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OPTIMAL ALLOTMENT POLICY IN THE EUROSYSTEM'S MAIN REFINANCING OPERATIONS[1]

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Abstract

On several occasions during the period 2001-2003, the European Central Bank (ECB) decided to deviate from its "neutral" benchmark allotment rule, with the effect of not alleviating a temporary liquidity shortage in the banking system. This is remarkable because it implied the possibility of short-term interest rates raising significantly above the main policy rate. In the present paper, we show that when the monetary authority cares for both liquidity and interest rate conditions, the optimal allotment policy may entail a *discontinuous* reaction to initial conditions. More precisely, we prove that there is a threshold level for the accumulated aggregate liquidity position in the banking system prior to the last operation in a given maintenance period, so that the benchmark allotment is optimal whenever liquidity conditions are above the threshold, and a tight allotment is optimal whenever liquidity conditions are below the threshold.

JEL CODES: E43, E52

KEYWORDS: euro, monetary policy instruments, operational framework, refinancing operations

Non-technical summary

On several occasions during the past three years, short-term interest rates in the euro area departed more visuably than usual from the mid of the interest rate corridor. These deviations all followed occurrences of so-called underbidding, meaning that a number of major commercial banks, for speculative reasons, reduced their bids in one of the Eurosystem's main refinancing operation. In all cases, the European Central Bank decided to not fully compensate for the liquidity shortage created by the underbidding, providing the banking system with only partial relief from the tight conditions in the money market. Is this observation consistent with optimal central bank behavior?

To address this question, we propose a model that captures some of the main institutional aspects of the Eurosystem's operational framework. The liquidity (i.e., short-term credit) required by the banking system is provided by the central bank through weekly main refinancing operations. Banks demand liquidity, inter alia, to meet reserve requirements on average over a four-week maintenance period. After the last operation in any given period, a liquidity shock may affect the aggregate liquidity position in the banking system, and counterparties may adjust their reserve balances by having recourse to the central bank's standing deposit and lending facilities. Within this model, we consider the central bank's problem of choosing allotments in a sequence of main refinancing operations.

One central result of the analysis says that when the monetary authority cares for both interest rates and for outstanding liquidity, then the optimal intertemporal allotment strategy in the relevant parameter range implies a *discontinuous* reaction to initial conditions. Specifically, we prove that there is a threshold level, so that the "neutral" benchmark allotment rule is optimal only when the aggregate liquidity position is above the threshold, whereas a significantly tighter allotment is optimal when the liquidity position is below the threshold.

The result relies on the averaging provision for central bank reserves over the maintenance period. When the volume of the penultimate tender in a maintenance period has been insufficient then, because the last tender is close to the end of the period, the "neutral" benchmark allotment in the last tender would be very large. Injecting the benchmark allotment would therefore imply that also the aggregate central bank credit outstanding to the banking system is very large. On the other hand, being tight would induce interest rates to rise to the upper end of the corridor. It turns out that the described trade-off between liquidity and interest-rate smoothing is typically non-convex, which causes the described effect.

1 Introduction

From January 1st, 1999, the euro has replaced the individual currencies of altogether 11 European countries. Greece followed by joining the euro area in the beginning of 2001, and there are currently 13 further sovereign countries that are considered as candidates to join the world's second largest currency union in the near future. The creation of the new currency and the persuance of a monetary policy implied the design of a new policy framework for monetary policy implementation and liquidity management.[4] The aim of the present paper is to contribute to the understanding of this operational framework, especially concerning the economics that link the decisions of the monetary authority and the behavior of short-term interest rates in the interbank market. To this aim, we propose a model that captures some of the main institutional aspects of the Eurosystem's operational framework, where we focus on the active role of the central bank as a provider of liquidity to the banking system.[5]

Our model has the following structure. Banks have to satisfy reserve requirements as an average of the reserve account balance over a four-week maintenance period.[6] The required liquidity is provided by the monetary authority through weekly main refinancing operations. Credits that are allocated in these operations have a maturity of two weeks.[7] After the last operation in any given period, a liquidity shock may affect the aggregate liquidity position in the banking system, and counterparties may have recourse either to the marginal lending facility or to the deposit facility.

Within this framework, we consider the central bank's problem of choosing allotments in a sequence of main refinancing operations. Interest rates are steered indirectly via an end-of-period liquidity effect. It is assumed that the central bank's objective function is a discounted weighted sum of squared interest rate and liquidity deviations over all future maintenance periods. Under these assumptions, the monetary authority faces in general a trade-off between being

[4]The European framework is different from the framework established by the U.S. Federal Reserve system, but also has some commonalities (see Borio [5] for a comparison). The Eurosystem's operational framework is documented in ECB [7]. For a description of the framework as an element of the ECB's overall monetary strategy, see Manna, Pill, and Quiros [17].

[5]The Eurosystem is composed of the European Central Bank (ECB) and the national central banks of the 12 countries that have adopted the new currency.

[6]This assumption is made for convenience. In reality, the maintenance period is usually one month.

[7]As from March 2004, the operational framework will be somewhat modified. In particular, under the new regime, operations will not overlap, and will not hang over into the subsequent maintenance period. The conclusions are not affected. See Section 6.

closer to the interest rate target and being closer to the liquidity target. We will study the resulting dynamic programming problem in the first part of the paper, and compare the optimal policy with the so-called benchmark allotment rule that is normally applied by the ECB in its main refinancing operations.

The second part of the paper is concerned with the optimal allotment in the respective last operation in the reserve maintenance period. This allotment decision is most decisive for monetary policy implementation because it determines to a large extent the liquidity conditions at the end of the period. Figure 1 exhibits the EONIA money market index in the year 2001. It can be seen that the overnight interest rate spiked upwards at the end of the maintenance periods in February, April, and October. These conditions have been caused by occurrences of a phenomenon usually referred to as "underbidding," meaning that the largest banks in the market, for speculative reasons, reduced their bids in a central bank operation (cf. Ewerhart [11] or Nyborg, Bindseil and Strebulaev [18]). Similar patterns can be seen in the data for the subsequent years 2002 and 2003. Our question is why the ECB did not fully compensate the shortfall in liquidity created by the underbidding. We therefore study in somewhat more depth the optimal allotment in the last tender.[8]

To our knowledge, the issue of an optimal intertemporal allotment policy has not been addressed so far in the literature. Most of the established theoretical literature on the interbank market for reserves, as originated by work of Poole [19], Ho and Saunders [15], Campbell [6], Spindt and Hoffmeister [20], and Hamilton [13], has tended to abstract from the active role of the central bank, with only a few recent exceptions. Bartolini, Bertola and Prati [3] develop an intertemporal model of the market for Federal Funds allowing for daily central bank intervention. Without intervention, the expected variance of the market rate is increasing over the maintenance period. With unconstrained intervention, however, the central bank may implement its target interest rate in all but the final day of the maintenance period. The model thereby allows a positive analysis of the consequences of various central bank policies on the time-series properties of the Federal Funds rate. Bartolini, Bertola and Prati [2] offer an explanation for the empirical observation that banks in the U.S. tend to hold more reserves on settlement days than on other days of the maintenance

[8]The obvious explanation is that the increased rates at the end of the period should make underbidding unprofitable. However, this argument can be valid only for the initial episodes of underbidding. Central bankers realized very soon that, unexpectedly, the two-week swap rate continued to fall below the minimum bid rate on the day of a critical operation, despite the substantial threat (see [9]).

period. It is assumed that banks have to satisfy reserves on average over a two-day reserve maintenance period. It is shown that under uncertainty about reserve requirements, and in the presence of transaction costs, when the Federal Reserve leaves the market rate constant over the maintenance period, demand for reserves is higher on the second day than on the first. While the paper does not write out an objective function for the central bank, it discusses informally the trade-off between a higher interest rate vis-à-vis increased reserves on settlement day from the point of view of the central bank. Ayuso and Repullo [1] study the demand in fixed and variable rate tenders, and consider in this context also the static allotment problem of a central bank. It is assumed that the central bank's loss function penalizes market rates below the target more severely than market rates above the target. This induces the central bank to choose a tight allotment volume. As a consequence, allotments in fixed-rate tenders (where banks pay the target rate) are profitable, generating overbidding. However, in a variable-rate tender with pre-announced liquidity injection, there is a bidding equilibrium without excess demand.[9]

The rest of the paper is structured as follows. Section 2 introduces the model. In section 3, we derive the optimal intertemporal allotment policy. In sections 4 and 5, we discuss the optimal allotment in the last operation of a given period. Section 6 discusses the validity of our results under the recently released changes to the operational framework. Section 7 concludes. The appendix contains details on calibration and simulation as well as technical derivations.

2 A model of the operational framework

We consider a time horizon consisting of an infinite number of consecutive maintenance periods $t = 0, 1, 2, ...$, each of which corresponds to an interval $[t; t+1]$.[10] In each maintenance period t, there are four main refinancing operations or tenders A, B, C, and D at times

$$t + \tau_A < t + \tau_B < t + \tau_C < t + \tau_D,$$

where $\tau_A > 0$ and $\tau_D < 1$. Maturities are assumed to be overlapping, i.e, credit allocated in a given tender matures at the date of the tender after next. E.g.,

[9] More recently, Bindseil [4] has argued that the ECB has tended to create loose liquidity conditions at the end of maintenance periods during the overbidding episode, which would contrast with the objective function assumed by Ayuso and Repullo.

[10] To denote points in continuous time, we will use the greek letter τ, while the latin t refers to discrete points in time or to maintenance periods.

as Figure 2 illustrates, the penultimate tender C in period $t-1$ allocates credit maturing at time τ_A of period t, the last tender D in period $t-1$ allocates credit maturing at time τ_B, and so on.

The aggregate amount of central bank credit allocated to the banking sector in period t's tender A is denoted by $Y_{t,A}$, and so on for the other tenders. The resulting liquidity supply to the banking system is characterized by the property that at any point in time, transactions allocated at two subsequent weeks contribute to the fulfilment of the reserve requirements. Formally, the *instantaneous* liquidity conditions, i.e., the outstanding volume of central bank credit at time τ in period t is given by

$$L_\tau = \begin{cases} Y_{t-1,D} + Y_{t,A} & \text{for } t+\tau_A < \tau \le t+\tau_B \\ Y_{t,A} + Y_{t,B} & \text{for } t+\tau_B < \tau \le t+\tau_C \\ Y_{t,B} + Y_{t,C} & \text{for } t+\tau_C < \tau \le t+\tau_D \\ Y_{t,C} + Y_{t,D} & \text{for } t+\tau_D < \tau \le t+1+\tau_A \end{cases} . \tag{1}$$

The market rate does not depend on the instantaneous, but on the (expected) *average* liquidity conditions in period t, which is the integral over instantaneous conditions

$$Z_t = \int_t^{t+1} L_\tau d\tau.$$

It is not difficult to check that under the assumptions on timing made above, we have

$$\begin{aligned} Z_t \;&=\; \tau_A Y_{t-1,C} + \tau_B Y_{t-1,D} + (\tau_C - \tau_A) Y_{t,A} + \\ &\quad + (\tau_D - \tau_B) Y_{t,B} + (1-\tau_C) Y_{t,C} + (1-\tau_D) Y_{t,D}. \end{aligned} \tag{2}$$

Thus, altogether six operations contribute to the fulfilment of reserve requirements in any given maintenance period t, two of which hang over from the previous period $t-1$.

Aggregate liquidity demand during period t is equal to the sum of exogenous aggregate reserve requirements $\overline{R}$, assumed for simplicity to be unchanged over time, and stochastic autonomous liquidity factors A_t. Changes to autononous factors such as banknotes in circulation and government balances in the accounts of some national central banks cause liquidity flows between the banking system and the non-bank sector which are (by definition) outside of the control of the central bank's liquidity management.[11] The random variables A_t are assumed to be independently and identically distributed with a differentiable cumulative distribution function $G(A)$. Shocks are measured in euro days, that

[11] For a more exhaustive description of automonous factors in the euro area, see [8].

is, they are weighted according to their duration. In each period t, the liquidity shock A_t is assumed to realize after the last refinancing operation in that period, at time $t + \tau_s$, where $\tau_D < \tau_s < 1$.[12]

Considered as a total over the period, we assume that demand for liquidity is perfectly inelastic with respect to interest rate conditions. Counterparties in the euro area usually prefer to have recourse to the marginal lending facility over being sanctioned for not complying with the minimum reserve obligation. Moreover, and in contrast to the Federal reserve system, excess reserves do not play a significant role in the euro area in relative terms.[13] Note also that the demand for reserves is typically elastic *within* the period. Autonomous factors are in general inelastic in the short run, but may be cross-financed by reserve holdings.

Standing facilities offer individual banks the opportunity to borrow and lend an arbitrary amount overnight at the marginal lending rate r^L and the deposit rate $r^D < r^L$, respectively.[14] Because of the averaging provision of reserve requirements over the maintenance period, standing facilities are rationally used only at the end of the maintenance period, i.e., after the realization of the autonomous factor shock.

The market rate prevailing in period t after the realization of the autonomous factor shock is denoted by r_t^s. From the inelasticity of demand at the end of the period and from the availability of the standing facilities, it follows that the market rate at the end of the last day of the maintenance period is a step function of excess liquidity at the end of the period, i.e.,[15]

$$r_t^s = \begin{cases} r^L & \text{if } Z_t - A_t < \overline{R} \\ r^D & \text{if } Z_t - A_t > \overline{R} \end{cases} .$$

Thus, in the model the market rate reaches the marginal lending rate when the aggregate average liquidity supply is below aggregate demand, and analogously the market rate drops to the deposit rate when supply exceeds demand. Therefore, as has been noted before, under the martingale hypothesis, the level of the

[12] This assumption is made for simplicity. In reality, autonomous factors resemble a continuous-time stochastic process.

[13] Absolute figures for average excess reserves are in fact not that different (USD 1 bn in the U.S. vs EUR 0.65 bn in the euro area).

[14] The ECB requires *all* central bank credits to be collateralized, including any recourse to the marginal lending facility. Since this requirement has never become binding on an aggregate level, we will ignore it in the model.

[15] We do not specify the behavior of the market rate in the zero-probability case of a perfect match between supply and demand. Any specification, e.g., $r_t^s = (r^D + r^L)/2$, will work.

market rate within the period can be expressed as a weighted average of the rates of the standing facilities, where the weights are given by the respective probabilities that the upper and lower boundaries of the interest rate corridor are reached at the end of the last day of the period.[16]

Proposition 1. *Let Z_t denote the average liquidity supply expected for period t. Then the market rate at any time $t \leq \tau < t + \tau_s$ in period t is given by*

$$r_\tau = (1 - G(Z_t - \overline{R}))r^L + G(Z_t - \overline{R})r^D, \tag{3}$$

where $1 - G(Z_t - \overline{R})$ is the probability of ending the maintenance period with recourse to the marginal lending facility, and $G(Z_t - \overline{R})$ is the probability of ending the maintenance period with recourse to the deposit facility.

Proof. See text above. □

The relevance of Proposition 1 comes from the fact that it provides a link between the central bank's allotment policy and the market rate prevailing on the interbank market. E.g., if the market expects the allotment policy to be restrictive, or else that autonomous factors will drain the banking system after the last operation in a given period, then the market rate will increase above the mid of the corridor. In fact, the existing experience with the framework (e.g., during the episodes of non-accomodating central bank behavior) suggests that this argument captures a first-order determinant for the EONIA money market index in the euro area.

3 Optimal allotment policy

Proposition 1 suggests that the monetary authority, in order to implement its interest rate target $r^* \in (r^D; r^L)$, would have to provide for average liquidity conditions Z^* such that

$$r^* = (1 - G(Z^* - \overline{R}))r^L + G(Z^* - \overline{R})r^D. \tag{4}$$

Solving for Z^* gives the neutral average liquidity as a sum of aggregate reserve requirements and a percentile of the autonomous factor distribution

$$Z^* = \overline{R} + G^{-1}(\frac{r^L - r^*}{r^L - r^D}).$$

[16] It is essentially undisputed in the empirical literature that the EONIA follows approximately a martingale process. See Ewerhart et al. [12] for details.

E.g., in the typical case where the corridor is symmetric around the target rate, i.e., when $r^* = (r^L + r^D)/2$, the liquidity target is just the sum of reserve requirements and the median of the autonomous factor shock distribution.

It will be assumed in this paper that the central bank wishes to implement its interest rate target in a *smooth manner*. By this formulation, we mean that the monetary authority will have a preference for keeping fluctuations both of the market rate and in the supply of liquidity to the banking sector at a minimum. The obvious target value for the instantaneous liquidity is $L^* := Z^*$. Formally, the objective function of the central bank is then given by

$$U = -E[\int_{t_0}^{\infty} \delta^{\tau - t_0} \left((r_\tau - r^*)^2 + \mu (L_\tau - L^*)^2 \right) d\tau], \qquad (5)$$

where δ is the discount factor, and $\mu > 0$ is the relative weight given to the objective of liquidity smoothing vis-à-vis the objective of interest rate smoothing.

With this objective function, we suppose that the central bank should care also about the instantaneous volume of outstanding credit. This assumption can be disputed by the claim that the liquidity target does not matter for implementation of monetary policy, as long as the average liquidity position at the end of the period, and thereby the interest rate target is met. However, in our opinion, it seems plausible that a central bank wishes to ensure a constant level of *satiation* with liquidity. E.g., deviations from the target level to the downside might create situations where liquidity demand is no longer dominated by the purpose of fulfilling reserve requirements, but affected also by the less predictable needs of daily banking business. This might then engender undesirable effects in the money market such as temporary market power, widened bid-ask spreads, or increased price dispersion. These effects may similarly come about when the outstanding volume of liquidity is significantly larger than the pro-rata share, potentially leaving some individual banks overrunning with liquidity. We conclude that fluctuations in the outstanding central bank credit may have undesirable consequences that either did not become visible in the data due to the ECB's neutral liquidity management, or for which the EONIA is not a sufficient statistics. Introducing an implicit target for the outstanding central bank credit is a way of taking account of these consequences.[17]

[17] An alternative motive for the central bank to keep fluctuations of instantaneous liquidity at a low level might be that overnight balances on different days are not perfect substitutes under all circumstances. E.g., it has been validated empirically that the EONIA exhibits a weak, but measurable response to an on average increased liquidity demand at the end of the calender month, which is due to window dressing activities (cf. [12]). A deviation from neutral conditions might amplify these effects. Finally, there is some empirical evidence, albeit limited, for a non-zero liquidity effect before the last operation.

Looking at the problem of determining a sequence of optimal liquidity injections starting with the beginning of a maintenance period t_0, the choice variables for the monetary authority become the normalized allotment vectors

$$y_t = \begin{pmatrix} \Delta Y_{t,A} \\ \Delta Y_{t,B} \\ \Delta Y_{t,C} \\ \Delta Y_{t,D} \end{pmatrix}$$

for all periods $t \in \{t_0, t_0+1, ...\}$, where the normalization $\Delta Y := Y - L^*/2$ helps to simplify the notation. The normalization of tender volumes is with respect to $L^*/2$ because at any point in time, there is credit allocated in two subsequent operations outstanding to the banking system. The state variables are the volumes of the tenders C and D

$$x_t = \begin{pmatrix} \Delta Y_{t-1,C} \\ \Delta Y_{t-1,D} \end{pmatrix},$$

that hang over from the previous maintenance period. The (trivial) law of motion is given by

$$x_{t+1} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} y_t, \tag{6}$$

which is the formal counterpart of the notion that the volume of the tenders in the previous period must be taken as given. Clearly, initial conditions are given by the exogenous variables $Y_{t_0-1,C}$ and $Y_{t_0-1,D}$.

A number of simplifications are made in the subsequent analysis. Firstly, we will assume that the monetary authority faces no problem whatsoever with injecting its chosen allotments into the banking system. In reality, the assumption is typically satisfied, due to significant excess demand in the main refinancing operations, and to the neutral allotment policy chosen by the ECB.[18] The operations for which the Eurosystem has not been able to allocate the desired amount of liquidity were just those subjected to strategic underbidding. Given that we consider the theoretically optimal allotment policy, which is not necessarily equivalent to the benchmark allotment rule in the definition of the ECB (more on this at the end of this section), it is not apparent how restrictive the assumption is. Nevertheless, we believe that useful insights can be gained even under this simplifying assumption.

Another simplification is that we will consider the limit case of τ_s being close to 1. This means to ignore deviations from the target values that occur after

[18] In main refinancing operations, banks are allowed to submit up to ten bids, each of which consists of an interest rate and a volume. All bids specifying an interest rate above the marginal rate are satisfied, and bids on the marginal rate are prorated. By excess demand, we mean then the sum of unsatisfied bids with interest rates at or below the marginal rate.

the realization of the autonomous factor shock. The general conclusions are not affected. E.g., in the most relevant case when the target rate lies in the center of the corridor, then having τ_s strictly below 1 is equivalent to a linear transformation of the interest rate criterion, i.e., in a change in μ.

Finally, we will use a linear approximation for the liquidity effect. The first-order Taylor expansion of the right-hand side of equation (3) with respect to Z_t around Z^* reads

$$r_\mathcal{T} \approx r^* - \rho(Z_t - Z^*),$$

where we have used (4), and where

$$\rho := G'(Z^* - \overline{R})(r^L - r^D)$$

is a measure of the liquidity effect.[19] With the help of these simplifications, the objective function of the central bank, viewed as a function of the individual allotments, becomes an infinite sum of quadratic functions, as the following auxiliary result shows.

Lemma 1. *Using a linear approximation for the liquidity effect, we obtain*

$$\lim_{\tau_s \to 1} U(\tau_s) = \frac{1}{|\ln \delta|} \sum_{t=t_0}^{\infty} \delta^{t-t_0} F(\Delta Y_{t-1,C}, \Delta Y_{t-1,D}, \Delta Y_{t,A}, \Delta Y_{t,B}, \Delta Y_{t,C}, \Delta Y_{t,D}),$$

where $F(.)$ is the quadratic form

$$\begin{aligned} & F(\Delta Y_{t-1,C}, \Delta Y_{t-1,D}, \Delta Y_{t,A}, \Delta Y_{t,B}, \Delta Y_{t,C}, \Delta Y_{t,D}) \\ = \; & x_t' K x_t + y_t' Q y_t + 2 y_t' H x_t, \end{aligned}$$

for matrices K, Q and H that are functions of the exogenous parameters of the model.

[19] The first-order approximation replaces the typically sigmoid liquidity effect (cf. Appendix A) by a piecewise linear function with kinks at the marginal lending and deposit rates. This raises two questions. The first concerns the validity of the subsequently used dynamic programming techniques in view of the boundary conditions (the kinks due to the standing facilities) in our model. However, as the simulations show, the respective deviations from interest rate and liquidity targets are declining over the maintenance periods. Thus, when the boundary conditions are not binding in the initial maintenance period, they also do not bind in subsequent periods. This argument suggests that the shape of the feasibility set is not affected by the boundary conditions unless binding in the initial maintenance period. Another question is what would happen to our results when the kinks are smoothed out. The answer here can be best illustrated using Figure 9. Start with the diagram on the left-hand side. With a sigmoid liquidity effect, the trade-off between liquidity and interest-rate deviation becomes concave left of L^*, and convex right of L^*. The change of coordinates brings us to the diagram on the right-hand side. The graphs suggest that the convex-shaped curve between points A and B in this diagram would bend in and become less convex, or would even become concave over some region. Thus, it appears that a smoothing-out of the kinks in our model would even amplify the non-convexity of the feasibility set, so that we are on the safe side here as well.

Proof. See the appendix. □

This reformulation of the problem allows to apply the tools of linear-quadratic dynamic programming, and we obtain a formula for the optimal allotment policy.

Theorem 1. (Optimal dynamic policy) *The optimal allotment policy is of the form*

$$y_t = -(Q + \delta B' P B)^{-1} H x_t,$$

where P *is a constant matrix of dimension* (2×2) *and* B *is the matrix of dimension* (2×4) *on the right-hand side of equation (6).*[20]

Proof. See the appendix. □

The simulations of our calibrated model (see the appendix for details on this) indicate that the central bank's problem is characterized by the following economics. There is a motive to inject an amount of liquidity in the four tenders of each maintenance period so that the target rate will be approximated as closely as possible. At the same time, this comes at the cost of missing the instantaneous liquidity target. The optimal policy will therefore keep the respective sum of the sizes of two consecutive tenders approximately equal to the instantaneous tender liquidity in the market immediately before the respective tender (only approximately because of the discount factor), yet will deviate from this principle in tenders A and D. In tender A, the idea is to obtain a better overall level of liquidity than the one that resulted as an overlap from the last period. In tender D, the monetary authority seeks also to achieve closer-to-the-stationary-level conditions for the next maintenance period. The optimal policy is a result of balancing all these trade-offs (cf. Figure 3). Similar considerations are possible in the case of an initial excess liquidity (cf. Figure 4). Of course, there is no trade-off under normal conditions.

The ECB follows a specific "neutral" or benchmark allotment rule, which means that there is a formula that allows to calculate, prior to any main refinancing operation, the proper allotment (cf. [8]). The term *neutral* refers to the fact that the allotment brings the aggregate accumulated liquidity position in the banking system to its pro-rata value within the maintenance period at the time of the subsequent tender in the same period (or, if there is no subsequent tender within the same period, at the end of the maintenance period).

[20] With some effort, one can even show that the matrix P has four identical entries, but this fact has no obvious economic implications, so that we omit the proof.

The analysis reveals the notable fact that *the benchmark allotment rule in the definition of the ECB is not equivalent to the theoretically optimal allotment policy.* E.g., in the example exhibited in Figure 3, the benchmark allotment in tender A would be somewhat larger than the theoretically optimal allotment. This is so because the benchmark allotment in tender A would be chosen so as to compensate fully the initially red position of the banking system by the time of tender B. That is, in the absense of interim shocks, the benchmark allotment would generate a balanced aggregate liquidity position at time $t+\tau_B$. In contrast, the theoretically optimal allotment in tender A (as shown in Figure 3) is smaller, putting part of the balancing of the liquidity shortage to the subsequent tenders B, C, and D. The theoretically optimal allotment thereby takes account of the fact that a smaller deviation from the liquidity target, albeit for a somewhat longer duration, is preferable for a central bank optimizing our suggested objective function (5). The benchmark allotment rule thereby appears "more proactive" than the theoretically optimal policy.

4 Allotment in the last tender

The decision over the allotment in the last operation in a given maintenance period is typically considered to be the most important allotment decision, because it determines the liquidity conditions at the end of the period, and thereby the level of the prevailing interest rates. In the absence of shocks, there is no trade-off between the criteria of interest rate and liquidity smoothing, and the central bank may simply allot the benchmark amount, and may thereby reach both interest-rate and liquidity target. However, in general, this will not always be the case.

The trade-off will be especially pronounced after an occurrence of underbidding in the penultimate operation. The term *underbidding* refers to a situation where the total of the incoming bids in a central bank operation is lower than the benchmark allotment. As in the given operation, the central bank can only allocate liquidity by satisfying incoming bids, the central bank's allotment is bound to be below the benchmark allotment. In this sense, underbidding precludes the implementation of the the benchmark allotment rule, and generates a trade-off between liquidity and interest rate smoothing.

The formal set-up is as follows (cf. Figure 5). The information available to both the market and the central bank at the time of tender D are the allotments in the tenders up to tender C in period 0. The aggregate liquidity counting for the fulfilment of reserve requirements in period 0, and provided by the central bank in the operations prior to tender C, can be summarized in the term

$$\Lambda_{0,B} := \tau_A \Delta Y_{-1,C} + \tau_B \Delta Y_{-1,D} + (\tau_C - \tau_A)\Delta Y_{0,A} + (\tau_D - \tau_B)\Delta Y_{0,B}.$$

The volume $Y_{0,C}$ of the penultimate tender C in period 0 is also exogenous. Underbidding in tender C is formally defined as

$$\Lambda_{0,B} + (\tau_D - \tau_C)\Delta Y_{0,C} < 0.$$

Under this condition, the banking system has on aggregate build up a red position at the time of tender D, i.e., in order to satisfy reserve requirements as an average over the maintenance period, the outstanding central bank credit after tender D must exceed L^*.

In contrast to the previously studied setting, choice variables for the central bank are now the volume $Y_{0,D}$ of the last tender in period 0, and the allotment vector y_t for all subsequent periods $t \in \{1, 2, ...\}$. The resulting liquidity and interest rate conditions are determined as before. The central bank's objective function is

$$\widehat{U} = -E[\int_{\tau_D}^{\infty} \delta^{\tau - \tau_D} \left((r_\tau - r^*)^2 + \mu (L_\tau - L^*)^2 \right) d\tau],$$

with choice parameters $Y_{0,D}$ and $(y_t)_{t=1,2,...}$.

There are two focal allotment sizes. The first is the benchmark amount Y^I that guarantees that the overall liquidity position at the end of the maintenance period is "neutral," so that the market rate reaches both the marginal lending and the deposit rate with equal probability. After an operation C that suffered from insufficient demand, the benchmark amount is typically large, so that the allotment causes a sizable deviation from the aggregate liquidity target. The benefit, however, is that the interest rate target is reached without any deviation. The second focal allotment Y^L is the one that generates, from the settlement day of tender D onwards, an outstanding central bank credit of L^*. We will refer to Y^L as the *liquidity-refill allotment*. Here the cost is that the interest rate will rise and potentially reach the marginal lending rate. It is not difficult to check that only allotments that lie within the interval defined by these two extreme positions can be optimal for a central bank that maximizes a weighted sum of the two errors.

Simulations have been performed by first choosing $Y_{0,D}$ and using subsequently the optimal policy derived in the previous section (for $t_0 = 1$). Somewhat surprisingly at first sight, we found that the feasibility set in the two-dimensional error space, defined by allotment sizes between Y^I and Y^L, is essentially concave rather than convex (cf. Figure 6). As a consequence, when the central bank minimizes a weighted sum of quadratic deviations from both liquidity and interest rate targets, then the optimal choices are the liquidity-refill allotment Y^L and an amount that is slightly smaller than the benchmark allotment Y^I.

The simulations suggest the following two comparative statics exercises. Firstly, the more severe the underbidding, the more costly will it be for the central bank to make the benchmark allotment. However, the cost in terms of an interest rate deviation is typically not further increased due to the marginal lending facility. Thus, ceteris paribus, the more pronounced the underbidding, the more appropriate appears the liquidity-refill amount (cf. Figure 7). As this prediction essentially corresponds to a non-accomodating central bank strategy that is conditional on underbidding, this argument shows that the behavior observable in the data is consistent with an optimal allotment policy.[21] A similar comparative statics result is valid concerning the timing of the operations. Specifically, the later the tenders in the period, the less costly is the interest rate deviation for the central bank, and the more appropriate is the tightness at the end of the period (cf. Figure 8).

It appeared to us that the mechanics of the model do not depend on the infinite horizon of the planning problem. In fact, the simulations performed on the basis of the intertemporal model suggest that the impact of the allotment in tender D on the interest rate is strongest in the current maintenance period, and that the impact on the outstanding liquidity is strongest before tender A in the subsequent period. To discuss the optimal allotment in tender D, it may therefore be sufficient, for most practical purposes, to ignore any deviations from the interest rate target after the end of the current period, and any deviation from the liquidity target after the time of tender A in the subsequent period. In the next section, we will therefore consider a boiled-down version of the model with the intention to reveal the first-order mechanics underlying the simulation results.

[21]The reason for this somewhat surprising result may be the fact that the assumed objective function ignores volumes. Even if the shadow cost of further tightening supply may be zero, the banking sector would "feel" the additional tightness in the form of higher and not remunerated funding costs.

5 A first-order approximation of the model

We consider the same situation as in the previous section (see Figure 6). The following parameters are exogenous. Let Λ_B denote the interim excess liquidity accumulated immediately after tender B, let Y_C be the volume of tender C, and let ρ denote the end-of-period elasticity (i.e., the liquidity effect). Again we would like to answer the following question: How to choose the volume of tender D so as to minimize the weighted quadratic deviations for both interest rate and liquidity targets?

In order to reach an explicit solution to the problem, we will make the following approximations. Firstly, as discussed above, ignore any deviation from the interest rate target after the end of the current period, and any deviation from the liquidity target after the time of tender A in the subsequent period. According to the simulations performed, these deviations add little to the total deviation. Next, ignore the discounting. This is no major loss given the reduced horizon. Finally, as before, assume that the liquidity shock occurs very close to the end of the period. Under these assumptions, the central bank's problem simplifies as follows:[22]

$$\min_{Y_D} \ (r - r^*)^2 + \widehat{\mu}(L - L^*)^2 \tag{7}$$

s.t.

$$r - r^* = -\rho(\Lambda_B + (1 - \tau_C)(Y_C - \frac{L^*}{2}) + (1 - \tau_D)(Y_D - \frac{L^*}{2})) \tag{8}$$

$$L - L^* = (Y_C - \frac{L^*}{2}) + (Y_D - \frac{L^*}{2}) \tag{9}$$

The trade-off is that a lower Y_D leads to an increased market rate, while the outstanding liquidity is lowered. We wish to draw the feasibility set, i.e., the set of combinations of quadratic interest rate and liquidity deviations that result from allotments that lie below the sum of total bids in the ultimate tender.

As in the previous section, there are two allotments that jump into the eyes of the observer. The first is the benchmark amount, i.e., the allotment that ends the maintenance period with an average liquidity position that allows banks to satisfy reserve requirements in a regular way: the characteristic of this amount

[22]The relative weight of the liquidity criterion is now

$$\widehat{\mu} = \mu \frac{1 - \tau_D}{1 + \tau_A - \tau_D}$$

because the central bank cares for the deviation from the liquidity target between tender D and next period's tender A.

is that it implies that the market rate is equal to the target, i.e., $r = r^*$. Using (8), this is equivalent to

$$\Lambda_B + (1 - \tau_C)(Y_C - \frac{L^*}{2}) + (1 - \tau_D)(Y_D - \frac{L^*}{2}) = 0.$$

Rearranging gives

$$Y_D = \frac{L^*}{2} - \frac{1}{1 - \tau_D}\{\Lambda_B + (1 - \tau_C)(Y_C - \frac{L^*}{2})\}.$$

This formula is a theoretical counterpart to the formula for the benchmark allotment published in ECB [8]. With this allotment, the interest rate target is met, but the liquidity target is missed by

$$\begin{aligned} Y - Y^* &= Y_C + Y_D - Y^* \\ &= -\frac{\tau_D - \tau_C}{1 - \tau_D}(Y_C - \frac{L^*}{2}) - \frac{1}{1 - \tau_D}\Lambda_B. \end{aligned}$$

The second focal allotment size is the liquidity refill that matches the target $L = L^*$ for the outstanding central bank credit. Clearly, (9) implies

$$Y_D = L^* - Y_C,$$

and the corresponding deviation from the interest rate target is given by

$$r - r^* = \rho(\Lambda_B + (\tau_D - \tau_C)(Y_C - \frac{L^*}{2})).$$

What is the shape of the feasibility set between these two points? To answer this question, we express $(r - r^*)$ as a function of $(L - L^*)$. From (8) and (9) we obtain

$$r - r^* = -\rho(\Lambda_B + (\tau_D - \tau_C)(Y_C - \frac{L^*}{2}) + (1 - \tau_D)(L - L^*)).$$

Rearranging gives the feasibility constraint for the monetary authority:[23]

$$(r - r^*) + \rho(1 - \tau_D)(L - L^*) = -\rho(\Lambda_B + (\tau_4 - \tau_3)(Y_C - \frac{L^*}{2})) \qquad (10)$$

After an occurrence of underbidding, the right-hand side of this budget equality is strictly positive (by definition). The monetary authority therefore faces a linear trade-off between interest rate and liquidity smoothing. Figure 9 shows how this trade-off translates into a convex feasibility set in the plane of quadratic deviations by a change of parameters. In particular, in the neighborhood of the benchmark allotment, the feasibility set has locally the shape of a parabola.

[23] This constraint is binding when the central bank has chosen not to have recourse to additional operations.

Introducing the standing facilities means to essentially cut off the upper part of the feasibility set. More precisely, the standing facilities restrict the deviation of the market rate from the target rate in both directions: We have

$$\begin{aligned}(r-r^*)^2 &\leq (r^L-r^*)^2 \text{ if } r>r^* \\ (r-r^*)^2 &\geq (r^D-r^*)^2 \text{ if } r<r^*.\end{aligned}$$

Figure 10 shows schematically the shape of the feasibility set in the case where the target rate lies in the center of the corridor, i.e., $r^* = (r^L + r^D)/2$, and the underbidding has been sufficiently pronounced.[24]

Critical role of the liquidity situation at the last tender. Optimal is either the liquidity-refill amount or an interior solution that is typically close to, and somewhat smaller than, the benchmark amount. If an interior solution is optimal, it must solve problem (7), which ignores the standing facilities. Plugging the constraints (8) and (9) into the central bank's objective function and differentiating with respect to Y_D yields the optimal allotment in the case of an interior solution

$$\Delta Y_D = -\frac{(1-\tau_D)\rho^2}{(1-\tau_D)\rho^2+\widehat{\mu}}\Lambda_B - \frac{(1-\tau_C)(1-\tau_D)\rho^2+\widehat{\mu}}{(1-\tau_D)^2\rho^2+\widehat{\mu}}\Delta Y_C.$$

The resulting interest rate is

$$r - r^* = -\beta\{\Lambda_B + (\tau_D-\tau_C)\Delta Y_C\},$$

where

$$\beta = \frac{\widehat{\mu}\rho^2}{(1-\tau_D)^2\rho^2+\widehat{\mu}}.$$

Thus, the central bank's problem possesses an interior solution if and only if the aggregated liquidity position at the time of tender D is moderate in the sense that it satisfies

$$\beta^{-1}(r^D - r^*) < \Lambda_B + (\tau_D-\tau_C)\Delta Y_C < \beta^{-1}(r^L-r^*).$$

These inequalities characterize the areas of interest. To start with, if the aggregate liquidity position at the time of tender D is below the lower threshold, i.e., if

$$\Lambda_B + (\tau_D-\tau_C)\Delta Y_C < -\beta^{-1}|r^D-r^*|, \tag{11}$$

then we had a significant demand reduction in tender C that did not allow the central bank to inject the benchmark amount.

[24]Obviously, if the underbidding is mild, so that even the liquidity-neutral allotment would not cause the market rate to reach the marginal lending rate, then the trade-off between liquidity and interest-rate smoothing is convex, and an interior solution is optimal.

In this case, the benchmark amount in tender D would be too large, so that the liquidity target cannot be met. The result is that the central bank optimally relieves only part of the tightness in the money market.[25] The comparative statics results are now easy to obtain. As β does not depend on ΔY_C, the boundary solution is more likely to be optimal when ΔY_C is small (cf. again Figure 7). Similarly, when the timing parameters τ_C and τ_D are increased marginally by the same amount then $\widehat{\mu}$ increases, and the right-hand-side of the inequality (11) increases, so that a boundary solution becomes more likely (cf. again Figure 8). Thus, the simplified model confirms the results of the simulation study.[26]

In the other polar case, the aggregate liquidity position is above the upper threshold, i.e.,

$$\Lambda_B + (\tau_D - \tau_C)\Delta Y_C > \beta^{-1}(r^L - r^*), \tag{12}$$

which would mean that the monetary authority has decided to allot in tender C significantly more than the benchmark amount. In reality, the probably more likely case is that a liquidity-providing shock occurs between tenders C and D. The reader will note that this scenario can be captured by the current model by adding the shock to the left-hand-side of equation (12). There are again two focal allotment sizes. The first is obviously the benchmark amount Y^I, that guarantees that the overall liquidity position at the end of the maintenance period is such that the market rate reaches both the marginal lending and the deposit rate with equal probability. The benchmark would typically be small in this scenario.

The second focal allotment Y^L, typically larger in this scenario, is the one that generates, from the settlement day of tender D onwards, an outstanding central bank credit that corresponds to the target L^*. Allotting Y^L means here to flood the market with liquidity, so that the market rate would drop to the deposit rate. However, in contrast to the scenario of an undersized tender C, the theoretically optimal allotment may not be feasible due to insufficient demand. One can show that the shape of the feasibility set is very similar to the underbidding case. In particular, the monetary authority faces a non-convex

[25] Apparently, the non-accomodating behavior has been anticipated by the majority of banks, inducing them to bid more aggressively in the respective last tenders, documented by a significant deviation of the marginal rate from the minimum bid rate.

[26] The discussion reveals a potential rationale for the neutral allotment policy, that is chosen by the ECB also for tenders before the last tender: e.g., in the penultimate tender, when the corridor is symmetric, and there is an additional liquidity shock between tender C and D, then making the neutral allotment implies that the probability of an interior solution, i.e., of not reaching the boundary of the corridor, is maximized.

trade-off.[27] The comparative statics is also intuitive. When the excess liquidity at the time of tender D increases, then the trade-off more often will favor the boundary solution, i.e., the (small) liquidity-refill allotment. Likewise, if the tenders C and D are closer to the end of the maintenance period, then the benchmark will ceteris paribus be smaller, and the boundary solution will be optimal more often.

6 The changes to the operational framework

The formal analysis in this paper has focused on the operational framework, as it has been in place from January 1999. As already mentioned in the introduction, the operational framework for montary policy implementation in the euro area will be slightly changed from March 2004 onwards. The new framework will rely on non-overlapping transactions with a maturity of one week only. Transactions will also not hang over into the subsequent period, and the Governing Councel confines himself to making policy decisions only at the beginning of maintenance periods (see [9]).

Clearly, there is no fundamental difficulty in modifying the model so that it can be used to analyze the optimal allotment policy in the new framework. To see this, assume that we have still four tenders per period, and that transactions allotted in a tender mature at the time of the next tender. Moreover, assume that $\tau_A = 0$, meaning that the first tender is at the beginning of the maintenance period and that the transactions allocated in the last tender mature at the end of the maintenance period. It is clear that under the new framework, the intertemporal nature of the problem disappears, so that we can restrict ourselves to a discussion of one period. We may therefore drop the index t for the maintenance period in the sequel.

Proposition 1 is valid independent of the timing and maturities of the operations, when we take account of the fact that the formula for $Z_t = Z$ changes. Specifically, we would get that

$$Z = \tau_B Y_A + (\tau_C - \tau_B) Y_B + (\tau_D - \tau_C) Y_C + (1 - \tau_D) Y_D.$$

The changes to the other formulas are similarly straightforward, so they are omitted. Instead, we will confine ourselves to a brief discussion as to whether the

[27] The feasibility constraint of the central bank remains the same, only that the right-hand-side of equation (10) is negative. As the deviations from interest-rate and liquidity target are squared, this leads to the same shape for the feasibility set as in the case of underbidding.

conclusions of this paper would remain valid. To start with, it does not appear that the less proactive nature of the optimal allotment policy, when compared to the ECB's benchmark allotment rule depends on the specific timing of the operations. Even with one-week maturities, and without overlap, we would expect that a liquidity imbalance would be compensated only gradually under the optimal policy, so that the benchmark allotment rule would continue to appear more "proactive" than the optimal policy.

Second, concerning the optimal allotment in the last tender after a liquidity imbalance, it is of course less likely under the new regime that underbidding occurs. However, in principle, a liquidity shock during the maintenance period has the same effect. Then again, we would get a trade-off between liquidity and interest-rate smoothing, in fact not very different from the one that we discussed in Section 5. As a consequence, the discontinuity in the reaction function will arise. Thus, the conclusions of this paper apparently do not depend on the timing and maturity of the operations in the maintenance period.

7 Conclusion

We have derived the optimal intertemporal allotment policy in a model that captures some of the institutional features of the Eurosystem's operational framework for monetary policy implementation. It was shown that the theoretically optimal allotment policy is not equivalent to the ECB's benchmark allotment rule. Specifically, it was found that the benchmark allotment rule rebalances the liquidity situation, intuitively speaking, at the first opportunity, while the theoretically optimal policy would typically compensate the same imbalance more gradually.

The formal analysis suggests also that, somewhat surprisingly, creating a substantial monetary tightness at the end of the maintenance period after an occurrence of underbidding may be consistent with pursuing an optimal allotment policy. The rationale for this conclusion is that when the volume of the penultimate tender in a maintenance period has been insufficient, and the last tender is close to the end of the period, then the benchmark allotment would be very large. Injecting the benchmark allotment would therefore imply that the outstanding central bank credit is temporarily much larger than usual. On the other hand, once the allotment is chosen to be tight, there will be no further deviation from the interest rate target by a somewhat tighter allotment. As a consequence, it may be optimal to allocate only the amount that aligns aggregate outstanding credit with the target level. In fact, it follows from these arguments that the optimal reaction of the monetary authority will be a discontinuous function of the prevailing liquidity conditions, which would provide a possible explanation for the Eurosystem's recent deviations from the benchmark allotment rule.

8 Appendix A. Calibration and simulation

In this appendix, we will briefly describe the performed calibration and the simulation methods.

Calibration. Most parameters can be set to realistic values without difficulty. This applies to the key policy rates r^*, r^L, r^D, to the reserve requirements $\overline{R}$, to the timing parameters $\tau_A, \tau_B, \tau_C, \tau_D$, and to the initial conditions $Y_{-1,C}, Y_{-1,D}$. The discount rate was set very low, reflecting our hypothesis that the central bank cares about interest-rate and liquidity conditions prevailing in the current period much more than about the corresponding conditions in the subsequent period.

The only parameter that requires some care is the liquidity effect.[28] In order to measure ρ, we have collected daily liquidity data as well as the liquidity forecasts that are published by the ECB prior to the main refinancing operations, covering the period January 1, 1999 to October 9, 2002. We have generated from the data set the historical distribution of the autonomous factor shock accumulated between the last tender and the final day of the maintenance period. By autonomous factor shock, we mean here the actual shock minus the figure that has been predicted in the ECB's liquidity forecast. The historical distribution has then been transformed into the curve depicted in Figure 11 in the following way.

Start with a definite value for the liquidity surplus or shortage created by the allotment in the last tender, say, $\Delta L_t = 5$ bn euro days in excess. We wish to determine the expected value for the market rate at the end of the period. Presupposing equation (3), we have

$$E[r_t^s] = r^D + \text{prob}\{\Delta L - A_t < 0\}(r^L - r^D).$$

[28]See Hamilton (1997) for a quantification of the liquidity effect in the market for U.S. Federal Funds.

Thus, it suffices to estimate the probabilities that the end of the period will be tight. This will be the case if and only if the autonomous factor shock absorbes a liquidity of 5 bn euro days or more. So we derived from the data the relative likelihood that a shock after the last operation will have this property. It turns out that in about 25% of the historical cases, we had a shock absorbing 5 bn euro days or more after the last operation. Taking expectations, and assuming a ± 1 percent width of the interest rate corridor, yields a market rate that lies 0.5 percentage points below the mid of the corridor. The other diamonds in Figure 11 are calculated analogously.

The curve is obviously non-linear, but when a rough linear approximation is chosen, then we have a slope of approximately 8 basis points per bn euro days. That is, if a liquidity-absorbing shock of, say 3 bn euro days occurs after the last operation in the period, then the expected rate at the end of the maintence period would (in the linear approximation) be 24 basis points above the midpoint between the marginal lending facility and the deposit facility. The interpretation of this figure is that if banks use their information about the autonomous factor shock, then a tightness of 3 bn euro days, created by a smaller-than-neutral allotment in the last tender, would drive up market rates by the beforementioned 24 basis points.[29]

Simulation. Having calibrated the parameters, we calculated the optimal policy numerically for a number of inititial conditions. It turned out to be useful that the policy matrix P can be numerically approximated as the limit of the sequence $P^{(k)}$, defined by

$$
\begin{aligned}
P^{(0)} &= 0 \\
P^{(k)} &= K - H'(Q + \delta B' P^{(k-1)} B)^{-1} H,
\end{aligned}
$$

which allows a numerical computation of the optimal allotment policy. The iterative solution to the Riccati equation has been found by explicitly calculating 20 rounds of the approximation. Convergence was very fast, so that the chosen number of iteration steps was much higher than needed in all calculated examples. Further details on calibration and simulation are available from the first-named author.

[29] Our estimate lies in the range of other quantifications. See Ejerskov et al. [10], as well as references given therein.

9 Appendix B. Proofs

This appendix contains the proofs of Lemma 1 and Theorem 1.

Proof of Lemma 1. We rewrite the central bank's objective function as an infinite sum over integrals

$$\begin{aligned} U(\tau_s) &= -E[\int_{t_0}^{\infty} \delta^{\tau - t_0} \left((r_\tau - r^*)^2 + \mu(L_\tau - L^*)^2\right) d\tau] \\ &= -\sum_{t=t_0}^{\infty} \delta^{t-t_0} E[\int_{t}^{t+1} \delta^{\tau} \left((r_\tau - r^*)^2 + \mu(L_\tau - L^*)^2\right) d\tau] \end{aligned}$$

In the limit $\tau_s \to 1$, the market rate is constant in each maintenance period, and L_τ is given by (1). Using

$$\int_{\tau_0}^{\tau_1} \delta^{\tau} d\tau = \frac{1}{|\ln \delta|}(\delta^{\tau_0} - \delta^{\tau_1}),$$

we obtain

$$\begin{aligned} & \lim_{\tau_s \to 1} U(\tau_s) \\ = \; & -\frac{1}{|\ln \delta|} \sum_{t=t_0}^{\infty} \delta^{t-t_0} \{(1-\delta)(r_t - r^*)^2 \\ & +\mu\{(1-\delta^{\tau_A})(\Delta Y_{t-1,C} + \Delta Y_{t-1,D})^2 \\ & +(\delta^{\tau_A} - \delta^{\tau_B})(\Delta Y_{t-1,D} + \Delta Y_{t,A})^2 + (\delta^{\tau_B} - \delta^{\tau_C})(\Delta Y_{t,A} + \Delta Y_{t,B})^2 \\ & +(\delta^{\tau_C} - \delta^{\tau_D})(\Delta Y_{t,B} + \Delta Y_{t,C})^2 + (\delta^{\tau_D} - \delta)(\Delta Y_{t,C} + \Delta Y_{t,D})^2\}. \end{aligned}$$

Since

$$\begin{aligned} & r_t - r^* = -\rho\{\tau_A \Delta Y_{t-1,C} + \tau_B \Delta Y_{t-1,D} \\ & +(\tau_C - \tau_A)\Delta Y_{t,A} + (\tau_D - \tau_B)\Delta Y_{t,B} \\ & +(1-\tau_C)\Delta Y_{t,C} + (1-\tau_D)\Delta Y_{t,D}\}, \end{aligned}$$

we obtain

$$\begin{aligned} & \lim_{\tau_s \to 1} U(\tau_s) \\ = \; & -\frac{1}{|\ln \delta|} \sum_{t=t_0}^{\infty} \delta^{t-t_0} F(\Delta Y_{t-1,C}, \Delta Y_{t-1,D}, \Delta Y_{t,A}, \Delta Y_{t,B}, \Delta Y_{t,C}, \Delta Y_{t,D}), \end{aligned}$$

where

$$\begin{aligned} & F(\Delta Y_{t-1,C}, \Delta Y_{t-1,D}, \Delta Y_{t,A}, \Delta Y_{t,B}, \Delta Y_{t,C}, \Delta Y_{t,D}) \\ = \; & (1-\delta)\rho^2\{\tau_A \Delta Y_{t-1,C} + \tau_B \Delta Y_{t-1,D} \end{aligned}$$

$$
\begin{aligned}
&+(\tau_C-\tau_A)\Delta Y_{t,A}+(\tau_D-\tau_B)\Delta Y_{t,B}\\
&+(1-\tau_C)\Delta Y_{t,C}+(1-\tau_D)\Delta Y_{t,D}\}^2\\
&+\mu\{(1-\delta^{\tau_A})(\Delta Y_{t-1,C}+\Delta Y_{t-1,D})^2+(\delta^{\tau_A}-\delta^{\tau_B})(\Delta Y_{t-1,D}+\Delta Y_{t,A})^2\\
&+(\delta^{\tau_B}-\delta^{\tau_C})(\Delta Y_{t,A}+\Delta Y_{t,B})^2+(\delta^{\tau_C}-\delta^{\tau_D})(\Delta Y_{t,B}+\Delta Y_{t,C})^2\\
&+(\delta^{\tau_D}-\delta)(\Delta Y_{t,C}+\Delta Y_{t,D})^2\}.
\end{aligned}
$$

Rewriting the quadratic form $F(.)$ yields

$$
\begin{aligned}
& F(\Delta Y_{t-1,C},\Delta Y_{t-1,D},\Delta Y_{t,A},\Delta Y_{t,B},\Delta Y_{t,C},\Delta Y_{t,D})\\
= \; & x_t'Kx_t+y_t'Qy_t+2y_t'Hx_t,
\end{aligned}
$$

where the matrices are given by

$$
\begin{aligned}
K &= (1-\delta)\rho^2\begin{pmatrix}\tau_A\\ \tau_B\end{pmatrix}\begin{pmatrix}\tau_A\\ \tau_B\end{pmatrix}'\\
&\quad+\mu\begin{pmatrix}1-\delta^{\tau_A} & 1-\delta^{\tau_A}\\ 1-\delta^{\tau_A} & 1-\delta^{\tau_B}\end{pmatrix},
\end{aligned}
$$

$$
\begin{aligned}
Q &= (1-\delta)\rho^2\begin{pmatrix}\tau_C-\tau_A\\ \tau_D-\tau_B\\ 1-\tau_C\\ 1-\tau_D\end{pmatrix}\begin{pmatrix}\tau_C-\tau_A\\ \tau_D-\tau_B\\ 1-\tau_C\\ 1-\tau_D\end{pmatrix}'\\
&\quad+\mu\begin{pmatrix}\delta^{\tau_A}-\delta^{\tau_C} & \delta^{\tau_B}-\delta^{\tau_C} & 0 & 0\\ \delta^{\tau_B}-\delta^{\tau_C} & \delta^{\tau_B}-\delta^{\tau_D} & \delta^{\tau_C}-\delta^{\tau_D} & 0\\ 0 & \delta^{\tau_C}-\delta^{\tau_D} & \delta^{\tau_C}-\delta & \delta^{\tau_D}-\delta\\ 0 & 0 & \delta^{\tau_D}-\delta & \delta^{\tau_D}-\delta\end{pmatrix},
\end{aligned}
$$

and

$$
H=(1-\delta)\rho^2\begin{pmatrix}\tau_C-\tau_A\\ \tau_D-\tau_B\\ 1-\tau_C\\ 1-\tau_D\end{pmatrix}\begin{pmatrix}\tau_A & \tau_B\end{pmatrix}+\mu\begin{pmatrix}0 & \delta^{\tau_A}-\delta^{\tau_B}\\ 0 & 0\\ 0 & 0\\ 0 & 0\end{pmatrix}.
$$

This proves Lemma 1. □

Proof of Theorem 1. Consider the optimal linear regulator problem where the objective function is to maximize

$$
\sum_{t=0}^{\infty}\delta^t\{x_t'Kx_t+y_t'Qy_t+2y_t'Hx_t\}
$$

subject to the law of motion

$$
x_{t+1}=By_t,
$$

where

$$B = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The maximization is with respect to sequences $\{y_t, x_t\}_{t=0}^{\infty}$. Then, the theory of quadratic linear dynamic programming (see problem 4.1 in Ljungqvist and Sargent, [16]) predicts that the optimal policy has the form

$$y_t = -(Q + \delta B'PB)^{-1}Hx_t,$$

where P solves the algebraic matrix Riccati equation

$$P = K + H'(Q + \delta B'PB)^{-1}H.$$

This proves the assertion. □

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Figure 1. The development of the EONIA within the interest rate corridor during the year 2001.

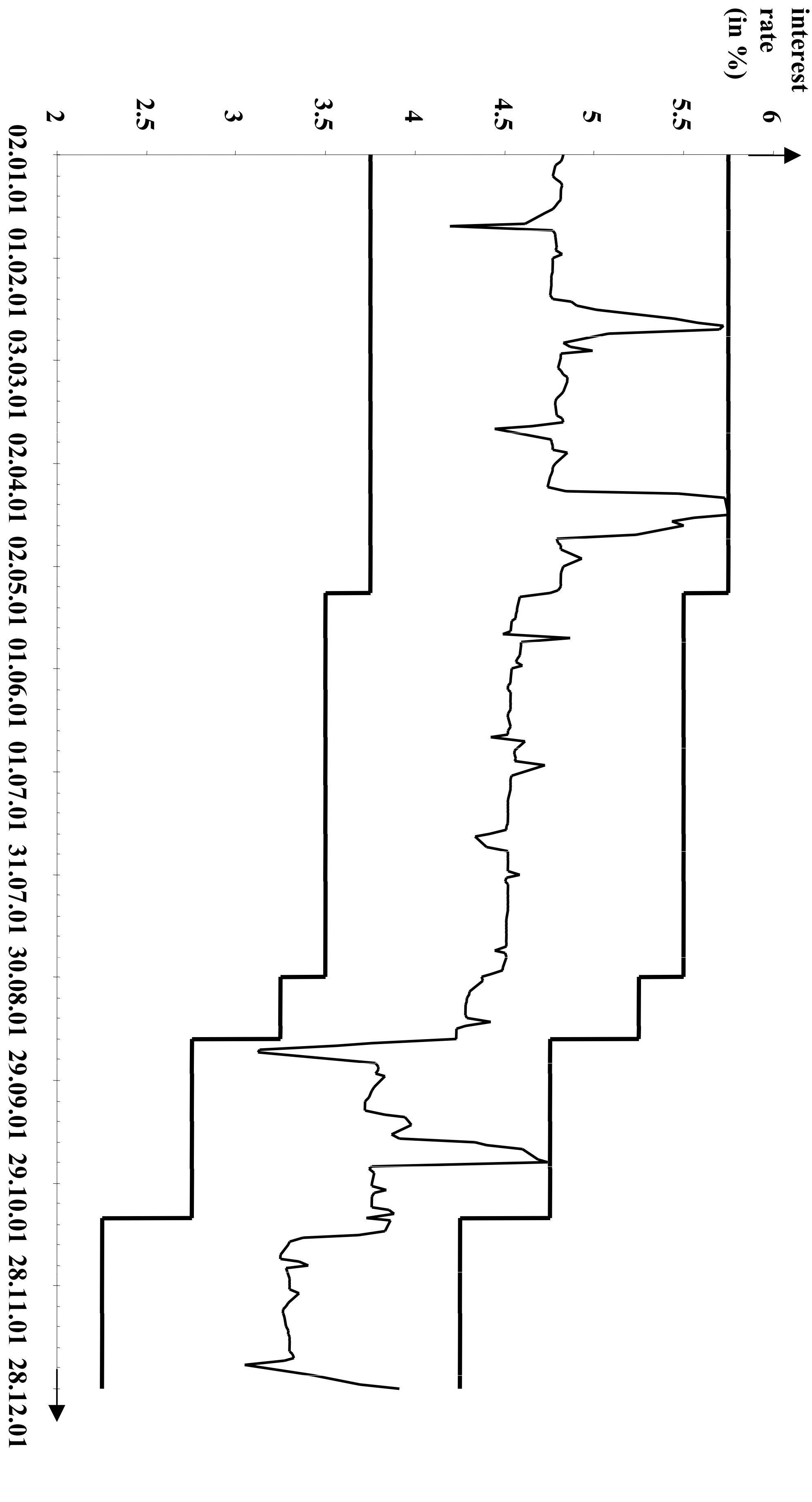


Figure 2. Time structure of the model: four tenders per maintenance period, overlapping maturities, autonomous factors, and standing facilities.

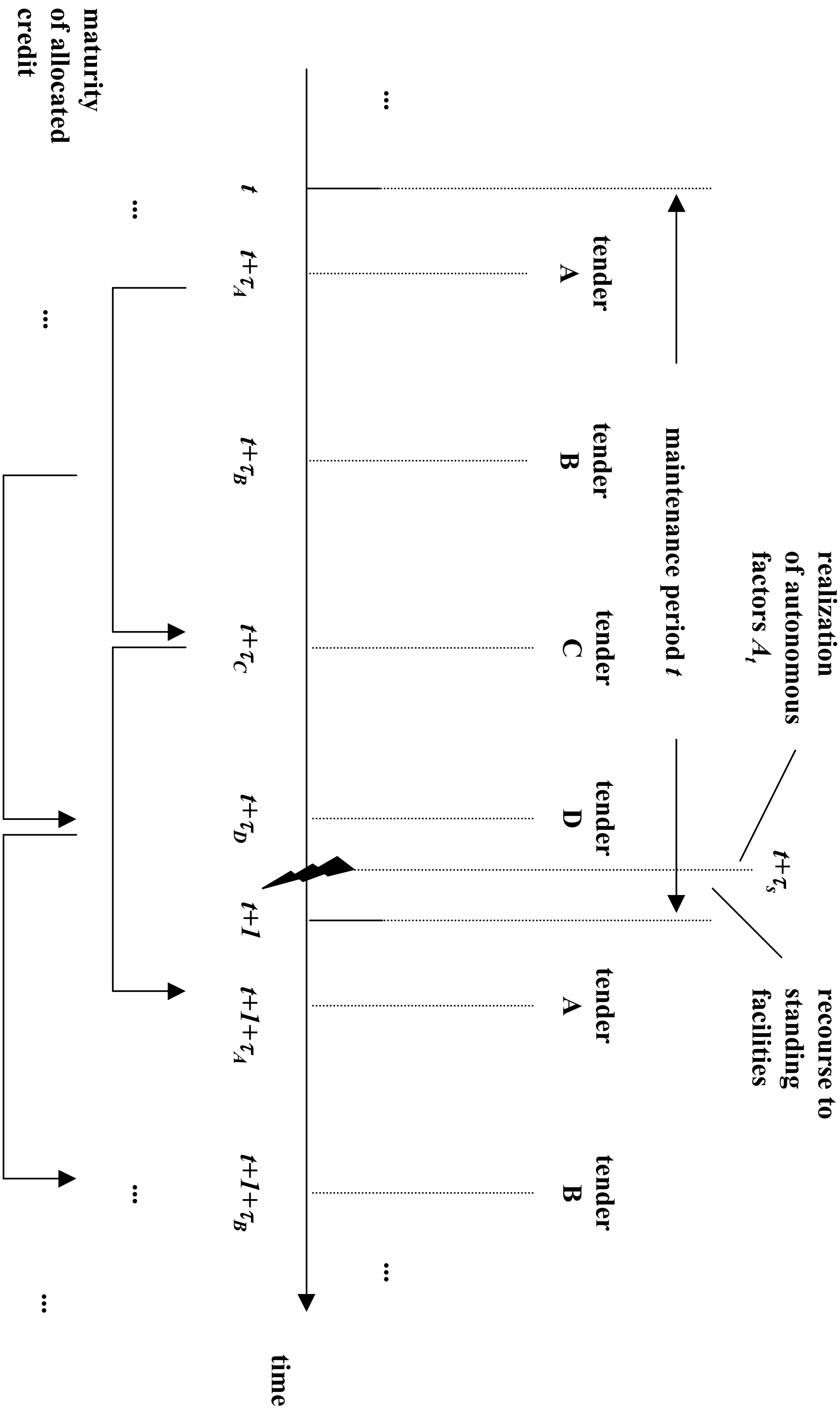


Figure 3. Smoothing response of the central bank to an initial liquidity deficit (we write t for t_0 for simplicity).

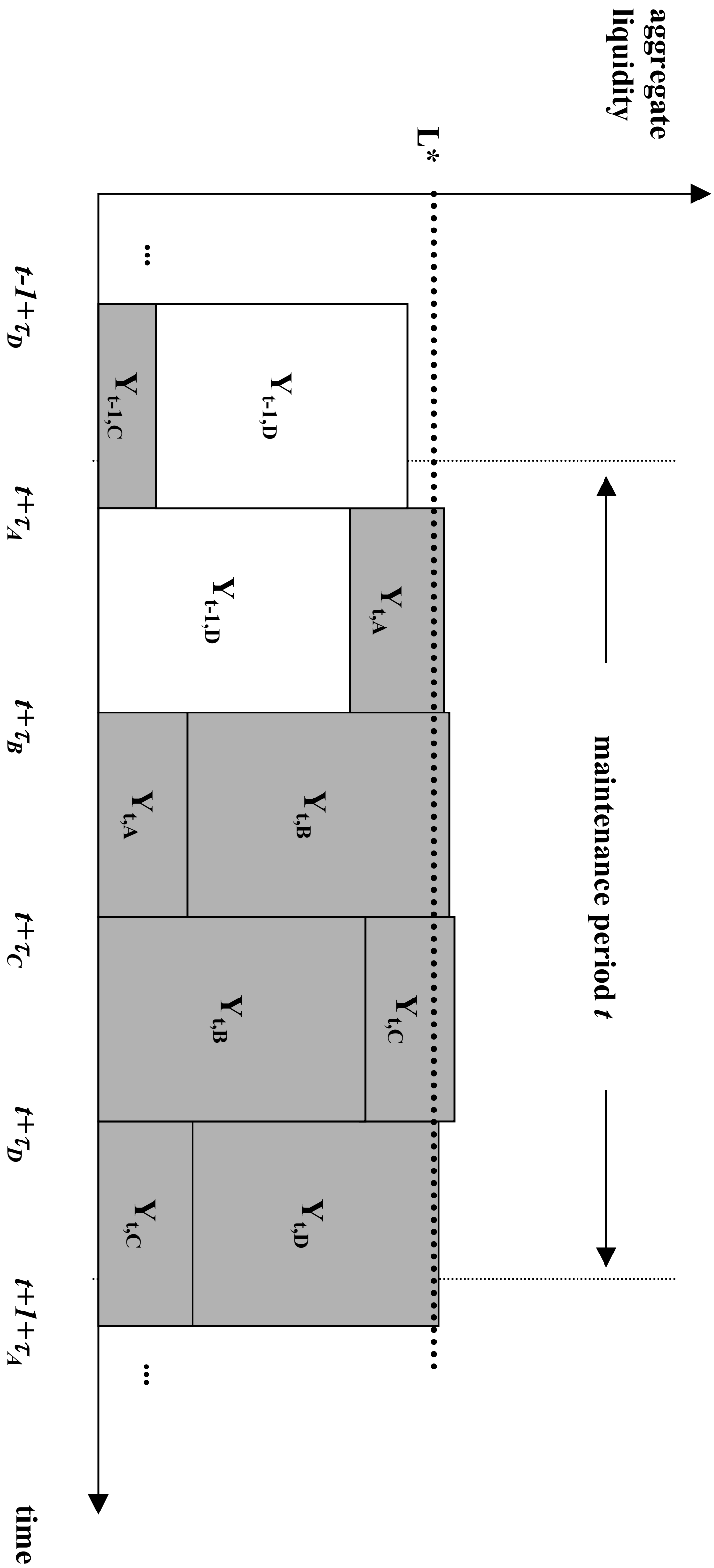


Figure 4. Smoothing response of the central bank to an initial excess liquidity.

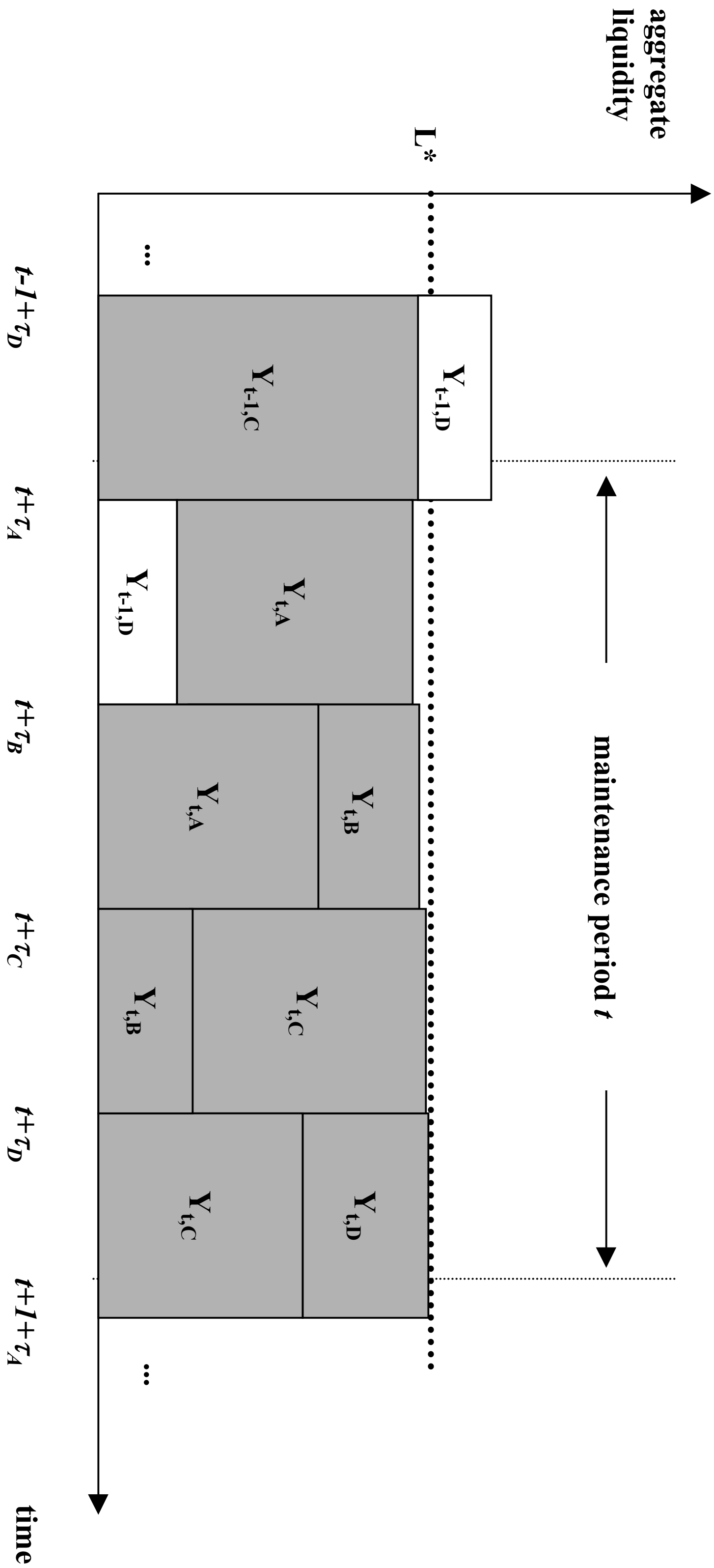


Figure 5. Choosing the allotment in tender D.

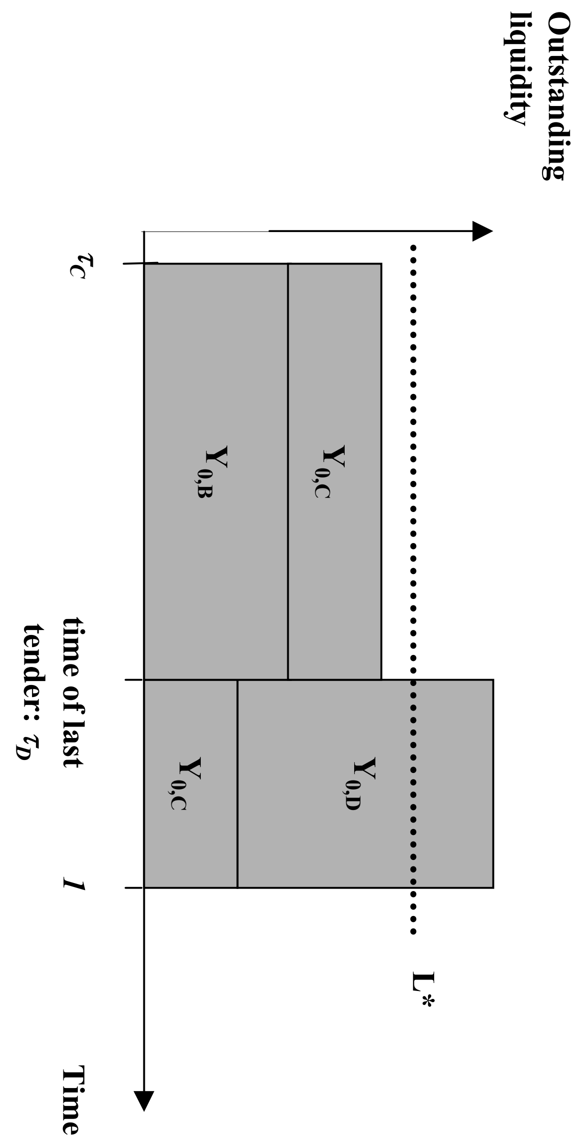


Figure 6. Tightness in response to demand reduction is consistent with an optimal allotment policy; the figure shows an calibrated example, assuming 20% underbidding in tender C.

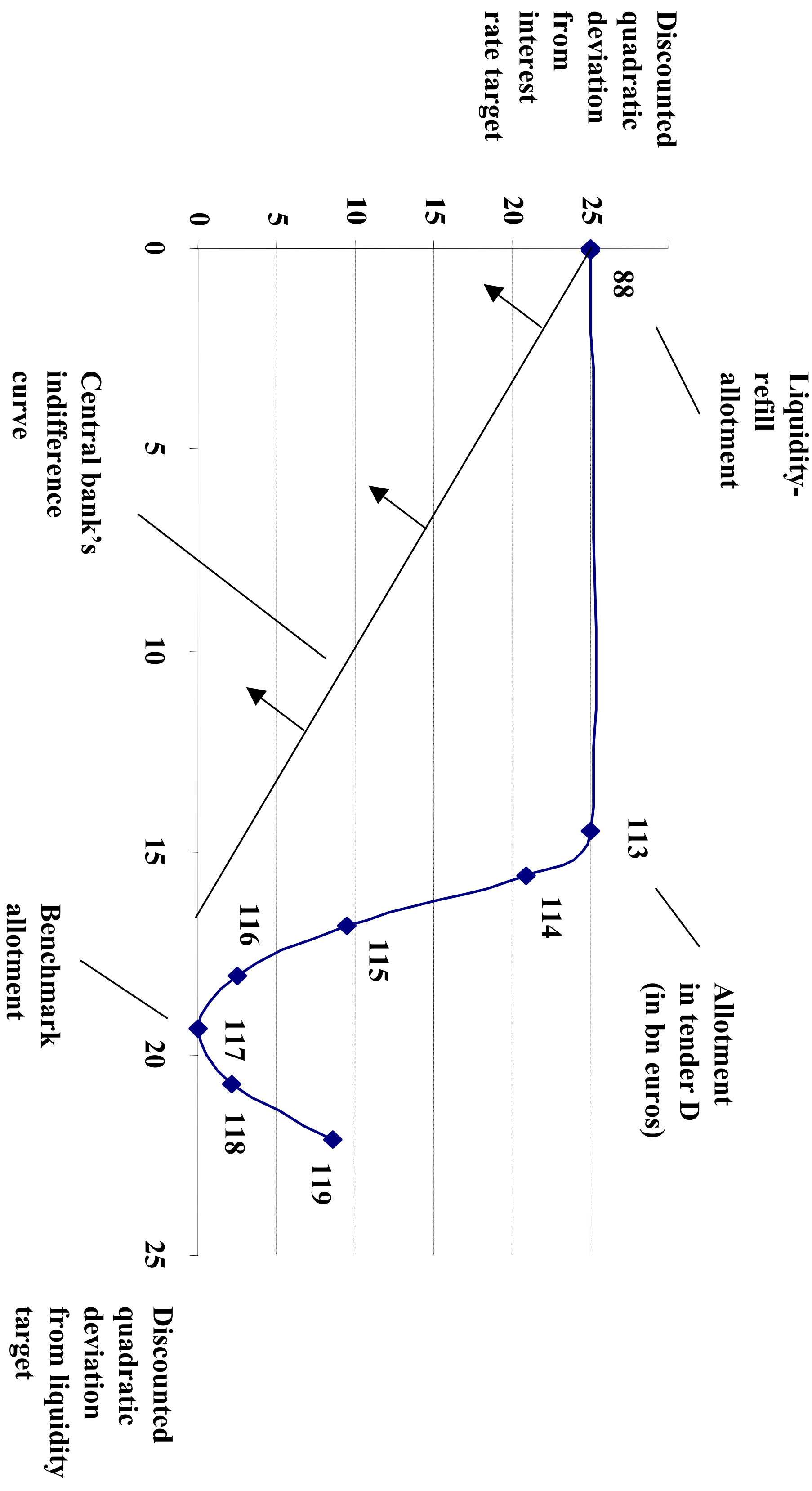


Figure 7. Strong underbidding suggests tight liquidity provision. Comparative statics with respect to the volume of tender C.

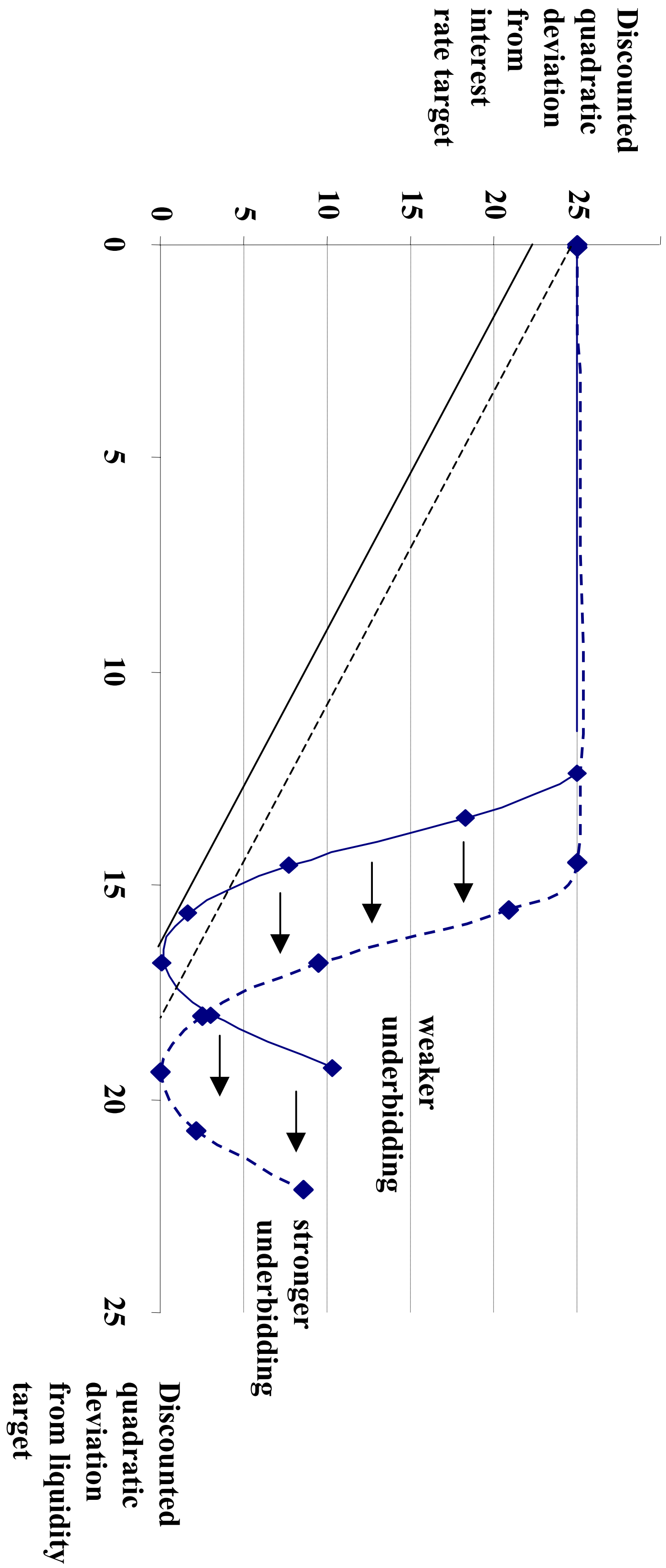


Figure 8. Later operations also suggest tightness. Comparative statics with respect to the timing parameters.

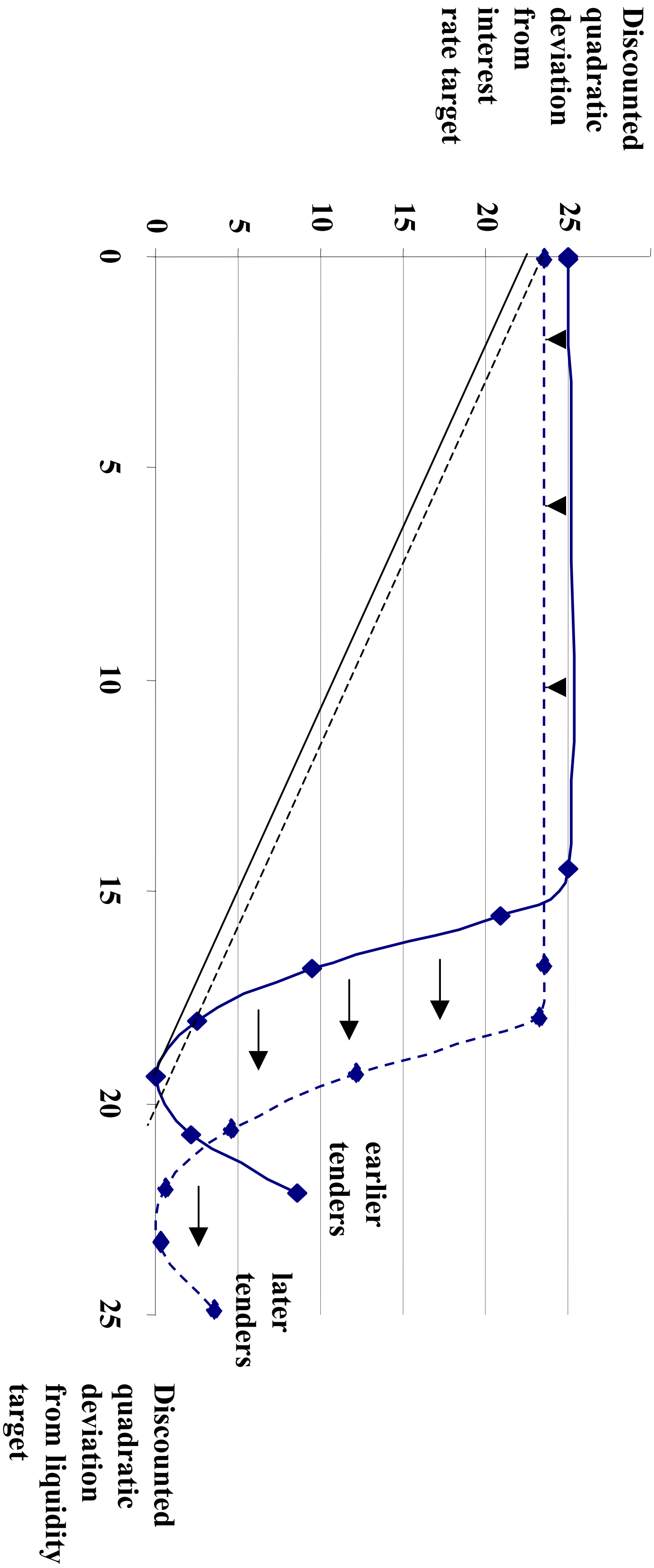


Figure 9. The shape of the feasibility set.

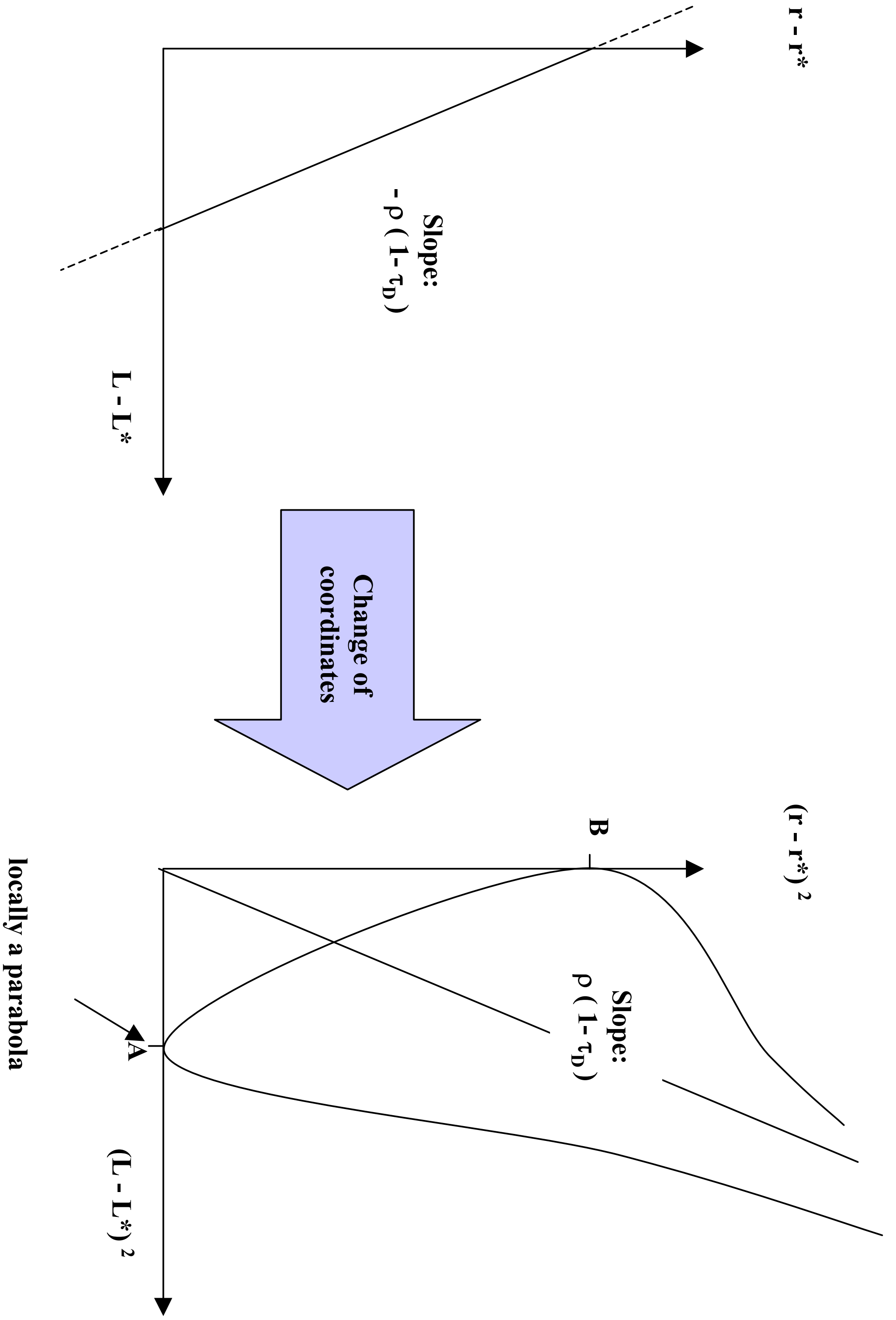


Figure 10. The arrows indicate a more restrictive allotment decision.

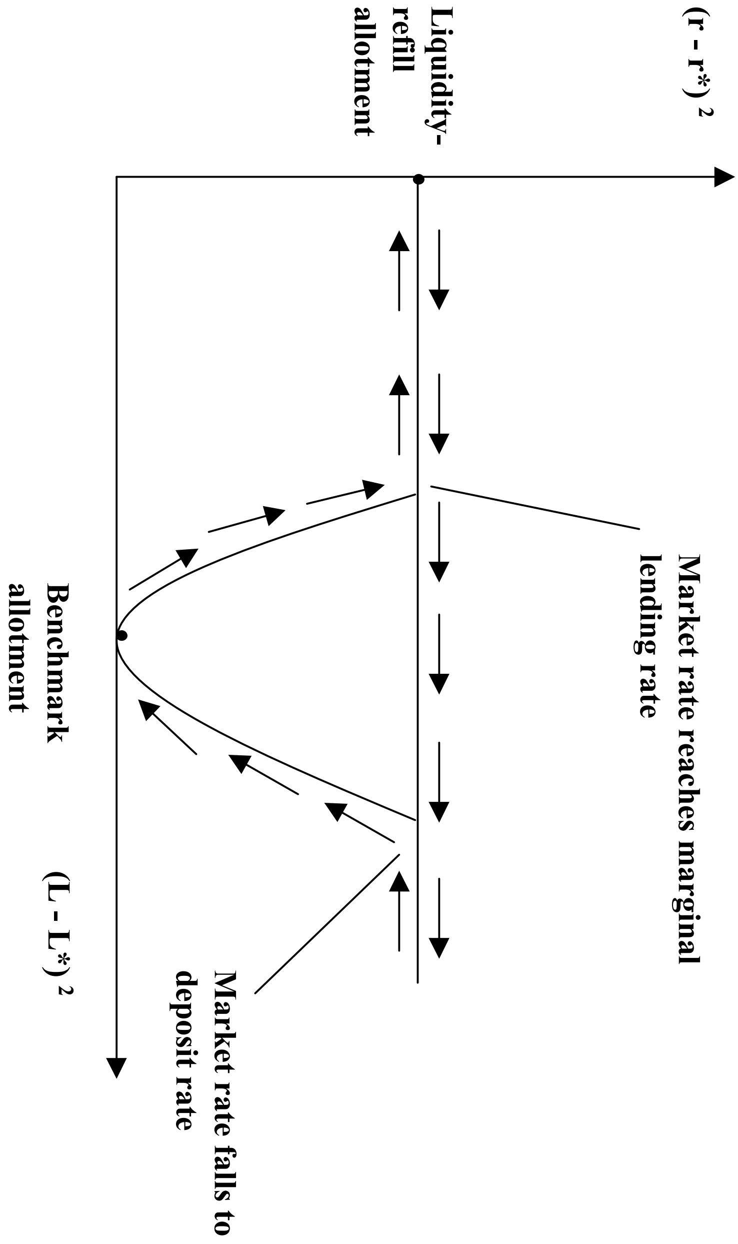
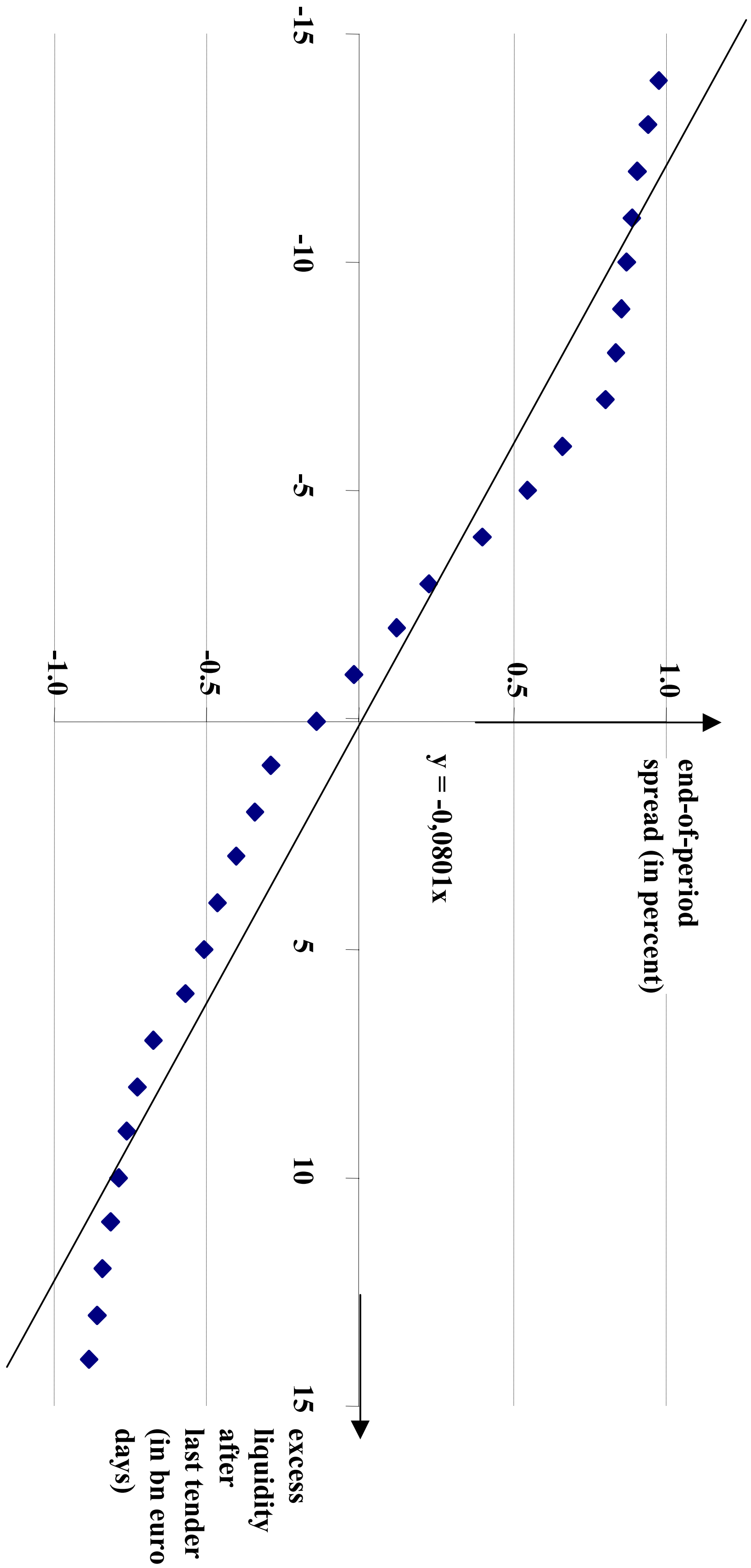


Figure 11. The figure shows the expected spread, caused by a liquidity imbalance of a given size, between the end-of-period rate and the midpoint of the interest rate corridor.



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