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INFLATION FORECAST-BASED-RULES AND INDETERMINACY

A PUZZLE AND A RESOLUTION

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In 2006 all ECB publications will feature a motif taken from the €5 banknote.

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Abstract

We examine an interesting puzzle in monetary economics between what monetary authorities claim (namely to be forward-looking and pre-emptive) and the poor stabilization properties routinely reported for forecast-based rules. Our resolution is that central banks should be viewed as following 'Calvo-type' inflation-forecast-based (IFB) interest rate rules which depend on a discounted sum of current and future rates of inflation. Such rules might be regarded as both within the legal frameworks, and potentially mimicking central bankers' practice. We find that Calvo-type IFB interest rate rules are first: less prone to indeterminacy than standard rules with a finite forward horizon. Second, for such rules in difference form, the indeterminacy problem disappears altogether. Third, optimized forms have good stabilization properties as they become more forward-looking, a property that sharply contrasts that of standard IFB rules. Fourth, they appear data coherent when incorporated into a well-known estimated DSGE model of the Euro-area.

JEL Classification: E52, E37, E58

Keywords: Inflation-forecast-based interest rate rules, Calvo-type interest rate rules, indeterminacy

Non Technical Summary

All modern central banks stress the importance of forward-looking policy and many the notion that interest rates should be based on future inflation expectations. The basis for such 'forecast-based' rules is that, by anchoring expectations, they improve the credibility and transparency of monetary policy as well as allowing policy to be pre-emptive. However, one concern is that typical forward-looking monetary policy rules (such as Taylor-type rules) may to lead to real indeterminacy which implies that when a shock displaces the economy from its equilibrium, there are an infinite number of possible paths for the real variables leading back to equilibrium. Such 'sunspot equilibria' are of interest because sunspot fluctuations – i.e. persistent movements in inflation and output that materialize even in the absence of shocks to preferences or technology – are typically welfare reducing and potentially quite large. Whether policy rules lead to real indeterminacy depends on whether feedback parameters are insufficiently, or indeed overly, aggressive as well as depending on the length of the forecast horizon itself.

Consequently, there would appear to an interesting puzzle in monetary economics between what policy makers claim (namely to be forward-looking and pre-emptive) and the poor stabilization properties routinely reported for forecast-based rules. The purpose of this paper is to suggest a resolution. We propose viewing central banks as following 'Calvo-type' inflation-forecast-based interest rate rules which depend on a discounted sum of current and all future rates of inflation. Such rules, it turns out, are less prone to indeterminacy than standard ones with a finite forward horizon and, if formulated in difference form, the indeterminacy problem disappears altogether.

Indeed, we show that optimized Calvo-type rules have good stabilization properties as they become more forward-looking, a property that sharply contrasts that of standard IFB rules. Finally, when taken to the data, Calvo-type monetary policy rules appear to behave at least as well as and sometimes better than more standard monetary-policy reaction function.

1 Introduction

All modern central banks stress the importance of forward-looking policy and many the notion that interest rates should be based on future inflation expectations. Well-known examples include the central banks of Canada and New Zealand but this is also true by implication of the practice of other central banks as well. For example, the monetary policy strategy of the European Central Bank states that "price stability is to be maintained over the medium term", (ECB (1999), p. 47) which precisely suggests a forward-looking non-inflationary strategy. The basis for inflation 'forecast-based' rules is that, by anchoring expectations, they improve the credibility and transparency of monetary policy as well as allowing policy to be pre-emptive. However, such rules have been criticized on various fronts. One concerns the result that typical forward-looking monetary policy rules (such as Taylor-type rules) tend to lead to real indeterminacy (Woodford (2003), chapter 4). This implies that when a shock displaces the economy from its equilibrium, there are an infinite number of possible paths for the real variables leading back to equilibrium. Such 'sunspot equilibria' are of interest because sunspot fluctuations – i.e. persistent movements in inflation and output that materialize even in the absence of shocks to preferences or technology – are typically welfare reducing and potentially quite large. Whether policy rules lead to real indeterminacy depends on whether feedback parameters are insufficiently, or indeed overly, aggressive as well as depending on the length of the forecast horizon itself (Levin *et al.* (2003), Batini and Pearlman (2002), and Batini *et al* (2004a, b)).

Consequently, there would appear to an interesting puzzle in monetary economics between what policy makers claim (namely to be forward-looking and pre-emptive) and the poor stabilization properties routinely reported for forecast-based rules. The purpose of this paper is to suggest a resolution. We propose viewing central banks as following 'Calvo-type' inflation-forecast-based (IFB) interest rate rules which depend on a *discounted* sum of current and all future rates of inflation. Such rules, it turns out, are less prone to indeterminacy than standard ones with a finite forward horizon and, if formulated in difference form, the indeterminacy problem disappears altogether. Indeed, we show that optimized Calvo-type rules have good stabilization properties as they become more forward-looking, a property that sharply contrasts that of standard IFB rules. Finally, when taken to the data, they appear to behave at least as well as and sometimes better than more standard monetary-policy reaction function.

Abstracting from such technical characteristics, moreover, the Calvo-type rules we examine might also be regarded as both within the legal framework of, and potentially mimicking central bankers' practice. Such rules, however, raise the following tension.

On the one hand, we know that the authorities frequently tailor policy and communication strategies to forward-looking outcomes. On the other, we further know that forward-looking policy rules are susceptible to indeterminacy. Central banks, moreover, themselves generate expectations for future-dated outcomes but do so in a chronically uncertain environment. Accordingly, we might conjecture that whilst policy makers will want to incorporate forecasts into their decision strategies, they may be reluctant to treat them commensurate with realized outcomes. This "chronically uncertain environment" faced by policy makers takes many forms. Consider a few examples.

First, macroeconomic time series tend to be actively revised in the quarters following their publication; thus, rule-based policy prescriptions derived from realized data may depart significantly from its real-time counterpart (e.g. Orphanides (2001)). It goes without saying that forecasting in such a 'noisy' data environment complicates the policy process considerably; authorities might then take recourse to contemporaneous or backward-looking rules, or else persevere with strategies that explicitly incorporate but potentially downplay (i.e., discount) forward-looking information.

Second, and more fundamentally, central-banks often employ strong conditioning assumptions in these very forecasts such as constant projections of financial variables, shock processes and external assumptions. Potentially, therefore, forecasts (particularly medium-term ones) might be considered more a benchmark for scenario analysis and discussion than a specific expected outturn. In line with this, forecasts are often wrapped around confidence intervals or "fan-charts" whose widths are necessarily increasing in the forecast horizon. Moreover, with every new forecast round, data, assumptions, expert judgment and risks are updated such that 'forecasts' may themselves be heavily revised over time and differ markedly across institutions (e.g., Artis and Marcellino (2001)). Again, in such circumstances, central banks may wish to incorporate these forecasts in their information set and policy strategy but weigh them accordingly. Similarly, one might consider other germane examples based on the various forms of model and judgmental uncertainty (e.g., Onatski and Stock (2002), Svensson (2005)) and its consequences for attenuated or non-attenuated policy making.

Summing up we might say that whilst forward-looking policies require forecasts, the very nature of the policy process – i.e., forecast and judgmental revisions, real-time data problems, model uncertainty etc – constrains policy makers to treat such information in a manner different from realized outcomes. Our solution – to think of policy as a Calvo-type IFB rule – though simple, is quite powerful: policy makers target future outcomes (such as future inflation rates) in a geometrically discounted manner. We show that this precludes indeterminacy for a number of cases and, indeed, appears potentially data coherent when appended to an estimated DSGE model.

The paper proceeds as follows. Section 2 sets out a model chosen for its tractability and summarizes the analytical findings of the literature on IFB rules in their standard form. In the analysis we focus exclusively on 'pure' inflation targeting without a feedback on the output gap. We do this for two reasons: first, the problems associated with measuring the output gap and therefore implementing rules of the 'Taylor' type. Second since simplicity per se is regarded as a positive aspect of monetary rules, it is of interest to study the stabilizing performance of rules which are indeed as simple and transparent as possible. The new contribution to the IFB literature is in section 3 which provides results for 'Calvo-type' interest rate rules. Section 4 illustrates the relative empirical performance of our chosen rule when incorporated into the well known Smets-Wouters DSGE model of the euro area. Section 5 concludes.

2 The Model and Previous Results for IFB Rules

We adopt a standard New Keynesian model popularized notably by Clarida *et al.* (1999) and Woodford (2003).

$$\begin{aligned}
\pi_t &= \beta E_t \pi_{t+1} + \lambda mc_t && (1)\\
mc_t &= -(1+\phi)a_t + \sigma c_t + \phi y_t && (2)\\
c_t &= E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) && (3)\\
y_t &= c_y c_t + g_y g_t && (4)\\
a_t &= \rho_a a_{t-1} + \epsilon_{a,t} && (5)\\
g_t &= \rho_g g_{t-1} + \epsilon_{g,t} && (6)
\end{aligned}$$

In (1) and (3), π_t is the inflation rate, β is the private sector's discount factor, $E_t(\cdot)$ is the expectations operator, y_t is output, c_t is consumption and the slope of the Phillips curve λ can be expressed in terms of the average contract length of Calvo-type price contracts. mc_t given by (2) is the marginal cost, where a_t is a technology shock and ϕ is the Frisch parameter. (1) is derived as a linearized form of staggered price setting about a zero-inflation steady state and (3) is a linearized Euler equation with i_t the nominal interest rate and σ the risk aversion parameter. (4) is a linearized aggregate equilibrium relation where g_t is a government spending shock and c_y and g_y are consumption and government spending shares respectively, in the steady state. According to (5) and (6), shocks follow AR(1) processes. All variables are expressed as deviations about the steady state, π_t and i_t as absolute deviations and c_t, y_t and g_t as proportional deviations.

To close the model we need an interest rate rule. The cornerstone of much of the monetary-policy literature is the well-known Taylor (1993) rule, a generalized version of which is,

$$\begin{aligned} \rho \in [0,1) \quad &: \quad i_t = \rho i_{t-1} + (1-\rho)\left[\pi_t^* + \theta_\pi E_t(\pi_{t+j} - \pi_{t+j}^*) + \theta_y E_t(y_{t+k} - \hat{y}_{t+k})\right] \\ \rho = 1 \quad &: \quad i_t = i_{t-1} + [\Theta_\pi E_t(\pi_{t+j} - \pi_t^*) + \Theta_y E_t(y_{t+k} - \hat{y}_{t+k})] \end{aligned} \tag{7}$$

where i_t is the nominal interest rate, π_t is inflation over the interval $[t-1, t]$, $\hat{y}$ is potential output, y_t actual output so $y - \hat{y}_t$ is the 'output gap'. Variables i_t, $\hat{y}_t$ and y_t are measured in deviation form about a zero-inflation state-state and π_t^* is the inflation target. Integers j, k are the policymaker's forecast horizons, which is a feedback on single-period inflation over the interval $[t+j-1, t+j]$ and a feedback on the output gap over the period $t+k$. Thus, this specification of an interest rate rule accommodates not only 'outcome-based' rules ($j, k \leq 0$) but also 'forecast-based' ones (with $j, k > 0$). Finally, $\theta_\pi, \theta_y > 0$ and $\Theta_\pi, \Theta_y > 0$ are feedback parameters: the larger are the values of these parameters, the faster is the pace at which the central bank acts to eliminate the gap between expected inflation and the expected output gap and their target values.

The parameter $\rho \in [0,1]$ measures the degree of interest rate smoothing. If $\rho = 1$ we have an *integral (or difference) rule* that is equivalent to the interest rate responding to a *price-level target*.[1] For $\rho < 1$, (7) can be written as $\Delta i_t = \frac{1-\rho}{\rho}[\theta_\pi E_t(\pi_{t+j} - \pi_{t+j}^*) + \theta_y E_t(y_{t+k} - \hat{y}_{t+k}) - i_t]$ which is a partial adjustment to a static IFB rule $i_t = \theta_\pi E_t(\pi_{t+j} - \pi_{t+j}^*) + \theta_y E_t(y_{t+k} - \hat{y}_{t+k})$.

For reasons already discussed our analysis focuses on standard IFB rules without an output gap target ($\theta_y = 0$) and with a zero inflation target $\pi_t^* = 0$.[2] Then writing $\theta_\pi \equiv \theta$ and $\Theta_\pi \equiv \Theta$, (7) becomes

$$\begin{aligned} i_t &= \rho i_{t-1} + \theta(1-\rho)E_t\pi_{t+j}\,;\ \rho \in [0,1),\, \theta > 0 \\ &= i_{t-1} + \Theta E_t \pi_{t+j}\,;\ \rho = 1,\ \Theta > 0 \end{aligned} \tag{8}$$

Stability and indeterminacy of a dynamic system are associated with the roots of the system's characteristic equation, or equivalently, the eigenvalues of its state-space setup. If the number of unstable roots (outside the unit circle) exactly matches the number of

[1] Unlike its non-integral counterpart, an integral rule responding to inflation does not require observations of the steady state (natural) rate of interest, about which i_t is expressed, to implement. The merits of price-level versus inflation targeting are examined in Svensson (1999) and Vestin (2003).

[2] Another form of IFB rule found in the literature targets *average* inflation over a specified time horizon, as investigated by Levin *et al.* (2003) and Batini and Pearlman (2002). This is represented as: $i_t = \rho i_{t-1} + \theta(1-\rho)\frac{E_t \sum_{r=0}^{j} \pi_{t+r}}{1+j}$. As indicated after result 4 below, these rules roughly have the same determinacy properties as a standard IFB rule with half the horizon j.

non-predetermined variables, there is a unique solution path. Too few unstable roots then leads to indeterminacy, while too many leads to instability.

The mechanism through which indeterminacy arises can be illustrated in the context of a simplified version of our model. On the demand side, we replace the Keynes-Ramsey condition (3) with an ad hoc IS curve $y_t = -\alpha(i_t - E_t\pi_{t+1})$, and assume $y_t = c_t$ and $\beta = 1$. Moreover, suppose that the central bank employs a non-integral rule without interest rate smoothing ($\rho = 0$) so that (7) becomes $i_t = \theta E_t\pi_{t+1}$. Substituting out for y_t and i_t we arrive at the following process for inflation

$$E_t(\pi_{t+1}) = \frac{1}{1 - \lambda\alpha(\theta - 1)}\pi_t \tag{9}$$

Consider the case in which private sector expectations are driven by a non-fundamental shock process and anticipate that inflation next period will be equal to 1. This will lead to an increase in real interest rates, with a consequent reduction in demand of $\alpha(\theta - 1)$. Given (9), price-setting behaviour will thus imply a current inflation rate of $1 - \lambda\alpha(\theta - 1)$, which we define as π_0.

Now assume that θ is chosen so that $0 < \pi_0 < 1$, which is the case if $1+1/\left(\lambda\alpha\right) > \theta > 1$. If we then lead equation (9) forward in time and take expectations, consistency requires that the sequence of successive inflationary expectations is given by $1, 1/\pi_0, 1/\pi_0^2, 1/\pi_0^3, \ldots$. However, these inflation expectations tend to infinity – a solution that clashes with private-sector expectations. Thus, the unique possible solution is $\pi_t = y_t = i_t = 0$ for all $t > 0$. On the other hand, suppose that the central bank is *not aggressive*, and $\theta < 1$. In this case, $\pi_0 > 1$, and hence the sequence of inflationary expectations tends to zero – a solution that fulfils private-sector expectations making these 'self-fulfilling'. Now suppose the central bank is *over-aggressive* such that $\pi_0 < -1$. This happens when $\theta > 1+\frac{2}{\lambda\alpha}$. In this case, the economy experiences cycles of positive and negative inflation but again the sequence of inflationary expectations tends to zero and fulfils private-sector expectations. Self-fulfilling expectations implies that any initial private-sector expectation leads to an acceptable path for inflation – hence indeterminacy. Furthermore, if these (non-fundamental) shocks to private-sector expectations follow a stochastic process, then 'sunspot equilibria' are generated. These are typically welfare-reducing because they induce increased volatility in the system.

The main results for this form of IFB rule from Batini and Pearlman (2002), Batini *et al.* (2004) and Batini *et al.* (2006) can be summarized as follows:

***Result 1*: For an *integral rule* feeding back on current inflation ($j = 0$), $\Theta > 0$ is a necessary and sufficient condition for stability and determinacy. For higher feedback horizons ($j \geq 1$), $\Theta > 0$ is a necessary but not sufficient condition for stability and determinacy.**

***Result 2*: For j-period ahead *integral IFB rules*, $j \geq 1$, there exists a range $\Theta \in [0, \bar{\Theta}(j)]$ with $\bar{\Theta}(j) > 0$ such that the model is stable and determinate.**

***Result 3*: For a *non-integral rule* feeding back on current inflation ($j = 0$), $\theta > 1$ is a necessary and sufficient condition for stability and determinacy. For higher feedback horizons ($j \geq 1$), $\theta > 1$ is a necessary but not sufficient condition for stability and determinacy.**

***Result 4*: For j-period ahead *non-integral IFB rules*, $j \geq 1$, there exists some lead J such that for $j > J$ there is indeterminacy *for all values of* θ.[3] J is given by**

$$J = \frac{1}{1-\rho} + \frac{(1-\beta)\sigma}{\lambda} \tag{10}$$

To get a feel for these results we now provide numerical values for threshold values $\bar{\theta}$ for non-integral rules and $\bar{\Theta}$ for integral rules. In Figure 1, based on Table 1, parameter estimates are taken from Batini *et al.* (2006).[4] For non-integral rules we set $\rho = 0.8$.

Threshold	ρ	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
$\bar{\theta}(j)$	0.8	102	12	3.4	1.7	1.0	indeterminacy
$\bar{\Theta}(j)$	1	23	3.6	1.2	0.68	0.46	0.34

Table 1. Critical upper bounds for $\bar{\theta}(j)$ and $\bar{\Theta}(j)$.

These numerical results corroborate the analytical results summarized above. The indeterminacy problem becomes more acute as the horizon j increases imposing a tighter constraint on the range of IFB rules available. For non-integral rules with $\rho = 0.8$, the maximum horizon J is just over 5 quarters. In accordance with result 2, for integral rules

[3]Strictly, there are some mild conditions on the parameters that a plausible calibration easily satisfies for this result to hold – see Batini and Pearlman (2002). For the average inflation rule of the type set out in the previous footnote, the corresponding lead $\hat{J}$ is given by $\hat{J} = 2J - 1$

[4]Parameter values are: $\lambda = 0.27$, $\beta = 0.99$, $\sigma = 3.91$, $\phi = 2.16$, $\rho_a = \rho_g = 0.9$, $sd(\epsilon_g) = 2.75$, $sd(\epsilon_a) = 0.59$ found using Bayesian methods and US data. It should be noted we choose this simple NK model for its tractability: alternative variants with consumption and inflation persistence mechanisms greatly improved the fit of the model and were preferred in the Bayesian sense.

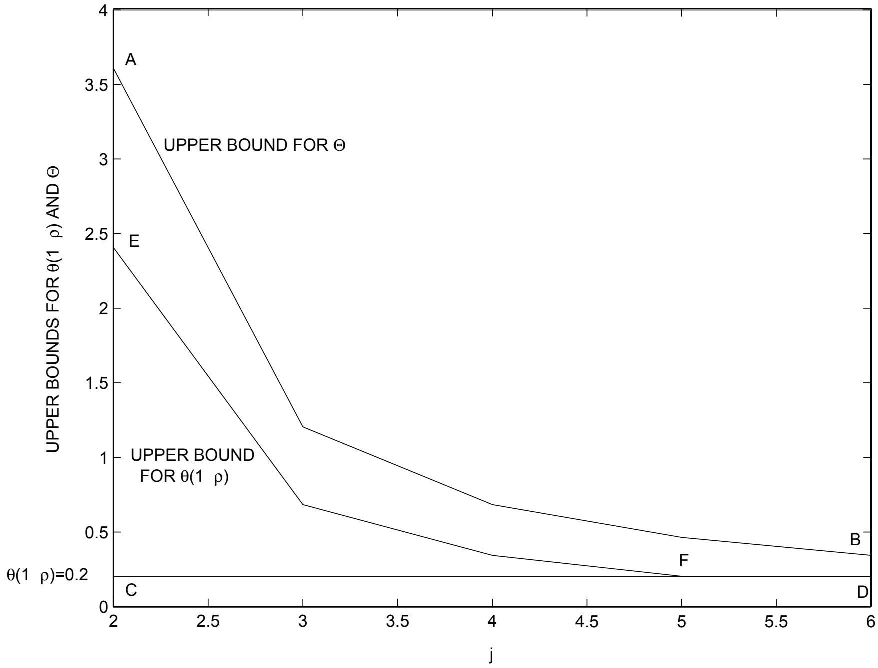

Figure 1: **Critical Upper Bounds for $(1-\rho)\theta$ and Θ.**

as j increases there is always some feedback coefficient on expected inflation $0 < \Theta < \bar{\Theta}$ such that the IFB rule yields stability and determinacy. For non-integral rules the area of determinacy in $(j, (1-\rho)\theta)$ space is EFC. For integral rules the corresponding space in (j, Θ) space is ABDC.[5]

3 Calvo-Type Interest Rate Rules

We now turn to the main focus of this paper, which is on an alternative way of thinking about IFB rules, referred to in the Introduction as *Calvo-type interest rate rules*.[6] To formulate this first define the discounted sum of future expected inflation rates as

$$\Theta_t = (1-\varphi)E_t(\pi_t + \varphi\pi_{t+1} + \varphi^2\pi_{t+2} + \cdots)\,;\ \varphi \in (0,1) \tag{11}$$

[5]Further insight into these results can be provided by writing the expected value of future inflation approximately as $E_t\pi_{t+j} = (\lambda^{max})^j\pi_t$ where λ^{max} is the largest stable eigenvalue of the system under control. Then as j increases, $(\lambda^{max})^j$ decreases, so that the feedback effect becomes negligible, and the system exhibits indeterminacy similar to the $\theta < 1$ type.

[6]We use this terminology since they have the same structure as Calvo-type price or wage contracts. (Calvo (1983)). One can think of the rule as a feedback from expected future inflation which continues in any one period with probability φ and is switched off with probability $1-\varphi$. The probability of the rule lasting for just j periods is then $(1-\varphi)\varphi^j$ and the mean lead horizon is therefore $(1-\varphi)\sum_{j=1}^{\infty} j\varphi^j = \frac{\varphi}{1-\varphi}$.

Then

$$\varphi E_t \Theta_{t+1} - \Theta_t = -(1-\varphi)\pi_t \tag{12}$$

With this definition, a rule of the form

$$\begin{aligned} i_t &= \rho i_{t-1} + \theta(1-\rho)\Theta_t\,;\ \rho \in [0,1),\, \theta > 0 \\ &= i_{t-1} + \Xi\Theta_t\,;\ \rho = 1,\ \Xi > 0 \end{aligned} \tag{13}$$

emerges which describes feedback on forward-looking inflation with mean lead horizon $\frac{\varphi}{1-\varphi}$. Thus with $\varphi = 0.5$, for example, we have a Calvo-type rule that compares with (7) with a horizon $j = 1$.

Consider first *non-integral* rules. With a Calvo-type rule, writing (1), (3), (12) and (13) in matrix form, the characteristic equation of the system can be shown to be

$$(1-\varphi z)(z-\rho)((\beta z-1)(z-1)-\frac{\lambda}{\sigma}z)+\theta(1-\varphi)(1-\rho)\frac{\lambda}{\sigma}z=0 \tag{14}$$

where z is the forward operator (i.e., $zx_t \equiv x_{t+1}$). Noting that the system (1), (3) and (13) has only one lag term, the condition for stability and indeterminacy of the system is that exactly one root of (14) must lie within the unit circle. Accordingly, we investigate (14) using the root locus method.[7] The root locus diagram for (14) is shown Figure 2, which depicts the complex plane, and is a generic shape for all parameter values of the system. The root locus starts out at the roots of (14) for $\theta = 0$; these roots are denoted on the diagram by $\bullet$. Note that one root, $z = 1/\varphi$, is outside the unit circle, while another, $z = \rho$ is inside the unit circle, and it is easy to show that $(\beta z - 1)(z - 1) - \frac{\lambda}{\sigma}z = 0$ has one root outside and one root inside the unit circle. The arrows then show how the four roots change as θ changes. Note in particular that the smallest root has a branch from it leading to $z = 0$, while the largest has a branch leading to $z = \infty$. Of the other two roots, one of them has a branch passing through $z = 1$, and where their branches meet, they both branch into the complex plane and head to infinity at an angle of 60° asymptotically to the real line.[8]

There are several things to note about this root locus diagram. First when $\theta = 1$, then $z = 1$ as well; this is immediate from (14).[9] Second, the diagram therefore implies that the system has a single stable root for all values of $\theta > 1$, no matter what are the other parameter values. However this apparently general result needs some explanation, and

[7] See Appendix A for a brief guide to the root-locus method.

[8] The root locus diagram would look qualitatively the same if $1/\varphi$ were the largest real root, or ρ the smallest real root for $\theta = 0$.

[9] Note that the Taylor principle, that interest rate should react by more than one-to-one to expected or current or past inflation means that $\theta > 1$ for non-integral rules.

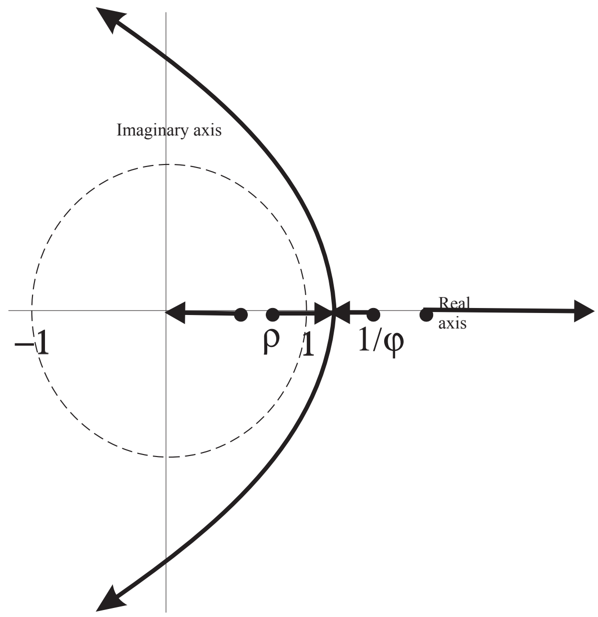

Figure 2: **Position of Zeroes as θ Changes from 0 to ∞.**

indeed some slight qualification. We summarise the main results as follows, and provide a proof in Appendix B:

Result 5

A sufficient condition for the system (1)-(3) with the Calvo interest rate rule (13) to be determinate for all $\theta > 1$ is that $\rho > \varphi$.

Estimated interest-rate rules (including our estimates in section 5) suggest substantial smoothing with typically $\rho > 0.95$. The condition in this last result is therefore that $\varphi < 0.95$ or in other words the mean lag must be less than 19 quarters. A final observation is that as the interest rate smoothing increases, at the limit where we have an integral rule, result 5 *always* holds. Thus the result for integral (price-level) rules is an immediate corollary of Result 5.

Result 6

The system (1)-(3) with the Calvo integral (i.e., price-level) interest rate rule is determinate for all $\Xi > 0$.

The proof follows once one has replaced $(1-\rho)\theta$ by Ξ, so that the same argument follows as for the proof of the previous result (with $z = 1$ when $\Xi = 0$) and in addition $1 > \varphi$.

Thus according to result 6 the indeterminacy problem *disappears altogether* (in the context of our simple model) if the authorities target a weighted average of present and future price levels with geometrically declining weights.

The Calvo IFB rule (13) is not completely forward-looking as it includes a reaction to current inflation with weight unity. Does the improved determinacy properties of this rule compared with standard IFB rules crucially depend on the presence of current inflation?[10] We can see this is not the case because average inflation rules, which also react to current inflation over a finite time horizon j referred to in footnotes 2 and 3, have similar indeterminacy properties to single-period IFB rules with horizon $\frac{j+1}{2}$ if j is odd and $\frac{j}{2}$ if even. Further suppose that the Calvo rule involves only forward-looking inflation, so that (12) contains the term $E_t\pi_{t+1}$ instead of π_t, so that the characteristic equation (14) becomes

$$(1-\varphi z)(z-\rho)((\beta z-1)(z-1)-\frac{\lambda}{\sigma}z)+\theta(1-\varphi)(1-\rho)\frac{\lambda}{\sigma}z^2=0 \qquad (15)$$

One can show using the root locus technique, that provided $\rho > \varphi$, there is indeterminacy only for values of θ beyond that value at which $z=-1$. Thus the critical value of θ for indeterminacy is that which satisfies (15) at $z=-1$. It is easy to see that this critical value is given by $f(\rho)(1+\varphi)/(1-\varphi)$, where $f(\rho)$ is a function of ρ (and the other parameters), is an increasing function of φ. Thus for the Calvo-rule as φ and therefore the expected horizon increases, the proneness to indeterminacy actually *falls*. Furthermore the function $f(\rho)\to\infty$ as $\rho\to 1$, so with Calvo *integral* rules that are purely forward-looking, the critical value for θ above which there is indeterminacy becomes infinite and result 6 holds.

We end this section by noting the contrast between Result 5 and Result 3, which can be illustrated by the root locus diagrams of Figure 3 for IFB rules. These depict the cases for interest rates depending on either (a) inflation 3 periods ahead, or (b) average inflation over the current period, and up to 3 periods ahead. Both these demonstrate that there may be a range of $\theta > 1$ for which there is determinacy (exactly one stable root), but for θ too large, there is indeterminacy.

4 Optimal Monetary Policy

4.1 Utility-Based Welfare

In the simple model of this paper a quadratic approximation to the utility of the household that underlies the model takes the form

$$\Omega_0 = E_0\left[\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[(y_t-\hat{y}_t)^2+w_\pi\pi_t^2+w_i i_t^2\right]\right] \qquad (16)$$

[10]We are grateful to an anonymous ECB working paper referee for posing this question.

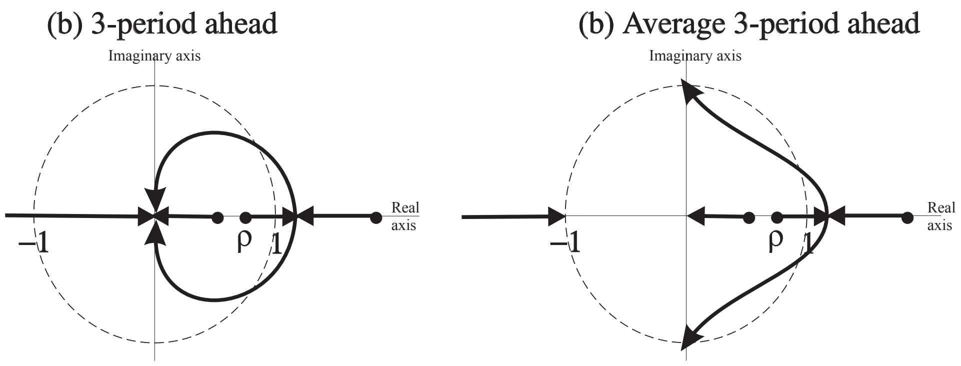

Figure 3: **Position of Zeroes for (a) forward-looking rules (b) average forward-looking rules as θ Changes from 0 to ∞.**

where $\hat{y}_t$ is potential output achieved when prices are flexible ($mc_t = 0$ in (1)) and

$$w_\pi = \frac{\zeta}{\lambda(\sigma+\phi)} = \frac{\zeta\xi}{(1-\xi)(1-\beta\xi)(\sigma+\phi)} \tag{17}$$

In (17), $1-\xi$ is probability of a price optimization for each firm, σ is the risk aversion parameter, $1+\phi$ is the elasticity of disutility with respect to hours worked and ζ is the elasticity of substitution of differentiated goods making up aggregate output.[11] For estimated or calibrated parameter values reported in Batini *et al.* (2006), this gives $w_\pi = 1.826$. Based on an annual inflation rate this is equivalent to $w_\pi = \frac{1.826}{16} = 0.11$ which is at the lower end of commonly used weights.

4.2 Optimal Policy with and without Commitment

We first compute the optimal policies where the policy maker can commit, and the optimal discretionary policy where no commitment mechanism is in place. To obtain the weight on interest rate variance, w_i, we first compute the optimal commitment rule with the interest rate responding only to current inflation (see below). We impose an approximate *zero lower bound* on the nominal interest rate by experimenting with w_i so it is sufficiently high so as to ensure $i_t > 0$ with almost unit probability 0.99%; i.e., (assuming a normal distribution), $sd(i_t) < \frac{i}{2.33}$ where $i = (\frac{1}{\beta} - 1) \times 100$ is the natural rate of interest. A weight $w_i = 0.5$ was necessary to achieve this condition.

[11]See Woodford (2003), chapter 6.

4.3 Optimized IFB Rules

We now turn to optimized IFB rules. The general form of the rule that covers both integral and non-integral rules is given by

$$i_t = \rho i_{t-1} + \Xi E_t \pi_{t+j}\,; \quad \rho \in [0,1],\, \Xi\, j \geq 0 \tag{18}$$

The corresponding Calvo-type rules are given by

$$i_t = \rho i_{t-1} + \Xi E_t \Theta_t\,; \quad \rho \in [0,1],\, \Xi,\, j \geq 0 \tag{19}$$

Given the estimated variance-covariance matrix of the white noise disturbances, an optimal combination (Ξ, ρ) can be found for each rule defined by the time horizon $j \geq 0$.

4.4 Numerical Results

We first focus on the optimal commitment rule, the optimal discretionary (time-consistent) rule and an optimized current inflation rule. Results for the three types of rules are summarized in table 2. Figures 4-11 compare the responses under the three rules following an an unanticipated productivity shock ($a_0 = 1$) and an unanticipated government spending shock ($g_0 = 1$). The output and inflation equivalent welfare differences compared with the optimal commitment policy are computed as follows. Suppose the welfare loss difference is X. This is equivalent to a permanent output gap of y_e if $\frac{1}{2(1-\beta)} y_e^2 = X$, and to a permanent inflationary bias of π_e if $\frac{1}{2(1-\beta)} b\pi_e^2 = X$; i.e.,

$$y_e = \sqrt{2(1-\beta)X} \tag{20}$$

$$\pi_e = \sqrt{\frac{2(1-\beta)X}{b}} \tag{21}$$

Comparing the three types of rules, there are two notable results that. First, Clarida *et al.* (1999) stress the existence of stabilization gains from commitment in New-Keynesian models: we show that in our simple model that these are substantial, amounting to an permanent output equivalent of 1.02% or an inflationary bias of 0.75% per quarter or 3% per year. The source of this time-inconsistency problem is from pricing and consumption behaviour together. Following a shock which diverts the economy from its steady state, given expectations of inflation, the opportunist policy-maker can increase or decrease output by reducing or increasing the interest rate which increases or decreases inflation. Consider the case where the economy is below its steady-state level of output. A reduction in the interest rate then causes consumption demand to rise. Firms who are locked into price contracts respond to an increase in demand by increasing output and increasing the price according to their indexing rule. Those who can re-optimize only increase their price. These changes are for *given* inflationary expectations and illustrate the incentive to inflate when the output gap increases. In a non-commitment equilibrium, however, the incentive is anticipated and the result is greater inflation variability as compared with the commitment case. This contrast between the commitment and discretionary cases is seen clearly in the figures.

The second notable results concerns the optimized current inflation rule. We find that most of the gains from commitment (in fact over 80%) can be achieved by this very simple optimized rule without an output gap feedback and the cost of simplicity is only 0.11% of output or an inflationary bias of 0.08 per quarter or 0.32% per year. In the figures we see how the optimized current inflation rule closely mimics the optimal commitment rule.

Rule	ρ	Ξ	Loss Function	y_e	π_e
Minimal Feedback on π_t	1	0.001	49.01	0.97	0.72
IFB0	1	2.035	2.509	0.11	0.08
IFB1	1	12.00	2.676	0.12	0.09
IFB2	1	3.570	4.574	0.23	0.17
IFB3	1	1.216	32.02	0.78	0.57
IFB4	1	0.675	208.1	2.03	1.50
Calvo IFB ($\varphi = 0.5$)	1	2.203	2.602	0.12	0.09
Calvo IFB ($\varphi = 0.67$)	1	2.351	2.636	0.12	0.09
Calvo IFB ($\varphi = 0.75$)	1	2.444	2.653	0.12	0.09
Calvo IFB ($\varphi = 0.875$)	1	2.616	2.678	0.13	0.09
Calvo IFB ($\varphi = 0.917$)	1	2.683	2.689	0.13	0.09
Optimal Commitment	n.a.	n.a	1.896	0	0
Optimal Discretion	n.a.	n.a	53.55	1.02	0.75

Table 2. Welfare-Based Optimal Rules and Optimized IFB Rules Compared.

Turning now to IFB rules we compute the optimized standard rules with future horizon $j = 0, 1, 2, 3, 4$, denoted by $IFBj$ and compare these with Calvo rules with probability of survival $\varphi = 0.5, 0.67, 0.75, 0.875$ and 0.917 corresponding to an average future horizon of $\frac{\varphi}{1-\varphi} = 1, 2, 4, 7$ and 11 quarters respectively. Our results first confirm a finding of Batini *et al.* (2006): that the stabilization performance of standard optimized $IFBj$ rules deteriorates sharply as the horizon j increases. Our new result that follows from the stability analysis of Calvo-type rules and the absence of an 'indeterminacy constraint' is that *this sharp deterioration is not a feature of Calvo-type optimized IFB rules.* Even optimized rules with an expected future horizon of 3 years performs almost as well as the current inflation rule.

5 Calvo-Interest Rate Rules: A DSGE Model Illustration

In this section, we implement the afore-mentioned Calvo interest rules in a benchmark model of the euro area, namely that of Smets and Wouters (2003). First we provide a brief description of the model.

5.1 The Smets-Wouters Model

The Smets-Wouters (SW) model in an extended version of the standard New-Keynesian DSGE closed-economy model with sticky prices and wages estimated by Bayesian techniques. The model features three types of agents: households, firms and the monetary policy authority. Households maximize a utility function with two arguments (goods and leisure) over an infinite horizon. Consumption appears in the utility function relative to a time-varying external habit-formation variable. Labour is differentiated over households, so that there is some monopoly power over wages, which results in an explicit wage equation and allows for the introduction of sticky nominal Calvo-type wages contracts. Households also rent capital services to firms and decide how much to accumulate given certain capital adjustment costs. Firms produce differentiated goods, decide on labour and capital inputs, and set Calvo-type price contracts. Wage and price setting is augmented by the assumption that those prices and wages that can not be freely set are partially indexed to past inflation. Prices are therefore set as a function of current and expected real marginal cost, but are also influenced by past inflation. Real marginal cost depends on wages and the rental rate of capital. The short-term nominal interest rate is the instrument of monetary policy. The stochastic behaviour of the model is driven by ten exogenous shocks: five arising from technology and preferences, three cost-push shocks and two monetary-policy shocks. Consistent with the DSGE set up, potential output is defined as the level of output that would prevail under flexible prices and wages in the absence of cost-push shocks.

5.2 Monetary Policy Reaction Functions

In line with the empirical approach to monetary rules in SW, we modify the monetary-policy reaction functions for the standard and Calvo IFB rules as respectively,

$$
\begin{aligned}
i_t &= \rho i_{t-1} + (1-\rho)[\bar{\pi}_t + \theta_\pi E_t(\pi_{t+j} - \bar{\pi}_{t+j}) + \theta_y \tilde{y}_t] + \theta_{\Delta\pi}(\pi_t - \pi_{t-1}) + \theta_{\Delta y}(\tilde{y}_t - \tilde{y}_{t-1}) \\
i_t &= \rho i_{t-1} + (1-\rho)[\bar{\pi}_t + \theta_\pi \Theta_t(\varphi) + \theta_y \tilde{y}_t] + \theta_{\Delta\pi}(\pi_t - \pi_{t-1}) + \theta_{\Delta y}(\tilde{y}_t - \tilde{y}_{t-1}) \qquad (22)
\end{aligned}
$$

where $\tilde{y}_t$ is the output gap. Then in the SW model $j = -1$ and the interest rate feeds back on lagged inflation. To incorporate this rule as a special case we also modify (11) to become

$$\Theta_t = (1-\varphi)E_t[\pi_{t-1} - \pi^*_{t-1} + \varphi(\pi_t - \pi^*_t) + \varphi^2(\pi_{t+1} - \pi^*_{t+1}) + \cdots];\ \varphi \in (0,1) \qquad (23)$$

so (12) now becomes

$$\varphi E_t \Theta_{t+1} - \Theta_t \quad = \quad -(1-\varphi)(\pi_{t-1} - \pi^*_{t-1}) \qquad (24)$$

This modified Calvo rule reduces to the past inflation rate rule in SW as a special case by putting $\varphi = 0$. The mean lead horizon is now given by $\frac{1}{1-\varphi} - 2$.[12]

5.3 Results

Thus, we re-estimate by Bayesian methods the SW model with the policy rule replaced by (22) and the model supplemented with (23); where, to repeat, the special case of φ=0 retrieves the default, backward-looking SW policy rule.[13] Table 3 reports the parameters of the policy reaction function for each model variant, from IFB-1 to IFB4 to the Calvo case.[14] As standard, two sets of parameter results are presented. First, the estimated posterior mode of the parameters, which is obtained by directly maximizing the log of the posterior distribution with respect to the parameters (and a standard error based on the corresponding Hessian). Second, the 5th and 95th percentile of the posterior distribution of the parameters obtained through the Metropolis-Hastings sampling algorithm (using 100,000 draws from the posterior, 3 parallel chains and an average acceptance rate of around 0.25) for the various model variants. The models are estimated using the Dynare software, Juillard (2004). Note, in re-estimation, we used identical priors to those used in SW. For the additional parameter, φ moreover, we assumed a Beta distribution with a prior mean of 0.8 (corresponding to a mean lead horizon of 3 quarters), with a standard error of 0.1.

Turning to the results themselves, we see that the Calvo rule yields a φ value centred at an implied mean lead horizon of 3 to 4 quarters in the policy rule. As shown in the last row of Table 3 which reports the model odds, this rule beats all contemporaneous and forecast-based rules in marginal likelihood terms without leading to any deterioration

[12]It is straightforward to show that the results of section 3 still hold with this modification.

[13]We are grateful to Gregory De Walque and Raf Wouters for providing the SW model in Dynare code.

[14]Results for the other parameters (as well as the Dynare files to replicate our results) are available on request from the authors. Notably, the full set of parameter values appeared very well identified and stable across the model variants.

Rule		IFB(-1)	IFB(0)	IFB(1)	IFB(4)	Calvo IFB
ρ	Mode	0.969	0.969	0.965	0.951	0.958
		(0.012)	(0.013)	(0.015)	(0.025)	(0.021)
	Mean	0.965	0.967	0.958	0.940	0.951
		[0.943:0.984]	[0.951:0.986]	[0.932:0.983]	[0.891:0.977]	[0.918:0.982]
θ_π	Mode	1.700	1.701	1.700	1.700	1.702
		(0.099)	(0.100)	(0.100)	(0.099)	(0.099)
	Mean	1.697	1.698	1.700	1.703	1.707
		[1.535:1.853]	[1.531:1.860]	[1.542:1.873]	[1.541:1.867]	[1.539:1.868]
θ_y	Mode	0.121	0.121	0.117	0.111	0.120
		(0.045)	(0.045)	(0.046)	(0.049)	(0.045)
	Mean	0.117	0.123	0.109	0.107	0.120
		[0.045:0.186]	[0.057:0.193]	[0.034:0.178]	[0.028:0.171]	[0.053:0.186]
$\theta_{\Delta\pi}$	Mode	0.146	0.146	0.111	0.118	0.121
		(0.052)	(0.052)	(0.049)	(0.049)	(0.049)
	Mean	0.155	0.110	0.116	0.119	0.126
		[0.070:0.239]	[0.034:0.195]	[0.034:0.200]	[0.041:0.196]	[0.048:0.212]
$\theta_{\Delta y}$	Mode	0.154	0.154	0.152	0.147	0.151
		(0.023)	(0.023)	(0.022)	(0.022)	(0.021)
	Mean	0.152	0.152	0.146	0.139	0.146
		[0.120:0.191]	[0.115:0.187]	[0.112:0.183]	[0.099:0.178]	[0.109:0.181]
φ	Mode	-	-	-	-	0.8398
		-	-	-	-	(0.103)
	Mean	-	-	-	-	0.797
		-	-	-	-	[0.646:0.956]
Prob.		0.289	0.096	0.158	0.224	0.234

Table 3. Calvo and Standard IFB Rules Compared.

IFBj Rule: $i_t = \rho i_{t-1} + (1-\rho)[\pi_t^* + \theta_\pi E_t(\pi_{t+j} - \pi_{t+j}^*) + \theta_y \tilde{y}_t] + \theta_{\Delta\pi}(\pi_t - \pi_{t-1}) + \theta_{\Delta y}(\tilde{y}_t - \tilde{y}_{t-1})$

Calvo IFB (φ): $i_t = \rho i_{t-1} + (1-\rho)[\pi_t^* + \theta_\pi \Theta_t(\varphi) + \theta_y \tilde{y}_t] + \theta_{\Delta\pi}(\pi_t - \pi_{t-1}) + \theta_{\Delta y}(\tilde{y}_t - \tilde{y}_{t-1})$ where $\varphi E_t \Theta_{t+1} - \Theta_t = -(1-\varphi)(\pi_{-t} - \pi_{t-1}^*)$. Hessian Standard errors are in parenthesis and 5th and 95th percentiles in squared brackets. Log marginal likelihood of IFB(-1) equals -298.65.

in the parameter values. Comparing the likelihood values of the Calvo rule with the backward-looking, IFB (-1), rule there is clearly a very close data coherence. Indeed, in terms of Bayesian odds ratio (0.234/0.289 = 0.81), we effectively could not discriminate between these two types or rules.[15] Summing up, one might say that whilst not conclusive, these results do suggest that a Calvo-type rule is perfectly competitive with more conventional monetary policy rules.

6 Conclusions

The large literature on IFB rules now strongly suggests that rules that target future inflation with a specified time horizon are prone to indeterminacy and have poor stabilization properties. This raises an interesting puzzle of why many central banks insist on forward-looking inflation targets. Our paper proposes a resolution of this puzzle by suggesting that the policy process of central banks may in fact be best modelled in the form of a Calvo-type rule that targets a discounted infinite sum of future expected inflation.

Our main findings are first: Calvo-type IFB interest rate rules are less prone to indeterminacy than standard ones with a finite forward horizon. Second, for such rules in integral (i.e., difference) form the indeterminacy problem disappears altogether. In this case the Calvo rule takes the form of a weighted average future price-level target with geometrically declining weights. Third, as a consequence of these results, optimized Calvo-type rules have good stabilization properties as they become more forward-looking, which sharply contrasts with the substantial deterioration in the corresponding performance of standard IFB rules. Fourth, in terms of data coherence in the context of the SW model, a Calvo-type rule with a mean forward horizon of just less than one year is perfectly competitive with more conventional monetary policy rules.

[15] As discussed in Geweke (1999), the Bayesian approach to estimation allows a formal comparison of different models based on their marginal likelihoods. The marginal likelihood of Model M_i is given by,

$$p(Y \mid M_i) = \int_{\Xi} p(\xi \mid M_i)\, p(Y \mid \xi, M_i)\, d\xi$$

where $p(\xi \mid M_i)$ is the prior density for model M_i and $p(Y \mid \xi, M_i)$ is the data density for model M_i given the parameter vector ξ and the data vector Y. Then the posterior odds ratio is given by

$$PO_{ij} \equiv \frac{p(M_i|Y)}{p(M_j|Y)} = \frac{p(Y|M_i)p(M_i)}{p(Y|M_j)p(M_j)} = \frac{p(Y|M_i)}{p(Y|M_j)}$$

assuming equal prior model probabilities ($p(M_i) = p(M_j)$). The posterior model probabilities are reported in Table 3.

A A Topological Guide to The Root Locus Technique

Here we present a brief guide to how to use the root locus technique. We start by some standard 'rules' as provided in control theory textbooks, and then apply them to the specific example of the paper.

The idea is to track the roots of the polynomial equation $f(z) + \theta g(z) = 0$ as θ moves from 0 to ∞. Clearly for $\theta = 0$, the roots are those of $f(z) = 0$, whereas when $\theta \to \infty$, the roots are those of $g(z) = 0$. The root locus then connects the first set of roots to the second set by a series of lines and curves. We shall assume without loss of generality that the coefficient of the highest power of f is negative and that of g is positive. Since the roots of a polynomial may be complex, the root locus must be plotted in the complex plane.

There are a number of different ways of stating the standard 'rules' that underlie the technique. One popular way (see Evans (1954)) of sketching the root locus by hand involves just 6 steps:

1(a). Define $n(f)$ = no. of zeros of $f(z)$, $n(g)$ = no. of zeros of $g(z)$. For our case, $n(f) = 4, n(g) = 1$.

1(b). Loci start at the zeros of $f(z)$, and end at the zeros of $g(z)$ and at ∞ if $n(f) > n(g)$.

2. Number of loci must be equal to $\max(n(f), n(g)) = 4$, in our case.

3. A point on the real axis is on the root locus if the number of zeros of f and g on the real axis to its left is odd.

4. Loci ending at ∞ do so at angles to the positive real axis given by $2k\pi/(n(f)-n(g))$, where the integer k ranges from 0 to $(n(f) - n(g)) - 1$. In our case, these angles are $0,\ 2\pi/3,\ 4\pi/3$.

5. If all coefficients of f and g are real, then the root locus is symmetric about the real axis.

6. Loci leave the real axis where $\partial\theta/\partial z = 0$.

B Proof of Result 5

We prove this in two steps. Firstly, we need to show that the branch point into the complex plane near to $z = 1$ is to the right of $z = 1$. Secondly, we have to show that the branches of the root locus do not cross the unit circle twice (otherwise there are too many stable roots over a certain range of values of θ greater than 1, and hence indeterminacy over this range).

Step 1. The branch point is to the right of $z = 1$ provided that the root locus passes through this point from left to right as θ increases. But this means that we require $\partial z/\partial\theta > 0$ at $z = \theta = 1$. By implicit differentiation of (14) we find that

$$[(1-\beta)(1-\rho)(1-\varphi) + \frac{\lambda}{\sigma}(1-2\varphi+\rho\varphi)]\left.\frac{\partial z}{\partial\theta}\right|_{\theta=1} = (1-\rho)(1-\varphi)$$

It is easy to see that a sufficient condition for $\partial z/\partial\theta\big|_{\theta=1} > 0$ is $\rho > \varphi$.

Step 2. We now investigate those points on the root locus that lie on the unit circle. These are of course characterized by $z = e^{i\phi} = cos\phi + isin\phi$. To solve for ϕ, the easiest approach is to substitute $z = e^{i\phi}$ directly, and then multiply (14) through by $e^{-i\phi}$. Then the imaginary part of this expression is independent of θ, and can be written as

$$\varphi\beta sin3\phi - [\beta(1+\varphi\rho)+\varphi(1+\beta+\lambda/\sigma))]sin2\phi + [(1+\varphi\rho)(1+\beta+\lambda/\sigma)+\varphi+\rho\beta-\rho]sin\phi = 0 \tag{B.1}$$

Using the substitutions $sin2\phi = 2sin\phi\cos\phi$, $sin3\phi = (4cos^2\phi - 1)sin\phi$, it is clear that one solution to (B.1) is $sin\phi = 0$, which corresponds to $\phi = 0$ (z=1) and $\phi = \pi$ (z=-1, which is technically a solution when $\theta < 0$). It follows that the other solutions are given by

$$4\varphi\beta cos^2\phi - 2[\beta(1+\varphi\rho)+\varphi(1+\beta+\lambda/\sigma))]cos\phi + (1+\varphi\rho)(1+\beta+\lambda/\sigma)+\varphi+\rho\beta-\rho-\varphi\beta = 0$$

Provided that $\rho > \varphi$, it is easy to show that the coefficient of $cos\phi$ is more than twice that of $cos^2\phi$; it follows that at least one of the solutions to $cos\phi$ is greater than 1. But this means that there is no more than one real solution for ϕ, so that there cannot be a double crossing of the unit circle for $\rho > \varphi$.

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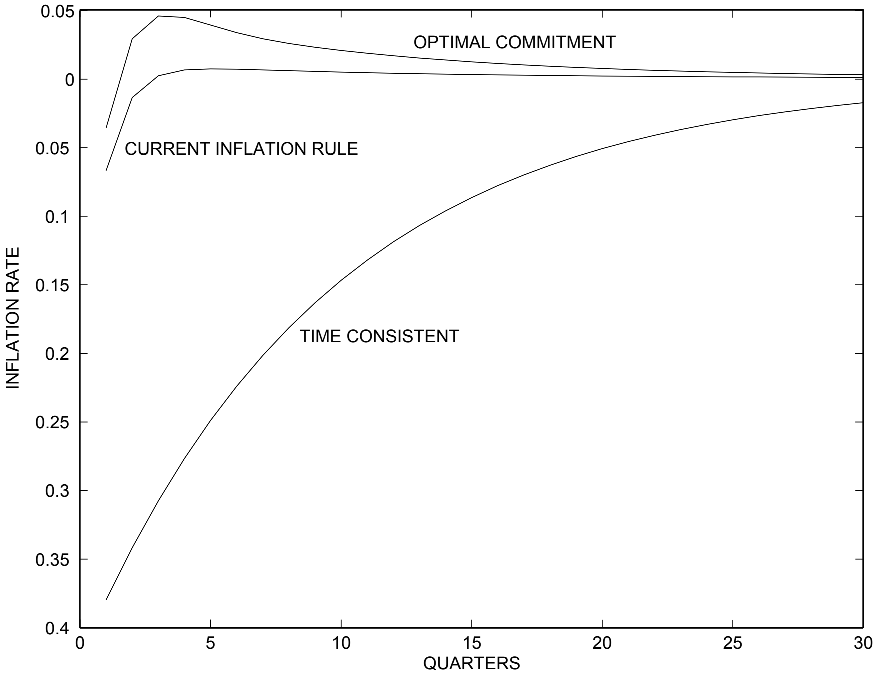

Figure 4: Inflation following Shock $a_0 = 1$.

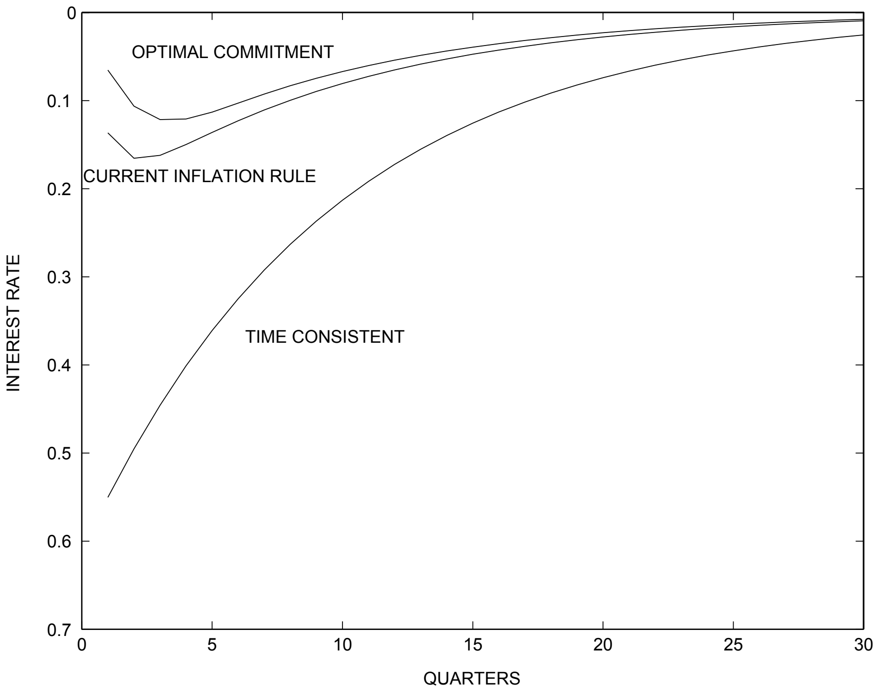

Figure 5: Interest Rate following Shock $a_0 = 1$.

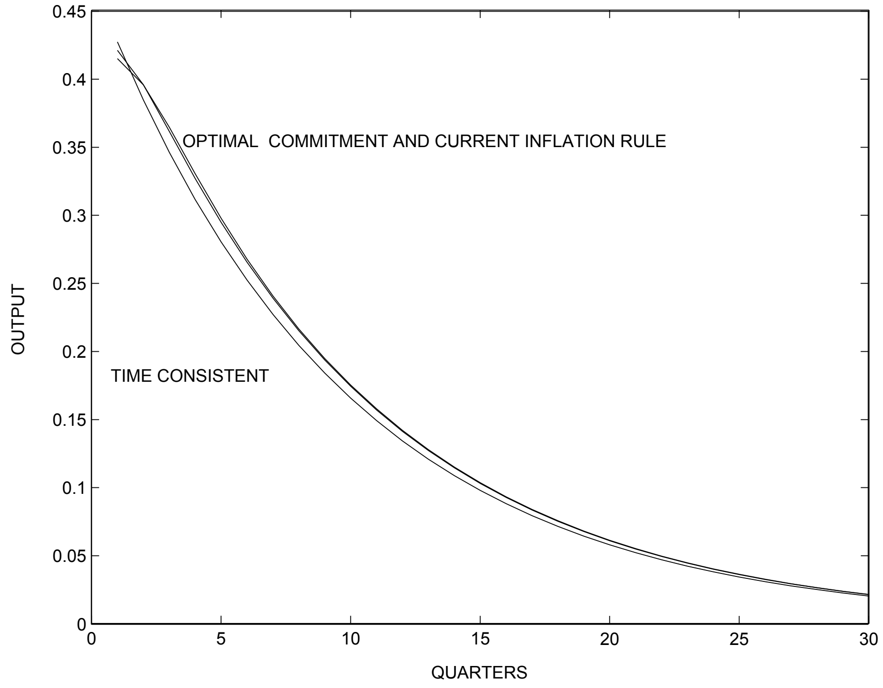

Figure 6: Output following Shock $a_0 = 1$.

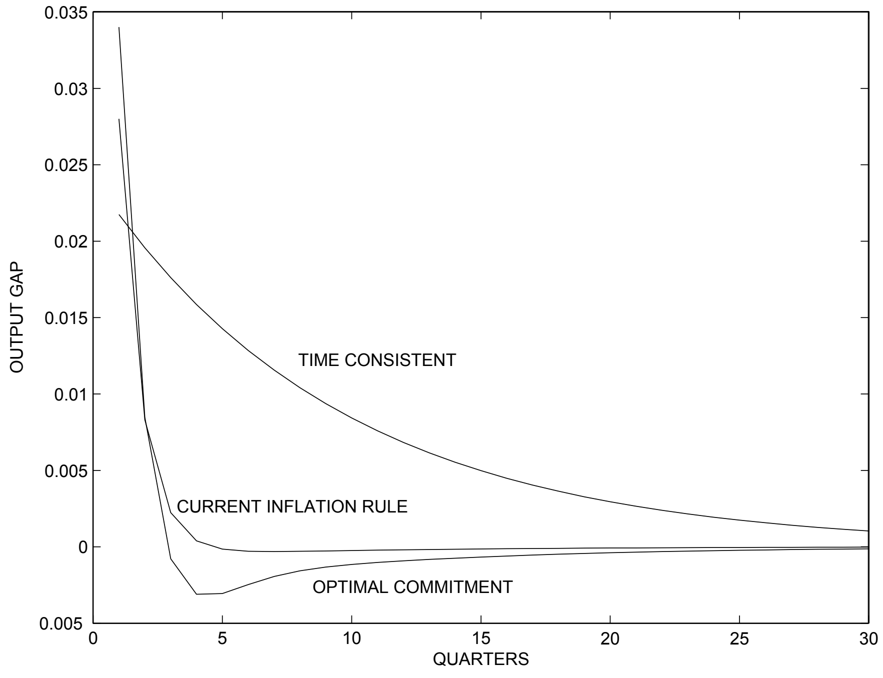

Figure 7: Output Gap following Shock $a_0 = 1$.

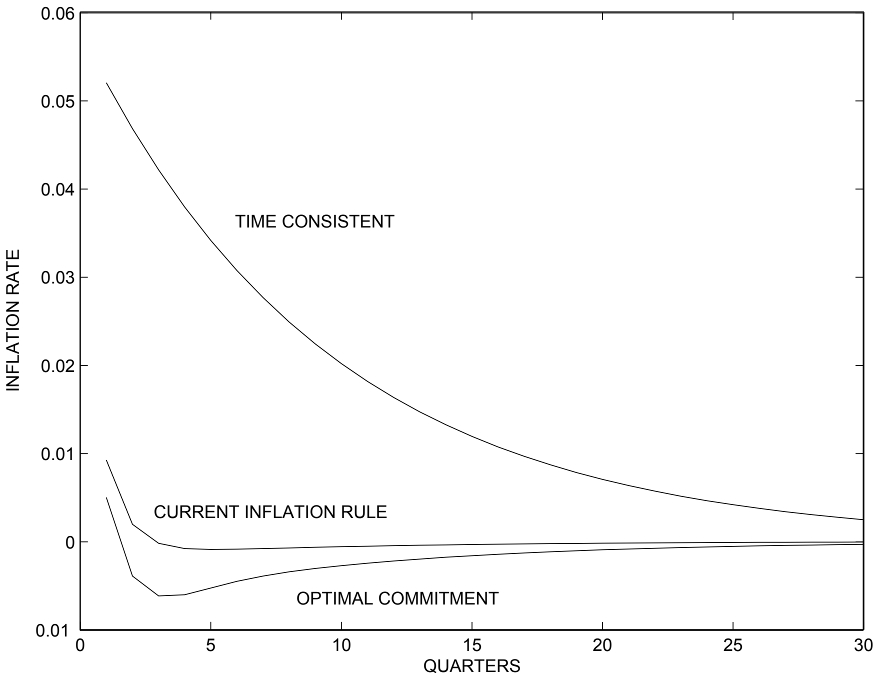

Figure 8: Inflation following Shock $g_0 = 1$.

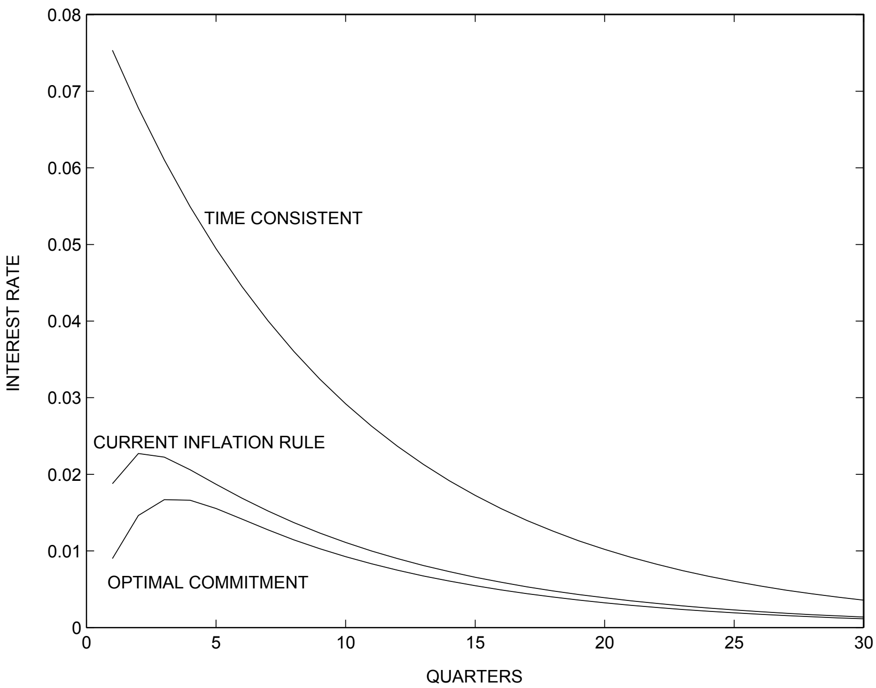

Figure 9: Interest Rate following Shock $g_0 = 1$.

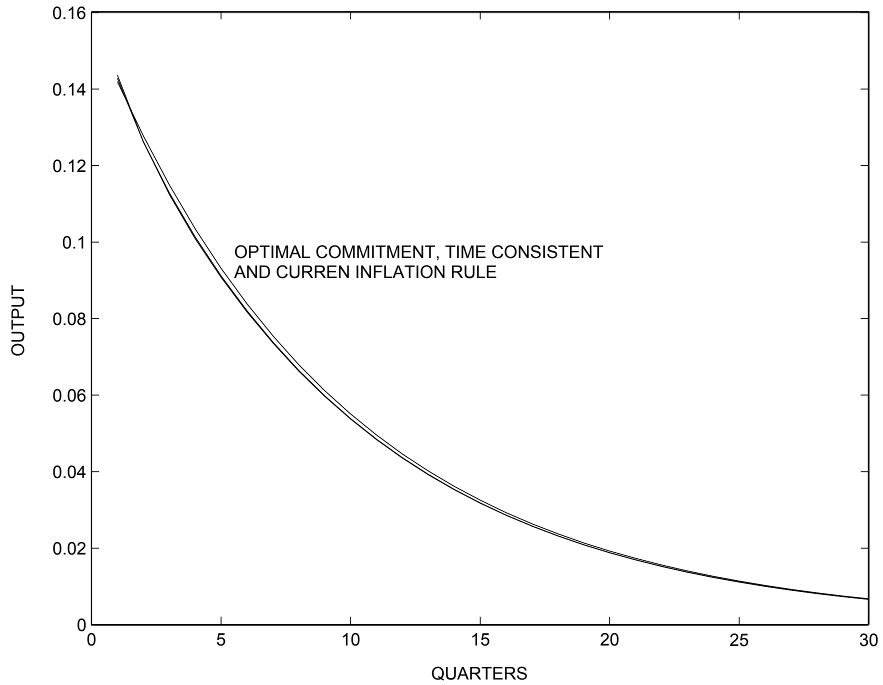

Figure 10: Output following Shock $g_0 = 1$.

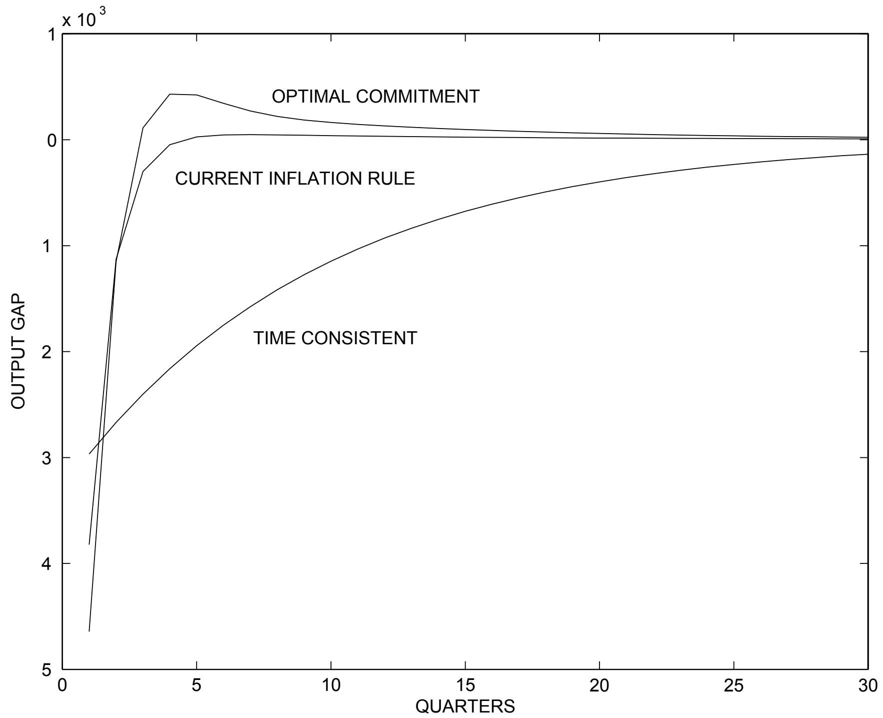

Figure 11: Output Gap following Shock $g_0 = 1$.

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