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Abstract

We reconsider the well-established paradigm of a rational individual's choice of a consumption schedule, building on the idea that human beings devote resources to withstand their desire for immediate consumption, i.e to become more patient, thereby making less remote the pleasure derived from deferred consumption. We construct an infinite-horizon model of a small open economy, in which individuals can accumulate a stock of personal capital that reduces the discount on future consumption. Personal capital captures the effect of a consumer's past experience and choices on his future utilities. Our main results are: i) when individuals are heterogenous with respect to ability to become patient all individuals exhibit the same rate of time preference in the long run; ii) effort is rewarded in the long run to the extent that individuals who need to make more effort to become patient are wealthier and enjoy a higher level of utility in the steady state. The latter result stems from the complementarity between personal capital and deferred consumption.

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Non-technical summary

The habit of distinctly realizing the future and providing for it, writes Marshall in his *Principles of Economics*, has developed itself slowly and fitfully in the course of man's history. In economic models, this habit is usually captured by a time preference parameter – the rate of time preference, which relates the utility of future consumption stream to current utility. Most of the time, economic modelling considers this habit as an exogenous, time-independent characteristic of consumers in clear contrast with Marshall's statement. However, individuals devote a lot of effort, especially through education and training activities enhancing self-control, to withstand their desire for immediate consumption.

Our modelling strategy is based on an assumption that individuals can build up a stock of personal capital, which is oriented towards the future and affects their future discount rates. Personal capital includes past experience, training, education and habits that make individual less impatient, and is costly to accumulate. Good habits are usually costly to accumulate. It would be all the more true were we to reinterpret our model of infinitely-lived individuals in an intergenerational context. Parents for example devote a lot of time and effort to teach children to resist their desire for immediate consumption of sweets or to consider the consequences of their acts on their future well-being. All these efforts aim at increasing children's ability to take account of the future, thereby decreasing their rate of time preference.

In growth theory, there is a link between the rate of time preference and the steady state and equilibrium dynamics of the economy. More patient individuals, for example, are predicted to save more and to enjoy a higher level of welfare in the long run. How does endogenous discount factors affect the economic equilibrium? In a small open economy, we consider individuals who differ with respect to their innate ability to reduce their discount rate. Individuals are *innately more patient* if they are endowed with a higher ability to reduce their discount rate. Since the interest rate is given, individuals' rates of time preference adjust to the interest rate. Provided that the steady state is interior for all individuals, all individuals therefore have the same rate of time preference in the long run, equal to the rate of interest. Interestingly, the model show that individuals who are innately more patient have a lower level of wealth and utility in the long run. The latter result stems from the complementarity between personal capital and future consumption.

"The habit of distinctly realizing the future and providing for it has developed itself slowly and fitfully in the course of man's history." Marshall (1920)

1 Introduction

We reconsider the well-established paradigm of a rational individual's choice of a consumption schedule, building on the idea that human beings devote resources to withstand their desire for immediate consumption and thereby make less remote the pleasure derived from deferred consumption.

Economic theory has long been concerned with explaining why nations or individuals accumulate wealth[1]. Thriftiness has been associated with wealth and even considered as a moral virtue conducive to economic growth and enhanced welfare. Marshall (1920) believes, as indicated by the excerpt, that human societies have evolved towards more prospectiveness and better faculty to realise the future. Fisher (1930) also postulates a positive relationship between wealth and patience at the level of both individuals and nations. Interestingly, he points out the existence of a relationship between preferences, thriftiness or improvidence, and the price of deferred consumption, the rate of interest. Although Fisher has the opinion that certain population groups are more impatient than others and that low income or low social status are associated with improvidence, he believes that the causes leading to improvidence can be influenced through training in self-control, habit formation, better health care and education and incentives for taking better care of children.

Which conclusions can we draw from Marshall's and Fisher's insights? First, thriftiness or improvidence, patience or impatience, are characteristics of certain social groups or of certain societies. Second, these characteristics appear to evolve over time. If we refer to these characteristics as preferences, we come to the conclusion that preferences are not

[1] For an extensive survey on the motives for saving and major economists' views of patience, we refer the reader to Warneryd (1993).

given for ever, which may come as no surprise, but casts doubt on the validity of certain results one can draw from models of economic growth based on exogenous rates of time preference. In fact, the relationship between savings and patience has been investigated at the very early stages of growth theory, and we find in Ramsey's (1928) celebrated paper some support for our concern. Ramsey examines the behaviour of an economy consisting of households endowed with different rates of time preference, and conjectures that in the long run the most patient household will own the whole wealth of the economy. He argues[2] that "equilibrium would be attained by a division of society into two classes, the thrifty enjoying bliss and the improvident at the subsistence level". This extreme result blurs the conclusion one can draw from models of infinitely-lived representative agents. The assumption that the consumer's side of the economy discounts future utilities from consumption at a given constant rate may be problematic. The conclusions of the infinite horizon model are clearly not robust to the introduction of heterogeneity with respect to rates of time preference, unless one is ready to admit the long run division of society into two (not three) social classes. In addition, the small open economy version of infinite horizon models requires that the rate of time preference equals the world rate of interest; otherwise the economy would accumulate or run down wealth forever (see Blanchard and Fischer 1989, note 32, page 85).

Another research avenue assumes that the rate at which future utility is discounted depends upon the utility levels of present and future consumption and thereby endogenises the rate of time preference. This was first explored by Uzawa (1968); stability in the Uzawa model requires that the rate of time preference increases with consumption, an assumption that stands in contradiction with conventional wisdom according to which the rich are more likely to be patient. Lucas and Stokey (1984) and Epstein (1987) assume recursive (non additive) preferences; with such a specification, the rate of time preference varies

[2]This conjecture has been proved by Becker (1980) in a dynamic general equilibrium model in which consumers differ with respect to rates of time preference. See Vidal (1996, 2000) for applications to the case of intergenerational altruism.

with the consumption path where it is evaluated. They find that individuals who are more patient (in some sense) are wealthier in the steady state, in line with economic intuition. Here we follow a different approach and focus on the human capacity to make effort to increase prospectiveness and to enhance self-control. A neglected research direction for growth theorists, already mentioned by, among others, Fisher, is indeed that individuals or societies devote effort and resources to overcome their desire for immediate consumption. This approach[3] has been recently revived by Becker and Mulligan (1997). Our main contribution here is to investigate this assumption in an infinite horizon model and to characterise the long-run rate of time preference, the long-run distribution of wealth and the dynamics, when individuals can devote resources to become more patient.

In this paper, we analyse the dynamics and steady state of a small open economy consisting of a set of consumers who differ with respect to their ability in resisting the desire for immediate consumption. A consumer's rate of time preference is assumed to depend on a stock of personal capital, which is costly to accumulate. Personal capital includes personal experience (e.g., education, training, activities enhancing self-control, etc...) that affects current and future utilities. Unlike the small open economy version of infinite horizon models, there is no need to assume that the economy's rate of time preference equals the world rate of interest, since each consumer's rate of time preference adjusts to economic conditions. We give a sufficient condition under which consumers' discount rates converge to a unique steady-state level. In the long run, all consumers have the same equilibrium rate of time preference; the rich are neither more patient nor more impatient than the poor. Morever, each consumer's steady-state rate of time preference equals the world rate of interest. These findings challenge models that assume consumers differing with respect to exogenous rates of time preference.

[3] A closely related strand of literature deals with endogenous altruism, see Mulligan (1997) or Vidal (1999). The approach in this paper clearly differs from the assumption that the rate of time preference depends on the level of consumption, as in Chakrabarty (2001) for example.

We then inquire into the relationship between consumption, wealth, welfare and innate ability to become patient. Since all individuals are equally patient in the long run, our model sheds a new light on the relationship between patience, savings and wealth. We find that effort is rewarded in the long run; those who need to make more effort to become patient and have thus been less gifted by Mother Nature are more likely to be wealthier and to enjoy a higher level of welfare in the long run. This is mainly owing to the complemetarity between personal capital and deferred consumption, whereby a higher level of future-oriented personal capital triggers higher saving.

The rest of the paper is organised as follows. In section 2, we present the main assumptions of the model and set up a consumer's programme. Section 3 is devoted to the existence of a solution to a consumer's programme. In section 4, we examine the dynamics of utility and write a consumer's utility in a recursive way. The optimality conditions are given in section 5. The steady state and dynamics of this economy are analysed in section 6. We inquire into the relationship between ability to become patient and steady-state utility in section 7. Section 8 contains our concluding remarks. Technical details are given in the appendix.

2 The Economy

The economy consists of a set of infinitely-lived individuals indexed by $i \in I$. Time is discrete with $t = ..., 0, 1, ...$. For simplicity we assume that there is no population growth. The key assumption here is that consumers make efforts to become more patient; a consumer's discount rate is therefore the solution to a rational individual's maximisation programme. The economy is a small open economy that operates in a perfectly competitive world. The world rate of interest is stationary at a level $r > 0$.

2.1 Technology and prices

Production occurs according to a constant-returns-to-scale technology using two inputs, capital, K_t, and labour, L_t. The output produced at time t, Y_t, is: $Y_t = F(K_t, L_t) \equiv L_t f(k_t)$, where $k_t \equiv K_t/L_t$ is the capital per efficiency unit of labour. Markets are competitive so that production factors are paid their marginal product:

$$r_t = f'(k_t) - \eta \text{ and } w_t = f(k_t) - k_t f'(k_t)$$

where r_t is the rate of interest, w_t the wage rate per efficiency unit of labour, and $\eta \in [0,1]$ the rate of depreciation of capital. Since the small open economy[4] allows unrestricted lending and borrowing, its rate of interest is set equal to the world rate of interest: $r_t = r$. The wage rate per efficiency unit of labour is therefore constant and equal to w.

2.2 Consumers

Each consumer i is infinitely-lived; his preferences[5] (as from period $t = 0$) are defined over his consumption path $(c_t^i)_{t\geq 0}$ and are represented by the utility function:

$$\mathcal{U}_0^i = \sum_{t=0}^{+\infty} \Gamma_t^i u\left(c_t^i\right) \tag{1}$$

where $u(.)$ is the period-t felicity function and Γ_t^i a time discount factor that maps period-t felicity from period-t consumption into period-0 utility.

Assumption 1. The felicity function $u : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is continuous, twice continuously differentiable on $\mathbb{R}_+^*$, and satifies: $u(0) = 0$ and $\forall c > 0$, $u(c) > 0$, $u'(c) > 0$, $u''(c) < 0$, $\lim_{c\rightarrow 0} u'(c) = +\infty$.

From now on, we omit the upperscript i unless necessary. The period-$t+1$ discount factor depends on both the period-t discount factor and the period-$t+1$ stock of personal capital, z_{t+1} (on the concept of personal capital, see Becker 1996, chapter 1):

$$\Gamma_{t+1} = \gamma(z_{t+1})\Gamma_t, \quad \Gamma_0 = 1 \tag{2}$$

[4] Alternatively, one could assume that the production technology is linear: $Y_t = rK_t + wL_t$, under the additional restriction that the stock of capital is positive, $K_t \geq 0$.

[5] Preferences as defined here are given and also encompass the endogenous choice of time preference.

where, without loss of generality, we normalise the initial discount factor, $\Gamma_0 = 1$, and $\gamma(.)$ satisfies the following assumption:

Assumption 2. The function $\gamma : \mathbb{R}_+^* \to \mathbb{R}_+^*$ is twice continuously differentiable, and satisfies: $\forall z > 0$, $\gamma(z) > 0$, $\gamma'(z) > 0$, $\gamma''(z) < 0$.

Personal capital includes experience that affects current and future utility. It is oriented towards the future and reduces the discount rate one applies to future utility. Training and education, efforts to resist current consumption and to save, such as adhering to a well-tempered life-style, and all activities enhancing self-control bear implications for both current and future utility through habit formation. Efforts are less costly when they become an habit. In our model, for instance, a consumer's efforts to resist his desire for immediate consumption today reduce the cost of saving in the future by reducing his future discount rates[6].

In each period, the consumer is endowed with one unit of time. He supplies a fraction $l_t \in [0,1]$ of his time endowment to the labour market and devotes the remainder, $1 - l_t$, to accumulating personal capital. The consumer's wealth, x_t, evolves according to:

$$x_{t+1} = (1+r)\,x_t + wl_t - c_t, \quad x_0 \text{ given} \tag{3}$$

The stock of personal capital falls over time owing to psychological depreciation of efforts made in the past, whereby an habit is not acquired forever but requires protracted efforts, and is assumed to evolve according to:

$$z_{t+1} = (1-\delta)\,z_t + g\,(1-l_t)\,, \quad z_0 \text{ given} \tag{4}$$

where $\delta \in \,]0,1[$ is the rate of depreciation of personal capital. g maps time devoted to accumulating personal capital into personal capital.

Assumption 3. The function $g : [0,1] \to \mathbb{R}_+^*$ is twice continuously differentiable, and satisfies: $\forall y > 0$, $g(y) > 0$, $g'(y) > 0$, $g''(y) \leq 0$, and $g(0) = 0$.

[6]Becker and Mulligan (1997) document the relationship between education, a factor influencing personal capital, and consumption growth.

2.3 A consumer's discounted wealth

A *consumer's discounted initial wealth*, X_0, is the sum of his initial non-human wealth, x_0, and his human wealth which is the discounted sum of his future potential earnings[7]:

$$X_0 \equiv x_0 + \sum_{t=0}^{+\infty} \frac{w}{(1+r)^{t+1}} = x_0 + \frac{w}{r} \tag{5}$$

In each period, the period-t wealth discounted at $t = 0$, X_t, decreases; the discounted values of consumption and time devoted to accumulating personal capital must be substracted from the period-t discounted wealth to obtain the next period's discounted wealth:

$$X_{t+1} = X_t - (1+r)^{-t-1} (c_t + (1-l_t) w), \quad X_0 \text{ given} \tag{6}$$

Following Arrow and Kurz (1970) we write a consumer's solvency constraint:

$$\forall t, \; X_t \geq 0 \tag{7}$$

It says that, in each period, a consumer can reimburse his debt, $-x_t$, with his future potential earnings (see appendix 1). Since the sequence $(X_t)_{t\geq 0}$ is monotonic non-increasing, the condition (7) is equivalent to:

$$\lim_{t\to+\infty} X_t \geq 0 \tag{8}$$

2.4 A consumer's program

We can now set up the maximisation program of a consumer. It is convenient to use a consumer's total wealth discounted at $t = 0$ instead of current wealth. A consumer maximises his utility (1) subject to (2), (4), (5), (6), $l_t \in [0,1]$, and the non-negativity constraints:

$$\max_{(c_t,l_t)_{t\geq 0}} \sum_{t=0}^{+\infty} \Gamma_t u(c_t) \tag{9}$$

[7] A consumer's initial wealth is discounted at the beginning of period 0, i.e. before the current rate of interest is paid.

subject to:

$$\begin{array}{c} z_{t+1} = (1-\delta)\, z_t + g\,(1-l_t) \\ X_{t+1} = X_t - (1+r)^{-t-1}\,(c_t + (1-l_t)\,w) \\ \Gamma_{t+1} = \gamma\,(z_{t+1})\,\Gamma_t \\ c_t \geq 0,\ X_t \geq 0,\ l_t \in [0,1] \end{array} \tag{10}$$

The initial stocks are given: $X_0 > 0$, $z_0 > 0$, and $\Gamma_0 = 1$.

3 Existence of solutions to a consumer's maximisation program

A sequence $(c_t, l_t, X_t, z_t, \Gamma_t)$ is feasible if it satisfies the set of constraints (10). To prove existence we proceed as follows. First, we show that the set of feasible sequences belongs to a compact set with respect to the product topology; second, we show that the objective function is continuous for this topology (see Becker and Boyd 1997).

Compactness. For any $t \geq 0$, every element of the sequences (c_t), (l_t), (X_t), (z_t), and (Γ_t) is bounded from above and from below. Indeed, we have (for any $t \geq 0$):

$$0 \leq l_t \leq 1$$

$$0 \leq X_t \leq X_0$$

$$0 \leq c_t \leq (1+r)^{t+1}\, X_t \quad \text{(see expression (6))}$$

For z_t, we have:

$$\underline{z}_t \leq z_t \leq \overline{z}_t$$

where $\underline{z}_t \equiv (1-\delta)^t\, z_0$ and $\overline{z}_t \equiv (1-\delta)^t\,(z_0 - g\,(1)\,/\delta) + g\,(1)\,/\delta$. $(\underline{z}_t)$ denotes the sequence of stocks of personal capital in the absence of effort to accumulate personal capital, while $(\overline{z}_t)$ denotes the sequence of stocks of personal capital in the case where a consumer devotes his full time endowment to such effort. Since $\gamma\,(.)$ is continuous and non-decreasing, we also have:

$$\underline{\Gamma}_t \leq \Gamma_t \leq \overline{\Gamma}_t$$

where $\underline{\Gamma}_{t+1} = \underline{\Gamma}_t \gamma\,(\underline{z}_{t+1})$, $\overline{\Gamma}_{t+1} = \overline{\Gamma}_t \gamma\,(\overline{z}_{t+1})$, and $\underline{\Gamma}_0 = \overline{\Gamma}_0 = 1$.

Hence the set of feasible sequences is closed and bounded, i.e. compact (with respect to the product topology).

Continuity of the objective function. Since the felicity function, $u(.)$, is continuous, any finite sum $\sum_{t=0}^{T} \Gamma_t u(c_t)$ is continuous. For continuity, it is sufficient that the infinite sum converges uniformly. Proposition 1 results from the following inequalities:

$$0 \leq \Gamma_t u(c_t) \leq \overline{\Gamma}_t u\left((1+r)^{t+1} X_0\right)$$

Proposition 1. *If the infinite sum* $\sum_{t=0}^{+\infty} \overline{\Gamma}_t u\left((1+r)^{t+1} X_0\right)$ *converges, there exists a solution to the representative consumer's maximisation program.*

Example. Consider the CES felicity function: $u(c) = c^a$, $0 < a < 1$. We have:

$$\frac{\overline{\Gamma}_{t+1} u\left((1+r)^{t+2} X_0\right)}{\overline{\Gamma}_t u\left((1+r)^{t+1} X_0\right)} = \gamma(\overline{z}_{t+1})(1+r)^a$$

and

$$\lim_{t \to +\infty} \overline{z}_t = g(1)/\delta$$

If $\gamma(g(1)/\delta)(1+r)^a < 1$, there exists a solution to a consumer's maximisation program.

4 The dynamics of utility

We prove that the objective function of a consumer can be equivalently rewritten as the initial state of a dynamic equation of utility. Consider a sequence of consumption (c_t) and a sequence of personal capital (z_t). Let us define the time discount factor as from period t_0:

$$\Gamma_{t_0,t_0} = 1 \text{ and } \forall t \geq t_0,\ \Gamma_{t_0,t+1} = \gamma(z_{t+1})\,\Gamma_{t_0,t}$$

A consumer's utility evaluated at date t_0 is given by:

$$\mathcal{U}_{t_0} = \sum_{t=t_0}^{+\infty} \Gamma_{t_0,t} u(c_t)$$

The discount factors as from date 0, $\Gamma_{0,t} = \Gamma_t$, satisfy $\Gamma_{0,t_0}\Gamma_{t_0,t} = \Gamma_t$ (for $0 \leq t_0 \leq t$). Hence:

$$\Gamma_{t_0}\mathcal{U}_{t_0} = \sum_{t=t_0}^{+\infty} \Gamma_t u(c_t) = \Gamma_{t_0} u(c_{t_0}) + \Gamma_{t_0+1}\mathcal{U}_{t_0+1}$$

The sequence of utility $(\mathcal{U}_t)_{t\geq 0}$ satisfies:

$$\forall t \geq 0,\ \mathcal{U}_t = u(c_t) + \gamma(z_{t+1})\mathcal{U}_{t+1} \tag{11}$$

This describes the dynamics of a consumer's utility. His period-t utility is equal to the sum of his period-t felicity derived from consumption and his next period's discounted utility.

Let us now consider the converse. Let $\left(\widetilde{\mathcal{U}}_t\right)_{t\geq 0}$ be a sequence of positive real numbers that satisfies (11). Clearly, if we have:

$$\Gamma_{t_0}\widetilde{\mathcal{U}}_{t_0} = \sum_{t=t_0}^{T-1} \Gamma_t u(c_t) + \Gamma_T \widetilde{\mathcal{U}}_T \tag{12}$$

then by substituting $\widetilde{\mathcal{U}}_T = u(c_T) + \gamma(z_{T+1})\widetilde{\mathcal{U}}_{T+1}$ into (12) we obtain the same expression at date $T+1$. Taking the limit of (12) as T goes to infinity we get:

$$\Gamma_{t_0}\widetilde{\mathcal{U}}_{t_0} = \sum_{t=t_0}^{+\infty} \Gamma_t u(c_t) + \lim_{T\to+\infty} \Gamma_T \widetilde{\mathcal{U}}_T$$

We therefore have the following proposition:

Proposition 2. *A sequence of positive real numbers* $\left(\widetilde{\mathcal{U}}_t\right)_{t\geq 0}$ *that satisfies:* $\forall t \geq 0$, $\mathcal{U}_t = u(c_t) + \gamma(z_{t+1})\mathcal{U}_{t+1}$, *is a sequence of utility associated with the sequences* $(c_t)_{t\geq 0}$ *and* $(z_t)_{t\geq 0}$ *if and only if:*

$$\lim_{T\to+\infty} \Gamma_T \widetilde{\mathcal{U}}_T = 0 \tag{13}$$

Two remarks are here in order. First, this restriction is clearly necessary, since any sequence $(\mathcal{U}_t + b/\Gamma_t)_{t\geq 0}$ is a solution to (11) if $(\mathcal{U}_t)_{t\geq 0}$ solves (11). Second, given the upper bound $\overline{\Gamma}_t$ defined in section 3, it is sufficient that the sequence $\left(\widetilde{\mathcal{U}}_t\right)_{t\geq 0}$ satisfies:

$$\lim_{T\to+\infty} \overline{\Gamma}_T \widetilde{\mathcal{U}}_T = 0$$

We will use this sufficient condition when analysing the optimality conditions in the next section.

5 Optimality conditions

The results of section 4 and appendix 1 allow us to rewrite a consumer's problem as follows:

$$\max_{(c_t,l_t)_{t\geq 0}} \mathcal{U}_0$$

subject to:

$$\begin{gathered}(1+r)\,x_t - c_t + wl_t - x_{t+1} = 0\\ (1-\delta)\,z_t + g\,(1-l_t) - z_{t+1} = 0\\ u\,(c_t) + \gamma\,(z_{t+1})\,\mathcal{U}_{t+1} - \mathcal{U}_t = 0\\ c_t \geq 0,\ 0 \leq\ l_t \leq 1\\ \lim_{t\to+\infty} (1+r)^{-t}\,x_t \geq 0 \text{ and } \lim_{t\to+\infty} \overline{\Gamma}_t\widetilde{\mathcal{U}}_t = 0\end{gathered}$$

The initial stocks are given: $x_0 \geq 0$ and $z_0 > 0$. The last two constraints, which ensure the uniqueness of the sequence of utility and the solvency of the representative consumer, play no role in the marginal optimality conditions (these conditions are obtained by maximising on a finite horizon with fixed endpoints).

We assume in the remainder that the condition stated in proposition 1 holds. To obtain the marginal optimality conditions we set up the following Lagrangian:

$$\begin{aligned} L \ =\ & \mathcal{U}_0 + \sum_{t=0}^{+\infty} \{\lambda_{t+1}\,((1+r)\,x_t - c_t + wl_t - x_{t+1}) \\ & +\mu_{t+1}\,((1-\delta)\,z_t + g\,(1-l_t) - z_{t+1}) + \nu_{t+1}\,(u\,(c_t) + \gamma\,(z_{t+1})\,\mathcal{U}_{t+1} - \mathcal{U}_t)\} \end{aligned}$$

where λ_{t+1}, μ_{t+1}, and ν_{t+1} are the Lagrangian multipliers associated to the dynamic equations. The optimal control variables, c_t^* and l_t^*, satisfy:

$$\nu_{t+1}u'\,(c_t^*) = \lambda_{t+1} \tag{14}$$

$$w\lambda_{t+1} - \mu_{t+1}g'\,(1-l_t^*) \begin{cases} \leq 0 \text{ if } l_t^* = 0 \\ \geq 0 \text{ if } l_t^* = 1 \\ = 0 \text{ if } l_t^* \in \,]0,1[\end{cases} \tag{15}$$

Maximising with respect to the state variables, $x_{t+1,}$ z_{t+1} and $\mathcal{U}_{t+1}$, gives for all $t \geq 0$:

$$\lambda_{t+1} = (1+r)\,\lambda_{t+2} \tag{16}$$

$$\mu_{t+1} = (1-\delta)\,\mu_{t+2} + \nu_{t+1}\gamma'\left(z_{t+1}^*\right)\mathcal{U}_{t+1}^* \tag{17}$$

$$\nu_{t+1}\gamma\left(z_{t+1}^*\right)=\nu_{t+2} \tag{18}$$

Moreover, maximising with respect to $\mathcal{U}_0$ leads to $\nu_1=1$.

The condition (18) together with (2) implies $\nu_{t+1}=\Gamma_t^*$. The marginal optimality conditions can be expressed with a unique multiplier $\theta_{t+1}\equiv\mu_{t+1}/\nu_{t+1}$, which is the marginal value (in terms of utility) of the stock of personal capital.

Proposition 3. *The marginal optimality conditions of the representative consumer's problem are:*

$$u'\left(c_t^*\right)=(1+r)\gamma\left(z_{t+1}^*\right)u'\left(c_{t+1}^*\right) \tag{19}$$

$$\theta_{t+1}=(1-\delta)\gamma\left(z_{t+1}^*\right)\theta_{t+2}+\gamma'\left(z_{t+1}^*\right)\mathcal{U}_{t+1}^* \tag{20}$$

$$wu'\left(c_t^*\right)-\theta_{t+1}g'\left(1-l_t^*\right)\begin{cases}\leq 0 \text{ if } l_t^*=0\\ \geq 0 \text{ if } l_t^*=1\\ =0 \text{ if } l_t^*\in\left]0,1\right[\end{cases} \tag{21}$$

Proof. (14) and (16) imply (19). (17) gives (20) with $\theta_{t+1}\equiv\mu_{t+1}/\nu_{t+1}$. l_t^* satisfies (21), since $w\lambda_{t+1}-\mu_{t+1}g'\left(1-l_t^*\right)=\nu_{t+1}\left(wu'\left(c_t^*\right)-\theta_{t+1}g'\left(1-l_t^*\right)\right)$. Conversely, the marginal optimality conditions are obtained from (19), (20) and (21) with $\nu_1=1$, (18), $\lambda_{t+1}=\nu_{t+1}u'\left(c_t^*\right)$ and $\mu_{t+1}=\theta_{t+1}\nu_{t+1}$.

The first[8] of these conditions equates the marginal rate of substition between period-t and period-$t+1$ consumption, $u'\left(c_t^*\right)/\gamma\left(z_{t+1}^*\right)u'\left(c_{t+1}^*\right)$, to the technical rate of substitution, $1+r$. The second condition states that the implicit value of period-$t+1$ personal capital is equal to the sum of its utility gain, $\gamma'\left(z_{t+1}^*\right)\mathcal{U}_{t+1}^*$, and its residual value in terms of future utility gain, $(1-\delta)\gamma\left(z_{t+1}^*\right)\theta_{t+2}$. The third condition states that the value of one additional unit of labour at period t is equal to the difference between the gain it allows in terms of current felicity, $wu'\left(c_t^*\right)$, and the loss in terms of future utility that results from the reduction in the stock of personal capital, θ_{t+1}. For an interior solution, $l_t^*\in\left]0,1\right[$, this value is nil.

[8] This condition relates the marginal rate of substitution between consumptions in two successive periods to the rate of time preference and the rate of interest. See Becker and Barro (1988) who find a formally similar condition in the case of endogenous fertility.

For an interior solution, both l_t^* and l_{t+1}^* belong to $]0,1[$, and the expressions (20) and (21) can be combined:

$$\frac{wu'\left(c_t^*\right)}{g'(1-l_t^*)} = \frac{(1-\delta)\,\gamma\left(z_{t+1}^*\right)wu'\left(c_{t+1}^*\right)}{g'(1-l_{t+1}^*)} + \gamma'\left(z_{t+1}^*\right)\mathcal{U}_{t+1}^*$$

6 Steady state and dynamics

From this section onwards, we assume that consumers differ with respect to their innate ability to reduce discount rates, which is represented by the function $\gamma_i(.)$, $i \in I$. Since the price formation mechanism is not related to consumers' decisions in the small open economy setting, we consider the steady state and dynamics of each consumer i independently. First, we define a consumer's steady state. Second, we derive a condition under which a consumer's steady state is interior. Third, we examine the dynamics and provide a sufficient condition for stability in the saddle-point sense.

An *optimal interior steady state* for consumer i is a vector $(c^i, l^i, z^i, x^i, \mathcal{U}^i)$ that satisfies:

$$c^i = rx^i + wl^i \tag{22}$$

$$\delta z^i = g\left(1-l^i\right) \tag{23}$$

$$(1+r)\,\gamma_i\left(z^i\right) = 1 \tag{24}$$

$$wu'\left(c^i\right) = (1-\delta)\,\gamma_i\left(z^i\right)wu'\left(c^i\right) + \gamma_i'\left(z^i\right)g'\left(1-l^i\right)\mathcal{U}^i \tag{25}$$

$$\mathcal{U}^i = \frac{u\left(c^i\right)}{1-\gamma_i\left(z^i\right)} \tag{26}$$

The first two conditions result from the dynamic equations of current wealth and of the stock of personal capital in steady state. The third condition says that in a steady state the economy is at the modified golden rule; here the rate of interest is given and the stock of personal capital adjusts to the economic conditions. The fourth condition that results from (20) and (21) says that, for an interior solution, the cost of accumulating one additional unit of personal capital equals the sum of its next period's residual value and the utility gain. The fifth condition determines the steady-state utility.

Proposition 4. *There exists an interior optimal steady state for consumer i if and only if:*

$$\gamma_i(0+) < \frac{1}{1+r} < \gamma_i(g(1)/\delta) \tag{27}$$

where $\gamma_i(0+) = \lim_{z \to 0} \gamma_i(z)$.

Proof. The steady-state stock of personal capital[9] of consumer i is determined uniquely by (24), $z^i = \gamma_i^{-1}(1/(1+r))$. The steady-state labour supply l^i satisfying (23) belongs to $]0,1[$ if and only if $0 < z^i < g(1)/\delta$, and we then have: $l^i = 1 - g^{-1}(\delta z^i)$. (25) and (26) give:

$$\Phi\left(c^i\right) \equiv \frac{u(c^i)}{u'(c^i)} = \frac{(1-\gamma_i(z^i))\,w\,(1-(1-\delta)\,\gamma_i(z^i))}{\gamma_i'(z^i)\,g'(1-l^i)} \tag{28}$$

This equation determines a unique level of steady-state consumption, since $u(c)/u'(c)$ is an increasing one-to-one function (see assumption 1); we have:

$$c^i = \Phi^{-1}\left(\frac{wr(r+\delta)}{(1+r)^2}\frac{1}{\gamma_i'(z^i)\,g'(1-l^i)}\right) \tag{29}$$

Moreover, the steady state levels of wealth and utility are given by (22) and (26).

Interestingly, if the world rate of interest lies inside the interval $[\gamma_i(0+), \gamma_i(g(1)/\delta)]$, then there is a steady state. Only outside this interval[10] does consumer i either accumulate forever or run down his wealth as far as he can. In standard infinite horizon models, this interval degenerates and it is usually assumed that the exogenous rate of time preference equals the world interest rate. Allowing for the endogenous determination of each

[9] $\lim_{z \to 0} \gamma_i(z)$ exists and is non negative since: $\forall z > 0$, $\gamma_i(z) > 0$, $\gamma_i'(z) > 0$.

[10] For the dynamics, we have (see (19)):

$$\frac{u'(c_t^i)}{u'(c_{t+1}^i)} = \gamma_i(z_{t+1}^i)(1+r)$$

If $\gamma_i(0+) \geq \frac{1}{1+r}$, $\frac{u'(c_t^i)}{u'(c_{t+1}^i)} > \gamma_i(0+)(1+r) \geq 1$ and $\lim c_t^i = +\infty$.

If $\gamma_i(g(1)/\delta) < \frac{1}{1+r}$, $\limsup \frac{u'(c_t^i)}{u'(c_{t+1}^i)} < 1$ (since $\limsup z_{t+1}^i \leq \frac{g(1)}{\delta}$) and $\lim c_t^i = 0$.

consumer's time preference requires milder assumptions, namely, one only has to assume that (27) holds. This leaves room to analyse the effect of a shock on the world interest rate in a small open economy; in particular, one doesn't need to assume that the economy's rate of time preference varies with a change in the world interest rate.

In the case of an interior steady state, we analyse the six-dimensional equilibrium dynamics in appendix 2. We find that a sufficient condition for the steady state to be stable in the saddle-point sense is that $u''(c)$ is sufficiently close to zero. This condition is illustrated by a CES felicity function of the form $u(c) = c^a$, $a \in]0, 1[$.

7 Heterogeneity and time preference

From now on we further assume that the condition (27) in proposition 4 holds for all consumers. An important feature of this model is that there is no heterogeneity of time preference in the long run, while individuals remain heterogenous with respect to wealth, consumption, labour supply and utility levels.

Proposition 5. *Assume that, for all $i \in I$, we have: $\gamma_i(0+) < \frac{1}{1+r} < \gamma_i(g(1)/\delta)$. All consumers have the same rate of time preference in the long run, regardless of their innate ability to reduce their discount rates. This rate of time preference equals the rate of interest.*

Proof. If $\forall i \in I$, $\gamma_i(0+) < \frac{1}{1+r} < \gamma_i(g(1)/\delta)$, we have $\forall i \in I$, $\gamma_i(z^i) = \frac{1}{1+r}$.

Regardless of their ability to reduce discount rates, consumers adjust their own discount rate to economic conditions and ultimately end up with a common rate of time preference. Abilities, however, determine the steady-state levels of consumption, labour supply, wealth and utility. Are individuals who are more able to reduce their discount rate wealthier in the long run? How do heterogenous abilities to withstand the desire for immediate consumption impinge upon the long-run distributions of consumption, labour supply, wealth and utility?

Let us consider two individuals i and j and assume that $\gamma_i(0) > \gamma_j(0)$ and $\forall z > 0$, $\gamma_i'(z) \geq \gamma_j'(z)$. This implies that for all $z > 0$, we have $\gamma_i(z) > \gamma_j(z)$. In the remainder of this section, we shall refer to the individual i as the *innately more patient* individual. In a steady state, the endogeneous rate of time preference of each individual is equal to the rate of interest and we have:

$$\gamma_j\left(z^j\right) = \frac{1}{1+r} = \gamma_i\left(z^i\right) > \gamma_j\left(z^i\right) \Rightarrow z^j > z^i$$

As also illustrated by figure 1, the innately more patient individual (i) needs to accumulate a lower stock of personal capital than the innately less patient individual (j).

Insert figure 1 here

We then have:

$$\gamma_i'\left(z^i\right) \geq \gamma_j'\left(z^i\right) > \gamma_j'\left(z^j\right) \text{ (since } \gamma_j'' < 0)$$

$$g\left(1-l^j\right) = \delta z^j > g\left(1-l^i\right) = \delta z^i \text{ and } 1-l^j > 1-l^i$$

Owing to the concavity of g, we have:

$$g'\left(1-l^j\right) \leq g'\left(1-l^i\right)$$

Thus, we obtain the following inequality:

$$g'\left(1-l^j\right)\gamma_j'\left(z^j\right) < g'\left(1-l^i\right)\gamma_i'\left(z^i\right)$$

which together with (29) implies $c^j > c^i$. In the long run the innately more patient individual has a lower level of consumption. From (22) and (26), we see that the innately more patient individual also has a lower level of wealth and utility in the long run.

Proposition 6. *Assume that individual i is innately more patient, i.e. is endowed with a higher ability to reduce discount rates, than individual j in the following sense:*

$$\gamma_i(0) > \gamma_j(0) \text{ and } \forall z > 0,\ \gamma_i'(z) \geq \gamma_j'(z). \tag{30}$$

In the long run, the innately less patient individual consumes more, has a higher level of wealth and attains a higher level of utility.

An individual who is endowed with a higher ability to become patient needs to accumulate a lower level of personal capital to reach his long-run rate of time preference than one endowed with a lower ability. There are complementarities between personal capital and future consumption: a higher level of future-oriented personal capital reinforces the utility derived from deferred consumption and triggers higher saving. Intuitively, consumers who need make a lot of efforts to reduce their discount rate become used to future-oriented behaviours. As a result they save more and end up with higher levels of wealth and utility.

8 Conclusion

In the long run, all consumers have the same (endogenous) rate of time preference even though the distributions of wealth, labour supply, consumption and utility are not trivial. Interestingly, efforts to become patient are rewarded in the long run, as individuals endowed with a lower innate ability to become patient end up with a higher utility level.

Our findings point to other research directions. First, we have considered the case of a small open economy. Endogenising prices by adding a standard neoclassical production sector to our model would not alter the substance of our results, as the long-run equilibrium would be characterised by some modified Golden Rule condition ($f'(k)\gamma_i(z^i) = 1$ where f is the production function); however, the precise conditions for existence and the dynamic properties of such an equilibrium still have to be derived, and are a bit more involved. Second, the infinitely-lived representative agent is often thought of as a dynasty of altruistic individuals linked to each other through a chain of positive bequests. Endogenous intergenerational altruism would, in that respect, be the equivalent of endogenous time preference. The additional constraint according to which bequests must be non-negative (see Abel 1987, Weil 1987 and Thibault 2000) would modify our long-run results, since some dynasties may be constrained with respect to bequests and therefore prevented from accumulating the same level of concern for their offspring (altruism). Such a model would clearly allows us to understand better the effects of fiscal policy on savings and the distribution of wealth.

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Appendix 1

By definition of the consumer's discounted wealth, we have:

$$X_0 = x_0 + \frac{w}{r} \tag{31}$$

$$X_{t+1} = X_t - (1+r)^{-t-1} \left(c_t + (1 - l_t)\, w\right) \tag{32}$$

(31) and (32) imply:

$$X_t = x_0 + \frac{w}{r} - \sum_{s=0}^{t-1} \frac{1}{(1+r)^{s+1}} \left(c_s + (1 - l_s)\, w\right)$$

Using $c_s + (1 - l_s)\, w = (1+r)\, x_s + w - x_{s+1}$ we obtain:

$$\begin{aligned} X_t &= x_0 + \frac{w}{r} - \sum_{s=0}^{t-1} \frac{1}{(1+r)^{s+1}} \left((1+r)\, x_s + w - x_{s+1}\right) \\ &= \frac{1}{(1+r)^t} \left(x_t + \frac{w}{r}\right) \end{aligned}$$

Hence:

$$X_t \geq 0 \Leftrightarrow \frac{w}{r} \geq -x_t$$

The condition $\lim_{t \to +\infty} X_t \geq 0$ is equivalent to $\lim_{t \to +\infty} (1+r)^{-t} x_t \geq 0$, since $\lim_{t \to +\infty} (1+r)^{-t} \frac{w}{r} = 0$.

Appendix 2

The shadow prices evolve according to:

$$\lambda_{t+1} = (1+r)\,\lambda_{t+2}$$

$$\lambda_{t+1} = \nu_{t+1} u'(c_t)$$

$$w\lambda_{t+1} = \mu_{t+1} g'(1-l_t)$$

$$\mu_{t+1} = (1-\delta)\,\mu_{t+2} + \nu_{t+1}\gamma'(z_{t+1})\,\mathcal{U}_{t+1}$$

$$\nu_{t+2} = \gamma(z_{t+1})\,\nu_{t+1}$$

The state variables evolve according to:

$$x_{t+1} = (1+r)\,x_t - c_t + wl_t$$

$$z_{t+1} = (1-\delta)\,z_t + g(1-l_t)$$

$$\mathcal{U}_t = u(c_t) + \gamma(z_{t+1})\,\mathcal{U}_{t+1}$$

Let us define:

$$V_t = \frac{\mathcal{U}_t}{\lambda_{t+1}}\nu_{t+1},\ \pi_t = \frac{\mu_{t+1}}{\lambda_{t+1}} \text{ and } g'^{-1} \equiv h$$

to obtain the following dynamical system of dimension five.

$$x_{t+1} = (1+r)\,x_t - c_t + w\left(1 - h\left(\frac{w}{\pi_t}\right)\right)$$

$$V_t = \frac{u(c_t)}{u'(c_t)} + \frac{1}{1+r}V_{t+1}$$

$$z_{t+1} = (1-\delta)\,z_t + g \circ h\left(\frac{w}{\pi_t}\right)$$

$$u'(c_t) = (1+r)\,\gamma(z_{t+1})\,u'(c_{t+1})$$

$$\pi_t = \frac{1-\delta}{1+r}\pi_{t+1} + \frac{1}{1+r}\frac{\gamma'(z_{t+1})}{\gamma(z_{t+1})}V_{t+1}$$

Together with $\nu_{t+1} = \gamma(z_t)\nu_t$, which is stable, this dynamical system is equivalent to the previous one. x_0, z_o and $\nu_1 = 1$ are the initial conditions. The stability of this system (in the saddle-point sense) therefore requires 3 unstable and 2 stable roots.

Consider a n-dimensional first-order dynamical system:

$$BY_{t+1} + AY_t = C$$

Differentiation yields:

$$BdY_{t+1} + AdY_t = 0$$

Looking for linear solutions ($dY_{t+1} = \xi dY_t$) gives:

$$(B\xi + A)\, dY_t = 0$$

The eigenvalues of this system are the solutions ξ to $\det(B\xi + A) = 0$. The $(B\xi + A)$ matrix of our dynamical system is:

$$\begin{matrix} 1+r-\xi & 0 & 0 & -1 & \frac{w^2}{\pi^2}h' \\ 0 & \xi-(1+r) & 0 & (1+r)\frac{u'^2-u\,u''}{u'^2} & 0 \\ 0 & 0 & 1-\delta-\xi & 0 & -\frac{w^2}{\pi^3}h' \\ 0 & 0 & \frac{(1+r)u'\gamma'}{u''}\xi & \xi-1 & 0 \\ 0 & \frac{\xi}{1+r}\frac{\gamma'}{\gamma} & \frac{\xi V}{1+r}\frac{\gamma\gamma''-\gamma'^2}{\gamma^2} & 0 & \frac{1-\delta}{1+r}\xi-1 \end{matrix}$$

The characteristic polynomial of this matrix is:

$$P(\xi) = (1+r-\xi) \times Q(\xi)$$

where

$$Q(\xi) = (\xi-(1+r))\left[(1-\delta-\xi)(\xi-1)\left(\frac{1-\delta}{1+r}\xi-1\right) + \frac{w^2}{\pi^3}h'(\xi-1)\frac{\xi V}{1+r}\frac{\gamma\gamma''-\gamma'^2}{\gamma^2}\right]$$
$$+\frac{\xi}{1+r}\frac{\gamma'}{\gamma}\left[(1+r)\frac{u'^2-u\,u''}{u'^2}\frac{w^2}{\pi^3}h'\frac{(1+r)\,u'\gamma'}{u''}\xi\right]$$

There is an unstable eigenvalue $\xi_1 = 1+r > 1$. Note that the limit of $Q(\xi)$ when ξ tends to $\mp\infty$ is $-\infty$. We have:

$$Q(0) = -(1+r)(1-\delta) < 0$$

$$Q(1) = \frac{1}{1+r}\frac{\gamma'}{\gamma}\left[(1+r)\frac{u'^2 - u\,u''}{u'^2}\frac{w^2}{\pi^3}h'\frac{(1+r)\,u'\gamma'}{u''}\right] > 0$$

There thus exists a stable eigenvalue $\xi_2 \in]0,1[$ and an unstable one $\xi_3 > 1$.

$$\begin{aligned} &Q(-1) = \\ &-(2+r)\left[2(2-\delta)\left(\frac{1-\delta}{1+r}+1\right)+2\frac{w^2}{\pi^3}h'\frac{V}{1+r}\frac{\gamma\gamma''-\gamma'^2}{\gamma^2}\right] \\ &+\frac{1}{1+r}\frac{\gamma'}{\gamma}\left[(1+r)\frac{u'^2-u\,u''}{u'^2}\frac{w^2}{\pi^3}h'\frac{(1+r)\,u'\gamma'}{u''}\right] \end{aligned}$$

Using $\frac{V}{1+r} = \frac{1}{r}\frac{u}{u'}$ we rewrite $Q(-1)$ as follows:

$$Q(-1) = -C - D\frac{u}{u'} - E\frac{u'}{u''}\left(1 - \frac{u''u}{u'^2}\right)$$

where

$$C = 2(2+r)\left(\frac{1-\delta}{1+r}+1\right) > 0$$

$$D = 2(2+r)\frac{\gamma\gamma''-\gamma'^2}{\gamma^2}\frac{w^2}{\pi^3}\frac{h'}{r} > 0 \text{ since } \gamma'' < 0 \text{ and } h' < 0$$

$$E = -(1+r)\frac{w^2}{\pi^3}\frac{h'\gamma'^2}{\gamma} > 0$$

A sufficient condition for stability in the saddle-point sense is $Q(-1) > 0$, since there then exists two additional eigenvalues such that $0 < \xi_4 < -1 < \xi_5$. This condition is verified if u'' is sufficiently close to zero. We now look at this condition in the case of a CES felicity function: $u(c) = c^a$, $a \in]0,1[$. Steady-state consumption is given by: $c = a.F$, where:

$$F = \frac{wr(r+\delta)}{(1+r)^2}\frac{1}{\gamma'(z)\,g'(1-l)} > 0$$

The expression of $Q(-1)$ gives:

$$Q(-1) = -C - D.F + \frac{E.F}{1-a}$$

C, D, E, and F are independent from a. We therefore have:

$$\lim_{a\to 1} Q(-1) = +\infty$$

This ensures that the steady state is saddle-point stable if a is sufficiently close to 1.

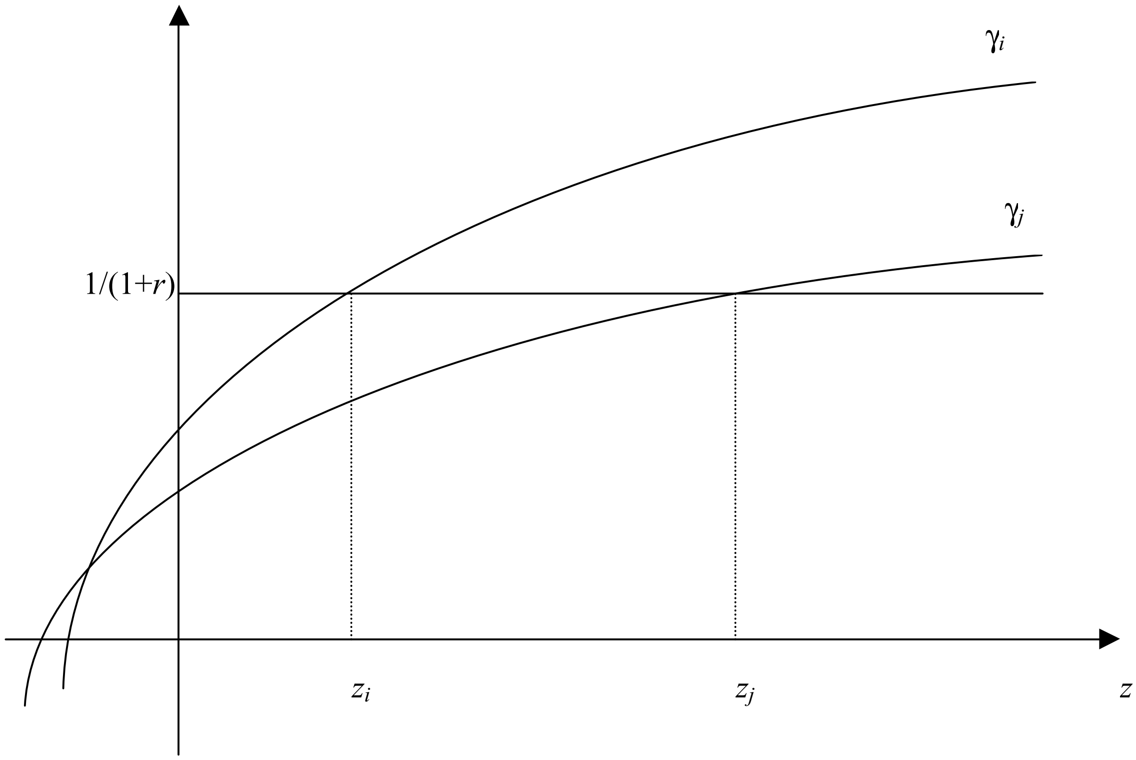

Figure 1: Steady state levels of personal capital

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