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DECLINING VALUATIONS AND EQUILIBRIUM BIDDING IN CENTRAL BANK REFINANCING OPERATIONS

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In 2006 all ECB publications will feature a motif taken from the €5 banknote.

1 This paper is a revised version of the draft "Optimal Bidding in Variable Rate Tenders with Collateral Requirements." The paper has been presented at the ECB, at the Banque de France, and very recently, on the conference "Microstructure of Financial and Money Markets" in Paris. We would like to thank Vish Viswanathan, the Editorial Board of the ECB Working Paper Series, and an anonymous referee for very helpful suggestions. Insightful comments made by Caroline Jardet, Cornelia Holthausen, and Philippe Février are gratefully acknowledged. The paper has also benefited from discussions with Steen Ejerskov, Wolfgang Köhler, and Marco Lagana. Of course, the opinions and views expressed herein are those of the authors alone, and do not necessarily reflect the views of the European Central Bank or of the Banque de France.

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CONTENTS

Abstract. It is argued that bidders in liquidity-providing central bank operations should typically possess declining marginal valuations. Based on this hypothesis, we construct an equilibrium in central bank refinancing operations organised as variable rate tenders. In the case of the discriminatory pricing rule, bid shading does not disappear in large populations. The predictions of the model are shown to be consistent with the data for the euro area.

JEL classification codes: D44, E52

Keywords: Open market operations, uniform price auction, discriminatory auction, Eurosystem.

Non-technical summary

The present paper aims at contributing to the microeconomic modelling of bidding behaviour in central bank operations. We are especially interested in improving our understanding of incentives shaping bidding behaviour in the main refinancing operations that have been conducted by the Eurosystem since June 2000 as so-called variable rate tenders with the discriminatory pricing rule.[5]

A review of the theoretical literature shows that, traditionally, the theory of auctions of divisible goods has considered the so-called constant returns set-up. In this set-up, it is assumed that bidders participating in the primary market have access to a perfect secondary market after the auction, where the price level in the secondary market is not known at the time of the auction. Indeed, constant returns appear to be appropriate for the study of many specific examples of share auctions, including the case of treasury auctions.

As we argue in the present paper, however, the assumptions underlying the set-up with constant returns should typically not be satisfied in the case of liquidity-providing central bank operations. Indeed, the necessity for the respective lender to limit the financial risks associated with the funding transaction establishes a close link between the liquidity transferred to the borrower and the specific measures taken to manage the risk. Ignoring this link may lead to an incorrect description of the incentives underlying central bank operations.

For example, in a liquidity-providing operation of the Eurosystem, the borrower has a discretion, within the lists of eligible collateral established by the Eurosystem, concerning the assets that are transferred to the central bank to cover the funding operation. This discretion may matter if central bank and market participants have different needs for controlling the risks of funding

[5] In a discriminatory auction, each bidder may submit a demand schedule, and a cut-off price is determined by equating demand and supply. Each bidder receives then an allotment corresponding to her demand at the cut-off price, where a rationing rule is applied at the margin, if necessary.

transactions. In particular, while a less liquid asset may justify a higher interest rate in the interbank market because the lending counterparty may be forced to liquidate the asset quickly, this type of consideration is much less relevant when the central bank is the lender.

Following this example further, the different pricing of liquidity risk by the private market and the central bank should imply a preference, on the part of the bidders, for illiquid collateral to be used in central bank operations. With an increasing allotment, the counterparty must forward more liquid types of collateral, which makes the primary market increasingly less attractive compared to the interbank market. This consideration offers support for a set-up with declining marginal valuations for liquidity providing central bank operations.

The second part of the paper develops a theory of the variable rate tender under the assumption of declining returns. This theory is consistent with the view that there is trade after the auction. We consider first an auction of shares with uniform pricing and uncertain supply, structurally similar to a more general model analysed by Klemperer and Meyer [35].[6] Our theoretical analysis extends their model to allow for the discriminatory pricing rule. To our knowledge, this construction provides the first complete theoretical description of equilibrium bidding in the variable rate tenders of the Eurosystem.

The analysis of the equilibrium in the tender predicts *strategic bid shading* by individual banks, both for uniform and discriminatory pricing. Here, bid shading means that a bidder in an open market operation demands a quantity at one or several interest rate levels that is strictly below her true demand. This type of behaviour can be optimal when there is uncertainty about the endogenous realisation of the stop-out rate. E.g., with uniform pricing, the bidder pays the stop-out rate on all winning bids, i.e., on her allotment. In this situation, shading of bids, especially at relatively low interest rates, is

[6] The difference between uniform and discriminatory auctions lies in the pricing rule. With uniform pricing, each bidder pays the cut-off price, while with discriminatory pricing, each bidder pays her own bid.

profitable in expected terms because the interest rate payment for the whole amount allotted will be reduced. As a consequence of this consideration, optimal bid schedules are steeper than the underlying true demand in the case of the uniform pricing rule. This result holds even for a heterogeneous population of banks.[7]

For the case of the discriminatory pricing rule, the model predicts bid schedules that are *flatter* than the underlying true demand. Indeed, under discriminatory pricing, a successful bidder pays her bid, so that there is an obvious incentive to reduce demand. More importantly, the incentive for bid shading becomes the more pronounced the higher and less likely the interest rate. The analysis of optimal bidding therefore suggests the submission of bids on relatively few interest rate levels. We also find that in the case of the discriminatory pricing rule, bid shading should be observable even when the number of bidders is large. This result is shown to be in line with the empirical evidence for the euro area.

More specifically, our results suggest that the difference in prices in the primary and secondary markets for interbank liquidity, if we use these simplifying terms, is due to two factors. First, there are different standards and uses of funding instruments in the primary and secondary market. Second, there is also optimizing behaviour on the part of the counterparties that participate in open market operations.

To see this point, consider a typical bidder who is active both in the primary and in the secondary market. Given that the Eurosystem accepts also collateral that is less liquid than general collateral (GC) and that can be used in the market only against a spread, there is an incentive to replace interbank funding transactions based on GC by central bank funding based on less liquid types of collateral. The resulting difference in the composition of collateral used in the primary and secondary markets leads ceteris paribus

[7] Bid shading characterises the behaviour of an individual bidder, and is therefore not tantamount to "underbidding" which refers to the event that aggregate demand in an open market operation, i.e., the total of incoming bids, is lower than the amount appropriate to guarantee a smooth fulfilment of reserve requirements over time.

to an increase of demand and thereby of funding costs in the primary market vis-à-vis the secondary market. However, this is just one effect. The second effect works against the first, and is driven by the counterparty's uncertainty about the stop-out rate. This uncertainty implies incentives for bid shading, as explained above. Submitting a more aggressive bid schedule is not attractive for the bidder because the marginal increase in the interest rate that must be paid would be larger than the additional value from the resulting marginal increase in the allotment.

These considerations are reflected in the data for the euro area. The marginal rate in the weekly main refinancing operations of the Eurosystem is statistically not different from the GC repo rate. This is surprising at first sight because an interbank repo that would be collateralised by a less liquid asset class (i.e., typical for the primary market) would involve an interest rate that is strictly above the GC repo rate. A natural explanation of this observation is offered by our analysis of bidding behaviour in the discriminatory auction format.

I. Introduction

In many modern currency areas, monetary policy is implemented by steering short-term interest rates indirectly through the provision of more or less liquidity to the banking system. Besides outright trading, one established method of creating a flow of liquidity between the central bank and the market is to perform a tender or auction in which commercial banks first submit bids or bid schedules, and the central bank subsequently determines, usually with a certain degree of discretion, the flow of liquidity to be effected to or from any commercial bank. E.g., in the euro area, the European Central Bank (ECB) employs tender procedures in this way to auction off repurchase agreements and collateralized loans with an average face value of more than 200 bn euro on a weekly basis. Understanding the incentives shaping bidding behaviour in these auctions therefore appears to be of critical importance for the effectiveness of monetary policy implementation.

From a theoretical perspective, open market operations are auctions of a perfectly divisible good or auctions of shares (Wilson [55]).[8] It has been noted in the literature that these auctions are economically quite different from single-unit auctions, so there has been a need for an independent study of auctions of shares.[9] One of the more prominent examples of a share auction is constituted by the mechanisms used for the sale of treasury notes (see Bikhchandani and Huang [8]). Similar to central bank operations, these auctions take place in the anticipation of a secondary market in which bidders may trade the good after the auction at a common market price. In line with this institutional feature, the theoretical literature has traditionally assumed constant marginal valuations for the good to be auctioned (see, e.g., Back

[8]Wilson proved that auctions of shares may be subject to severe collusion. A number of papers have stressed the role of endogenous supply in avoiding collusion. See Hansen [30], Back and Zender [3], Lengwiler [39], McAdams [43], and LiCalzi and Pavan [40]. As shown by Kremer and Nyborg [36, 37], collusion can also be reduced by modifying the standard rationing scheme.

[9]E.g., when bidders demand more than one unit in a multi-unit auction with uniform pricing, then truthful bidding will typically be suboptimal. This finding stands in contrast to the theoretical optimality of truthful bidding in the single-unit second price auction, of which the auction with uniform pricing is a natural generalisation.

and Zender [2]). However, as we will argue in this paper, the assumption of constant marginal valuations may not be appropriate for studying central bank operations.

As will be explained more carefully in Section II, open market operations differ from treasury auctions in particular because of the need to handle financial risks associated with the funding transaction. Indeed, the necessity for the respective lender to limit those risks establishes a close link between the liquidity transferred to the borrower on the one hand and the specific measures taken to manage the risk on the other. Ignoring this link may lead to an inaccurate description of the incentives underlying participation decisions and bidding behaviour in central bank operations.

For example, in a liquidity providing operation of the Eurosystem, the borrowing counterparty has a substantial discretion concerning the assets that are transferred to the central bank to cover the funding operation. This discretion may matter if central bank and market participants have different needs for controlling the risks associated with funding transactions. In particular, while a less liquid asset may justify a higher interest rate in the interbank market because the lending counterparty may be forced to liquidate the asset quickly, this type of consideration should be much less relevant when the central bank is the lender. The different pricing of liquidity risk by the private market and the central bank should imply a preference of bidders for illiquid collateral to be used in central bank operations. Moreover, with an increasing allotment in the central bank tender, the counterparty would have to forward more liquid types of collateral, which makes the primary market increasingly less attractive compared to the interbank market. This consideration offers support for a set-up with declining marginal valuations for liquidity-providing central bank operations. Consistent with this view, we will assume throughout that marginal valuations in the auction are strictly declining.[10]

[10] Another difference lies in the objectives that are pursued with individual operations. In the case of treasury auctions, the theoretical literature has emphasised the objective of revenue maximisation besides, of course, allocational efficiency. In central bank opera-

Of course, we are not first in making this assumption. Most notably, Klemperer and Meyer [35] consider a reverse auction in which profit-maximising oligopolists select supply functions in the presence of uncertainty about market demand. Our discussion of auctions with *uniform pricing*[11] in Section III is closely related to their analysis, but allows for a heterogeneous population of bidders. We also offer a more explicit treatment of the non-negativity requirement for allotments. Section IV in our paper can be understood as an adaptation of Klemperer and Meyer's [35] model to the case of the *discriminatory pricing* rule. Ausubel and Cramton [1] assume declining marginal valuations and show in particular that the incentives for differential bid shading cause an allocative inefficiency in the uniform price auction. Our findings illustrate their analysis by discussing specific equilibria with downward-sloping demands in both the uniform-price and the discriminatory auction.

There is also a closely related literature that investigates simultaneous auctions of a finite number of identical objects to a population of bidders with multi-unit demand. Noussair [47] provides necessary and sufficient conditions for bid functions to describe a Bayesian Nash equilibrium in a uniform-price auction offered to bidders with two-unit demand. It is shown that there is an incentive to lower the bid placed on the second unit. Engelbrecht-Wiggans and Kahn [27, 28] characterise equilibra in uniform-price and discriminatory

tions, however, revenue maximisation plays a subordinate role. Objectives pursued with individual central bank operations are in general less clear-cut but seem to entail quantitative elements, the signalling of the monetary policy stance, and an equal treatment of bidders.

[11] The scientific debate on auctions of shares and multi-unit auctions has focused mostly on the two types of auctions, the uniform-price auction and the discriminatory auction. These are auction formats that have been used both in treasury auctions and in central bank operations. E.g., the Eurosystem has relied on variable rate tenders with discriminatory pricing in its regular main refinancing operations since June 2000. In both procedures, each bidder may submit a demand schedule, and a cut-off price is determined by equating demand and supply. Each bidder receives then an allotment corresponding to his or her demand at the cut-off price, where a rationing rule is applied at the margin, if necessary. The difference between uniform and discriminatory auctions lies in the pricing rule. With uniform pricing, each bidder pays the cut-off price, while with discriminatory pricing, each bidder pays her own bid. Back and Zender [2] and Wang and Zender [54] establish a revenue inequivalence between uniform and discriminatory auctions for constant marginal valuations. See Maskin and Riley [42] for an analysis of optimal multi-unit auctions.

auctions of two identical units. They also predict bid shading, and find in addition that discriminatory pricing may imply the submission of identical bids for both units, despite decreasing returns. In these papers, the stop-out price under uniform pricing is assumed to be the highest losing bid. Draaisma and Noussair [16] derive necessary conditions for a Bayesian equilibrium in a uniform price auction where the stop-out price is the lowest winning bid. More general existence results for indivisible objects have recently been obtained by Jackson and Swinkels [33] and McAdams [44].

We believe that the present paper may contribute to the theoretical literature in two ways. First, we construct equilibria in the discriminatory share auction to an arbitrary number of bidders with decreasing marginal valuations.[12] To obtain explicit results, we have chosen to consider as the simplest possible set-up a specification with linear marginal valuations, and uniformly distributed uncertainty about supply. There is no principal difficulty in extending the present model to non-linear demand functions. As it turns out, the linear set-up leads to piecewise linear bid schedules also in the discriminatory tender so that the bidding behaviour in the two tender formats can be studied and compared in a very explicit way.[13]

The second contribution of this paper is the result that for large populations of bidders, bid shading may be present in discriminatory auctions (but not

[12]Characterisation of bidding behaviour in the primary market that, while different in interpretation, are structurally similar to our results have recently been given by Biais, Martimort, and Rochet [6] in the context of adverse selection and by Viswanathan and Wang [53] in the context of risk aversion. Our analysis goes beyond these contributions by considering explicitly the possibility of trade between the bidders after the auction.

[13]It has been acknowledged in the literature that there has been a lack of specific set-ups of auctions of shares with decreasing marginal valuations that are fully tractable on the one hand and rich enough on the other to be empirically testable, at least in principle. The reason for this deficiency may be that, in an auction of shares, each bidder selects a whole *demand function* in a strategic way. The sufficiency conditions in the calculus of variations, however, may be non-trivial to check, so that most existing models, especially those allowing for incomplete information about demand, have been solved employing a "first-order approach." Hortaçsu [32] derives an explicit solution of a share auction with discriminatory pricing for two bidders and an exponential distribution of types. He remarks, however, that the Euler condition employed is only necessary. Chakraborty [15] offers sufficient conditions for a Bayesian equilibrium in a discriminatory auction of two identical units.

in uniform price auctions). Indeed, as will become clear, as long as aggregate uncertainty about the allotment of the central bank exists, a bidder in the discriminatory auction has an incentive to shade her complete bid schedule. We discuss this point and show that the conclusion is consistent with the evidence for the euro area.

The rest of the paper is structured as follows. In Section II, we argue that bidders in central bank refinancing operations should typically have declining valuation functions. Section III derives the equilibrium of the variable rate tender for the case of the uniform pricing rule. Section IV treats the case of the discriminatory pricing rule. Section V concludes. The Appendix contains technical proofs.

II. Demand in refinancing operations

As described in the Introduction, open market operations (or simply central bank operations) are used by central banks in particular in order to create a flow of liquidity between the central bank and the banking system. E.g., if autonomous liquidity factors such as banknotes cause a flow of liquidity from the banking system into the non-bank sector, then a central bank may decide to compensate the liquidity outflow by injecting additional money into the banking system. In this section, we argue that marginal valuations of bidders in liquidity-providing central bank operations should typically be strictly decreasing.

A model of dual funding. Our hypothesis of declining marginal valuations (or simply declining returns) for central bank operations can be obtained from the following *model of dual funding*.[14] Consider a single commercial bank seeking a given amount q of funding. There are two independent sources of funds, called for simplicity primary and secondary market, with market rates r^P and r^S, respectively. Both markets are perfect, with two qualifications.

[14] The current framework may also help to shed light on the likely consequences of having a single list of eligible collateral (cf. ECB [21]), and on the recent discussion (cf. ECB [26]) regarding counterparties' incentives to use collateral of a given type (in particular, lower-rated government debt).

The first qualification is that the access to the primary market is one-sided only, i.e., there is no possibility to deposit liquidity in the primary market. The second qualification is the existence of a premium on interest rates that needs to be paid for the participation in the two markets. This point will be made more precise below.[15]

In the institutional realities of the money market, demand q will roughly correspond to a sum of the bank's requirements on reserve holdings and precautionary demand resulting from idiosyncratic liquidity needs (cf. in particular Poole [48] and Baltensperger [4]). The primary market corresponds roughly to central bank supply of interbank liquidity. Indeed, since we are proposing the model of dual funding as a means to determine the bank's marginal valuation, or equivalently, the bank's demand at a given price, the format of the tender procedure applied in the primary market does not enter the consideration at this stage. Details on the procedures used for the handling of collateral in the context of lending from the Eurosystem can be found in ECB [20, 23, 25]. The secondary market is a shortcut for the euro money market including all forms of unsecured and secured interbank lending including the exploitation of hedging possibilities provided by the market for derivatives in short-term interest rates.

Our modelling of the funding activities is motivated by the fact that the transfer of liquidity from one counterparty to another involves either the reverse transfer of collateral or the taking of credit risk by one lender. To avoid case distinctions in the formal discussion, we will interpret unsecured lending as the usage of a credit line that is collateralized with an intangible "asset" tantamount to the confidence of the other counterparty in the bank's repayment of the debt. For simplicity, we refer to these types of assets as *confidence assets*. Following this convention, all lending both in the primary and in the secondary markets takes place against some collateral. We may

[15] Our assumption of a perfect secondary market reflects the general perception that the euro money market is very efficient (see, e.g., Hartmann, Manna, and Manzaranes [31]). That is to say that premia on the individual transactions are caused mainly by the need to price in the financial risk associated with the transaction, and less by the remaining market imperfections.

therefore change the perspective and put the collateral in the focus of our consideration.

Let A denote the range of all asset types available for the bank, including the confidence assets. The bank's balance sheet will be represented by a measure with density $x(a) \geq 0$ (cf. Billingsley [9], p. 213). We assume that the bank's collateral is sufficient to cover the necessary funding. As haircuts are not considered in the basic set-up, this is tantamount to the constraint

$$\int_A x(a)da \geq q. \tag{1}$$

Indeed, if this condition was violated, the bank could not find the necessary funding, and would be considered to be at least illiquid.[16]

Eligibility. Primary and secondary markets may differ in the type of transactions that are accepted by the other counterparty, be it the respective national central bank or some market participant. Our argument is valid in particular in a scenario where central bank and market participants accept the same range of funding operations. To capture the realities of the market place, however, we will be somewhat more general in the sequel. E.g., all provision of liquidity by the Eurosystem has to be protected by collateral (cf. ECB [23]), which restricts transactions involving unsecured lending to the secondary market. The Eurosystem does also not take part in tri-party repos, i.e. repurchase agreements in which a private agent exchanges cash or collateral for at least one of the counterparties. On the other hand, the interbank market typically does not accept highly illiquid assets (such as bank loans) as collateral, which restricts this type of funding transaction to the primary market. Certain types of transactions are feasible both in the primary and in the secondary market. A simple example for the euro area may be a repurchase agreement over a one week maturity secured by a euro area government bond.[17]

[16] The integral representation is used for the convenience of the reader. When the number of asset types is finite, then (1) becomes a sum over all positions of the bank's balance sheet, and marginal valuations will be weakly declining staircase functions.

[17] The list of collateral eligible for transactions with the Eurosystem is maintained by the ECB, and can be downloaded from its website http://www.ecb.int.

To make the above-described distinction between asset types explicit in the model, we assume that there are three mutually disjoint categories of collateral

$$A = P \cup S \cup PS,$$

where, as usual, $\cup$ denotes the set-theoretic union operator. Category P collateral is accepted by the primary market only. Category S collateral is accepted by the secondary market only. Category PS collateral can be used in both markets. Above, we have provided examples that show that all three sets P, S, and PS should be nonempty in the case of the Eurosystem. To further illustrate the meaning of these categories, we recall that in the course of the establishment of a single list in the euro area, equities have been taken from the list of eligible collateral, while foreign debt instruments have been allowed as eligible collateral (see Bayle [5] and ECB [21, 22]). In the present framework, this would mean to shift the set of asset types A^e corresponding to equities from SP to S, and to shift the set of asset types A^f corresponding to foreign debt instruments from S to SP.

Premia on market prices. The basic benchmark rate in the secured market is without doubt the repo rate against highly liquid interest rate instruments standardised in the market under the heading general collateral (GC). To fix ideas, we will refer to this rate as the *"risk-free" rate.* Given this benchmark, a convenient way to specify an interest rate level for an individual transaction is to add premia that reflect the difference between the benchmark rate and actual costs of funding.[18]

On the risk-free rate a *risk premium* is added for any risk that the lender must take. The premium may have various motivations. Most obviously, in the case of an unsecured transaction, the premium on the risk-free rate is the risk cost which is an interest rate margin that reimburses the lender for the expected loss caused by the *credit risk* inherent to the transaction.[19] But also

[18] When the funding aspect of a repurchase transaction becomes dominated by the demand for a specific security, then repo rates on such "specials" drop below the GC rate. See Duffie [18] and Jordan and Jordan [34].

[19] For an analytical model capturing this consideration, see Bierman and Hass [7] or Yawitz [56].

the conditions on secured transactions (e.g. a repurchase agreement) may differ significantly. One main factor affecting prices for secured transactions is the liquidity of the collateral offered by the borrower. Indeed, while the market value of the collateral will typically somewhat exceed the value of the transaction, and additional risk control measures such as margining may be in place, the acceptance of less liquid assets as collateral may mean a *liquidity risk* for the lender. That is to say, if the borrower defaults and the lender is forced, for whatever reason, to liquidate the collateral quickly, then the liquidation value may be substantially lower than the market value that could be realised with more time. This specific risk justifies a higher interest rate spread on the loan that is collateralized with a less liquid asset.[20]

The logic of the risk premium is consistent with market experience. For instance, Santillán, Bayle, and Thygesen [49] report that market participants contacted in the context of market surveys tended to distinguish collateral along a dimension that could be termed "expensiveness." According to this study, one of the most frequently mentioned reasons for the heterogeneity has been the liquidity of the assets. Välimäki [52] studies bidding in central bank operations under the assumption of constant returns. However, he acknowledges the strength of the assumption (see p. 35 in his thesis): "For simplicity, we abstract from the fact that the interest rates of different instruments carry different premia over the risk-free yield curve." Our view is also consistent with the observation that the US Federal Reserve System conducts tenders in parallel for three types of collateral: Treasury, Agencies, and Mortgage Backed. The spread between the most liquid collateral and the least liquid collateral can be as high as 6-7 basis points.

In addition to the interest rate payment, i.e., the sum of risk-free rate and the risk premium, the borrower may still have *transaction costs*, at least in a secured operation. The necessity to deliver the collateral can be costly, for instance, when the collateral is held in one country and needs to be delivered

[20] More generally, the risk of having to resolve a position in an illiquid market will depress the current valuation of that position. For a formal treatment, see the intriguing analysis of Diamond and Verrechia [17].

in another country of the euro area. While both the Eurosystem and the market have developed solutions for this type of international transaction (see ECB [25]), the fees and efforts necessary to accomplish the transaction may be still non-negligible at present, as reflected in the ongoing discussion about harmonisation of the landscape for securities clearing and settlement in the euro area. In addition to the costs of transferring collateral, there may be administrative costs of handling the assets (e.g., because of income flows, ownership reporting, voting rights, etc).

Finally, there may be *opportunity costs* for the borrower associated with the use of collateral for funding purposes. By definition, opportunity costs do not differ between primary and secondary markets, so they do not influence the decision between funding markets, conditional on the usage of the collateral for funding purposes. For instance, if the collateral pool of the bank is small, then the collateral used in a funding operation is no longer available for securing other transactions. Moreover, potential fees obtainable for securities lending cannot be realised. There may also be changes in the usage of risk capital and in regulatory charges, which can be understood as factors increasing or decreasing the costs associated with an individual funding transaction.

Optimal funding. According to this simple terminology, the total funding costs are a sum of the risk free rate, premia for credit and liquidity risks, the transaction costs, and the opportunity costs of collateral. In the model, we have to abstract from the different dimensions that may determine the costs of an individual funding operation. To each collateral/asset a, we therefore assume given data on average premia (expressed in basis points) for eligible uses in the respective markets. This provides a transaction-specific premium $t^P(a) \geq 0$ for collateral $a \in P \cup PS$ and similarly a premium $t^S(a) \geq 0$ for collateral $a \in S \cup PS$. The effective cost of an individual funding transaction for the counterparty is the sum of the prevailing interest rate and the transaction-specific premium.

The bank's funding policy (x^P, x^S) determines the amounts $x^P(a)$ and $x^S(a)$ of collateral of each asset type a to be used in primary and secondary market funding. Several restrictions need to be satisfied. The first restriction says that the bank obtains the necessary funding in the first place. This condition can be formalized as follows:

$$\int_A \{x^P(a) + x^S(a)\} da \geq q. \tag{2}$$

Another natural restriction on the funding policy is given by the bank's balance sheet. Both borrowing or lending of collateral is not considered in the basic model. It is then obvious that the bank's funding policy (x^P, x^S) has to satisfy

$$x^P(a) + x^S(a) \leq x(a), \tag{3}$$

i.e., the total collateral of a given asset type used in the primary and secondary market must not exceed the total available in the bank's balance sheet. The reader will note that the consistency of conditions (2) and (3) is ensured by our assumption (1) that the bank is not illiquid. Similarly to (3), we also need to impose

$$x^P(a) \geq 0 \tag{4}$$

$$x^S(a) \geq 0 \tag{5}$$

for all assets $a \in A$. The final restriction incorporates collateral requirements and eligibility criteria expressed by the partition of A into the sets P, S, and PS:

$$x^P(a) = 0 \text{ for } a \in S, \text{ and } x^S(a) = 0 \text{ for } a \in P. \tag{6}$$

For simplicity, the basic model excludes the possibility of intermediation, i.e., on-lending of reserves in the secondary market.[21] The problem of the bank can then be formulated as follows. The task is to determine the least costly funding policy that guarantees a funding of at least q, and which does, for any asset type a, not exceed the available collateral, i.e.,

$$\min_{(x^P, x^S)} \int_A x^P(a)\{r^P + t^P(a)\} + x^S(a)\{r^S + t^S(a)\} da \tag{7}$$

s.t. (2), (3), (4), (5) and (6).

[21] See, however, Neyer and Wiemers [46] for a model of intermediation with $PS = \varnothing$.

We impose that the bank's balance sheet contains a sufficiently broad range of assets with a heterogeneous cost structure.[22] Under this condition, the solution to the linear programme (7) has a very natural structure. There are three cases, depending on the relative levels of primary and secondary market rates, and on the structure of the bank's balance sheet. We consider only the most interesting case, the internal solution, in which market rates differ by not too much between markets and the bank is active in both primary and secondary markets. See Figure 1 for illustration. In this case, there will be critical values of premia t_*^P and t_*^S in the primary and secondary markets, respectively, satisfying

$$r^P + t_*^P = r^S + t_*^S, \tag{8}$$

such that a category P collateral a is used for funding purposes (i.e., $x^P(a) = x(a)$ and $x^S(a) = 0$) if and only if $t^P(a) \leq t_*^P$, a category S collateral a is used for funding purposes (i.e., $x^S(a) = x(a)$ and $x^P(a) = 0$) if and only if $t^S(a) \leq t_*^S$. Moreover, a category PS collateral a is used for funding in the primary market if and only if

$$t^P(a) \leq t_*^P \text{ and } r^P + t^P(a) \leq r^S + t^S(a). \tag{9}$$

Analogously, a category PS collateral a is used for funding in the secondary market if and only if

$$t^S(a) \leq t_*^S \text{ and } r^S + t^S(a) \leq r^P + t^P(a). \tag{10}$$

The critical values are determined implicitly by the condition that the nominal value of the collateral used by the bank for funding purposes equals just q. The formal description of this condition is given in the proof of Proposition 1 which can be found in the Appendix.

But from this solution, it is immediate that an increase in r^P typically leads to a strictly lower use of collateral in the primary market, and to a strictly higher use of collateral in the secondary market. Indeed, as Figure 1 illustrates, if the rate in the primary market increases ceteris paribus, then

[22] In technical terms, this means that the measures induced by the balance sheet function $x(.)$ via the three mappings $t^P : P \to \mathbb{R}_+$, $t^S : S \to \mathbb{R}_+$, and $(t^P, t^S) : PS \to \mathbb{R}_+^2$ have densities that vanish nowhere.

the most expensive type of collateral used in the primary market will not be used for funding purposes anymore. Instead, secondary market funding will increase by using the cheapest collateral that has not been used beforehand. Moreover, collateral that implied similar premia in the primary and secondary markets will be used in the secondary market. If there is a sufficiently broad range of collateral with heterogeneous cost structure, then an increase in r^P can only lead to a reduction of demand in the primary market.

Proposition 1. *Assume that the bank's balance sheet contains a sufficiently broad range of collateral with a heterogeneous cost structure. Then the demand function in the primary market will not react perfectly elastic to conditions in the secondary market, i.e., marginal valuations of bidders in the primary market will be strictly declining.*

The proposition says that if collateral is sufficiently heterogeneous with respect to the premia caused by it in transactions vis-à-vis either central bank or market participants, then the demand function of an individual counterparty in the primary market will react smoothly to changes in the secondary market rate. Thus, the marginal valuation of an individual commercial bank in open market operations may be strictly declining due to the necessity to handle the financial risks associated with the funding transaction, and due to the fact that central bank and market participants have different needs for managing these risks.[23]

Comparison with treasury auctions. In the example of governmental treasury auctions, the differences in the premia between primary and secondary markets are mainly driven by differences in liquidity between on-the-run and off-the-run securities (cf. Bikhchandani and Huang [8]). In the model of dual funding, this would correspond to a situation with an essentially degenerated measure on A. Just one type of transaction $a^* \in PS$ is

[23] The reader will note that the conclusion of Proposition 1 does not depend on the differences of collateral requirements between primary and secondary markets. To see why, assume that $P = S = \varnothing$. Then the argument goes through under the realistic assumption that the transaction premia of individual transactions differ at least somewhat between primary and secondary markets.

feasible, with a liquidity premium $t^P(a^*)$ in the primary market, and $t^S(a^*)$ in the secondary market. Demand in the primary auction should therefore respond almost perfectly elastically at $r^P = r^S + t^S(a^*) - t^P(a^*)$, which is tantamount to essentially constant marginal valuations for participants in treasury auctions.

We return now to our discussion of central bank refinancing operations. In particular, we will assume strictly declining marginal valuations for the rest of the paper.

III. Uniform pricing

An auctioneer puts up for sale a random quantity, the total allotment $\widetilde{Q} \geq 0$, of a perfectly divisible good. There are $i = 1, ..., n$ bidders. The bidders do not observe the total allotment prior to the submission of bids. Neither does the auctioneer exploit his information about the incoming bid schedules to affect the distribution of $\widetilde{Q}$. There are two alternative interpretations for uncertainty about the aggregate allotment. First, the central bank may possess a superior knowledge of the liquidity conditions in the banking system. Second, there may be a fraction of non-strategic bidders.[24] In practice, one might expect that both effects contribute to the uncertainty about the residual supply perceived by the individual bidder.

We will focus on the case of linear equilibria and will therefore assume that $\widetilde{Q}$ has full support on $[0; \overline{Q}]$ for some $\overline{Q} > 0$. To obtain a piecewise linear equilibrium also under the discriminatory pricing rule, we will have to assume that $\widetilde{Q}$ is uniformly distributed. Moreover, it will be assumed throughout that marginal valuations are linearly decreasing from a maximum valuation $\overline{v} > 0$ that is common to all bidders.[25] Thus, bidder i's marginal valuations

[24] E.g., in the bidding data for the Eurosystem, there is a clearly distinguishable subpopulation of bidders who place their bids at surprisingly high rates. These bidders need not be non-strategic in the literal sense, but may be severely credit constrained as described in Section II.

[25] In the model of dual funding developed in Section II, the interest rate $\overline{v}$ corresponds to the smallest value of the primary market rate r^P at which its is optimal for the counterparty to rely exclusively on secondary market funding.

for quantities $q_i \geq 0$ are formally given by

$$v_i(q_i) = \overline{v} - B_i^{-1} q_i,$$

for an exogenous parameter $B_i > 0$. By a symmetric set-up, we mean a parameter constellation satisfying $B_1 = ... = B_n$. The interpretation is that $\overline{v}$ is the interest rate in the unsecured market, and that the slope parameter B_i reflects the structure of counterparty i's pool of collateral.

The tender mechanism asks each bidder i to submit a bid schedule that specifies, for any price $p \geq 0$, the amount $x_i(p) \geq 0$ that bidder i is willing to buy at p. A schedule $x_i(.)$ is called *admissible* if $x_i(.)$ is non-increasing, left-continuous, and if $x_i(p) = 0$ for any sufficiently high p. Let $x(p) = \sum_{i=1}^{n} x_i(p)$ denote the total demand at price p, and $P^*(\widetilde{Q}) = \{p \geq 0 | x(p) \leq \widetilde{Q}\}$ the set of prices at which total demand can be satisfied with the quantity $\widetilde{Q}$. It is straightforward to check that $P^*(\widetilde{Q})$ is non-empty for any $\widetilde{Q} \geq 0$ and for any vector of admissible bid schedules $\{x_i(.)\}_{i=1,...,n}$. We may therefore define the *stop-out price* as the infimum $p^*(\widetilde{Q}) = \inf P^*(\widetilde{Q})$ of such prices. Only admissible bid schedules are accepted by the auctioneer.

Individual allotments are determined by satisfying all bids strictly above the stop-out price, and by applying rationing at the margin, if necessary.[26] Formally, if $x(0) \leq \widetilde{Q}$, then all bids are satisfied, so that the allotment to bid i amounts to $q_i^*(\widetilde{Q}) = x_i(0)$. Otherwise, define $x_i^+(p^*) = \lim_{p \to p^*, p > p^*} x_i(p)$ as bidder i's demand at a price just above p^*, and let $x^+(p^*) = \sum_{j=1}^{n} x_j^+(p^*)$ denote the corresponding aggregate. In this case, bidder i obtains an allotment

$$q_i^*(\widetilde{Q}) = x_i^+(p^*(\widetilde{Q})) + \frac{x_i(p^*(\widetilde{Q})) - x_i^+(p^*(\widetilde{Q}))}{x(p^*(\widetilde{Q})) - x^+(p^*(\widetilde{Q}))}\{\widetilde{Q} - x^+(p^*(\widetilde{Q}))\}$$

in state $\widetilde{Q}$. Thus, when demand exceeds supply, the allotment is composed of a complete allocation of the part of the bid schedule that lies above the stop-out price, and a pro-rata allocation of any flat segment of the bid schedule

[26] Our results remain valid without modification for all alternative rationing rules with the property that bids strictly above the stop-out price are fully satisfied. As pointed out by Kremer and Nyborg [36], rules with this characteristic are used predominantly in practice.

that lies at the stop-out price. The tuple $(p^*, q_1^*, ..., q_n^*)$ consisting of the stop-out price and the individual allotments will be referred to as the outcome of the tender.

In both tender formats, bidders are assumed to maximize expected profits. In the uniform-price tender, bidder i pays the stop-out price p^* per marginal unit, so that bidder i's profits from an outcome $(p^*, q_1^*, ..., q_n^*)$ is given by

$$\Pi_i^u = \int_0^{q_i^*} \{v_i(q_i) - p^*\} dq_i.$$

Figure 2 illustrates bidder i's profit under the uniform pricing rule as the shaded area between marginal valuation and stop-out price. The crucial element driving bidding behaviour in this type of auction is the residual supply, i.e., the supply diminished by the allotments made to the other bidders at a given price. In our model, supply is perfectly inelastic, while the stated demand of the other bidders is downward sloping. Thus the residual supply, being the horizontal difference between supply by the central bank and demand by the other bidders, must be increasing in the price. Indeed, the higher the price, the lower the aggregate demand of the other bidders, so that the residual supply must be increasing.

The figure suggests that it is clearly dominated to demand a strictly positive quantity at a price $p > \overline{v}$. Similarly, it will be intuitively clear that a zero bid at any price $p < \overline{v}$ will be dominated. Therefore, given the linear quadratic set-up, a natural candidate for an equilibrium strategy is to scale the underlying true demand function $d_i(p) = B_i \max\{\overline{v} - p; 0\}$ by a constant factor, so that the candidate equilibrium attains the form

$$x_i(p) = B_i^u \max\{\overline{v} - p; 0\}, \tag{11}$$

where $B_i^u > 0$ is a constant. In the context of a uniform-price auction, we will refer to an equilibrium in which all bidders $i = 1, ..., n$ use some schedule of the form (11) as a linear equilibrium.

Proposition 2. *Assume $n \geq 3$, and that $\overline{Q}$ is not too large. Then there exists a linear equilibrium (11) in the tender with uniform pricing. In fact,* *the equilibrium is unique within the class of linear equilibria. When compared to the underlying true demand, bids are shaded, i.e., $B_i^u < B_i$ for all i. Moreover, in any equilibrium with heterogeneous bidders, shading of bids is monotonic, i.e., for all $i \neq j$ we have $B_i^u < B_j^u$ if and only if $B_i < B_j$. In the symmetric set-up, the equilibrium is given by $B_i^u/B_i = (n-2)/(n-1)$ for all i.*

The proof as well as the full characterization of the equilibrium strategies can be found in the Appendix. There, we follow the approach used by Kyle [38], Klemperer and Meyer [35], and Back and Zender [2], which relies on the intuition that if the quantity to be transacted is uncertain for the bidders, then the optimal bidding strategy for the uniform pricing rule can essentially be found by a state-by-state optimisation against the ex-post residual supply curve.

The prediction of the model is consistent with general studies of bidding behaviour in uniform-price auctions such as Ausubel and Cramton [1]. Specifically, the main prediction is differential bid shading, i.e., that bidders will state their demand sincerely only for the initial marginal unit, and shade their bids at all strictly positive quantities. Indeed, intuitively, the bidder has an incentive to shade demand because the stop-out price will apply not only to the marginal unit, but to the whole allotment to bidder i.

The conditions imposed in Proposition 2 appear necessary to obtain the result. In particular, there is no linear equilibrium with strictly decreasing bid schedules for just two bidders. Indeed, it is straightforward to check that, in this case, the slope of the residual supply for the individual bidders is too steep to allow convergence of the dynamics of mutual best responses. Similarly, the requirement on $\overline{Q}$ ensures that the tender price does not fall to zero in equilibrium with strictly positive probability. If this condition is violated with strictly positive probability, then the established equilibrium breaks down in the present model. Intuitively, if the stop-out price is zero with strictly positive probability, then bidders would like to overstate demand at price zero in anticipation of the rationing. With continuous prices, however, this cannot be an optimal strategy. In the proof, we allow for the possibility that the stop-out price drops to zero as the result of a deviation from equilibrium behaviour.

Proposition 3. *Consider a family of tenders* $\{T(n)\}_{n=3,4,5,...}$ *with uniform pricing, in which a random quantity not larger than* $\overline{Q}(n) = n\overline{Q}$ *is auctioned off to n bidders. Assume that the slope parameters* $\{B_i(n)\}_{i=1,...,n}$ *are uniformly bounded, i.e., there are* $\overline{B} > \underline{B} > 0$ *such that* $\underline{B} \leq B_i(n) \leq \overline{B}$ *for all* $n \geq 3$ *and for all* $i = 1, ..., n$*. Assume also that* $\overline{Q} < \underline{B}\overline{v}$*. Then* $\lim_{n\to\infty} B_i^u(n)/B_i(n) = 1$ *for all i.*

The proof can be found in the Appendix. Proposition 3 extends a special case of Klemperer and Meyer's Proposition 8a (linear demand with $m = 0$) by allowing for bidder heterogeneity. It also suggests the robustness of a very general result by Swinkels [51] for auctions of a finite number of identical units. Intuitively, Proposition 3 says that bid shading disappears under the uniform pricing rule in large populations of bidders provided that relative marginal valuations do not vary too widely. The reason is that, when the number of bidders increases and each new bidder adds non-negligible demand, then the residual supply curve faced by an individual bidder becomes flatter and flatter in the (q, p) diagram. As a consequence, the effect that an individual bidder will have on the price realised in the tender will be smaller and smaller, leading to a higher quantity submitted at a given price. In the limit, the residual supply is essentially a horizontal line, and induces price-taking behaviour on the part of the individual bidder. Thus, under the uniform pricing rule, bid shading vanishes in the limit population.

IV. Discriminatory pricing

Bid schedules can be formally described in two natural ways, one expressing demand at given prices (the one used so far), and the other attaching prices to given quantities. The discriminatory pricing rule requires the payment of the individual bidder's own price bid on the allotted quantities. It is therefore natural to work, rather than with the bid schedule $x_i(p)$ itself, with the *inverse* schedule $b_i(q_i) = \inf\{p \geq 0 | x_i(p) \leq q_i\}$. The figure $b_i(q_i)$ can be understood as the stated willingness to pay (the "bid") for the marginal unit at quantity q_i, contrasting the true willingness to pay for the marginal unit (the "valuation"), which is given by $v_i(q_i)$. Similarly as in the definition of the stop-out price, one can check that $b_i(q_i)$ is well-defined for admissible bid schedules $x_i(.)$. Moreover, $b_i(q_i)$ is a non-increasing and right-continuous function.

Under discriminatory pricing, the bidder i pays his own bid $b_i(q_i)$ for any marginal unit, so that the resulting profit from an outcome $(p^*, q_1^*, ..., q_n^*)$ amounts to

$$\Pi_i^d = \int_0^{q_i^*} \{v_i(q_i) - b_i(q_i)\} dq_i. \tag{12}$$

The shaded area in Figure 3 depicts bidder i's profit under the discriminatory pricing rule. Thus, in contrast to the case of the uniform-price auction, the whole bid schedule above the realized stop-out price p^* determines the bidder's profit, not just the quantity placed at p^*. This feature of the discriminatory pricing rule makes the general characterisation of the equilibrium more involved so that we have to restrict ourselves to the symmetric set-up.[27]

Proposition 4. *Assume $n \geq 2$, and that bidders $i = 1, ..., n$ have identical marginal valuations $v_i(q_i) = \overline{v} - B^{-1}q_i$. Assume also that $\overline{Q} < n\overline{v}B$. Then there exists an equilibrium in the tender with discriminatory pricing in which bidder i submits the piecewise linear bid schedule*

$$x_i(p) = \begin{cases} 0 & \text{for } p > \overline{v}^d \\ B^d(\overline{v}^d - p) & \text{for } p_{\min} < p \leq \overline{v}^d \\ B(\overline{v} - p) & \text{for } p \leq p_{\min} \end{cases} \tag{13}$$

[27] In the asymmetric case, marginal conditions do not describe an equilibrium, which illustrates a potential problem with the first-order approach to the analysis of multi-unit auctions. Indeed, one can show that an equilibrium in pure strategies does not exist in the asymmetric case. The reason is that first-order conditions for an equilibrium imply crossovers and flat sections on bid schedules, which in turn make non-marginal deviations attractive. The impossibility of an equilibrium in a related bidding game with an asymmetric population of bidders has been shown by Menezes and Monteiro [45].

for $i = 1, ..., n,$ *where*

$$\overline{v}^d = \overline{v} - \frac{\overline{Q}}{(2n-1)B} \tag{14}$$

$$B^d = \frac{2n-1}{n-1}B \tag{15}$$

$$p_{\min} = \overline{v} - \frac{\overline{Q}}{nB}, \tag{16}$$

are the maximum price bid, the slope of the inverse bid schedule, and the minimum stop-out price, respectively.

The proof can be found in the Appendix. Figure 3 illustrates the bidding behaviour in the discriminatory auction. The intersection point between the individual bid schedule and the residual supply would determine both the allotment to the counterparty and the marginal rate in the operation. Moving from this intersection point upwards until the marginal valuation is found would deliver an interest rate level that should correspond to the marginal interbank repo rate $r^S + t^S(a)$. This marginal interbank repo rate is associated with the asset type a for which a counterparty would be indifferent between using it either in the primary or in the secondary market, or not at all for funding purposes (cf. Section II).

Compared to the uniform-pricing rule, the shading is of a different nature. In a uniform tender, the slope of the strategic bid schedule is steeper than the true demand. This is because in a uniform-price auction, each bidder pays the stop-out price for the whole allotment. As the stop-out price is determined on the basis of the relevant part of his bid schedule, shading of bids should be expected for any strictly positive quantity. Moreover, the shading of bids should become more pronounced for larger quantities because the relative benefit from shading the bid schedule increases for larger quantities.

In an auction with discriminatory pricing, however, the strategic bid curve is flatter than the true demand. There is little shading of bids at larger quantities, and more shading at smaller quantities. As a consequence, with discriminatory pricing, there is shading in the intercept of the demand function. The reason for this different form of bid shading is that the price bid placed at a given quantity will appear to be too high from an ex-post perspective only if the allotment was large enough. The placement of an honest price bid at low quantities is therefore more likely to lead to a decrease in expected profits than the placement of an honest price bid at high quantities. This leads to a more pronounced shading of bids for small quantities.

Proposition 4 makes also clear predictions about the shape of the bid schedules if n is large. Specifically, assume that as n goes to infinity, the quantity allotted is $\overline{Q}(n) = n\overline{Q}$. Then the above result predicts that the maximum price at which a bid is placed will converge against

$$\lim_{n\to\infty} \overline{v}^d(n) = \overline{v} - \frac{\overline{Q}}{2B}. \tag{17}$$

Thus, strategic behaviour does not disappear in the limit.[28] As we will explain now, this may be an explanation for an unexpected empirical observation that has been made for euro area.

Evidence of bid shading. Since June 2000, the Eurosystem has chosen to conduct its regular main refinancing operations in the form of variable rate tenders with discriminatory pricing. Our model of equilibrium bidding in variable rate tenders, even though it is somewhat stylised, can be interpreted in a way that corresponds to the realities of these operations.

Indeed, the formal model predicts that even in a large auction, the marginal rate in the tender lies strictly below the marginal interbank repo rate, as a consequence of bid shading.[29] This prediction is in line with the data for the euro area. To see why this is the case, recall that the general collateral

[28] Given that the uncertainty about aggregate supply does *not* disappear in the limit, this finding is consistent with existing theoretical results for large auctions obtained by Swinkels [51]. See also Swinkels [50].

[29] The marginal rate is the lowest rate at which bids are allotted, and corresponds intuitively to the *marginal cost* of funding in the primary market. Expectations about this rate determine the optimal split of funding between primary and secondary markets. The marginal rate differs from the so-called weighted average rate, which is the volume-weighted average of the interest rates of all winning bids, and which would intuitively correspond to the *average cost* of funding in the primary market. In the discriminatory auction, the weighted average rate lies always weakly above the marginal rate.

is a collection of higher-rated securities that serve as a standard in interbank lending. The criteria for general collateral are comparably strict. In fact, they are stricter than the eligibility criteria that are applied by the Eurosystem for collateral used in open market operations. As mentioned before, counterparties tend to use the less expensive illiquid collateral in central bank operations. The natural benchmark for the marginal rate in the central bank operation is therefore not the GC repo rate but the (higher) rate required for a repurchase agreement in the market against the illiquid collateral.

As a consequence, the GC repo rate should be on average lower than the hypothetical rate at which the market would price repurchase agreements based on marginal asset types used as collateral in the refinancing operations of the Eurosystem. However, surprisingly at first sight, one cannot reject the null hypothesis of the equality between the market repo rate and the marginal rate in the main refinancing operations of the Eurosystem (cf. Cassola, Ravazzolo, and Würtz [14]). The analysis therefore suggests strategic bid shading as a possible explanation for the marginal rate in the auction being lower than predicted by arbitrage considerations.

V. Conclusion

In this paper, we have discussed some aspects of the economics of monetary refinancing operations. We have argued that the auctioning of central bank reserves differs from the auctioning of other divisible goods such as treasury securities. Refinancing operations are special because they involve the management of the financial risks associated with the refinancing transaction. E.g., in the case of the regular refinancing operations conducted by the Eurosystem, counterparties have some discretion concerning the choice of eligible collateral. The individual bidder will therefore exploit comparably cheap (i.e., illiquid) collateral first in central bank operations. As the private market requires a premium on illiquid assets, this suggests declining marginal valuations for interbank liquidity. This point is crucial because marginal valuations are the main determinant of optimal bidding behaviour in central bank operations.

To evaluate incentives for bidding in central bank operations under this assumption, we have considered a model in the tradition of Klemperer and Meyer [35], assuming linearly decreasing marginal valuations and uncertain supply. Our contribution here is the explicit construction of an equilibrium in the discriminatory auction. The construction provides an explicitly solvable model with declining marginal valuations.[30]

The analysis of equilibrium bidding behaviour supports the view that counterparties have an incentive to adjust their bid schedules strategically in response to the uncertainty about the stop-out rate. The adjustments lead throughout to lowered demand, i.e., to bid shading. For instance, under the uniform pricing rule, the bidder has an incentive to understate her quantity demanded especially at relatively low interest rates, because this will on average lower the stop-out rate applied on all her winning bids. Therefore, in the case of the uniform pricing rule, counterparties should be expected to submit bid schedules that are steeper than the underlying demand.

These findings have served as a reference point for our analysis of the more relevant discriminatory pricing rule, applied by the Eurosystem in its main refinancing operations since June 2000. Here, the model unambiguously predicts that optimal bid schedules in the discriminatory auction will be *flatter* than the underlying demand. This is intuitive, because under the discriminatory pricing rule, a winning bidder not only has an obvious incentive for reducing demand at all interest rates, but also for a more pronounced bid shading at higher and less likely interest rates. This would suggest the optimal submission of bid schedules that a concentrated on relatively few interest rates.

Our theory also predicts that the extent of bid shading under the discriminatory pricing rule does not disappear in large auctions. This suggests an explanation of the fact that the spread between the marginal rate in the main refinancing operations of the Eurosystem and the general collateral repo rate

[30] For an empirical test of the model, see Cassola, Ewerhart, and Morana [13].

in the euro money market is statistically zero, despite the broader range of collateral eligible for the Eurosystem when compared to the repo market.

The point to note here is that the Eurosystem accepts also collateral that is less liquid than general collateral (GC), which leads to a difference in the composition of collateral between primary and secondary markets. In the absence of strategic bidding, funding costs should therefore depend on whether liquidity is obtained from the central bank or in the secured segment of the euro money market. The difference should amount to a few basis points. However, there is only a statistically insignificant difference between the GC repo rate and the marginal rate obtained in the weekly open market operations. Our analysis of optimal bidding behaviour in the context of a secondary market provides a natural explanation of this observation.[31]

Appendix. Proofs.

Proof of Proposition 1. Consider an interior solution (t_*^P, t_*^S) of problem (7) with $t_*^P > 0$, as depicted in Figure 1. The volume of funding in the primary market is given by

$$q^P = \int_{P_0(t_*^P)} x(a)da + \int_{P_1(t_*^P, t_*^S)} x(a)da,$$

where

$$P_0(t_*^P) = \{a \in P | t^P(a) \leq t_*^P\},$$

and $P_1(t_*^P, t_*^S)$ is the subset of SP characterised by (9). We wish to show that q^P is decreasing continuously in r^P. Assume therefore that r^P increases marginally, ceteris paribus. In Figure 1, the solid diagonal would shift slightly to the left. To show that q^P decreases, assume to the contrary that q^P

[31] As pointed out by many writers, it would be very desirable to have a full-fledged model of the discriminatory auction incorporating heterogeneity of bidders, incomplete information, and non-linear valuations. This might allow to explain further interesting empirical observations as reported, e.g., in ECB [19], Bindseil, Nyborg, and Strebulaev [10], Breitung and Nautz [11], Linzert, Nautz, and Bindseil [41], and Bruno, Ordine, and Scalia [12]. In Ewerhart, Cassola, and Valla [29], we develop such a model for the case of the fixed rate tender.

either stays constant or increases. Then, by the definition of the sets $P_0(t_*^P)$ and $P_1(t_*^P, t_*^S)$, the critical value t_*^P either stays constant or increases. In particular, it follows from (8) that t_*^S increases strictly. But then, because there is a wide range of collateral, the volume of secondary market funding

$$q^S = \int_{S_0(t_*^S)} x(a)da + \int_{S_1(t_*^P, t_*^S)} x(a)da,$$

with

$$S_0(t_*^S) = \{a \in P | t^S(a) \leq t_*^S\},$$

and the set $S_1(t_*^P, t_*^S)$ characterised by (10), increases strictly. However, because (2) must be satisfied with equality in the cost-minimizing case, we have $q^P + q^S = q$. As q^P and q^S cannot both increase, we obtain a contradiction. Therefore q^P must decrease. It is now obvious from the implicit function theorem that q^P cannot jump when the bank's balance sheet is sufficiently heterogeneous. This proves Proposition 1. $\square$

Proof of Proposition 2. Keep i fixed, and assume that bidders $j \neq i$ use a bid schedule $x_j(p) = B_j^u \max\{\overline{v} - p; 0\}$ for some $B_j^u > 0$. Let $B_{-i}^u = \sum_{j \neq i} B_j^u$. Assume that bidder i uses an admissible bid schedule $x_i(p)$, and consider state $\widetilde{Q}$. It will be shown first that *any price-quantity combination* $(p; q_i) = (p^*(\widetilde{Q}); q_i^*(\widetilde{Q}))$ *resulting from* $(x_i(.), x_{-i}(.))$ *in state* $\widetilde{Q}$ *under the uniform pricing rule satisfies precisely one of the following three conditions*:

(i) $p > \overline{v}$ and $q_i = \widetilde{Q}$

(ii) $0 < p \leq \overline{v}$ and $q_i = \widetilde{Q} - (\overline{v} - p)B_{-i}^u \geq 0$

(iii) $p = 0$ and $0 \leq q_i \leq \widetilde{Q} - \overline{v}B_{-i}^u$

Clearly, if $x(0) \leq \widetilde{Q}$, then $p^*(\widetilde{Q}) = 0$ and condition (iii) is satisfied. Assume therefore $x(0) > \widetilde{Q}$. Since bidder i is the only bidder with a potentially discontinuous bid schedule,

$$\begin{aligned} q_i^*(\widetilde{Q}) &= x_i^+(p^*(\widetilde{Q})) + \widetilde{Q} - x^+(p^*(\widetilde{Q})) \\ &= \widetilde{Q} - \sum_{j \neq i} x_j^+(p^*(\widetilde{Q})) \\ &= \widetilde{Q} - \sum_{j \neq i} x_j(p^*(\widetilde{Q})). \end{aligned}$$

This implies that either (i), (ii), or (iii) will be satisfied. This proves the assertion. Next, we show that *the schedule (11) with*

$$B_i^u = \frac{B_i B_{-i}^u}{B_i + B_{-i}^u} \tag{18}$$

is ex-post optimal for bidder i, provided that

$$\widetilde{Q} < \overline{v}(B_i^u + B_{-i}^u). \tag{19}$$

Fix $\widetilde{Q}$. Under the linear specification, bidder i's ex-post profit is given by

$$\Pi_i^u(p^*, q_i^*) = \int_0^{q_i^*} \{v_i(q_i) - p^*\} dq_i = q_i^* \{\overline{v} - p^* - \frac{q_i^*}{2B_i}\}. \tag{20}$$

Selecting a point (p, q_i) satisfying (i) obviously cannot yield a strictly positive profit. Moreover, from (19) it follows that among the points (p, q_i) satisfying condition (iii), the profit-maximising alternative would entail a quantity $q_i^* = \widetilde{Q} - \overline{v}B_{-i}^u$. We are therefore essentially reduced to case (ii), where $q_i^* + (\overline{v} - p^*)B_{-i}^u = \widetilde{Q}$. Implicit differentiation delivers $\partial q_i^* / \partial p^* = B_{-i}^u$. Using this in the first-order condition resulting from (20) yields the assertion. Now, we show that *the system (18), for $n \geq 3$ and for $i = 1, ..., n$, has the unique solution*

$$B_i^u = B_i + \frac{B_*^u}{2} - \sqrt{B_i^2 + (\frac{B_*^u}{2})^2}, \tag{21}$$

where the parameter B_*^u is the unique strictly positive root of the equation

$$\frac{B_*^u}{2} - \frac{1}{n-2} \sum_{i=1}^{n} \{ \sqrt{B_i^2 + (\frac{B_*^u}{2})^2} - B_i \} = 0. \tag{22}$$

Define $B_*^u = \sum_{i=1}^n B_i^u$. Using this notation, condition (18) can be rewritten as $(B_i - B_i^u)(B_*^u - B_i^u) = B_i B_i^u$. Solving for B_i^u yields

$$B_i^u = B_i + \frac{B_*^u}{2} \pm \sqrt{(B_i + \frac{B_*^u}{2})^2 - B_*^u B_i}.$$

However, only the negative root is economically relevant because otherwise $B_i^u > B_i$, which would contradict (18). This delivers (21). Summing up over all $i = 1, ..., n$, and rearranging yields (22). To see why (22) has a unique strictly positive root, note that the equation is certainly satisfied for $B_*^u = 0$. The left-hand side of (22) has a strictly positive first derivative in $B_*^u = 0$, and is strictly concave for all $B_*^u \geq 0$, so there is at most one root $B_*^u > 0$. Using (21), this proves uniqueness. On the other hand, for $B_*^u \to \infty$, the left-hand side of (22) follows the asymptotic

$$\frac{B_*^u}{2} - \frac{1}{n-2}\sum_{i=1}^{n}\{\sqrt{B_i^2 + (\frac{B_*^u}{2})^2} - B_i\} \sim -\frac{1}{n-2}\{B_*^u - \sum_{i=1}^{n} B_i\},$$

which eventually becomes negative for a sufficiently large B_*^u. Invoking the intermediate value theorem proves existence, and thereby the assertion. Clearly, we have $B_i^u < B_i$. Moreover, the right-hand side of (21) is strictly increasing in B_i, which proves the monotonicity property of equilibrium bids in heterogeneous populations of bidders. The assertion concerning the symmetric case is immediate from (21) and (22). □

Proof of Proposition 3. Fix n. Without loss of generality, $B_1(n) \leq B_i(n)$ for all $i = 1, ..., n$. From the proof of Proposition 2, we know that the slope parameters $B_1^u(n), ..., B_n^u(n)$ are characterised uniquely by (18). Monotonicity implies $B_1^u(n) \leq B_i^u(n)$ for all $i = 1, ..., n$. In particular, one has $B_{-i}^u(n) \geq (n-1)B_1^u(n)$ for any i. Using (18), one obtains

$$\frac{B_1^u(n)}{B_1(n)} = 1 - \frac{B_1^u(n)}{B_{-1}^u(n)} \geq \frac{n-2}{n-1}.$$

But then,

$$B_{-i}^u(n) \geq (n-1)B_1^u(n) \geq (n-2)B_1(n) \geq (n-2)\underline{B}.$$

Using (18) again, one finds

$$\frac{B_i^u(n)}{B_i(n)} \geq 1 - \frac{B_i^u(n)}{(n-2)\underline{B}} \geq 1 - \frac{B_i(n)}{(n-2)\underline{B}} \geq 1 - \frac{\overline{B}}{(n-2)\underline{B}} = 1 - \varepsilon_n,$$

where $\varepsilon_n \to 0$ for $n \to \infty$. But then, for n sufficiently large,

$$\overline{Q}(n) = n\overline{Q} < n\overline{v}\frac{\underline{B}}{1-\varepsilon_n} \leq \overline{v}\sum_{i=1}^{n}\frac{B_i(n)}{1-\varepsilon_n} \leq \overline{v}\sum_{i=1}^{n} B_i^u(n),$$

so that Proposition 1 guarantees the existence of the linear equilibrium. Considering the limit for $n \to \infty$ yields the assertion. □

Proof of Proposition 4. Keep an individual bidder i fixed. Denote bidder i's bid schedule by $x_i(p)$, and the resulting inverse bid schedule by $b_i(q_i)$. Assume that bidders $j \neq i$ use the bid schedule given by (13). We will show now that the optimal response of bidder i is of the same form. Expected profit for bidder i is given by

$$E[\Pi_i^d] = \frac{1}{\overline{Q}} \int_0^{\overline{Q}} \int_0^{q_i^*(\widetilde{Q})} (v_i(q_i) - b_i(q_i)) dq_i d\widetilde{Q}.$$

Changing the order of integration yields

$$E[\Pi_i^d] = \frac{1}{\overline{Q}} \int_0^{\overline{Q}} \int_{Q(q_i)} (v_i(q_i) - b_i(q_i)) d\widetilde{Q} dq_i, \tag{23}$$

where $Q(q_i) = \{\widetilde{Q} \in [0; \overline{Q}] | q_i^*(\widetilde{Q}) \geq q_i\}$ is the set of total allotments such that the allotment to bidder i is at least q_i. Rewriting (23) delivers

$$E[\Pi_i^d] = \int_0^{\overline{Q}} \mathrm{pr}\{Q(q_i)\}(v_i(q_i) - b_i(q_i)) dq_i, \tag{24}$$

where $\mathrm{pr}\{Q(q_i)\}$ denotes the probability that bidder i receives an allotment of at least q_i. Given that the bid schedules of bidders $j \neq i$ are continuous, it is a straightforward exercise to check that

$$\begin{aligned} \mathrm{pr}\{Q(q_i)\} &= \mathrm{pr}\{q_i^*(\widetilde{Q}) \geq q_i\} \qquad (25) \\ &= \frac{1}{\overline{Q}} \begin{cases} \overline{Q} - q_i & \text{if } b_i(q_i) > \overline{v}^d \\ \overline{Q} - q_i - (n-1)(\overline{v}^d - b_i(q_i))B^d & \text{if } p_{\min} \leq b_i(q_i) \leq \overline{v}^d \\ \overline{Q} - q_i - (n-1)(\overline{v} - b_i(q_i))B & \text{if } b_i(q_i) < p_{\min} \end{cases} \end{aligned}$$

for $q_i \in [0; \overline{Q}/n]$. The explicit calculation is helpful as it shows that $\mathrm{pr}\{Q(q_i)\}$ does not depend on the whole bid schedule, but only on $b_i(q_i)$. We will search now first for a pointwise maximiser $b_i^*(q_i)$ of the integrand

$$I(b_i, q_i) = \mathrm{pr}\{Q(q_i)\}(v_i(q_i) - b_i)$$

in (24), and check then that the thereby obtained inverse bid schedule $b_i^*(q_i)$ results from the conjectured bid schedule $x_i(p)$. Let $q_i \geq 0$ be given. Clearly, we have $b_i^*(q_i) \leq \overline{v}^d$ for $q_i < \overline{Q}$ because otherwise, lowering $b_i^*(q_i)$ marginally would increase the integrand. We maximise the integrand now assuming the second case in (25), ignoring the restrictions for the moment. This yields

$$\begin{aligned} b_i^*(q_i) &= \arg\max_{b_i}(\overline{Q} - q_i - (n-1)(\overline{v}^d - b_i)B^d)(\overline{v} - B^{-1}q_i - b_i) \\ &= \frac{\overline{v} - B^{-1}q_i + \overline{v}^d}{2} - \frac{\overline{Q} - q_i}{2(n-1)B^d}. \end{aligned} \tag{26}$$

Evaluating this expression at $q_i = 0$ delivers (14), and plugging the obtained value for $\overline{v}^d$ back into (26) leads to (15). It is suboptimal to choose the boundary value $b_i(q_i) = \overline{v}^d$. To see why this is true, one compares the value of the integrand at the boundary, i.e.,

$$I(\overline{v}^d, q_i) = \frac{\{\overline{Q} - (2n-1)q_i\}(\overline{Q} - q_i)}{\overline{Q}B(2n-1)}$$

with the expected profit from the interior solution

$$I(b_i^*(q_i), q_i) = \frac{(\overline{Q} - nq_i)^2}{\overline{Q}B(2n-1)}.$$

It is straightforward to check that the interior solution $b_i^*(q_i)$ is always preferred to $\overline{v}^d$. A deviation to a bid $b_i(q_i) < p_{\min}$ is also not optimal. To see why, note that from (25), in this case

$$I(b_i, q_i) = \{\overline{Q} - q_i - (n-1)(\overline{v} - b_i)B\}(v_i(q_i) - b_i).$$

For q_i given, the maximising argument of this expression is

$$b_i^{\#}(q_i) = \overline{v} - \frac{\overline{Q}}{2(n-1)B} - \frac{q_i}{2B}\frac{n-2}{n-1}.$$

But for $q_i \leq \overline{Q}/n$, a straightforward calculation shows that $b_i^{\#}(q_i) \geq p_{\min}$. As $I(b_i, q_i)$ is concave in b_i, the optimum bid must satisfy $b_i^*(q_i) \geq p_{\min}$. Therefore, lowering $b_i^*(q_i)$ below $p_{\min}$ cannot improve bidder i's payoff at $q_i < \overline{Q}/n$. But then, the minimum stop-out price that is realized in the auction is

$$\overline{v}^d - \frac{\overline{Q}}{nB^d} = \overline{v} - \frac{\overline{Q}}{nB} = p_{\min},$$

provided that $\overline{Q} < n\overline{v}B$. The bidder is indifferent about the bid schedule for quantities $q_i > \overline{Q}/n$, so we may set $b_i^*(q_i) = \overline{v} - B^{-1}q_i$ for these values. We have found a pointwise maximiser of (24). Clearly, this function $b_i^*(q_i)$ results from the bid schedule given in (13). $\square$

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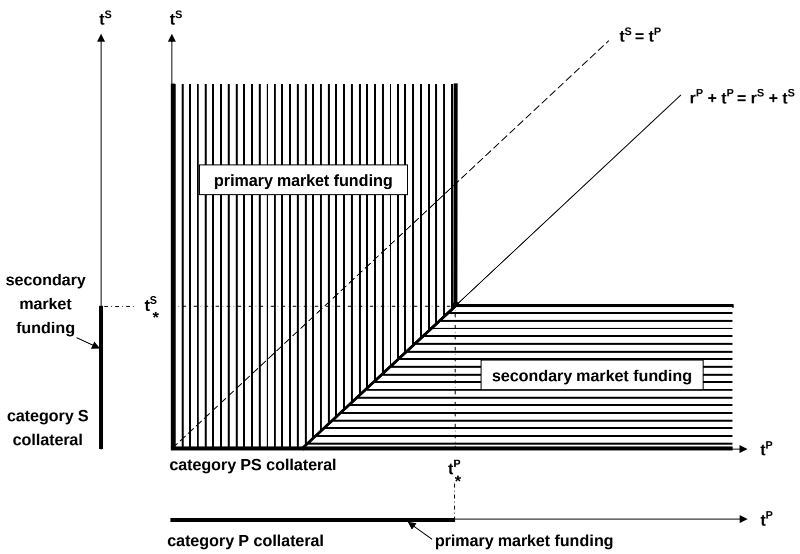

Figure 1

1

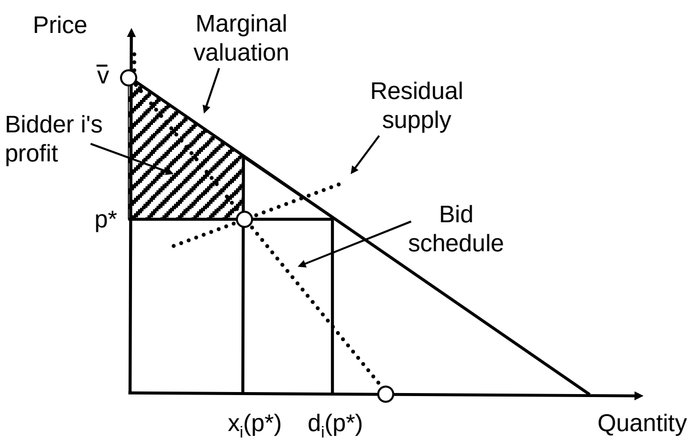

Figure 2

2

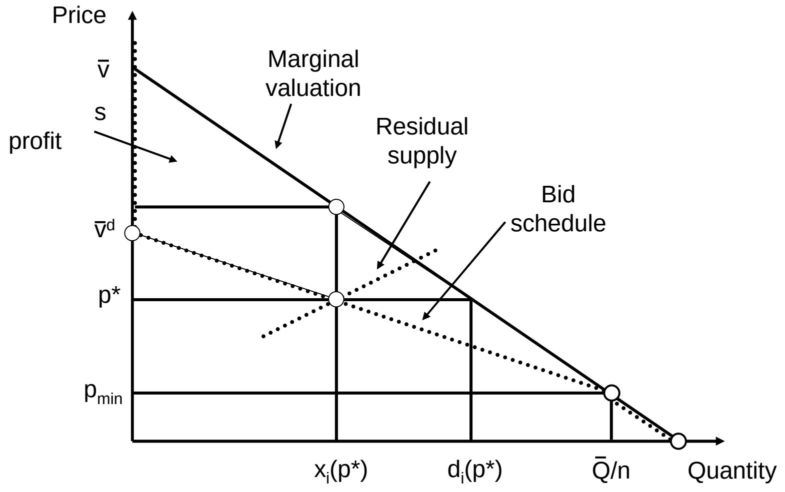
Price
v̄
s
profit
Marginal valuation
Residual supply
Bid schedule
v̄d
p*
pmin
xi(p*)
di(p*)
Q̄/n
Quantity

Figure 3

3

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