LIQUIDITY, INFORMATION, AND THE OVERNIGHT RATE

by Christian Ewerhart, Nuno Cassola, Steen Ejerskov and Natacha Valla
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Abstract

We model the interbank market for overnight credit with heterogeneous banks and asymmetric information. An unsophisticated bank just trades to compensate its liquidity imbalance, while a sophisticated bank will exploit its private information about the liquidity situation in the market. It is shown that with positive probability, the liquidity effect (Hamilton, 1997) is reversed, i.e., a liquidity drainage from the banking system may generate an overall decrease in the market rate. The phenomenon does not disappear when the number of banks increases. We also show that private information mitigates the effect of an unexpected liquidity shock on the market rate, suggesting a conservative information policy from a central bank perspective.

Keywords: Liquidity effect, asymmetric information, monetary policy implementation
JEL classification: G14, G21, E52
Non-technical summary

The liquidity effect (Hamilton, 1997) predicts a negative correlation between liquidity in the interbank market and short-term interest rates. This paper is concerned with the possibility of a reversal of the liquidity effect in the money market. This means that a low overnight rate may be associated with a scarce liquidity situation, or correspondingly that a high overnight rate may be associated with ample liquidity in the interbank market.

The formal analysis considers a model in the Kyle (1985) tradition. There are two types of credit institutions, unsophisticated and sophisticated banks. An unsophisticated bank is assumed to trade away its current account imbalance, while a sophisticated bank acts strategically in the money market. As a robust phenomenon, we find that liquidity drainage from the banking system may cause a decrease in the market rate, and that a liquidity inflow into the banking system may cause an increase in the market rate. The effect is driven by incentives for sophisticated banks to delay the balancing of their reserve accounts. Somewhat unexpectedly, the phenomenon is present also in arbitrarily large markets.

It is shown that the reversal of the liquidity effect reduces the expected quadratic deviation from the target rate. In this sense, the informational inefficiency of the market can be supportive to the objective of steering interest rates to a specific target. Our findings suggest that the publication of aggregate liquidity information, if not in the context of a refinancing operation, could be detrimental to the objectives of monetary policy implementation.
1. Introduction

Cash market parlance as well as empirical evidence suggest the existence of a negative correlation between the daily rate for overnight credit in the interbank market on the one hand and the aggregate outstanding liquidity in the banking system on the other. This effect, known as the liquidity effect (Hamilton [7]), will be recognized as a variation of the more conventional theme that relatively scarcer commodities tend to be traded at relatively higher prices.

In this paper, we point out the theoretical possibility of a liquidity effect bearing a reversed sign. In fact, that short-term interest rates do not always reflect liquidity conditions in the interbank market is a phenomenon that, while still undocumented in the empirical literature, apparently is not new to central bankers in various currency areas. E.g., in the case of the euro area, the final days of the reserve maintenance period March 24 to April 23, 2003 were characterized by ample liquidity conditions. This can be inferred from the 4 bn euro recourse to the deposit facility on the last day of the maintenance period, shown in Figure 1. Still, as exhibited in Figure 2, the money market index EONIA stayed well above the target rate of 2.50 % for most of the last two days. The reader will realize that if the market rate would reflect liquidity conditions properly, this should not be feasible: the market must be informationally inefficient (Grossman and Stiglitz [6]).

In the formal analysis, we consider an interbank market with heterogeneous banks and asymmetric observability of individual liquidity positions. It is shown that an aggregate liquidity inflow into (drainage from) the banking system may be associated by an increase (decrease) in the market rate. As the analysis shows, this possibility is created by adverse incentives for some banks in the market. Specifically, there is an incentive to actively speculate on the market rate in order to exploit valuable private information about liquidity flows. As a consequence of such strategic behavior, the impounding of information into prices is delayed, and the overnight rate may give an incorrect signal of the aggregate liquidity situation in the market.
The present paper can be considered as an application of a line of research that was originated by Kyle [9]. This literature tackled the conceptual difficulties of modeling markets with asymmetrically informed participants by using the metaphor of a market maker who is observing only an aggregate of the order flow and who takes the residual position to clear the market. What is new in our approach is that it takes account of the specific information structure in the interbank market arising from the fact that banks can usually observe only liquidity flows that run through their own balance sheet.

The first model that incorporated asymmetric information in the context of reserve management was Campbell [3], who studied the announcement effect of macro data on the Federal funds rate. The paper considers a finite population of commercial banks which have to satisfy reserve requirements on average over a two-day statement period. Individual banks may have a preference for early fulfilment. However, this information is not observable for the other banks. Campbell argues that under informational asymmetry, individual banks mistake the stronger demand caused by higher reserve requirements (with should affect rates on different days in the same way) for a widespread preference towards early fulfilment (which affects rates on different days in a heterogeneous manner). As a consequence, the public release of information about aggregate reserve requirements may affect the market rate stronger under informational asymmetry than under complete information.4

Our set-up differs from Campbell’s in that we allow for a heterogeneous population of commercial banks. The assumption will be that some banks are less professional in managing their reserves than others. Specifically, it is assumed that “unsophisticated” banks ignore the averaging condition of reserve requirements and trade in the interbank market to immediately adjust

---

4The literature on optimal reserve management by commercial banks, initiated by contributions by Poole [10], Ho and Saunders [8], and others, is sometimes confounded in a potentially misleading way with another strand of literature that focusses on issues such as the insurance motive of interbank trading, the public good property of holding liquid assets, and the problem of systemic risk. Early contributions in this vein are Bhattacharya and Gale [2], and Bhattacharya and Fulghieri [1]. See DeBandt and Hartmann [4] for a survey of this literature.
their reserve balance to the required level. By following this simple rule, these banks never build up positions, and avoid any speculation on short-term interest rates. With some banks managing their reserves in a more defensive way than others, however, a liquidity shock affecting the whole system may generate heterogeneous reactions by individual banks. The purpose of this paper is to study the consequences of this behavioral heterogeneity on the statistics of the market rate.

The rest of the paper is structured as follows. Section 2 deals with the case of a single sophisticated bank. Section 3 discusses volatility and variance of the market rate. In Section 4, we extend our analysis to the case of an arbitrary number of sophisticated banks. Section 5 summarizes and concludes. The appendix contains proofs of the formal results.

2. A single sophisticated bank

The model follows the Kyle [9] tradition, yet with a number of modifications that reflect the institutional specifics of liquidity management. As will become apparent, the main difference to the established framework is the information structure. The interpretation also differs slightly from the traditional framework. In particular, the traditional noise traders in the microstructure tradition have here the interpretation of behaviorally unsophisticated banks. To ease exposition, we will start with the case of a single sophisticated bank. In fact, while this example allows an interpretation with finitely many unsophisticated banks, we will, for the sake of simplicity, also assume only one unsophisticated bank.5

The example has the following set-up. Three counterparties participate in the trading protocol of the money market, bank A, bank B, and a market maker. There are three dates. On day 0, the market rate is \( r_0 \). In the morning of day 1, the liquidity managers of bank A and B are individually informed about their idiosyncratic liquidity positions, and choose an order

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5The choice of the Kyle framework is made mainly for analytical convenience. There should be no major difficulty replicating our results in a Campbell-style model.
volume. The market maker observes the aggregate order volume, sets a price for day 1, and clears the market to break even. Finally, on day 2, the liquidity situation becomes public information.

Between dates 0 and 1, there is a liquidity flow between each individual bank and the non-bank sector caused by the autonomous deposit and withdrawal decisions by individual bank customers. In addition, there is a flow between bank A and B caused by money transfers. We denote the flow from bank A to the non-bank sector by $z_A$, the flow from bank B to the non-bank sector by $z_B$, and the flow from bank A to bank B by $y_{AB}$, where flows can be negative. It is assumed that the components of the shock, i.e., $z_A$, $z_B$, and $y_{AB}$ are normally and independently distributed, with expected values

$$E[z_A] = E[z_B] = 0, E[y_{AB}] = \overline{y},$$

and respective variances $\sigma_A^2$, $\sigma_B^2$, and $\sigma_y^2$. Fixing the expected flows between individual commercial banks and the non-bank sector to zero is a mere normalization. On the other hand, a systematic flow of liquidity $\overline{y}$ between heterogeneous banks is consistent e.g. with the well-documented empirical fact that smaller bank tend to be net providers of liquidity in the interbank market (cf. [8]). Aggregate autonomous factors are given by

$$z = z_A + z_B,$$

where, following a widely used convention, a positive sign indicates a liquidity drainage from the banking system, and a negative sign a liquidity inflow.

We assume that under symmetric and complete information about aggregate liquidity conditions $z$, market participants expect the overnight rate to be $r(z)$, where $r'(z) > 0$.\textsuperscript{6} To keep the model tractable, we will use the first-order approximation

$$r(z) = r_0 + \rho z,$$

(1)

\textsuperscript{6}E.g., in a corridor system, the liquidity situation at any given point of time after the last open market operation in a given reserve maintenance period will be indicative about the relative likelihood of reaching the top or bottom of the interest rate corridor. Invoking the martingale hypothesis then generates the suggested behavior (cf. Woodford [12]).
where $\rho > 0$ is an exogenous parameter measuring the liquidity effect.

The realization of the liquidity shock at time 1 is observed by an individual banks as a change to its respective reserve account balance. Thus, bank A observes a liquidity outflow of

$$\tilde{z}_A = z_A + y_{AB},$$

while bank B observes

$$\tilde{z}_B = z_B - y_{AB}.$$

Banks are heterogeneous. Specifically, bank A is assumed to possess a sophisticated liquidity management, and to choose an order volume $x_A$ so as to minimize net funding costs. In contrast, bank B is unsophisticated, and just trades away any temporary imbalance on the reserve account. Thus, the order volume of bank B will amount to

$$x_B = z_B - y_{AB}.$$

This kind of heterogeneous behavior is suggested by descriptive studies of the money market (cf. [5], [11]).

An alternative justification is that banks may have position targets, which enter the individual bank’s objective function (cf. Campbell [3]). A natural target position for a bank could be defined in terms of having a balanced reserve account. The relative weight that the objective of staying close to the target obtains in the bank’s objective function reflects then the willingness to trade in the money market for speculative reasons. Our model replaces the continuum of possible relative weights by just two extreme cases: the sophisticated bank gives zero weight to the position target, while the unsophisticated bank gives full weight to the position target.

\footnote{In an alternative interpretation, bank B represents an aggregate of several unsophisticated banks, and the liquidity variables $z_B$ and $y_{AB}$ represent the respective aggregate net liquidity flows.}
Aggregate liquidity demand is then $x = x_A + x_B$. The market maker observes $x$, and determines the competitive zero-profit market rate\(^8\)

$$\tilde{r}(x) = E[r_0 + \rho z|x].$$

**Proposition 1.** An equilibrium in the interbank market for overnight credit is constituted by the strategies

$$x_A(\tilde{z}_A) = \beta(\tilde{z}_A - \overline{y})$$

$$\tilde{r}(x) = r_0 + \lambda(x + \overline{y}),$$

where $\beta \in (0; 1)$, and

$$\lambda = \frac{\beta \sigma^2_A + \sigma^2_B}{\beta^2 \sigma^2_A + \sigma^2_B + (1 - \beta)^2 \sigma^2_B}.$$

**Proof.** See the appendix. \(\Box\)

It can be seen from Proposition 1 that the sophisticated bank never fully accommodates its liquidity demand, i.e., $\beta < 1$. Instead, it hides some of its excess liquidity or some of its liquidity deficit in order to not fully reveal its liquidity situation to the market. In addition, the sophisticated bank takes account of the fact that its reserve balance is distorted by the expected flow to bank B. We will now derive conditions under which the strategic behavior of the sophisticated bank implies a reversal of the liquidity effect.\(^9\)

A shock $(z_A, z_B, y_{AB})$ to the banking system will be referred to as *liquidity-absorbing* if $z_A + z_B > 0$. (Recall our earlier sign convention for autonomous

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\(^8\)The reader might ask himself why we have to replace the general price mechanism by the somewhat specific institution of a market maker. Note, however, that under asymmetric information, individual traders have little incentive to reveal their true willingness to trade. As a consequence, the Walrasian auctioneer, announcing prices in order to elicit demand and supply, may be unable to clear the market in the usual tatonnement process.

\(^9\)While we have not stressed this point so far, the model is perfectly consistent with an interpretation where individual banks have to satisfy reserve requirements on average over a two-day statement period. To see why, note that at the very end of day 2, typically only one side of the market can be satisfied, and the residual demand or supply will be cleared by usage of central bank facilities. E.g., if aggregate demand exceeds supply, all banks may deposit excess liquidity in the interbank market. However, some banks may end up with a reserve deficit, and will have recourse to the central bank’s lending facility.
factors). It should be clear that the interbank flow \( y_{AB} \) does not appear in this definition because it does not affect aggregate liquidity conditions. From Proposition 1, we know that a shock induces a decline in the equilibrium rate if the aggregate order volume is below average, i.e., if

\[
x_A(z_A) + x_B(z_B) < -\beta.
\]

This latter condition is equivalent, again by Proposition 1, to

\[
\beta(z_A - \beta) + z_B < -\beta,
\]

or

\[
\beta z_A + z_B - (1 - \beta)(y_{AB} - \beta) < 0.
\]

We have shown in Proposition 1 that \( \beta \in (0; 1) \). Thus, for a liquidity-absorbing shock \((z_A, z_B, y_{AB})\) to induce the market rate to fall, it suffices to simultaneously satisfy the following conditions:

\[
\begin{align*}
    z_A &> 0 \\
    -z_A &< z_B < -\beta z_A \\
    y_{AB} &\geq \beta
\end{align*}
\]

As these conditions describe a set of strictly positive measure in the three-dimensional euclidean space of liquidity shocks, we have shown a special case of the central result of our paper:

**Proposition 2.** *For all parameter values of the model, there is a positive probability that a liquidity shock will cause both a liquidity drainage and a decreasing market rate.*

**Proof.** See text above. \( \Box \)

Proposition 2 says that the path that the market rate takes in response to a newly established liquidity situation need not be monotonous: It may happen that the initial development of the market rate goes into the direction opposite to the one predicted by the liquidity effect. Of course, a completely analogous derivation shows the robust possibility of a liquidity-providing shock.
to cause the market rate to increase. As we now want to show, this effect mitigates the expected uncertainty of short-term interest rates.

3. Volatility and variance

As the benchmark case, we consider a scenario where both banks act non-strategically, i.e., the banks trade in the market with the sole intention of rebalancing their reserve accounts immediately. In this case, total order volume turns out to be

\[ x = x_A(\tilde{z}_A) + x_B(\tilde{z}_B) = z_A + y_{AB} + z_B - y_{AB} = z, \]

i.e., equal to the liquidity imbalance caused to the overall banking system. The market maker therefore obtains complete information about the aggregate liquidity situation of the banking system vis-à-vis the non-bank sector, and sets the price on day 1 equal to the full-information rate \( r(z) \). Thus, in the benchmark setting with unsophisticated behavior by both banks, we obtain a change in the market rate between day 0 and day 1 equaling \( r(z) - r_0 \), and no change between day 1 and day 2 (see Figure 3). The volatility of the sequence of market rates (i.e., the ex-ante expected average standard deviation of the price increment between two consecutive trading days) in the absence of strategic behavior is therefore given by

\[ v^u = \frac{1}{2} STD(r(z) - r_0). \]

We compare this benchmark volatility with the volatility in the original set-up with one sophisticated and one unsophisticated bank. Here, the price on day 1 is \( \bar{r}(x) \), where

\[ x = x_A(\tilde{z}_A) + x_B(\tilde{z}_B), \]

and the price on day 2 is \( r(z) \). Thus, the volatility in a market with strategic behavior is given by

\[ v^s = \frac{1}{2}(STD(\bar{r}(x) - r_0) + STD(r(z) - \bar{r}(x))). \]
It turns out that strategic money trading increases the volatility of the price process.

**Proposition 3.** For all parameter values of the model, $v^s > v^u$.

**Proof.** See the appendix.¶

The idea of the proof is to show that with sophisticated traders in the market, the day-to-day changes of the overnight rate must be uncorrelated. Intuitively, if these changes were correlated, this would mean a predictable pattern in the price path, leaving room for profitable arbitrage opportunities. With strategic traders in the market, this cannot be the case. In contrast, when all banks in the market care only for a balanced reserve position at the end of day 1, then this will produce autocorrelation in the price path, which reduces the volatility measure.

We will now turn to the question of how the procrastinated trading of some banks will affect monetary policy implementation. The interpretation will be that the central bank has installed neutral liquidity conditions on day 0 (e.g., by making the benchmark allotment in the last tender of the maintenance period), and that the market rate $r_0$ on that day corresponds to the target rate. With this interpretation in mind, we will now define the expected quadratic average from the target rate on day 1. Day 2 can be neglected in the discussion of interest rate targeting because the full-information rate prevails by assumption. In the benchmark case of two unsophisticated banks, the rate on day 1 will be the full-information rate, i.e., $r(z)$, so that the expected quadratic deviation from the target rate is given by

$$V^u = E[(r(z) - r_0)^2].$$

In contrast, with one sophisticated bank, the partial-information rate $\tilde{r}(x)$ will prevail on day 1, so that the quadratic deviation amounts to

$$V^s = E[(\tilde{r}(x) - r_0)^2].$$

Using the previous results, we can show that strategic behavior lowers the expected deviation from the interest rate target.
Proposition 4. For all parameter values of the model, $V^* < V^u$.

Proof. See the appendix.¶

Thus, and in mild contrast to the above finding on the volatility, it turns out that the average quadratic deviation from the interest rate $r_0$ is smaller in the presence of sophisticated behavior. If $r_0$ is interpreted as the central bank’s target rate then this finding says that the informational inefficiency may in fact be supportive to the objectives of monetary policy implementation. This leads us to the conclusion that the provision of public information about aggregate liquidity conditions after the last refinancing operation may in fact be detrimental to monetary policy implementation.

4. Extension to $N$ sophisticated and $M$ unsophisticated banks

Consider now the general case of $N \geq 0$ sophisticated banks $i = 1, \ldots, N$ and of $M \geq 1$ unsophisticated banks $i = N + 1, \ldots, N + M$. See Figure 4 for illustration. We will use the convention that unless indicated otherwise, the parameter $i$ runs over all $N + M$ banks. Denote the liquidity flows from bank $i$ to the non-bank sector by $z_i$, and the liquidity flow from bank $i$ to bank $j$ by $y_{ij}$, where $y_{ij} = -y_{ji}$. The expected value of the liquidity outflow from an individual firm to the non-bank sector is assumed to be $E[z_i] = 0$ for simplicity. For the expected interbank flow from bank $i$ to bank $j$, we will write

$$E[y_{ij}] = \bar{y}_{ij}.$$ 

For reasons of tractability, it will turn out to be useful to impose certain symmetry restrictions on the variances of the involved liquidity flows.\footnote{Dropping these restrictions leads to a generic system of $N$ quadratic equations in $N$ variables, which typically does not allow an explicit solution.}

Specifically, the variances of the flows from individual banks to the non-bank sector are assumed to be identical within the groups of sophisticated and unsophisticated banks, respectively, i.e.,

$$VAR(z_i) = \begin{cases} \sigma^2_s & \text{for } i = 1, \ldots, N \\ \sigma^2_u & \text{for } i = N + 1, \ldots, N + M. \end{cases}$$
Moreover, the variances of the flows between two individual banks are assumed to be the same if either both banks are sophisticated, or both banks are unsophisticated or one bank is, and the other is not. Thus, we assume

\[ \text{VAR}(y_{ij}) = \begin{cases} 
\sigma^2_{ss} & \text{for } i, j \in \{1, ..., N\} \\
\sigma^2_{uu} & \text{for } i, j \in \{N + 1, ..., N + M\} \\
\sigma^2_{su} & \text{otherwise}
\end{cases} \]

As before, bank \( i \) observes the balance of its reserve account, i.e.,

\[ z_i = z_{i0} + \sum_{j \neq i} y_{ij}. \]

(Recall our earlier convention that says here that the sum runs over all banks \( j = 1, ..., N + M \), leaving \( i \) out). For ease of notation, we will write

\[ y_i = \sum_{j \neq i} y_{ij} \]

for the total liquidity flow from bank \( i \) to other banks in the system. Clearly, the expected liquidity imbalance for bank \( i \) is

\[ E[\tilde{z}_i] = \sum_{j \neq i} y_{ij} =: \gamma_i. \]

When aggregating over flows, we find that the total flow of liquidity from the sophisticated to the non-sophisticated banks is given by

\[ \gamma := \sum_{i=1}^{N} y_i = -\sum_{i=N+1}^{N+M} y_i. \]

This statistics will play a certain role in the subsequent analysis. The problem of a sophisticated bank \( i = 1, ..., N \) is to maximize expected profits from speculation

\[ \pi_i(x_i) = E[(r(z) - \tilde{r}(x))x_i|\tilde{z}_i]. \]

Individual order flow for bank \( i \) is denoted by \( x_i(\tilde{z}_i) \). Aggregate order flow is then

\[ x = \sum_{i=1}^{N} x_i(\tilde{z}_i) + \sum_{i=N+1}^{N+M} \tilde{z}_i. \]
With these specifications, the equilibrium analysis of the model generalizes in a straightforward way as follows.

**Proposition 5.** An equilibrium in the interbank market for overnight credit is constituted by strategies

\[ x_i(\tilde{z}_i) = \beta(\tilde{z}_i - \bar{y}_i) \]  

for \( i + 1, \ldots, N \), and

\[ \tilde{r}(x) = r_0 + \lambda(x + \bar{y}) \]

where \( \beta \in (0; 1) \) and

\[ \lambda = \rho \frac{N \beta \sigma_s^2 + M \sigma_u^2}{N \beta^2 \sigma_s^2 + M \sigma_u^2 + (1 - \beta)^2 NM \sigma_{zu}^2}. \]

**Proof.** See the appendix. \[\]

We continue with the discussion of the reversal of the liquidity effect in the case of finitely many sophisticated banks. Generalizing our earlier definition, we will say that a shock \((\{z_i\}_{i=1,\ldots,N+M}, \{y_{ij}\}_{i>j})\) to the banking system is *liquidity-absorbing* if

\[ z = \sum_{i=1}^{N+M} z_i > 0. \]

According to Proposition 5, the market rate will fall in consequence of a liquidity shock if and only if the aggregate order flow is smaller than its expected value. Formally, this conditions is true if \( x < -\bar{y} \). As total order flow is the sum of sophisticated and unsophisticated demand, this is tantamount to

\[ \beta \sum_{i=1}^{N} (\tilde{z}_i - \bar{z}_i) + \sum_{i=N+1}^{N+M} \tilde{z}_i < -\bar{y}. \]

Using the definition of \( \tilde{z}_i \) and rearranging gives

\[ \beta z_s + z_u + (1 - \beta)(y - \bar{y}) < 0, \]
where

$$z_s = \sum_{i=1}^{N} z_i,$$

$$z_u = \sum_{i=N+1}^{N+M} z_i,$$

are the flows of liquidity to the non-bank sector aggregated about sophisticated and unsophisticated banks, respectively.

As in the case of a single sophisticated bank, the effect is driven by $\beta \in (0; 1)$. E.g., for a liquidity-absorbing shock ($\{z_i\}_{i=1,...,N+M}, \{y_{ij}\}_{i>j}$) to induce the market rate to fall, it suffices to simultaneously satisfy the following conditions:

$$z_s > 0$$

$$-z_s < z_u < -\beta z_s$$

$$y \leq \bar{y}.\quad (8)$$

These conditions specify again a subset in the space of liquidity shocks of strictly positive measure, so that we have generalized Proposition 2 to an arbitrary number of sophisticated banks. In fact, Propositions 3 and 4 extends likewise in a straightforward manner to the generalized set-up when we note that the definitions of volatility and variance of the market rate are well-defined also in the generalized model. We summarize our findings as follows.

**Proposition 6.** With finitely many strategic banks, there is a positive probability that a liquidity drainage causes the market rate to fall. On average, the volatility of the market rate is larger, while the quadratic deviation from the neutral rate $r_0$ is smaller in the presence of sophisticated liquidity management.

**Proof.** See the appendix.¶

With potentially many commercial banks persuing essentially independent operations with their respective clients, the question arises as to whether the
reversal of the liquidity effect would disappear if the number of banks operating in the currency area is only sufficiently large. It turns out that this is not the case. To see why, let the size $N + M$ of the banking population go to infinity, keeping approximately constant fractions $\alpha$ and $1 - \alpha$ of sophisticated and unsophisticated banks, respectively. Clearly, any enlargement of the population reduces the informational role that the market order of an individual bank has on the overnight rate. Thus, any manipulative motive of a single bank for strategic trading is marginalized as the population grows. However, for an individual bank, the informational advantage is still of some value. On average, speculating on this information yields a positive profit so that the individual bank takes a position in its own interest even when the population is large. This makes the reversal of the liquidity effect a robust phenomenon that can be present also in very large markets.

**Proposition 7.** *In the limit economy with a constant fraction $\alpha$ of sophisticated banks, sophisticated banks behave strategically with $\beta^\infty \in (0; 1)$, and there is a strictly positive probability for a reversal of the liquidity effect.*

**Proof.** See the appendix.¶

### 5. Conclusion

We have modified the Kyle [9] framework to capture some institutional aspects of the interbank market for overnight liquidity. Main assumptions included heterogeneous levels of sophistication in commercial banks’ liquidity management, as well as an asymmetric information distribution resulting from a decentralized realization of an autonomous factor shock. It has been shown that under these conditions, the liquidity effect may be overthrown in the sense that a liquidity drainage from the banking system may induce the market rate to decrease. The reason is that banks with a sophisticated liquidity management exploit the averaging condition on reserve holdings and procrastinate their balancing of true liquidity needs, so that information is impounded into prices only with a certain delay. As a consequence, the quadratic deviation from the target rate is on average smaller in the absence
of aggregate information about market conditions when compared to a full
information set-up. This suggests the conclusion that a conservative infor-
mation policy may indeed be supportive for the implementation of monetary
policy.

Appendix

Proof of Proposition 1. The proof has three steps. We first check the
optimality of bank A’s strategy, given the linear pricing rule and the unso-
phisticated behavior of bank B. Profits for bank A, conditional on observing
the realized liquidity imbalance \( \tilde{z}_A \), are given by

\[
E[(r(z) - \tilde{r}(x))x_A|\tilde{z}_A] = E[\{\rho z - \lambda(x + \tilde{y})\}x_A|\tilde{z}_A] \\
= \{\rho E[z|\tilde{z}_A] - \lambda(x_A + E[x_B|\tilde{z}_A + \tilde{y}]\}x_A,
\]

where, by the projection theorem for normally distributed random variables,

\[
E[z|\tilde{z}_A] = \frac{\sigma^2_A}{\sigma^2_A + \sigma^2_y}(\tilde{z}_A - \tilde{y}) \\
E[x_B|\tilde{z}_A] = -\tilde{y} - \frac{\sigma^2_y}{\sigma^2_A + \sigma^2_y}(\tilde{z}_A - \tilde{y}).
\]

The corresponding first-order condition is

\[
x_A(\tilde{z}_A) = \frac{\rho E[z|\tilde{z}_A]}{2\lambda} - \frac{1}{2}(E[x_B|\tilde{z}_A] + \tilde{y}).
\]

Using the explicit expressions for the conditional expectations gives

\[
x_A(\tilde{z}_A) = \beta(\tilde{z}_A - \tilde{y}),
\]

where

\[
\beta = \frac{\rho}{2\lambda} \frac{\sigma^2_A}{\sigma^2_A + \sigma^2_y} + \frac{1}{2} \frac{\sigma^2_y}{\sigma^2_A + \sigma^2_y}.
\] (9)

We continue by checking the zero-profit or no-arbitrage condition for the
market maker, assuming a linear strategy for bank A, and liquidity-balancing
for bank B. Under these conditions,

\[
E[r(z)|x] = r_0 + \rho E[z|x],
\]
where, by another application of the projection theorem,

\[
E[z|x] = \frac{COV(z, \beta z_A + z_B - (1 - \beta)y_{AB})}{VAR(\beta z_A + z_B - (1 - \beta)y_{AB})} (x + \overline{y})
\]

\[
= \frac{\beta \sigma_A^2 + \sigma_B^2}{\beta^2 \sigma_A^2 + \sigma_B^2 + (1 - \beta)^2 \sigma_y^2} (x + \overline{y}).
\]

We show now that \( \beta \in (0; 1) \). Note first that \( \lambda > 0 \) by the second-order condition for the sophisticated bank’s problem. From (9) then it follows that \( \beta > 0 \). It therefore remains to be shown that \( \beta < 1 \). Plugging (2) into (9) and rearranging yields the quadratic equation

\[
\beta^2 + \beta(2\zeta + \xi) - (\zeta + \xi) = 0, \tag{10}
\]

where \( \zeta = \sigma_B^2 / \sigma_A^2 \) and \( \xi = \sigma_y^2 / (\sigma_A^2 + \sigma_y^2) \). This equation possesses a unique positive root, given by

\[
\beta(\zeta, \xi) = -\left(\zeta + \frac{\xi}{2}\right) + \sqrt{\left(\zeta + \frac{\xi}{2}\right)^2 + \zeta + \xi}.
\]

If \( \beta \geq 1 \), then the left-hand side of (10) is strictly positive, so we must have \( \beta < 1 \).

**Proof of Proposition 3.** We start from the obvious triangle decomposition

\[
r(z) - r_0 = (r(z) - \tilde{r}(x)) + (\tilde{r}(x) - r_0).
\]

Hence,

\[
VAR(r(z) - r_0) = VAR(r(z) - \tilde{r}(x)) + VAR(\tilde{r}(x) - r_0)
+ 2COV(r(z) - \tilde{r}(x), \tilde{r}(x) - r_0).
\]

We will focus for the moment on the covariance term. Using

\[
r(z) - r_0 = \rho(z_A + z_B)
\]
\[
\tilde{r}(x) - r_0 = \lambda(\beta z_A + z_B - (1 - \beta)(y_{AB} - \overline{y})),
\tag{11}
\]
and using the independence of $z_A$, $z_B$, and $y_{AB}$, we obtain

$$COV(r(z) - \tilde{r}(x), \tilde{r}(x) - r_0) = \lambda COV(\rho(z_A + z_B) - \lambda(\beta z_A + z_B - (1 - \beta)(y_{AB} - \bar{y})),$$

$$\beta z_A + z_B - (1 - \beta)(y_{AB} - \bar{y}))$$

$$= \lambda\{\beta(\rho - \beta \lambda)\sigma^2_A + (\rho - \lambda)\sigma^2_B - \lambda(1 - \beta)^2 \sigma^2_y\}$$

$$= 0,$$

where we used (2) in the last equation. This yields

$$VAR(r(z) - r_0) = VAR(r(z) - \tilde{r}(x)) + VAR(\tilde{r}(x) - r_0).$$

Thus,

$$4(v^s)^2 = VAR(\tilde{r}(x) - r_0) + VAR(r(z) - \tilde{r}(x))$$

$$+ 2STD(\tilde{r}(x) - r_0)STD(r(z) - \tilde{r}(x))$$

$$> VAR(\tilde{r}(x) - r_0) + VAR(r(z) - \tilde{r}(x))$$

$$= VAR(r(z) - r_0)$$

$$= 4(v^u)^2,$$

proving the assertion. \\

**Proof of Proposition 4.** By definition,

$$V^u = E[(r(z) - r_0)^2]$$

$$= \rho^2 E[(z_A + z_B)^2]$$

$$= \rho^2(\sigma^2_A + \sigma^2_B).$$

On the other hand, by (11) and (2), we get

$$V^s = E[(\tilde{r}(x) - r_0)^2]$$

$$= \lambda^2 E[\{(\beta z_A + z_B - (1 - \beta)(y_{AB} - \bar{y})\}^2]$$

$$= \lambda^2\{\beta^2 \sigma^2_A + \sigma^2_B + (1 - \beta)^2 \sigma^2_y\}$$

$$= \rho^2 \frac{\beta \sigma^2_A + \sigma^2_B + (1 - \beta)^2 \sigma^2_y}{\beta^2 \sigma^2_A + \sigma^2_B} V^u.$$
The assertion then follows from $\beta < 1$.\[\]

**Proof of Proposition 5.** The proof follows the lines of the proof of Proposition 1. The details are as follows. Assuming that the market maker’s price-setting behavior (4) is common knowledge, expected profits for bank $i$, for $i = 1, ..., N$, are given by

$$
\pi_i(x_i) = E[(r(z) - \tilde{r}(x))x_i|\tilde{z}_i] \\
= E[(\rho z - \lambda(x + \bar{y}))x_i|\tilde{z}_i] \\
= \{\rho E[z|\tilde{z}_i] - \lambda(x_i + E[x_{-i}|\tilde{z}_i] + \bar{y})\} x_i,
$$

where

$$
x_{-i} = \sum_{j \neq i} x_j.
$$

The corresponding first-order condition is

$$
x_i(\tilde{z}_i) = \frac{\rho}{2\lambda} E[z|\tilde{z}_i] - \frac{1}{2} E[x_{-i}|\tilde{z}_i] - \frac{\bar{y}}{2}, \quad (12)
$$

for $i = 1, ..., N$. We will now calculate the two expected values in (12). For $i = 1, ..., N$, we have by the projection theorem that

$$
E[z|\tilde{z}_i] = \frac{COV(z, \tilde{z}_i)}{VAR(\tilde{z}_i)} (\tilde{z}_i - E[\tilde{z}_i]) \\
= \frac{\sigma_i^2}{\sigma_i^2 + \sum_{j \neq i} \sigma_{ij}^2} (\tilde{z}_i - \bar{y}_i).
$$

The second expected value is given by

$$
E[x_{-i}|\tilde{z}_i] = E[x_{-i}] + \frac{COV(x_{-i}, \tilde{z}_i)}{VAR(\tilde{z}_i)} (\tilde{z}_i - \bar{y}_i),
$$

where

$$
E[x_{-i}] = \sum_{j=N+1}^{N+M} \bar{y}_j = -\bar{y},
$$
and the covariance and variance terms are given by

\[ COV(x_{-i}, \tilde{z}_i) = \sum_{j=1}^{N} COV(x_j, \tilde{z}_i) + \sum_{j=N+1}^{N+M} COV(x_j, \tilde{z}_i) \]

\[ = \sum_{j=1}^{N} COV(\beta_j(z_j + \sum_{k\neq j} y_{jk}), z_i + \sum_{l \neq i} y_{li}) + \]

\[ + \sum_{j=N+1}^{N+M} COV(z_j + \sum_{k \neq j} y_{jk}, z_i + \sum_{l \neq i} y_{li}) \]

\[ = \sum_{j=1}^{N} \beta_j COV(y_{ji}, y_{ij}) + \sum_{j=N+1}^{N+M} COV(y_{ji}, y_{ij}) \]

\[ = -\sum_{j=1}^{N} \beta_j \sigma_{ij}^2 - \sum_{j=N+1}^{N+M} \sigma_{ij}^2, \]

and by

\[ VAR(\tilde{z}_i) = \sigma_i^2 + \sum_{j \neq i}^{} \sigma_{ij}^2. \]

Thus, from (12), we get (3), where the vector \((\beta_1, ..., \beta_N)\) is the solution of the system of equations

\[ 2\beta_i = \frac{1}{\sigma_i^2} \left\{ \rho \sigma_i^2 + \sum_{j=1}^{N} \beta_j \sigma_{ij}^2 + \sum_{j=N+1}^{N+M} \sigma_{ij}^2 \right\}, \quad (13) \]

for \(i = 1, ..., N\). In the symmetric set-up, to which we refined ourselves earlier above, this leads to

\[ 2\beta = \frac{\sigma_i^2 (\rho/\lambda) + (N-1)\beta \sigma_{ss}^2 + M \sigma_{su}^2}{\sigma_s^2 + (N-1)\sigma_{ss}^2 + M \sigma_{su}^2}. \quad (14) \]

Next, we check the zero-profit condition for the market maker. We find that

\[ E[r(z)]|x] = r_0 + \rho E[z|x] \]

\[ = r_0 + \rho \frac{COV(z, x)}{VAR(x)} (x - E[x]), \quad (15) \]
where

\[ COV(z, x) = COV(\sum_{i=1}^{N+M} z_i, \sum_{j=1}^N x_j(z_j)) + COV(\sum_{i=1}^{N+M} z_i, \sum_{j=N+1}^{N+M} z_j) \]

\[ = COV(\sum_{i=1}^{N+M} z_i, \sum_{j=1}^N \beta_j z_j) + COV(\sum_{i=1}^{N+M} z_i, \sum_{j=N+1}^{N+M} z_j) \]

\[ = \sum_{i=1}^N \beta_i \sigma_i^2 + \sum_{i=N+1}^{N+M} \sigma_i^2, \]

and

\[ VAR(x) = VAR(\sum_{i=1}^N x_i(\tilde{z}_i) + \sum_{i=N+1}^{N+M} \tilde{z}_i) \]

\[ = VAR(\sum_{i=1}^N \beta_i (z_i + \sum_{j \neq i} y_{ij}) + \sum_{i=N+1}^{N+M} (z_i + \sum_{j \neq i} y_{ij})) \]

\[ = \sum_{i=1}^N \beta_i^2 \sigma_i^2 + \sum_{i=N+1}^{N+M} \sigma_i^2 + VAR(\sum_{i=1}^N \beta_i \sum_{j \neq i} y_{ij} + \sum_{i=N+1}^{N+M} \sum_{j \neq i} y_{ij}) \]

\[ = \sum_{i=1}^{N+M} \beta_i^2 \sigma_i^2 + \sum_{i>j} (\beta_i - \beta_j)^2 \sigma_{ij}^2, \]

where we let \( \beta_i := 1 \) for \( i = N + 1, ..., N + M \). Moreover, we have

\[ E[x] = E[\sum_{i=1}^N \beta_i (z_i + \sum_{j \neq i} (y_{ij} - \bar{y}_{ij}) + \sum_{i=N+1}^{N+M} (z_i + \sum_{j \neq i} y_{ij})) \]

\[ = \sum_{i+N+1}^{N+M} \sum_{j \neq i} \bar{y}_{ij} = -\bar{y}. \]

Using this information, (15) implies (4), where

\[ \lambda = \rho \frac{\sum_{i=1}^{N+M} \beta_i \sigma_i^2}{\sum_{i=1}^{N+M} \beta_i^2 \sigma_i^2 + \sum_{i>j} (\beta_i - \beta_j)^2 \sigma_{ij}^2}. \]

The symmetric set-up implies (5). Combining (14) and (5) yields the quadratic equation

\[ \beta^2 + (A + B)\beta - B = 0, \quad (16) \]
for constants $A > 0$ and $B > 0$. Thus, as in the proof of Proposition 1, there is a unique positive root $\beta < 1$. \\

**Proof of Proposition 6.** The first assertion is proved in the text before the Proposition. For the second assertion, recall from the proof of Proposition 2 that it suffices to show that

$$\text{COV}(r(z) - \bar{r}(x), \bar{r}(x) - r_0) = 0.$$ 

From (1) and (4), we obtain

$$\text{COV}(r(z) - \bar{r}(x), \bar{r}(x) - r_0)$$

$$= \text{COV}(\rho z - \lambda x + \lambda y, \lambda (x + y))$$

$$= \text{COV}(\rho \sum_{i=1}^{N+M} z_i - \lambda \sum_{i=1}^{N+M} \beta_i(z_i + y_i), \lambda \sum_{i=1}^{N+M} \beta_i(z_i + y_i))$$

$$= \lambda \text{COV}\{\sum_{i=1}^{N+M} (z_i - \lambda \beta_i) - \lambda \beta_i y_i, \sum_{i=1}^{N+M} \beta_i(z_i + y_i)\}$$

$$= \lambda \{ \sum_{i=1}^{N+M} (\rho - \lambda \beta_i) \beta_i \sigma^2 - \lambda \sum_{i>j} (\beta_i - \beta_j)^2 \sigma^2_{ij} \}$$

$$= \lambda \{ N(\rho - \beta \lambda) \beta \sigma^2 + M(\rho - \lambda) \sigma^2_u - NM(1 - \beta) \sigma^2_{su} \}$$

$$= 0,$$

where we used (5) in the last equation. This proves the assertion concerning the volatility. As for the quadratic deviation from the neutral rate $r_0$, we
have

\[ V^* = E[(\hat{r}(x) - r_0)^2] \]

\[ = \lambda^2 E\left[ \left( \sum_{i=1}^{N} \beta(z_i + y_i - y) \right) + \sum_{i=N+1}^{N+M} (z_i + y_i - y)^2 \right] \]

\[ = \lambda^2 \left\{ N\beta^2 \sigma_s^2 + M\sigma_u^2 - (1 - \beta)^2 NM\sigma^2_{su} \right\} \]

\[ = \rho^2 \frac{N\beta^2 \sigma_s^2 + M\sigma_u^2}{N\sigma_s^2 + M\sigma_u^2} \frac{N\beta^2 \sigma_s^2 + M\sigma_u^2}{N\sigma_s^2 + M\sigma_u^2} V^u \]

\[ > V^u, \]

where we have used

\[ V^u = E[(r(z))^2 - r_0)^2] \]

\[ = \rho^2 (N\sigma_s^2 + M\sigma_u^2) \]

and \( \beta \in (0, 1) \). This proves the assertion concerning the variance, and thereby the Proposition. \( \Box \)

**Proof of Proposition 7.** Let \( \beta^{N,M} \) be the solution to the quadratic equation (16) in a model with \( N \) sophisticated and \( M \) unsophisticated banks. A short calculation shows that the parameters in the corresponding quadratic equation are given by

\[ A^{N,M} = \frac{M\sigma_s^2 \sigma_u^2 + M(N-1)\sigma_u^2 \sigma_{ss}^2 + M^2 \sigma_u^2 \sigma_{su}^2}{N\sigma_u^2 + N(N-1)\sigma_u^2 \sigma_{ss}^2 + NM\sigma_u^2 \sigma_{su}^2} \]

\[ B^{N,M} = \frac{M\sigma_s^2 \sigma_u^2 + M\sigma_u^2 \sigma_{ss}^2 + M^2 \sigma_u^2 \sigma_{su}^2}{N\sigma_u^2 + N(N-1)\sigma_u^2 \sigma_{ss}^2 + NM\sigma_u^2 \sigma_{su}^2}. \]

Thus, letting \( N \) and \( M \) going simultaneously to infinity, ensuring

\[ \frac{N}{M} \approx \frac{\alpha}{1 - \alpha} \]

we obtain limit values

\[ A^\infty = \frac{(1 - \alpha)\sigma_u^2}{\alpha \sigma_s^2} \]

\[ B^\infty = \frac{(1 - \alpha)\sigma_u^2}{\alpha \sigma_s^2} \frac{\alpha \sigma_s^2 + (1 - \alpha)\sigma_u^2}{\alpha \sigma_{ss}^2 + (1 - \alpha)\sigma_{su}^2}. \]
Thus, the limit of the roots $\beta^\infty = \lim \beta^{N,M}$ is strictly contained in the open interval $(0; 1)$. To prove the second part of the assertion, note that the individual distributions of the aggregate, the independent flows

$$z_{a}^{N,M} := \sum_{i=1}^{N} z_{i},$$
$$z_{u}^{N,M} := \sum_{i=N+1}^{N+M} z_{i}$$

are both normally distributed with expectation value zero. Standard deviations are given by

$$\text{STD}(z_{s}^{N,M}) = \sqrt{N}\sigma_{s} \approx \sqrt{\alpha} \sqrt{N + M}\sigma_{s},$$
$$\text{STD}(z_{u}^{N,M}) = \sqrt{M}\sigma_{u} \approx \sqrt{1 - \alpha}\sqrt{N + M}\sigma_{u}.$$ 

As the population grows, i.e., when $N + M$ increases, keeping the respective proportions of sophisticated and unsophisticated banks approximately constant, the joint density of the random vector $(z_{s}^{N,M}, z_{u}^{N,M})$ is merely rescaled by the common factor $1/\sqrt{N + M}$. Thus, as the sufficient conditions (6) through (8) for a reversal of the liquidity effect are invariant with respect to re-scaling for a fixed $\beta := \beta^\infty$, we find that the probability of a shock satisfying these conditions remains bounded away from zero for a growing population size. ¶
References


Figures

Figure 1. Net marginal lending (deposit if negative) during the example period.
Figure 2. Intraday-behavior of the overnight rate on the last two days of the example period.
Figure 3. Volatility of the market rate; example: liquidity drainage.

**Non-strategic behavior of bank A**

![Graph showing non-strategic behavior](image1)

**Strategic behavior of bank A**

![Graph showing strategic behavior](image2)
Figure 4. Visualization of the generalized model.
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