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A NO-ARBITRAGE FACTOR-AUGMENTED VAR APPROACH

by Emanuel Mönch

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Abstract

This paper suggests a term structure model which parsimoniously exploits a broad macroeconomic information set. The model does not incorporate latent yield curve factors, but instead uses the common components of a large number of macroeconomic variables and the short rate as explanatory factors. Precisely, an affine term structure model with parameter restrictions implied by no-arbitrage is added to a Factor-Augmented Vector Autoregression (FAVAR). The model is found to strongly outperform different benchmark models in out-of-sample yield forecasts, reducing root mean squared forecast errors relative to the random walk up to 50% for short and around 20% for long maturities.

Keywords: Affine term structure models, Yield curve, Dynamic factor models, FAVAR

JEL codes: C13; C32; E43; E44; E52

Non-technical Summary

This paper studies forecasts of government bond yields on the basis of information from many macroeconomic variables by combining recent advances in no-arbitrage modeling of the yield curve and factor analysis for large datasets. It shows that interest rate forecasts can be significantly improved using a broad macroeconomic information set instead of only a few variables.

A growing body of research is currently focusing on the interaction between the yield curve and other economic variables. A central feature of most joint models of the term structure of interest rates and the macroeconomy is a monetary policy rule that relates the short-term interest rate to a small set of output and inflation measures. The assumption implicit in such models is that the central bank sets the monetary policy instrument based on the information in only a few key aggregates. Yet, central banks are known to actively monitor a variety of economic time series variables and there is growing empirical evidence that they react to economic information beyond output and inflation.

This paper explicitly incorporates into a term structure model the assumption that monetary policy makers base their decisions on large macroeconomic information sets, i.e. they act in a "data-rich environment" (see Bernanke and Boivin (2003)). Precisely, the common components of a large number of times series of different economic categories and the short-term interest rate are employed as predictors for yields. The cross-sectional coherence of the model-implied interest rates is guaranteed by assuming the absence of arbitrage opportunities. Following a common practice in the class of affine yield curve models, this implies the definition of a stochastic discount factor as a function of time-varying risk premia which are themselves linearly related to the macroeconomic factors that summarize the state of the economy.

In an application to US data, the paper first provides evidence that the federal funds rate is indeed better explained using a large macroeconomic dataset than information on inflation and the output gap alone. Moreover, the common components of many macroeconomic time series are shown to explain yields quite well in-sample. The good performance of the suggested term structure model carries over to the prediction of yields: in a recursive out-of-sample forecast exercise, the model is shown to strongly outperform a set of different benchmark models. The improvement is particularly pronounced at the short end of the yield curve but still significant for very long maturities.

The paper's results reconfirm the well-documented usefulness of dynamic factor models for forecasting economic time series in an application related to the bond market. Given the increasing availability of macroeconomic data and the straightforwardness of the associated estimation method, the presented model may thus serve as a complement to the tools currently used for yield curve forecasts.

I Introduction

In this paper, I suggest a term structure model which parsimoniously exploits a broad macroeconomic information set. The model does not incorporate latent yield curve factors, but instead uses the common components of a large number of macroeconomic variables and the short rate as explanatory factors.

Traditional models of the term structure of interest rates are built upon decompositions of yields into latent factors using one or another statistical method (e.g. Nelson and Siegel (1987), Knez, Litterman, Scheinkman (1994), Duffie and Kan (1996)). While the fit of these models is usually rather good, their economic meaning is somewhat limited since they have relatively little to say about the relationship between observable economic variables and interest rates of different maturities. Yet, it is of importance not only for traders in bond markets but also for central banks and government agencies to understand how the yield curve reacts to macroeconomic shocks. To explore this issue, one therefore needs to construct models which jointly describe macro and term structure dynamics.

In a seminal paper, Ang and Piazzesi (2003) augment a standard three-factor affine term structure model with two macroeconomic factors. They find that the included macroeconomic variables improve yield forecasts, accounting for up to 85 % of the variation in interest rates. Inspired by this finding, a vivid literature has emerged lately which explores different approaches of jointly modelling the term structure and the macroeconomy. Hördahl, Tristani, and Vestin (2005), for example, build a small structural model for the joint evolution of output, inflation, and short-term interest rates, to which they add the term structure. They find their model to outperform Ang and Piazzesi's (2003) model as well as traditional latent factor models in terms of out-of-sample forecast performance. Diebold, Rudebusch, and Aruoba (2005) estimate a model which allows for correlated latent and observed macroeconomic factors and find that macroeconomic variables have strong effects on future movements of the yield curve, while latent interest rate factors have a relatively small impact on macroeconomic variables. Further examples of recent papers which jointly model term structure and macro dynamics include e.g. Dewachter and Lyrio (2004), Wu (2002), Rudebusch and Wu (2003), and Dai and Philippon (2004).

Using different models and methodologies, all these papers conclude that macroeconomic variables are useful for explaining and/or forecasting government bond yields. A common feature of these studies, though, is that only very small macroeconomic information sets are being exploited for the analysis. Commonly, the models include a measure of the output gap and a measure of inflation, plus at most two other variables and one or more latent yield curve factors. The main reason for this informational limitation is that state-of-the-art affine term structure models imply the estimation of a large number of parameters, thereby considerably restricting the number of explanatory variables one can include in the model. Yet, by restricting the analysis to only a few variables, potentially useful macroeconomic information is being neglected.

A recent strand of the macroeconomic literature advances the use of dynamic factor models to incorporate large macroeconomic information sets in economic analysis (e.g. Stock and Watson (2002), Forni, Hallin, Lippi, and Reichlin (2003)). Such models break down the cross-sectional information contained in large panels of economic time series into common and series-specific components, and thereby enable the researcher to separate out aggregate and idiosyncratic shocks. A number of studies have found that dynamic factor models are particularly powerful in forecasting economic time series, especially measures of output and inflation.

In this paper, I examine the usefulness of factors extracted from large macroeconomic datasets for explaining and forecasting the term structure of interest rates. This exercise is basically motivated by three observations. First, it has recently been argued by some authors that central banks actively monitor a large number of macroeconomic time series, and that monetary policy decisions would thus be based on the information contained in not only a few key aggregates but many economic variables. Loosely speaking, the central bank sets interest rates in a "data-rich environment" (Bernanke and Boivin (2003)). Accordingly, dynamic factors which parsimoniously summarize the information contained in a large number of time series variables should prove useful in explaining interest rates set by central banks. Bernanke and Boivin (2003), Favero, Marcellino and Neglia (2005) and Belviso and Milani (2005) consistently provide empirical evidence supporting this claim. By comparing standard Taylor rules with specifications based on dynamic factors, these papers show that the latter exhibit information beyond output and inflation that helps explaining monetary policy. Moreover, Giannone, Reichlin, and Sala (2004) show that factor-based forecasts of the federal funds rate perform as good as market-based forecasts. Second, as argued above, dynamic factor models have been shown to perform well in forecasting measures of output and inflation (see, e.g. Stock and Watson (2002)). Since both expected output and expected inflation are likely to have an impact on bond yields, this delivers another argument for using them in a term structure model. Finally, Mönch (2004) provides evidence that dynamic factors proxy for systematic sources of risk that explain the cross-section of equity returns and may thus be used to explain risk premia. Overall, since the prices and yields of non-defaultable government bonds are driven by expectations about future short-term interest rates, expected future inflation and risk premia, the evidence pointed to above suggests that factors extracted from large panels have explanatory power also for the yield curve.

What is the appropriate modelling framework for incorporating a broad macroeconomic information set into term structure analysis through the use of dynamic factors? In a recent paper, Bernanke, Boivin, and Eliasz (2005) suggest to combine the advantages of factor analysis and structural VAR analysis by estimating a joint vector-autoregression of factors extracted from a large cross-section of time series and perfectly observable economic variables such as the short-term interest rate. They find their approach which they label "factor-augmented VAR (FAVAR)" to be a useful tool for properly identifying the monetary policy transmission mechanism. The FAVAR model provides a dynamic characterization of short-term interest rates set by the central bank in response to the main economic shocks which are summarized by a few common factors. As a by-product, it delivers a path of expected future short rates conditional on a broad macroeconomic information set. On the other hand, given a short rate equation, affine term structure models provide a tool to build up the entire yield curve subject to no-arbitrage restrictions. It is thus an obvious next step to combine a factor-augmented VAR model with the standard affine setup by using the FAVAR as the state equation in an essentially affine term structure model. This is done in the present paper.

Estimation of the model is in two steps. First, I extract a few common factors from a large macroeconomic dataset using standard static principal components and estimate the parameters governing their joint dynamics with the monetary policy instrument in a VAR. Second, I estimate a no-arbitrage vector autoregression of yields of different maturities on the exogenous pricing factors. Specifically, I obtain the price of risk parameters by minimizing the sum of squared fitting errors of the model following the nonlinear least squares approach suggested by Ang, Piazzesi, and Wei (2005). Since my model does not include latent yield curve factors, the parameters governing the dynamics of the state variables can be estimated separately by standard OLS. Hence, estimation is fast which makes the model particularly useful for recursive out-of-sample forecasts which are the main focus of this paper.

The results of the paper can be summarized as follows. A term structure model including as factors the short rate and four common components which together explain the bulk of variation in a large panel of monthly macroeconomic time series variables for the US, provides a good in-sample fit of the term structure of interest rates. Preliminary regressions show that factors extracted from a large macroeconomic dataset contain information useful for explaining the federal funds rate beyond output and inflation. Moreover, the model factors are highly significant explanatory variables for yields. Compared to a model which incorporates the short rate and four individual measures of output and inflation as factors, there is a clear advantage in using the larger macroeconomic information set. The results from out-of-sample forecasts of yields underpin this finding. The term structure model based on common factors clearly outperforms the model based on individual variables for all maturities at all horizons. Moreover, in forecasts beyond the one-month ahead horizon, the model outperforms various yield-based benchmark models including the random walk, a standard three-factor affine model and the model recently suggested by Diebold and Li (2005) which has been documented being particularly powerful in out-of-sample yield forecasts over longer horizons. Moreover, the relation of the macro factors to level, slope and curvature of the yield curve is studied.

The remainder of this paper is summarized as follows. In section II, the affine term structure model based on common dynamic macro factors is motivated and its exact parametrization discussed. Section III briefly sketches the method used to estimate the model. In section IV, I first provide some preliminary evidence on the usefulness of factors extracted from large panels to explain yields and then discuss the results of the out-of-sample forecasts in section V. Section VI concludes the paper.

II The Model

Monetary policy decisions are commonly assumed to be based on the information contained in not not only a few key aggregates but many economic variables. Yet, since it is infeasible to empirically model the policy reaction function as to depend on a large number of individual variables, economists customarily map short term interest rates to a few variables, including mostly a measure of the output gap and inflation. A convenient way of keeping track of a plethora of information without including too many variables into a model, however, is to think of all macroeconomic variables as being driven by a few common factors and an idiosyncratic component. In such a setup, the reaction of the monetary policy maker to shocks affecting different categories of economic variables can be modelled by relating the short-term interest rate to factors which by construction capture the common response of a large number of individual variables to the economy-wide shocks. This kind of framework thus allows to considerably reduce the dimensionality of the policy problem in a "data-rich" environment (Bernanke and Boivin (2003)).

A State dynamics and short rate equation

More formally, assume there is a large number of macroeconomic time series that are each driven by the monetary policy instrument r, a small number of unobserved common factors F and an idiosyncratic component e, i.e.

$$X_t = \Lambda_f F_t + \Lambda_r r_t + e_t, \tag{1}$$

where X_t is a $M \times 1$ vector of period-t observations of the variables in the panel, Λ_f and Λ_r are the $M \times k$ and $M \times 1$ matrices of factor loadings, r_t is the short-term interest rate, F_t is the $k \times 1$ vector of period-t observations of the common factors, and e_t is an $M \times 1$ vector of idiosyncratic components.[1] Note that equation (1) can also be written in a way that allows X_t to depend on current and lagged values of the fundamental factors. Stock and Watson (2002) show, however, that the static formulation is not restrictive since F_t can be interpreted as including an arbitrary number of lags of the fundamental factors. Accordingly they refer to the model above - without the observable r_t - as a dynamic factor model.

Economists typically think of the economy as being affected by monetary policy through the short term interest rate, r_t. On the other hand, the central bank is assumed to set interest rates in response to the overall state of the economy, characterized e.g. by the deviations of inflation and output from their desired levels. As has been discussed by Bernanke, Boivin, and Eliasz (2005), theoretical macroeconomic aggregates as inflation and output might not be perfectly observable neither to the policy-maker nor to the econometrician. More realistically, the macroeconomic time series observed by the central bank or the econometrician will in general be noisy measures of broad economic concepts such as output and inflation. Accordingly, these variables should be treated as unobservable in empirical work so as to avoid confounding measurement error or idiosyncratic dynamics with fundamental economic shocks. Bernanke et al. (2005) therefore suggest to extract a few common factors from a large number of macroeconomic time series variables and to study the mutual dynamics of monetary policy and the key economic aggregates by estimating a joint VAR of the factors and the policy instrument, an approach which they label "Factor-Augmented VAR" (FAVAR).

The term structure model suggested here is built upon the assumption that yields are driven by movements of short term interest rates as well as the main shocks hitting the economy. The latter are proxied for by the factors which capture the bulk of common variation in a large number of macroeconomic time series variables. The joint dynamics of these factors and the monetary policy instrument are modelled in a vector autoregression. I thus employ the FAVAR model suggested by Bernanke et al. (2005) as a central building block for my term structure model. In addition, restrictions are imposed on the parameters governing the impact of the state variables on the yields in order to ensure no-arbitrage. Accordingly, I will term the approach pursued here a "No-Arbitrage Factor-Augmented

[1]The idiosyncratic components may display some slight cross- and serial correlation, see Stock and Watson (2002) for a detailed discussion of this issue.

Vector Autoregression". Overall, the dynamics of the economy are described by

$$\begin{pmatrix} F_t \\ r_t \end{pmatrix} = \tilde{\mu} + \tilde{\Phi}(L) \begin{pmatrix} F_{t-1} \\ r_{t-1} \end{pmatrix} + \tilde{\nu}_t, \tag{2}$$

where $\tilde{\mu} = (\tilde{\mu}_f', \tilde{\mu}_r)'$ is a $(k+1) \times 1$ vector of constants, $\tilde{\Phi}(L)$ is a $(k+1) \times (k+1)$ matrix of order-p lag polynomials and $\tilde{\nu}_t$ is a $(k+1) \times 1$ vector of reduced form shocks with variance covariance matrix $\tilde{\Omega}$. To summarize, equation (2) says that the factors capturing the common variation in many economic time series variables are driven partly by their own dynamics, partly by monetary policy through the short term rate, and partly by exogenous shocks.

Let us have a closer look at the policy reaction function implied by this model. Since the short term interest rate is included in the state vector, the dynamics of the policy instrument are completely characterized by the last equation in the VAR above, i.e.

$$r_t = \tilde{\mu}_r + \tilde{\phi}_f(L)F_{t-1} + \tilde{\phi}_r(L)r_{t-1} + \tilde{\nu}_t^r. \tag{3}$$

Hence, in the FAVAR model the short-term interest rate set by the central bank is characterized by a response to the lagged observations of the main economic driving forces, $\tilde{\phi}_f(L)F_{t-1}$, by some partial adjustment element, $\tilde{\phi}_r(L)r_{t-1}$, and a monetary policy shock orthogonal to the former two components. The policy reaction function is thus purely backward looking. Yet, since the evolution of r and the main economic driving forces are jointly characterized by a Factor-augmented VAR model, the implied dynamics of the short term interest rate are potentially much richer than in standard affine term structure models where the short rate is an affine function of contemporaneous observations of the factors whose dynamics are described independently of changes in monetary policy. Hence, the no-arbitrage FAVAR model studied here explicitly allows for feedback from monetary policy to the macroeconomy, a feature missing e.g. from the model in Ang and Piazzesi (2003) who assume macroeconomic and term structure factors to be orthogonal. The approach pursued in this paper is thus closer in spirit to the work by Hördahl et al. (2005) who jointly model the evolution of output, inflation, and short-term interest rates within a structural economic model. As in their paper I expect the richer dynamic structure of the FAVAR model to improve forecast performance.

To facilitate notation in the sequel, I rewrite the VAR in equation (2) in companion form as

$$Z_t = \mu + \Phi Z_{t-1} + \nu_t, \tag{4}$$

where $Z_t = (F_t', r_t, F_{t-1}', r_{t-1}, \ldots, F_{t-p+1}', r_{t-p+1})'$, and where μ denotes a vector of constants and zeros, Φ the respective companion form matrix of VAR coefficients, and Ω

the companion form variance covariance matrix of the model factors. Accordingly, the short rate r_t can be expressed in terms of Z_t as $r_t = \delta' Z_t$ where $\delta' = (0_{1\times k}, 1, 0_{1\times(k+1)(p-1)})$.

In the present model, the vector of state variables Z only comprises the macro driving factors, F, and the short term rate, r. Notice that this assumption could in principle be relaxed by augmenting the state vector with latent yield factors as in Ang and Piazzesi (2003). In this case, however, the two-step estimation method would no longer be feasible, and one would have to resort to standard maximum-likelihood techniques as the one put forward by Chen and Scott (1993) that are commonly employed in the affine term structure literature. Moreover, the number of parameters that would have to be estimated jointly would be considerably higher and thus estimation speed lower.

B Pricing Kernel

To model the dynamics of the pricing kernel, I follow the arbitrage-free term structure literature initiated by Duffie and Kan (1996), which has also been applied, among others, by Ang and Piazzesi (2003) and Hördahl et al. (2005). These authors define the nominal pricing kernel as $M_t = \exp(-r_t)\frac{\psi_{t+1}}{\psi_t}$, where ψ_t denotes the Radon-Nikodym derivative which converts the risk-neutral into the true data-generating distribution. ψ is assumed to follow the lognormal process $\psi_{t+1} = \psi_t \exp(-\frac{1}{2}\ \lambda_t'\Omega\lambda_t - \lambda_t'\nu_{t+1})$ and is thus driven by the shocks ν driving the state variables. Accordingly, the nominal pricing kernel M is given by

$$\begin{aligned} M_{t+1} &= \exp(-r_t - \frac{1}{2}\lambda_t'\Omega\lambda_t - \lambda_t'\nu_{t+1}), \\ &= \exp(-\delta' Z_t - \frac{1}{2}\lambda_t'\Omega\lambda_t - \lambda_t'\nu_{t+1}). \end{aligned} \tag{5}$$

The vector λ_t denotes the market prices of risk. Following Duffee (2002) these are commonly assumed to be affine in the underlying state variables Z, i.e.

$$\lambda_t = \lambda_0 + \lambda_1 Z_t. \tag{6}$$

In order to keep the model parsimonious I restrict the prices of risk to depend only on current observations of the model factors. Obviously, there is some arbitrariness in this restriction. In principle, one can also think of theoretical models that give rise to market prices of risk which depend on lagged state variables. However, since the dimensionality of the problem requires to make some identification restrictions, assuming that market prices of risk depend only on current observations of the states seems to be a plausible compromise. Note that since the state vector Z_t includes current and lagged observations of the macro factors and the short rate, this choice implies a set of obvious zero restrictions on the parameters λ_0 and λ_1.[2] In an arbitrage-free market, today's price of a n-months to maturity zero-coupon bond must equal the expected discounted value of the price of an (n-1)-months to maturity bond tomorrow:

$$P_t^{(n)} = E_t[M_{t+1} P_{t+1}^{(n-1)}].$$

Assuming that yields are affine in the state variables, bond prices $P_t^{(n)}$ are exponential linear functions of the state vector:

$$P_t^{(n)} = \exp\left(A_n + B_n' Z_t\right),$$

where the scalar A_n and the coefficient vector B_n depend on the time to maturity n. Following Ang and Piazzesi (2003), I show in appendix A that no-arbitrage is guaranteed by computing coefficients A_n and B_n according to the following recursive equations:

$$A_n = A_{n-1} + B_{n-1}' (\mu - \Omega\lambda_0) + \frac{1}{2} B_{n-1}' \Omega B_{n-1}, \tag{7}$$

$$B_n = B_{n-1}' (\Phi - \Omega\lambda_1) - \delta' \tag{8}$$

Given the price of an n-months to maturity zero-coupon bond, the corresponding yield is thus obtained as

$$\begin{aligned} y_t^{(n)} &= -\frac{\log P_t^{(n)}}{n} \\ &= a_n + b_n' Z_t, \end{aligned} \tag{9}$$

where $a_n = -A_n/n$ and $b_n' = -B_n'/n$.

III Estimation of the Term Structure Model

Prior to estimating the term-structure model, the common factors have to be extracted from the panel of macro data. This is achieved using standard static principal components following the approach suggested by Stock and Watson (2002). Precisely, let V denote the eigenvectors corresponding to the k largest eigenvalues of the $T \times T$ cross-sectional variance-covariance matrix of the data XX'. Then, subject to the normalization $F'F/T =$

[2] In particular, $\lambda_0 = (\tilde{\lambda}_0', 0_{1\times(k+1)(p-1)})'$ where $\tilde{\lambda}_0$ is a vector of dimension $(k+1)$ and $\lambda_1 = \begin{pmatrix} \tilde{\lambda}_1 & 0_{(k+1)\times(k+1)(p-1)} \\ 0_{(k+1)(p-1)\times(k+1)} & 0_{(k+1)(p-1)\times(k+1)(p-1)} \end{pmatrix}$ where $\tilde{\lambda}_1$ is a $(k+1)\times(k+1)$ matrix.

I_k, estimates $\hat{F}$ of the factors and $\hat{\Lambda}$ the factor loadings are given by[3]

$$\hat{F} = \sqrt{T-1}\,V \quad \text{and}$$
$$\hat{\Lambda} = \sqrt{T-1}\,X'V,$$

i.e. the common factors are estimated as $\sqrt{T-1}$ times the eigenvectors corresponding to the k largest eigenvalues of the variance-covariance matrix XX'. Given the factor estimates, estimation of the term structure model is performed using a consistent two-step approach following Ang, Piazzesi and Wei (2005). First, estimates of the parameters (μ, Φ, Σ) governing the dynamics of the model factors are obtained by running a VAR(p) on the estimated factors and the short term interest rate. Second, given the estimates from the first step, the parameters $\tilde{\lambda}_0$ and $\tilde{\lambda}_1$ which drive the evolution of the state prices of risk, are obtained by minimizing the sum of squared fitting errors of the model. That is, for a given set of parameters the model-implied yields $\hat{y}_t^{(n)} = \hat{a}_n + \hat{b}_n' Z_t$ are computed and then for the N yields used in the estimation

$$S = \sum_{t=1}^{T} \sum_{n=1}^{N} (\hat{y}_t^{(n)} - y_t^{(n)})^2 \tag{10}$$

is minimized with respect to $\tilde{\lambda}_0$ and $\tilde{\lambda}_1$ given the estimates of the VAR parameters μ, Φ, and Ω. Although being possibly less efficient than a joint estimation of all model parameters in a one-step maximum likelihood procedure, the two-step approach has the clear advantage that it is fast and thus much better suited for an recursive out-of-sample forecast exercise.[4]

Due to the recursive formulation of the bond pricing parameters, the sum of squared fitting errors is highly nonlinear in the underlying model parameters. It is thus helpful to find good starting values to achieve fast convergence. This is done in the following way. I first estimate the parameters $\tilde{\lambda}_0$ under the assumption that risk premia are constant

[3] To account for the fact that r is an observed factor which is assumed unconditionally orthogonal to the unobserved factors F in the model (1), its effect on the variables in X has to be concentrated out prior to estimating F. Here, this is achieved by simply regressing all variables in X onto r and extracting principal components from the variance-covariance matrix of residuals of these regressions. Note that Bernanke et al. (2005) use a slightly a slightly more elaborate approach in order to identify monetary policy shocks within their FAVAR model.

[4] Nonetheless, it would be interesting to estimate the latent macro factors and the parameters characterizing their impact on yields jointly within a one-step estimation procedure. The cross-equation restrictions of the yield curve model would then put additional structure on the estimation of the factors, thereby potentially sharpening up our understanding of the macroeconomic driving forces behind the yield curve. In a recent paper, Law (2004) uses a similar idea to study the extent to which variation in bond yields can be explained by macroeconomic fundamentals.

but nonzero, i.e. I set to zero all elements of the matrix $\tilde{\lambda}_1$ which governs the time-varying component of the market prices of risk. Then, I take these estimates as starting values in an estimation step that allows for variation in the market prices of risk, i.e. I let all elements of $\tilde{\lambda}_0$ and $\tilde{\lambda}_1$ be estimated freely. Finally, to enhance tractability of the model I follow the common practice in the affine term structure model literature and re-estimate the model after setting to zero those elements of $\tilde{\lambda}_1$ which are insignificant. Standard errors of the prices of risk parameters reported in section IV are computed via the numerical gradient of the sum of squared fitting errors function S. The standard errors of the state equation parameters are unadjusted OLS standard errors.[5]

IV Empirical Results

A Data

I estimate the model using the following data. The macroeconomic factors are extracted from a dataset which contains about 160 monthly time series of various economic categories for the US. Among others, it includes a large number of time series related to industrial production, more than 30 employment-related variables, around 30 price indices and various monetary aggregates. It further contains different kinds of survey data, stock indices, exchange rates etc. This dataset has been used by Giannone et al. (2004) to forecast US output, inflation, and short term interest rates.[6] Stock and Watson's (2002) principal components estimation of the common factors in large panels of time series requires stationarity. I therefore follow Giannone et al. (2004) in applying different preadjustments to the time series in the dataset.[7] Finally, I standardize all series to have mean zero and unit variance.

I use data on zero-coupon bond yields of maturities 1, 3, 6, and 9 months, as well as 1, 2, 3, 4, 5, 7, and 10 years. All interest rates are continuously-compounded smoothed Fama-

[5]Notice that Ang, Piazzesi and Wei (2005) compute standard errors using GMM to adjust for the two-stage estimation process. However, since the no-arbitrage FAVAR model involves estimation of a VAR of lag order higher than 1, a large number of moment conditions would be needed to identify the state equation parameters via GMM and thus computation would be burdensome. Hence, since the focus here is on forecast performance rather than in-sample fit, I do not follow the approach of Ang et al. (2005).

[6]I am grateful to Lucrezia Reichlin who generously provided me with this dataset. Note that I exclude all interest rate related series from the original panel and instead include the zero-coupon yields used in the term structure model. For a detailed description of the data, the reader is referred to the paper by Giannone et al (2004).

[7]Though with a slight difference as regards the treatment of price series: instead of computing first differences of quarterly growth rates I follow Ang and Piazzesi (2003) and compute annual inflation rates.

Bliss yields and have been constructed from US treasury bonds using the method outlined in Bliss (1997).[8] I estimate and forecast the model over the post-Volcker disinflation period, i.e. from 1983:01 to the last available observation of the macro dataset, 2003:09.

B Model Specification

In the first step of the estimation procedure, I extract common factors from the large panel of macroeconomic time series using static principal components following Stock and Watson (2002). Together, the first 10 factors explain about 70% of the total variance of all variables in the dataset. The largest contribution is accounted for by the first four factors, however, which together explain more than 50% of the total variation in the panel. Interestingly, a look at the correlation patterns of all 10 factors with yields of all maturities and their lags, reveals that it is the first four factors that are most highly correlated with yields.

The number of factors I can include in my term-structure model is limited due to parameterization constraints imposed by the market prices of risk specification. If no additional restrictions are imposed on the market prices of risk, the number of parameters to estimate in the second step of the estimation procedure increases quadratically with the number of factors. For the sake of parsimony I thus restrict the number of factors to the first four principal components extracted from the large panel of monthly time series and the short rate. Unreported results with smaller and larger number of factors have shown that this specification seems to provide the best tradeoff between estimability and model fit. A similar choice has to be made regarding the number of lags to include in the factor-augmented VAR which represents the state equation of my term structure model. Standard information criteria indicate an optimal lag length of 4 for the joint VAR of factors and the short rate so I use this particular specification of the state equation.

C Factor Estimates

Due to the well-known rotational indeterminacy problem in factor analysis, structural interpretation of the factors is difficult. In fact, unless strong identification assumptions are imposed on the factor loadings, a potentially infinite number of linear rotations of the factors can be found that all explain the same amount of total variation in the panel but imply different sets of factor loadings. Here, the factors are first extracted form the large panel of macro data and then treated as observable in the estimation of the term structure model. Implicitly, they are indexes summarizing the information in many time series with weights chosen such that the sum of squared idiosyncratic components in equation (1)

[8] I am grateful also to Robert Bliss who provided me with the programs and raw data to construct the Fama-Bliss yields.

is minimized. Hence, in order to obtain some understanding of what type of economic information the estimated factors capture, it is feasible to regress them onto the individual variables in the panel. Table I lists for each of the four factors those five series with which it exhibits the strongest correlation. It turns out from these results that the first factor clearly is closely linked to business cycle variables such as measures of employment and industrial production. In contrast, the second factor is most strongly correlated with different measures of consumer price inflation. Hence, without any rotation, there is a clear dichotomy between a real and a nominal factor as the two main driving forces behind a large number of various economic time series.[9] The third factor loads most strongly on leading indicators of the business cycle such as M1, inventories and loans and securities series. Finally, the fourth factor is most strongly correlated with measures of money supply and producer prices. A plot of the factor time-series together with some important real and nominal variables is provided in figure 1.

D Preliminary Evidence

Before estimating the term structure model subject to no-arbitrage restrictions, I run a set of preliminary regressions to check whether the extracted macro factors are useful explanatory variables in a term structure model. In section D.1, I use a simple encompassing test to assess whether a factor-based policy reaction function provides a better explanation of monetary policy decisions than a standard Taylor-rule based on individual measures of output and inflation. In section D.2, I then perform unrestricted regressions of yields on the model factors.

D.1 Test of "Excess Policy Response"

The use of dynamic factors instead of individual macroeconomic variables to forecast yields has been justified with the argument that central banks react to larger information sets than just individual measures of output and inflation. Whether this conjecture holds true empirically can be tested by comparing the fit of a standard Taylor-rule policy reaction function with that of a policy reaction function based on dynamic factors. Bernanke and Boivin (2003) present evidence for an "excess policy reaction" of the Fed by including the fitted value of the federal funds rate from a factor-based reaction function into an otherwise standard Taylor-rule and checking the significance of its coefficient. An alternative approach amounts to separately estimating the two competing policy reaction functions and then to perform an encompassing test *à* la Davidson and MacKinnon (1993). This

[9]Using the same dataset, Giannone et al. (2005) find that the dynamic dimension of the US economy is two, i.e. they identify a real and a nominal shock which explain the bulk of variation in all time series contained in the panel.

is the strategy adopted by Belviso and Milani (2005). Here I follow these authors and compare a standard Taylor rule with partial adjustment[10],

$$r_t = \rho r_{t-1} + (1-\rho)(\phi^{\pi}\pi_t + \phi^y y_t),$$

with a policy reaction function based on the four factors which I use as state variables in my term structure model,

$$r_t = \rho r_{t-1} + (1-\rho)\phi^{F'}F_t.$$

The results from both regressions are summarized in tables II and III. As indicated by the regression R^2s of 0.967 and 0.970, the factor-based policy rule seems to fit the data slightly better than the standard Taylor rule. The Davidson-MacKinnon (1993) encompassing test can now be used in order to asses whether this improvement in model fit is statistically significant. I thus regress the federal funds rate onto the fitted values from both alternative specifications. This yields the following result:

$$\begin{aligned} r_t &= \alpha\, \hat{r}_t^{Taylor} + (1-\alpha)\, \hat{r}_t^{Factors} + \epsilon_t \\ &= 0.119\, \hat{r}_t^{Taylor} + 0.881\, \hat{r}_t^{Factors} \\ &= (0.173) \qquad\quad (0.173) \end{aligned}$$

Hence, the coefficient on the standard Taylor rule is insignificant whereas the coefficient on the factor-based fitted federal funds rate is highly significant.[11] I interpret this result as evidence in favor of the hypothesis that the Fed reacts to a broad macroeconomic information set.

D.2 Unrestricted Estimation

To obtain a first impression whether the factors extracted from the panel of macro variables also capture predictive information about yields of higher maturities, table IV summarizes the mutual correlations between the yields and various lags of the factors used for estimating the model. As one can see in this table, the short-term interest rate ($y^{(1)}$) shows strongest contemporaneous correlation with yields of any other maturity. Yet, all four macro factors extracted from the panel of monthly US time series, are also strongly

[10] Inflation π is defined as the annual growth rate of the GDP implicit price deflator (GDPDEF). The output gap is measured as the percentage deviation of log GDP (GDPC96) from its trend (computed using the Hodrick-Prescott filter and a smoothing parameter of 14400). Both quarterly series have been obtained from the St. Louis Fed website and interpolated to the monthly frequency using the method described in Mönch and Uhlig (2005). For the interpolation of GDP I have used industrial production (INDPRO), total civilian employment (CE16OV) and real disposable income (DSPIC96) as related monthly series. CPI and PPI finished goods have been employed as related monthly series for interpolating the GDP deflator.

[11] This result is robust to alternative specifications of both reaction functions using a larger number of lags of the policy instrument and the macro variables or factors.

correlated with yields of different maturities. The first factor, which closely tracks the business cycle (see also table I), is positively correlated with yields. The second factor, which clearly captures inflation movements, is also strongly positively correlated with yields of all maturities. The third factor which is most closely related to leading indicators, is uncorrelated with yields of shorter maturities, but positively correlated with longer maturity yields. Finally, the fourth factor, being negatively correlated with business cycle variables such as employment measures, is also positively correlated with yields of all maturities. Correlating lagged factors with yields, one can see that the strong impact of the short rate on yields of all maturities decreases for the benefit of the macro factors. In particular, the correlation between lagged observations of the business cycle related first and third factor and yields increases with the lag length. This gives a first indication that the macro factors should prove useful in forecasting yields.

To explore further the question whether the models' factors have explanatory power for yields, table V provides estimates of an unrestricted VAR of yields of different maturities onto a constant, the four macro factors and the federal funds rate, i.e. it estimates the pricing equation for yields,

$$Y_t = A + BZ_t + u_t,$$

where no cross-equation restrictions are imposed on the coefficients A and B.

The first observation to make is that the R^2 of these regressions are all very high. Together with the short rate, the four factors explain more than 95% of the variation in short yields, and still about 90% of the variation in longer yields. Not surprisingly, the federal funds rate is the most highly significant explanatory variable for short maturity yields. However, in the presence of the macro factors its impact decreases strongly towards the long end of the maturity spectrum.

E Estimating the Term Structure Model

E.1 In-Sample Fit

In this section, I report results obtained from estimating the FAVAR model subject to the cross-equation restrictions (7) and (8) implied by the no-arbitrage assumption as outlined in section II. The model fits the data surprisingly well given that it does not make use of latent yield curve factors. Table VI reports the first and second moment of observed and model-implied yields and 1-year holding period returns, respectively. These figures indicate that on average the no-arbitrage FAVAR model fits the yield curve almost exactly. Figure 3 provides a visualization of this result by showing average observed and model-implied yields across the maturity spectrum. Notice that the model seems to be missing some of the variation in longer maturities since the standard deviations of fitted interest rates are slightly lower than the standard deviations of the observed yields, especially at the long end of the curve. This can also be seen in figure 2 which plots the time series for a selection of observed and model-implied yields. While the fit is very good at the short end of the yield curve, the model does not perfectly capture all the variation at the long end of the maturity spectrum. Accordingly, observed and model-implied holding period returns are almost identical on average whereas the fitted returns exhibit standard deviations slightly smaller than the observed returns. Yet, the difference amounts to only a few basis points and is thus fairly small.

Overall, the no-arbitrage FAVAR model is able to capture the cross-sectional variation of government bond yields quite well, with a slightly better in-sample fit at the short end of the curve. As we will see further below, this has an impact also on the forecast results obtained from the model. Indeed, the improvement over latent-factor based term structure models is more pronounced at the short than at the long end of the yield curve. Yet, as has been discussed above, estimating a TSM without latent yield factors considerably facilitates estimation of the model and thus makes recursive out-of-sample forecasts feasible.

E.2 Parameter Estimates

Table VII reports the parameter estimates and associated standard errors of the no-arbitrage FAVAR model. The upper panel shows parameter estimates of the Factor-augmented VAR that represents the state equation of the model, the second panel provides the estimates of the state prices of risk which constitute the remaining components of the recursive bond pricing parameters A and B.

As the diagonal elements of the first lag's coefficient matrix indicate, all five model factors are relatively persistent, a feature that is needed to explain time-variation in yields which are themselves highly persistent time series processes. Since the model factors are by construction unconditionally uncorrelated only few of the off-diagonal elements of the autoregression coefficients in Φ are significant, however.

As the second panel of table VII shows, all elements of the vector $\tilde{\lambda}_0$ governing the unconditional mean of the market prices of risk are large and highly significant. This indicates that risk premia are characterized by a large constant component. As indicated by the size and significance of the estimates $\tilde{\lambda}_1$, there is also some significant amount of time variation in risk premia over the sample period considered. It is difficult to interpret individual elements in the estimated prices of risk matrix, however. Indeed, unreported results from alternative model specifications varying e.g. the number of factors, the number of lags in the state equation or the sample period have shown that the price of risk estimates are quite sensitive to changes in model specification. Hence, economic reasoning based on the significance of individual parameters governing the state prices of risk is unwarranted. Instead, in order to visualize the relation between risk premia and the model factors, figure 5 provides a plot of model-implied term premia for the 1-year and 5-year yield. As indicated by these plots, term premia at the short end of the yield curve are more closely related to the business cycle as proxied by the first macro factor whereas premia for longer yields seem to track inflation which is represented by the second factor.

Figure 4 shows a plot of the loadings b_n of the yields onto the contemporaneous observations of the model factors. The signs of these loadings are consistent with those obtained from regressing yields onto the model factors without imposing no-arbitrage restrictions, summarized in table V. By construction of my arbitrage-free model, the loading of the 1-month yield onto the short rate factor equals unity and those for the macro factors are zero. However, the impact of the short rate on longer yields strongly decreases with maturity and almost approaches zero at the very long end of the maturity spectrum. Hence, movements in the short-term interest rate only have a marginal direct effect on long-term interest rates. These are almost entirely driven by macroeconomic factors. Most importantly, the inflation-related second factor has a strongly increasing impact on yields going up the maturity spectrum. In contrast, the business cycle related first factor has an equally strong impact on yields of medium and longer maturities. The third factor which is leading the business cycle with a reversed sign has an increasingly positive impact on yields of longer maturities and a negative but small impact on very short maturities. This result is consistent with the well-documented procyclicality of the slope of the yield curve.

V Out-of-Sample Forecasts

In this section, I compare the out-of-sample forecast performance of the no-arbitrage FAVAR with that of the no-arbitrage VAR model, a VAR(1) on yield levels, the Diebold-Li (2005) version of the Nelson-Siegel (1987) three-factor model, an essentially affine latent factor only model ($A_0(3)$), and a simple random walk. The latter three models are expected to be the most challenging competitors. Diebold and Li (2005) have shown their model to outperform a variety of yield forecasting models including different specifications of forward regressions, AR and VAR models for yields and the random walk. Moreover, Duffee (2002) has shown that the essentially affine latent factor only model has strong out-of-sample forecast performance. Finally, the random walk is often reported as being difficult to beat in out-of-sample forecasts of interest rates.

A The Competitor Models

Precisely, the forecasts for the different competitor models are computed as follows.

1. **No-Arbitrage FAVAR model**:

$$\hat{y}^{(n)}_{t+h|t} = \hat{a}_n + \hat{b}_n \hat{Z}^{FAVAR}_{t+h|t}$$

where Z^{FAVAR} contains the four factors explaining the bulk of variation in the panel of monthly time series for the US, and the 1-month yield. The coefficients $\hat{a}_n$ and $\hat{b}_n$ are obtained recursively according to equations (7) and (8), using as input the estimates $\hat{\mu}$, $\hat{\Phi}$, and $\hat{\Sigma}$ obtained by running a VAR(1) on the states, as well as the estimates $\hat{\lambda}_0$ and $\hat{\lambda}_1$ obtained by minimizing the sum of squared fitting errors of the model. Forecasts $\hat{Z}^{FAVAR}_{t+h|t}$ are obtained from a VAR(1) fitted to the companion form state vector, i.e.

$$\hat{Z}^{FAVAR}_{t+h|t} = \hat{\Phi}^h Z^{FAVAR}_t + \sum_{i=0}^{h-1} \hat{\Phi}^i \hat{\mu}$$

2. **No-Arbitrage VAR**:

$$\hat{y}^{(n)}_{t+h|t} = \hat{a}_n + \hat{b}_n \hat{Z}^{VAR}_{t+h|t}$$

where Z^{VAR} contains the quarterly growth rate of IP, the help-wanted index, the annual growth rates of CPI and PPI, and the 1-month yield. The coefficients $\hat{a}_n$ and $\hat{b}_n$ are obtained recursively according to equations (7) and (8) and guarantee the absence of arbitrage opportunities. The specification and estimation of the model is the same as for the no-arbitrage FAVAR model.

3. **VAR(1) on Yield Levels**:

$$\hat{y}_{t+h|t} = \hat{c} + \hat{\Gamma} y_t$$

where $y_t = \{y^{(1)}, y^{(3)}, \ldots, y^{(120)}\}$ and $\hat{c}$ and $\hat{\Gamma}$ are obtained by regressing the vector y_t onto a constant and its h-months lag.

4. **Diebold-Li (2005)**:

$$\hat{y}^{(n)}_{t+h|t} = \hat{\beta}_{1,t+h|t} + \hat{\beta}_{2,t+h|t} \left(\frac{1 - e^{-\lambda n}}{\lambda n} \right) + \hat{\beta}_{3,t+h|t} \left(\frac{1 - e^{-\lambda n}}{\lambda n} - e^{-\lambda n} \right)$$

where

$$\hat{\beta}_{t+h|t} = \hat{c} + \hat{\Gamma} \hat{\beta}_t$$

Diebold and Li (2005) obtain estimates of the factors β by fixing λ to 0.0609 and then simply regressing yields onto the factor loadings 1, $(\frac{1-e^{-\lambda n}}{\lambda n})$, and $(\frac{1-e^{-\lambda n}}{\lambda n} - e^{-\lambda n})$.[12] Note that Diebold and Li find better forecasting performance of their model when the factor dynamics are estimated by fitting simple AR(1) processes instead of a VAR(1). With the data and sample period used here, however, I find that their model performs better when the latent factor dynamics are estimated using a VAR as specified above.

5. **Essentially Affine Latent Factor Only Model** ($A_0(3)$):

$$\hat{y}^{(n)}_{t+h|t} = \hat{a}_n + \hat{b}_n \hat{Z}^{A_0(3)}_{t+h|t}$$

where $Z^{A_0(3)}$ is composed of three latent yield factors, backed out from the yields using the method by Chen and Scott (1993). In particular, it is assumed that the 1-month, 1-year and 10-year yield are observed without error. Otherwise the model setup is the same as for the no-arbitrage FAVAR model, but only one lag of the state vector enters the state equation. Moreover, the transition matrix Φ in the state equation is assumed to be lower-triangular and the variance-covariance matrix Ω to be an identity matrix so as to ensure exact identification of the model (see Dai and Singleton (2000) for a discussion of the identification issue in affine TSM). Following Duffee (2002), prices of risk are affine in the state variables $Z^{A_0(3)}$ and not assumed to be driven by the factor volatility. Duffee (2002) provides evidence that this "essentially affine" model yields the best out-of-sample forecast results among a set of different affine term structure model specifications. Moreover, Dai and Singleton (2002) show that risk premia are best captured by the essentially affine model. Notice that since estimating the model involves backing out the latent factors from the yields, estimation is tedious and takes considerably longer than estimation of the no-arbitrage FAVAR and VAR models where the parameters of the state equation are estimated in a first stage of the estimation via OLS.

6. **Random Walk**:

$$\hat{y}^{(n)}_{t+h|t} = y^{(n)}_t$$

Assuming a random walk model for interest rates implies a simple "no-change" forecast of individual yields. Hence, in this model the h-months ahead prediction of an n-maturity bond yield in period t is simply given by its time t observation.

[12] The particular value of λ chosen by Diebold and Li maximizes the curvature loading for a maturity of 30 months. For more details on this choice, the reader is referred to Diebold and Li's paper.

B Forecast Results

The forecasts are carried out over the time period 2000:01-2003:09. The affine models are first estimated over the period 1983:01 - 1999:12 to obtain starting values for the parameters. All models are then estimated recursively using data from 1983:01 to the time that the forecast is made, beginning in 2000:01.

Table VIII summarizes the root mean squared errors obtained from these forecasts. Three main observations can be made. First, the no-arbitrage FAVAR model clearly outperforms the no-arbitrage VAR model for all maturities at all forecast horizons. This implies strong support for the use of a broad macroeconomic information set when forecasting the yield curve based on macroeconomic variables only. Second, at the one month horizon, the essentially affine latent factor only model and the random walk outperform the macro-based FAVAR and VAR models for yields of all maturities, with the random walk being slightly superior for medium and longer maturities (2 to 10 years) and the $A_0(3)$ model performing best for short maturities. Third and most importantly, at the six months and twelve months ahead horizons, the no-arbitrage FAVAR model strongly outperforms all considered benchmark models in forecasting bond yields of all maturities. Hence, modelling macroeconomic and short-rate dynamics jointly within a factor-augmented VAR subject to no-arbitrage restrictions seems to considerably enhance out-of-sample forecasts of yields of all maturities.

The improvement in terms of root mean squared forecast errors is particularly pronounced for short and medium term maturities as table IX documents. It reports RMSEs of all considered models relative to the random walk forecast. At the one-month forecast horizon, all yield-based models outperform the affine models based on macro variables. However, at forecast horizons beyond one month, the no-arbitrage FAVAR model strongly outperforms all other models across the entire spectrum of maturities. Relative to the random walk, the no-arbitrage FAVAR model reduces root mean squared forecast errors up to 50% at the short end of the yield curve and still improves forecast performance of long yields about 20%. Compared to the best performing competitor model, the essentially affine latent factor only model, the improvement is still remarkable. This shows that combining a broad macroeconomic information set and parameter restrictions implied by no-arbitrage within one model delivers a promising tool for forecasting bond yields.[13]

[13]Notice that unreported results from a version of the no-arbitrage FAVAR model with only one lag in the transition equation have shown a slightly worse performance. In particular, this model specification has been outperformed by the random walk at the very long end of the yield curve. Hence, allowing for a relatively rich specification of the joint dynamics of macro factors and the short rate appears to considerably enhance forecast accuracy.

In order to formally assess whether the improvement of the FAVAR model over the benchmark models in terms of forecast error is significant, I apply White's (2000) "reality check" test. This test can be used to evaluate superior predictive ability of a model with respect to one or more benchmark models. Here, I test whether the no-arbitrage FAVAR model has superior predictive accuracy with respect to the five considered benchmark models. The test statistics are reported in table X. Negative figures indicate that the average squared forecast loss of the no-arbitrage FAVAR model is smaller than that of the respective competitor model while positive test statistics indicate the opposite. White (2000) shows how to derive the empirical distribution of the test statistic by means of a block bootstrap of the forecast error series. I perform 1000 block-bootstrap resamples from the prediction error series to compute the significance of forecast improvement.

As we have seen above, at the one-month forecast horizon the FAVAR model outperforms the VAR model, but performs worse than the yield-based forecast models. However, the improvement over the VAR model is significant at almost all maturities. At the 6-months ahead forecast horizon, the no-arbitrage FAVAR model beats all benchmark models at all horizons. Moreover, the improvement with respect to the benchmarks is significant at the 5% level for all maturities. This underlines the observation made above that the model performs considerable better than the best performing competitor, the essentially affine latent factor only model ($A_0(3)$). A similar pattern is found for the 12-months ahead forecasts. The forecast loss of the no-arbitrage FAVAR model is significantly smaller than those of all considered benchmarks at all maturities. Altogether, the evidence suggests that the no-arbitrage FAVAR model is particularly useful in forecasting yields of all maturities at forecast horizons beyond one month, the advantage over benchmark models being particularly strong at the short end of the curve. Hence, augmenting a Factor-Augmented VAR model with tight parameter restrictions implied by the no-arbitrage assumption may lead to significantly improved yield forecasts. Moreover, the fact that the no-arbitrage FAVAR model outperforms a model based on a VAR of four individual macro variables plus the short rate which is otherwise identically specified strongly underscores the usefulness of incorporating a broad macroeconomic information set into term structure analysis.

To summarize, the no-arbitrage FAVAR model exhibits strong relative advantages over a variety of benchmark models which have been documented powerful tools in forecasting the yield curve. The improvement is particularly pronounced for short and medium term maturity yields. Notice that I have not compared the model to alternative affine term structure models which incorporate macro factors such as the models by Ang and Piazzesi (2003) or Hördahl et al (2005). Simultaneously including macro and latent yield curve factors, these models are considerably more cumbersome to estimate than the model presented in this paper and thus a comparison based on recursive out-of-sample forecasts is infeasible. As has already been discussed above, the no-arbitrage FAVAR model has the advantage that the state equation parameters are obtained in a separate step of the estimation procedure, a feature that considerably enhances estimation speed and thus might make the approach more suitable for application in practice.

C How are the Macro Factors Related to Latent Yield Factors?

In order to better understand the source of the strong forecast performance of the no-arbitrage FAVAR model, it is interesting to relate the macro factors to the traditional latent decomposition of yields into level, slope, and curvature. In this section, I thus regress estimates of latent factors onto the macro factors and the short rate. The latent yield factors are computed as the first three principal components of the yields used to estimate the term structure model. Similar to results from previous studies, the first three principal components explain about 90.8%, 6.4% and 1.6% of the total variance of the panel. Following a common practice in the term structure literature I label them "level", "slope", and "curvature". The first three columns of table XI summarize the results of these regressions. The four macro factors and the short-term interest rate explain almost all of the variation in the yield level. The main contribution comes from the short rate, the business cycle related first factor and the inflation-related second factor, but also the remaining macro factors are significant explanatory variables for the yield level. Almost 80% of the variation in the slope of the yield curve are explained by the macro factors. Both the business cycle related first and the inflation-related second factor are positively linked with the slope of the yield curve. This is consistent with the fact that short-term interest rates are expected to rise relative to long-term interest rates in an inflationary environment. Moreover, the short rate has a strongly significant negative coefficient in the slope equation which is consistent with the intuition that rises in the short rate lead to a decreasing yield curve slope. Finally notice that only about 48% of the variation in the curvature of the yield curve are explained by the macro factors. Hence, variations in the relative size of short, medium and long-term yields seem to be the least related to macroeconomic news.

VI Conclusion

This paper presents a model of the term structure of interest rates which is entirely built upon observable macroeconomic information. Instead of relying on a latent factor-based decomposition of interest rates, yields are modelled as affine functions of the short rate and a few factors which capture the bulk of variation in a large number of macroeconomic time series variables. This particular modelling approach which I label a“No-Arbitrage Factor-Augmented Vector Autoregression” is motivated by recent evidence which suggests that factors extracted from large macro panels are powerful predictors of short-term interest rates and measures of output and inflation. Moreover, since monetary policy decisions are likely based on the developments in a variety of economic time series, it is straightforward to model interest rates as a function of the factors which by construction summarize the main sources of economic fluctuation.

The model is estimated in two steps. First, the factors are extracted from a large panel of macroeconomic time series using the principal components-based approach suggested by Stock and Watson (2002) and the parameters governing their joint dynamics with the short-term interest rate are estimated in a VAR. In a second step, the price of risk parameters of the affine term structure model specification are obtained by minimizing the sum of squared fitting errors of the model. This consistent two-step approach makes estimation fast and allows to carry out a recursive out-of-sample forecasting exercise.

Preliminary regressions show that the factors of the model contain information for explaining the monetary policy instrument which is not captured by individual measures of output and inflation. Moreover, unrestricted regressions of yields on the model factors show that common components extracted from the large panel of macroeconomic time series are highly significant explanatory variables for yields. Accordingly, an affine term structure model built upon these factors and the short rate provides a good in-sample fit of the term structure of interest rates. Compared to a model which incorporates the short rate and four individual measures of output and inflation as factors, there is an advantage in using the larger macroeconomic information set. The results from out-of-sample forecasts of yields underpin this finding. The term structure model based on common factors clearly outperforms the model based on individual variables for all maturities at all horizons. Moreover, in forecasts beyond one month ahead the model strongly outperforms a set of yield-based forecast models including the one by Diebold and Li (2005) that has been documented particularly powerful in out-of-sample forecasts over longer horizons, a standard three latent factor essentially affine model, and the random walk. At forecast horizons of six and twelve months ahead the reduction in terms of root mean squared forecast errors relative to the random walk amounts up to 50% for short yields and still is about 20% for very long yields. The improvement in forecast accuracy is shown to be statistically significant for all maturities.

A number of potential extensions to the work carried out in this paper are conceivable. First, since financial markets are assumed to respond quickly to macroeconomic news, the forecast exercise could be done using real-time data. Unfortunately, however, real-time macroeconomic datasets of the size necessary for the use of large-scale factor models are still scarce. Second, to improve on the interpretability of the model, a more structural factor model approach could be applied. Instead of extracting factors from a large cross-section of macroeconomic time series, Belviso and Milani (2005) have recently suggested to extract factors from groups of variables of the same economic category and to use this structural factor-augmented FAVAR model to assess the effect of monetary policy. In such a framework, particular emphasis could be given to factors summarizing agents' expectations of inflation and output developments which have been documented important determinants of long-term yields (see, e.g., Dewachter and Lyrio (2004)). Finally, in principle the model setup employed in this paper can also nicely be used as a tool to disentangle the effects of specific economic shocks on risk premia and the risk-adjusted future path of expected short-term rates.

References

[1] Ang, Andrew and Monika Piazzesi, 2003, A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables, *Journal of Monetary Economics* 50(4), 745787

[2] Ang, Andrew, Monika Piazzesi, and Min Wei, 2005, What does the Yield Curve Tell us about GDP Growth?, *Journal of Econometrics*, forthcoming

[3] Bekaert, Gert, and Robert J. Hodrick, 2001, Expectations Hypotheses Tests, *Journal of Finance* 56, 1357-1394

[4] Belviso, Francesco, and Fabio Milani, 2005, Structural Factor-Augmented VAR (SFAVAR) and the Effects of Monetary Policy, *mimeo* Princeton University

[5] Bernanke, Ben S., and Jean Boivin, 2003, Monetary Policy in a Data-Rich Environment, *Journal of Monetary Economics* 50(3), 525-546

[6] Bernanke, Ben S., Jean Boivin, and Piotr Eliasz, 2005, Measuring the Effects of Monetary Policy: A Factor-Augmented Vector Autoregressive (FAVAR) Approach, *Quarterly Journal of Economics* 120(1), 387-422

[7] Boivin, Jean and Serena Ng, 2005, Understanding and Comparing Factor-Based Forecasts, mimeo, Columbia University

[8] Bliss, Robert, 1997, Testing Term Structure Estimation Methods, *Advances in Futures and Options Research* 9, 197231

[9] Chen, RR, and L. Scott, 1993, Maximum Likelihood Estimation for a Multi-factor Equilibrium Model of the Term Structure of Interest Rates, *Journal of Fixed Income* 3, 14-31

[10] Cochrane, John Y., and Monika Piazzesi, 2005, Bond Risk Premia, *American Economic Review* 95(1), forthcoming

[11] Dai, Qiang, and Thomas Philippon, 2004, Fiscal Policy and the Term Structure of Interest Rates, mimeo, New York University

[12] Davidson, Russell and James G. MacKinnon, 1993, Estimation and Inference in Econometrics, New York, Oxford University Press.

[13] Dewachter, Hans, and Marco Lyrio, 2004, Macro Factors and the Term Structure of Interest Rates, *Journal of Money, Credit, and Banking*, forthcoming

[14] Diebold, Francis X., and Canlin Li, 2005, Forecasting the Term Structure of Government Bond Yields, *Journal of Econometrics*, forthcoming

[15] Diebold, Francis X., Glenn D. Rudebusch, and S. Boragan Aruoba, 2005, The Macroeconomy and the Yield Curve: A Dynamic Latent Factor Approach,*Journal of Econometrics*, forthcoming

[16] Duffie, Donald, and Rui Kan, 1996, A Yield-Fcator Model of Interest Rates, *Mathematical Finance* 6, 379-406

[17] Favero, Carlo A., Marcellino, Massimiliano, and Francesca Neglia, 2005, Principal Components at Work: The Empirical Analysis of Monetary Policy with Large Data Sets, *Journal of Applied Econometrics*, forthcoming

[18] Forni, M., M. Hallin, M. Lippi, and L. Reichlin, 2000, The Generalized Factor Model: Identification and Estimation, *The Review of Economics and Statistics* 82, 540-554

[19] Forni, M., M. Hallin, M. Lippi, and L. Reichlin, 2003, The Generalized Dynamic Factor Model One-Sided Estimation and Forecasting, *mimeo*, Universit Libre de Bruxelles

[20] Giannone, Domenico, Lucrezia Reichlin, and Luca Sala, 2004, Monetary Policy in Real-Time, *NBER Macroeconomics Annual*

[21] Hördahl, Peter, Oreste Tristani, and David Vestin, 2005, A joint econometric model of macroeconomic and term structure dynamics, *Journal of Econometrics*, forthcoming

[22] Knez, P.J., R. Litterman, and J. Scheinkman, 1994, Explorations Into Factors Explaining Money Market Returns, *Journal of Finance* 49(5), 1861-1882

[23] Law, Peyron, 2004, Macro Factors and the Yield Curve, *mimeo*, Stanford University

[24] Mönch, Emanuel, 2004, Explaining the Cross-Section of Returns with Macroeconomic Factors, mimeo, Humboldt University Berlin

[25] Mönch, Emanuel and Harald Uhlig, 2005, Towards a Monthly Business Cycle Chronology for the Euro Area, *Journal of Business Cycle Measurement and Analysis* 2(1), 43-69.

[26] Rudebusch, Glenn D. , and Tao Wu, 2003, A Macro-Finance Model of the Term Structure, Monetary Policy, and the Economy, *Federal Reserve Bank of San Francisco Working Paper* No. 03-17

[27] Stock, James H., and Mark W. Watson, 2002, Macroeconomic Forecasting Using Diffusion Indexes, *Journal of Business and Economic Statistics* 20, 147-162

[28] White, Halbert, 2000, A Reality Check for Data Snooping, *Econometrica* 68(5), 1097-1126

[29] Wu, Tao, 2002, Macro Factors and the Affine Term Structure of Interest Rates, *Federal Reserve Bank of San Francisco Working Paper* No. 02-06

Bond Pricing Parameters

The absence of arbitrage between bonds with different maturities implies the existence of the stochastic discount factor M such that

$$P_t^{(n)} = E_t[M_{t+1}\ P_{t+1}^{(n-1)}],$$

i.e. the price of a n-months to maturity bond today must equal the expected discounted price of an $(n-1)$-months to maturity bond tomorrow. Following Ang and Piazzesi (2003), the derivation of the recursive bond pricing parameters starts with assuming that the nominal pricing kernel M takes the form

$$M_{t+1} = \exp(-r_t - \frac{1}{2}\lambda_t'\Omega\lambda_t - \lambda_t'\nu_{t+1}),$$

and by guessing that bond prices P are exponentially affine in the state variables Z, i.e.

$$P_t^{(n)} = \exp(A_n + B_n' Z_t).$$

Plugging the above expressions for P and M into the first relation, one obtains

$$\begin{aligned} P_t^{(n)} &= E_t[M_{t+1}\ P_{t+1}^{(n-1)}] \\ &= E_t\left[\exp(-r_t - \frac{1}{2}\lambda_t'\Omega\lambda_t - \lambda_t'\nu_{t+1})\exp(A_{n-1} + B_{n-1}'Z_{t+1})\right] \\ &= \exp(-r_t - \frac{1}{2}\lambda_t'\Omega\lambda_t + A_{n-1})\ E_t\left[\exp(-\lambda_t'\nu_{t+1} + B_{n-1}'(\mu + \Phi Z_t + \nu_{t+1}))\right] \\ &= \exp(-r_t - \frac{1}{2}\lambda_t'\Omega\lambda_t + A_{n-1} + B_{n-1}'\mu + B_{n-1}'\Phi Z_t)\ E_t\left[\exp((-\lambda_t' + B_{n-1}')\nu_{t+1})\right] \end{aligned}$$

Since the innovations ν of the state variable process are assumed Gaussian with variance-covariance matrix Ω, it is obvious that

$$\begin{aligned} \ln E_t\left[\exp((-\lambda_t' + B_{n-1}')\nu_{t+1})\right] &= E_t\left[\ln(\exp((-\lambda_t' + B_{n-1}')\nu_{t+1}))\right] + \\ &\quad \frac{1}{2}Var_t\left(\ln(\exp((-\lambda_t' + B_{n-1}')\nu_{t+1}))\right) \\ &= \frac{1}{2}\left[\lambda_t'\Omega\lambda_t - 2B_{n-1}'\Omega\lambda_t + B_{n-1}'\Omega B_{n-1}\right] \\ &= \frac{1}{2}\lambda_t'\Omega\lambda_t - B_{n-1}'\Omega\lambda_t + \frac{1}{2}B_{n-1}'\Omega B_{n-1}. \end{aligned}$$

Hence, $E_t\left[\exp((-\lambda_t' + B_{n-1}')\nu_{t+1})\right] = \exp(\frac{1}{2}\lambda_t'\Omega\lambda_t - B_{n-1}'\Omega\lambda_t + \frac{1}{2}B_{n-1}'\Omega B_{n-1})$ and thus

$$P_{t+1}^{(n)} = \exp(-r_t - \frac{1}{2}\lambda_t'\Omega\lambda_t + A_{n-1} + B_{n-1}'\mu + B_{n-1}'\Phi Z_t + \frac{1}{2}\lambda_t'\Omega\lambda_t - B_{n-1}'\Omega\lambda_t + \frac{1}{2}B_{n-1}'\Omega B_{n-1}).$$

Using the relations $r_t = \delta' Z_t$ and $\lambda_t = \lambda_0 + \lambda_1 Z_t$ and matching coefficients thus yields

$$P_{t+1}^{(n)} = \exp(A_n + B_n' Z_t)$$

where

$$\begin{aligned} A_n &= A_{n-1} + B_{n-1}'(\mu - \Omega\lambda_0) + \frac{1}{2}B_{n-1}'\Omega B_{n-1} \quad \text{and} \\ B_n &= B_{n-1}'(\Phi - \Omega\lambda_1) - \delta'. \end{aligned}$$

Tables and Figures

Table I: **Factor Loadings**

This table summarizes R-squares of univariate regressions of the factors extracted from the panel of macro variables on all individual variables. For each factor, I report the five variables that it is most highly correlated with. Notice that the series have been transformed to be stationary prior to extraction of the factors, i.e. for most variables the regressions correspond to regressions on growth rates. The four factors together explain more than 50% of the total variation in the large panel of macroeconomic time series. The first factor clearly is closely related to output and the second to inflation. The third loads on variables of different economic categories and seems to be leading the business cycle. The fourth factor is again most strongly correlated with inflation and money supply.

Factor 1 - 24.9 % of total variance	R^2
Employment on nonag payrolls: Manufacturing	0.79
Employment on nonag payrolls: Goods-producing	0.77
Capacity Utilization: Total (NAICS)	0.76
Index of IP: Non-energy excl CCS and MVP (NAICS)	0.76
Index of IP: Total	0.76
Factor 2 - 13.3 % of total variance	
CPI: all items (urban)	0.79
CPI: all items less medical care	0.76
CPI: all items less food	0.74
CPI: all items less shelter	0.69
PCE chain weight price index: Total	0.69
Factor 3 - 7.6 % of total variance	
M1 (in mil of current $)	0.49
CPI: medical care	0.47
Inventories: Mfg and Trade: Mfg, durables (mil of chained 96$)	0.41
Loans and Securities @ all comm banks: Securities, U.S. govt (in mil of $)	0.36
Inventories: Mfg and Trade: Mfg (mil of chained 96$)	0.36
Factor 4 - 5.4 % of total variance	
Employment on nonag payrolls: Financial activities	0.33
PPI: finished goods excl food	0.27
PPI: finished consumer goods	0.24
CPI: transportation	0.23
M3 (in mil of current $)	0.23

Table II: **Policy rule based on individual variables**

This table reports estimates for a policy rule with partial adjustment based on individual measures of output and inflation, i.e.

$$r_t = c + \rho r_{t-1} + (1-\rho)(\phi^y y_t + \phi^\pi \pi_t),$$

where r denotes the federal funds rate, y the deviation of log GDP from its trend, and π the annual rate of GDP inflation. The sample period is 1983:01 to 2003:09. Standard errors are in parentheses. The R^2 of this regression is 0.967.

c	ρ	ϕ_y	ϕ_π
-0.011	0.955	1.332	2.592
(0.078)	(0.017)	(0.627)	(0.850)

Table III: **Policy rule based on factors**

This table reports estimates for a policy rule with partial adjustment based on the four factors extracted from a large panel of macroeconomic variables, i.e.

$$r_t = c + \rho r_{t-1} + (1-\rho)(\phi^{F1} F1_t + \phi^{F2} F2_t + \phi^{F3} F3_t + \phi^{F4} F4_t),$$

where r again denotes the federal funds rate and $F1$ to $F4$ the four macro factors extracted from a panel of about 160 monthly time series for the US. The sample period is 1983:01 to 2003:09. Standard errors are in parentheses. The R^2 of this regression is 0.97.

c	ρ	ϕ_{F1}	ϕ_{F2}	ϕ_{F3}	ϕ_{F4}
0.564	0.902	0.174	0.160	-0.004	0.050
(0.152)	(0.025)	(0.031)	(0.049)	(0.025)	(0.030)

Table IV: **Correlation of Model Factors and Yields**

This table summarizes the mutual correlation patterns between the yields and factors used for estimating the term structure model. $F1, F2, F3$ and $F4$ denote the macro factors extracted form the large panel of monthly economic time series for the US. $y^{(1)}$ denotes the federal funds rate.

	$y^{(1)}$	$y^{(3)}$	$y^{(6)}$	$y^{(9)}$	$y^{(12)}$	$y^{(24)}$	$y^{(36)}$	$y^{(48)}$	$y^{(60)}$	$y^{(84)}$	$y^{(120)}$
Correlation of observable factors and yields											
F1	0.392	0.440	0.478	0.499	0.514	0.539	0.545	0.547	0.546	0.545	0.541
F2	0.723	0.733	0.725	0.718	0.712	0.698	0.688	0.678	0.671	0.659	0.649
F3	0.025	0.016	0.014	0.020	0.031	0.093	0.151	0.194	0.223	0.260	0.289
F4	0.296	0.279	0.272	0.268	0.266	0.260	0.254	0.247	0.241	0.232	0.223
$y^{(1)}$	1.000	0.991	0.982	0.975	0.969	0.947	0.925	0.905	0.889	0.865	0.843
Correlation of 1 months lagged observable factors and yields											
F1(-1)	0.441	0.486	0.520	0.539	0.550	0.566	0.567	0.565	0.562	0.557	0.551
F2(-1)	0.706	0.711	0.701	0.693	0.688	0.676	0.668	0.661	0.654	0.644	0.634
F3(-1)	0.004	-0.000	0.001	0.008	0.020	0.085	0.145	0.189	0.220	0.258	0.288
F4(-1)	0.272	0.254	0.250	0.248	0.248	0.246	0.242	0.236	0.231	0.222	0.215
$y^{(1)}(-1)$	0.983	0.979	0.970	0.962	0.956	0.934	0.913	0.894	0.879	0.855	0.832
Correlation of 3 months lagged observable factors and yields											
F1(-3)	0.515	0.551	0.577	0.590	0.596	0.598	0.589	0.580	0.573	0.562	0.552
F2(-3)	0.661	0.664	0.651	0.643	0.638	0.632	0.629	0.626	0.623	0.617	0.611
F3(-3)	-0.024	-0.022	-0.015	-0.006	0.008	0.077	0.139	0.184	0.216	0.254	0.283
F4(-3)	0.244	0.232	0.228	0.227	0.228	0.230	0.227	0.222	0.216	0.207	0.198
$y^{(1)}$(-3)	0.944	0.946	0.935	0.926	0.920	0.902	0.885	0.869	0.856	0.835	0.815
Correlation of 6 months lagged observable factors and yields											
F1(-6)	0.576	0.607	0.627	0.635	0.638	0.632	0.616	0.601	0.589	0.572	0.556
F2(-6)	0.591	0.583	0.567	0.559	0.555	0.558	0.566	0.571	0.575	0.578	0.580
F3(-6)	-0.057	-0.047	-0.035	-0.023	-0.008	0.063	0.125	0.170	0.201	0.239	0.267
F4(-6)	0.221	0.219	0.218	0.217	0.217	0.215	0.209	0.201	0.195	0.185	0.175
$y^{(1)}$(-6)	0.894	0.886	0.872	0.863	0.857	0.849	0.841	0.832	0.823	0.807	0.790
Correlation of 9 months lagged observable factors and yields											
F1(-9)	0.638	0.662	0.675	0.679	0.679	0.663	0.641	0.621	0.606	0.586	0.568
F2(-9)	0.514	0.493	0.473	0.463	0.460	0.473	0.493	0.507	0.517	0.529	0.536
F3(-9)	-0.066	-0.040	-0.019	-0.003	0.014	0.083	0.140	0.180	0.209	0.244	0.271
F4(-9)	0.177	0.180	0.181	0.182	0.182	0.181	0.175	0.166	0.157	0.142	0.127
$y^{(1)}$(-9)	0.815	0.802	0.788	0.781	0.778	0.782	0.786	0.785	0.782	0.774	0.764
Correlation of 12 months lagged observable factors and yields											
F1(-12)	0.656	0.672	0.676	0.675	0.671	0.647	0.621	0.600	0.583	0.560	0.540
F2(-12)	0.431	0.406	0.384	0.376	0.375	0.403	0.436	0.459	0.475	0.492	0.502
F3(-12)	-0.073	-0.037	-0.009	0.009	0.024	0.082	0.129	0.165	0.192	0.227	0.255
F4(-12)	0.169	0.170	0.178	0.184	0.189	0.196	0.191	0.182	0.173	0.159	0.146
$y^{(1)}$(-12)	0.732	0.710	0.696	0.691	0.692	0.710	0.727	0.735	0.738	0.738	0.733

Table V: **Unrestricted VAR of Yields on Factors**

This table summarizes the results of an unrestricted VAR of yields of different maturities on the four macro factors extracted from the panel of economic time series, and the short term interest rate. The estimation period is 1983:01 to 2003:09. t-values are in brackets.

	$y^{(3)}$	$y^{(6)}$	$y^{(12)}$	$y^{(24)}$	$y^{(36)}$	$y^{(48)}$	$y^{(60)}$	$y^{(84)}$	$y^{(120)}$
cst	1.084	1.697	2.458	3.735	4.683	5.348	5.825	6.452	6.985
	[12.081]	[14.331]	[16.281]	[19.931]	[22.204]	[23.459]	[24.178]	[24.818]	[24.955]
F1	0.253	0.429	0.614	0.792	0.885	0.947	0.992	1.055	1.113
	[13.252]	[17.038]	[19.097]	[19.853]	[19.728]	[19.520]	[19.353]	[19.073]	[18.680]
F2	0.314	0.470	0.626	0.824	0.957	1.052	1.124	1.225	1.319
	[10.966]	[12.444]	[12.974]	[13.762]	[14.210]	[14.455]	[14.610]	[14.758]	[14.759]
F3	0.026	0.045	0.108	0.285	0.435	0.540	0.615	0.710	0.787
	[1.806]	[2.399]	[4.470]	[9.505]	[12.878]	[14.811]	[15.945]	[17.056]	[17.577]
F4	0.091	0.149	0.217	0.309	0.369	0.409	0.438	0.476	0.510
	[5.189]	[6.418]	[7.312]	[8.389]	[8.905]	[9.137]	[9.251]	[9.324]	[9.278]
$y^{(1)}$	0.861	0.795	0.718	0.574	0.459	0.376	0.315	0.235	0.166
	[58.071]	[40.613]	[28.766]	[18.529]	[13.164]	[9.965]	[7.909]	[5.460]	[3.592]
$\bar{R}^2$	0.99	0.98	0.98	0.96	0.95	0.94	0.93	0.92	0.91

Table VI: **In-sample Fit: Observed and Model-Implied Yields and Returns**

This table summarizes empirical means and standard deviations of observed and fitted yields and model-implied 1-year holding period returns. Yield are reported in percentage terms and holding period returns are stated in basis points. The first and second row in each panel report the respective moment of observed yields and fitted values implied by the no-arbitrage FAVAR model. The third and fourth row in each panel report the respective moment of observed and model-implied 1-year holding period returns. Note that these are stated in basis points whereas yields are reported in percentage terms.

	1	3	6	9	12	24	36	48	60	84	120
						Mean					
$y^{(n)}$	5.22	5.44	5.62	5.77	5.90	6.31	6.58	6.76	6.89	7.04	7.17
$\hat{y}^{(n)}$	5.22	5.45	5.61	5.75	5.90	6.33	6.57	6.76	6.90	7.04	7.17
$rx^{(n)}$	-	-	-	-	-	6.91	7.75	8.35	8.85	17.00	11.08
$\hat{rx}^{(n)}$	-	-	-	-	-	6.92	7.67	8.37	8.95	16.84	10.78
						Standard Deviation					
$y^{(n)}$	2.12	2.18	2.26	2.30	2.33	2.31	2.28	2.25	2.24	2.23	2.24
$\hat{y}^{(n)}$	2.12	2.12	2.18	2.24	2.28	2.31	2.26	2.21	2.18	2.16	2.17
$rx^{(n)}$	-	-	-	-	-	2.79	3.93	5.08	6.23	8.73	12.45
$\hat{rx}^{(n)}$	-	-	-	-	-	2.67	3.44	4.11	4.76	6.80	8.62

Table VII: **Parameter Estimates for no-Arbitrage FAVAR model**

$$\text{State dynamics}: \; Z_t = \mu + \Phi_1 Z_{t-1} + \ldots \Phi_4 Z_{t-4} + \nu_t, \quad E[\nu_t \nu_t'] = \Omega$$

	μ	Φ_1					Φ_2				
F1	0.084	1.149	0.211	0.025	0.039	-0.007	0.132	-0.271	0.034	0.148	0.078
	(0.128)	(0.108)	(0.153)	(0.114)	(0.062)	(0.053)	(0.165)	(0.237)	(0.145)	(0.083)	(0.072)
F2	-0.104	0.179	1.200	0.007	-0.057	0.006	-0.235	-0.238	-0.053	0.025	0.023
	(0.083)	(0.070)	(0.099)	(0.074)	(0.040)	(0.035)	(0.107)	(0.154)	(0.095)	(0.054)	(0.047)
F3	0.132	-0.213	-0.056	0.900	0.023	-0.054	0.040	-0.098	0.158	0.017	-0.023
	(0.094)	(0.079)	(0.113)	(0.084)	(0.045)	(0.039)	(0.122)	(0.174)	(0.107)	(0.061)	(0.053)
F4	-0.216	-0.384	-0.792	-0.142	0.893	0.041	0.058	0.650	0.057	-0.268	-0.139
	(0.164)	(0.138)	(0.197)	(0.146)	(0.079)	(0.069)	(0.212)	(0.304)	(0.187)	(0.107)	(0.093)
$y^{(1)}$	0.428	0.341	0.451	0.075	0.045	0.929	-0.094	-0.581	-0.361	0.057	-0.120
	(0.148)	(0.125)	(0.177)	(0.132)	(0.071)	(0.062)	(0.192)	(0.274)	(0.169)	(0.096)	(0.084)
		Φ_3					Φ_4				
F1		-0.621	0.113	-0.055	-0.119	0.035	0.251	-0.046	0.062	-0.018	-0.120
		(0.163)	(0.235)	(0.146)	(0.084)	(0.072)	(0.122)	(0.158)	(0.103)	(0.059)	(0.052)
F2		0.142	-0.018	0.128	-0.033	-0.047	-0.016	-0.000	-0.102	0.027	0.037
		(0.106)	(0.153)	(0.095)	(0.054)	(0.047)	(0.079)	(0.103)	(0.067)	(0.038)	(0.034)
F3		0.217	0.235	-0.432	0.053	0.066	-0.120	0.034	0.299	-0.014	-0.018
		(0.120)	(0.173)	(0.108)	(0.062)	(0.053)	(0.090)	(0.116)	(0.076)	(0.044)	(0.039)
F4		0.283	-0.309	0.139	-0.129	-0.020	0.206	0.367	-0.067	0.329	0.153
		(0.210)	(0.302)	(0.187)	(0.108)	(0.093)	(0.156)	(0.203)	(0.132)	(0.076)	(0.067)
$y^{(1)}$		0.038	0.368	0.246	-0.007	-0.130	-0.117	-0.097	-0.024	-0.049	0.233
		(0.189)	(0.272)	(0.169)	(0.097)	(0.084)	(0.141)	(0.183)	(0.119)	(0.069)	(0.061)
		Ω									
F1		0.086									
		(0.008)									
F2		-0.036	0.036								
		(0.004)	(0.003)								
F3		0.036	-0.027	0.047							
		(0.005)	(0.003)	(0.004)							
F4		-0.062	0.009	-0.013	0.142						
		(0.008)	(0.005)	(0.005)	(0.013)						
$y^{(1)}$		0.005	-0.000	-0.002	-0.003	0.116					
		(0.006)	(0.004)	(0.005)	(0.008)	(0.011)					

$$\text{Market prices of risk}: \; \lambda_t = \lambda_0 + \lambda_1 Z_t$$

$\tilde{\lambda}_0$	$\tilde{\lambda}_1$				
-29.535	1.536	-1.241	-1.701	-	-3.701
(0.038)	(0.724)	(0.172)	(0.624)	-	(1.550)
-290.060	-1.420	-4.239	-1.202	-0.347	-1.076
(0.034)	(0.266)	(0.044)	(0.113)	(0.076)	(0.061)
-141.987	-2.407	-	1.217	-	3.649
(0.018)	(1.078)	-	(0.266)	-	(0.964)
-52.033	-	-	-1.821	1.090	-5.523
(0.013)	-	-	(0.146)	(0.712)	(0.010)
-3.113	-	-	-	-	-
(0.081)	-	-	-	-	-

Table VIII: **Out-of-sample Yield Forecasts : RMSEs**

This table summarizes the root mean squared errors of out-of-sample yield forecasts. The models have been estimated using data from 1983:01 until 1999:12. The forecasting period is 2000:01-2003:09. "No-A FAVAR" refers to an essentially affine term structure model using as states four factors extracted from a large macro panel and the short rate; "No-A VAR" refers to an essentially affine model with IP growth, the index of help-wanted adds in newspapers, CPI growth, PPI growth and the short rate in the state vector. "VAR yields" refers to a VAR(1) on yield levels, "Diebold-Li" denotes the Diebold-Li version of the three-factor Nelson-Siegel model, "$A_0(3)$" the essentially affine three latent factor only model, and "Random Walk" refers to a simple no-change random walk forecast.

	no-A FAVAR	no-A-VAR	VAR yields	Diebold-Li	$A_0(3)$	Random Walk
			Panel A: 1-month ahead forecasts			
1	0.759	0.784	0.340	0.363	**0.336**	0.412
3	0.650	0.607	0.223	0.298	**0.218**	0.267
6	0.654	0.667	0.231	0.353	**0.207**	0.255
9	0.619	0.659	0.263	0.410	**0.237**	0.268
12	0.624	0.669	0.289	0.436	**0.270**	0.282
24	0.612	0.844	0.332	0.394	0.351	**0.313**
36	0.587	0.963	0.351	0.367	0.434	**0.331**
48	0.596	0.957	0.367	0.371	0.460	**0.347**
60	0.609	0.952	0.383	0.385	0.451	**0.361**
84	0.564	0.907	0.410	0.419	0.422	**0.384**
120	0.532	0.895	0.441	0.464	0.407	**0.407**
			Panel B: 6-month ahead forecasts			
1	**0.561**	0.699	1.065	1.213	0.789	1.202
3	**0.493**	0.698	1.123	1.240	0.851	1.147
6	**0.565**	0.777	1.219	1.316	0.916	1.127
9	**0.645**	0.884	1.288	1.369	0.973	1.112
12	**0.692**	0.989	1.322	1.383	1.001	1.095
24	**0.711**	1.116	1.262	1.262	0.930	1.012
36	**0.721**	1.195	1.164	1.144	0.856	0.955
48	**0.736**	1.236	1.105	1.091	0.841	0.929
60	**0.735**	1.251	1.075	1.073	0.848	0.921
84	**0.740**	1.252	1.051	1.073	0.848	0.924
120	**0.716**	1.203	1.045	1.088	0.950	0.937
			Panel C: 12-month ahead forecasts			
1	**0.995**	1.343	2.116	2.095	1.626	2.093
3	**1.056**	1.508	2.301	2.141	1.730	2.122
6	**1.185**	1.587	2.476	2.267	1.816	2.140
9	**1.321**	1.735	2.561	2.346	1.867	2.120
12	**1.345**	1.850	2.572	2.366	1.873	2.069
24	**1.226**	1.860	2.321	2.178	1.641	1.787
36	**1.181**	1.802	2.054	1.976	1.419	1.584
48	**1.139**	1.804	1.887	1.859	1.324	1.478
60	**1.086**	1.810	1.788	1.796	1.302	1.425
84	**1.120**	1.808	1.688	1.742	1.280	1.386
120	**1.098**	1.780	1.627	1.715	1.417	1.376

Table IX: **RMSEs Relative to Random Walk**

This table summarizes the root mean squared errors of out-of-sample yield forecasts relative to the simple random walk forecasts. The models have been estimated using data from 1983:01 until 1999:12. The forecasting period is 2000:01-2003:09. "No-A FAVAR" refers to an essentially affine term structure model using as states four factors extracted from a large macro panel and the short rate; "No-A VAR" refers to an essentially affine model with IP growth, the index of help-wanted adds in newspapers, CPI growth, PPI growth and the short rate in the state vector. "VAR yields" refers to a VAR(1) on yield levels, "Diebold-Li" denotes the Diebold-Li version of the three-factor Nelson-Siegel model and "$A_0(3)$" the essentially affine three latent factor only model.

	no-A FAVAR	no-A-VAR	VAR yields	Diebold-Li	$A_0(3)$
Panel A: 1-month ahead forecasts					
1	1.842	1.904	0.827	0.881	**0.816**
3	2.437	2.277	0.837	1.117	**0.818**
6	2.559	2.611	0.904	1.381	**0.811**
9	2.307	2.456	0.978	1.526	**0.883**
12	2.210	2.369	1.024	1.544	**0.957**
24	1.959	2.698	**1.063**	1.260	1.123
36	1.773	2.910	**1.061**	1.108	1.310
48	1.717	2.758	**1.057**	1.069	1.326
60	1.685	2.637	**1.059**	1.064	1.247
84	1.467	2.361	**1.067**	1.090	1.097
120	1.309	2.202	1.084	1.142	**1.000**
Panel B: 6-month ahead forecasts					
1	**0.467**	0.582	0.886	1.009	0.656
3	**0.430**	0.608	0.979	1.082	0.742
6	**0.501**	0.689	1.081	1.168	0.812
9	**0.579**	0.795	1.158	1.230	0.874
12	**0.632**	0.904	1.208	1.264	0.914
24	**0.702**	1.103	1.248	1.247	0.919
36	**0.755**	1.252	1.219	1.199	0.897
48	**0.792**	1.330	1.189	1.174	0.905
60	**0.798**	1.359	1.167	1.165	0.920
84	**0.801**	1.354	1.137	1.161	0.918
120	**0.764**	1.284	1.115	1.161	1.014
Panel C: 12-month ahead forecasts					
1	**0.476**	0.642	1.011	1.001	0.777
3	**0.498**	0.711	1.085	1.009	0.816
6	**0.554**	0.742	1.157	1.059	0.848
9	**0.623**	0.818	1.208	1.107	0.881
12	**0.650**	0.894	1.243	1.143	0.905
24	**0.686**	1.041	1.299	1.219	0.918
36	**0.746**	1.138	1.297	1.247	0.896
48	**0.771**	1.221	1.277	1.258	0.896
60	**0.762**	1.270	1.255	1.261	0.914
84	**0.808**	1.305	1.218	1.257	0.923
120	**0.799**	1.294	1.183	1.247	1.030

Table X: **White's reality check test statistics**

This table summarizes "Whites Reality Check" test statistics based on a squared forecast error loss function. I choose the no-arbitrage FAVAR model as the benchmark model and compare it bilaterally with the competitor models. Negative test statistics indicate that the average squared forecast loss of the "no-A FAVAR" model is smaller than that of the respective competitor model. Bold figures indicate significance at the 5% interval. Significance is checked by comparing the average forecast loss differential with the 5% percentile of the empirical distribution of the loss differential series approximated by applying a block bootstrap with 1000 resamples and a smoothing parameter of 1/12. Bold figures highlight significance at the 5% level.

	VAR	VARylds	DL	$A_0(3)$	RW
			Panel : 1-month ahead forecasts		
1	**-0.218**	3.064	2.967	3.088	2.708
3	0.401	2.537	2.276	2.554	2.392
6	-0.091	2.543	2.063	2.611	2.462
9	**-0.292**	2.177	1.515	2.259	2.154
12	**-0.355**	2.108	1.397	2.175	2.130
24	**-2.244**	1.812	1.515	1.728	1.892
36	**-3.862**	1.553	1.478	1.134	1.640
48	**-3.710**	1.554	1.534	1.057	1.645
60	**-3.563**	1.566	1.554	1.204	1.667
84	**-3.340**	1.091	1.038	1.028	1.224
120	**-3.417**	0.671	0.520	0.856	0.859
-0.723	**-0.726**				
			Panel : 6-month ahead forecasts		
1	**-1.064**	**-4.996**	**-7.109**	**-1.881**	**-6.938**
3	**-1.491**	**-6.198**	**-7.971**	**-2.944**	**-6.570**
6	**-1.747**	**-7.102**	**-8.705**	**-3.184**	**-5.825**
9	**-2.245**	**-7.578**	**-8.969**	**-3.253**	**-5.022**
12	**-3.070**	**-7.725**	**-8.818**	**-3.209**	**-4.395**
24	**-4.574**	**-6.621**	**-6.639**	**-2.194**	**-3.156**
36	**-5.606**	**-5.066**	**-4.798**	**-1.285**	**-2.369**
48	**-6.079**	**-4.121**	**-3.930**	**-0.984**	**-1.939**
60	**-6.324**	**-3.729**	**-3.701**	**-1.071**	**-1.863**
84	**-6.279**	**-3.386**	**-3.656**	**-1.031**	**-1.858**
120	**-5.748**	**-3.537**	**-4.081**	**-2.371**	**-2.229**
-3.243	**-2.847**				
			Panel : 12-month ahead forecasts		
1	**-4.491**	**-19.400**	**-19.110**	**-9.214**	**-18.973**
3	**-6.448**	**-23.301**	**-19.471**	**-10.487**	**-18.904**
6	**-6.236**	**-26.380**	**-20.876**	**-10.553**	**-17.702**
9	**-7.098**	**-26.890**	**-20.971**	**-9.717**	**-15.335**
12	**-9.065**	**-26.910**	**-21.130**	**-9.487**	**-13.841**
24	**-11.105**	**-21.837**	**-18.076**	**-6.705**	**-9.563**
36	**-10.617**	**-15.928**	**-13.999**	**-3.519**	**-6.357**
48	**-11.224**	**-12.776**	**-12.047**	**-2.614**	**-5.073**
60	**-12.038**	**-11.404**	**-11.446**	**-2.961**	**-4.884**
84	**-11.619**	**-9.019**	**-9.949**	**-2.191**	**-3.840**
120	**-11.323**	**-8.159**	**-9.711**	**-4.536**	**-3.954**

Table XI: **Regression of Latent Yield Factors on the Model Factors**

This table summarizes the results obtained from a regression of level, slope, and curvature yield factors onto the factors of the FAVAR model. Level, slope, and curvature are computed as the first three principal components extracted from the yields used to estimate the term structure model. They explain 90.8%, 6.4% and 1.6% of the total variance of all yields, respectively. The sample period is 1984:01-2003:9. t-statistics are in brackets.

	Level	Slope	Curvature
C	0,040	0,244	-0,145
	[22.769]	[18.712]	[-7.103]
F1	0,007	0,032	-0,058
	[20.133]	[11.828]	[-13.783]
F2	0,008	0,038	-0,049
	[14.400]	[9.215]	[-7.491]
F3	0,004	0,037	0,009
	[13.880]	[16.737]	[2.431]
F4	0,003	0,016	-0,017
	[8.832]	[6.309]	[-4.245]
$y^{(1)}$	0,005	-0,041	0,024
	[15.617]	[-18.643]	[7.011]
$\bar{R}^2$	0,959	0,786	0,481

Figure 1: **Plots of Model Factors**

This figure provides a plot of the factors used in the no-arbitrage FAVAR model. Each factor is confronted with an individual macroeconomic variable in order to show the close correspondence to the real and the nominal side of the economy.

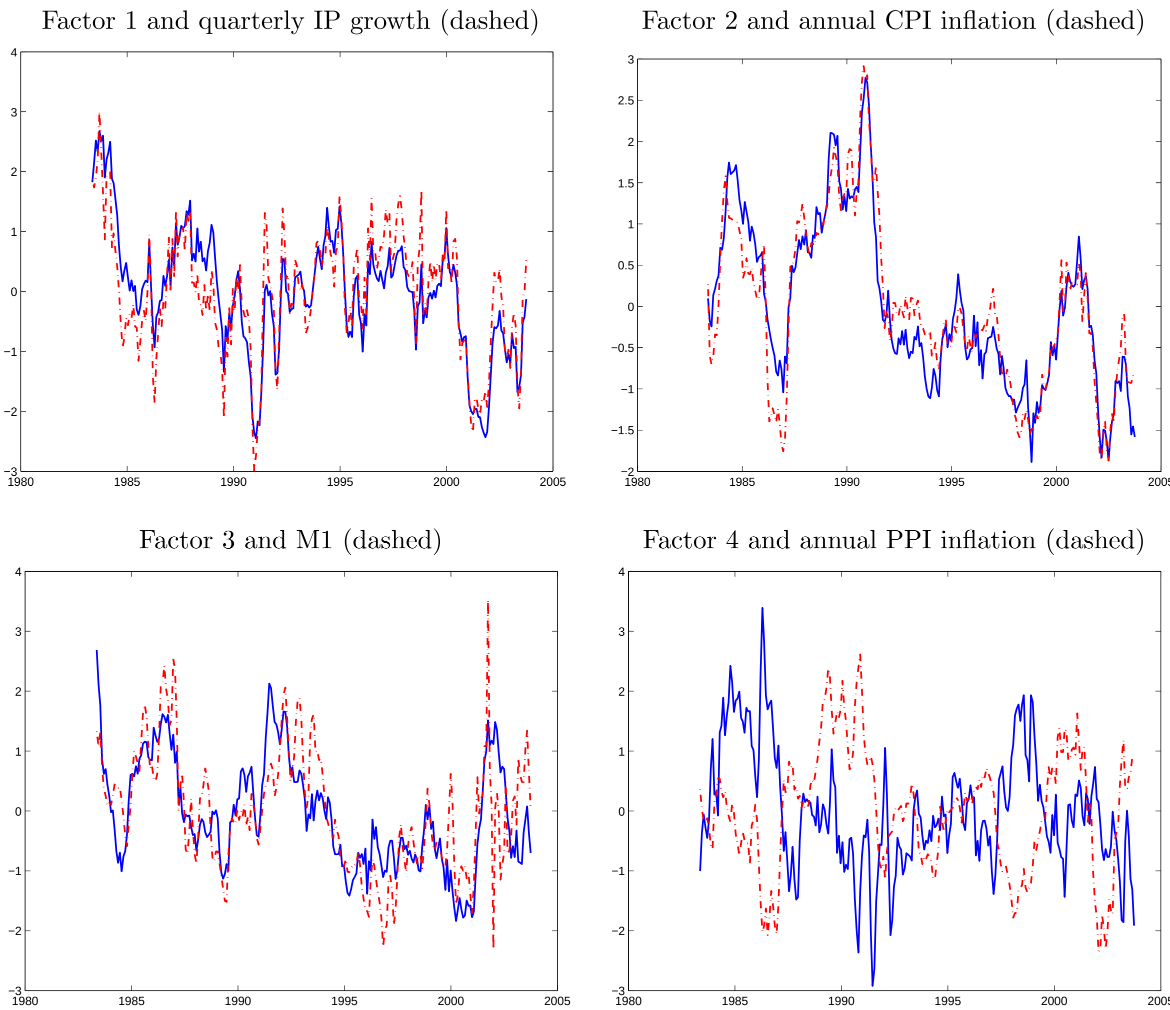


Figure 2: **Observed and Model-Implied Yields**

This table provides plots of the observed and model-implied time series for four selected interest rates, the 6-months yield, the 12-months yield and the 3 and 10-years yields.

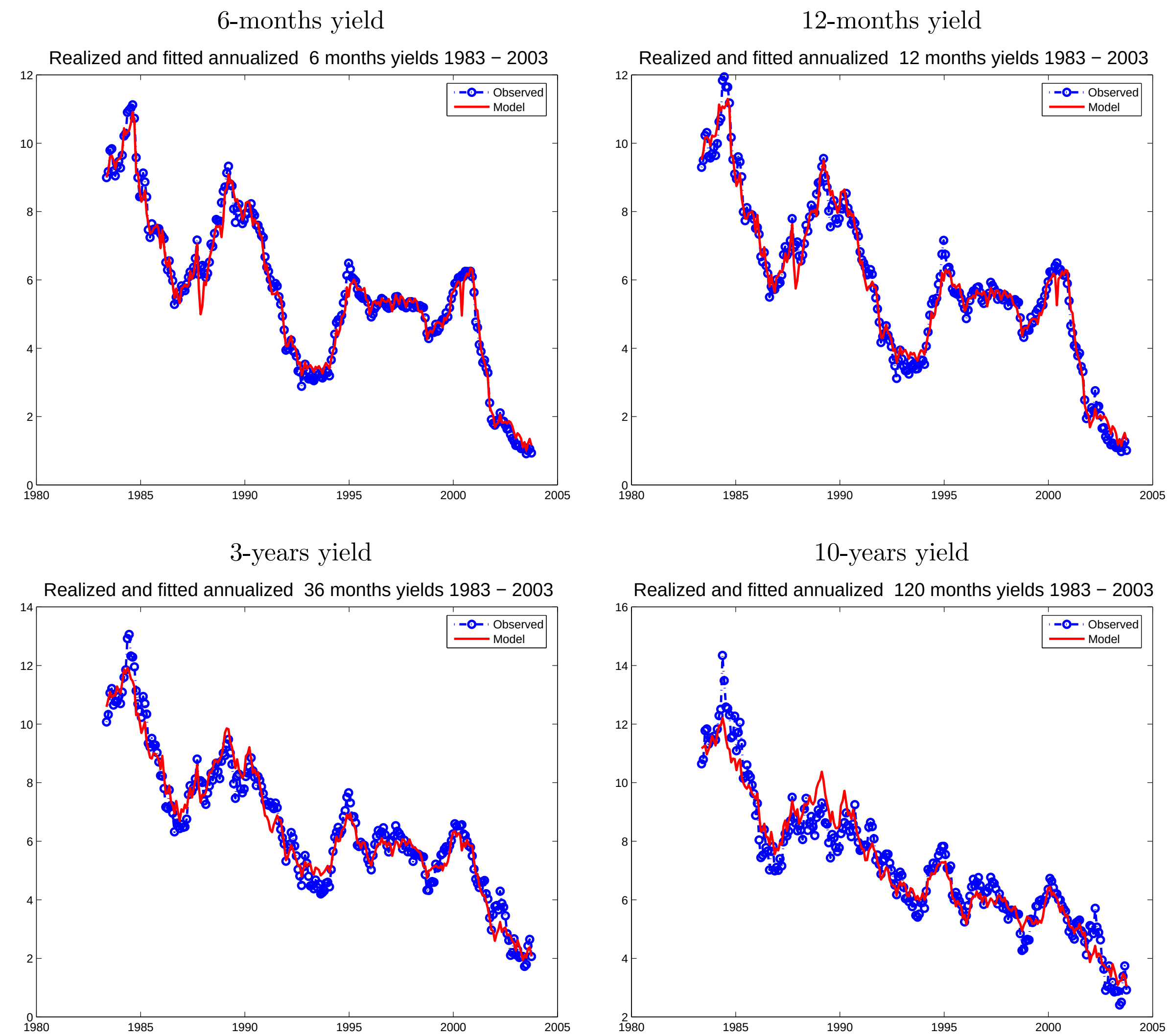


Figure 3: **Observed and Model Implied Average Yield Curve**

This figure provides a plot of observed yields (averaged across time) against those implied by the no-arbitrage FAVAR model. Visibly, the model gives a good fit to the actual yield curve.

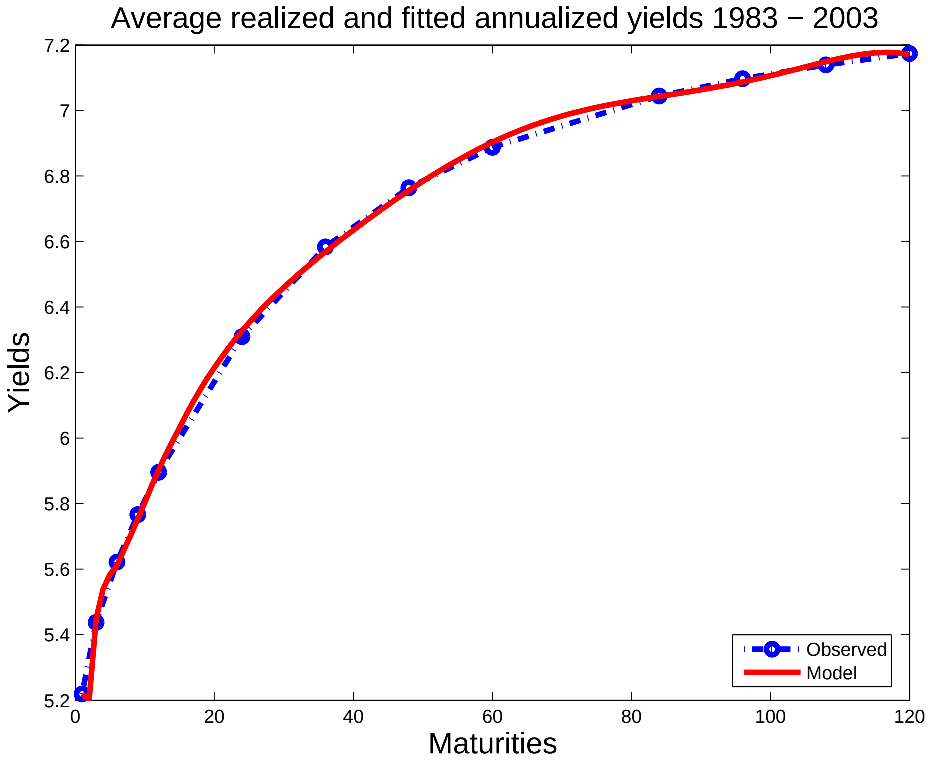


Figure 4: **Implied Yield Loadings**

This figure provides a plot of the yield loadings b_n implied by the no-arbitrage FAVAR model. The coefficients can be interpreted as the response of the n-month yield to a contemporary shock to the respective factor.

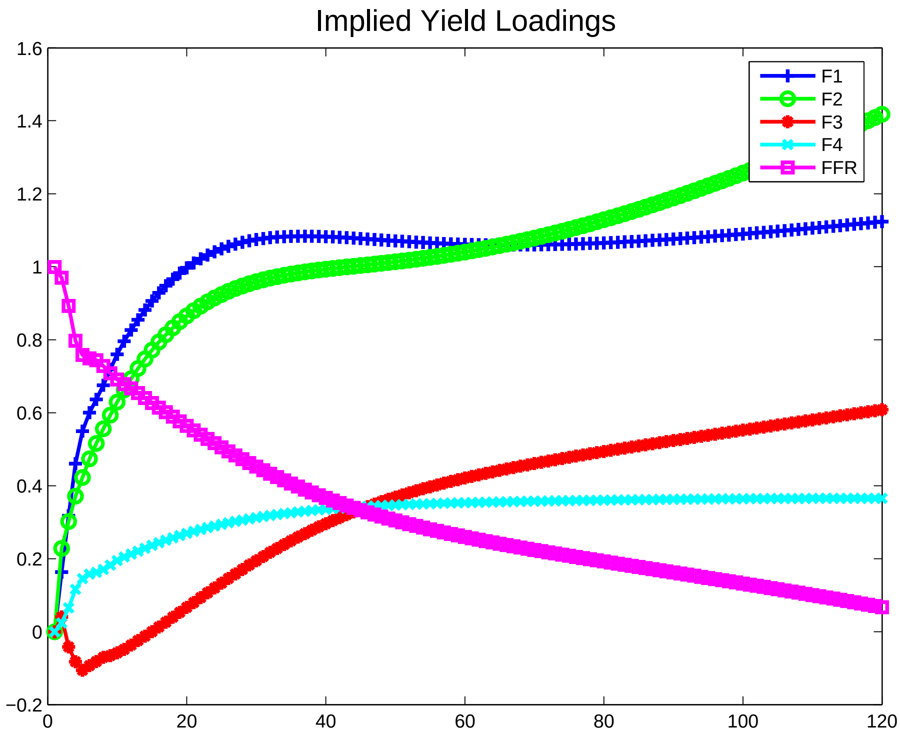
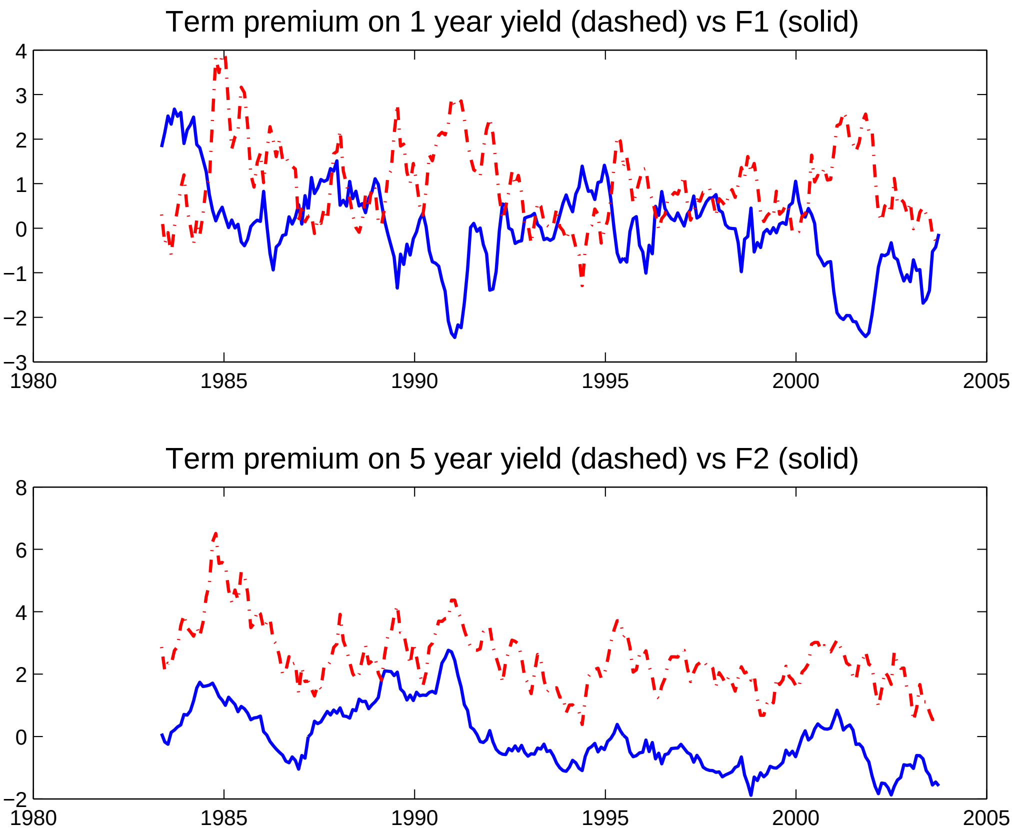


Figure 5: **Risk Premia Dynamics**

This figure provides a plot of the term premia for 2-year and 5-year yields as implied by the no-arbitrage FAVAR model. Both are related to the first and second model factor, respectively.



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