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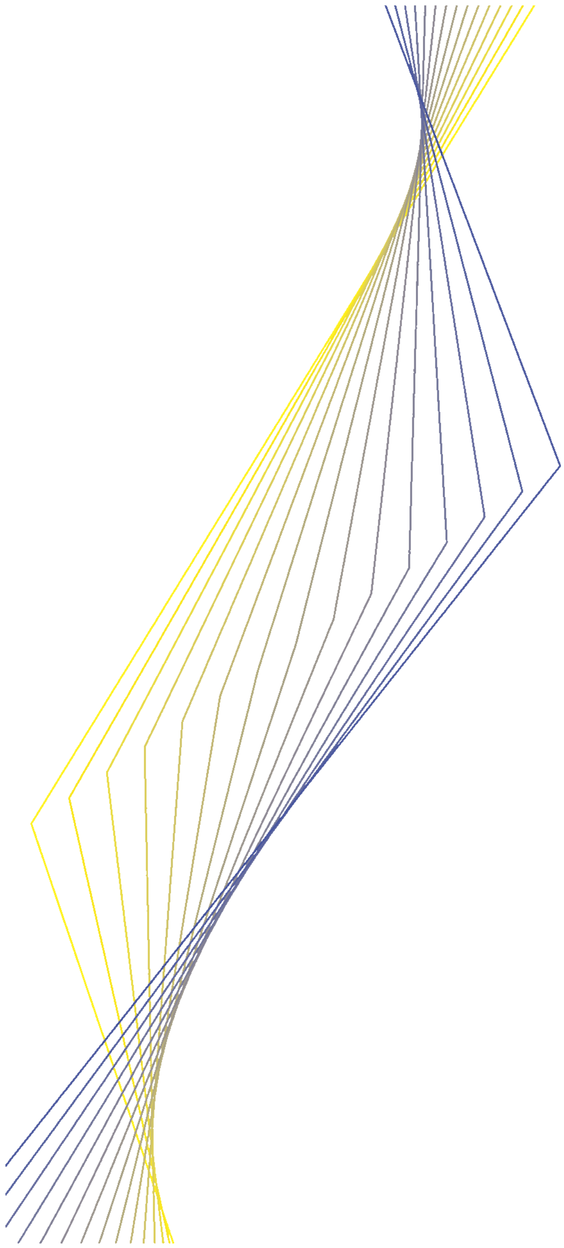

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A MODEL OF THE EUROSYSTEM'S OPERATIONAL FRAMEWORK FOR MONETARY POLICY IMPLEMENTATION[1]

BY CHRISTIAN EWERHART[2]

November 2002

1 *Postal address for correspondence: Economic Theory III, Adenauerallee 24-26, D-53113 Bonn, Germany. E-mail: ewerhart@wiwi.uni-bonn.de.* *This is a revised version of my earlier paper "A Model of the ECB Monetary Operations - with Application to the Problem of Underbidding." The research contained in this paper was carried out whilst the author was visiting the Directorate General Research as part of the ECB Research Visitors Programme 2002. I thank the Editorial Board of the ECB Working Paper Series as well as an anonymous referee for many insightful suggestions. For useful discussions and comments, I thank Ignazio Angeloni, Ulrich Bindseil, Nuno Cassola, Steen Ejerskov, Vitor Gaspar, Hans-Peter Grüner, Philipp Hartmann, Martin Hellwig, Oliver Kirchkamp, Benny Moldovanu, Cyril Monnet, Clara Martin Moss, Kjell Nyborg, Ulrich Schlieper, Livio Stracca, Natacha Valla, and Juan Luis Vega. I am indebted to Mathias Meisel, Head of Fixed Income at Zurich Group Invest, for extensive discussions that initiated this project. Financial support by the Deutsche Forschungsgemeinschaft through the SFB 504 is gratefully acknowledged. The opinions expressed herein are those of the authors and do not necessarily represent those of the European Central Bank.*

2 *University of Bonn, Department of Economics*

Address	**Kaiserstrasse 29**
	D-60311 Frankfurt am Main
	Germany
Postal address	**Postfach 16 03 19**
	D-60066 Frankfurt am Main
	Germany
Telephone	**+49 69 1344 0**
Internet	**http://www.ecb.int**
Fax	**+49 69 1344 6000**
Telex	**411 144 ecb d**

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Contents

Abstract

This paper offers a game theoretic model of liquidity provision through repeated central bank tenders, in the spirit of the operational framework of the Eurosystem. Banks are required to satisfy reserve requirements subject to an averaging provision over individual maintenance periods, and transactions may hang over into the respective subsequent period. It is shown that liquidity shocks are absorbed by the system by exponentially declining oscillations around the stationary equilibrium. When a policy rate cut is expected, bidders strategically reduce demand prior to the decision, which may unbalance the system. The anticipation of strategic behavior may generate an oscillation even before the maintenance period in which the decision is expected. When the recently released ECB proposal is implemented in the model, then the bidders' strategic motives are effectively eliminated. It is shown that, alternatively, bidding behavior can be corrected using a simple reimbursement scheme.

JEL CODES: E51, G28

KEYWORDS: euro, monetary policy instruments, money market, operational framework, refinancing operations, reserve requirements, underbidding, volatility

Non-technical summary

This paper offers a game theoretic model of liquidity provision through repeated central bank tenders. The basic features of the framework correspond to those of the main refinancing operations conducted by the European Central Bank since the switch to variable-rate tenders in June 2000. The model allows to formally discuss the impact of liquidity shocks and changing policy rate expectations on the dynamics of bidding volumes, tender conditions, and money market rates. In particular, it is feasible to evaluate the theoretical effectiveness of specific policy proposals.

The model has the following structure: There are infinitely many maintenance periods. Each of a finite number of banks is required to satisfy an exogenously given reserve requirement. The reserve requirement is subject to an averaging condition, and needs to be fulfilled in all maintenance periods. Refinancing is feasible either by participation in the central bank tenders or by borrowing money in the competitive overnight market.

It is assumed that in each maintenance period, there are two tender operations in which transactions are allotted to the participating banks. Transactions allotted in the first tender of a given maintenance period mature at the time of the second tender of the same maintenance period, while transactions allotted in the second tender mature at the time of the first tender in the subsequent period.

The stationary equilibrium is characterized by balanced bidding volumes, stable tender conditions, and flat money market rates. Unexpected liquidity shocks are absorbed by the system by exponentially declining oscillations around the stationary equilibrium. The speed of convergence depends on two factors which are, firstly, the extent to which the transactions of the second tender hang over into the subsequent period, and secondly, the relative importance of the second tender as a means of refinancing the banking system.

To study the impact of rate change expectations on the operational framework, a monetary policy decision is introduced between the two tender operations in one of the maintenance periods. When a policy rate cut is expected, bidders will strategically reduce demand prior to the decision, when compared to the stationary level. This unbalances the system in several ways. One effect is that, after the occurrence of underbidding, the system behaves as in the case of a significant liquidity shock. The reason is that underbidding in the first tender

generates a strong demand in the second tender, which in turn yields excess liquidity at the beginning of the subsequent period.

Another consequence of underbidding is that in the period immediately prior to the one in which the rate cut is expected, bidders will forecast the lowered overnight rates at the beginning of the subsequent period, and therefore reduce their demand for those central bank transactions that hang over into the subsequent period. As a consequence of these mechanics, the release of information about a potential change of the policy rates in the future may affect even short-term interest rates.

The analysis proceeds by deriving the equilibrium prediction for the measures that have recently been put forward by the European Central Bank for public consultation. It is shown that in the model, implementing the proposed changes resolves the underbidding problem, and that the suggested combination of the individual measures concerning the timing of the maintenance period is the most effective approach to reduce speculative motives on the part of the bidders.

As an alternative means for reducing the extent of strategic underbidding, the paper studies also an insurance scheme that reimburses non-strategic bidders in the case of a policy rate cut. The incentive effect of this scheme is that the risk of a policy decision is shifted from the banks that participate in the open market operations to the central bank, which corrects bidding behavior in the prospect of changing policy rates.

A possible implementation of the suggested reimbursement scheme uses variable rate transactions, where the reference rate would be the base rate at the end of the respective maintenance period. E.g., when the minimum bid rate of a given tender is 4.00%, and the accepted bid specifies a rate of 4.10% then, following a rate cut of 50 basispoints in the current maintenance period, all interest payments would be based on the rate 4.10% - 0.50% = 3.60%. However, when policy rates remain unchanged, then the interest payments would still be based on the rate 4.10%. Thus, under the proposed scheme, banks would effectively submit spreads on the minimum bid rate rather than absolute rates.

The proposed implementation of the reimbursement scheme via variable rate transactions symmetrically requires banks to effectively pay rate increments following an ex post increase in the minimum bid rate, and therefore eliminates also (less harmful) incentives for overbidding in times of increasing policy rates.

1. Introduction

This paper proposes a theoretical framework for the analysis of liquidity provision through repeated central bank tenders. The basic features of the model correspond to those of the main refinancing operations conducted by the Eurosystem[2] since the introduction of the variable-rate tender in June 2000. It is the objective of the present study to contribute to the understanding of the mechanics of monetary policy implementation in the euro area, and to formally discuss policy proposals that support and guarantee the smooth functioning of the operational framework even under unfavorable market conditions.

From the empirical literature on the Eurosystem's operational framework and its interplay with the euro money market, three contributions deserve special emphasis. To start with, Nyborg, Bindseil and Strebulaev (2002) offer a variety of stylized facts on bidder behavior in the Eurosystem's refinancing operations. One of their findings is that private information seems to be of only minor importance in these operations. Indeed, theory suggests that in a common value auction, more uncertainty should amplify the winner's curse, and therefore should lead to lower bids. However, it turns out that the volatility of money market rates is positively correlated with the level of bids in the refinancing operations, which seems to contradict the theoretical prediction.[3]

Another finding of the above study is that bidders tend to use what Nyborg et al. call "dampened cycling" strategies. That is, bidder participation is not constant over time, but varies significantly within maintenance periods. This behavior is attributed by Nyborg et al. to the existence of collateral requirements. Their study provides many further empirical insights, and we strongly recommend the paper to the interested reader.

Hartmann, Manna and Manzanares (2001) provide an empirical examination of the microstructure of the euro money market. This study finds in particular that market expectations of interest rate changes were on average relatively precise during the sample period. Market power and adverse selection seemed to be of minor importance.

Gaspar, Pérez-Quirós and Sicilia (2001) consider the performance of monetary policy implementation by the Eurosystem from a money market perspective.

[2] The Eurosystem consists of the European Central Bank (henceforth ECB) and the 12 national central banks of the euro area.

[3] A potential explanation might be that much of the private information is already reflected in the market rates (I am grateful to Benny Moldovanu for pointing this out). More work seems desirable to clarify this interesting point.

One of the main findings in their study is that monetary policy decisions by the Governing Council have not significantly affected the volatility of the overnight rates in the euro money market. Moreover, as in the previously mentioned paper, it is found that market participants had on average correct expectations about the change in the policy rate.

The theoretical literature on tender operations in the Eurosystem is relatively recent. Recurring topics in this literature are the so-called overbidding in fixed-rate tenders, the design of the tender operations (fixed-rate vs. variable-rate), and the pre-announcement of the liquidity injection. Some contributions also discuss briefly the underbidding issue. Ayuso and Repullo (2000) explain the overbidding phenomenon in fixed-rate tenders as a consequence of an asymmetric loss function of the central bank. They argue that variable-rate tenders can remedy the overbidding problem when the intended liquidity injection is preannounced. Nautz and Oechssler (2001) offer a model without equilibrium in which the increasing extent of overbidding in fixed-rate tenders is a consequence of a myopic best-response adaptation by the banks participating in the tenders. Bindseil (2002) discusses a model with a two-day maintenance period, and analyzes the performance of specific tender procedures with respect to a number of central bank objective functions. Bindseil (2001) considers the impact of the publication of liquidity forecasts on the overnight rate in the euro banking system. Välimäki (2002) argues that with overlapping maturities, the overnight rate will increase above the target level even in expectation of a policy rate cut. He derives in particular that the variable-rate tender is superior to alternative procedures. In contrast to our set-up, operations do not hang over in Välimäki's model. Catalão-Lopes (2001) argues that variable-rate tenders mitigate the overbidding problem. In a somewhat different vein, Pérez-Quirós and Mendizábal (2000) offer an argument explaining why the martingale hypothesis has been met more closely after January 1999.

The literature on the US market for federal funds has a somewhat longer tradition. Ho and Saunders (1985) offer a model of the market for federal funds that explains the spread between funds rate and other short-term interest rates. Campbell (1987) documents features of the federal funds rate day-to-day behavior and offers a theoretical explanation of these features. Spindt and Hoffmeister (1988) consider a model in the tradition of Garman (1976) that derives the increasing variance of the federal funds rate towards the end of the maintenance period as a consequence of boundedly rational behavior on the part of the reserve managers. Spindt and Hoffmeister also provide a very detailed description of reserve accounting in the US. Hamilton (1996) rejects the martingale hypothesis for the market for federal funds. He conjectures that transaction costs are responsible for his empirical finding. Hamilton (1997) quantifies the effect of additional liquidity on money market rates.

The present paper wishes to contribute to the theoretical literature on monetary operations by analyzing the impact of liquidity shocks and changing policy rate expectations on the dynamics of bidding volumes, tender conditions, and market rates. To illustrate our results, we will use the Eurosystem in the year 2001 as a case study. A prominent feature of the bidding data for that year has been that banks did not always demand sufficient liquidity from the Eurosystem in response to changing expectations about the development of policy rates. Altogether, one could observe four occurrences of "underbidding" in the Eurosystem in the year 2001.[4] We will provide more details on the sequence of events in section 2.

There are a number of reasons why underbidding is not desirable. The most obvious reason is that, as a consequence of insufficient liquidity provision, market participants have to have recourse to the standing facilities to a larger extent, which increases the volatility of short-term interest rates. Another problem is that underbidding generates uneven tender volumes, which may reduce the central bank's ability to tighten liquidity conditions when necessary. Also, an unbalanced refinancing system increases uncertainty, and thereby makes forecasting and bidding more difficult for market participants. Last but not least, underbidding may impair the communication between central bank authorities and financial markets.[5]

The formal discussion suggests a number of strategies that can be employed by the central bank in order to reduce strategic motives for underbidding in the prospect of decreasing policy rates. It is shown in particular that the proposal that has recently been released by the ECB for public consultation effectively realigns the incentives of bidders in the main refinancing operations with the objectives of smooth liquidity provision and stable short-term market rates. Moreover, the individual measures of the solution can be seen to complement

[4] Underbidding is usually understood in the sense that aggregate bids in the tender operation amount to less than the neutral liquidity allocation, which is the forecasted amount necessary to guarantee that credit institutions, in aggregate, fulfil their reserve requirements in a regular way accross the reserve maintenance period (cf. ECB, 2002b).

[5] An example is Bundesbank president Ernst Welteke's comment on the dangers of inflation prior to the main refinancing operation on November 7, which has been interpreted as an attempt to avoid the imminent underbidding (cf. Financial Times Deutschland, 5.11.2001).

each other in the sense that an individual measure may be less effective than the proposed combination.

An alternative strategy that seemingly has not been considered before is to reimburse non-strategic bidders in the case of a policy rate cut. The incentive effect of such an insurance scheme is that it shifts the risk of a policy decision from the individual bank to the central bank, which corrects bidding behavior in the prospect of changing policy rates.

A possible implementation of this scheme employs transactions with variable rate interest payments. For an example, consider a situation where the main policy rate is 4.00% at the beginning of a reserve maintenance period, and a bank obtains a repo at 4.10% in the initial tender. Then, following a rate cut of 50 basispoints, the interest rate for the repo would be lowered, ex-post, to the rate 4.10% - 0.50% = 3.60%. When policy rates remain unchanged, however, then the interest rate for the repo would remain at 4.10%. By conditioning the transaction rate in this way on the base rate that prevails at the end of the maintenance period, the reimbursement scheme eliminates the speculative motive that underlies the underbidding phenomenon, and thereby realigns the incentives of individual banks with the central bank's objective of smooth liquidity provision.

The proposed policy scheme has an analogy in the theoretical literature on the Coase conjecture (see, e.g., Tirole, 1992). There, a monopolist trying to sell units of a homogeneous good in a number of subsequent periods faces the problem that potential buyers will wait in order to profit from declining prices. One possible strategy for the monopolist is a money-back guarantee, which can be considered as an equivalent to the proposed reimbursement strategy.[6] However, it will be noted that the averaging provision of reserve requirements makes the analogy imperfect, which suggests a separate formal derivation of the incentive compatible scheme in the case of central bank tenders.

The rest of the paper is structured as follows. Section 2 provides some background on the institutional framework, and on the sequence of events during 2001. Section 3 presents our model. In section 4, we derive underbidding as an equilibrium outcome in an intertemporal model of liquidity provision. Section 5 contains a discussion of existing policy proposals. In section 6, we derive the

[6] An illustrative example is the marketing of an innovative computer gadget to consumers who are aware of the fact that prices will fall in the near future. The retailer may then be able to improve sales numbers when he promises to reimburse early buyers in case of a later cutback of the price.

incentive compatible reimbursement scheme. Section 7 discusses the robustness of the results, and section 8 concludes. Technical proofs are collected in the appendix.

2. Background on the institutional setting and the sequence of events

Operational framework. For the operational implementation of its monetary policy and liquidity management, the ECB uses a collection of procedures and instruments that, according to a useful taxonomy, can be classified as open market operations, standing facilities, and reserve requirements.

By open market operations, liquidity is provided to the banking sector at the initiative of the ECB. Among these operations, the so-called *main refinancing operations (MROs)* play a central role.[7] The operation determines the volume of the transaction between central bank and individual banks. The typical transaction type is a so-called repo or securities repurchase transaction, but collaterized loans are also in use.[8] The maturity of the weekly main refinancing operations lies in a close range around 14 days, so that there is an overlap of maturities between any two consecutive tenders.

On June 8, 2000, the ECB announced that the main refinancing operations of the Eurosystem would be conducted as *variable-rate tenders*, starting from the operation to be settled on June 28, 2000. The switch to a variable repo has been considered as having no direct implication on monetary policy.[9] While the ECB did not rule out that main refinancing operations of the Eurosystem may be conducted as fixed-rate tenders in the future, all tender operations since then have been variable-rate tenders.

Before bids have to be submitted, the ECB provides the market with a forecast from which banks can calculate the anticipated aggregate liquidity needs of

[7] During the year 2001, average refinancing volume was approximately Euro 222 bn. Thereof 161 bn was provided by MROs, 57 bn by longer-term operations, 3 bn by fine-tuning and structural operations, and 0.8 bn by the use of the marginal lending facility.

[8] In a repo transaction, it is agreed that the central bank buys certain collateral securities from the respective counterparty and sells it back to the counterparty after two weeks, which provides this counterparty with liquid means for the intermediate period. The counterparty pays an interest on this de facto credit. Repo transactions have been used by the Bundesbank already since 1979.

[9] Such an implication could have been derived from the differential between the minimum bid rate and assignment rates due to aggressive bidding behavior in the expectation of increasing base rates. However, the 50 basis points hike with effect from June 9 for standing facilities and from June 15 for main refinancing operations cleared the market from those expectations, and made a smooth transition to the variable rate procedure feasible (cf. Deutsche Bank, 2000).

the banking system. In variable-rate tenders, eligible banks are permitted to submit up to ten individual bids, each consisting of an interest rate and a euro amount of liquidity desired at that interest rate. Under normal circumstances, several hundred banks in the euro area submit bids. The ECB then allocates a certain amount of liquidity which is determined on the basis of a number of factors, including, but not limited to the liquidity forecast.

Bids that fall short of the *minimum bid rate* are discarded. This policy rate has been designed by the central bank to play the role performed until then by the pre-announced rate in fixed-rate tender operations, i.e., serving the purpose of signaling the stance of monetary policy. In allocating liquidity, the bids with the highest interest rates are considered with priority until either the desired amount of liquidity has been fully allocated, or there are no eligible bids left. The lowest rate at which liquidity is allocated is called the *marginal rate* of allotment. When the central bank decides not to satisfy all bids at the marginal rate, then proportional rationing is applied. The price determination is discriminatory or *American* (rather than uniform or *Dutch*), i.e., each counterparty pays a price corresponding to the volume-weighted interest rates of her successfully placed bids.

The liquidity provision to the banking system is complemented by the standing facilities, which essentially allow a counterparty to either acquire liquidity with overnight maturity at the so-called *marginal lending rate*, which has been typically 1 percent above the minimum bid rate, or to deposit spurious central bank money at the *deposit rate*, which symmetrically has been 1 percent below the minimum bid rate. These facilities are used under normal circumstances only at the end of the maintenance period when all tender operations have already been conducted, and the market's short-term demand and supply for liquidity become very inelastic.

The bulk of the demand for liquidity in the banking sector is generated by the *reserve requirements*.[10] All required reserves are remunerated according to a rate that equals the time-weighted average of the MRO rates.[11]

[10]During 2001, the Euro area credit institutions' aggregate reserve requirement was on average Euro 124 bn, while autonomous factors (chiefly banknotes in circulation, central government deposits in the Eurosystem, and central banks' foreign reserve assets) amounted to 98 bn.

[11]According to ECB (2002a), the rationale for having reserve requirements in the first place is twofold, namely the stabilization of interest rates, and the creation or enlargement of a structural liquidity shortage that increases the effectiveness of monetary policy measures. See Manna, Pill and Quirós (2001) for discussion.

Reserve requirements need to be satisfied on average within a so-called reserve requirement or *maintenance period.* A maintenance period in the Eurosystem usually lasts one month, ending with the 23rd of the respective calender month. If a bank obtains a positive volume in a tender at the end of a maintenance period, the respective transactions typically mature only in the next period. In that case, liquidity obtained in one period also serves to fulfil the reserve requirements in the subsequent period, and therefore becomes a substitute for liquidity acquired in the subsequent period.

The euro money market. The underlying good traded in the euro money market is alternatively referred to as cash, central bank money, liquidity, or reserves. A useful definition characterizes the money market as dealing with credit of maturity below one year. The market can be divided into a cash market and a derivative market. The cash market consists of three segments, which are markets for unsecured deposits (concentrated on overnight maturity), the repo market, and the foreign exchange swap market. Typical money market derivatives are interest rate swaps and futures, and these are traded over-the-counter ("OTC") as well as exchange-based.

The main reference rate in the euro money market is the EONIA (euro overnight index average). It is computed as a weighted average of overnight unsecured lending transactions undertaken in the interbank market, initiated within the euro area by the contributing panel banks. Given that the Eurosystem provides standing facilities for both lending and borrowing at 1% below and above the minimum bid rate, respectively, money market rates have a clear defined range in which they may fluctuate. However, since overnight transactions are uncollateralized, while both tender offers and standing facilities require collateral, the correspondence is imperfect, and it may therefore happen, e.g., that the money market rate slightly exceeds the marginal lending rate.

The cash market is characterized by the concentration of intermediation activity on a comparably small number of major institutions. Apart from these global players, small to medium-size banks remain more nationally oriented. Because credit is uncollateralized, lenders typically extend a credit line to a prospective lender, before money is transferred.

For further institutional details and empirical investigations of the operational framework and the euro money market, we refer the reader to ECB (2000, 2001, 2002a).[12] Aggregate data on the tender auctions is available from the

[12]See also Würtz (2002), and references given therein.

ECB Monthly Bulletin.

Underbidding. With the growing expectation of base rate cuts in the euro area, instances of demand reduction have interfered with the smooth provision of liquidity to the banking system. Below, we give a brief account of the four incidences of underbidding in the Eurosystem during the year 2001. Underbidding had occurred once before on April 7, 1999, whereas under a fixed-rate tender regime, and with less costly effects for market participants.

February 2001. The first instance of underbidding in 2001 occurred on February 14, one day prior to an ECB Governing Council meeting. The banks submitted total bids of only euro 65.3 bn, which was the lowest level since the introduction of the variable rate tender, and less than half of the previous average bidding volume (see Figure 1). Offers were concentrated on few rates close to the minimum bid rate. The number of bidders was also lower than under normal circumstances. As Figure 1 shows, the ECB satisfied all bids completely for the first time, and prior allotments totalling almost 100 percent of the submitted bids heralded the imminent problem. Counterparties were also more reluctant to bid in long-term refinancing operations.

The MRO following the ECB Council meeting, on February 21, was the last in the maintenance period, which ended on February 23. The total of bids exceeded euro 200 bn, and a record amount of euro 155 bn was allocated, but the liquidity situation remained tight, so that the market rate temporarily reached almost the marginal lending rate (cf. Figure 2). A second problem was that the tender volumes on February 14 and February 21 had a very different size. This affected the volumes of the subsequent tenders in a way that led to alternating allotment amounts, which were either too low or too high, continuing over the whole of March and even into April.

April 2001. Before allotment sizes could balance out, on April 11, immediately prior to the Council meeting on the same day, demand dropped again, this time to a record low of less than euro 25 bn. One week later, on April 19, after money market rates had reached the marginal lending rate for the first time, banks submitted record bids of over euro 250 bn in the last MRO prior to the end of the maintenance period, and a new record volume of euro 172 bn was allocated. To re-balance tender volumes, the Eurosystem run an additional operation with a maturity of seven days on April 30.

On May 10, the Governing Council of the ECB decided to lower the minimum bid rate on the MROs by 0.25 percentage points to 4.50%, and to lower both the marginal lending rate and the deposit facility correspondingy. This measure reduced speculative motives concerning interest rates, and stabilized both tenders volume and market rate for the subsequent months. However, as Figure 1 shows, excess demand in the tender operations began again to decline in the summer of 2001.

October 2001. The MRO operation on October 10 also led to comparatively low bids by the market participants. While all bids were satisfied by the ECB, the total amount of available liquidity was too low to guarantee a smooth fulfilment of the reserve requirements. At the end of the maintenance period, the use of marginal lending facility was higher than usual. However, since autonomous factors turned out to be unexpectedly low, unusual movements in the overnight rate were prevented.

November 2001. Finally, on November 7, counterparties submitted bids of only euro 38.4 bn, obviously in anticipation of the base rate cut at the ECB Council meeting at the next day. While banks did not have to have recourse to the marginal lending facility in order to fulfil their reserve requirements, the period ended with two tenders of very different size, so that the ECB decided to conduct another one-week-maturity operation in the subsequent period to equalize the volumes of the two tenders. To reduce the possibilities for speculation in connection with underbidding, the Governing Council adopted a rule in November by which monetary policy decisions would henceforth be made only at the Council's respective first meeting in each month.

The above incidents suggest that the smooth operation of the operational framework is affected by underbidding in times in which the market expects decreasing interest rates. In the next section, we develop a model that captures demand reduction as an equilibrium outcome between agents that assign positive probability to the possibility that a base rate cut will occur either within the current period or in one of the subsequent periods.

3. The model

There are n counterparties or banks. Each bank i is required to hold average reserves of $\overline{H}^i$ in each maintenance period. Assume that bank i has guaranteed in each period an average net cash position of H_0^i. This position will be positive for banks that attract sizeable amounts of deposits such as saving banks, and negative for banks with extensive lending, e.g., banks specialized in real estate financing.[13] We assume for simplicity that $\overline{H}^i$ and H_0^i are exogenously given and constant. Then bank i has an expected net position of required liquidity

$$\overline{L}^i = \overline{H}^i - H_0^i \tag{1}$$

in each maintenance period. If $\overline{L}^i > 0$, then bank i is in demand of liquidity, and this demand may be satisfied using a combination of funds acquired either in the respective operations or in the money market. If $\overline{L}^i < 0$, then the bank possesses excess liquity, which it may lend out in the money market. Of course, a bank could decide both to participate in the tender operations and to lend out money if market spreads make this sufficiently attractive.

The time structure is as follows (see Figure 3). Maintenance periods are denoted by $t = 0, 1, 2, ...$ We normalize the continuous time scale such that the initial maintenance period corresponds to the time interval $[0; 1]$. At some time $\tau_1 \in [0; 1]$, central bank money is auctioned off to the banks in a first tender. Then, still in period 0, at time $\tau_2 > \tau_1$, liquidity is provided via a second tender. At time 1, the initial maintenance period ends, and the second maintenance period commences. From then onwards, we have two tender operations in each period t, one at $t + \tau_1$, and another at $t + \tau_2$.

For simplicity, we assume that the transactions allotted in the first tender in each period t mature at the time of the second tender, that is at time $t+\tau_2$. Liquidity purchased in this tender therefore does not contribute to the fulfilment of reserve requirements in later periods. However, transactions alloted in the second tender of period t are assumed to mature at time $t + 1 + \tau_1$, and therefore contribute to the fulfilment of the reserve requirement in the subsequent maintenance period $t + 1$. We also assume that there are no other refinancing operations, i.e., no longer-term, fine-tuning, or structural operations.

All interest rates are normalized so that they correspond to the length of one maintenance period.

It is assumed that the central bank exerts its influence on the liquidity provision by the conditions it allows in the individual tenders. Specifically, let $\underline{r} > 0$ denote the *minimum bid rate*. Write L^i for the liquidity acquired by bank i in a given tender. Total liquidity provision is then $L = \sum_{i=1}^n L^i$. The marginal *assignment rate* that has to be paid by bank i is given by the inverse supply

[13] Considered as an aggregate over the banking system in the Euro aera, this position has been negative, and thereby added to the structural liquidity shortage created by the central bank.

function $\underline{r} + s(L)$, for some continuous mark-up function $s(L)$. We assume that $s(L) \geq 0$ is strictly increasing, so that an individual bank can obtain more liquidity only at a higher average rate, and each bank obtains worse conditions when other banks increase their demand. Let $L^i_{t,A}$ and $L^i_{t,B}$, respectively, denote the liquidity acquired by bank i in the first and second tender of period t.

The chosen set-up suggests that the higher the bid rates, the more liquidity the central bank will be willing to allot. While this assumption is not likely to affect the conclusions of the analysis (cf. our discussion in section 7), it should be noted that there is probably not too much empirical support for it. For example, during the phase of rate hike expectations and variable rate tenders, i.e., essentially during the second half of the year 2000, elements of such a policy were only observed to a very limited extent, and seem to have discontinued after a while.[14] However, using a model with elastic supply may nevertheless be desirable because it implies the realistic feature of a tender volume that is gradually declining when the money market rate comes close to the minimum bid rate.

The model will allow banks unrestricted access to interbank liquidity at the current market rate at all times.

Consider first the no-arbitrage conditions that arise from the possibility of participating in the main refinancing operations. In each period $t \geq 0$, central bank money acquired in the first tender operation is a perfect substitute to money from the interbank market, hence

$$\underline{r}_t + s(L_{t,A}) = r_t, \tag{2}$$

where $\underline{r}_t$ and r_t are minimum bid and market rate in period t, respectively, and

$$L_{t,A} = \sum_{i=1}^{n} L^i_{t,A}. \tag{3}$$

A similar consideration for the second tender suggests that for any $t \geq 0$,

$$(1 - \tau_2 + \tau_1)(\underline{r}_t + s(L_{t,B})) = (1 - \tau_2)r_t + \tau_1 r_{t+1}, \tag{4}$$

where $L^i_{t,B}$ is the liquidity acquired by bank i in the second tender of period t, and

$$L_{t,B} = \sum_{i=1}^{n} L^i_{t,B}. \tag{5}$$

[14] We are very grateful to the anonymous referee for pointing this out.

The above equations capture the reserve manager's decision between refinancing in the money market and participation in the tender. E.g., in the second operation of a given period, auction rates may be more attractive, but because central bank funds hang over into the subsequent maintenance period, where market rates may be lower, the reserve manager may be ready to pay a higher interest on overnight credit than in the tender transaction. As a consequence, market rates may differ from tender rates in the second auction.[15]

As a third equilibrium condition, we have the aggregate reserve requirement

$$\tau_1 L_{t-1,B} + (\tau_2 - \tau_1) L_{t,A} + (1 - \tau_2) L_{t,B} = \overline{L} \tag{6}$$

for all periods $t \geq 0$, where

$$\overline{L} = \sum_{i=1}^{n} \overline{L}^i. \tag{7}$$

This aggregate condition follows from the individual reserve requirements and from the continuous-time market clearing condition for the money market.

The reader will note that the model does not explicitly consider standing facilities. This is for simplicity of exposition only. In section 7, we outline an extension of the model that takes account of the marginal lending and deposit facilities.

To derive a reference point for our subsequent analysis of underbidding, assume now that the minimum bid rate remains stable over time. Specifically, assume $\underline{r}_t = \underline{r}$, $r_t = r$, $L_{t,A} = L_A$, and $L_{t,B} = L_B$, for all t. Then from (2) and (4),

$$\underline{r} + s(L_A) = \underline{r} + s(L_B) = r, \tag{8}$$

so that $L_A = L_B$. From (6),

$$(\tau_2 - \tau_1) L_A + (1 - \tau_2 + \tau_1) L_B = \overline{L}. \tag{9}$$

Thus, the smooth operation of the open market operation described by

$$r^* = \underline{r} + s(\overline{L}), \text{ and} \tag{10}$$

$$L_A^* = L_B^* = \overline{L}, \tag{11}$$

[15] The model of the money market requires a brief theoretical discussion (I am grateful to Martin Hellwig for pointing this out). Assuming a Walrasian equilibrium in a market presupposes to allow a hypothetical auctioneer to determine the market price on the basis of demand and supply. However, strictly speaking, in our model, after the second tender, both demand and supply are perfectly inelastic. To avoid this problem, one may assume that market participants trade on perfect forward markets just before the respective tender. At that stage, both demand and supply are still elastic, and tatonnement identifies a unique market-clearing price.

is a stationary equilibrium in which the overnight rate is equal to the tender rates, and liquidity provision is smooth.

Liquidity shocks. We turn first to the impact of an initial liquidity shock on the stationary equilibrium. The key question will be which interest rate will result in maintenance period t, and in particular, in the initial period. It is clear that the market rate depends on the expectations that individuals hold on the rates in the next period. More precisely, demand in the second refinancing operation in any period t depends, via the no-arbitrage requirement, on the conditions in the market in the subsequent period $t+1$. These conditions, however, depend again on market conditions in the subsequent period $t+2$, and so on. While the resulting infinite regress would allow in principle many continuation paths for excess liquidity and liquidity spread, it turns out that under a minor additional assumption, there is a unique path that is consistent with rational expectations.

For the sake of notational simplicity, it will be useful to consider the case where there is a non-equilibrium amount of central bank funds from the second tender of the previous period $t = -1$ in the market. The reader will note that because of the averaging provision on the reserve requirement, any initial liquidity shock can be expressed in that way. Therefore, the initial period can without loss of generality be assumed to start with the liquidity position

$$E_0 = L_{-1,B} - \overline{L}, \tag{12}$$

where a strictly positive E_0 indicates excess liquidity in the initial maintenance period, while a strictly negative E_0 indicates a liquidity shortage in this period.

In reality, the liquidity shortage created by the central bank is large enough to ensure that liquidity shocks due to autonomous factors are relatively small. We assume therefore that $|E_0|$ is not too large when compared to the overall liquidity shortage $\overline{L}$.[16]

It will be useful to have the following notation available. For any $t > 0$, let

$$E_t = L_{t-1,B} - \overline{L} \tag{13}$$

[16] It is not difficult to see what would happen when we drop this assumption: E.g., when $E_0 > \overline{L}$, then the tender operations would become superfluous for refinancing purposes, which appears as a rather unlikely event. On the other hand, when $E_0/\overline{L} << 0$, then liquidity demand in the initial period would be very high, and in very extreme cases, the resulting tender volumes would satisfy liquidity requirements already in the next period just from the maturity overlap. While theoretically feasible, also this case appears less likely in practice.

denote the difference between the actual and the stationary liquidity position. Similarly, let

$$\rho_t = r_t - \underline{r}_t - s(\overline{L}) \tag{14}$$

denote the interest rate spread between the stationary and the actual overnight rate. We will refer to ρ_t as the liquidity spread in period t.

To make the model tractable, we consider from now on a linear approximation, i.e., we assume

$$s(L) = \alpha L \tag{15}$$

for some constant $\alpha > 0$. The parameter α can be interpreted as a measure for the tightness of liquidity provision. If α is small, then the central bank is ready to provide sufficient liquidity to the market at rates that are close to the minimum bid rate. In contrast, if α is large, then additional liquidity is made expensive. While α is in principle a choice variable for the central bank, we will treat it here as an exogenous constant.

By a *rational expectations equilibrium*, we mean a triple of processes

$$(r_t, L_{t,A}, L_{t,B})_{t\geq 0} \tag{16}$$

that satisfies (2), (4), and (6) for all integers $t \geq 0$, where $L_{t-1,A}$ and $L_{t-1,B}$ are exogenous constants. It should be clear that an equilibrium path can be alternatively be described by the pair $(\rho_t, E_t)_{t\geq 0}$ of processes describing liquidity spread and excess liquidity, respectively.

To simplify the analysis, we will assume throughout the paper that market participants do not hold too extreme expectations about the development of money market rates. Specifically, we will say that market expectations are *conservative* if banks expect the money market rate to lie above the minimum bid rate at the time of the first tender in all periods, i.e., $r_t \geq \underline{r}_{t,A}$. Given the absense of interim liquidity shocks in our model, this assumption is implied by strictly positive demand in the respective first refinancing operation in all periods, which appears as a relatively mild restriction when the structural liquidity deficit generated by the central bank is sufficiently large.[17]

Our first main result says that an unexpected liquidity shock is absorbed by the system by exponentially declining oscillations around the stationary equilibrium (cf. Figure 4).

[17]While we have not further elaborated on this point, we conjecture that this assumption can be substantially relaxed without affecting uniqueness.

Theorem 1. *The unique rational expectations equilibrium with conservative market expectations is given by*

$$\rho_0 = -\lambda\alpha E_0, \quad (17)$$

$$\rho_t = \varepsilon^t \rho_0, \text{ and } E_t = \varepsilon^t E_0, \text{ for } t > 0, \quad (18)$$

where

$$\lambda = \frac{\tau_1 + (1-\tau_2)\varepsilon}{\tau_2 - \tau_1} > 0, \quad (19)$$

and $\varepsilon = \varepsilon(\tau_1, \tau_2) \in [-1; 0]$. *That is, an initial liquidity shortage moves the money market rate above the rate of the second tender, and excess liquidity lets the money market rate drop below this tender rate. Moreover, in all subsequent periods, there is alternatingly excess liquidity or liquidity shortage, with market rates moving into the respective opposite direction. In absolute terms, both excess liquidity and liquidity spread converge to zero at an exponential rate.*

Proof. See the appendix. □

The rate of convergence to the stationary equilibrium depends in an intuitive way on the parameters of the model. It is clear that, when $|\varepsilon|$ is small, then convergence will be fast, while if $|\varepsilon|$ is close to 1, convergence will be slow. Moreover, as we show in the appendix,

$$\frac{\partial|\varepsilon|}{\partial\tau_1} > 0 \text{ and } \frac{\partial|\varepsilon|}{\partial\tau_2} < 0. \quad (20)$$

Thus, convergence to the stationary equilibrium will be the faster, the smaller the overlap of transactions from the second tender operation into the subsequent period, and the shorter the term of the transactions allocated in the second tender, i.e., the smaller the relative importance of the second tender as a means to satisfy aggregate reserve requirements.

An assumption underlying Theorem 1 is that the central bank follows a full allotment policy. However, in the case of the euro area, the ECB would always react to liquidity shocks by mechanically adjusting its liquidity supply (cf. ECB, 2002b). This is why the prediction of Theorem 1 is not reflected in the corresponding time series for the Eurosystem, at least not for the interest rate part.[18] Still, the above result shows that it is important that the central bank neutralizes exogenous liquidity shocks through its open market operations,

[18] Varying tender volumes are indeed observable, as Figure 1 shows, yet appear to be rather a consequence of the active liquidity management of the ECB and the overlapping term structure of the MRO transactions.

because otherwise, oscillations in both interest rates and tender volumes might be a consequence.

4. Falling interest rates

The model is now extended by allowing a base rate change between the first and the second tender in some arbitrary period t_0. Formally, let τ_d be such that $\tau_1 < \tau_d < \tau_2$ and assume that at time $t_0 + \tau_d$, there is a Governing Council meeting at which the policy rates may be adapted.

The uncertainty about the base rate change after the Council meeting is represented by a random variable d with distribution function $F(d)$ where, by way of convention, a positive d corresponds to a lowering of the base rate. Thus, the minimum bid rate in periods $t \neq t_0$ will be

$$\underline{r}_t = \begin{cases} \underline{r} & \text{if } t < t_0 \\ \underline{r} - d & \text{if } t > t_0 \end{cases} . \tag{21}$$

In period t_0, a minimum bid rate of $\underline{r}_{t_0,A} = \underline{r}$ will apply in the first tender, and a minimum bid rate of $\underline{r}_{t_0,B} = \underline{r} - d$ in the second tender. The distribution of d is assumed to be common knowledge. In the sequel, we will focus mostly on the case that the market expects a rate cut, i.e., that $E[d] > 0$. The case of increasing policy rates is discussed in section 7.

The notation of variables in periods $t < t_0$ remains unaffected. For variables in period t_0, we will use the following conventions. Let $r_{t_0,A}$ denote the money market rate in maintenance period t_0, before the Council meeting, and let $r_{t_0,B}(d)$ denote the money market rate in period t_0, after the Council meeting, and given that the base rate has been lowered by d. Consistent with this notation, we write

$$\rho_{t_0,A} = r_{t_0,A} - \underline{r} - \alpha\overline{L}, \text{ and} \tag{22}$$

$$\rho_{t_0,B}(d) = r_{t_0,B}(d) - \underline{r} + d - \alpha\overline{L}. \tag{23}$$

Let, as before, $L_{t_0,A}$ denote the tender volume in the first operation in period t_0. Let $L_{t_0,B}(d)$ and $L_{t_0,B}$ denote the respective tender volumes in the second operation in period t_0 after a rate cut of d, and when the policy rate remains unchanged. For periods $t > t_0$, all variables become functions of the realized rate cut. E.g., $r_t(d)$ denotes the money market rate in period t after a rate cut of d. We will write r_t for $r_t(0)$, and use an analogous notation for the other variables of the model.

The modifications of the equilibrium conditions are straightforward. Conditions for periods $t < t_0$ remain unaffected, with the only exception that, in the no-arbitrage condition for the second tender (4) in period $t_0 - 1$, the rate r_{t_0} should to be read as $r_{t_0,A}$. The conditions for period $t = t_0$ are as follows. First, the interest to be paid on funds acquired in the first tender must equal the average interest to be paid on the money market for the same term. Hence,

$$(\tau_2 - \tau_1)(\underline{r} + \alpha L_{t_0,A}) = (\tau_d - \tau_1)r_{t_0,A} + (\tau_2 - \tau_d)E[r_{t_0,B}(d)], \tag{24}$$

where $E[.]$ denotes the expected value operator. Similarly, the rates obtainable in the second tender must be arbitrage-free with respect to money market rates, i.e.,

$$(1 - \tau_2 + \tau_1)(\underline{r} - d + \alpha L_{t_0,B}(d)) = (1 - \tau_2)r_{t_0,B}(d) + \tau_1 r_{t_0+1}(d) \tag{25}$$

for any realized value of d. The money market rates within period t_0 before and after the Council meeting are linked by the additional equilibrium requirement

$$r_{t_0,A} = E[r_{t_0,B}(d)]. \tag{26}$$

Indeed, if (26) is not satisfied, then risk-neutral reserve managers could decrease expected interest payments by satisfying reserve requirements either earlier or later within the maintenance period. Finally, the aggregate reserve requirement for period t_0 is

$$\tau_1 L_{t_0-1,B} + (\tau_2 - \tau_1)L_{t_0,A} + (1 - \tau_2)L_{t_0,B}(d) = \overline{L}. \tag{27}$$

For all periods $t > t_0$, the symbols r_t, $L_{t,A}$, and $L_{t,B}$ in equations (2), (4), and (6) will be replaced by $r_t(d)$, $L_{t,A}(d)$, and $L_{t,B}(d)$, and the equilibrium conditions in these periods need to be satisfied for all possible realization values of d.

We will now analyze the equilibrium under rate cut expectations. Note first that $L_{t_0,B}(d)$ does not depend on the realization of d because reserve requirements make demand perfectly inelastic after the Council meeting (cf. (27)). Hence, the liquidity shortage $E_{t_0+1} = L_{t_0,B} - \overline{L}$ at the beginning of period $t_0 + 1$ is also independent of the rate cut. According to Theorem 1, rational expectations after the Council meeting imply

$$\rho_{t_0+1}(d) = -\lambda\alpha E_{t_0+1}, \tag{28}$$

i.e., the liquidity spread is also unaffected by the rate change. It is clear therefore that in the periods after the Council meeting, the development of market rates and bidding volumes does not depend on the level of the base rate. More precisely, we have

$$r_t(d) = r_t - d,\ L_{t,A}(d) = L_{t,A},\ \text{and}\ L_{t,B}(d) = L_{t,B} \tag{29}$$

for all $t > t_0$. Using (25), this implies in particular that the money market rate after the Council meeting reacts without distortion to the change in the policy rate, i.e.,

$$r_{t_0,B}(d) = r_{t_0,B} - d, \tag{30}$$

and consequently, from (26),

$$r_{t_0,A} = E[r_{t_0,B}(d)] = r_{t_0,B} - E[d]. \tag{31}$$

The above derivation leads us to our second main result which says that an expected base rate cut may generate underbidding, and that this underbidding generates oscillations in subsequent periods unless remedied by the central bank (cf. Figure 5).

Theorem 2. *Assume that at the beginning of period t_0, there is a net liquidity position of E_{t_0}. Then equilibrium quantities and spreads in period t_0 are as follows:*

$$L_{t_0,A} = \overline{L} - \lambda E_{t_0} - |\varepsilon| \frac{1-\tau_2}{\tau_1} \frac{E[d]}{\alpha} \tag{32}$$

$$L_{t_0,B} = \overline{L} - |\varepsilon| E_{t_0} + |\varepsilon| \frac{\tau_2 - \tau_1}{\tau_1} \frac{E[d]}{\alpha} \tag{33}$$

$$\rho_{t_0,A} = -\lambda\alpha E_{t_0} - |\varepsilon| \frac{1-\tau_2}{\tau_1} E[d] \tag{34}$$

$$\rho_{t_0,B} = -\lambda\alpha E_{t_0} + \lambda \frac{\tau_2 - \tau_1}{\tau_1} E[d] \tag{35}$$

That is, an expected rate cut leads to reduced demand for central bank money in the tender before the Council meeting and to an increased demand for central bank money in the tender after the Council meeting. After the Council meeting, market rates exceed tender rates. Moreover, in the periods following the Council meeting, bidding volumes and market rates behave as if the system was hit by an unexpected liquidity shock.

Proof. See the appendix. □

Our analysis stresses the following rationale for underbidding. Since cash before and after the Council meeting are perfect substitutes, a profit-maximizing bank will defer the refinancing to the second tender that is likely to offer better conditions. However, since all banks in the market reason in this way, demand will be insufficient in the first tender. As a consequence, the second tender will be more "crowded" than the first, so that the rate differential with respect to the minimum bid rate will be higher in the second tender.

Theorem 2 captures the impact of both excess liquidity and base rate cut expectations on the intertemporal equilibrium. In particular, as (34) shows, base rate cut expectations will lower the overnight rates before the meeting. However, as can be seen from

$$|\varepsilon|\frac{1-\tau_2}{\tau_1} = 1 - \lambda\frac{\tau_2-\tau_1}{\tau_1} < 1, \tag{36}$$

the market only partially reflects these expectations (because of the anticipated liquidity shortage after the Council meeting).

Impact on prior periods. An anticipated interest rate cut will not only affect bidding behavior in the current period and in all subsequent periods, but also in the periods before the policy decision. To see this, consider now the periods $t < t_0$. Our next result predicts an oscillation with increasing amplitude in anticipation of the underbidding (cf. Figure 5).

Theorem 3. *Let E_0 denote the net liquidity position at the beginning of period 0. Assume that a base rate cut of $E[d]$ is expected to come into effect between the two tenders in period t_0. Then, for periods $t \leq t_0$, excess liquidity amounts to*

$$E_t = \varepsilon^t E_0 + \frac{\varepsilon^{t_0-t} - \varepsilon^{t_0+t}}{1-\varepsilon^{-2}}\frac{\tau_2-\tau_1}{\alpha\tau_1}E[d], \tag{37}$$

and the overnight rate is given by

$$\rho_t = -\lambda\alpha\varepsilon^t E_0 + \frac{\lambda\varepsilon^{t_0+t} - \lambda'\varepsilon^{t_0-t}}{1-\varepsilon^{-2}}\frac{\tau_2-\tau_1}{\tau_1}E[d], \tag{38}$$

where

$$\lambda' = \frac{\tau_1 + (1-\tau_2)\varepsilon^{-1}}{\tau_2-\tau_1}. \tag{39}$$

For periods $t > t_0$, the equilibrium path follows the prediction of Theorem 1.

Proof. See the appendix. □

Equations (37) and (38) show that the joint development of excess liquidity and liquidity spread can be decomposed into two additive component effects. The first effect is induced by the initial excess liquidity (which may be zero), and moves the excess liquidity in the subsequent periods up and down. The second effect captures the increasing reluctance to bid in the second tender in periods with an odd distance to t_0, and the increasing willingness to bid in the second tender in periods with an even distance to t_0. The effect is caused by the fact that money from the second tender immediately prior to period t_0 earns lower interest, on average, on the money market when compared to money from the first tender in the same period $t_0 - 1$. Moreover, the unbalanced volumes of the tenders in period t_0 affect, via a straightforward backward induction logic, also the volumes in prior periods.

The reader will note that in the case of the Eurosystem, as before, the active liquidity management of the ECB is likely to eliminate most of the predicted oscillations.

5. Existing policy proposals

A number of policy measures to address the underbidding problem have been proposed in the literature as well as in the media. We briefly discuss some of these measures in the sequel.

Symmetric interest rate target. Nautz and Oechssler (2001, p. 21) conclude their discussion of the underbidding problem with the suggestion of "replacing the asymmetric minimum bid rate by a symmetric interest rate target similar to the Federal Funds rate target in the U.S."[19] A related proposal has recently been made by Dredner Bank economist Claudia Henke (2002). However, abandoning the minimum bid rate is sometimes viewed as implying the loss of the central bank's most effective instrument for steering short-term market rates. Moreover, it should be noted that the tendency to delay reserve holdings in expectation of falling policy rates appears to be a general feature of liquidity provision through repeated open market operations. E.g., in their discussion of the US reserve accounting method, Spindt and Hoffmeister's (1988, p. 404) argue that "a manager expecting the Fed to ease before settlement could build up a big 'red' position [...] early in the period by selling funds, and then buy back his position cheaply when rates fall as expected." While it is also argued that in the US, reserve management tends to be conservative, and speculation is limited, it is not obvious that the removal of the minimum bid rate in the operational framework of the Eurosystem would ceteris paribus eliminate the problem.

[19] In the US, the federal system provides liquidity to the commercial banks using daily open market operations. The main policy rate is a daily target rate, the Federal Funds target rate. Cf., e.g., Borio (2000).

More frequent auctions. It has been suggested in the media that modern technology should make it possible to have many auctions with low volumes. However, it is not clear that a larger number of small auctions before and after the policy announcement can really resolve the problem. After all, when policy rates are expected to fall, then in all small auctions prior to the policy anouncements, counterparties will be reluctant to bid for liquidity, and in all small auctions after the announcement, there will be a stronger demand. So even daily or hourly auctions are not likely to mitigate the underbidding phenomenon.

Coordination of Council meetings and operations. At the beginning of October 2002, the ECB opened a process of public consultation on a number of possible modifications of the operational framework. The proposed changes address also the underbidding experience, and can be briefly summarized as follows. To start with, the scheduling of maintenance periods and Governing Council meetings would be coordinated, so that, as a rule, any policy rate adaptation would be implemented only with the beginning of a new maintenance period. Then, the maturity of the MROs would be reduced to one week, and any reserve maintenance period would start on the settlement day of some MRO, so that MRO transactions would no longer hang over into the following period. Finally, longer-term refinancing operations would be suspended.[20]

It will be noted that the above proposal, by separating the economics of any pair of subsequent maintenance periods, and by implementing rate adaptations "between" rather than within periods, resolves the underbidding problem in the theoretical framework. In fact, the analysis suggests that the measures described above complement each other, especially the first two, that concern the timing of the maintenance period. To see the point, assume that only the policy decision is shifted to the beginning of maintenance period t_0, but that transactions may still hang over into the subsequent period, i.e., assume $\tau_d = 0$ and $\tau_1 > 0$. Then, incentives to underbid in the first tender of period t_0 would indeed be eliminated. However, bidders may still be reluctant to bid in the second tender of period $t_0 - 1$ because the overnight rate in the subsequent period t_0 would be expected to be lower, making the second tender (which hangs over into the subsequent period) less attractive for refinancing purposes in period $t_0 - 1$. Analogous considerations can be made when the Council meeting is shifted to some other point in maintenance period t_0. In all cases, underbidding would result in the operation immediate before the policy decision. Thus, a coordination of the individual policy decision with

[20]For a more comprehensive description of the proposal, see ECB (2002c).

the beginning of a maintenance period alone may be less effective than the proposed combination of measures. Similarly, it can be seen that if only the maturity overlap into the subsequent maintenance period is eliminated, i.e., if $\tau_1 = 0$, but there is no coordination of the policy decision with the maintenance period, then underbidding may still occur in the first tender. Thus, from a theory perspective, it appears that the correction of bidding incentives via a coordination of dates requires to have indeed both proposed changes at the same time.[21]

6. Reimbursement

We have seen above that the measures proposed in the ECB consultation document may effectively reduce the bidders' speculative motives in view of decreasing policy rates. While this would in principle solve the underbidding problem, it nevertheless appears of some value to consider also alternative proposals. We will therefore complement the preceding analysis and derive in this section a policy scheme that is not based on the coordination of dates, but rather on the idea that the incentives of counterparties could alternatively be corrected by an appropriately chosen reimbursement scheme. The basic format of the resulting concept is that of a guarantee of compensation provided by the central bank to those banks that submit bids for interbank liquidity even in the expectation of decreasing policy rates.

To derive the incentive compatible payment structure, we will compare the equilibrium conditions in the absence of rate change expectations to the corresponding conditions in the presence of such expectations. It will be noted that only two equilibrium conditions are directly affected by the expectations on d. These conditions concern the respective decisions of how much liquidity to acquire, firstly, in the second tender of period $t_0 - 1$, and secondly, in the first tender of period t_0.

Consider first the decision of a counterparty on how much liquidity to acquire in the second tender of period $t_0 - 1$. To make arbitrage infeasible, the average

[21] The careful reader will note that the framework predicts no underbidding in the case where the MROs are realigned with the maintenance period, i.e., $\tau_1 = 0$, and the policy decision comes into effect shortly before the end of the maintenance period, i.e., $\tau_2 < \tau_d < 1$. The reason is that in our simple model without explicit modeling of the standing facilities, there is no difference between adapting the policy rates after the second tender and adapting the rates with the beginning of the new maintenance period. When standing facilities are modeled explicitly, however, as suggested in section 7, then an adaptation of the standing facility rates before the end of the maintenance period would affect expectations about the market rate at the end of the maintenance period, and thereby distort bidding behavior before (and after) the policy decision.

interest earned on money acquired in this tender, with the provision that the reimbursement scheme is in effect, must equal the expected average interest earned from money market credit with the same maturity. Hence, writing X for the amount of necessary reimbursement on funds acquired in the second tender of period $t_0 - 1$, we must have

$$(1 - \tau_2 + \tau_1)(\underline{r} + \alpha L_{t_0-1,B}) - X = (1 - \tau_2) r_{t_0-1} + \tau_1 r_{t_0,A}. \tag{40}$$

From (31), we know that the overnight rate before the Council meeting equals the expected rate after the meeting, i.e.,

$$r_{t_0,A} = r_{t_0,B} - E[d]. \tag{41}$$

This yields

$$(1 - \tau_2 + \tau_1)(\underline{r} + \alpha L_{t_0-1,B}) = (1 - \tau_2) r_{t_0-1} + \tau_1 r_{t_0,B} - (\tau_1 E[d] - X). \tag{42}$$

Thus, in order to eliminate speculative underbidding in the second tender of period $t_0 - 1$, we need to set

$$X := \tau_1 E[d]. \tag{43}$$

Note that this condition can be satisfied by using a conditional reimbursement of the base rate cut for that part of the contract term that falls into period t_0.

Consider now the decision of a counterparty on how much liquidity to acquire in the first tender of period t_0. Again, the average interest earned on money acquired in this tender, under the provision that the reimbursement scheme is in effect, must be equal to the expected average interest earned from money market lendings with the same maturity. Writing Y for the necessary amount of reimbursement on moneys acquired in the first tender of period t_0, incentive compatibility implies

$$(\tau_2 - \tau_1)(\underline{r} + \alpha L_{t_0,A}) - Y = (\tau_d - \tau_1) r_{t_0,A} + (\tau_2 - \tau_d)(r_{t_0,B} - E[d]). \tag{44}$$

Plugging (41) into (44) yields

$$Y := (\tau_2 - \tau_1) E[d]. \tag{45}$$

As above, this condition can be satisfied by paying the counterparty the rate cut differential in case of a base rate cut, and now, in contrast to above, for the whole term of the transaction.

Thus, in order to generate incentive compatible behavior even under rate change expectations, the central bank could promise to reimburse, conditional on a base rate cut of d, the base rate differential to the participating bidders. Specifically, the interest margin d would have to be paid back on all transactions allotted in the second tender of period $t_0 - 1$ on a pro rata basis for the time interval $[t_0, t_0+\tau_1]$. Moreover, the interest margin d would also have to be reimbursed on all transactions allotted in the first tender of period t_0, and for these transactions over the whole transaction term $[t_0 + \tau_1; t_0 + \tau_2]$.

We have seen in Theorems 2 and 3 that money market volatility decomposes into a component caused by liquidity shocks, and another that is due to expectations on changes of base rate. We call the second component "speculative volatility" in period t. We have shown:

Theorem 4. *Assume that the central bank uses the above described reimbursement scheme. Then counterparties submit bidding volumes as under neutral expectations. Speculative volatility in all periods $t \neq t_0$ will be zero. In period t_0, money market rates will reflect expectations about the change in the policy rate in the current period (cf. Figure 6). If, in addition, the policy rate decision is made at the beginning of a maintenance period, then money market rates are completely unaffected by interest rate expectations, and the reimbursement is restricted to transactions contributing to the fulfillment of reserve requirements in more than one period.*

Proof. See the text before the theorem. □

There would probably be several ways of integrating a reimbursement scheme into the operational framework of the Eurosystem. One route of realization would suggest the use of variable rate MRO transactions. More precisely, the terms of the contract underlying a two-week repo transaction would be modified in the sense that the respective rate for interest payments for a given maintenance period would be indexed on the base rate prevailing at the end of that maintenance period. The spread between contract rate and end-of-period base rate is determined here as the difference between bid rate and base rate at the time of the tender operation that initiated the transaction. Note that this form of indexation would mean a symmetric form of reimbursement that guarantees a balanced demand also in times of increasing interest rates.[22]

[22]The incentive-compatibility of the reimbursement scheme relates our discussion to the question of optimal risk allocation in an economy (cf. Hellwig, 1994). Specifically, under the present operational framework, the risks resulting from the uncertainty about the policy rate will be borne by the banks. With variable rate transactions, however, those risks would be shifted to the central bank.

7. Robustness

In this section, we briefly discuss the extent to which the simplification made in the formal analysis might affect the derived predictions.

Remuneration. In reality, all credit institutions in the euro area obtain a remuneration on their required reserve holdings by the central bank. This remuneration is calculated as a time-weighed average of the rates in the main refinancing operations. The reader will note that the no-arbitrage conditions (2) and (4) appear to abstract from this issue. However, remuneration can be easily incorporated in our model by making a mild additional assumption. Specifically, assume that all counterparties understand that any marginal refinancing obtained in the money market will ultimately generate a higher demand in the corresponding central bank operation. Under this assumption, the remuneration payment is unaffected by the reserve manager's individual decision about how to refinance, which justifies the use of the no-arbitrage conditions even when remuneration is taken into account.

Auction micro structure. The model abstracts from the auction method, and replaces the determination of tender volume and average rate by a simple central bank supply function $s(.)$, which raises two issues. Firstly, there could be imperfect and heterogeneous information of individual bidders about the demand of the respective other bidders. However, as we have already mentioned in the introduction, asymmetric information does not seem to play a major role as a determinant of bidding behavior in the refinancing operations. Another issue is that in reality, the central bank has in principle full discretion about the liquidity provided in the operations. I.e., while the liquidity forecast will usually give market participants an indication about the intended liquidity injection, the central bank has no obligation whatsoever to behave in line with those expectations. This would leave bidders uncertain about the policy stance of monetary authorities. However, we deem it rather likely that, over time, bidders will learn the central bank's monetary policy stance and will therefore be able to predict better and better the liquidity provided and the bids that will win.

Elastic supply. We believe that the assumption of elastic supply, which has been motivated in section 3, is not likely to severely affect the results. After all, a higher money market rate is in our stylized model always concomitant with a scarcity of liquidity. So while an elastic supply function implies higher allotments when bid rates are higher, this feature of the model should be understood in the sense that a more pronounced scarcity of liquidity (which reveals itself in the form of increased market rates) induces the central bank to inject more liquidity in a given operation. This, however, is intuitively not very different from the neutral allotment policy that is actually used by the ECB, so that, as a consequence, we believe that conclusions drawn from our analysis should not depend too much on the assumption of an elastic supply function.

Increasing interest rates. When market participants expect increasing interest rates, then demand for liquidity will tend to be too high prior to the interest rate decision. This has not been a major problem for the ECB in the considered period, due to the introduction of the variable rate tender procedure in June 2000. The claim that increasing interest rates do not unbalance the refinancing system has been proved formally for the symmetrical implementation via indexation in section 6. In fact, the mechanics of the argument can easily be seen not to depend on the sign of $E[d]$. It is less clear, however, that also the implementation using outright reimbursement can be made fully compatible with phases of increasing interests, even though the central bank can always decide to limit the amount of liquidity that is provided to the market.

Standing facilities and interim liquidity shocks. An extension of the model would analyze the consequences of fluctuations in the liquidity demand that arise during the maintenance period, such as those effected by autonomous factors. For this, assume that at the beginning of each maintenance period, liquidity needs are still uncertain, and that there is a point of time $\tau_s > \tau_d$ within each maintenance period, after the second tender, at which the true liquidity needs will be revealed to market participants. Given inelastic supply and demand, the market rate at the end of the maintenance period would either approach the marginal lending facility or the deposit facility, depending on the total liquidity position in the market. This effect will make the demand for liquidity prior to τ_s elastic, and each counterparty would choose a net liquidity position L_s^i prior to τ_s such that the expected marginal cost for holding L_s^i would equal the money market rate in that period. Aggregate liquidity demand, i.e., the right hand side of (6), would therefore depend on the money market rate prior to the interim shock. Specifically, the higher the overnight rate, the more willing would counterparties be to take the risk of being squeezed at the end of the period, and the smaller would be demand for interim liquidity. While we have not persued this line of investigation any further, we strongly believe that if interim shocks are not too large in expectation, then the main conclusions of the analysis would not be affected.[23]

Demand elasticity. We have assumed in the analysis that the total liquidity demand within a given maintenance period is perfectly inelastic with respect to price conditions. Yet, at least in principle, counterparties could delay some of their liquidity needs that result from their business relationships with non-banks. E.g., a bank might agree with a creditor to pay out a tranche a week later than scheduled and pay a certain fee in response. Alternatively, equity holders may step in and provide additional cash if necessary. A generalized version of the model would therefore have to capture that liquidity demand is completely inelastic in the short-term, but probably somewhat more elastic in the longer term. While we believe that allowing for an elastic demand for liquidity would not severely affect our conclusions, more work on this question seems desirable.

Risk aversion. For risk-averse market participants, speculation is more costly. As a consequence, we would expect that underbidding is less pronounced for risk-averse bidders. On the other hand, the benefit of the reimbursement scheme to market participants is higher because the risks originating from the potential change in the policy rates is taken away from the individual counterparties.

Large banks. In the analysis, we have abstracted away restrictions resulting from collateral requirements and credit limits in the interbank market. Incorporating these issues might contribute to the understanding of the role of large banks in the underbidding outcome, as suggested by Nyborg, Bindseil, and Strebulaev (2002). Specifically, it appears plausible that, since money market refinancing depends on extended credit lines, only large banks may have the capacity of distributing their demand unevenly over the refinancing operations in the first place. In a similar vein, since strategic delay of demand in tender operations presupposes a sufficient degree of deliberation over a significant amount of eligible collateral, again only sufficiently large banks may be able to speculate on decreasing interest rates. This might explain that, as reported in the above-mentioned study, larger bidders seem to reduce their bids to a larger extent than smaller banks in the expectation of falling policy rates.

Four tender operations and maturity overlap. The predictions of the model would probably be more accurate if one would consider four tender operations and an overlapping term structure. However, the corresponding predictions are not difficult to guess. Namely, since both the third and the ultimate tender

[23] Cf. also footnote 21.

in each maintenance period allocate transactions that run into the respective subsequent period, the tender rate would be distorted to a smaller extent in the third tender, and to a larger extent in the ultimate tender. Also concerning this modification, we believe that the main conclusions of the present analysis would not be affected.

8. Concluding remarks

In this paper, we propose a model of liquidity provision through repeated central bank tenders, where transactions may hang over into subsequent maintenance periods. The framework allows a formal discussion of policy proposals made in relation with the operational framework of the Eurosystem. Three main results are obtained. First, liquidity shocks are absorbed by the system by exponentially declining oscillations around the stationary equilibrium. Second, when a policy rate cut is expected, bidders will strategically reduce demand prior to the date of the policy decision, which may unbalance the dynamic system of bidding volumes, tender conditions, and money market rates before and after the decision. Finally, when either the ECB proposal or the reimbursement scheme is implemented, then speculation becomes unprofitable, and banks bid as in the stationary equilibrium.

A. Appendix: Proofs

Proof of Theorem 1. Assume that the minimum base rate is kept constant by the central bank over time, i.e., $\underline{r}_t = \underline{r}$. We claim that, when boundary conditions are ignored for the moment, excess liquidity and liquidity spread evolve according to the following recursive system of equations

$$\begin{aligned} \rho_{t+1} &= -\frac{1-\tau_2+\tau_1}{1-\tau_2}\alpha E_t \qquad (46) \\ &\quad -\frac{(1-\tau_2+\tau_1)(\tau_2-\tau_1)+(1-\tau_2)^2}{\tau_1(1-\tau_2)}\rho_t \\ E_{t+1} &= -\frac{\tau_1}{1-\tau_2}E_t - \frac{\tau_2-\tau_1}{\alpha(1-\tau_2)}\rho_t. \qquad (47) \end{aligned}$$

To prove this claim, note that from (14) and (4), we have

$$\begin{aligned} \rho_{t+1} &= r_{t+1} - \underline{r} - \alpha\overline{L} \qquad (48) \\ &= \frac{1}{\tau_1}\{(1-\tau_2+\tau_1)(\underline{r}+\alpha L_{t,B}) - (1-\tau_2)r_t\} - \underline{r} - \alpha\overline{L}. \qquad (49) \end{aligned}$$

Using (6) for replacing $L_{t,B}$ yields

$$\begin{aligned}\rho_{t+1} &= \frac{1}{\tau_1}\{(1-\tau_2+\tau_1)(\underline{r}+\frac{\alpha}{1-\tau_2}(\overline{L}-\tau_1 L_{t-1,B}-(\tau_2-\tau_1)L_{t,A})) \\ &\quad -(1-\tau_2)r_t\}-\underline{r}-\alpha\overline{L}.\end{aligned} \tag{50}$$

Eliminating $L_{t,A}$ using (2) and simplifying yields (46). Similarly, from (6) we may derive

$$\begin{aligned}L_{t+1} &= L_{t,B}-\overline{L} \quad (51)\\ &= \frac{1}{1-\tau_2}(\overline{L}-\tau_1 L_{t-1,B}-(\tau_2-\tau_1)L_{t,A}). \quad (52)\end{aligned}$$

Using (2) and simplifying yields (47). This proves the above claim.

A brief calculation shows now that the linear transformation A specified by (46) and (47) viewed as a mapping from the real plane into itself, possesses determinant

$$\det A = 1, \tag{53}$$

and trace

$$\operatorname{tr} A = -\frac{1-\tau_1}{\tau_1}-\frac{\tau_2}{1-\tau_2}<0. \tag{54}$$

This shows that the eigenvalues of A are ε and ε^{-1} for some real number $\varepsilon < 0$. Without loss of generality, we require that $|\varepsilon| \leq 1$. Clearly,

$$\varepsilon+\varepsilon^{-1} = \operatorname{tr} A. \tag{55}$$

Consider now a pair (ρ_0, E_0). For any (ρ_t, E_t), to be part of a rational expectation equilibrium, we would have to require

$$(\rho_t, E_t) = A^t(\rho_0, E_0), \tag{56}$$

where A^t denotes the t-th power of the linear transformation A. Therefore, for any nonzero vector $(\rho_0, E_0) \in \Re^2$, we have that $|(\rho_t, E_t)|$ exceeds all bounds for sufficiently large t unless (ρ_0, E_0) is a scalar multiple of the eigenvector belonging to ε. However, we assumed that market participants expect the market rate to stay above the minimum bid rate in the first tender.

The eigenvector to ε is the solution of the equation

$$(A-\varepsilon I)(\rho_0, E_0) = (0,0), \tag{57}$$

where I denotes the identity mapping. This implies

$$\frac{\tau_2-\tau_1}{\alpha(1-\tau_2)}\rho_0+(\frac{\tau_1}{1-\tau_2}+\varepsilon)E_0 = 0, \tag{58}$$

and thereby (17). □

Proof of equation (20). Total differentiation of (55) from the proof of Theorem 1 yields

$$(1-\frac{1}{\varepsilon^2})d\varepsilon = \frac{1}{\tau_1^2}d\tau_1 - \frac{1}{(1-\tau_2)^2}d\tau_2. \tag{59}$$

Hence the assertion. □

Proof of Theorem 2. From (24) and (31), we find

$$\underline{r} + \alpha L_{t_0,A} = r_{t_0,B} - E[d]. \tag{60}$$

Solving for $L_{t_0,A}$ and plugging the result into the aggregate reserve requirement (27), one obtains

$$\tau_1 L_{t_0-1,B} + \frac{\tau_2-\tau_1}{\alpha}(r_{t_0,B} - E[d] - \underline{r}) + (1-\tau_2)L_{t_0,B} = \overline{L}. \tag{61}$$

Rearranging yields

$$r_{t_0,B} = \underline{r} + E[d] + \frac{\alpha}{\tau_2-\tau_1}(\overline{L} - \tau_1 L_{t_0-1,B} - (1-\tau_2)L_{t_0,B}). \tag{62}$$

When one plugs (62) and (28) in the no-arbitrage condition (25), and solves for $L_{t_0,B}$, then a straightforward calculation using (55) gives

$$L_{t_0,B} = (1-\varepsilon)\overline{L} + \varepsilon L_{t_0-1,B} - \varepsilon\frac{\tau_2-\tau_1}{\alpha\tau_1}E[d], \tag{63}$$

and therefore equation (33). Plugging this into (27) and solving for $L_{t_0,A}$ yields

$$L_{t_0,A} = \overline{L} - (L_{t_0-1,B} - \overline{L})\frac{\tau_1+(1-\tau_2)\varepsilon}{\tau_2-\tau_1} + \frac{(1-\tau_2)\varepsilon}{\tau_1}\frac{E[d]}{\alpha}, \tag{64}$$

and thereby equation (32). Plugging (63) into (62), one obtains

$$\begin{aligned} r_{t_0,B} \quad = \quad & \underline{r} + \alpha\overline{L} - \frac{\tau_1+(1-\tau_2)\varepsilon}{\tau_2-\tau_1}\alpha(L_{t_0-1,B} - \overline{L}) + \\ & + \frac{\tau_1+(1-\tau_2)\varepsilon}{\tau_1}E[d], \end{aligned} \tag{65}$$

hence (35), and, using (31), we also have (34). □

Proof of Theorem 3. The idea of the proof is to assume a liquidity spread ρ_0 and to check whether the expectations about period t_0 with are necessary to rationalize this liquidity spread correspond to our previous findings. Write the initial conditions as a linear combination of the eigenvectors of the linear transformation A (cf. Theorem 1):

$$\begin{pmatrix} \rho_0 \\ E_0 \end{pmatrix} = \gamma\begin{pmatrix} -\lambda\alpha \\ 1 \end{pmatrix} + \gamma'\begin{pmatrix} -\lambda'\alpha \\ 1 \end{pmatrix}, \tag{66}$$

where λ' is defined as in (39). Then, for any $t \in \{0, 1, ..., t_0\}$, writing ρ_{t_0} for $\rho_{t_0,A}$,

$$\begin{pmatrix} \rho_t \\ E_t \end{pmatrix} = \varepsilon^t \gamma \begin{pmatrix} -\lambda\alpha \\ 1 \end{pmatrix} + \varepsilon^{-t}\gamma' \begin{pmatrix} -\lambda'\alpha \\ 1 \end{pmatrix}. \tag{67}$$

This holds in particular for $t = t_0$. On the other hand, from Theorem 2,

$$\begin{pmatrix} \rho_{t_0} \\ E_{t_0} \end{pmatrix} = \begin{pmatrix} -\lambda\alpha E_{t_0} - \dfrac{1-\tau_2}{\tau_1}|\varepsilon|E[d] \\ E_{t_0} \end{pmatrix} \tag{68}$$

$$= \beta \begin{pmatrix} -\lambda\alpha \\ 1 \end{pmatrix} + \beta' \begin{pmatrix} -\lambda'\alpha \\ 1 \end{pmatrix}, \tag{69}$$

where

$$\beta = E_{t_0} - \frac{\tau_2 - \tau_1}{\alpha\tau_1} \frac{1}{1-\varepsilon^{-2}} E[d] \tag{70}$$

$$\beta' = \frac{\tau_2 - \tau_1}{\alpha\tau_1} \frac{1}{1-\varepsilon^{-2}} E[d]. \tag{71}$$

As the eigenvectors form a basis, we have

$$\beta = \varepsilon^{t_0}\gamma \tag{72}$$

$$\beta' = \varepsilon^{-t_0}\gamma'. \tag{73}$$

Hence,

$$E_0 = \gamma + \gamma' \tag{74}$$

$$= \varepsilon^{-t_0}\beta + \varepsilon^{t_0}\beta' \tag{75}$$

$$= \varepsilon^{-t_0} E_{t_0} - \frac{\varepsilon^{-t_0} - \varepsilon^{t_0}}{1-\varepsilon^{-2}} \frac{\tau_2 - \tau_1}{\alpha\tau_1} E[d]. \tag{76}$$

Solving for E_{t_0} yields

$$E_{t_0} = \varepsilon^{t_0} E_0 - \frac{1-\varepsilon^{2t_0}}{1-\varepsilon^{-2}} \frac{\tau_2 - \tau_1}{\alpha\tau_1} E[d]. \tag{77}$$

From (67), for any $t \in \{0, 1, ..., t_0\}$, we have

$$E_t = \varepsilon^t \gamma + \varepsilon^{-t}\gamma' \tag{78}$$

$$= \varepsilon^{t-t_0}\beta + \varepsilon^{t_0-t}\beta' \tag{79}$$

$$= \varepsilon^{t-t_0} E_{t_0} - \frac{\varepsilon^{t-t_0} - \varepsilon^{t_0-t}}{1-\varepsilon^{-2}} \frac{\tau_2 - \tau_1}{\alpha\tau_1} E[d]. \tag{80}$$

Using (77), this yields

$$E_t = \varepsilon^t E_0 + \frac{\varepsilon^{t_0-t} - \varepsilon^{t_0+t}}{1-\varepsilon^{-2}} \frac{\tau_2 - \tau_1}{\alpha\tau_1} E[d]. \tag{81}$$

This proves (37). We determine the liquidity spread. For $t \in \{0, 1, ..., t_0\}$, we have

$$\begin{aligned}
\rho_t &= -\lambda\alpha\varepsilon^t\gamma - \lambda'\alpha\varepsilon^{-t}\gamma' && (82)\\
&= -\lambda\alpha\varepsilon^{t-t_0}\beta - \lambda'\alpha\varepsilon^{t_0-t}\beta' && (83)\\
&= -\lambda\alpha\varepsilon^t E_0 + (\lambda\varepsilon^{t_0+t} - \lambda'\varepsilon^{t_0-t})\frac{\tau_2-\tau_1}{\tau_1}\frac{1}{1-\varepsilon^{-2}}E[d]. && (84)
\end{aligned}$$

Using (34) from Theorem 2 again yields (38). □

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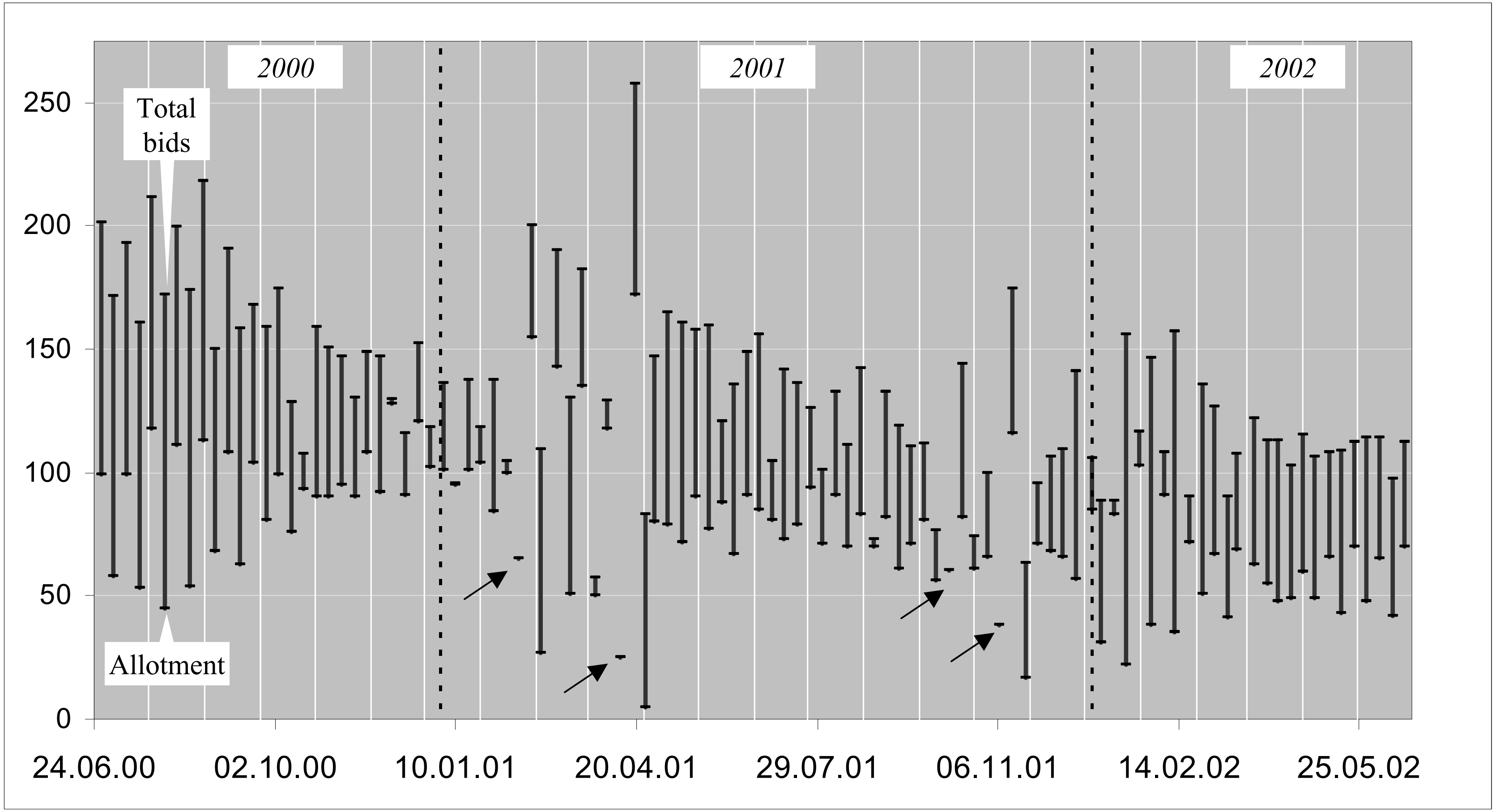
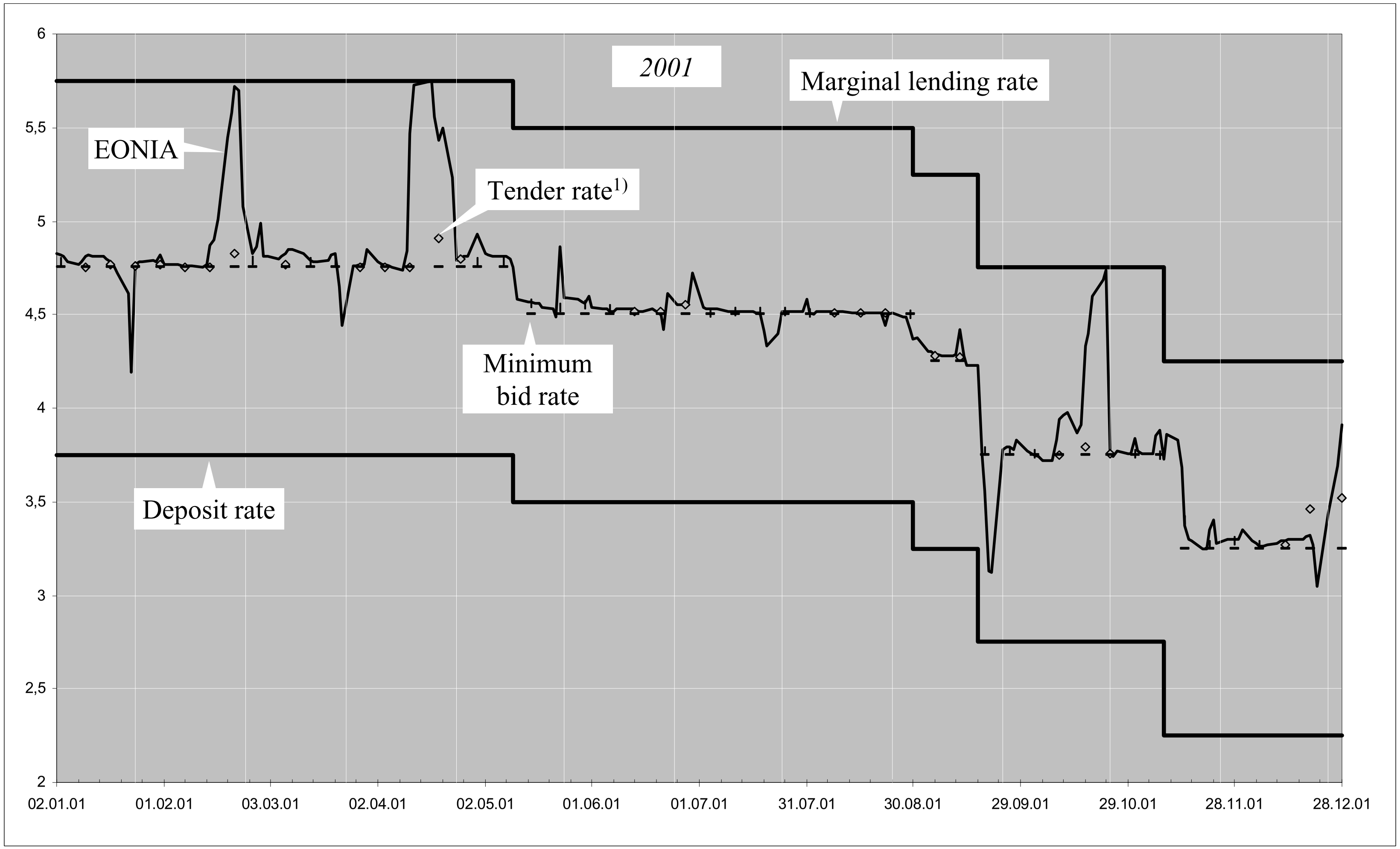


Source: ECB

Figure 1. Bids and allotment in the ECB main refinancing operations (in euro bn) since the introduction of the variable-rate tender; the arrows indicate the four occurrences of underbidding in the year 2001



1) Weighted average MRO rate

Source: Bloomberg, ECB, Deutsche Bundesbank

Figure 2. The development of policy rates and interest rates (in percent) during the year 2001

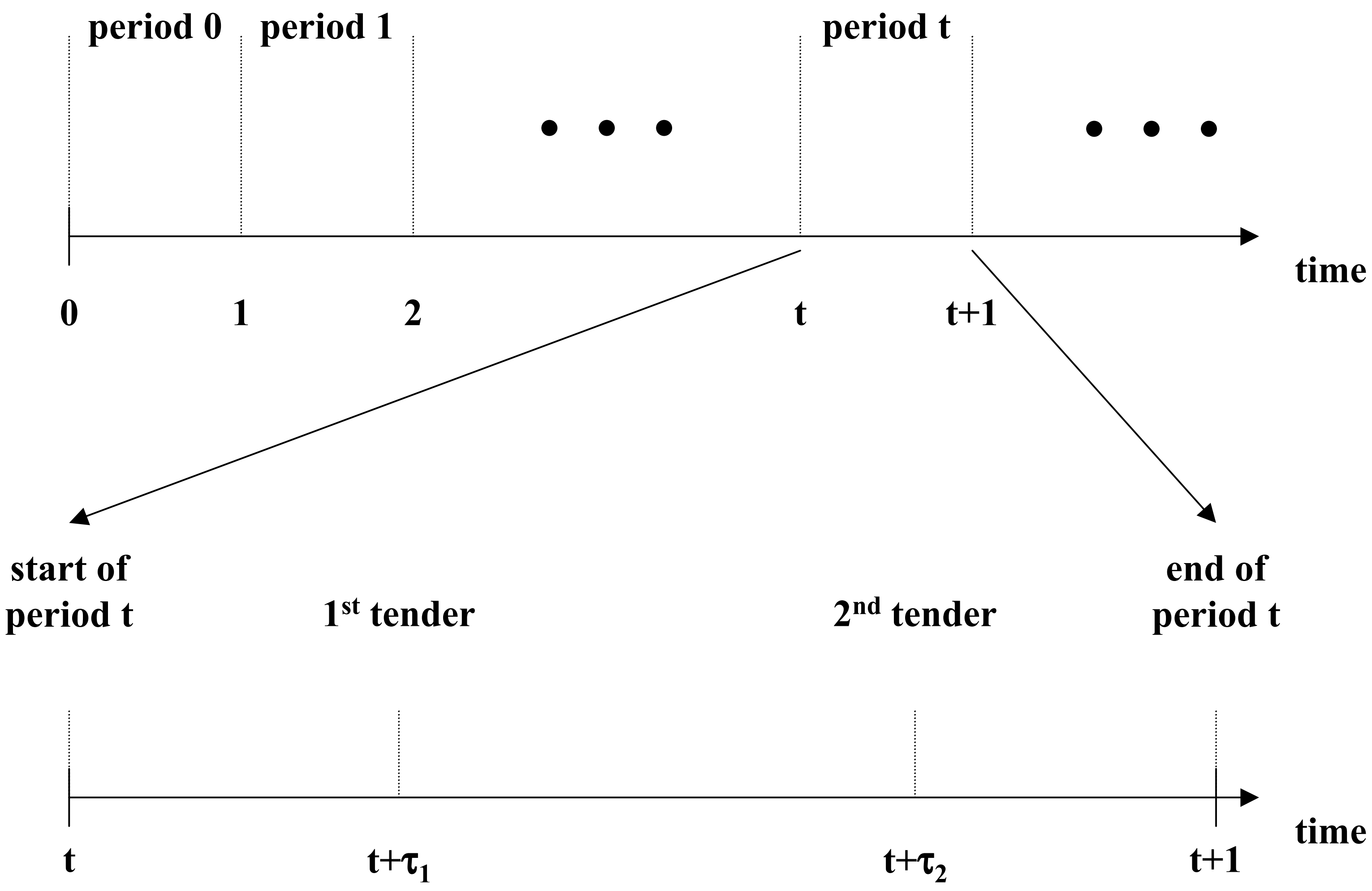


Figure 3. Time structure

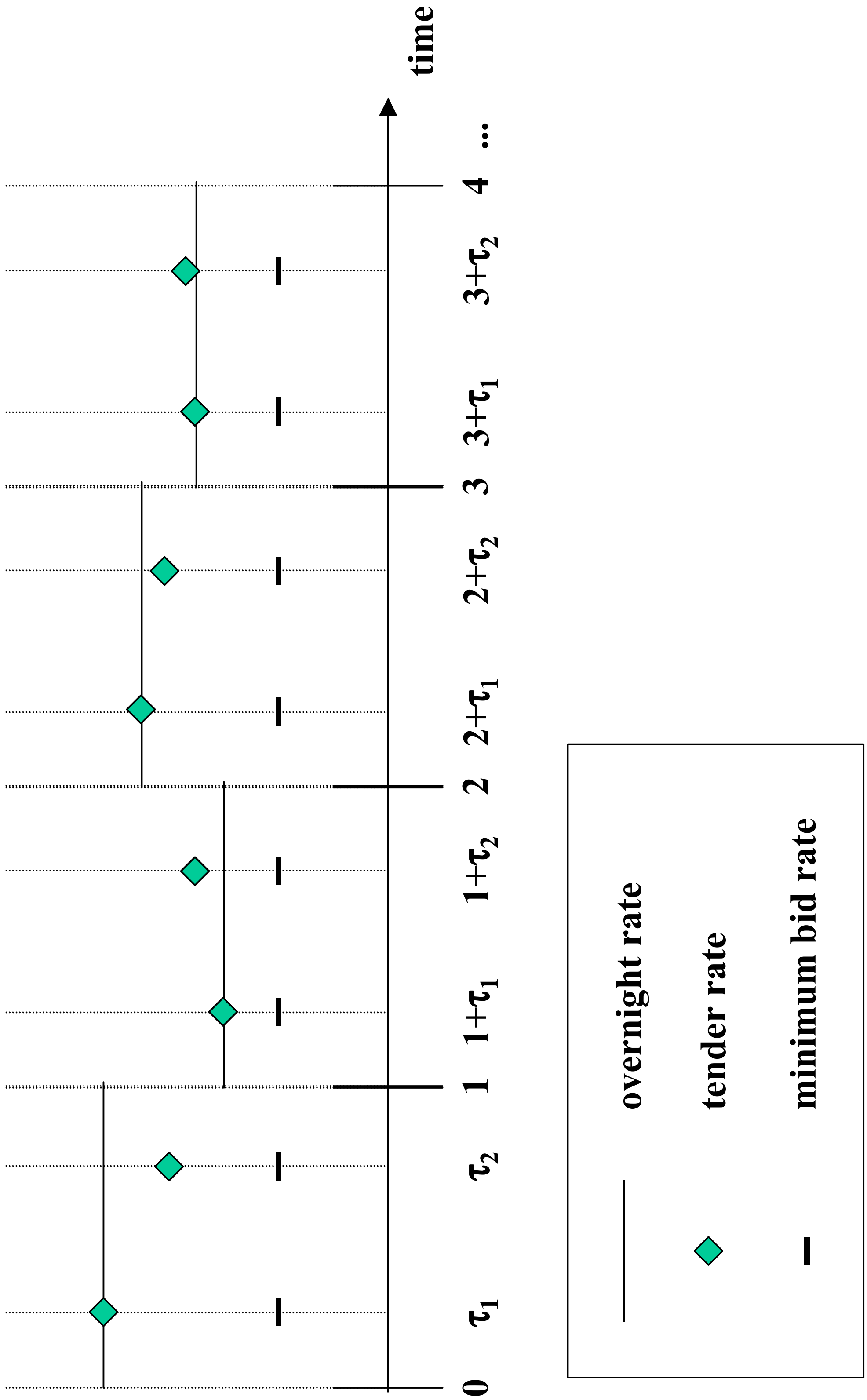


Figure 4. Initial shortage of liquidity

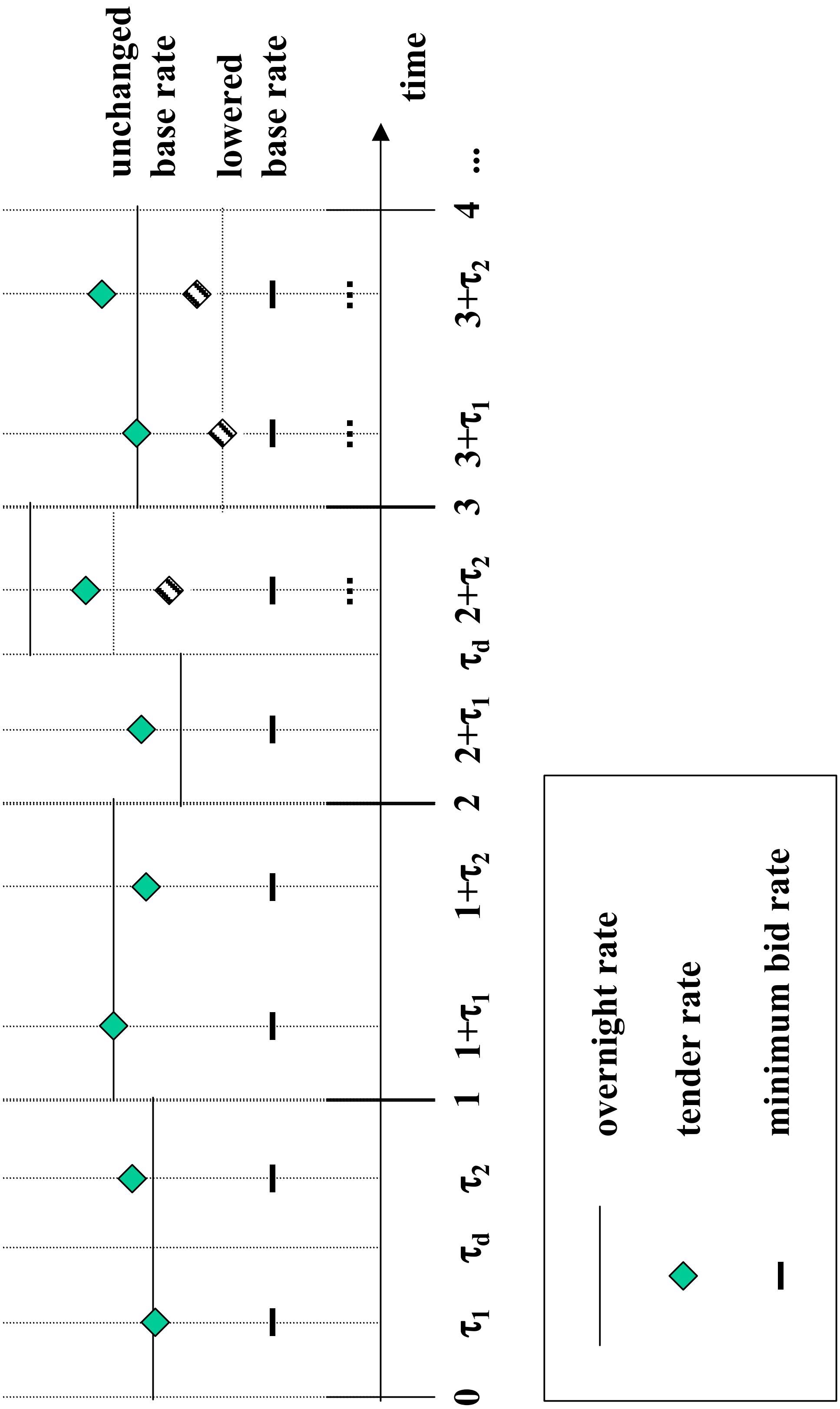


Figure 5. Expected but unrealized rate cut in period 2

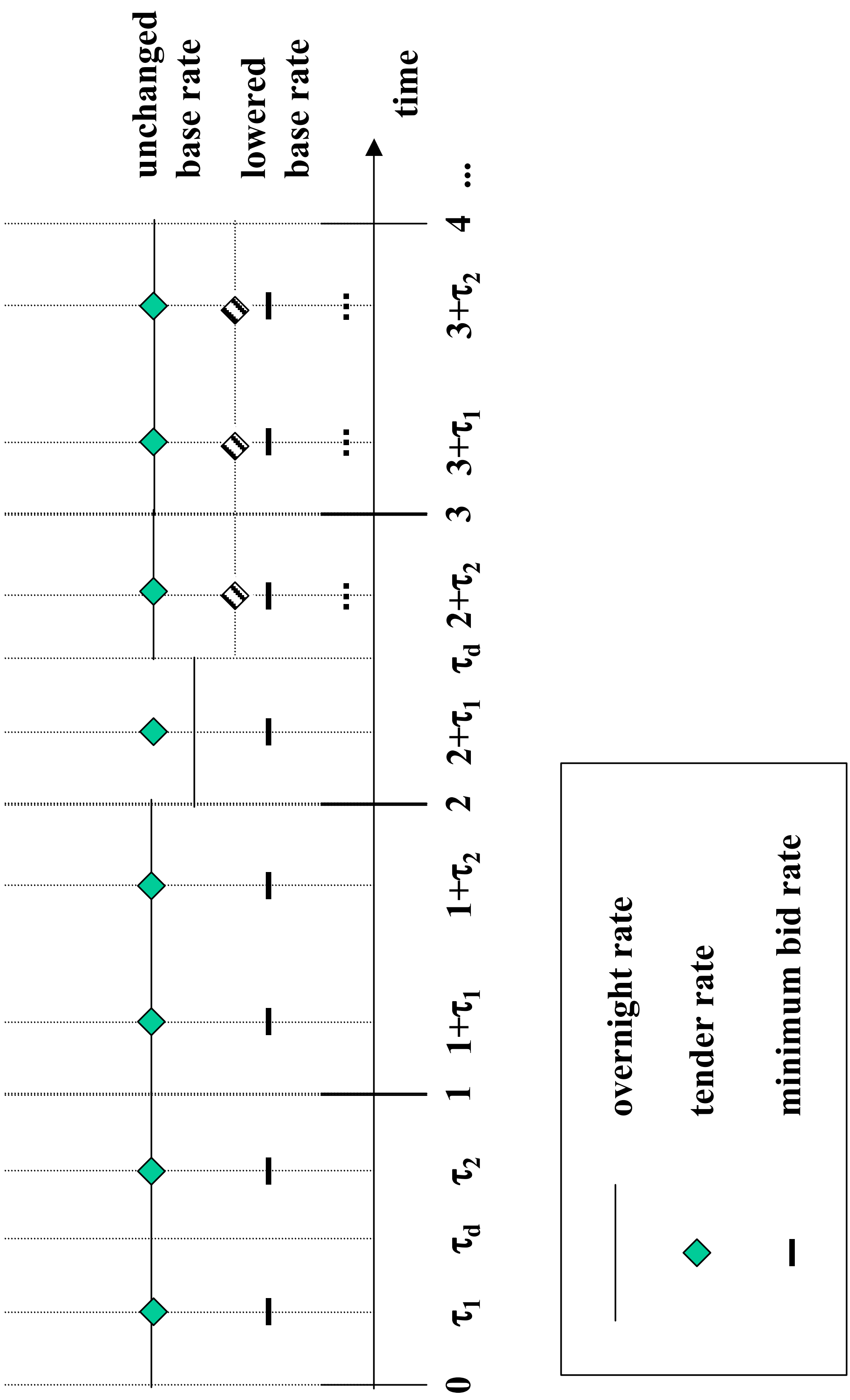


Figure 6. Reimbursement

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