints of the competitive economy. The next task is to select the relations that represent the relevant constraints in the planner's optimal policy problem. This amounts to describing the competitive equilibrium in terms of a minimal set of relations involving only real allocations, in the spirit of the primal approach described in Lucas and Stokey (1983). There is a fundamental difference, though, between that classic approach and the one followed here, which stems from the impossibility, in the presence of sticky prices and matching frictions, of reducing the planner's problem to a maximization only subject to a single implementability constraint<sup>8</sup>.

<sup>8</sup>See also Khan, King and Wolman (2003), Schmitt-Grohe and Uribe (2002) and Faia (2005).

The first order conditions of the consumers can be summarized as follows:

$$c_t^{-\sigma} = \beta(1 + r_t^n)E_t \left\{ c_{t+1}^{-\sigma} \frac{p_t}{p_{t+1}} \right\} \quad (25)$$

The first order conditions and the constraints of the production sector can be summarized as follows:

$$1 - \psi(\pi_t - 1)\pi_t + \beta E_t \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} [\psi(\pi_{t+1} - 1)\pi_{t+1} \frac{y_{t+1}}{y_t}] = (1 - mc_t)\varepsilon \quad (26)$$

$$\frac{\kappa}{m} \theta_t^\xi = E_t \left\{ \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} (1 - \rho) [(1 - \varsigma)mc_{t+1}z_{t+1} - \varsigma\theta_{t+1}\kappa - (1 - \varsigma)b + \frac{\kappa}{m} \theta_{t+1}^\xi] \right\} \quad (27)$$

where  $\theta_t \equiv \frac{v_t}{u_t}$  and  $u_t = 1 - n_t$ . Finally need to include the resource constraint and the employment dynamic:

$$n_t z_t = c_t + g_t + \kappa v_t + \frac{\psi}{2} (\pi_t - 1)^2 y_t \quad (28)$$

$$n_t = (1 - \rho)(n_{t-1} + v_{t-1} m u_t^\xi v_t^{1-\xi}) \quad (29)$$

### 3.1 The Role of Frictions in This Economy

As discussed previously this economy is characterized by three main frictions: price stickiness, monopolistic competition and matching frictions in the labor market. A monetary policy maker endowed with a single instrument is not able eliminate all the three distortions but can only trade-off among them. To better understand the type of trade-offs present in this model economy it is useful to discuss the role of each friction singularly and the level of the policy instrument required to offset them.

I start by analyzing the role of matching frictions in the labor market since they provide the novel aspect for the design of the optimal policy in the context of new keynesian models. To understand how search externality distort the competitive equilibrium let's assume for simplicity that those are the only frictions characterizing the model economy and let's derive the conditions for *constrained pareto efficiency*. The optimal policy problem of the planner in this context (and in absence of government expenditure) can be written as follows:

$$Max E_t \left\{ \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right\} \quad (30)$$

s. to

$$z_t n_t - \kappa v_t = c_t \quad (31)$$

$$n_t = (1 - \rho)(n_{t-1} + v_{t-1}q(\theta_{t-1})) \quad (32)$$

where  $\theta_t \equiv \frac{v_t}{u_t}$ ,  $u_t = 1 - n_t$  and  $q(\theta) = m\theta_t^{-\xi}$ . Let's define as  $\psi_{1,t}$  and  $\psi_{2,t}$  respectively the lagrange multipliers on constraints (31) and (32). First order conditions for this problem are as follows:

- $c_t$  :

$$c_t^{-\sigma} - \psi_{1,t} = 0 \quad (33)$$

- $n_t$  :

$$\psi_{1,t}z_t - \psi_{2,t+1}(1 - \rho)mv_{t+1}^{1-\xi}\xi(1 - n_{t+1})^{\xi-1} + \psi_{2,t+1} = 0 \quad (34)$$

- $v_t$  :

$$-\psi_{1,t}\kappa - \psi_{2,t}m(1 - n_t)^{\xi-1}(1 - \rho)(1 - \xi)v_t^{-\xi} = 0 \quad (35)$$

After rearranging and merging the above first order conditions we obtain the following:

$$\frac{\kappa}{m}\theta_t^\xi = E_t\left\{\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}(1 - \rho)[(1 - \xi)mc_{t+1}z_{t+1} - \xi\theta_{t+1}\kappa - (1 - \xi)b + \frac{\kappa}{m}\theta_{t+1}^\xi]\right\} \quad (36)$$

Notice that equation (36) is equivalent to (27) when  $\xi = \varsigma$ . This is exactly the condition that Hosios (1990) suggests to achieve constrained pareto efficiency in an economy with matching frictions. Efficiency requires workers' bargaining power being equivalent to their share in the matching technology. When workers bargaining power is too low ( $\xi \leq \varsigma$ ) firms find too profitable to form a match thereby inducing excessive vacancy creation. Viceversa, when workers' bargaining power is too high there is excessive unemployment. If the condition for efficiency are not met an obvious solution is to endow the policy maker with a complementary subsidy that forces  $\xi = \varsigma$ . As we shall see below in absence of such subsidy the monetary authority can use inflation to control for the optimal level of unemployment in the economy.

Let's now consider the role of price stickiness and monopolistic competition. *Price stickiness* induces a gap with the flexible price allocation since part of resources are wasted in the activity of adjusting prices,  $(\pi_t - 1)^2$ . Finally monopolistic competition reduces the level of economic activity by decreasing optimal demand, hence the marginal cost to firms (see also Schmitt-Grohe and Uribe (2004)).

We are now in the position to determine the level of the policy instrument which can offset each distortion at the time. Obviously the cost of adjusting prices can be eliminated by setting  $\pi_t = 1$  at all times, thereby following a strict price stability policy. From the Phillips curve it is obvious that an increase in the marginal cost (hence a reduction of mark-up and an increase

in demand) can be achieved by setting positive inflation. Both price stickiness and monopolistic competition can be directly affected by the inflation level. As for the matching frictions the policy maker can have only an indirect impact on those externalities by using inflation. Consider the marginal discounted value of a match for a firm:

$$V_t^J = mc_t z_t - w_t + E_t\left\{\beta \frac{\lambda_{t+1}}{\lambda_t}\right\}[(1 - \rho)V_{t+1}^J] \quad (37)$$

After solving recursively we obtain:

$$V_t^J = \sum_{j=0}^{\infty} E_{t+j}\left\{\beta \frac{\lambda_{t+j+1}}{\lambda_{t+j}}(1 - \rho)\Pi_{t+j}\right\} \quad (38)$$

where  $\Pi_{t+j} = mc_{t+j}z_{t+j} - w_{t+j}$ . Since the discounted value of a match must equate the cost of posting vacancy discounted by the probability of filling a vacancy we can also conclude that:

$$\frac{\kappa}{q(\theta_t)} = \sum_{j=0}^{\infty} E_{t+j}\left\{\beta \frac{\lambda_{t+j+1}}{\lambda_{t+j}}(1 - \rho)\Pi_{t+j}\right\} \quad (39)$$

A reduction in the mark-up (or an increase in demand and marginal costs) achieved through an increase in inflation can increase unitary profits,  $\Pi_{t+j} = mc_{t+j}z_{t+j} - w_{t+j}$ , at each point in time. An increase in unitary profits, for given cost of posting vacancy  $\kappa$ , can reduce labor market tightness,  $q(\theta_t)$ , thereby reducing the congestion externality. Positive inflation can therefore have an indirect impact on the probability of forming matches through its impact on the demand for varieties.

To summarize we have on the one side two distortions, monopolistic competition and search externality, that call for positive inflation while we have on the other side a third distortion, price stickiness, which calls for zero net inflation. Since the policy maker should trade-off among those three distortions we expect the optimal policy to deviate from strict price stability.

### 3.2 The Optimal Policy Problem Under Commitment

I now turn to the specification of a general set-up for the optimal policy conduct.

**Definition 2.** Let  $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}\}_{t=0}^{\infty}$  represent sequences of Lagrange multipliers on the constraints (25), (26), (27), (28) and (29) respectively. Then for given stochastic processes  $\{z_t, g_t\}_{t=0}^{\infty}$  and for given  $B_0$  plans for the control variables  $\{c_t, n_t, v_t, \pi_t, r_t^n, mc_t\}_{t=0}^{\infty}$  and for the co-state variables  $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}\}_{t=0}^{\infty}$  represent a first best constrained allocation if they solve the following maximization problem:

Choose  $\Lambda_t^n \equiv \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}\}_{t=0}^{\infty}$  and  $\Xi_t^n \equiv \{c_t, n_t, v_t, \pi_t, r_t^n, mc_t\}_{t=0}^{\infty}$  to

$$\text{Min}_{\{\Lambda_t^n\}_{t=0}^\infty} \text{Max}_{\{\Xi_t^n\}_{t=0}^\infty} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t E_t \{ U(C_t, N_t) \right. \quad (40)$$

$$\begin{aligned} & + \lambda_{1,t} \left[ c_t^{-\sigma} - \beta(1+r_t^n) E_t \left\{ c_{t+1}^{-\sigma} \frac{1}{\pi_{t+1}} \right\} \right] + \\ & + \lambda_{2,t} [1 - \psi(\pi_t - 1)\pi_t y_t c_t + \beta E_t (c_{t+1})^{-\sigma} [\psi(\pi_{t+1} - 1)\pi_{t+1} y_{t+1}] - (1 - mc_t)c_t y_t \varepsilon] + \\ & + \lambda_{3,t} \left[ \frac{\kappa}{m} \theta_t^\xi - E_t \left\{ \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} (1 - \rho) [(1 - \varsigma) m c_{t+1} z_{t+1} - \varsigma \theta_{t+1} \kappa - (1 - \varsigma) b + \frac{\kappa}{m} \theta_{t+1}^\xi] \right\} \right] + \\ & + \lambda_{4,t} \left[ n_t z_t - \kappa v_t - \frac{\psi}{2} (\pi_t - 1)^2 y_t - c_t - g_t \right] + \\ & + \lambda_{5,t} \left[ n_t - (1 - \rho)(n_{t-1} + v_{t-1} m u_t^\xi v_t^{1-\xi}) \right] \} \end{aligned}$$

where  $\theta_t \equiv \frac{v_t}{u_t}$  and  $u_t = 1 - n_t$

### 3.2.1 Non-recursivity and Initial Conditions

As a result of the constraint (25), (26) and (27) exhibiting future expectations of control variables, the maximization problem as spelled out in (40) is intrinsically non-recursive.<sup>9</sup> As first emphasized in Kydland and Prescott (1980), and then developed by Marcet and Marimon (1999), a formal way to rewrite the same problem in a recursive stationary form is to enlarge the planner's state space with additional (pseudo) co-state variables. Such variables, that I denote  $\chi_{1,t}$ ,  $\chi_{2,t}$  and  $\chi_{3,t}$  for (25), (26) and (27) respectively, bear the crucial meaning of tracking, along the dynamics, the value to the planner of committing to the pre-announced policy plan. Another aspect concerns the specification of the law of motion of these lagrange multipliers. For in this case both constraints feature a simple one period expectation, the same co-state variables have to obey the laws of motion<sup>10</sup>:

$$\begin{aligned} \chi_{1,t+1} &= \lambda_{1,t} \\ \chi_{2,t+1} &= \lambda_{2,t} \\ \chi_{3,t+1} &= \lambda_{3,t} \end{aligned} \quad (41)$$

<sup>9</sup>See Kydland and Prescott (1977), Calvo (1978). As such the system does not satisfy per se the principle of optimality, according to which the optimal decision at time t is a time invariant function only of a small set of state variables.

<sup>10</sup>The laws of motion of the additional costate variables would take a more general form if the expectations horizon in the forward looking constraint(s) featured a more complicated structure, as, for instance, in the case of constraints in present value form. See Marcet and Marimon (1999).

Appendix A shows the stationary Lagrangian problem which is indeed recursive in the state space  $\{z_t, g_t, \chi_{1,t}, \chi_{2,t}, \chi_{3,t}\}$ . To avoid time consistency problems and consistently with a *timeless perspective* I set the values of the three co-state variables at time zero equal to their solution in the steady state. I will return on this point in the next subsection.

### 3.3 Calibration

**Preferences.** Time is measured in quarters. I set the discount factor  $\beta = 0.99$ , so that the annual interest rate is equal to 4 percent. The parameter on consumption in the utility function is set equal to 2.

**Production.** Following Basu and Fernald (1997) I set the value added mark-up of prices over marginal cost to 0.2. This generates a value for the price elasticity of demand,  $\varepsilon$ , of 6. I set the cost of adjusting prices  $\psi = 50$  so as to generate a slope of the log-linear Phillips curve consistent with empirical and theoretical studies.

**Labor market frictions parameters.** The matching technology is homogenous of degree one function and is characterized by the parameter  $\xi$ . Consistently with estimates by Blanchard and Diamond (1989) I set this parameter to 0.4. I set the steady state firm matching rate,  $q(\theta)$ , to 0.7 which is the value used by denHaan, Ramsey and Watson (1997). The probability for a worker of finding a job,  $\theta q(\theta)$ , is set equal to 0.6, which implies an average duration of unemployment of 1.67 as reported in Cole and Rogerson (1996). With those values it is possible to determine the number of vacancies as well as the vacancy/unemployment ratio. The exogenous separation probability,  $\rho$ , is set to 0.08 consistently with estimates from Hall (1995) and Davis et al. (1996); this value is also compatible with those used in the literature which range from 0.7 (Merz (1995)) to 0.15 (Andolfatto (1996)). The degree of wage rigidity,  $\lambda$ , is set equal to 0.6 and is compatible with estimates from Smets and Wouters (2003). The value for  $b$  is set so as to generate a steady state ratio,  $\frac{b}{w}$ , of 0.5 which corresponds to the average value observed for industrialized countries (see Nickell and Nunziata (2001)). The steady state scale parameter,  $m$ , is obtained using the observation that steady state number of matches is given by  $\frac{\rho}{1-\rho}(1-u)$ . The bargaining power of workers,  $\varsigma$ , is set to 0.5 as in most papers in the literature, while the value for the cost of posting vacancies is obtained from the steady state version of labour market tightness evolution.

**Exogenous shocks and monetary policy:** The process for the aggregate productivity shock,  $z_t$ , follows an AR(1) and based on the RBC literature is calibrated so that its standard deviations is set to 0.008 and its persistence to 0.95. Log-government consumption evolves according to the following exogenous process,  $\ln\left(\frac{g_t}{g}\right) = \rho_g \ln\left(\frac{g_{t-1}}{g}\right) + \varepsilon_t^g$ , where the steady-state share of government consumption,  $g$ , is set so that  $\frac{g}{y} = 0.25$  and  $\varepsilon_t^g$  is an i.i.d. shock with standard

deviation  $\sigma_g$ . Empirical evidence for the US in Perotti (2004) suggests  $\sigma_g = 0.008$  and  $\rho_g = 0.9$ .

## 4 Steady State Optimal Policy

Before turning to the optimal stabilization policy in response to shocks we need to characterize the long-run optimal policy, which is the one to which the policy maker would like to converge. To develop an analogy with the Ramsey-Cass-Koopmans model, this amounts to computing the *modified golden rule* steady state. To determine the long-run inflation rate associated to the optimal policy problem above, one needs to solve the steady-state version of the set of efficiency conditions. Notice in particular that the first order condition with respect to inflation reads as follows:

$$(\lambda_{2,t} - \chi_{2,t})c_t^{-\sigma}\psi(2\pi_t - 1)y_t - \lambda_{4,t}\psi(\pi_t - 1)y_t = 0 \quad (42)$$

For the whole set of optimality conditions of the Ramsey plan to be satisfied in the steady state a necessary condition is that equation (42) is satisfied in the steady state. In that steady-state, we have  $\lambda_{2,t} = \lambda_{2,t-1} = \chi_{2,t}$ . Hence condition (42) immediately implies:

$$\lambda_4\psi(\pi - 1)y = 0 \quad (43)$$

Since  $\lambda_4 > 0$  (the resource constraint must hold with equality),  $y > 0$  and  $\psi > 0$  (we are not imposing *a priori* that the steady-state coincides with the flexible price allocation), in turn (43) must imply  $\pi = 1$ . Hence the Ramsey planner would like to generate an average (net) inflation rate of zero. The intuition for why the long-run optimal inflation rate is zero is simple. Under commitment, the planner cannot resort to ex-post inflation as a device for eliminating the inefficiency related to market power in the goods market. Hence the planner aims at choosing that rate of inflation that allows to minimize the cost of adjusting prices, and summarized by the quadratic term  $\frac{\theta}{2}(\pi_t - 1)^2$ .

One may wonder why the search externality does not apparently exert any influence on the desired optimal long-run inflation rate. In light of our considerations above, the desire of reducing the congestion externality by increasing firms profits has been shown to be a sufficient motive for inducing the planner to deviate from choosing a constant markup allocation. However, since the policy maker can exert only an indirect effect on the search externality via a reduction in mark-up and since it cannot resort to ex-post inflation as a device for eliminating the inefficiency related to market power, we conclude that it is not possible under commitment to deviate from strict price stability.

## 5 Optimal Stabilization Policy and Welfare

Since we are mostly interested in the analysis of optimal policy along the business cycle we can now analyze the optimal dynamic of variables in response to shocks. We focus on productivity and government expenditure shocks. To solve for the optimal stabilization policy I compute second order approximations<sup>11</sup> of the first order conditions of the Lagrangian problem described in definition 2. Technically I compute the stationary allocations that characterize the deterministic steady state of the first order conditions to the Ramsey plan. I then compute a second order approximation of the respective policy functions in the neighborhood of the same steady state. This amounts to implicitly assuming that the economy has been evolving and policy has been conducted around such a steady already for a long period of time (under timeless perspective).

Before proceeding with the quantitative analysis of the optimal policy it is worth noticing that the competitive economy of the present model generates a volatility of unemployment which is higher than the one featured by a standard new keynesian model and that the Beveridge curve holds. Overall the model is able to account fairly well for the main stylized facts characterizing the labor market<sup>12</sup>.

### 5.1 Dynamic of the Optimal Policy in Response to Shocks

Figure (1) shows impulse response of selected variables to productivity shocks. An increase in productivity induces an increase in output. Optimal policy also features an increase in inflation (and prices) to allow the economy to take full advantage of the higher productivity. Indeed a reduction in the mark-up (or an increase in demand and marginal costs) achieved through an increase in inflation can increase unitary profits,  $\Pi_{t+j} = mc_{t+j}z_{t+j} - w_{t+j}$ , at each point in time. An increase in unitary profits, for given cost of posting vacancy  $\kappa$ , increases vacancy posting and reduces labor market tightness,  $q(\theta_t)$ , thereby squeezing the congestion externality. As a consequence there is an increase in employment as well. Positive inflation can therefore have an indirect positive impact on the probability of forming matches through its impact on the demand for varieties. This is beneficial on consumption and employment as well.

Figure (2) shows the response of the same set of variables to government expenditure shocks. Optimal monetary policy implies in this case a fall in consumption and in the price level. The government will want to have less consumption when government purchases are high since this

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<sup>11</sup>Second order approximation methods have the particular advantage of accounting for the effects of volatility of variables on the mean levels of the same. See Schmitt-Grohe and Uribe (2004a,b) among others.

<sup>12</sup>In this context it is important to stress that the qualitative results concerning optimal policy (mostly the deviations from price stability) remain the same independently from the calibration of the labor market parameters.

makes the state contingent claims value of the public spending high, making it easier to satisfy monopoly producers. This argument is valid when the utility of the representative agent is separable so that the price of the state contingent security only depends on consumption<sup>13</sup>. In order to generate a fall in consumption the government increases the nominal interest rate and this also implies a fall in the price level.

Importantly deviations from price stability are significant in response to both shocks. This is in sharp contrast with the conclusions reached by previous studies of optimal policy which had shown that deviations from price stability were nil or negligible mostly in response to productivity shocks. In general it had been established that, in a context with monopolistic competition and sticky prices, the constrained pareto optimum is reached by replicating the flexible price allocation and that monetary policy should be neutral in response to productivity shocks. On the contrary in our context optimal policy is in pro-cyclical in response to productivity shocks. With search frictions indeed the level of employment is inefficiently low and a trade-off exists between inflation and employment/output stabilization. In presence of such trade-off the monetary authority should strike a balance between reducing the cost of adjusting prices and increasing employment.

A crucial determinant of the size and the direction of the congestion externality is the distance between the worker's bargaining power and the share of unemployed people in the matching technology. As mentioned before, Hosios (1990) had shown that efficiency can be achieved in a matching model when workers' bargaining power is equal to the share of unemployed people in the matching technology. In different cases the labor market is tight, the probability of forming a match is low and employment is inefficiently low. In general whenever the labor market features an inefficiency the monetary authority has an incentive to intervene and deviate from price stability. However when the bargaining power is high (for given share of unemployed people in the matching technology), workers' share of surplus is high (and firms' share is low) and vacancies have little profitability. In this case there is an excess of searching workers compared to the number of vacancies. This increases labor market tightness and unemployment. Under those circumstances optimal policy should be pro-cyclical since an increase in inflation, achieved through a reduction of the nominal interest rate, increases demand and unitary firms' profits which in turn increase vacancy profitability and employment. On the contrary when workers' bargaining power is low (firms' share of surplus is high) there is an excessive vacancy creation hence the monetary authority should be countercyclical.

Figure (3) shows the dynamic of the Ramsey allocation in response to productivity shocks and for different values of the bargaining power (0.2 vs. 0.6) and for given share of unemployed people

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<sup>13</sup>See Khan, King and Wolman (2000).

in the matching technology,  $\xi = 0.4$ . When  $\varsigma = 0.2$  ( $< \xi = 0.4$ ) optimal policy is countercyclical. In this case there is excessive vacancy creation hence a reduction in inflation by increasing the mark-up reduces demand and vacancy profitability. The contrary is true for  $\varsigma = 0.6$ . In both cases however the result is an increase in consumption, output and employment due to the reduction in the search externality.

## 5.2 Comparison Between Ramsey Policy and Policy Rules

Some interesting observations come from the comparison between the dynamic under the Ramsey policy and the dynamic generated by some simple operational policy rules. Figure (4) shows the dynamic of selected variables in response to productivity shocks and under three different regime. The first regime is represented by a Taylor rule of the following type:

$$\ln\left(\frac{1+r_t^n}{1+r^n}\right) = \left(\phi_\pi \ln\left(\frac{\pi_t}{\bar{\pi}}\right) + \phi_y \ln\left(\frac{y_t}{y}\right)\right) \quad (44)$$

with  $\phi_\pi = 1.5$  and  $\phi_y = 0.5$ . The second regime is represented by a rule that targets unemployment of this form:

$$\ln\left(\frac{1+r_t^n}{1+r^n}\right) = \left(\phi_\pi \ln\left(\frac{\pi_t}{\bar{\pi}}\right) + \phi_u \ln\left(\frac{u_t}{u}\right)\right) \quad (45)$$

with  $\phi_\pi = 3$  and  $\phi_u = 0.3$ . finally the third regime is given by the Ramsey policy.

Figure (4) shows the dynamic of selected variables in response to productivity shocks and under the three regimes described. It stands clear that the Ramsey policy implies a much higher volatility compared to the other two rules. This is so since under both rules, the Taylor rule and the unemployment targeting, the monetary authority is only concerned with stabilization and for this reason behaves countercyclically (inflation goes down in response to productivity shocks). On the contrary the Ramsey planner has the incentive to take full advantage of the productivity increase thereby exploiting all the benefits of the expansionary phase.

## 5.3 Optimal Volatility of Inflation

To fully analyze the properties of the optimal policy along the cycle we are obliged to study the path of the optimal inflation volatility for different values of the bargaining power. Once again the level of the bargaining power is an indicator of the size and the direction of the congestion externality. As before we expect the optimal policy to be pro-cyclical when the number of searching workers is too high compared to the number of vacancies (e.g. when workers' bargaining power is higher than the share of unemployed people in the matching technology) and viceversa. However in any case inflation volatility is increasing whenever the bargaining power gets distant from the number

of unemployed people in the matching technology. Indeed the incentive of the monetary authority to intervene and reduce the search externality are higher when the distance between the current employment and the efficient one increase (e.g. when the distance between workers' bargaining power and share of unemployed workers in the matching technology increases independently from the direction). This is confirmed by figure (5) which shows that the optimal inflation volatility is indeed U-shaped with respect to  $\varsigma$  for given value of  $\xi = 0.4$ .

#### 5.4 Adding Real Wage Rigidity

Shimer (2003), Hall (2003) noticed that in a matching model a' la Mortensen and Pissarides wages are too volatile since little adjustment takes place along the employment margin. They also noticed that the introduction of real wage rigidity helps to resolve some of the puzzling features of the standard matching model. Thereby following Hall (2003) I assume that the individual real wage is a weighted average of the one obtained through the Nash bargaining process and the one obtained as solution to the steady state<sup>14</sup>:

$$w_t = \lambda[\varsigma(mc_t z_t + \theta_t \kappa) + (1 - \varsigma)b] + (1 - \lambda)w \quad (46)$$

Adding real wage rigidity does not alter any of the previous results<sup>15</sup> since it does not change the main policy trade-offs. Optimal monetary is still characterized by significant deviations from price stability and the optimal volatility of inflation is still a U-shaped function of the bargaining power. The only noticeable difference is that the introduction of real wage rigidity increases the volatility of all variables under the Ramsey policy. Intuitively real wage rigidity tends to exacerbate the inflation/unemployment trade-off thereby calling for stronger intervention on the side of the monetary authority. The latter result seems consistent with Blanchard and Gali' (2005).

## 6 Conclusion

This paper derives optimal monetary policy in a model with monopolistic competition and sticky prices, matching frictions and real wage rigidity in the labour market. In response to both productivity and government expenditure shocks optimal policy features significant deviations from price stability. This is so since search externalities generate an unemployment/inflation trade-off which induces the monetary authority to strike a balance between reducing the cost of adjusting prices and increasing an inefficiently low employment.

<sup>14</sup>Notice that the results in this paper remain valid when the wage is set as a weighted average of current and past values.

<sup>15</sup>Results are not reported for brevity but are available upon request.

In response to productivity shocks optimal policy is pro-cyclical when the worker's bargaining power is higher than the share of unemployed people in the matching technology and viceversa. This is so since when the workers' share of surplus is high (firms' share is low) there is an excessive number of searching workers and few vacancies. In this case the monetary authority has an incentive to increase vacancy creation by increasing their profitability. It does that by reducing the interest rate, therefore increasing demand and inflation. The opposite is true when the workers' share of surplus is low: in this case the congestion externality is generated by an excessive vacancy creation that the monetary authority tries to discourage.

## 7 Appendix A - The Stationary Lagrangian Problem

Let  $\Lambda_t^n \equiv \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}\}_{t=0}^\infty$  and  $\Xi_t^n \equiv \{c_t, n_t, v_t, \pi_t, r_t^n, mc_t\}_{t=0}^\infty$  to

$$\begin{aligned} & \text{Min}_{\{\Lambda_t^n\}_{t=0}^\infty} \text{Max}_{\{\Xi_t^n\}_{t=0}^\infty} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t E_t \{ W(c_t, n_t, v_t, \pi_t, mc_t) \right. \\ & + \lambda_{1,t} \left[ \frac{c_t^{-\sigma}}{(1+r_t^n)} \right] + \\ & + \lambda_{2,t} [1 - \psi(\pi_t - 1)\pi_t y_t c_t - (1 - mc_t)c_t y_t \varepsilon] + \\ & + \lambda_{3,t} \left[ \frac{\kappa}{m} \theta_t^\xi c_t \right] + \\ & + \lambda_{4,t} \left[ n_t z_t - \kappa v_t - \frac{\psi}{2} (\pi_t - 1)^2 y_t - c_t - g_t \right] + \\ & \left. + \lambda_{5,t} \left[ n_t - (1 - \rho)(n_{t-1} + v_{t-1} m u_t^\xi v_t^{1-\xi}) \right] \right\} \end{aligned} \quad (47)$$

where:

$$\begin{aligned} W(c_t, n_t, v_t, \pi_t, mc_t) = & \\ & U(c_t) - \chi_{1,t} \beta E_t \{ c_t^{-\sigma} \pi_t \} + \chi_{2,t} \beta E_t (c_t)^{-\sigma} [\psi(\pi_t - 1)\pi_t y_t] \\ & - \chi_{3,t} E_t \{ \beta (c_t)^{-\sigma} (1 - \rho) [(1 - \varsigma) m c_t z_t - \varsigma \theta_t \kappa - (1 - \varsigma) b + \frac{\kappa}{m} \theta_t^\xi] \} \end{aligned}$$

$$\theta_t \equiv \frac{v_t}{u_t} \text{ and } u_t = 1 - n_t.$$

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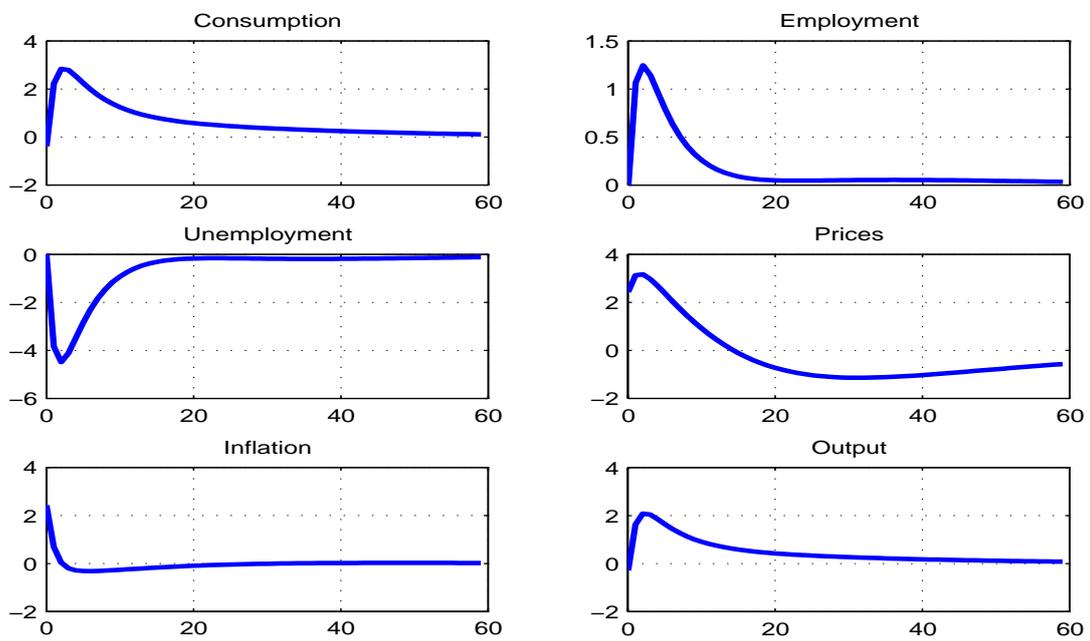


Figure 1: Impulse responses under Ramsey allocation to productivity shocks.

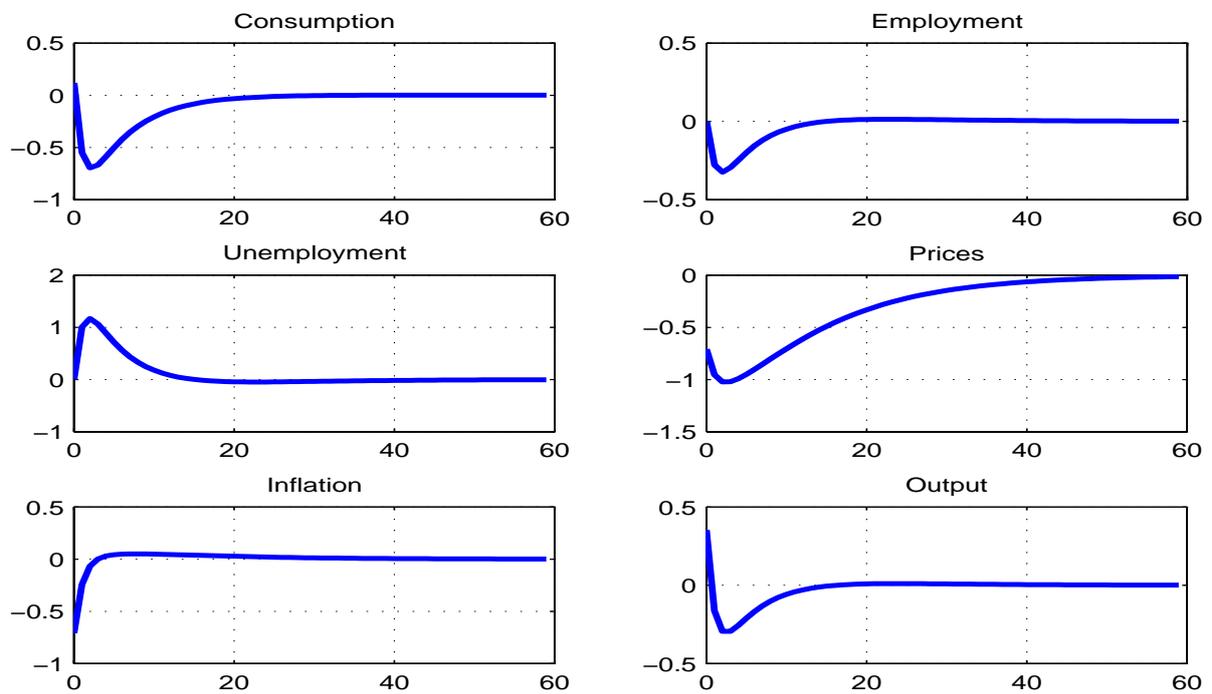


Figure 2: Impulse responses under Ramsey allocation to government expenditure shocks.

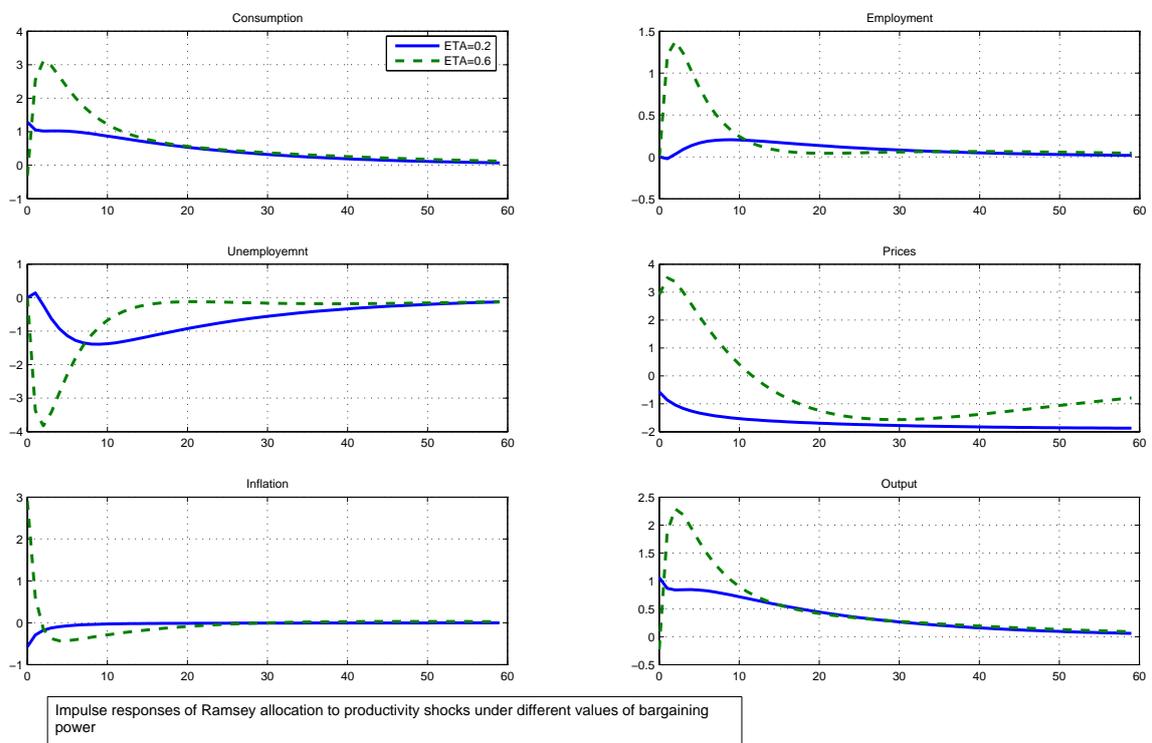
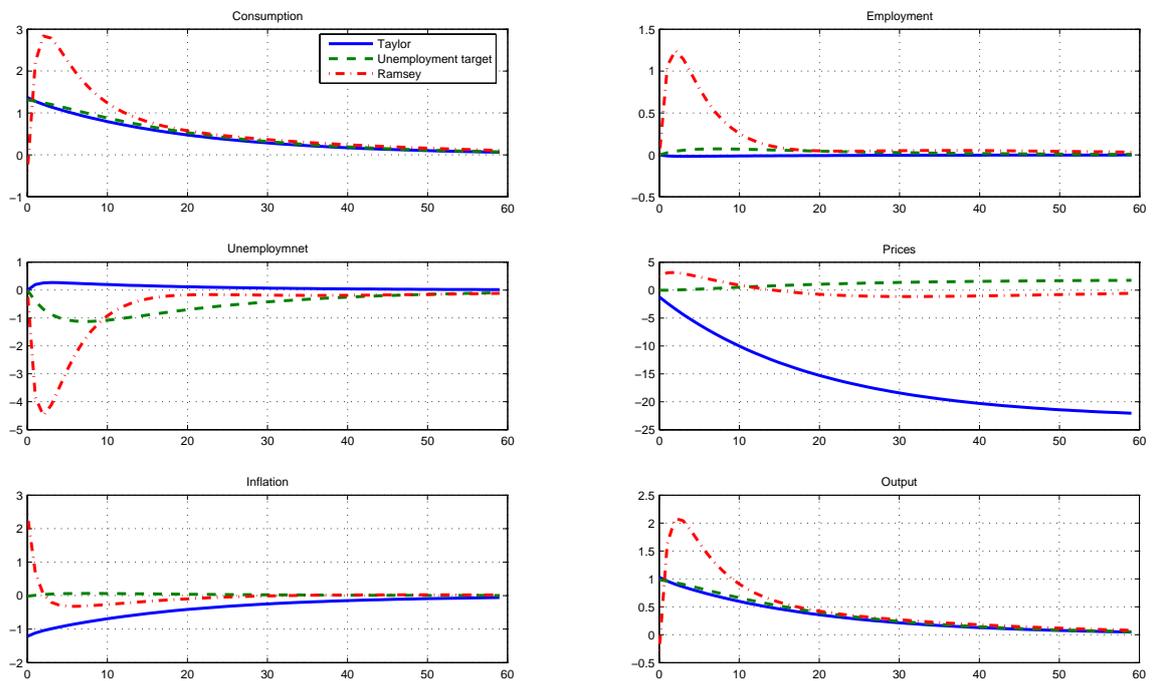


Figure 3:



Impulse responses to productivity shocks under three different regimes: Taylor rules, unemployment targeting and Ramsey policy.

Figure 4:

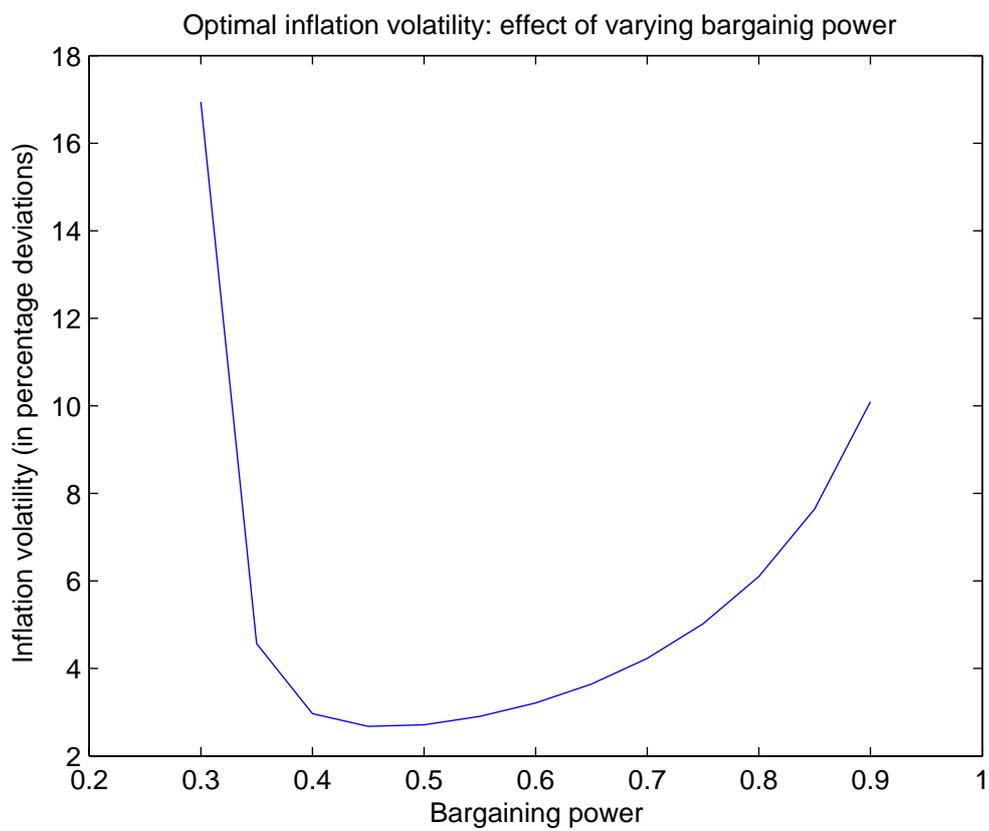


Figure 5:

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