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Credit market competition and liquidity crises

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Abstract

We develop a model where banks invest in reserves and loans, and trade loans on the interbank market to deal with liquidity shocks. Two types of equilibria emerge, depending on the degree of credit market competition and the level of aggregate liquidity risk. In one equilibrium, all banks keep enough reserves and remain solvent. In the other, some banks default with positive probability. The latter equilibrium exists when competition is not too intense and high liquidity shocks are not too likely. The model delivers several implications concerning the severity of crises and credit availability along the business cycle.

JEL Classifications: G01, G20, G21.

Keywords: Systemic crises, interbank market, cash-in-the-market pricing, price volatility.

Non-Technical Summary

Competition in the banking sector has often been blamed for banks' excessive risk-taking and the resulting instability. The recent financial crisis has contributed to renew the interest about this topic, given banks' increased appetite for risk taking in the years preceding the crisis and the fact that economies characterized by similar levels of competition differed significantly in terms of their resilience to the crisis.

The existing debate has focused on banks' asset side and, specifically, on the effect that competition has on the credit risk banks are exposed to. However, while certainly important, credit risk is not the only source of risk banks face. As the recent crisis has shown, liquidity risk also represents a major source of instability in the banking sector due to the mismatched maturity between banks' assets and liabilities and the role that they play as liquidity providers to depositors. The exposure to potentially large and unexpected withdrawals can become a crucial source of risk if banks cannot raise liquidity on demand.

When faced with large deposits withdrawals, banks with insufficient liquidity holdings have to sell illiquid assets in the market to raise liquidity at short notice. The level of asset prices determines the amount of liquidity that they can raise and thus their ability to withstand large withdrawals. When liquidity in the market is scarce relative to the amount of assets on sale, banks are forced to sell assets at fire sale prices. As a consequence, they may be unable to raise all necessary liquidity, thus becoming insolvent.

How does competition affect banks' ability to withstand liquidity shocks? Is a competitive banking system more or less subject to liquidity crises than a more monopolistic market? Does competition influence only the return on loans or also asset prices and thus the ability of banks to raise liquidity on demand? Does competition interact with other factors, such as the level of liquidity risk, in determining the emergence of liquidity crises? Does it have an impact also on the severity of these crises?

This paper tackles these questions, by developing a theoretical model where banks are subject to liquidity risk and can meet their liquidity demands by holding liquid assets initially and/or selling loans in the interbank market. Credit market competition plays a role for the occurrence of liquidity crises as it determines the return of loans, thus affecting individual banks' initial liquidity holding and, in turn, the amount of aggregate liquidity in the interbank market. Beside these effects, competition also affects asset prices. The effect on asset prices is twofold, as they depend both on the amount of liquidity available in the market and on the return (i.e., the fundamental value) of the loans. The overall effect of competition on stability and banks' exposure to liquidity risk is then determined by the combination of these effects that competition has on the return from loans and on asset prices.

Our analysis provides three main results about the relationship between competition, stability and liquidity risk. First, regarding the effect of credit market competition on stability and the occurrence of systemic liquidity crises, our paper shows that competition is beneficial for stability, as crises only occur when competition is not intense. This result suggests, consistently with the empirical evidence in Beck, Demirgüç-Kunt and Levine (2006, 2007), that policies fostering competition -such as lower barriers to bank entry and fewer restrictions on bank activities and bank operations in general- may reduce banking system fragility.

Second, regarding the link between aggregate liquidity risk and systemic crises, our model shows that crises can only emerge if large liquidity shocks are not too likely. Moreover, the lower is the level of aggregate liquidity risk, the more severe crises are in that they involve a larger number of banks defaulting and they entail a larger credit provision. These results suggest that in periods of booms, when the level of aggregate liquidity risk is small, banks tend to take more risk, thus being more exposed to adverse shocks. Moreover, they are consistent with the evidence in Schularick and Taylor (2012) that credit growth is a powerful predictor of financial crises.

Finally, concerning the relationship between aggregate liquidity risk and the effect of competition on financial stability, our analysis suggests that the beneficial effect of competition on stability can be smaller in economies characterized by a lower probability of large liquidity shocks. This result suggests that economies characterized by similar level of banking competition may differ in terms of financial stability, thus reconciling the mixed empirical evidence about the effect of competition on stability. In turn, this suggests the importance of incorporating the determinants of liquidity risk, as, for example, banks' funding structure, the institutional and regulatory framework and the structure and characteristics of guarantees scheme, into the analysis of the relationship between competition and stability.

1 Introduction

Competition in the banking sector has often been blamed for banks' excessive risk-taking and the resulting instability. The recent financial crisis has contributed to renew the interest about this topic, given banks' increased appetite for risk taking in the years preceding the crisis and the fact that economies characterized by similar levels of competition differed significantly in terms of their resilience to the crisis.

So far the debate among policymakers and academics has focused on the effect that competition has on the credit risk banks are exposed to. In the academic literature, two opposing views can be distinguished depending on how competition affects banks' and borrowers' risk taking behavior. One view is that, by reducing banks' franchise value, competition reduces banks' incentives to behave prudently (e.g., Keeley, 1990, Hellman, Murdoch and Stiglitz, 2000, and Allen and Gale, 2004a). An opposite view is that the low loan rates associated with intense competition induce borrowers to take less risk, thus reducing banks' portfolio risk (e.g., Boyd and De Nicoló, 2005).[1]

While certainly important, credit risk is not the only source of risk banks are exposed to.[2] As well known by the seminal paper by Diamond and Dybvig (1983) and the large subsequent literature on financial stability, banks are also subject to liquidity risk due to the mismatched maturity between their assets and liabilities and their role as liquidity providers to depositors. As the recent crisis has shown, these features can become a crucial source of risk if banks cannot raise liquidity on demand. When faced with large withdrawals, banks with insufficient liquidity holdings have to sell illiquid assets in the market to raise liquidity at short notice. The level of asset prices determines the amount of liquidity that they can raise and thus their ability to withstand large withdrawals. When liquidity in the market is scarce relative to the amount of assets on sale, asset prices are low and banks may be unable to withstand the liquidity shock, thus becoming insolvent.

While liquidity crises have been widely studied in the literature on financial stability (see, for example, Allen and Gale, 2007 and the papers cited therein), no study so far has addressed the potential role of credit market competition in the emergence of liquidity crises. Is a competitive banking system more or less subject to liquidity crises than a more monopolistic market? Does competition influence only the return on loans or also asset prices and thus the ability of banks to raise liquidity on demand? Does competition interact with other factors, such as the level of (exogenous) liquidity risk, in determining the emergence of liquidity crises?

[1] An intermediate view is proposed by Martinez-Miera and Repullo (2010), who find a U-shape relationship between banks' asset risk and competition. See also, the surveys by Carletti (2008) and Carletti and Vives (2009).

[2] A recent paper by Freixas and Ma (2015) shows that competition has different effects on stability depending on the type of risk considered (i.e., portfolio risk, insolvency risk, liquidity risk and systemic risk).

Does it have an impact also on the severity of these crises? These questions are important also in light of the recent financial crisis, during which economies with banking systems characterized by similar degrees of competition showed a very different resiliency to external shocks due, at least partly, to very different banks' funding structures.

To tackle these questions, in this paper we develop a standard banking model where banks are subject to liquidity risk and their initial portfolio allocations, as well as asset prices on the interbank market are affected by the degree of credit market competition. This framework allows us to endogenously determine the occurrence of systemic liquidity crises as well as their severity in terms of number of failing banks and credit availability as a function of the degree of credit market competition and the level of liquidity risk banks face. We then introduce credit risk and analyze the role of competition in a broader context where both liquidity and credit risk are present.

Our framework builds on a standard two periods banking model where banks raise funds from risk-averse consumers in exchange of demandable deposit contracts, and invest in a liquid asset (reserves) and/or loans to entrepreneurs. Because of the demandable feature of their liabilities, at the interim date banks face (aggregate) uncertainty relative to their demand for liquidity: There is a good state with a small fraction of early depositors, and a bad state where the fraction of early depositors is larger. We refer to the probability of the good state as our measure of aggregate liquidity risk.

Following Allen and Gale (2004b) and Allen, Carletti and Gale (2009), we consider that banks can meet their liquidity demands by holding reserves initially and/or selling loans in a (competitive) interbank market, where only banks can trade and prices are endogenously determined by the supply and demand for liquidity. The former is determined in the initial period by banks' initial portfolio allocations; while the latter is determined at the interim date after the liquidity shock realizes. Since the liquidity supply is inelastic in the interim period while the liquidity demand varies across the two states of the world, the model features price volatility and the possibility of cash-in-the-market pricing (fire sales). This in turn introduces the possibility of bank default if banks are unable to raise enough liquidity from the sale of their assets to honor the promised payments to depositors.

Given this framework, banks can choose between two strategies: They can choose a portfolio investment and deposit contract so that they behave *safe* and remain solvent in all states or so that they behave *risky* and default in one state of the world. Their choice critically depends on the profitability associated with either strategy. We show that two types of equilibrium can emerge depending on the degree of credit market competition and the level of aggregate liquidity risk. When both of these are low, the model features an (unique) equilibrium where a group of banks defaults in the state where a large liquidity shock realizes. Otherwise, only an equilibrium where all banks remain solvent in both states of the world is possible.

This result suggests a number of important observations about the richness and the contribution of our model. First, liquidity crises emerge in equilibrium only if two conditions are satisfied: large withdrawals are not too likely **and** credit market competition is not too intense. Only in these circumstances some banks find it optimal to over-invest in loans, grasp the high returns from loans in the good state of the world and default in the bad state when they are forced to liquidate all their loans and close down prematurely. This implies that banking systems build up a large amount of liquidity risk in good times, consistent with the recent crisis.

Second, because asset prices are endogenous and there is price volatility, the equilibrium with default is asymmetric in terms of banks' optimal strategies: *some* banks choose to default in the bad state of the world while the others choose to always remain solvent. As in equilibrium all banks have to make the same expected profits, the result suggests that the different types of banks realize different profits in the two states of the world: risky banks make higher profits in the good state when they can grasp larger returns from loans, while safe banks make higher returns in the bad state when they use their excess liquidity to buy the loans of risky banks at fire sale prices. This asymmetric feature of the equilibrium is not attainable in more standard models where banks' assets can be sold or liquidated for an exogenous value.

Third, the degree of credit market competition and the level of aggregate liquidity risk are substitute in determining the emergence of liquidity crises and their severity. In particular, we show that, as the risk of large liquidity shocks decreases, liquidity crises start emerging in increasingly more competitive banking systems and entail a larger group of risky banks.

Fourth, credit market competition affects banks' strategies in a non-trivial way. As standard in the literature, credit market competition affects the return that banks accrue from loans. In particular, the less intense competition is, the larger is such return, consistently with the charter value literature (e.g., Keeley, 1990, Hellman, Murdoch and Stiglitz, 2000, and Allen and Gale, 2004a). Besides this, competition affects asset prices and thus banks' profitability when trading in the interbank market. It is the combination of the effects of competition on loan rates and interbank prices that drives our key result that, for a given level of aggregate liquidity risk, more competitive banking systems are more stable in terms of not featuring liquidity crises. This result is in line with the result in Boyd and De Nicolò (2005) that banks become safer as competition increases because entrepreneurs choose less risky portfolios, and it is in contrast with the charter value literature mentioned above, where increased competition makes the financial system more fragile. Importantly, however, our result is derived in a setting where, as in Keeley (1990), Hellman, Murdoch and Stiglitz (2000), and Allen and Gale (2004a), entrepreneurs are passive and only banks decide the risk of their portfolios.

Lastly, competition and liquidity risk affect the amount of credit available in the economy. In particular, when large liquidity shocks are not very likely, banks grant more loans to entrepreneurs in aggregate in the equilibrium with default relative to the case when they all hold more liquid assets to remain solvent.

Our main results remain valid when we enrich the baseline model to account for the possibility that banks compete also in the deposit and interbank markets and that there are interactions across these markets. We show that our main result of a positive relationship between competition and stability when large liquidity withdrawals are not too likely remains valid, although the range of parameters where the equilibrium with default emerges is different from the one in the baseline model. In particular, the equilibrium with default is more likely than in the baseline model when deposit market competition is considered, and it is instead less likely when banks compete in the interbank market.

We then perform a number of robustness exercises of our baseline model. We first consider alternative consumers' utility functions and the presence of deposit insurance. We show that, as the degree of risk aversion increases, risky banks start to hold positive amounts of liquid assets to satisfy the greater demands by early withdrawing depositors. Despite this, the main insights of the baseline model remain valid both with risk adverse depositors and with deposit insurance although, again, the range of parameters for which default emerges in equilibrium changes relative to the baseline model.

As a final robustness exercise, we extend the baseline model to include risky loans and bank monitoring in order to address the relationship between competition, liquidity and credit risk. Accounting for this extra source of risk complicates the analysis substantially as loan rates are no longer safe and their probability of success depends on banks' monitoring efforts, which may differ across banks in equilibrium. Moreover, banks are now subject to two different sources of risk, and thus to the possibility of defaulting at the interim date for liquidity reasons or at the final date due to credit risk.

We first show that, as standard in the literature (e.g., Boot and Greenbaum, 1992) competition worsens banks' monitoring incentives, but only for those banks that choose to hold enough liquidity to withstand all liquidity shocks. We then show that the result of the baseline model that greater competition improves financial stability still remains valid as long as the positive effect of greater competition in terms of reducing liquidity risk dominates the negative effect in terms of greater credit risk for the safe banks.

The paper has a number of empirical implications. First, our results point to a positive relationship between competition and stability, consistently with recent empirical evidence. Second, the model suggests that systemic liquidity crises can emerge only if large liquidity shocks are not too likely, as only in this case banks find it profitable to behave risky when competition is not too intense. Third, it predicts that competition and liquidity risk are substitute in generating liquidity crises as the threshold level of credit market competition below which an equilibrium with default emerges increases as the probability of large liquidity shocks decreases. This suggests that, as observed in the recent financial crisis, countries with similar level of competition can differ in terms of their resilience to crises depending on banks' exposure to liquidity shocks. Fourth, our analysis implies that in economies characterized by lower probabilities of large liquidity shocks, crises entail a larger number of defaulting banks but also more credit availability. This last result is consistent with the idea that excessive credit growth is a good predictor of crises.

The novelty of the paper is to analyze the relationship between competition and liquidity risk in a framework where banks default when they experience a run by all their depositors and are forced to liquidate their long term assets at fire sales prices. In this sense, the paper is related to Allen and Gale (2004b) and Allen, Carletti and Gale (2009). The former provides a numerical example showing the possibility that price volatility and fire sales induce banks to choose different strategies in terms of exposure to liquidity driven runs. The latter shows the existence of excessive price volatility in the interbank market, thus justifying the need of open market operations conducted by a central bank. Although it uses the same features of asset price volatility and fire sales as causes of bank default, our paper differs from these contributions in important ways. Relative to Allen and Gale (2004b), we provide a formal characterization of the equilibrium of the model and fully characterize the range of parameters in which banks behave asymmetrically and some of them may default. In particular, we show how the degree of credit market competition and the structure of liquidity shocks affect the emergence of liquidity crises and their severity. In this respect, our paper differs substantially from Allen, Carletti and Gale (2009), who restrict their analysis to the range of parameters such that only the equilibrium without default exists.

A few papers have looked at the effect of competition on bank instability in terms of runs. The analysis of Rochet and Vives (2004) and Goldstein and Pauzner (2005) suggests that when banks offer higher repayments to early depositors (as it would be the case with more intense competition on the deposit market), bank runs are more likely to occur. Matutes and Vives (1996) show that deposit market competition does not have a clear effect on banks' vulnerability to runs, but higher promised repayments to depositors tend to make banks more unstable. Carletti, Hartmann and Spagnolo (2007) analyze the impact of credit market competition on banks' incentives to hold liquidity after a merger. They show that an increase in market power− as after a merger among large banks− increases banks' liquidity needs and thus the probability of liquidity crises. In contrast to these papers, we focus on the impact of credit market competition and banks' holdings in a framework where runs are due to the deterioration of asset prices rather than to depositors' coordination failures.

Our paper shows that competition is beneficial to financial stability but not necessarily to credit availability. Default can entail greater or smaller credit availability depending on the level of aggregate liquidity risk. In our model, systemic crises are associated with larger credit provision when the level of aggregate liquidity risk is small. In this respect, our results are related to papers analyzing banks' liquidity dynamics along the business cycle. Acharya, Shin and Yorulmazer (2011) show that bank liquidity is countercyclical as banks economize on reserves when the fundamentals of the economy are good.

Several recent contributions on financial stability have focused on crises generated from asset price volatility and fire sales losses. Examples are Acharya and Yorulmazer (2008), Diamond and Rajan (2011) and, in particular, Allen and Gale (1994) and Allen and Carletti (2006, 2008). We contribute to this literature by analyzing how competition affects asset prices, banks' risk taking behavior and thus the emergence of liquidity crises with partial default.

Finally, we show that the presence of competitive interbank markets supports the existence of a default equilibrium where some banks default in one state of nature and sell their loans to other banks in the interbank market. In this sense, the paper is related to some contributions that focus on the interbank market such as Flannery (1996), Freixas and Jorge (2008), Acharya, Gromb and Yorulmazer (2012) and Gale and Yorulmazer (2013). Similarly to our paper, Gale and Yorulmazer (2013) investigate allocations in a Diamond and Dybvig (1983) economy with aggregate liquidity risk and interbank market with endogenous prices. As in our framework, they show the existence of an equilibrium with both safe and risky banks. We add to this literature by investigating the effect of competition on the nature of the equilibrium. Our results prove that this is of first-order importance.

The paper proceeds as follows. Section 2 describes the model. Section 3 derives the equilibrium with and without default and characterizes the existence of either type as a function of the degree of competition and the level of aggregate liquidity risk. Section 4 analyzes the properties of the two equilibria. Section 5 extends the baseline model by considering the possibility that banks also compete in the deposit and interbank market. Section 6 checks the robustness of the baseline model with respect to consumers' risk aversion, deposit insurance and credit risk. Finally, Section 7 discusses the empirical implications of the model, while Section 8 concludes. All proofs are in the appendix.

2 The model

Consider a three date ($t = 0, 1, 2$) economy with banks, consumers and entrepreneurs. Banks raise funds from consumers in exchange for a demandable deposit contract, and invest in short assets and long term loans to entrepreneurs. Consumers are subject to liquidity shocks and banks can reshuffle liquidity in an interbank market if needed.

At date 0 each bank raises 1 unit of funds and invests a fraction R of them in a storage technology defined as reserves, and a fraction L in loans to entrepreneurs in exchange for a per-unit loan rate of r at date 2. Entrepreneurs invest the loan received in a (divisible) project yielding $V > 1$ at date 2 and have an opportunity cost equal to $\upsilon \in (0, \overline{\upsilon})$. Thus, the loan rate has to satisfy

$$V - r \geq \upsilon. \tag{1}$$

A higher υ forces the bank to reduce the loan rate r to ensure that entrepreneurs will accept the loan. As it corresponds to the minimum return that must accrue to entrepreneurs, we refer to the parameter υ as the degree of competition in the credit market.[3]

There is a continuum of identical consumers, each endowed with 1 unit of good at date 0 and nothing thereafter. At date 1, consumers receive a preference shock and become either early or late type. The former value consumption only at date 1; the latter value consumption only at date 2. Each consumer has a probability of being an early type λ_θ given by

$$\lambda_\theta = \begin{cases} \lambda_L & \text{w. pr. } \pi \\ \lambda_H & \text{w. pr. } (1-\pi), \end{cases}$$

with $\lambda_H > \lambda_L$. From the Law of Large Numbers λ_θ represents the fraction of early types at each bank. Since there is only aggregate uncertainty, the realization of λ_θ is the same for all banks. This implies that there are two states of nature, L and H, which we refer to as the *good* and the *bad* state, respectively.

[3] The idea is similar to Salop (1979), where the degree of competition is modelled with an exogenous transportation cost and results in a negative correlation between competition and loan rates as in the mainstream models of competition and as empirically found in Degryse, Kim and Ongena (2009).

The parameter π, representing the probability of the good state, can then be interpreted as a measure of aggregate (liquidity) risk. The lower π, the more the economy is subject to large aggregate liquidity shocks.

Consumers can either store for a return of 1 or deposit their endowment at the bank in exchange for a demand deposit contract promising a fixed repayment c_1 at date 1 or c_2 at date 2.[4] Depositors' utility function $u(c_t)$ is twice differentiable and satisfies the usual neoclassical assumptions: $u'(c) > 0$, $u''(c) < 0$ and $\lim_{c\to 0} u'(0) = \infty$.

At the beginning of date 1, the realized value of λ_θ becomes known and each consumer discovers privately whether he is an early or a late type. Then, an interbank market opens where banks can reshuffle liquidity if needed by selling and buying loans at a price P_θ. The market for loans is competitive and only banks can participate.[5] As we will explain later in detail, the price P_θ is endogenously determined by the aggregate demand and supply of liquidity in the market and, given that there are only two states, it can take at most two values. As typical in the literature on fire sales (e.g., Schleifer and Vishny, 1992, Allen and Gale, 1994, 2004b, and Acharya and Yorulmazer, 2008), the assumption of limited participation restricts the supply of liquidity in the interbank market, thus leading to cash-in-the-market pricing in equilibrium.

The timing of the model is as follows. At date 0, banks choose the deposit contract (c_1, c_2) and the initial portfolio allocation between reserves and loans (R, L) in order to maximize their expected profits. At the beginning of date 1, the uncertainty concerning consumers' types is resolved and the state $\theta = L, H$ is publicly observed. Early consumers withdraw at date 1 to meet their consumption needs, while late consumers can either wait till date 2 or claim to be early types and withdraw at date 1. In the former case, the bank continues to operate till date 2 and remains solvent. In the latter case, a run occurs and the bank is forced to sell all its loans, thus going bankrupt. Consumers obtain their promised payments c_1 and c_2 when the bank remains solvent, while they obtain a pro-rata share of the bank's resources in the case a run occurs.

3 Equilibrium

We start by characterizing the possible strategies of the banks in terms of portfolio investment decisions and deposit contract, and the market clearing conditions in the interbank market. Then, we derive the different equilibria of the model. As we will show in detail, two different equilibria emerge endogenously depending on

[4] Assuming that consumers have an alternative return of 1 implies that banks are monopolist in the deposit market. We relax this assumption in Section 5, where we analyze the case of imperfect competition in the deposit market.

[5] We relax the assumption of competitive interbank market in Section 5, where we consider the case of asset specificity.

the degree of credit market competition υ and the probability π of the good state. In one equilibrium, which we refer to as the equilibrium without default, all banks choose a portfolio allocation and a deposit contract that allows them to remain solvent in all states of the world. In the other, which we call the equilibrium with default, a group of banks chooses a portfolio allocation and a deposit contract so that they remain solvent in state L but default in state H.

A bank defaults at date 1 when late depositors decide to withdraw prematurely, thus generating a run.[6] This occurs when late consumers expect to obtain a higher utility from withdrawing at date 1 than from waiting till date 2, that is when the value of the bank's portfolio at date 2 does not suffice to repay them at least the consumption c_1 promised to early depositors. Formally, this is the case when

$$r\left(L+\frac{R-\lambda_\theta c_1}{P_\theta}\right)<(1-\lambda_\theta)c_1. \tag{2}$$

The LHS in (2) is the value of the bank's portfolio at date 2. This equals the return rL on the loans that the bank has initiated at date 0 plus the return/loss accrued by the bank in the interbank market at date 1. If $R-\lambda_\theta c_1>0$, the bank has excess resources at date 1 and will buy $\frac{R-\lambda_\theta c_1}{P_\theta}$ unit of loans at the price P_θ yielding a per unit return r at date 2. By contrast, if $R-\lambda_\theta c_1<0$, the bank has a liquidity shortage at date 1 and it needs to sell $\frac{R-\lambda_\theta c_1}{P_\theta}$ loans, thus losing the return r on each of them. The RHS is the minimum amount of resources that the bank has to repay to the $(1-\lambda_\theta)$ late depositors to prevent them from running at date 1.

As condition (2) shows, the default of a bank is endogenous in our model in that it depends on its initial decisions concerning its portfolio (R,L) and deposit contract (c_1,c_2), and on the market price P_θ. In other words, a bank **chooses** whether to default or remain solvent in all states depending on what strategy is more profitable. We then denote as *safe* a bank that chooses not to default, i.e., a bank that remains active till date 2 and is always able to pay the promised deposit payments in both states L and H, for given P_θ. By contrast, we define as *risky a* bank that chooses to default in at least one state. Furthermore, we denote with ρ and $1-\rho$ the measure of safe and risky banks, respectively. The case $\rho=1$ corresponds to the equilibrium where all banks play safe, while the case $\rho<1$ means that some banks will play risky in equilibrium and will default in one state.[7]

[6] Following Allen and Gale (2004b), we restrict our attention to fundamental runs.

[7] A risky bank can default only in one state of the world as otherwise it would always make zero profits and no bank would choose to behave risky. Moreover, $\rho>0$ must hold in equilibrium. Otherwise, if all banks were to default and sell their loans on the interbank market at date 0, there would be no bank that could acquire these loans. Prices would then be pushed to zero and some banks would find it profitable to deviate and hold enough liquidity to acquire these loans. Thus, the equilibrium must involve $0<\rho\leq 1$. No other equilibrium is possible, given the assumption of limited participation in the interbank market and the endogeneity of the interbank prices P_θ.

In what follows, for ease of exposition, we first describe the maximization problems of safe and risky banks for given prices P_θ, and the market clearing conditions that determine the equilibrium prices. Then, we characterize banks' equilibrium strategies as a function of the parameters υ and π.

In deriving the equilibria, we consider that banks make take-it-or-leave-it offers to entrepreneurs so that their participation constraint in (1) always holds with equality and the loan rate simply equals $r = V - \upsilon$. Moreover, we consider that consumers can deposit at one bank only, and observe whether a bank is safe or risky. This implies that safe and risky banks offer different deposit contracts to consumers, and that, in the equilibrium with default, consumers have to be indifferent between depositing in a safe or in a risky bank.

3.1 Safe Banks

Safe banks remain always solvent so that they repay the promised amount to both early and late consumers and don't experience any run. At date 1 they meet the liquidity demands of their λ_θ early depositors by using their reserve holdings and by trading loans on the interbank market, if needed. For given prices P_L and P_H, at date 0 each safe bank chooses its portfolio allocation (R^S, L^S) and a (non-contingent) deposit contract (c_1^S, c_2^S) to solve the following problem:

$$\underset{c_1^S, c_2^S, R^S, L^S}{Max} \Pi^S = rL^S + \pi \frac{(R^S - \lambda_L c_1^S)}{P_L} r + (1-\pi)\frac{(R^S - \lambda_H c_1^S)}{P_H} r - [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\, c_2^S \quad (3)$$

subject to

$$R^S + L^S = 1, \quad (4)$$

$$c_2^S \geq c_1^S, \quad (5)$$

$$r\left(L^S + \frac{R^S - \lambda_\theta c_1^S}{P_\theta}\right) \geq (1-\lambda_\theta)c_2^S \;\forall\; \theta = L, H, \quad (6)$$

$$\begin{aligned} E[u(c_1^S, c_2^S, \lambda_\theta)] &= \pi\left[\lambda_L u(c_1^S) + (1-\lambda_L)u(c_2^S)\right] + \\ &\quad +(1-\pi)\left[\lambda_H u(c_1^S) + (1-\lambda_H)u(c_2^S)\right] \geq u(1), \end{aligned} \quad (7)$$

$$c_1^S, c_2^S, R^S, L^S \geq 0 \quad (8)$$

The expected profits of safe banks are given by the expected returns from the loans minus the expected repayment to late depositors at date 2. The former are represented by the first three terms in (3): the bank receives the return r from the initial L^S loans plus(minus) the expected returns(losses) realized in the interbank market. With probability π, the banks has to satisfy $\lambda_L c_1^S$ early liquidity demands. If $R^S - \lambda_L c_1^S > 0$, it uses the excess liquidity $R^S - \lambda_L c_1^S$ to buy $\frac{(R^S - \lambda_L c_1^S)}{P_L}$ units of loans at the price P_L on which it receives $\frac{(R^S - \lambda_L c_1^S)}{P_L} r$ at date 2. By contrast, if $R^S - \lambda_L c_1^S < 0$ the bank sells $\frac{(R^S - \lambda_L c_1^S)}{P_L}$ units of loans at the price P_L to satisfy the early withdrawals, thus losing the return $\frac{(R^S - \lambda_L c_1^S)}{P_L} r$ at date 2. The same happens in state H, which occurs with probability $1 - \pi$: the bank faces $\lambda_H c_1^S$ early withdrawals and buys(sell) $\frac{(R^S - \lambda_H c_1^S)}{P_H}$ units of loans in the interbank market on which it gains(loses) $\frac{(R^S - \lambda_H c_1^S)}{P_H} r$ date 2. The last term in (3) represents the expected payments to late consumers at date 2. Given that the bank always remains solvent, all late consumers always obtain the promised repayment c_2^S, so that the expected payments for the bank are given by $[\pi(1 - \lambda_L) + (1 - \pi)(1 - \lambda_H)]\, c_2^S$.

Constraint (4) represents the budget constraint at date 0. Constraint (5) ensures that at date 0 late consumers are offered a repayment c_2^S at least equal to c_1^S. Constraint (6) is a resource constraint at date 2. It states that in each state $\theta = L, H$ a safe bank has enough resources to repay c_2^S to the $1 - \lambda_\theta$ late depositors. This constraint also ensures that the bank makes non-negative profits in both states L and H. Moreover, together with constraint (5), it ensures that safe banks never experience a run as the deposit contract (c_1^S, c_2^S) is incentive compatible at both dates 1 and 2. Constraint (7) is the consumers' participation constraint at date 0. It requires that depositors' expected utility $E[u(c_1^S, c_2^S, \lambda_\theta)]$ is at least equal to the utility $u(1)$ that consumers would obtain from storing. The last constraint (8) is simply a non-negative requirement on reserves, loans and consumption.

3.2 Risky Banks

Differently from safe banks, risky banks default in one state. Specifically, given $\lambda_H > \lambda_L$, they remain solvent in state L and go bankrupt in state H when late consumers start withdrawing early anticipating that condition (2) is satisfied. This forces risky banks to sell all their loans in that state, thus depressing the price P_H to a level that makes it impossible for them to remain solvent. Depositors receive then a pro-rata share of the bank's resources and banks make zero profits.

Formally, for given prices P_L and P_H, at date 0 each risky bank chooses its portfolio allocation (R^R, L^R) and deposit contract (c_1^R, c_2^R) so as to solve the following constrained maximization problem:

$$\underset{c_1^R, c_2^R, R^R, L^R}{Max} \Pi^R = \pi \left[rL^R + \frac{(R^R - \lambda_L c_1^R)}{P_L} r - (1 - \lambda_L) c_2^R \right] \tag{9}$$

subject to

$$R^R + L^R = 1 \tag{10}$$

$$c_2^R \geq c_1^R, \tag{11}$$

$$r\left(L^R + \frac{R^R - \lambda_L c_1^R}{P_L}\right) > (1-\lambda_L)c_2^R, \tag{12}$$

$$\begin{aligned} E[u(c_1^R, c_2^R, \lambda_\theta)] &= \pi\left[\lambda_L u(c_1^R) + (1-\lambda_L)u(c_2^R)\right] + \\ &+(1-\pi)u(P_H L^R + R^R) \geq u(1), \end{aligned} \tag{13}$$

$$c_1^R, c_2^R, R^R, L^R \geq 0 \tag{14}$$

The problem is similar to that of safe banks with the difference that risky banks make positive profits only in the good state. The first two terms in (9) represent the returns from the loans: risky banks obtain the return r on the L^R loans they have initially invested in plus(minus) the losses they obtain from accessing the interbank market where they buy(sell) $\frac{(R^R - \lambda_L c_1^R)}{P_L}$ units of loans on which they gain(lose) r at date 2. The last term in (9) is the repayment to the late depositors $(1-\lambda_L)c_2^R$.

Constraint (10) represents the budget constraint at date 0. Constraint (11) ensures that at date 0 late consumers are offered a repayment c_2^R at least equal to the one of the early consumers, c_1^R. Constraint (12) ensures that risky banks have enough resources at date 2 in state L to repay the promised consumption c_2^R to the late consumers. Together with (11), this constraint implies that the deposit contract is incentive compatible and that risky banks do not default in state L. Constraint (13) is the consumers' participation constraint at date 0. It requires that depositors' expected utility $E[u(c_1^R, c_2^R, \lambda_\theta)]$ is at least equal to the utility $u(1)$ that consumers would obtain from storing. Differently from the case of safe banks, depositors at the risky banks receive the promised repayments c_1^R and c_2^R only in the good state L, while they obtain a pro-rata share of the banks' portfolio value in the bad state H, as given by the proceeds $P_H L^R$ from liquidating the loans and the reserves R^R. The last constraint (14) is the usual non-negativity requirement on reserves, loans and consumption.

3.3 Market clearing

Having described the banks' maximization problem for given prices P_L and P_H, we can now turn to the market clearing conditions that determine the interbank prices P_L and P_H in equilibrium. Market clearing requires that prices adjust so that the aggregate demand for liquidity equals the aggregate supply of liquidity in all states of the world.

Consider first the good state L. Given $\rho \in (0,1]$ the fraction of safe banks and $1-\rho \in [0,1)$ the fraction of risky banks, market clearing in state L requires

$$\rho R^S + (1-\rho)R^R \geq \rho\lambda_L c_1^S + (1-\rho)\lambda_L c_1^R, \tag{15}$$

where the LHS is the aggregate supply of liquidity as given by the sum of the reserves ρR^S of the safe banks and $(1-\rho)R^R$ of the risky banks, and the RHS is the aggregate demand for liquidity as given by the demand for liquidity $\rho\lambda_L c_1^S$ of the safe banks and the demand for liquidity $(1-\rho)\lambda_L c_1^R$ of the risky banks.

In state H, market clearing requires

$$\rho R^S + (1-\rho)R^R \geq \rho\lambda_H c_1^S + (1-\rho)P_H L^R, \tag{16}$$

where again the LHS is the aggregate supply of liquidity of safe and risky banks as in (15) and the RHS is the aggregate demand of liquidity. Differently from state L, this is now given by the demand $\rho\lambda_H c_1^S$ of the safe banks and by the demand $(1-\rho)P_H L^R$ of the risky banks, which now default and sell all their loans L^R at the price P_H.

Conditions (15) and (16) imply that prices vary endogenously across the two states and that there is cash-in-the-market pricing when prices do not reflect fundamentals but are determined by the demand and supply of liquidity. The presence of price volatility derives from the characteristics of the demand and supply of liquidity. The supply of liquidity at date 1 is fixed and always equal to $\rho R^S + (1-\rho)R^R$. In contrast, the demand varies across states as it is determined by the realization of the liquidity shock λ_θ, the initial portfolio allocation and deposit contract, and whether default occurs in equilibrium. This implies that the market clears at different prices in the two states. As we will see in detail below, the presence of cash-in-the-market pricing and price volatility is an essential feature in our model for the characterization of the equilibrium, and in particular, for default to occur in equilibrium.

3.4 Characterization of the equilibrium

Having described the maximization problem for all types of banks and the market clearing conditions, we can now define and characterize the equilibrium of the model.

Definition 1 *An equilibrium consists of a price vector (P_L, P_H) and a proportion of safe banks $\rho \in (0,1]$ such that the deposit contract (c_1^i, c_2^i) and the portfolio allocation (R^i, L^i) chosen by each bank in each group $i = S, R$:*

i) maximize the bank's expected profits, for a given price P_θ;

ii) satisfy the market clearing conditions; and

iii) all banks make the same profits, irrespective of the portfolio allocation and the deposit contract they offer to consumers;

iv) all consumers obtain the same expected utility, irrespective of the type of bank they deposit their endowment at;

v) banks sell loans only if they need and do not have an incentive to invest in reserves at date 1.

Banks take their individual decisions of portfolio investment and deposit contract taking prices as given. Market prices are determined by the total demand and supply of liquidity in each state θ. In equilibrium, banks' decisions and market prices have to be consistent in that banks anticipate the equilibrium prices when taking their individual decisions.

Formally, the equilibrium is the solution to the first order conditions of banks' maximization problems with respect to c_1^i, c_2^i and R^i, the market clearing conditions and the condition that all banks make the same expected profits. This gives a system of nine equation and nine unknown variables. Given this complexity, we first derive banks' strategies as a function of ρ, and then determine the equilibrium value of ρ as a function of the parameters υ and π. In order to obtain closed form solutions, we focus in this section on the case where consumers have utility function $u(c_t) = \log(c_t)$. We will relax the assumption of logarithmic utility function in Section 6.1. We have the following result.

Proposition 1 *For a given* $\rho \in (0, 1]$*, the equilibrium features the following:*

1) Safe banks hold an amount of reserves

$$\rho R^S = \rho \lambda_H c_1^S + (1-\rho) P_H, \tag{17}$$

and offer consumers a deposit contract with

$$c_1^S = \left(\frac{P_L}{r} \frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (P_L - \pi)\lambda_H} \right)^{\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)} < 1 \tag{18}$$

and

$$c_2^S = \left(\frac{r}{P_L} \frac{\pi\lambda_L + (P_L - \pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H} \right)^{\pi\lambda_L+(1-\pi)\lambda_H} > 1. \tag{19}$$

2) Risky banks hold an amount of reserves

$$R^R = 0$$

and offer consumers a deposit contract with

$$c_1^R = \frac{P_L}{r} c_2^R \leq 1 \tag{20}$$

and

$$c_2^R = \frac{c_2^S - r(1-\pi)}{\pi} > 1. \tag{21}$$

3) The price in the good state is

$$P_L = \min(r, \widehat{P}_L)$$

where $\widehat{P}_L$ *solves* $E[u(c_1^R, c_2^R, \lambda_\theta)] = u(1)$ *as in (13), while the price in the bad state satisfies*

$$\pi \frac{r}{P_L} + (1-\pi) \frac{r}{P_H} = r, \tag{22}$$

and it is thus equal to

$$P_H = \frac{P_L(1-\pi)}{P_L - \pi}. \tag{23}$$

When $\rho = 1$, only safe banks exist. The total supply of liquidity, as given in (17), is just enough to cover the total demand of early consumers in state H, while, since $\lambda_H > \lambda_L$, it is strictly greater than the demand $\lambda_L c_1^S$ in state L. This excess of supply pushes up the price to $P_L = r$, and, from (23), $P_H = \frac{r(1-\pi)}{r-\pi} < 1$. At these prices, banks are indifferent between holding reserves or investing in loans at date 0 and can reshuffle liquidity at date 1 as needed, while still being able to offer the same deposit contract (c_1^S, c_2^S) to consumers. The individual bank's portfolio choice between loans and reserves becomes irrelevant as long as there is an initial aggregate investment of $\lambda_H c_1^S$ in liquid reserves. In this sense, price volatility has no implications for the equilibrium allocation in the absence of default because liquidity can be easily reshuffled among banks at date 1 and market prices do not influence either depositors' repayments or banks' profits.

When $\rho < 1$ both safe and risky banks exist and the equilibrium prices change significantly. In state L all banks remain solvent and the liquidity demands of early depositors at both safe and risky banks are satisfied. In state H risky banks experience a run and their depositors receive a pro-rata share of the value of the liquidated portfolios $P_H L^R$. As before, (23) indicates that P_H and P_L move in opposite directions. In equilibrium P_H must ensure that safe banks are willing to hold both reserves and loans at date 0 and P_L must induce safe banks to acquire the loans on sale at date 1 so that the market clears in both states. As with $\rho = 1$, there is still price volatility and cash-in-the-market pricing since $P_H < 1 < P_L \leq r$, but this now affects the equilibrium allocation as market prices affect depositors' repayments and banks' profits.

At the equilibrium prices, each safe bank holds an amount $\frac{(1-\rho)P_H}{\rho}$ of reserves in excess of its individual early liquidity demand $\lambda_H c_1^S$ in state H and uses it to acquire the loans sold by the risky banks at date 1. Risky banks find it optimal to invest everything in loans at date 0 and sell them in the interbank market at date 1 as needed. Since $P_L > 1$, in state L, the price is high enough that they can satisfy their early liquidity demands $\lambda_L c_1^R$ by selling only part of their loans and can thus continue operating till date 2. In state H, by contrast, the price is too low for them to remain solvent. Price volatility allows safe and risky banks to make different profits in the different states at date 1. In state L, safe banks buy loans at a price $P_L > 1$. This implies that risky banks make high returns from selling the loans, while safe banks do not from acquiring them. In state H, the situation is reversed. Safe banks are able to acquire loans cheaply since $P_H < 1$ and make high profits from acquiring them at date 1, while risky banks make zero profits. This difference in profits at date 1 for the different types of banks must be such that in an equilibrium with default banks are indifferent between behaving safe and risky in an equilibrium with $\rho < 1$. In this sense, price volatility and cash-in-the-market pricing are essential for default to occur in equilibrium.

Depositors at safe and risky banks obtain different deposit contracts but they are indifferent between depositing their endowment at either bank. Given that they remain solvent in both states, safe banks offer more volatile deposit payments than risky banks, that is $\frac{c_2^R}{c_1^R} < \frac{c_2^S}{c_1^S}$.

Now that we have characterized the equilibrium, we can focus on its existence. We have the following result, which holds as long as the difference between λ_H and λ_L is within a certain range.[8]

Proposition 2 *There exist a level of aggregate liquidity risk $\underline{\pi} < 1$ and a level of credit market competition $\upsilon^* < \overline{\upsilon}$ such that $\rho = \frac{\lambda_L c_1^R - P_H}{\lambda_L c_1^R - P_H + (\lambda_H - \lambda_L) c_1^S} < 1$ for $\pi \geq \underline{\pi}$ and $\upsilon \leq \upsilon^*$, and $\rho = 1$ otherwise.*

Insert Figure 1

The proposition, which is illustrated in Figure 1, suggests that default emerges in equilibrium only if two conditions are satisfied, namely if the probability of aggregate liquidity risk is low enough and if competition in the credit market is not too intense. Otherwise, that is if the risk of aggregate liquidity risk is high enough

[8] As shown in the proof of Proposition 2, the condition that the distance in the liquidity shocks, $\lambda_H - \lambda_L$, is not too large ensures that the difference in the bank's profits from behaving safe and behaving risky, i.e., $\Pi^S - \Pi^R$, is monotonically decreasing in the probability of the good state, π for any υ. This property implies the uniqueness of the equilibrium described in the proposition. If, by contrast, the difference $\lambda_H - \lambda_L$ is large enough, the difference $\Pi^S - \Pi^R$ may no longer be decreasing in π for any υ. If that is the case, there may exist multiple values of $\underline{\pi}$ above which $\Pi^R > \Pi^S$ when $\upsilon \to 0$ and a level of υ^* corresponding to each $\underline{\pi}$.

$(\pi < \underline{\pi})$ or if competition is intense ($\pi \geq \underline{\pi}$ but $\upsilon \geq \upsilon^*$), all banks behave safe and default does not emerge in equilibrium.

The intuition is simple but subtle at the same time because of the interrelation between credit market competition and market prices. Banks choose not to default if remaining solvent is a dominant strategy, that is if playing safe allows them to achieve strictly higher expected profits. By contrast, default emerges if banks are indifferent between remaining always solvent and defaulting in state H in terms of achieving the same expected profits from either strategy.

Both π and υ affect banks' profitability. Given that risky banks make positive profits only in the good state, an equilibrium with default can occur only if state L is likely enough. Otherwise, playing safe would always dominate. The minimum necessary threshold $\underline{\pi}$ equates the expected profits of safe and risky banks when banks enjoy monopoly power in the credit market, that is for $\upsilon = 0$ and $r = V$. For $\pi > \underline{\pi}$ safe and risky banks still make the same expected profits as long as υ is not too high, that is as long as holding loans is profitable enough. Otherwise, again, all banks find it profitable to be safe as this strategy allows them to achieve higher profits. In this sense, the requirement that $\pi \geq \underline{\pi}$ is a necessary and sufficient condition for the default equilibrium to emerge in equilibrium. For $\pi < \underline{\pi}$, behaving risky is a feasible strategy for banks, but it is never optimal because the good state is not likely enough.

Insert Figure 2

Figure 2 illustrates how υ affects the equilibrium. It plots the expected profit for a risky bank and a safe bank for a given $\pi > \underline{\pi}$ as a function of υ when $P_L = r = V - \upsilon$ as is the case when all banks are safe and $R^S > \lambda_L c_1^S$. As the graph shows, both $\Pi^S\big|_{P_L=r}$ and $\Pi^R\big|_{P_L=r}$ are decreasing in υ but $\Pi^R\big|_{P_L=r}$ is steeper and, therefore, it decreases more rapidly. This implies $\Pi^S - \Pi^R\big|_{P_L=r} > 0$ for $\upsilon > \upsilon^*$ and $\Pi^S - \Pi^R\big|_{P_L=r} < 0$ for $\upsilon < \upsilon^*$. Since P_L can be at most r, as otherwise no bank would be willing to buy loans in the interbank market at date 1, the equilibrium entails that all banks behave safe and $P_L = r$ for $\upsilon > \upsilon^*$ while a group of banks behave risky and $P_L < r$ so that $\Pi^S = \Pi^R$ holds for $\upsilon < \upsilon^*$.

This argument underlines the importance of credit market competition in the model. First, the parameter υ affects bank profits through the loan rate r accruing to banks at date 2. The lower is the level of competition, i.e., the smaller is υ, the more profitable are the loans and the less convenient is holding liquidity. Second, υ affects the interbank prices P_θ through changes in loan rates and in liquidity holdings. In particular, a smaller υ leads to a high P_L as it increases the "fundamental value" of the loans and, from (23), a smaller

P_H. Moreover, as a smaller υ reduces banks' incentives to hold liquidity, it also determines a reduction in prices when liquidity becomes scarce.

The considerations mean that competition affects both the market return of loans at date 1 for the risky banks that sell them and the date 2 return for the safe banks that acquire loans in the market at date 1. Banks find it profitable to behave risky when υ is low, since they can make high returns from holding loans in state L and selling them at date 1 when needed. Thus, as long as competition is not too intense (i.e., $\upsilon < \upsilon^*$), loans are sufficiently profitable in terms of returns both at date 1 and date 2 to sustain the default equilibrium and $P_L < r$. As υ increases, the profits in state L decrease and banks find it increasingly more convenient to behave safe, holding liquidity and buying at lower prices in the interbank market at date 1. As υ reaches υ^* the return on loans is just enough for the risky banks to be indifferent between maintaining no reserves and defaulting in state H and behave like a safe bank. As υ increases beyond υ^*, being a risky bank remains feasible but is no longer profitable and the only possible equilibrium is the equilibrium where all banks remain solvent and $P_L = r$.

The result of Proposition 2 implies that stronger competition in the credit market, as represented by lower loan rates, leads to a more stable financial system. The result of a positive relationship between competition and financial stability resembles the one in Boyd and De Nicoló (2005), although our underlying mechanism is quite different. In their framework, banks lend to entrepreneurs that choose the riskiness of the projects they invest in. More competition translates into safer portfolios because entrepreneurs appropriate a greater return of their projects and thus invest more conscientiously. By contrast, in our framework the entrepreneurs are completely passive, as in the charter value literature (e.g., Keeley, 1990, Hellman, Murdock and Stiglitz, 2000, Allen and Gale, 2004a). Banks choose the riskiness of their portfolios by choosing the initial allocation between short and long term assets and by trading on the interbank market at date 1. The interrelation between credit market competition, liquidity holdings and asset prices is the key to our result that banks become safer as competition increases even in a framework where they themselves determine the risk of their portfolios.[9]

It is worth noticing that the endogeneity of the interbank market prices and price volatility are essential for an equilibrium with partial default (i.e., $\rho < 1$). In a model with a fixed and exogenous liquidation value of loans at date 1, it would not be possible to have ex ante identical banks choosing different portfolio

[9] So far we have considered only the case where banks are subject to liquidity shocks. In Section 6.3, we extend the analysis to consider a framework where banks are also subject to credit risk.

strategies. Depending on the size of the liquidation value, banks would either always lose or gain from holding no reserves and defaulting in one state so that ρ would be either 0 or 1. In other words, investing in reserves at date 0 would always dominate investing in loans or viceversa. By contrast, when prices are endogenous and volatile across the two states of nature, as in our framework, banks can make the same expected profits from behaving differently. Prices are determined so that no asset dominates the other at date 0 and loans can be sold at date 1, and banks' gains and losses are different across the two states depending on whether they play safe or risky. When the good state realizes and competition is low enough, risky banks make higher profits than safe banks because both the loan price P_L and the date 2 loan return r are high. When the bad state materializes, the opposite is true: Safe banks make high returns from buying loans at date 1 and risky banks default. Thus, it is precisely the adjustment of market prices to the different levels of competition and different bank strategies that makes partial default possible and optimal for low enough degrees of competition.

Insert Figure 3

Figure 3 illustrates the equilibrium prices P_L and P_H as a function of the degree of credit market competition υ. In both the equilibrium with $\rho < 1$ and the one with $\rho = 1$, the price P_L decreases with υ, while P_H increases. As a consequence, the lower is the degree of credit market competition, the higher is price volatility. The reason is that, as required by condition (22), at date 0 the expected return banks accrue from holding loans and from holding reserves must be the same so that banks are willing to invest in both loans and reserves initially. Since P_L increases as competition decreases and, in turn, the date 2 loan return r increases, the price P_H must decrease so that (22) still holds.

4 Properties of the equilibrium

We now analyze some features of the equilibrium outlined above and, in particular, how the probability of the good state π affects the level of competition at which default starts to emerge in equilibrium and the severity of crises in terms of number of defaulting banks and amount of loans banks provide to entrepreneurs in equilibrium.

As shown in Propositions 2, given a probability $\pi > \underline{\pi}$, default exists in equilibrium if $\upsilon \leq \upsilon^*$, while all banks behave safe if $\upsilon \geq \upsilon^*$. At $\upsilon = \upsilon^*$ both types of equilibrium emerge, so that we have the coexistence of an equilibrium with $\rho < 1$ and one with $\rho = 1$.

To analyze the role of the parameter π, we now conduct some comparative statics exercise. We start by looking at how π affects the threshold υ^* determining the existence of the two equilibria for $\pi > \underline{\pi}$. We have the following result.

Proposition 3 *The threshold υ^* increases with π: $\frac{d\upsilon^*}{d\pi} > 0$.*

The proposition states that the equilibrium with default exists for a larger set of values of credit market competition as π increases. The reason is that, as the aggregate liquidity shock becomes less likely, behaving risky becomes more profitable for any given υ.

We now turn to analyze the effect of π on the severity of the crises, as expressed by the fraction of safe and risky banks in an equilibrium with default, and on the total amount of loans granted to entrepreneurs. We start by conducting both exercises at the level of credit market competition $\upsilon = \upsilon^*$ in order to compare the characteristics of the equilibrium with default and the equilibrium without default when they both exist. In deriving the results analytically, we focus on the special case where the difference between the liquidity shocks λ_H and λ_L is very small and, at the limit, $\lambda_H \to \lambda_L$.[10]

Proposition 4 *The number of safe banks ρ decreases with π at $\upsilon = \upsilon^*$: $\frac{d\rho}{d\pi} < 0$ at $\upsilon = \upsilon^*$.*

The result is intuitive. Since a higher probability of the good state increases banks' incentives to behave risky, a higher π reduces the fraction of safe banks in the economy at any given υ^*.

To see the effect of π on credit availability, we first define the total amount of loans banks grant to entrepreneurs TL as

$$TL = \rho L^S + (1-\rho)L^R,$$

where $L^S = 1 - R^S$ and $L^R = 1 - R^R$ are the amounts of loans granted by safe and risky banks, respectively. Using the results of Proposition 1, total lending is then equal to

$$TL_{(\rho=1)} = 1 - \lambda_H c_1^S \tag{24}$$

when $\rho = 1$ and all banks are safe, and

$$TL_{(\rho<1)} = \rho(1 - \lambda_H c_1^S) + (1-\rho)(1 - P_H) \tag{25}$$

when $\rho < 1$ so that default emerges in equilibrium. It follows:

[10] Focusing on the limit case where $\lambda_H \to \lambda_L$ is typical in the financial stability literature (e.g., Allen and Gale, 2004b). This assumption captures the idea of financial fragility where even very small shocks are sufficient to induce banks into default. Our results are however more general. Calculations available from the authors provide a sufficient condition for the results to hold when the difference $\lambda_H - \lambda_L$ is not too large. The numerical example provided below is also done for the more general case where $\lambda_H > \lambda_L$.

Lemma 1 *At $\upsilon = \upsilon^*$, $TL_{(\rho<1)} < TL_{(\rho=1)}$ if $P_H > \lambda_H c_1^S$, and $TL_{(\rho<1)} > TL_{(\rho=1)}$ otherwise.*

The lemma states that the impact of default on the total amount of loans granted or, in other words, on credit availability, depends on the comparison between the repayment to all consumers of a risky bank as represented by the price P_H and the promised repayment $\lambda_H c_1^S$ to the early consumers at a safe bank in state H. Banks decide their investment in loans and reserves at date 0 anticipating their needs for liquidity at date 1 and the market price P_θ. In aggregate, the total amount of reserves has to be enough to satisfy all consumers' liquidity demands. For this reason, the difference in the loan supply between the equilibrium without default and the equilibrium with default at $\upsilon = \upsilon^*$ depends only on the difference between $\lambda_H c_1^S$ and P_H. When $P_H > \lambda_H c_1^S$, the system needs more reserves in the equilibrium with default than in the one without default to repay all early withdrawing consumers in state H, thus implying a lower investment in loans in aggregate at date 0. The opposite is true if $P_H < \lambda_H c_1^S$.

The probability of the good state π affects the sign of the difference $P_H - \lambda_H c_1^S$ as follows.

Proposition 5 *Define $\overline{\pi}$ as the cutoff value of the probability of state L such that $P_H - \lambda_H c_1^S = 0$ when $P_L = r = V$. Then, if the difference $P_H - \lambda_H c_1^S$ is decreasing in π (i.e., $\frac{d(P_H - \lambda_H c_1^S)}{d\pi} < 0$), two cases can be distinguished:*

i) if $\overline{\pi} < \underline{\pi}$, $TL_{(\rho<1)} > TL_{(\rho=1)}$ for any $\pi \in (\underline{\pi}, 1)$;

ii) If $\overline{\pi} > \underline{\pi}$, there exists a value $\widehat{\pi} \in (\underline{\pi}, 1)$ such that at $\upsilon = \upsilon^*$ $TL_{(\rho<1)} > TL_{(\rho=1)}$ if $\pi > \widehat{\pi}$ and $TL_{(\rho<1)} < TL_{(\rho=1)}$ otherwise.

The proposition defines the conditions under which credit availability is higher (lower) in the equilibrium with default relative to the equilibrium without default at $\upsilon = \upsilon^*$ when the two equilibria coexist. The equilibrium with default (i.e., $\rho < 1$) entails more lending than the equilibrium without default (i.e., $\rho = 1$) when the probability of the good state π is high enough. Otherwise, total credit is lower in the equilibrium with default than in the one without default. The intuition hinges again on the fact that, as mentioned already, the profitability of being a risky bank increases with π as this corresponds to a higher probability of the good state where risky banks make positive profits. If $\frac{d(P_H - \lambda_H c_1^S)}{d\pi} < 0$, at date 1 early withdrawing depositors obtain relatively more from a safe bank than from a risky bank in state H as π increases. Thus, for π high enough, $P_H < \lambda_H c_1^S$ and the economy needs fewer reserves in an equilibrium with default than in an equilibrium without default.[11]

As highlighted in the proposition, whether total lending is higher in the equilibrium with default than in the one without default depends crucially on the threshold $\underline{\pi}$, that is the minimum level of π required for the existence of an equilibrium with default. When $\underline{\pi}$ is high enough (i.e., $\overline{\pi} < \underline{\pi}$), the range of π for which being risky is profitable does not include the low enough values that would guarantee that $P_H - \lambda_H c_1^S > 0$ holds, thus implying that the equilibrium with default always entails more credit than the equilibrium without default. By contrast, when $\underline{\pi}$ is low enough (i.e., $\overline{\pi} > \underline{\pi}$), the range of values of the probability of the good state for which the default equilibrium exists is sufficiently large and includes low enough values of π for which $P_H - \lambda_H c_1^S > 0$ and, in turn, total lending is lower in the equilibrium with default than in the one without default.

In Figures 5.a and 5.b below, we illustrate the existence of these two cases described in the proposition with a numerical example. We show that whether $\underline{\pi}$ is larger (smaller) than $\overline{\pi}$ depends crucially on the return of entrepreneurs' project V. In particular, the case $\overline{\pi} > \underline{\pi}$ exists when V is sufficiently high since both V and π positively affect the profitability of being a risky bank. Thus, the higher V, the lower is the minimum value of π, defined as $\underline{\pi}$, required for the equilibrium with default to exist.

Taken together, Propositions 3, 4 and 5 suggest that, as the probability of aggregate liquidity shocks decreases (i.e., π increases), crises emerge for a larger range of values of credit market competition and entail increasingly more risky banks and more credit availability.

We now compare the number of risky banks and total lending across equilibria with default and without default for a given level of credit market competition $\upsilon < \upsilon^*$ but different levels of aggregate liquidity risk. As illustrated in Figures 4, 5.a and 5.b, which plot the number of safe banks and total lending as a function of υ for different values of π, the same results stated in Propositions 3, 4 and 5 for $\upsilon = \upsilon^*$ hold for a given $\upsilon < \upsilon^*$ across different equilibria with default.

Insert Figure 4 and 5.a, 5.b

Figure 4 shows that a higher π leads to a larger υ^* and a lower fraction of safe banks ρ for any given

[11] Unfortunately, it is not easy to prove $\frac{d(P_H - \lambda_H c_1^S)}{d\pi} < 0$ analytically because, while $\frac{dc_1^S}{d\pi} > 0$,

$$\frac{dP_H}{d\pi} = -\frac{r(r-1)}{(r-\pi)^2} + \frac{\pi(1-\pi)}{(r-\pi)^2}\frac{d\upsilon^*}{d\pi}$$

is not monotonic in π since $r > 1$ but $\frac{d\upsilon^*}{d\pi} > 0$ as shown in Proposition 3. However, as long as $\frac{\pi(1-\pi)}{(r-\pi)^2}\frac{d\upsilon^*}{d\pi}$ is small enough, the monotonicity of $P_H - \lambda_H c_1^S$ is guaranteed.

$\upsilon < \upsilon^*$. Figure 5.a and 5.b show that a higher π leads to more credit availability for any given υ both in the equilibrium with default (i.e., $\upsilon < \upsilon^*$) and in the equilibrium without default (i.e., $\upsilon > \upsilon^*$). The intuition behind these results is similar to the case when $\upsilon = \upsilon^*$.

These results highlight that the structure of liquidity shocks, as represented by the parameter π, plays an important role in shaping the relationship between competition and stability. To the extent that the probability of aggregate liquidity shocks is affected by variables such as the regulatory and institutional environment or the funding structure of the banks, these results provide some interesting empirical implications of our model, which we discuss in detail in Section 7.

5 Extensions

In this section, we extend the baseline model to account for the possibility that banks also compete in the deposit and interbank market. We consider the two cases in turn in the next two subsections. For sake of brevity, we focus on how the thresholds υ^* and $\underline{\pi}$ characterizing the existence of an equilibrium with and without default change relatively to the baseline model.

5.1 Competition in the deposit market

So far we have considered that consumers can either deposit in a bank or store for a return of 1. In this section, we extend the analysis to the case where their opportunity cost is $\alpha \in (1, \overline{\alpha})$ so that consumers' participation constraint is now given by

$$E[u(c_1, c_2, \lambda_\theta)] \geq u(\alpha) > 0. \tag{26}$$

The higher α, the more banks have to promise to depositors in terms of higher c_1 and c_2. In this sense, the parameter α can be interpreted as representing the degree of competition in the deposit market, similarly to the parameter υ in the credit market.

We assume that the degree of credit market competition positively affects the degree of competition in the deposit market so that, as banks face more competition in the credit market, competition in the deposit market also increases. This could for example be the result of a change in the concentration in the banking sector affecting the competition in both the credit and the deposit market. Formally, we then define the degree of competition in the deposit market $\alpha(\upsilon)$ as an increasing function of credit market competition: $\frac{\partial \alpha(\upsilon)}{\partial \upsilon} \geq 0$, with $\alpha(\upsilon) \to 1$ when $\upsilon \to 0$ and $\alpha(\upsilon) \to \overline{\alpha}$ when $\upsilon \to \overline{\upsilon}$.

In this extended setting, the effect of the level of credit market competition on banks' incentives to behave risky becomes twofold. On the one hand, υ determines the wedge between the returns of loans and reserves, as in the baseline model. On the other hand, it affects the repayments that safe and risky banks need to offer to their depositors via a change in α, thus determining the split of surplus between banks and depositors. The equilibrium is then given by the interrelation of these two effects, as we show below.

Proposition 6 *When $\frac{\partial\alpha(\upsilon)}{\partial\upsilon} \geq 0$ is sufficiently small, there exist a level of aggregate risk $\underline{\pi}_\alpha = \underline{\pi}$ and a level of credit market competition $\upsilon^*_\alpha > \upsilon^*$ such that $\rho < 1$ for $\pi > \underline{\pi}_\alpha$ and $\upsilon \leq \upsilon^*_\alpha$ and $\rho = 1$ otherwise.*

The proposition shows that the main insights of Proposition 2 remain valid when we extend the model to consider the effect that a change in the degree of credit market competition has on the competition in the deposit market as long as the degree of competition in the deposit market is not too sensitive to the level of competition in the credit market. However, the set of parameters where default emerges in equilibrium is different from the one in the baseline model. The threshold $\underline{\pi}_\alpha$ is as in the baseline model, but υ^*_α is larger than υ^*. This result relies on the different effects that a greater α has on the promised repayments to depositors offered by safe and risky banks.

As α increases, both safe and risky banks need to increase the promised repayments to consumers to satisfy their participation constraints. However, risky banks are less affected by the increase in the consumers' repayment since with probability $1-\pi$ they default and all their depositors receive P_H, which is independent of α and increasing in υ as required by (23). By contrast, safe banks have to adjust depositors' promised consumption in both states as their promised repayment c_1^S is not contingent. For this reason, the derivative $\frac{\partial\alpha(\upsilon)}{\partial\upsilon}$ must be sufficiently small for the threshold υ^*_α to exist. When this is not the case, the equilibrium without default may no longer exist. In this sense, the proposition provides a sufficient condition for both the equilibrium with and without default to emerge in the model when the degree of competition in the credit and the deposit markets are positively correlated.

The following table illustrates the result of Proposition 6 with a numerical example. We assume $\alpha(\upsilon) = 1 + 0.01Log(1+\upsilon)$ and the following set of parameters: $V = 1.5$, $\pi = 0.8$, $\lambda_L = 0.8$, $\lambda_H = 0.81$ already used in the baseline model. We first compute the value of the threshold $\upsilon^*_\alpha = 0.359011 > \upsilon^* = 0.358859$ and show that at $\upsilon = \upsilon^*_\alpha$ safe and risky banks make the same profits so that the equilibrium with and without default coexist.[12] At $\upsilon = \upsilon^*_\alpha$, the equilibrium prices are $P_L = r = 1.14099$ and $P_H = 0.669223$ from (23).

[12] The threshold $\upsilon^* = 0.358859$ is computed for the same set of parameters $V = 1.5$, $\pi = 0.8$, $\lambda_L = 0.8$, $\lambda_H = 0.81$ but fixing $\alpha = 1$ as in the baseline model.

Then, we look at banks' expected profits at $\upsilon = \upsilon^*_\alpha - \varepsilon$ and $\upsilon = \upsilon^*_\alpha + \varepsilon$, where we set $\varepsilon = 0.01$, keeping the same level of prices $P_L = r$ and P_H from (23). When $\upsilon < \upsilon^*_\alpha$, being a safe bank is not profitable as $\Pi^S_{(\upsilon=\upsilon^*_\alpha-\varepsilon)} < \Pi^R_{(\upsilon=\upsilon^*_\alpha-\varepsilon)}$. This implies that for $\upsilon < \upsilon^*_\alpha$ an equilibrium with default (i.e., $\rho < 1$) arises. Prices have then to adjust to $P_H < P_L < r$ so that risky and safe banks make the same expected profits and (23) holds. When $\upsilon > \upsilon^*_\alpha$, being risky is not profitable as $\Pi^S_{(\upsilon=\upsilon^*_\alpha+\varepsilon)} > \Pi^R_{(\upsilon=\upsilon^*_\alpha+\varepsilon)}$. Then, being a safe bank is a dominant strategy and only the equilibrium without default ($\rho = 1$) exists for $\upsilon > \upsilon^*_\alpha$. We have:

	υ^*_α	$\Pi_{(\upsilon=\upsilon^*_\alpha)}$	$c_{1(\upsilon=\upsilon^*_\alpha)}$	$c_{2(\upsilon=\upsilon^*_\alpha)}$	$\Pi_{(\upsilon=\upsilon^*_\alpha-\varepsilon)}$	$\Pi_{(\upsilon=\upsilon^*_\alpha+\varepsilon)}$
Safe banks	0.359011	0.0248997	0.976973	1.11609	0.0270759	0.0227391
Risky banks	0.359011	0.0248997	1.10986	1.10986	0.0284914	0.0214598

Table 1: Competition in the deposit market

5.2 Asset specificity and interbank market

So far we have considered that the degree of competition in the credit market is determined by the value of entrepreneurs' outside opportunity υ, since this determines the loan rate r that banks obtain from loans. This way of modelling competition captures the negative relationship between competition and interest rates and, in this respect, it is similar to a model of spatial competition where the degree of competition is measured by the degree of differentiation among banks.

An alternative interpretation is to consider the parameter υ as representing the market power that banks enjoy over entrepreneurs because of private information. In this sense, υ can be interpreted as the degree of differentiation of banks' expertise in granting loans to entrepreneurs operating in a given sector or industry. The more banks differ in terms of their expertise, that is the lower is υ, the less intense is the competition in the credit market and thus the higher is r.

The main difference with this interpretation of υ is that credit market competition affects now also the sale of loans in the interbank market and thus market prices. When banks differ in terms of their expertise, each bank is the most efficient user of its loans. Thus, when loans are sold in the interbank market, they will yield only a fraction of the return r to the buyer at date 2. We denote this fraction as $\zeta(\upsilon) \in \left[\underline{\zeta}, 1\right]$ and assume it to be an increasing function of the degree of competition in the credit market υ: $\frac{\partial\zeta(\upsilon)}{\partial\upsilon} \geq 0$, with $\zeta(\upsilon) \to \underline{\zeta} > 0$ when $\upsilon \to 0$ and $\zeta(\upsilon) \to 1$ for $\upsilon \to \overline{\upsilon}$. This changes the market prices in the interbank market at date 1 since $P_\theta \leq r\zeta(\upsilon)$ must hold for banks to be willing to buy loans in the interbank market. This in turn changes the returns that banks can obtain from selling loans in the interbank market and thus their incentive to behave risky. We have the following result.

Proposition 7 *When $\frac{\partial \zeta(\upsilon)}{\partial \upsilon} \geq 0$ is sufficiently small, there still exist a level of aggregate risk $\underline{\pi}_{\zeta} > \underline{\pi}$ and a level of credit market competition $\upsilon^*_{\zeta} < \upsilon^*$ such that $\rho < 1$ for $\pi > \underline{\pi}_{\zeta}$ and $\upsilon \leq \upsilon^*_{\zeta}$, and $\rho = 1$ otherwise.*

The proposition states that the main insights of Proposition 2 remain valid when we extend the model to consider the effect that a change in the degree of credit market competition has on banks' interactions in the interbank market. Behaving risky is still profitable only when competition in the credit market is not intense and the level of aggregate risk is sufficiently low. However, since the price that banks are willing to pay to buy loans at date 1 is lower for any $\upsilon \in [0, \overline{\upsilon}]$, behaving risky is less profitable and the equilibrium with default exists in a smaller range of parameters relative to the baseline model, that is $\underline{\pi}_{\zeta} > \underline{\pi}$ and $\upsilon^*_{\zeta} < \upsilon^*$.

For the result in the proposition to hold, it must be the case that $\frac{\partial \zeta(\upsilon)}{\partial \upsilon}$ is sufficiently small so that the overall effect of a decrease in the degree of competition (i.e., a smaller υ) on the maximum return $r\zeta(\upsilon)$ that bank can obtain from selling loans on the interbank market remains positive. Otherwise, behaving risky may no longer be profitable for any υ.

The following table illustrates the result of Proposition 7 with a numerical example. We assume $\zeta(\upsilon) = 1 - 0.1(V - 1 - \upsilon)$ and the same set of parameters as in previous subsection: $V = 1.5$, $\pi = 0.8$, $\lambda_L = 0.8$, $\lambda_H = 0.81$ and $\varepsilon = 0.01$. As before, we first compute the value of the threshold $\upsilon^*_{\zeta} = 0.223545 < 0.358859 = \upsilon^*$ and show that at $\upsilon = \upsilon^*_{\zeta}$ safe and risky banks make the same profits so that the equilibrium with and without default coexist.[13] At $\upsilon = \upsilon^*_{\zeta}$, the equilibrium prices are $P_L = r\zeta(\upsilon) = 1.24117$ and $P_H = 0.521$, from the condition $r = \pi \frac{r\zeta(\upsilon)}{P_L} + (1 - \pi)\frac{r\zeta(\upsilon)}{P_H}$ guaranteeing that banks invest both in loans and reserves at date 0. Then, we look at banks' expected profits at $\upsilon = \upsilon^*_{\zeta} - \varepsilon$ and $\upsilon = \upsilon^*_{\zeta} + \varepsilon$, keeping the same level of prices $P_L = r\zeta(\upsilon)$ and P_H as given by the solution to $r = \pi \frac{r\zeta(\upsilon)}{P_L} + (1 - \pi)\frac{r\zeta(\upsilon)}{P_H}$. When $\upsilon < \upsilon^*_{\zeta}$, being a risky banks is more profitable than being a safe bank as $\Pi^S_{(\upsilon=\upsilon^*_{\zeta}-\varepsilon)} < \Pi^R_{(\upsilon=\upsilon^*_{\zeta}-\varepsilon)}$. Thus, the equilibrium features default (i.e., $\rho < 1$) and $P_L < r$. By contrast, for $\upsilon > \upsilon^*_{\zeta}$, being risky is less profitable as $\Pi^S_{(\upsilon=\upsilon^*_{\zeta}+\varepsilon)} > \Pi^R_{(\upsilon=\upsilon^*_{\zeta}+\varepsilon)}$. Thus the equilibrium features no default (i.e., $\rho = 1$). We have:

	υ^*_{ζ}	$\Pi_{(\upsilon=\upsilon^*_{\zeta})}$	$c_{1(\upsilon=\upsilon^*_{\zeta})}$	$c_{2(\upsilon=\upsilon^*_{\zeta})}$	$\Pi_{(\upsilon=\upsilon^*_{\zeta}-\varepsilon)}$	$\Pi_{(\upsilon=\upsilon^*_{\zeta}+\varepsilon)}$
Safe banks	0.223545	0.0581163	0.952413	1.21834	0.0604077	0.0558368
Risky banks	0.223545	0.0581163	1.17704	1.17704	0.0619402	0.054364

Table 2: Asset specificity

[13] The threshold $\upsilon^* = 0.358859$ is computed for the same set of parameters $V = 1.5$, $\pi = 0.8$, $\lambda_L = 0.8$, $\lambda_H = 0.81$ but fixing $\zeta = 1$ so that $P_L = r$ as in the baseline model.

6 Robustness of the baseline model

In this section, we test the robustness of the baseline model by relaxing some of its assumptions. First, we consider alternative consumers' utility functions to analyze the effect of risk aversion. Second, we study the case where deposits are insured. Third, we extend the model to incorporate credit risk and bank monitoring. As in Section 5, we restrict our attention to analyze the effect on the thresholds $\underline{\pi}$ and υ^* determining the equilibrium in the baseline model.

6.1 Risk aversion

So far we have assumed that depositors have a logarithmic utility function. This assumption has made it possible to solve the model analytically but, by implying a constant relative risk aversion coefficient equal to one, it has not allowed us to analyze the effect of risk aversion on the emergence of liquidity crises. To do this, we now assume that depositors have a constant relative risk aversion utility function (CRRA) given by

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where $\sigma \geq 1$ represents the degree of risk aversion.

All the rest of the model remains the same so that the maximization problem of safe and risky banks is still given by (3)-(8) and (9)-(14), respectively. We have the following result.

Proposition 8 *There exists a value of risk aversion $\overline{\sigma} > 1$ such that risky banks hold positive reserves ($R^R > 0$) for $\sigma \geq \overline{\sigma}$ and zero otherwise.*

The intuition behind the proposition is simple. A relative risk aversion greater than one implies that depositors require more liquidity insurance and thus larger promised repayments from their banks at date 1. This holds for both safe and risky banks. The former still repay depositors the promised consumptions in all states and never experience a run. The latter still sell part of their loans in state L and all of them in state H when they default. As σ increases, risky banks are forced to sell more loans in state L, thus depressing market prices. This reduces the profitability of loans relative to holding reserves. As σ increases beyond $\overline{\sigma}$, depositors' early demands at date 1 become so large that risky banks find it convenient to start holding a positive amount of reserves in order to sell fewer loans. This allows them to reduce the promised repayment to early consumers in state L and pay more to withdrawing depositors in state H, thus increasing their profits.

The main insights of Proposition 2 still hold when risky banks hold positive reserves. Given the complexity of the analytical expressions because of the CRRA utility function, we show this with the help of a numerical example. For the set of parameters $V = 6$, $\pi = 0.8$, $\lambda_L = 0.8$, $\lambda_H = 0.81$, $\sigma = 3.9$ and $\varepsilon = 0.1$, we obtain:[14]

	υ_σ^*	$\Pi_{(\upsilon=\upsilon_\sigma^*)}$	$R_{(\upsilon=\upsilon_\sigma^*)}$	$\Pi_{(\upsilon=\upsilon_\sigma^*-\varepsilon)}$	$\Pi_{(\upsilon=\upsilon_\sigma^*+\varepsilon)}$
Safe banks	0.784	0.602	0.827	0.62	0.585
Risky banks	0.784	0.602	0.457	0.632	0.572

Table 3: The equilibrium for $u(c)=\frac{c^{1-\sigma}}{1-\sigma}$ and $\sigma>\overline{\sigma}$.

The table shows that, given an enough high π, there still exists a level of credit market competition $\upsilon_\sigma^* = 0.784$ where safe and risky banks make equal expected profits so that both an equilibrium with $\rho = 1$ and an equilibrium with $\rho < 1$ coexist at $\upsilon = \upsilon_\sigma^*$. Both safe and risk banks hold positive reserves with $R^S > R^R$, and the market prices are given by $P_L = r = 5.216$ and, from (23), $P_H = 0.2362$.

As the table shows, keeping the same level of prices $P_L = r$ and P_H as given by (23), risky banks make higher expected profits than safe banks at $\upsilon = \upsilon_\sigma^* - \varepsilon$, while the opposite happens at $\upsilon = \upsilon_\sigma^* + \varepsilon$. This implies that an equilibrium with $\rho < 1$ and $P_L < r$ exists for $\upsilon < \upsilon_\sigma^*$, while an equilibrium with $\rho = 1$ and $P_L = r$ exists for $\upsilon > \upsilon_\sigma^*$. This confirms that the main result of Proposition 2 remains valid when consumers are risk averse and risky banks hold a positive amount of reserves.

6.2 Deposit Insurance

So far we have assumed that deposits are not insured. This implies that when a risky bank defaults in state H, its depositors receive the liquidation value P_H. We now consider the case where deposits are insured so that depositors at a risky bank are guaranteed to receive $c^I \in (P_H, c_1^R]$ in state H instead of P_H. The insurance scheme is assumed to be fairly priced. The deposit insurer transfers an amount $t = c^I - P_H$ to depositors of risky banks in state H in exchange for the payment of a premium C equal to

$$C = (1-\pi)t = (1-\pi)(c^I - P_H), \tag{27}$$

which reflects the fact that risky banks default only with probability $(1-\pi)$. For simplicity, we consider that the premium is paid ex post, but the same result holds if it is instead paid ex ante.[15]

The effect of deposit insurance on the threshold υ^* is not clear-cut. Safe banks are not affected by the presence of deposit insurance as they offer safe deposit contracts. By contrast, risky banks pay now a higher

[14] A higher relative risk aversion implies that depositors require greater liquidity insurance. Thus, V is now higher than in the previous examples in order to guarantee banks' profits are positive.

[15] Notice that this insurance scheme does not prevent the run at date 1, but it only entails a greater repayment for depositors at a risky bank when this defaults in state H. If the insurance scheme was able to prevent runs, then no bank would default any longer and the equilibrium with default would not occur.

repayment $c^I > P_H$ to depositors in state H and can thus reduce the promised repayments (c^R_{1di}, c^R_{2di}) to early and late depositors in state L. Whether this increases banks' incentives to behave risky depends on whether the advantage in terms of lower promised repayments to depositors in the good state dominates the premium C required for the insurance. We have then the following result.

Proposition 9 *With fairly priced deposit insurance, the range of parameters in which the equilibrium with* $\rho < 1$ *exists is smaller than that without deposit insurance (i.e.,* $\underline{\pi}_{di} > \underline{\pi}$ *and* $\upsilon^*_{di} < \upsilon^*$*) if*

$$(1-\pi)(c^I - P_H) > c^R_2 - c^R_{2di}, \tag{28}$$

and it is larger otherwise.

The proposition states that the introduction of fairly-priced deposit insurance reduces (enlarges) the range of parameters for which the default equilibrium exists if the premium $(1-\pi)(c^I - P_H)$ required for the provision of the insurance is larger (smaller) than the reduction $c^R_2 - c^R_{2di}$ in the promised repayments to consumers relative to the case without insurance. The result depends on whether the introduction of deposit insurance makes behaving risky more or less convenient relative to the case without deposit insurance. The result holds for any guaranteed repayment $c^I > P_H$.

6.3 Monitoring and credit risk

So far we have assumed that loans are safe and banks are only exposed to liquidity risk. We now modify this assumption by considering that loans are risky and banks can affect the risk of their loans through monitoring.

To do this analysis, we modify the baseline model in various directions. We start by assuming that loans are risky in that each entrepreneur's loan yields a stochastic return of the form

$$\widetilde{V} = \begin{cases} V > 1 & \text{w. pr. } q \\ \varepsilon < 1 & \text{w. pr. } (1-q), \end{cases}$$

with $q \in [0,1]$ being the probability of success and $\varepsilon \in (0,1)$ being the return in case of failure. Banks improve the success probability of their loans through monitoring. In particular, the monitoring effort m increases q as follows:

$$q = \begin{cases} q_H & \text{w. pr. } m \\ q_L & \text{w. pr. } (1-m), \end{cases}$$

with $q_H > q_L$. Thus, for a given level of monitoring m chosen by the bank, the expected probability of success is equal to

$$q = mq_H + (1-m)q_L. \tag{29}$$

Each unit of monitoring increases the success probability of loans by $\frac{\partial q}{\partial m} = q_H - q_L > 0$, but it entails a cost $c(m) = \frac{m^2}{2}$ to the bank.

Entrepreneurs still have an opportunity cost equal to $\upsilon \in (0, \overline{\upsilon})$ and their participation constraint is now

$$q(V - r) \geq \upsilon,$$

since with probability $(1 - q)$ the project fails and the return ε is appropriated by the bank. Considering again that banks make a take-it-or-leave-it offer to entrepreneurs, the loan rate is then given by

$$r(\upsilon, q) = V - \frac{\upsilon}{q}. \tag{30}$$

The loan rate decreases with υ as in the baseline model, but it now also depends positively on monitoring as $\frac{\partial r}{\partial m} = \frac{(q_H - q_L)}{q^2}\upsilon$. The reason is that greater monitoring makes the entrepreneur's participation constraint more slack for any given υ, thus allowing the bank to appropriate a larger fraction of the return V in the case the project succeeds.

The rest of the model remains the same. At date 0 banks choose their portfolio allocation (R, L) and deposit contract (c_1, c_2) and then the level of monitoring m.

Differently from the baseline model, there are now two sources of risk: liquidity risk at date 1 if the aggregate liquidity shock λ_H realizes and credit risk at date 2 if the loans fail. This complicates the framework as it introduces different types of default and the possibility that depositors run at the bank because of either liquidity risk or credit risk considerations. To simplify the analysis and keep the model tractable, we then proceed as follows. We first assume that the realization of the loan portfolio is observable at date 2 to both banks and depositors but not at date 1, when all agents still expect each bank to succeed at date 2 with probability q. This implies that banks invest in loans at date 0 and sell them at date 1 only to satisfy early depositors' liquidity demands and that runs can only occur in response to the realization of the aggregate liquidity shock λ_H. In other words, there is no adverse selection on the interbank market, and, as in the baseline model, runs and loan sales are determined by liquidity considerations.

Second, we assume that banks can hedge themselves against liquidity risk but not against credit risk. In other words, as in the baseline model, banks may find it optimal to hold enough reserves to protect themselves against the aggregate liquidity risk at date 1 but not against the risk of project failure at date 2. In line with this, we continue defining as *safe* the banks that do not experience a run at date 1 in both state L and H, and as *risky* those banks that instead experience a run in state H and are thus forced to sell all their loans at date 1. Both safe and risky banks are subject to credit risk and go into default at date 2 if their loans fail. This implies that, consistently with the existing literature (e.g., Boot and Greenbaum, 1992), the probability of success q, and in turn, the likelihood of not defaulting at date 2, depends on the level of monitoring the bank exerts.

Finally, in order to keep the analysis tractable, we now assume that safe banks can only buy loans on the interbank market from risky banks if they have an excess of reserves, but they do not trade loans among themselves for liquidity reasons. In other words, differently from the baseline model, we now exclude the possibility that a safe bank trades loans with another safe bank to satisfy its early liquidity demands. The reason is that now safe and risky banks exert different levels of monitoring in equilibrium and thus the loans they invest in entail different loan rates and carry a different level of credit risk. This would imply different market prices for a loan sold by a safe or by a risky bank. To avoid these complications, we simply consider that each safe bank holds an amount of reserves $R^S = \lambda_H c_1^S + \frac{(1-\rho)P_H L^R}{\rho}$ and uses the reserves in excess to buy the loans that the risky banks sell up to a maximum of $(1-\rho)P_H L^R$ in state H.

Despite the simplifying assumptions described above, the problem with credit risk is quite complicated. For example, the fact that safe and risky banks are subject to different levels of credit risk means that, for a given level of credit market competition υ, safe and risky banks obtain now a different loan return on their loans. This also implies that the return from holding a loan at date 2 for a safe bank depends on whether it invests in that loan at date 0 or whether it buys it at date 1 from a risky bank. Moreover, whereas all safe and risky banks behave alike and loans are correlated within each respective group, the loan returns are independent across the two types of banks. In other words, at date 2 the loans of safe banks can be successful while those of risky banks can fail or viceversa. Withstanding this, we proceed by assuming that a safe bank defaults at date 2 when its own loans fail at that date. This rules out the possibility that a safe bank can remain solvent at date 2 when its own loans default thanks to the success of the loans it buys from the risky banks at date 1.

To analyze the problem formally, we first define the date 2 return on a loan initiated at date 0 by a safe bank as

$$\begin{aligned} \widetilde{r}(\upsilon, q^S) &= q^S r(\upsilon, q^S) + (1-q^S)\varepsilon = \\ &= q^S V + (1-q^S)\varepsilon - \upsilon \end{aligned}$$

where, using (29), $q^S = q(m^S) = m^S q_H + (1-m^S) q_L$ and, from (30), $r(\upsilon, q^S) = V - \frac{\upsilon}{q^S}$, and the return on a loan initiated at date 0 by a risky bank as

$$\begin{aligned}\widetilde{r}(\upsilon, q^R) &= q^R r(\upsilon, q^R) + (1-q^R)\varepsilon = \\ &= q^R V + (1-q^R)\varepsilon - \upsilon\end{aligned}$$

where $q^R = q(m^R)$ and $r(\upsilon, q^R)$ are the same as for the safe banks once we use m^R instead of m^S.

Given this, the maximization problem of a safe bank becomes

$$\begin{aligned}\underset{c_1^S, c_2^S, m^S}{Max}\,\Pi^S &= q^S\left\{r(\upsilon, q^S)L^S + \left[\pi\frac{R^S - \lambda_L c_1^S}{P_L} + (1-\pi)\frac{R^S - \lambda_H c_1^S}{P_H}\right]\widetilde{r}(\upsilon, q^R) - \left[\pi(1-\lambda_L) - (1-\pi)(1-\lambda_H)\right]c_2^S\right\} + \\ &\quad - \frac{(m^S)^2}{2}\end{aligned} \tag{31}$$

subject to

$$\begin{aligned}E[u(c_1^S, c_2^S, q^S, \lambda_\theta)] &= \left[\pi\lambda_L + (1-\pi)\lambda_H\right]u(c_1^S) + \\ &+ q^S\left[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)\right]u(c_2^S) + \\ +(1-q^S)&\left[\pi(1-\lambda_L)u\left(\frac{\varepsilon L^S + \frac{R^S - \lambda_L c_1^S}{P_L}\widetilde{r}(\upsilon, q^R)}{1-\lambda_L}\right) + (1-\pi)(1-\lambda_H)u\left(\frac{\varepsilon L^S + \frac{R^S - \lambda_H c_1^S}{P_H}\widetilde{r}(\upsilon, q^R)}{1-\lambda_H}\right)\right] \geq u(1).\end{aligned} \tag{32}$$

The problem differs from the one defined in (3)-(8) in the baseline model in several respects. First, a safe bank makes positive profits now only with probability q^S. In this case, bank profits are given by the return $r(\upsilon, q^S)$ of the L^S loans that the safe bank gives at date 0 plus the expected return $\widetilde{r}(\upsilon, q^R)$ on the $\left[\pi\frac{R^S - \lambda_L c_1^S}{P_L} + (1-\pi)\frac{R^S - \lambda_H c_1^S}{P_H}\right]$ loans that the safe bank buys from the risky banks in state L and H, minus the promised repayment to late consumers. With probability $1 - q^S$, loans yield the return ε, the bank defaults at date 2 and makes zero profits. Irrespective of q, the bank pays the monitoring costs $\frac{(m^S)^2}{2}$.

Second, the possibility that a bank defaults at date 2 when the loans fail changes depositors' repayments, as it emerges from their participation constraint (32). While in the baseline model both early and late depositors always receive their promised consumptions (c_1^S, c_2^S), now this is only the case for the early depositors. Late depositors receive the promised repayment c_2^S only with probability q^S. With probability $1-q^S$, when the projects fail, late depositors receive a pro-rata share of bank's available resources at date 2, as given by a fraction $\frac{1}{1-\lambda_\theta}$ of the sum of the return on the loans εL^S the bank has initially invested in and the expected return $\widetilde{r}(\upsilon, q^R)$ on the loans that the safe bank buys in the interbank market from the risky banks using the excess reserves $\frac{R^S - \lambda_\theta c_1^S}{P_\theta}$ for $\theta = L, H$.

By contrast, the maximization problem of a risky bank is now given by

$$\underset{c_1^R, c_2^R, m^R}{Max} \Pi^R = q^R \pi \left[r(\upsilon, q^R) L^R - \frac{\lambda_L c_1^R - R^R}{P_L} r(\upsilon, q^R) - (1-\lambda_L) c_2^R \right] - \frac{(m^R)^2}{2} \tag{33}$$

subject to

$$E[u(c_1^R, c_2^R, q^R, \lambda_\theta)] = \pi \lambda_L u(c_1^R) +$$

$$+ q^R \pi (1-\lambda_L) u(c_2^R) + (1-q^R)\pi(1-\lambda_L) u\left(\frac{L^R - \frac{\lambda_L c_1^R - R^R}{P_L}}{1-\lambda_L} \varepsilon \right) + \tag{34}$$

$$+ (1-\pi) u(L^R P_H + R^R) \geq u(1)$$

As for the safe banks, also the problem of the risky banks differs from the baseline model in several respects. First, risky banks only obtain positive profits with probability $q^R \pi$ since they face a run with probability $1-\pi$ and default at date 2 as a consequence of the failure of their loans. Second, depositors' participation constraint is different because of bank credit risk. Early depositors obtain the promised repayment c_1^R in state L, which occurs with probability π, irrespective of whether the bank defaults at date 2, while late depositors receive the promised repayment c_2^R with probability $q^R \pi$ and a pro-rata share with probability $(1-q^R)\pi$ when the bank defaults at date 2. In this case, each of the $1-\lambda_L$ late depositors receives $\varepsilon \frac{L^R - \frac{\lambda_L c_1^R - R^R}{P_L}}{1-\lambda_L}$ at date 2, since loans only yield ε and the bank sells $\frac{\lambda_L c_1^R - R^R}{P_L}$ units of loans at date 1 to meet its demand for liquidity. Finally, as in the baseline model, all depositors obtain $L^R P_H + R^R$ with probability $1-\pi$, when the aggregate liquidity risk realizes and a run occurs.

Solving the model is complicated. We then restrict our attention to discuss the monitoring choice of safe and risky banks and the role of credit market competition on the emergence of liquidity crises and bank credit risk.

The following lemma characterizes the monitoring choice of safe and risky banks as a function of their portfolio investment and deposit contract.

Lemma 2 *Each safe bank chooses a level of monitoring*

$$\begin{aligned} m^S &= (q_H - q_L) \left[r(\upsilon, q^S) L^S + \left(\pi \frac{R^S - \lambda_L c_1^S}{P_L} + (1-\pi) \frac{R^S - \lambda_H c_1^S}{P_H} \right) \widetilde{r}(\upsilon, q^R) - [\pi(1-\lambda_L) - (1-\pi)(1-\lambda_H)] c_2^S \right] + \\ & \quad + (q_H - q_L) \frac{\upsilon}{q^S} L^S, \end{aligned} \tag{35}$$

while each risky bank chooses a level of monitoring

$$m^R = (q_H - q_L)\pi \left[r(\upsilon, q^R) L^R - \frac{\lambda_L c_1^R - R^R}{P_L} r(\upsilon, q^R) - (1-\lambda_L) c_2^R \right] + (q_H - q_L)\pi \frac{\upsilon}{q^R} \left(L^R - \frac{\lambda_L c_1^R - R^R}{P_L} \right). \tag{36}$$

The optimal level of monitoring m^i is increasing in the amount of loans L^i, with $i = S, R$. Moreover, the level of monitoring m^S exerted by safe banks is decreasing in the degree of competition υ, while the level of monitoring m^R does not change with υ. That is: $\frac{\partial m^i}{\partial L^i} > 0$ *and* $\frac{\partial m^S}{\partial \upsilon} < 0, \frac{\partial m^R}{\partial \upsilon} = 0$.

As stated in the lemma, bank monitoring effort increases with the amount of loans granted to entrepreneurs as a larger amount of loans increases the gains banks obtains from monitoring in terms of higher total loan returns. This effect holds both for safe and risky banks.

By contrast, the effect of competition on monitoring differs for the two types of banks. The reason is that competition has a twofold effect on banks' incentives to monitor. On the one hand, greater competition reduces banks' monitoring incentives as it lowers the returns $r(\upsilon, q^i)$ and $\widetilde{r}(\upsilon, q^R)$ that safe and risky banks obtain from loans, thus reducing their expected profits as measured in the first term in both (35) and (36). On the other hand, however, greater competition improves monitoring incentives as it increases the marginal effect of monitoring on the loan return as measured by the parameter υ in the last term in (35) and (36). This latter effect, which is new relative to the existing literature on competition and banks' risk-taking, results from the fact that the interest rate charged by banks to entrepreneurs depends on the monitoring effort as well as on the degree of market competition. In the case of risky banks, the two effects offset each other so that their monitoring level is not affected by the degree of competition. In the case of safe banks, in contrast, monitoring decreases with the degree of competition υ, as typical in the existing literature (e.g., Boot and Greenbaum, 1992). The reason is that, as (35) shows, the monitoring level exerted by a safe bank depends also on the monitoring exerted by the risky banks through the term $\widetilde{r}(\upsilon, q^R)$. This implies that the two mentioned effects of competition on monitoring and loan returns do no longer cancel out, and safe banks decreases their monitoring effort $-$thus having a higher probability of default at date 2$-$ as competition increases.

As the lemma suggests, safe and risky banks exert different monitoring levels. This implies that they are subject to different levels of credit risk and also that they obtain different returns from loans, for any given υ. If risky banks monitor more than safe banks, they obtain a return $r(\upsilon, q^R)$ from loans which is greater than the return $r(\upsilon, q^S)$ obtained by the safe banks, and viceversa if safe banks monitor more. This changes the profitability of holding loans relative to reserves and thus the incentives of behaving risky, for any given υ, relative to the baseline model.

Despite the introduction of credit risk and monitoring, the expected profits of both safe and risky banks are still decreasing in υ, as in the baseline model.[16] Whether there still exist a level $\underline{\pi}$ of probability of the good state and a level of credit market competition υ^* such that an equilibrium with risky banks ($\rho < 1$) for $\pi > \underline{\pi}$ and $\upsilon < \upsilon^*$ depends on whether the profits of the risky banks are steeper than the profits of the safe banks, as in the baseline model, for a price $P_L = \widetilde{r}(\upsilon, q^R)$. This is the case as long as the monitoring exerted by risky banks is not so high that behaving risky remains profitable for any level of competition υ. In this case, only an equilibrium with liquidity crises would emerge and ρ would always remain below 1.

As mentioned above, in a framework with credit risk default can occur at date 1 for liquidity reasons when $\rho < 1$ and at date 2 because of credit risk both when $\rho < 1$ and when $\rho = 1$. As competition increases (i.e., υ increases) the probability of default at date 2 increases for safe banks, since their monitoring decreases with υ, while it remains constant for risky banks. Despite this, an equilibrium with $\rho = 1$ may still entail a lower overall probability of default than an equilibrium with $\rho < 1$, although it emerges in more competitive credit markets where monitoring is lower, if the probability of liquidity crises, $(1 - \pi)$, dominates the higher credit risk of safe banks due to increased competition.

To sum up, in a model with both liquidity and credit risk, credit market competition has contrasting effects on banks' monitoring incentives, as well as on the probability of the different types of default −liquidity or credit risk driven−, so that its net effect on banks' overall probability of default depends on which of the various effects dominates. This potential ambiguity on the role of credit market competition in this more general framework depends on the fact that we have modelled bank asset risk as in the charter value literature: banks exert monitoring and their incentives to monitor may be decreasing in the degree of credit market competition as this reduces the return banks accrue from monitoring. The analysis would be different if we modelled bank asset risk as in the competition and stability view à la Boyd and De Nicolò (2005). In this case, entrepreneurs would exert monitoring and their incentives would increase with the intensity of credit market competition. This would imply that both liquidity and credit-driven crises would be decreasing in the level of competition.

7 Empirical predictions

The model provides several testable implications on the interaction between bank competition, aggregate liquidity risk and systemic crises, some of which are consistent with existing empirical evidence.

[16] The relationship between expected profits and credit market competition for the risky banks is as in the baseline model given that their level of monitoring does not depend on υ. For the safe banks there is instead the additional effect of the degree of credit market competition on monitoring, but, given this is negative, their expected profits are also still decreasing in υ.

One of the central results of our analysis is that crises are more likely in banking systems characterized by low degrees of credit market competition and low enough levels of aggregate liquidity risk. Crises are induced by liquidity shocks and, importantly, they are systemic in that they involve a group of banks and not just a single institution. The result remains valid when we consider competition in the deposit markets, asset specificity, different degrees of depositors' risk aversion and the presence of fairly-priced deposit insurance. Moreover, it extends to a more general framework with credit risk, as long as credit market competition has a stronger effect on banks' liquidity holdings than on banks' monitoring incentives.

A first implication of our analysis is the existence of a positive relationship between competition and stability. This is consistent with the evidence in Shaeck, Cihak and Wolfe (2009) that more competitive banking systems are less prone to experience systemic crises and in Beck, Demirgüç-Kunt, and Levine (2006, 2007) that policies fostering competition −such as lower barriers to bank entry and fewer restrictions on bank activities and bank operations in general− reduce banking system fragility.

A second implication concerns the link between aggregate liquidity risk and systemic crises. Our model suggests that crises can emerge only if large liquidity shocks are not too likely. Otherwise, banks do not make enough positive profits in the good state of nature and they all prefer to behave safely. Moreover, the lower the aggregate liquidity risk, the higher is the proportion of risky banks in the economy and the greater is credit availability (or in other words, the lower are banks' liquidity holdings). These results are consistent with the findings in Acharya, Shin and Yorulmazer (2011) that bank liquidity is countercyclical as banks economize on reserves when the fundamentals of the economy −as represented in our model by the parameter π− are positive. Moreover, extending our reasoning to a more dynamic framework, our results are also consistent with the observation, as reported in Castiglionesi, Feriozzi and Lorenzoni (2015), that banks choose to invest a smaller fraction of their portfolio in liquid assets, thus reducing market liquidity, during boom phases such as the one preceding the recent crisis, and with the evidence in Schularick and Taylor (2012) that credit growth is a powerful predictor of financial crises.

A third implication concerns the relationship between aggregate liquidity risk and the effect of competition on financial stability. Our analysis suggests that the threshold level of credit market competition below which an equilibrium with default emerges is negatively affected by the level of aggregate liquidity risk. Said it differently, the beneficial effect of competition on financial stability can be smaller in economies characterized by a lower probability of large liquidity shocks. This suggests the importance of incorporating the determinants of liquidity risk into the analysis of the relationship between competition and stability.

A number of recent empirical studies have moved in this direction. For example, consistently with our prediction, Beck, De Jonghe and Schepens (2013) find a larger impact of competition on banks' fragility in countries with lower systemic fragility and more generous deposit insurance.

8 Concluding remarks

In this paper we have developed a model where banks face liquidity shocks and can invest in liquid reserves and safe loans. The latter can be sold on an interbank market at a price that depends on the demand and supply of liquidity. We have shown that two types of equilibria exist, depending on the degree of credit market competition and the level of aggregate liquidity risk in the economy, as captured by the probability of large liquidity shocks.

When competition is not too intense and the probability of large liquidity shocks is small enough, the equilibrium features default in that a group of banks hold few reserves and go bankrupt when a large liquidity shock realizes. In this case, the interbank market is characterized by cash-in-the-market pricing in both states of nature and higher price volatility. In all other circumstances, the equilibrium features no default. Banks hold enough liquidity in aggregate and the interbank market works smoothly in reshuffling liquidity among banks when needed. As a consequence, all banks stay solvent. The model provides several insights on the relationship between competition, stability and liquidity risk.

The results are robust to a number of extensions and robustness exercises such as competition in the deposit market, asset specificity, different degrees of depositors' risk aversion and the presence of deposit insurance. Moreover, they remain valid in a more general framework where credit risk is introduced, as long as competition has a stronger effect on banks' portfolio choice between reserves and loans than on banks' monitoring incentives.

In our analysis we have assumed that consumers can observe the type of bank they deposit at. This implies that in the equilibrium with default safe and risky banks offer different deposit contracts. Removing this assumption would lead to a pooling in deposit contracts as all depositors would be promised the same deposit terms irrespective of their bank's type. Moreover, the non-observability of banks' type may introduce the possibility of contagion across types of banks as depositors would have fear for their own bank's solvency if they see other banks defaulting. In turn, this may affect banks' initial portfolio choices and thus the probability of systemic crises.

Appendix

Proof of Proposition 1: For a given $\rho \in (0,1]$, the equilibrium is characterized by the vector $\{R^S, L^S, c_1^S, c_2^S, R^R, L^R, c_1^R, c_2^R, P_L, P_H\}$, which corresponds to the solution to the maximization problem of safe and risky banks defined respectively in (3)-(8) and (9)-(14), the market clearing conditions (15) and (16) and the condition that all banks make the same expected profits (i.e., $\Pi^S = \Pi^R$).

We start from the maximization problem of the safe banks. The only binding constraint, besides the budget constraint (4), is depositors' participation constraint as given by (7). Constraint (6) cannot be binding. If it was binding in state L, safe banks would accrue zero profits in the good state $\theta = L$ and negative ones in state $\theta = H$. If it was binding in state H, safe banks would accrue zero profits in that state as it is the case for risky banks.

Using the Lagrangian, a safe bank's problem can be rewritten as

$$\mathcal{L}^S = \Pi^S - \mu^S \left\{[\pi\lambda_L + (1-\pi)\lambda_H]\ln(c_1^S) + [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\ln(c_2^S)\right\},$$

where Π^S is given by (3).

The first order conditions with respect to R^S, c_1^S, c_2^S and μ^S are as follows:

$$\frac{\pi}{P_L} + \frac{1-\pi}{P_H} = 1, \tag{37}$$

$$\left[\frac{\pi\lambda_L}{P_L} + \frac{(1-\pi)\lambda_H}{P_H}\right] r = -\frac{\mu^S}{c_1^S}\left[\pi\lambda_L + (1-\pi)\lambda_H\right] \tag{38}$$

$$c_2^S = -\mu^S \tag{39}$$

$$[\pi\lambda_L + (1-\pi)\lambda_H]\ln(c_1^S) + [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\ln(c_2^S) = 0. \tag{40}$$

Similarly for the risky banks, the only binding constraint is depositors' participation constraint as given by (13). If (12) was binding, risky banks would accrue zero profits also in the good state $\theta = L$.

Using the Lagrangian, a risky bank's problem can be written as follows:

$$\mathcal{L}^R = \Pi^R - \mu^R\left[\pi[\lambda_L\ln(c_1^R) + (1-\lambda_L)\ln(c_2^R)] + (1-\pi)[\ln(R^R + P_H(1-R^R))]\right],$$

where Π^R is given by (9).

The first order conditions with respect to R^R, c_1^R, c_2^R and μ^R are as follows:

$$-\pi r + \frac{\pi r}{P_L} = \frac{\mu^R(1-\pi)(1-P_H)}{R^R + P_H(1-R^R)} \tag{41}$$

$$\frac{r}{P_L} = -\frac{\mu^R}{c_1^R} \tag{42}$$

$$c_2^R = -\mu^R \tag{43}$$

$$\pi[\lambda_L\ln(c_1^R) + (1-\lambda_L)\ln(c_2^R)] + (1-\pi)[\ln(R^R + P_H(1-R^R))] = 0. \tag{44}$$

We solve the system by first using (37) to derive P_H as in (23). Substituting (23) into (38) and (39), and rearranging (40) gives c_1^S and c_2^S as in (18) and (19).

Using (42) and (43), we obtain c_1^R as in (20) and R^R from (41) as follows:

$$R^R = \frac{(1-\pi)c_1^R}{\pi(P_L - 1)} - \frac{P_H}{1-P_H}. \tag{45}$$

Substituting P_H from (23) into (45) and rearranging it gives

$$R^R = \frac{(1-\pi)}{\pi(P_L - 1)}(c_1^R - P_L) \leq 0$$

for any $P_L - 1 > 0$ and $c_1^R - P_L \leq 0$. The former follows from (23), as otherwise $P_H > 1 > P_L$, which cannot hold given the excess of liquidity in state L relative to state H. The latter follows from the fact that the profits of the risky banks must be non-negative in equilibrium. To see this, we rewrite (9) as follows:

$$\Pi^R = \pi\left[r + \left(\frac{1}{P_L} - 1\right)R^R r - (1-\lambda_L)c_2^R - \frac{\lambda_L c_1^R}{P_L} r\right]. \tag{46}$$

As $\left(\frac{1}{P_L} - 1\right)R^R r < 0$ for $P_L > 1$, $\Pi^R \geq 0$ requires

$$r - (1-\lambda_L)c_2^R - \frac{\lambda_L c_1^R}{P_L} r > 0.$$

Rewriting r as $\lambda_L r + (1-\lambda_L)r$ and rearranging the terms gives

$$(1-\lambda_L)(r - c_2^R) + \lambda_L r\left(\frac{P_L - c_1^R}{P_L}\right). \tag{47}$$

This is positive if $P_L - c_1^R > 0$ as this also implies that $r - c_2^R > 0$. Consider $P_L - c_1^R < 0$. Then, from $c_2^R = \frac{r}{P_L} c_1^R$, it is $c_2^R > r$ and (47) is negative. Hence, in equilibrium $P_L - c_1^R > 0$ must hold. It follows that $R^R = 0$ as in the proposition. Then, we use (16) to derive R^S as in (17).

To find c_2^R as in the proposition, we first rearrange $\Pi^S = \Pi^R$ using the expression for the profits of a safe and risky bank, as given by (3) and (9) respectively, as follows:

$$R^S\left[-1 + \frac{\pi}{P_L} + \frac{1-\pi}{P_H}\right]r - \left[\frac{\pi\lambda_L}{P_L} + \frac{(1-\pi)\lambda_H}{P_H}\right]rc_1^S + r(1-\pi) + \pi c_2^R - [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]c_2^S = 0.$$

From (37), $[-1 + \frac{\pi}{P_L} + \frac{1-\pi}{P_H}] = 0$ holds. From (38), it holds

$$\left[\frac{\pi\lambda_L}{P_L} + \frac{(1-\pi)\lambda_H}{P_H}\right]rc_1^S = [\pi\lambda_L + (1-\pi)\lambda_H]c_2^S.$$

Substituting these into the expression above for $\Pi^S - \Pi^R = 0$, we have c_2^R as in (21).

Finally from (44), which corresponds to depositors' participation constraint as given by (13), we have the expression pinning down $\widehat{P}_L$. The condition $P_L = \min\{r, \widehat{P}_L\}$ in the proposition follows from the fact that the price P_L takes different values depending on whether there is an excess of liquidity in the interbank market at date 1 or not. When there is an excess of liquidity in the interbank market, the market clearing (15) holds with strict inequality (as it is the case when $\rho = 1$). In this case, the price P_L raises to its maximum (i.e., $P_L = r$). Otherwise, the equilibrium price P_L is consistent with the market clearing (15) holding with equality and can be computed from (13) after substituting the expression for c_1^R, c_2^R, R^R and P_H. The proposition follows. $\square$

Proof of Proposition 2: The equilibrium without default (i.e., $\rho = 1$) exists when being safe is a dominant strategy. This is the case when no banks find it profitable to deviate by becoming risky. A deviation consists of a bank choosing a risky portfolio allocation (R^R, L^R) and a deposit contract (c_1^R, c_2^R) so that it defaults in state $\theta = H$ when all other banks play safe and, thus $P_L = r$. The condition $P_L = r$ holds because the deviation is computed under the assumption that all other banks behave safe (i.e. $\rho = 1$). In this case, as shown in Proposition 1, there is an excess of liquidity in the state $\theta = L$ so that the price raises to its maximum (i.e., $P_L = r$).

The portfolio allocation (R^R, L^R) and a deposit contract (c_1^R, c_2^R) of a risky bank deviating from the equilibrium without default corresponds to the solution to (41)-(44) evaluated at $P_L = r$ and $\rho = 1$ and are, thus, given by:

$$R^R = 0 \text{ and } c_1^R = c_2^R = c^R = (P_H)^{-\frac{(1-\pi)}{\pi}} = \left(\frac{r(1-\pi)}{r-\pi}\right)^{-\frac{(1-\pi)}{\pi}}. \tag{48}$$

Substituting R^R and $c_1^R = c_2^R = c^R$ into (9), we obtain that the expected profits of a risky bank when $P_L = r$ and $\rho = 1$ are equal to

$$\Pi^R\big|_{P_L=r} = \pi(r - c^R) = \pi\left[r - \left(\frac{r(1-\pi)}{r-\pi}\right)^{-\frac{(1-\pi)}{\pi}}\right]. \tag{49}$$

Adopting such a risky strategy, when all other banks in the economy behave safe (i.e., $\rho = 1$), is profitable if and only if the profits Π^R, as defined in (49), are larger than the profits that the bank can obtain by playing safe.

The portfolio allocation and deposit contract of a safe bank in the equilibrium without default are given by R^S, c_1^S and c_2^S obtained evaluating (17), (18) and (19) taking $\rho = 1$ and $P_L = r$. Substituting R^S, c_1^S and c_2^S into (3), we obtain that the profits of a safe bank in the equilibrium without default are equal to

$$\Pi^S\big|_{P_L=r} = r - c_2^S = r - \left(\frac{\pi\lambda_L + (r-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H}\right)^{\pi\lambda_L + (1-\pi)\lambda_H}. \tag{50}$$

Define $f(\pi, \upsilon) = \Pi^S\big|_{P_L=r} - \Pi^R\big|_{P_L=r}$. When $\upsilon \to \overline{\upsilon}$ and $r \to 1$, $f(\pi, u) \to 0$, since from (19) and (48), $c_2^S = c^R \to 1$. Differentiating $f(\pi, \upsilon)$ with respect to υ gives

$$\frac{\partial f(\pi,\upsilon)}{\partial \upsilon} = \frac{\partial\, \Pi^S\big|_{P_L=r}}{\partial \upsilon} - \frac{\partial\, \Pi^R\big|_{P_L=r}}{\partial \upsilon},$$

where

$$\frac{\partial\, \Pi^S\big|_{P_L=r}}{\partial \upsilon} = -(1 - \lambda_H c_1^S) < 0,$$

and

$$\frac{\partial\, \Pi^R\big|_{P_L=r}}{\partial \upsilon} = -\pi\left(1 - \frac{(1-\pi)c^R}{r(r-\pi)}\right) < 0.$$

The profits $\Pi^S\big|_{P_L=r}$ and $\Pi^R\big|_{P_L=r}$ are monotonically decreasing in υ. Moreover, the second derivatives of Π^S and Π^R with respect to υ are given by

$$\frac{\partial^2\, \Pi^S\big|_{P_L=r}}{\partial \upsilon^2} = \lambda_H^2 c_2^S \frac{[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]}{[\pi\lambda_L + (r-\pi)\lambda_H]} > 0,$$

and

$$\frac{\partial^2 \left.\Pi^R\right|_{P_L=r}}{\partial \upsilon^2} = \frac{\pi(1-\pi)c^R}{r^2\left(r-\pi\right)^2}\left[2r-1\right] > 0,$$

respectively. Both $\frac{\partial^2 \left.\Pi^S\right|_{P_L=r}}{\partial \upsilon^2}$ and $\frac{\partial^2 \left.\Pi^R\right|_{P_L=r}}{\partial \upsilon^2}$ are positive, which implies that both Π^S and Π^R are a convex function of υ.

It is easy to check that when $\upsilon \to \overline{\upsilon}$, $\frac{\partial f(\pi,\upsilon)}{\partial \upsilon} \to -(1-\lambda_H) < 0$. This means that, at $\upsilon \to \overline{\upsilon}$, Π^S crosses Π^R from above, that is $\Pi^S > \Pi^R$ as υ approaches to $\overline{\upsilon}$. Thus, a unique threshold $\upsilon^* \in (0, \overline{\upsilon})$, corresponding to the solution to $f(\pi,\upsilon) = \left.\Pi^S\right|_{P_L=r} - \left.\Pi^R\right|_{P_L=r} = 0$, exists if and only if $f(\pi,\upsilon) \leq 0$ for $\upsilon \to 0$. This is the case if π is sufficiently high. To see this, we first show that $f(\pi,\upsilon)$ is monotonically decreasing in π.

Differentiating $f(\pi,\upsilon)$ with respect to π gives

$$\frac{\partial f(\pi,\upsilon)}{\partial \pi} = -(r-c^R) - \frac{\partial c_2^S}{\partial \pi} + \pi\frac{\partial c^R}{\partial \pi}, \tag{51}$$

where

$$\frac{\partial c_2^S}{\partial \pi} = -(\lambda_H - \lambda_L)c_2^S\left[\ln\left(\frac{\pi\lambda_L + (r-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H}\right) - \frac{(r-1)\lambda_H}{[\pi\lambda_L + (r-\pi)\lambda_H]}\right], \tag{52}$$

and

$$\frac{\partial c^R}{\partial \pi} = -\frac{c^R}{\pi}\left[\frac{1}{\pi}\ln\left(\frac{r-\pi}{r(1-\pi)}\right) - \frac{r-1}{r-\pi}\right]. \tag{53}$$

The sign of $\frac{\partial c_2^S}{\partial \pi}$ is negative if the difference in the square bracket in (52) is positive. This difference is increasing in r and equal to zero when $r \to 1$ .Thus, $\frac{\partial c_2^S}{\partial \pi} < 0$ for any $r > 1$. Similarly, the sign of $\frac{\partial c^R}{\partial \pi}$ is negative if the difference in the square bracket in (53) is positive. This difference is increasing in r and it is equal to 0 for $r \to 1$. This implies that $\frac{\partial c^R}{\partial \pi} < 0$ for any $r > 1$.

The above arguments imply that $\frac{\partial f(\pi,\upsilon)}{\partial \pi}$ in (51) is negative as long as the positive term, $-\frac{\partial c_2^S}{\partial \pi}$, is dominated by the two other negative ones. This is the case when the difference $\lambda_H - \lambda_L$ is not too large. To see this, we first note that when $\lambda_H \to \lambda_L$, $\frac{\partial c_2^S}{\partial \pi} \to 0$ and thus $\frac{\partial f(\pi,\upsilon)}{\partial \pi} < 0$ for any $\upsilon \in (0, \overline{\upsilon})$. In the more general case when $\lambda_H > \lambda_L$, $\frac{\partial f(\pi,\upsilon)}{\partial \pi} < 0$ if $\left|\frac{\partial c_2^S}{\partial \pi}\right| < \pi\left|\frac{\partial c^R}{\partial \pi}\right|$. This sufficient condition is satisfied if, for any $\lambda_L \in (0,1)$, λ_H is lower than a certain threshold.

To find this threshold, we re-write $\left|\frac{\partial c_2^S}{\partial \pi}\right| < \pi\left|\frac{\partial c^R}{\partial \pi}\right|$ as

$$(\lambda_H - \lambda_L)c_2^S\left[\ln\left(\frac{\pi\lambda_L + (r-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H}\right) - \frac{(r-1)\lambda_H}{[\pi\lambda_L + (r-\pi)\lambda_H]}\right] < c^R\left[\frac{1}{\pi}\ln\left(\frac{r-\pi}{r(1-\pi)}\right) - \frac{r-1}{r-\pi}\right]. \tag{54}$$

The RHS is a positive constant independent of both λ_H and λ_L. The LHS changes with λ_H as follows:

$$\begin{aligned}\frac{\partial LHS}{\partial \lambda_H} &= \left[c_2^S + (\lambda_H - \lambda_L)\frac{\partial c_2^S}{\partial \lambda_H}\right]\left[\ln\left(\frac{\pi\lambda_L+(r-\pi)\lambda_H}{\pi\lambda_L+(1-\pi)\lambda_H}\right) - \frac{(r-1)\lambda_H}{[\pi\lambda_L+(r-\pi)\lambda_H]}\right] + \\ &+ (\lambda_H - \lambda_L)\, c_2^S \frac{\pi\lambda_L(r-1)}{\pi\lambda_L+(r-\pi)\lambda_H}\left[\frac{1}{\pi\lambda_L+(1-\pi)\lambda_H} - \frac{1}{\pi\lambda_L+(r-\pi)\lambda_H}\right] > 0\end{aligned},$$

since $\frac{\partial c_2^S}{\partial \lambda_H} = c_2^S\left[\frac{\pi\lambda_L(r-1)}{\pi\lambda_L+(r-\pi)\lambda_H} + \ln\left(\frac{\pi\lambda_L+(r-\pi)\lambda_H}{\pi\lambda_L+(1-\pi)\lambda_H}\right)(1-\pi)\right] > 0$ and $\left[\ln\left(\frac{\pi\lambda_L+(r-\pi)\lambda_H}{\pi\lambda_L+(1-\pi)\lambda_H}\right) - \frac{(r-1)\lambda_H}{[\pi\lambda_L+(r-\pi)\lambda_H]}\right] > 0$ from (52). This implies that the LHS in (54) increases with λ_H for $\lambda_H > \lambda_L$. Thus, there exists a threshold value $\widehat{\lambda}_H > \lambda_L$ such that the LHS and RHS in (54) are equal. It follows that, for any given λ_L, the inequality in (54) holds only for $\lambda_H < \widehat{\lambda}_H$. This, in turn, implies that $\frac{\partial f(\pi,\upsilon)}{\partial \pi} < 0$ for any $\lambda_H - \lambda_L \leq \widehat{\lambda}_H - \lambda_L$.

Given the monotonicity of $f(\pi,\upsilon)$ in π and given $f(\pi,\upsilon) < 0$ when $\pi \to 1$ and $f(\pi,\upsilon) > 0$ when $\pi \to 0$ for any υ, there exists a value $0 < \underline{\pi} < 1$ as represented by the solution to $f(\pi,\upsilon) = \left.\Pi^S\right|_{P_L=r} - \left.\Pi^R\right|_{P_L=r} = 0$

evaluated at $\upsilon \to 0$, such that $f(\pi, \upsilon) < 0$ for $\upsilon \to 0$ for any $\pi > \underline{\pi}$. It follows that it exists a value $\upsilon^* \in (0, \overline{\upsilon})$ above which $\Pi^S|_{P_L=r} > \Pi^R|_{P_L=r}$ and below which $\Pi^S|_{P_L=r} < \Pi^R|_{P_L=r}$.

Based on the analysis above, we have shown that behaving risky when all other banks are safe (i.e., when $\rho = 1$) is only profitable when $\pi \geq \underline{\pi}$ and $\upsilon \leq \upsilon^*$. Being safe is a dominant strategy otherwise. Thus, $\rho = 1$ exists if $\pi < \underline{\pi}$ for any υ and if $\pi \geq \underline{\pi}$ and $\upsilon \geq \upsilon^*$, as stated in the proposition.

To complete the proof, we need to show that the equilibrium with default (i.e., $\rho < 1$) exists when $\pi \geq \underline{\pi}$ and $\upsilon \leq \upsilon^*$. For the equilibrium with default to exist, safe and risky banks must make the same expected profits in equilibrium and the interbank market must clear in all states $\theta = L, H$. This is the case if safe banks are willing to buy loans on the interbank market (i.e., $P_L \leq r$). Hence, to prove the existence of the equilibrium with default when $\pi \geq \underline{\pi}$ and $\upsilon \leq \upsilon^*$, we need then to show that the condition $\Pi^S = \Pi^R$ holds for $P_L \leq r$ in the range $\upsilon \leq \upsilon^*$.

Above, we have shown that $\Pi^S|_{P_L=r} = \Pi^R|_{P_L=r}$ if $\pi \geq \underline{\pi}$ and $\upsilon = \upsilon^*$. This implies that at $\upsilon = \upsilon^*$, the equilibrium with default also exists since $\Pi^S = \Pi^R$ can be sustained with $P_L \leq r$.

Consider now a value of υ in the range $\upsilon < \upsilon^*$(still assuming $\pi \geq \underline{\pi}$). We have shown above that $\Pi^S|_{P_L=r} < \Pi^R|_{P_L=r}$ in this range. From (46), it is easy to see that Π^R increases with P_L for any given υ given that $\lambda_L c_1^R - R^R > 0$ holds. Using (22) and substituting (18), (19) and (23) into (3), we obtain

$$\Pi^S = r - c_2^S = r - \left(\frac{r}{P_L} \frac{\pi\lambda_L + (P_L - \pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H} \right)^{\pi\lambda_L + (1-\pi)\lambda_H}. \tag{55}$$

From (55), it is easy to see that the profits of a safe bank Π^S are decreasing in P_L since

$$\frac{\partial c_2^S}{\partial P_L} = (\pi\lambda_L + (1-\pi)\lambda_H) \left(\frac{r}{P_L} \frac{\pi\lambda_L + (P_L-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H} \right)^{(\pi\lambda_L + (1-\pi)\lambda_H)-1} \left[-\frac{r}{P_L^2} \frac{\pi\lambda_L + (P_L-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H} + \frac{r}{P_L} \frac{\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H} \right] =$$

$$= c_1^S \frac{r}{P_L} \left[\frac{-\pi\lambda_L - (P_L-\pi)\lambda_H + \lambda_H P_L}{P_L} \right] = \frac{c_1^S r (\lambda_H - \lambda_L)\pi}{P_L} > 0.$$

Then, since $\Pi^S|_{P_L=r} < \Pi^R|_{P_L=r}$ and $\frac{\partial \Pi^S}{\partial P_L} < 0$ and $\frac{\partial \Pi^R}{\partial P_L} > 0$, for $\Pi^R = \Pi^S$ to hold as required in the equilibrium with default, it must be $P_L < r$, which is consistent with safe banks providing liquidity in the interbank market at date 1. Thus, the equilibrium with default exists when $\pi \geq \underline{\pi}$ and $\upsilon \leq \upsilon^*$.

Finally, consider a value of υ in the range $\upsilon > \upsilon^*$. In this range $\Pi^S|_{P_L=r} > \Pi^R|_{P_L=r}$ holds. Since $\frac{\partial \Pi^S}{\partial P_L} < 0$ and $\frac{\partial \Pi^R}{\partial P_L} > 0$, for $\Pi^R = \Pi^S$ to hold, as required in the equilibrium with default, it must be $P_L > r$. At this price no safe bank will be willing to buy loans in the interbank market. Thus, the equilibrium with default cannot exist when $\upsilon > \upsilon^*$.

The proportion of safe banks ρ in the equilibrium with default is computed substituting (17) into (15). The proposition follows. □

Proof of Proposition 3: In the proof of Proposition 2, we derived υ^* as the solution to $f(\pi, \upsilon) = \Pi^S|_{P_L=r} = \Pi^R|_{P_L=r} = 0$, where $\Pi^S|_{P_L=r}$ and $\Pi^R|_{P_L=r}$ are given respectively by (50) and (49). The solution depends on π and we can use the implicit function theorem to compute the effect of a change in π on the threshold υ^*. Formally, we have that

$$\frac{d\upsilon^*}{d\pi} = -\frac{\partial f(\pi, \upsilon)/\partial \pi}{\partial f(\pi, \upsilon)/\partial \upsilon}.$$

The numerator is the same as (51), which is negative, as shown in the proof of Proposition 2, when the difference $\lambda_H - \lambda_L$ is not too large (i.e., when $\lambda_H < \widehat{\lambda}_H$ for any $\lambda_L \in (0, \lambda_H)$). So the sign of $\frac{d\upsilon^*}{d\pi}$ is given by the sign of the denominator $\partial f(\pi,\upsilon)/\partial\upsilon$. As shown in the proof of Proposition 2, $\partial f(\pi,\upsilon)/\partial\upsilon < 0$ for $\upsilon \to \bar{\upsilon}$, and $\partial f(\pi,\upsilon)/\partial\upsilon > 0$ as a consequence of the fact that both $\Pi^S\big|_{P_L=r}$ and $\Pi^R\big|_{P_L=r}$ are decreasing in υ and that the profits of a risky bank are steeper than those of a safe bank at $\upsilon = \upsilon^*$. Thus, $\frac{d\upsilon^*}{d\pi} > 0$, as in the proposition. □

Proof of Proposition 4: Denote as $f(\rho,\pi) = 0$ the condition pinning down ρ as in Proposition 2 as given by

$$\rho(\lambda_H - \lambda_L)c_1^S + (1-\rho)(P_H - \lambda_L c_1^R) = 0. \tag{56}$$

At $\upsilon = \upsilon^*$, since $P_L = r$ and $c_1^R = P_H^{-\frac{(1-\pi)}{\pi}} = \frac{r(1-\pi)}{r-\pi}^{-\frac{(1-\pi)}{\pi}}$, $f(\rho,\pi) = 0$ can be rewritten as follows:

$$\rho(\lambda_H - \lambda_L)\left(\frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (r-\pi)\lambda_H}\right)^{\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)} + \frac{r(1-\pi)}{r-\pi} - \lambda_L\left(\frac{r(1-\pi)}{r-\pi}\right)^{-\frac{(1-\pi)}{\pi}} = 0.$$

To compute the effect of π on ρ, we use the implicit function theorem. We obtain

$$\frac{d\rho}{d\pi} = -\frac{\frac{\partial f(\rho,\pi)}{\partial\pi}}{\frac{\partial f(\rho,\pi)}{\partial\rho}}.$$

The denominator $\frac{\partial f(\rho,\pi)}{\partial\rho}$ is equal to

$$\frac{\partial f(\rho,\pi)}{\partial\rho} = (\lambda_H - \lambda_L)c_1^S - (P_H - \lambda_L c_1^R) > 0,$$

since $P_H < \lambda_L c_1^R$ must hold in order for $\rho \in (0,1]$. This implies that the sign of $\frac{d\rho}{d\pi}$ is given by the opposite sign of $\frac{\partial f(\rho,\pi)}{\partial\pi}$ as given by

$$\frac{\partial f(\rho,\pi)}{\partial\pi} = \rho(\lambda_H - \lambda_L)\frac{\partial c_1^S}{\partial\pi} + (1-\rho)\frac{\partial(P_H - \lambda_L c_1^R)}{\partial\pi}. \tag{57}$$

The sign of $\frac{\partial f(\rho,\pi)}{\partial\pi}$ is determined by the sign of $\frac{\partial c_1^S}{\partial\pi}$ and $\frac{\partial(P_H-\lambda_L c_1^R)}{\partial\pi}$. The expression for $\frac{\partial c_1^S}{\partial\pi}$ is given by

$$\frac{\partial c_1^S}{\partial\pi} = (\lambda_H - \lambda_L)c_1^S\left[\ln\left(\frac{\pi\lambda_L+(1-\pi)\lambda_H}{\pi\lambda_L+(r-\pi)\lambda_H}\right) - \frac{\lambda_H(r-1)[\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)]}{[\pi\lambda_L+(r-\pi)\lambda_H][\pi\lambda_L+(1-\pi)\lambda_H]}\right] =$$

$$-(\lambda_H - \lambda_L)c_1^S\left[\ln\left(\frac{\pi\lambda_L+(r-\pi)\lambda_H}{\pi\lambda_L+(1-\pi)\lambda_H}\right) + \frac{\lambda_H(r-1)[\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)]}{[\pi\lambda_L+(r-\pi)\lambda_H][\pi\lambda_L+(1-\pi)\lambda_H]}\right] < 0$$

At the limit, when $\lambda_H \to \lambda_L$, $\frac{\partial c_1^S}{\partial\pi} \to 0$. To find the expression for $\frac{\partial(P_H-\lambda_L c_1^R)}{\partial\pi}$ we first look at the expressions for $\frac{\partial P_H}{\partial\pi}$ and $\frac{\partial c_1^R}{\partial\pi}$. We obtain

$$\frac{\partial P_H}{\partial\pi} = -\frac{r(r-1)}{(r-\pi)^2}, \tag{58}$$

and

$$\frac{\partial c_1^R}{\partial\pi} = -\frac{c_1^R}{\pi}\left[\frac{1}{\pi}\ln\left(\frac{r-\pi}{r(1-\pi)}\right) - \frac{r-1}{r-\pi}\right]. \tag{59}$$

It follows that $\frac{\partial P_H}{\partial\pi} < 0$ for any $r > 1$ and that $\frac{\partial c_1^R}{\partial\pi} < 0$ from the proof of Proposition 2 so that the sign of $\frac{\partial(P_H-\lambda_L c_1^R)}{\partial\pi}$ cannot be easily determined. However, from the proof of Proposition 2, we know that it exists $\rho \in (0,1)$ and that therefore, from the expression for ρ, $\lambda_L c_1^R - P_H > 0$ must hold. This implies that also

$\lambda_L \left|\frac{\partial c_1^R}{\partial \pi}\right| > \left|\frac{\partial P_H}{\partial \pi}\right|$ must hold. Given that, when $\pi \to 1$, $P_H \to 0$ and $c_1^R \to 0$, $\rho \to 0$, it follows that c_1^R must grow faster than P_H with π and thus that $\frac{\partial (P_H - \lambda_L c_1^R)}{\partial \pi} > 0$.

Given $\frac{\partial c_1^S}{\partial \pi} \to 0$ for $\lambda_H \to \lambda_L$ and $\frac{\partial (P_H - \lambda_L c_1^R)}{\partial \pi} > 0$, it follows that $\frac{\partial f(\rho,\pi)}{\partial \pi} > 0$ for $\lambda_H \to \lambda_L$ so that $\frac{d\rho}{d\pi} < 0$ as stated in the proposition. □

Proof of Lemma 1: The lemma follows immediately from the difference between (24) and (25) at $\upsilon = \upsilon^*$. □

Proof of Proposition 5: We proceed in steps. First, we characterize the threshold $\overline{\pi}$ in the proposition and compare it with the threshold $\underline{\pi}$ defined in Proposition 2. Then, we analyze how the difference $P_H - \lambda_H c_1^S\big|_{\upsilon=\upsilon^*}$ changes with π for any $\upsilon^* > 0$.

The cutoff value $\overline{\pi}$ corresponds to the solution to $P_H - \lambda_H c_1^S\big|_{P_L=r=V} = 0$. In order to characterize it, we first evaluate (23) and (18) given $P_L = r = V$, which gives:

$$P_H - \lambda_H c_1^S\big|_{P_L=r=V} = \frac{V(1-\pi)}{(V-\pi)} - \lambda_H \left(\frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (V-\pi)\lambda_H}\right)^{\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)}. \tag{60}$$

When $\pi = 0$, $P_H - \lambda_H c_1^S\big|_{P_L=r=V} = 1 - \lambda_H \left(\frac{1}{V}\right)^{1-\lambda_H} > 0$ since $V > 1$ and $0 < \lambda_H < 1$. When $\pi = 1$, $P_H - \lambda_H c_1^S\big|_{P_L=r=V} = 0 - \lambda_H \left(\frac{\lambda_L}{\lambda_L + (V-1)\lambda_H}\right)^{(1-\lambda_L)} < 0$. Differentiating $P_H - \lambda_H c_1^S\big|_{P_L=r=V}$, we obtain

$$\begin{aligned}\left.\frac{d(P_H - \lambda_H c_1^S)}{d\pi}\right|_{P_L=r=V} &= -\frac{V(V-1)}{(V-\pi)^2} - \lambda_H(\lambda_H - \lambda_L)c_1^S \left[\ln\left(\frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (V-\pi)\lambda_H}\right) + \right.\\ &\left. -\lambda_H \frac{[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)](V-1)}{[\pi\lambda_L + (1-\pi)\lambda_H][\pi\lambda_L + (V-\pi)\lambda_H]}\right].\end{aligned}$$

For $\lambda_H \to \lambda_L$, it holds that $\left.\frac{d(P_H - \lambda_H c_1^S)}{d\pi}\right|_{P_L=r=V} < 0$. Thus, there exists a unique solution $0 < \overline{\pi} < 1$ for the equation $P_H - \lambda_H c_1^S\big|_{P_L=r=V} = 0$ such that $P_H - \lambda_H c_1^S\big|_{P_L=r=V} > 0$ for $\pi < \overline{\pi}$ and $P_H - \lambda_H c_1^S\big|_{P_L=r=V} < 0$ for $\pi > \overline{\pi}$.

The condition $P_L = r = V$ holds true at $\upsilon = \upsilon^* = 0$, since $P_L = r$ at $\upsilon = \upsilon^*$ and $r = V$ when $\upsilon = 0$ from (1). From the proof of Proposition 2, $\underline{\pi}$ is defined as the probability of good state for which $\Pi^S\big|_{P_L=r} = \Pi^R\big|_{P_L=r} = 0$ when $\upsilon = 0$. In other words, when $\pi = \underline{\pi}$, $\upsilon^* = 0$. Formally, $\underline{\pi}$ is the solution to

$$\Pi^S - \Pi^R\big|_{\upsilon=0} = V - \left(\frac{\pi\lambda_L + (V-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H}\right)^{\pi\lambda_L + (1-\pi)\lambda_H} - \pi\left(V - \left(\frac{V(1-\pi)}{V-\pi}\right)^{-\frac{1-\pi}{\pi}}\right) = 0. \tag{61}$$

Comparing (60) with (61), two cases are possible: either $\underline{\pi} > \overline{\pi}$ or $\underline{\pi} < \overline{\pi}$. In order to determine whether $P_H - \lambda_H c_1^S\big|_{\upsilon=\upsilon^*} \lesseqgtr 0$ for any $\upsilon^* > 0$, we need first to determine the sign of $\left.\frac{d(P_H - \lambda_H c_1^S)}{d\pi}\right|_{\upsilon=\upsilon^*>0}$ and then consider the two cases $\underline{\pi} > \overline{\pi}$ and $\underline{\pi} < \overline{\pi}$, in turn.

Differentiating then $P_H - \lambda_H c_1^S$ with respect to π at any $\upsilon^* > 0$ after substituting (23) and (18) gives:

$$\begin{aligned}\left.\frac{d(P_H - \lambda_H c_1^S)}{d\pi}\right|_{\upsilon=\upsilon^*>0} &= -\frac{r(r-1) - \pi(1-\pi)\frac{d\upsilon^*}{d\pi}}{(r-\pi)^2} - \lambda_H(\lambda_H - \lambda_L)c_1^S \left[\ln\left(\frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (r-\pi)\lambda_H}\right) + \right.\\ &\left. -\lambda_H \frac{[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)](r-1)}{[\pi\lambda_L + (1-\pi)\lambda_H][\pi\lambda_L + (r-\pi)\lambda_H]}\right] + \\ &-\lambda_H^2 c_1^S \frac{[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]}{[\pi\lambda_L + (r-\pi)\lambda_H]}\frac{d\upsilon^*}{d\pi},\end{aligned}$$

where $\frac{d\upsilon^*}{d\pi} > 0$ from Proposition 3. For $\lambda_H \to \lambda_L$, the expression above simplifies to

$$\left.\frac{d(P_H - \lambda_H c_1^S)}{d\pi}\right|_{\upsilon=\upsilon^*>0} = -\frac{r(r-1)}{(r-\pi)^2} - \left(-\frac{\pi(1-\pi)}{(r-\pi)^2} + \lambda_H(1-\lambda_H)\frac{c_1^S}{r}\right)\frac{d\upsilon^*}{d\pi}.$$

Thus, assuming that $-\frac{\pi(1-\pi)}{(r-\pi)^2}\frac{d\upsilon^*}{d\pi}$ is sufficiently small, the condition $\frac{d(P_H-\lambda_H c_1^S)}{d\pi} < 0$ holds.

Having determined the sign of $\left.\frac{d(P_H-\lambda_H c_1^S)}{d\pi}\right|_{\upsilon=\upsilon^*>0}$, we consider the two possible cases $\underline{\pi} > \overline{\pi}$ and $\underline{\pi} < \overline{\pi}$.

When $\underline{\pi} > \overline{\pi}$, $P_H - \lambda_H c_1^S\big|_{\upsilon=\upsilon^*>0} < 0$ for any $\pi \in (\underline{\pi}, 1)$ as a consequence of $\left.\frac{d(P_H-\lambda_H c_1^S)}{d\pi}\right|_{\upsilon=\upsilon^*>0}$ and $P_H - \lambda_H c_1^S\big|_{P_L=r=V} < 0$ for $\pi > \overline{\pi}$. It follows that total credit is always larger in the equilibrium with default than in the one without default.

When $\underline{\pi} < \overline{\pi}$, $P_H - \lambda_H c_1^S\big|_{\upsilon=\upsilon^*=0} > 0$ as a consequence of the fact that $P_H - \lambda_H c_1^S\big|_{P_L=r=V} > 0$ for $\pi < \overline{\pi}$. As π increases, υ^* increases, as shown in Proposition 3, while $(P_H - \lambda_H c_1^S)\big|_{\upsilon=\upsilon^*>0}$ decreases. For $\pi = 1$, $(P_H - \lambda_H c_1^S)\big|_{\upsilon=\upsilon^*>0} = -\lambda_H\left(\frac{\lambda_L}{\lambda_L+(r-1)\lambda_H}\right)^{1-\lambda_L} < 0$. Then, there exists a value $\widehat{\pi} \in (\underline{\pi}, 1)$ such that $(P_H - \lambda_H c_1^S)\big|_{\upsilon=\upsilon^*>0} \geq 0$ for any $\pi \leq \widehat{\pi}$ and $(P_H - \lambda_H c_1^S)\big|_{\upsilon=\upsilon^*>0} < 0$ otherwise. The proposition follows.
□

Proof of Proposition 6: We divide the proof in two parts. We first show that there exist two thresholds $\underline{\pi}_\alpha$ and υ_α^* as in Proposition 2 when consumers' opportunity cost is $\alpha > 1$. Then, we study how these thresholds compare with $\underline{\pi}$ and υ^* characterized in Proposition 2.

As in the proof of Proposition 2, we compute the threshold υ_α^* as the solution to $\Pi_\alpha^S = \Pi_\alpha^R$, where Π_α^S and Π_α^R are the profits of a safe bank and a risky bank, respectively and they are both computed under the assumption that $\rho = 1$ and $P_L = r$. The idea is exactly as in the proof of Proposition 2, the threshold υ_α^* corresponds to the level of competition at which being safe is no longer a dominant strategy. In other words, at $\upsilon = \upsilon_\alpha^*$, it becomes profitable for a bank to deviate from the equilibrium without default by behaving risky.

The problem of a safe bank is as in (3)-(8) with the only difference that consumers' participation constraint is now given by (26). Following the same steps as in the proof of Proposition 1, we have $R_\alpha^S = \lambda_H c_{1a}^S$ and $c_{1\alpha}^S$ and $c_{2\alpha}^S$ are equal to

$$c_{1\alpha}^S = \alpha\left(\frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (r-\pi)\lambda_H}\right)^{\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)}, \tag{62}$$

$$c_{2\alpha}^S = \alpha\left(\frac{\pi\lambda_L + (r-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H}\right)^{\pi\lambda_L+(1-\pi)\lambda_H}, \tag{63}$$

given that $\rho = 1$ and $P_L = r$. The problem of a risky bank is also as in the baseline model with the only difference that consumers' participation constraint is now given by

$$\pi\left[\lambda_L u(c_{1\alpha}^R) + (1-\lambda_L)u(c_{2\alpha}^R)\right] + (1-\pi)u(R_\alpha^R + P_H L_a^R) \geq u(\alpha).$$

The risky bank then still chooses $R_\alpha^R = 0$ and, given $P_L = r$ promises depositors in state L

$$c_{1\alpha}^R = c_{2\alpha}^R = c_\alpha^R = \alpha^{\frac{1}{\pi}}(P_H)^{-\frac{1-\pi}{\pi}} = \alpha^{\frac{1}{\pi}}\left(\frac{r(1-\pi)}{r-\pi}\right)^{-\frac{1-\pi}{\pi}}. \tag{64}$$

Banks expected profits $\Pi_\alpha^S\big|_{P_L=r}$ and $\Pi_\alpha^R\big|_{P_L=r}$ are then still as in (50) and (49), respectively, but with $c_{2\alpha}^S$ and c_α^R as in (63) and (64).

We can then compute υ^*_α as the solution to

$$f(\pi,\upsilon,\alpha) = \Pi^S_\alpha\big|_{P_L=r} - \Pi^R_\alpha\big|_{P_L=r} = 0,$$

from which we have

$$f(\pi,\upsilon,\alpha) = r(1-\pi) - \alpha\left(\frac{\pi\lambda_L + (r-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H}\right)^{\pi\lambda_L+(1-\pi)\lambda_H} + \pi\alpha^{\frac{1}{\pi}}\left(\frac{r(1-\pi)}{r-\pi}\right)^{-\frac{1-\pi}{\pi}} = 0.$$

When $\upsilon \to \bar{\upsilon}$ and $r \to 1$, $f(\pi,\upsilon,\alpha) > 0$ since from (62) and (64), $c^S_{1\alpha} = c^S_{2\alpha} = 1$ and $c^R_\alpha \to \overline{\alpha}^{\frac{1}{\pi}}$. Both $\Pi^S_\alpha\big|_{P_L=r}$ and $\Pi^R_\alpha\big|_{P_L=r}$ are monotonically decreasing in υ. The expressions for the derivatives are similar to those in the proof of Proposition 2 and equal to

$$\frac{\partial\, \Pi^S_\alpha\big|_{P_L=r}}{\partial\upsilon} = -\left[1 - \lambda_H c^S_{1\alpha} + \frac{\partial\alpha}{\partial\upsilon}\left(\frac{\pi\lambda_L + (r-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H}\right)^{\pi\lambda_L+(1-\pi)\lambda_H}\right] < 0, \tag{65}$$

and to

$$\frac{\partial\, \Pi^R_\alpha\big|_{P_L=r}}{\partial\upsilon} = -\pi\left[1 - c^R_\alpha\frac{(1-\pi)}{r(r-\pi)} + \frac{\partial\alpha}{\partial\upsilon}\frac{c^R_\alpha}{\alpha\pi}\right] < 0, \tag{66}$$

since $\frac{\partial\alpha}{\partial\upsilon} > 0$.

For $\upsilon \to \bar{\upsilon}$, $f(\pi,\upsilon,\alpha) > 0$, which means that $\Pi^S_\alpha\big|_{P_L=r}$ lies above $\Pi^R_\alpha\big|_{P_L=r}$. Thus, there exists a unique threshold $\upsilon^*_\alpha \in (0,\bar{\upsilon})$ such that $f(\pi,\upsilon,\alpha) = \Pi^S_\alpha\big|_{P_L=r} - \Pi^R_\alpha\big|_{P_L=r} = 0$ if and only if $f(\pi,\upsilon,\alpha) \leq 0$ for $\upsilon \to 0$. This is the case if π is sufficiently high so that $\Pi^S_\alpha\big|_{P_L=r} < \Pi^R_\alpha\big|_{P_L=r}$ at $\upsilon = 0$. Given that $\alpha \to 1$ when $\upsilon \to 0$, $\underline{\pi}_\alpha \equiv \underline{\pi}$ as defined in proof of Proposition 2 and the result is like in the proposition.

Similarly to the baseline model, the uniqueness of the threshold υ^*_α is guaranteed when the two functions $\Pi^S_\alpha\big|_{P_L=r}$ and $\Pi^R_\alpha\big|_{P_L=r}$ are both convex. In this extended framework where the level of credit market competition υ affects the degree of competition in the deposit market α, this depends on the shape of the function $\alpha(\upsilon)$. As long as $\frac{\partial^2\alpha(\upsilon)}{\partial\upsilon^2} \leq 0$, $\Pi^S_\alpha\big|_{P_L=r}$ and $\Pi^R_\alpha\big|_{P_L=r}$ only cross once in the range $(0,\overline{\upsilon})$ and the uniqueness of the threshold υ^*_α is guaranteed.[17]

We now move to the comparison between υ^*_α and υ^* as defined in Proposition 2. In order for υ^*_α to be larger than υ^*, it must be that $f(\pi,\upsilon)$ is steeper than $f(\pi,\upsilon,\alpha)$ since $f(\pi,\upsilon) = f(\pi,\upsilon,\alpha) < 0$ when $\upsilon = 0$ and they are both increasing in υ $\Big($i.e., $\Pi^R\big|_{P_L=r}$ and $\Pi^R_\alpha\big|_{P_L=r}$ are steeper than $\Pi^S\big|_{P_L=r}$ and $\Pi^S_\alpha\big|_{P_L=r}$, respectively$\Big)$ when they cross zero. This is the case when

$$\left|\frac{\partial f(\pi,\upsilon)}{\partial\upsilon}\right| > \left|\frac{\partial f(\pi,\upsilon,\alpha)}{\partial\upsilon}\right|. \tag{67}$$

Using (65) and (66), we have:

$$\frac{\partial f(\pi,\upsilon,\alpha)}{\partial\upsilon} = -\left[1 - \lambda_H c^S_{1\alpha} + \frac{\partial\alpha}{\partial\upsilon}\left(\frac{\pi\lambda_L + (r-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H}\right)^{\pi\lambda_L+(1-\pi)\lambda_H}\right] + \pi\left[1 - c^R_\alpha\frac{(1-\pi)}{r(r-\pi)} + \frac{\partial\alpha}{\partial\upsilon}\frac{c^R_\alpha}{\alpha\pi}\right],$$

while, from Proposition 2, we have that

$$\frac{\partial f(\pi,\upsilon)}{\partial\upsilon} = -(1-\lambda_H c^S_1) + \pi\left(1 - \frac{(1-\pi)c^R}{r(r-\pi)}\right).$$

[17] Calculations available from the authors provide formal conditions on the function $\alpha(\upsilon)$ guaranteeing the existence of a unique threshold υ^*_α.

Rearranging the terms in the two expressions above, it follows that (67) holds if

$$\frac{\partial \alpha}{\partial \upsilon} < \frac{\alpha \left[\lambda_H (c^S_{1\alpha} - c^S_1) - \frac{\pi(1-\pi)}{r(r-\pi)} \left(c^R_\alpha - c^R \right) \right]}{c^S_{2\alpha} - c^R_\alpha}.$$

Thus, the proposition follows.□

Proof of Proposition 7: As for the previous proposition, we divide the proof in two parts. We first show that there exist two thresholds $\underline{\pi}_\zeta$ and υ^*_ζ as in Proposition 2 when we account for asset specificity and assume that banks accrue only a fraction $\zeta(\upsilon) < 1$ of the return r when buy an asset in the interbank market. Then, we study how these thresholds compare with $\underline{\pi}$ and υ^* characterized in Proposition 2.

As in the proof of Proposition 2, the thresholds υ^*_ζ and $\underline{\pi}_\zeta$ are computed from the condition equating the profits of a safe banks Π^S_ζ to those of a risky bank Π^R_ζ both evaluated when $\rho = 1$ and thus, when P_L raises up to its maximum. The idea is again to characterize υ^*_ζ as the level of competition at which being safe is no longer a dominant strategy and being risky represents a profitable deviation from the equilibrium without default.

Extending the model to account for asset specificity affects the determination of the equilibrium prices P_L, P_H. The upper bound for the price P_L is now given by

$$P_{L\zeta} = r\zeta(\upsilon), \tag{68}$$

as a loan bought in the interbank market will only yield the return $r\zeta(\upsilon)$ to the buyer. The price $P_{H\zeta}$ still needs to guarantee that banks invest both in reserves and loans at date 0. Thus, it satisfies

$$r = \pi \frac{r\zeta(\upsilon)}{P_{L\zeta}} + (1-\pi)\frac{r\zeta(\upsilon)}{P_{H\zeta}},$$

and, thus equal to

$$P_{H\zeta} = \frac{\zeta(\upsilon) P_{L\zeta}(1-\pi)}{P_{L\zeta} - \pi\zeta(\upsilon)} = \frac{r\zeta(\upsilon)(1-\pi)}{r-\pi}. \tag{69}$$

As in the baseline model, no banks default in equilibrium as long as there is enough liquidity in aggregate to repay depositors the promised consumptions. This implies that, given $\lambda_H > \lambda_L$, there is an excess of liquidity in the state $\theta = L$, which drives the price P_L to its maximum as defined in (68). However, unlike the baseline model, banks' initial portfolio allocation is no longer irrelevant. In the baseline model, the expected (per unit) gain accrued to a bank having an excess of reserves at date 1− as given by $\pi \frac{r}{P_L} + (1-\pi)\frac{r}{P_H} = r-$ was exactly equal to the (per unit) loss accrued to a bank having a shortage of liquidity as given by $\pi \frac{r}{P_L} + (1-\pi)\frac{r}{P_H} = r$. In this extended version of the model, these gains and losses are no longer the same. Having a shortage of liquidity entails a loss for a bank selling the loans equal to $\pi \frac{r}{P_{L\zeta}} + (1-\pi)\frac{r}{P_{H\zeta}} > r$, while buying a loan for a bank with excess reserves and buying loans in the interbank market entail a gain $\pi \frac{r\zeta(\upsilon)}{P_{L\zeta}} + (1-\pi)\frac{r\zeta(\upsilon)}{P_{H\zeta}} = r$. For this reason, no safe bank has an incentive to access the interbank market to sell loans to get additional liquidity. Hence, all safe banks invest $R^S_\zeta = \lambda_H c^S_{1\zeta}$, which implies that there is no trade on the interbank market when $\rho = 1$.

The promised repayments to consumers $(c^S_{1\zeta}, c^S_{2\zeta})$ are the solution to the following maximization problem

$$\underset{c^S_{1\zeta}, c^S_{2\zeta}}{Max}\, \Pi^S_\zeta = r(1 - \lambda_H c^S_{1\zeta}) + \pi(\lambda_H - \lambda_L)c^S_{1\zeta} - \left[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)\right] c^S_{2\zeta} \tag{70}$$

subject to

$$c^S_{2\zeta} \geq c^S_{1\zeta}, \tag{71}$$

$$\begin{aligned} E[u(c^S_{1\zeta}, c^S_{2\zeta}, \lambda_\theta)] &= \pi\left[\lambda_L u(c^S_{1\zeta}) + (1-\lambda_L)u(c^S_{2\zeta})\right] + \\ &+(1-\pi)\left[\lambda_H u(c^S_{1\zeta}) + (1-\lambda_H)u(c^S_{2\zeta})\right] \geq u(1), \end{aligned} \tag{72}$$

$$c^S_{1\zeta}, c^S_{2\zeta} \geq 0 \tag{73}$$

As in the baseline model, (72) is binding. Using the Lagrangian, the problem can be rewritten as follows:

$$\mathcal{L}^S_\zeta = \Pi^S_\zeta - \mu^S_\zeta\left[[\pi\lambda_L + (1-\pi)\lambda_H]\ln(c^S_{1\zeta}) + [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\ln(c^S_{2\zeta})\right],$$

where Π^S_ζ is given by (70).

The first order conditions with respect to $c^S_{1\zeta}$, $c^S_{2\zeta}$ and μ^S_ζ are as follows:

$$-r\lambda_H + \pi(\lambda_H - \lambda_L) = -\frac{\mu^S_\zeta}{c^S_{1\zeta}}[\pi\lambda_L + (1-\pi)\lambda_H] \tag{74}$$

$$c^S_{2\zeta} = -\mu^S_\zeta \tag{75}$$

$$[\pi\lambda_L + (1-\pi)\lambda_H]\ln(c^S_{1\zeta}) + [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]\ln(c^S_{2\zeta}) = 0. \tag{76}$$

Substituting (75) into (74) and then into (76), it gives

$$c^S_{1\zeta} = \left(\frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (r-\pi)\lambda_H}\right)^{\pi(1-\lambda_L)+(1-\pi)(1-\lambda_H)}, \tag{77}$$

and

$$c^S_{2\zeta} = \left(\frac{\pi\lambda_L + (r-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H}\right)^{\pi\lambda_L+(1-\pi)\lambda_H}. \tag{78}$$

Substituting the expression for $c^S_{1\zeta}$ and $c^S_{2\zeta}$ into (70), we obtain

$$\Pi^S_\zeta = r - c^S_2 = r - \left(\frac{\pi\lambda_L + (r-\pi)\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H}\right)^{\pi\lambda_L+(1-\pi)\lambda_H}.$$

The problem of a risky bank is the same as in the baseline model and still given by (9)-(14) with P_L and P_H as in (68) and (69). Using the first order conditions with respect to R^R_ζ, $c^R_{1\zeta}$, $c^R_{2\zeta}$ and μ^R_ζ, which are as in (41), (42), (43) and (44), we obtain:

$$R^R_\zeta = 0$$

$$c^R_{1\zeta} = c^R_{2\zeta} = c^R_\zeta = (P_{H\zeta})^{-\frac{(1-\pi)}{\pi}}\zeta^{-\lambda_L} = \left(\frac{r(1-\pi)}{r-\pi}\right)^{-\frac{(1-\pi)}{\pi}}\zeta^{-\lambda_L-\frac{(1-\pi)}{\pi}}, \tag{79}$$

and the profits of a risky bank are equal to

$$\Pi^R_\zeta = \pi(r - c^R_\zeta). \tag{80}$$

To characterize $\underline{\pi}_\zeta$ and υ_ζ^*, we follow the same steps as in the proof of Proposition 2. Define $f_\zeta(\pi, \upsilon) = \Pi_\zeta^S - \Pi_\zeta^R$. When $\upsilon \to \bar{\upsilon}$, $r \to 1$ and $\zeta \to 1$, thus $f_\zeta(\pi, \upsilon) \to 0$ since from (77), (78) and (79), $c_1^S = c_2^S = c_\zeta^R \to 1$. Differentiating $f(\pi, \upsilon)$ with respect to υ gives

$$\frac{\partial f_\zeta(\pi, \upsilon)}{\partial \upsilon} = \frac{\partial \Pi_\zeta^S}{\partial \upsilon} - \frac{\partial \Pi_\zeta^R}{\partial \upsilon},$$

where

$$\frac{\partial \Pi_\zeta^S}{\partial \upsilon} = -(1 - \lambda_H c_{1\zeta}^S) < 0,$$

and

$$\frac{\partial \Pi_\zeta^R}{\partial \upsilon} - \pi \left[1 - c_\zeta^R \frac{(1-\pi)}{r(r-\pi)} - \frac{c_\zeta^R}{\zeta}\left(\lambda_L + \frac{(1-\pi)}{\pi}\right)\frac{\partial \zeta}{\partial \upsilon}\right] < 0,$$

if $\frac{\partial \zeta}{\partial \upsilon}$ is not too large. Then, the profits Π_ζ^S and Π_ζ^R are monotonically decreasing in υ.

For $\upsilon \to \bar{\upsilon}$, $\frac{\partial f_\zeta(\pi,\upsilon)}{\partial \upsilon} \to -(1-\lambda_H) - \pi\left(\lambda_L + \frac{(1-\pi)}{\pi}\right)\frac{\partial \zeta}{\partial \upsilon} < 0$. Thus, there exists a unique threshold $\upsilon_\zeta^* \in (0, \bar{\upsilon})$ such that $f_\zeta(\pi, \upsilon) = \Pi_\zeta^S - \Pi_\zeta^R = 0$ at $\upsilon = \upsilon_\zeta^*$ if and only if $f_\zeta(\pi, \upsilon) \leq 0$ for $\upsilon \to 0$. This holds true when π is sufficiently high so that $\Pi_\zeta^S < \Pi_\zeta^R$ at $\upsilon = 0$. Denote as $\underline{\pi}_\zeta$, the level of π solving $\Pi_\zeta^S = \Pi_\zeta^R$ at $\upsilon = 0$. Then, it follows that the threshold $\upsilon_\zeta^* \in (0, \bar{\upsilon})$ exists if $\pi > \underline{\pi}_\zeta$.

Since Π_ζ^S are identical to the profits Π^S in the baseline model as defined in (50), while $\Pi_\zeta^R < \Pi^R$ for any υ given that $\zeta(\upsilon) < 1$, it follows that $\underline{\pi}_\zeta > \underline{\pi}$ as in the proposition.

To see how having $\zeta(\upsilon) < 1$ affects υ_ζ^*, we then simply compare the risky bank's expected profits in the case there is no effect of the degree of competition in the credit market on the interbank market (i.e., when $\zeta(\upsilon) = 1$ as in the baseline model) as given by (49) with those corresponding to the case when $\zeta(\upsilon) < 1$ as in (80). It follows that $\upsilon^* > \upsilon_\zeta^*$ since $c_\zeta^R > c^R$ as defined in (48) for any $\zeta(\upsilon) < 1$. The proposition follows. □

Proof of Proposition 8: The problem of a risky bank deviating from the equilibrium without default is again the same as the one of a risky bank in (9)-(14) with $P_L = r$ and P_H as in (23), while depositors' participation constraint is now given by

$$\pi\left[\lambda_L \frac{(c_1^R)^{1-\sigma}}{1-\sigma} + (1-\lambda_L)\frac{(c_2^R)^{1-\sigma}}{1-\sigma}\right] + (1-\pi)\left[\frac{R^R + P_H L^R}{1-\sigma}\right]^{1-\sigma} = 1.$$

Using the Lagrangian, the first order conditions with respect to R^R, c_1^R, c_2^R and μ^R are as follows:

$$-\pi r + \frac{\pi r}{P_L} = \mu^R(1-P_H)[R^R + P_H(1-R^R)]^{-\sigma} \tag{81}$$

$$(c_1^R)^\sigma \frac{r}{P_L} = -\mu^R \tag{82}$$

$$(c_2^R)^{-\sigma} = -\mu^R \tag{83}$$

$$\pi[\lambda_L (c_1^R)^{1-\sigma} + (1-\lambda_L)(c_2^R)^{1-\sigma}] + (1-\pi)(R^R + P_H L^R)^{1-\sigma} = 1. \tag{84}$$

Since $r = P_L$, from (82) and (83) it follows that

$$c_2^R = c_1^R = c^R.$$

Substituting it into (81) and using the expression for P_H from (23), we obtain

$$R^R = \frac{(r-\pi)}{\pi(r-1)}\left[(c^R)^\sigma - r\right]. \tag{85}$$

The sign of (85) depends on the sign of the square bracket. Given that $[(c^R)^\sigma - r]$ is monotonic in σ, it follows that there exists a threshold value of σ, defined as $\overline{\sigma}$ and given by

$$\overline{\sigma} = \frac{Log(r)}{Log(c^R)} > 1,$$

such that $R^R > 0$ for $\sigma > \overline{\sigma}$ and $R^R = 0$ for $\sigma \leq \overline{\sigma}$. The proposition follows. □

Proof of Proposition 9: The proof consists again of two parts. We first show that there exist two thresholds $\underline{\pi}_{di}$ and υ^*_{di} as in Proposition 2 when we introduce deposit insurance. Then, we study how these two thresholds υ^*_{di} and $\underline{\pi}_{di}$ compare with the thresholds υ^* and $\underline{\pi}$ characterized in the baseline model. As in the proof of Proposition 2, the thresholds υ^*_{di} and $\underline{\pi}_{di}$ are computed from the condition equating the profits of a safe banks Π^S_{di} to those of a risky bank Π^R_{di} both evaluated when $\rho = 1$ and thus, $P_L = r$. Again, the idea is to characterize υ^*_{di} as the level of competition at which being safe is no longer a dominant strategy and being risky represents a profitable deviation from the equilibrium without default.

Since they do not default, safe banks are not affected by the introduction of deposit insurance. This implies that $c^S_{1di} = c^S_1$ and $c^S_{2di} = c^S_2$ with c^S_1 and c^S_2 from (18) and (19) and the profits $\Pi^S_{di} = \Pi^S$, as given in (50). All variables are evaluated taking $\rho = 1$ and $P_L = r$.

The expected profits Π^R_{di} of a bank deviating from the equilibrium without default and defaulting in state H with deposit insurance are given by

$$\Pi^R_{di} = \pi\left[rL^R_{di} - r\left(\frac{\lambda_L c^R_{1di} - R^R_{di}}{P_L}\right) - (1-\lambda_L)c^R_{2di} - C\right] \tag{86}$$

where C is as in (27), while consumers' participation constraint is equal to

$$\pi\left[\lambda_L u(c^R_{1di}) + (1-\lambda_L)u(c^R_{2di})\right] + (1-\pi)u(c^I) = u(1).$$

The first order conditions are as in the baseline model besides (44), which becomes

$$\pi[\lambda_L \ln(c^R_{1di}) + (1-\lambda_L)\ln(c^R_{2di})] + (1-\pi)u(c^I) = 0.$$

As a consequence, the solutions to the maximization problem are the same as in the benchmark model without deposit insurance with the only difference that consumers promised repayments in state L are now

$$c^R_{1di} = c^R_{2di} = c^R_{di} = (c^I)^{-\frac{(1-\pi)}{\pi}}$$

instead of (48) so that (86) simplifies to

$$\Pi^R_{di} = \pi(r - c^R_{di} - C) = \pi\left[r - c^R_{di} - (1-\pi)(c^I - P_H)\right]. \tag{87}$$

To prove that υ^*_{di} exists, we follow the same steps as in the proof of Proposition 2. Define $f_{di}(\pi,\upsilon) = \Pi^S_{di} - \Pi^R_{di}$. When $\upsilon \to \overline{\upsilon}$ and $r \to 1$, $f_{di}(\pi,\upsilon) \to 0$ as $c^S_1 = c^S_2 = P_H \to 1$ and $c^I \to 1$, as $c^I \in \left(P_H, c^R_{di}\right]$. Differentiating $f_{di}(\pi,\upsilon)$ with respect to υ gives

$$\frac{\partial f_{di}(\pi,\upsilon)}{\partial\upsilon} = \frac{\partial \Pi^S_{di}}{\partial\upsilon} - \frac{\partial \Pi^R_{di}}{\partial\upsilon}$$

where

$$\frac{\partial \Pi^S_{di}}{\partial \upsilon} = -(1 - \lambda_H c^S_{1di}) < 0$$

and

$$\frac{\partial \Pi^R_{di}}{\partial \upsilon} = -\pi \left[\frac{(r-\pi)^2 - (1-\pi)^2 \pi}{(r-\pi)^2} \right] < 0.$$

The profits Π^S_{di} and Π^R_{di} are monotonically decreasing in υ. For $\upsilon \to \bar{\upsilon}$, $\frac{\partial f_{di}(\pi,\upsilon)}{\partial \upsilon} \to -(1-\lambda_H) < 0$. Thus, there exists a unique threshold $\upsilon^*_{di} \in (0, \bar{\upsilon})$ such that $f_{di}(\pi, \upsilon) = \Pi^S_{di} - \Pi^R_{di} = 0$ if and only if $f_{di}(\pi, \upsilon) \leq 0$ for $\upsilon \to 0$. This is the case when π is sufficiently high (i.e., $\pi > \underline{\pi}_{di}$). Define $\underline{\pi}_{di}$ as the solution to $\Pi^S_{di} = \Pi^R_{di}$ at $\upsilon = 0$. Then, it follows that the threshold $\upsilon^*_{di} \in (0, \bar{\upsilon})$ exists for $\pi > \underline{\pi}_{di}$.

To see how the introduction of deposit insurance affects the thresholds $\underline{\pi}_{di}$ and υ^*_{di}, we then simply compare a risky bank's expected profits without deposit insurance as given by (49) with that with deposit insurance as in (86), still assuming $P_L = r$. It follows that $\upsilon^* > \upsilon^*_{di}$ and $\underline{\pi}_{di} > \underline{\pi}$ when $\Pi^R > \Pi^R_{di}$ that is when

$$(1-\pi)(c^I - P_H) > c^R - c^R_{di}.$$

The proposition follows. □

Proof of Lemma 2: We characterize the optimal level of monitoring and its properties for a safe and a risky bank in turn. We start from a safe bank. The optimal level of monitoring is computed simply by deriving (31) with respect to m, given that $q(m) = mq_H + (1-m)q_L$. This gives m^S as in the lemma. To see how m^S varies with L^S and υ, we use the implicit function theorem. Denote as $f(m^S, L^S, \upsilon)$ the derivative of (31) with respect to m^S. The optimal level of monitoring is the solution to $f(m^S, L^S, \upsilon) = 0$. It follows that

$$\frac{dm^S}{dL^S} = -\frac{\frac{\partial f(m^S,L^S,\upsilon)}{\partial L^S}}{\frac{\partial f(m^S,L^S,\upsilon)}{\partial m^S}},$$

and

$$\frac{dm^S}{d\upsilon} = -\frac{\frac{\partial f(m^S,L^S,\upsilon)}{\partial \upsilon}}{\frac{\partial f(m^S,L^S,\upsilon)}{\partial m^S}}.$$

The denominator in both expressions is positive and equal to

$$\frac{\partial f(m^S, L^S, \upsilon)}{\partial m^S} = 1 - (q_H - q_L) \left[\frac{L^S \upsilon}{(q^S)^2}(q_H - q_L) - \frac{L^S \upsilon}{(q^S)^2}(q_H - q_L) \right] = 1 > 0.$$

Thus, the sign of $\frac{dm^S}{dL^S}$ and $\frac{dm^S}{d\upsilon}$ are equal to the opposite sign of $\frac{\partial f(m^S,L^S,\upsilon)}{\partial L^S}$ and $\frac{\partial f(m^S,L^S,\upsilon)}{\partial \upsilon}$, respectively.

We start from $\frac{\partial f(m^S,L^S,\upsilon)}{\partial L^S}$. We have that

$$\frac{\partial f(m^S, L^S, \upsilon)}{\partial L^S} = -(q_H - q_L) \left[r(\upsilon, q^S) - \widetilde{r}(\upsilon, q^R) \left(\frac{\pi}{P_L} + \frac{(1-\pi)}{P_H} \right) + \frac{\upsilon}{q^S} \right] < 0,$$

since $r(\upsilon, q^S) > \widetilde{r}(\upsilon, q^R) \left(\frac{\pi}{P_L} + \frac{(1-\pi)}{P_H} \right)$ in order to guarantee that at date 0, banks invest both in reserves and loans. From $\frac{\partial f(m^S,L^S,\upsilon)}{\partial L^S} < 0$, it follows that $\frac{dm^S}{dL^S} > 0$.

Consider now $\frac{\partial f(m^S,L^S,\upsilon)}{\partial \upsilon}$. We have that

$$\frac{\partial f(m^S, L^S, \upsilon)}{\partial \upsilon} = -(q_H - q_L) \left[-\frac{L^S}{q^S} - \left(\pi \frac{R^S - \lambda_L c^S_1}{P_L} + (1-\pi) \frac{R^S - \lambda_H c^S_1}{P_H} \right) + \frac{L^S}{q^S} \right] > 0,$$

which implies that $\frac{dm^S}{dv} < 0$ as in the lemma.

Let's now consider a risky bank. We compute the optimal level of monitoring simply deriving (33) with respect to m^R. This gives the expression as in the lemma. In order to compute the effect of L^R and υ on m^R, we use again the implicit function theorem. Denote as $f(m^R, L^R, \upsilon)$ the derivative of (33) with respect to m^R. Thus,

$$\frac{dm^R}{dL^R} = -\frac{\frac{\partial f(m^R,L^R,\upsilon)}{\partial L^R}}{\frac{\partial f(m^R,L^R,\upsilon)}{\partial m^R}},$$

and

$$\frac{dm^R}{d\upsilon} = -\frac{\frac{\partial f(m^R,L^R,\upsilon)}{\partial \upsilon}}{\frac{\partial f(m^R,L^R,\upsilon)}{\partial m^R}}.$$

The denominator in both expressions is positive and equal to

$$\frac{\partial f(m^R, L^R, \upsilon)}{\partial m^R} = 1-\pi(q_H-q_L)\left[\frac{\upsilon(q_H-q_L)}{(q^R)^2}\left(L^R - \frac{\lambda_L c_1^R - R^R}{P_L}\right) - \frac{\upsilon(q_H-q_L)}{(q^R)^2}\left(L^R - \frac{\lambda_L c_1^R - R^R}{P_L}\right)\right] = 1 > 0.$$

Thus, the sign of $\frac{dm^R}{dL^R}$ and $\frac{dm^R}{d\upsilon}$ are equal to the opposite sign of $\frac{\partial f(m^R,L^R,\upsilon)}{\partial L^R}$ and $\frac{\partial f(m^R,L^R,\upsilon)}{\partial \upsilon}$, respectively.

We start from $\frac{\partial f(m^R,L^R,\upsilon)}{\partial L^R}$. We have that

$$\frac{\partial f(m^R, L^R, \upsilon)}{\partial L^R} = -\pi(q_H - q_L)\left[r(\upsilon, q^R)\left(1 - \frac{1}{P_L}\right) + \frac{\upsilon}{q^R}\left(1 - \frac{1}{P_L}\right)\right] < 0,$$

since $P_L > 1$. From $\frac{\partial f(m^R,L^R,\upsilon)}{\partial L^R} < 0$, it follows that $\frac{dm^R}{dL^R} > 0$.

Consider now $\frac{\partial f(m^R,L^R,\upsilon)}{\partial \upsilon}$. We have that

$$\frac{\partial f(m^R, L^R, \upsilon)}{\partial \upsilon} = -\pi(q_H - q_L)\left[-\frac{L^R}{q^R} + \frac{1}{q^R}\frac{\lambda_L c_1^R - R^R}{P_L} + \frac{L^R}{q^R} - \frac{1}{q^R}\frac{\lambda_L c_1^R - R^R}{P_L}\right] = 0.$$

This implies that $\frac{dm^R}{d\upsilon} = 0$ and the lemma follows.□

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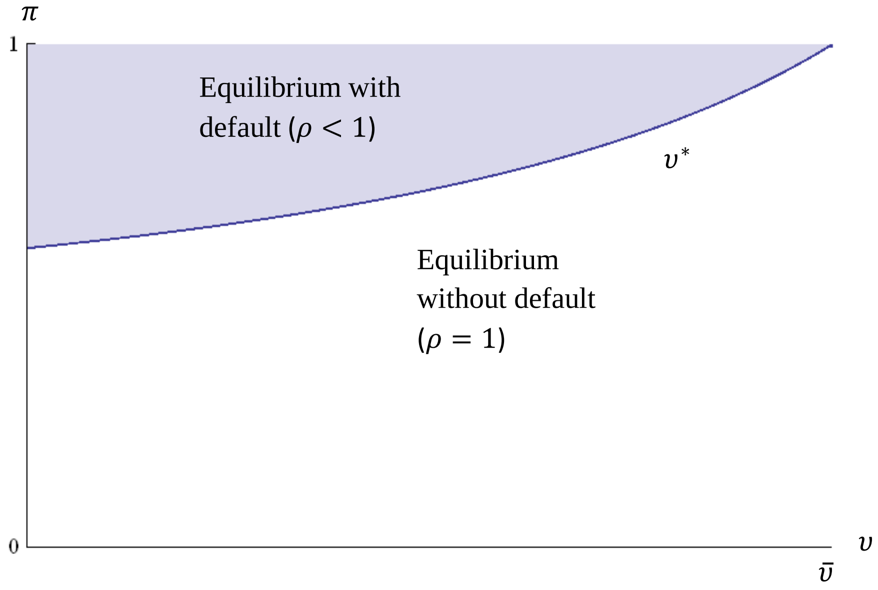

Figure 1: Equilibria as function of the level of aggregate risk and the degree of competition. The figure illustrates the ranges of π and υ where the equilibria with and without default exist. The equilibrium with default only exists if the level of aggregate liquidity risk and the degree of competition are low, as it is the case in the coloured region. Otherwise, only the equilibrium without default exists. The figure is drawn for the following set of parameters: $V = 1.5, \pi = 0.8, \lambda_L = 0.8, \lambda_H = 0.81$.

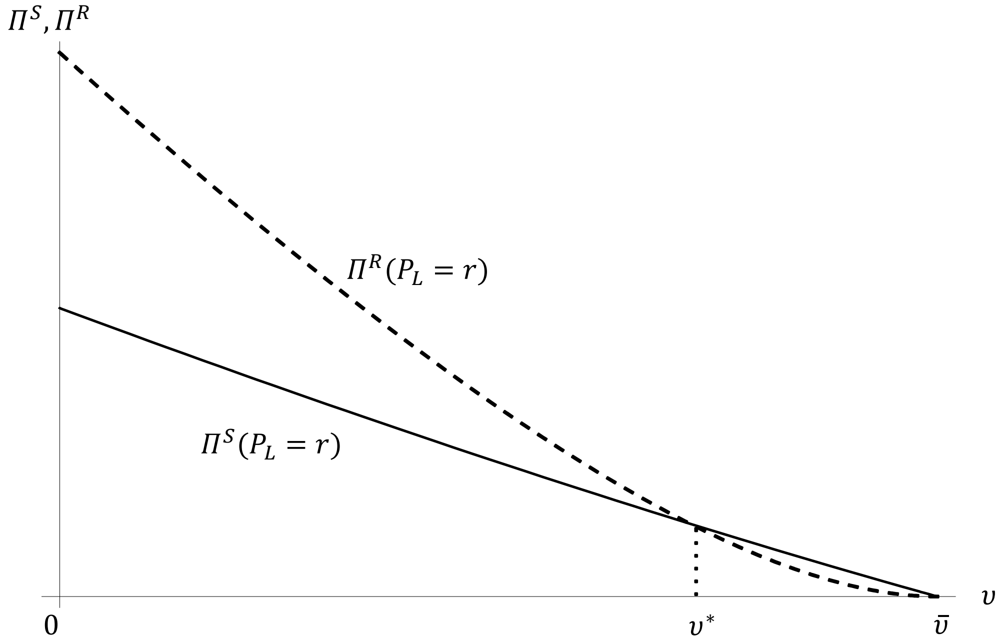

Figure 2: Effect of υ on the equilibrium. The figure illustrates how υ affects the equilibrium when $\pi \geq \underline{\pi}$. The solid line represents the profits of a safe bank Π^S, while the dashed one those of a risky bank Π^R in the case $\rho=1$ and $P_L = r$. When $\upsilon > \upsilon^*$, $\Pi^S(P_L = r) > \Pi^R(P_L = r)$ and only the equilibrium without default exists. When $\upsilon < \upsilon^*$, $\Pi^S(P_L = r) < \Pi^R(P_L = r)$ and only the equilibrium with default exists. At $\upsilon = \upsilon^*$, $\Pi^S(P_L = r) = \Pi^R(P_L = r)$ and the two equilibria coexist. The figure is drawn for the following set of parameters: $V = 1.5, \pi = 0.8, \lambda_L = 0.8, \lambda_H = 0.81$.

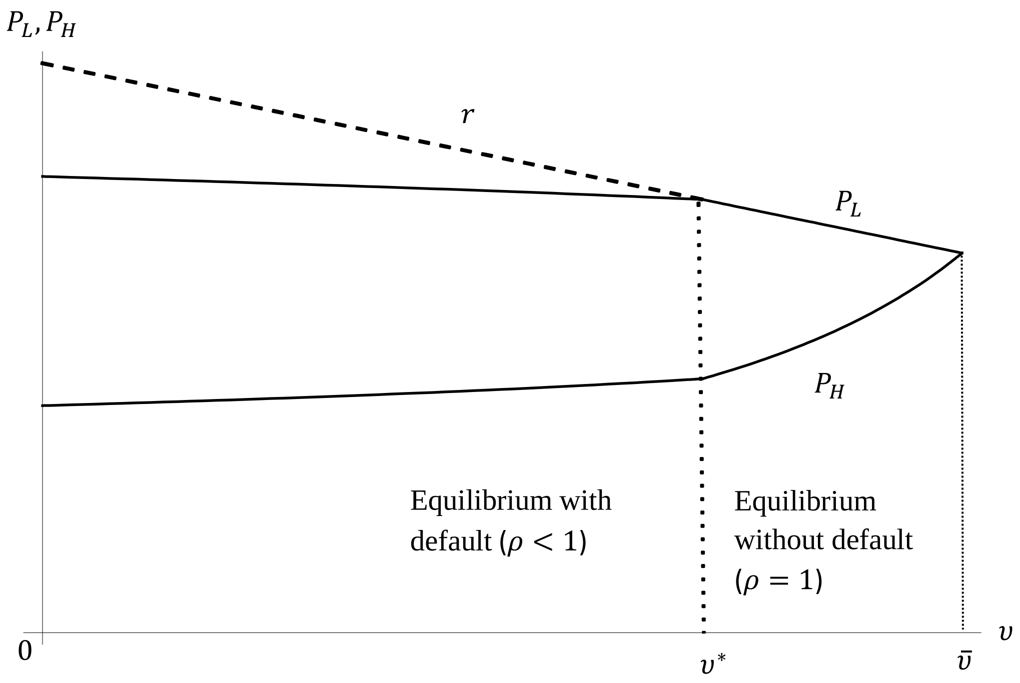

Figure 3: Equilibrium prices. The figure illustrates how the equilibrium prices (P_L, P_H) change with the degree of credit market competition υ in the equilibrium with default ($\rho < 1$) and in the one without default ($\rho = 1$). In both equilibria, the price P_L decreases with υ, while P_H increases. In the equilibrium without default ($\rho = 1$), $P_L = r$ holds, while P_L is strictly below r in the equilibrium with default ($\rho < 1$). The figure is drawn for the following set of parameters: $V = 1.5, \pi = 0.8, \lambda_L = 0.8, \lambda_H = 0.81$.

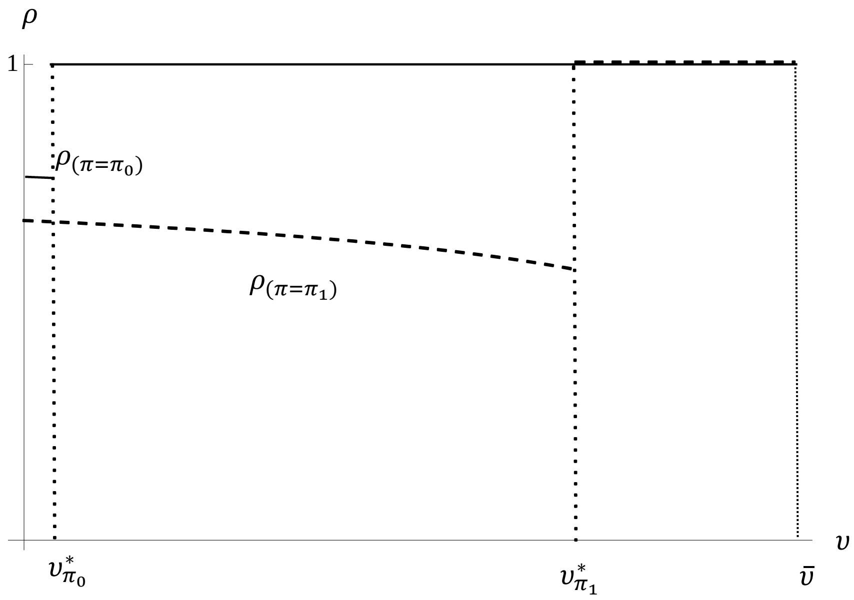

Figure 4: Proportion of safe banks as a function of υ and π. The figure shows how the proportion of safe banks ρ changes with the degree of credit market competition υ and the probability of the good state π. The solid line represents the proportion of safe banks ρ when $\pi = \pi_0$, while the dashed line is drawn for $\pi = \pi_1$, with $\pi_0 < \pi_1$. A higher π leads to fewer safe banks in the equilibrium with default. Irrespective of the level of π, the proportion of safe banks ρ is a discontinuous function of the degree of credit market competition υ. For $\upsilon < \upsilon^*$, the proportion of safe banks is strictly below one and decreases with competition. At $\upsilon = \upsilon^*$, it jumps to one and stays one for any $\upsilon > \upsilon^*$ as in this range only the equilibrium without default exists. The figure is drawn for the following set of parameters: $V = 1.5, \pi_0 = 0.6, \pi_1 = 0.8, \lambda_L = 0.8, \lambda_H = 0.81$. In order to clearly highlight the properties of the two curves, the axes origin in the figure has coordinates {0,0.9}.

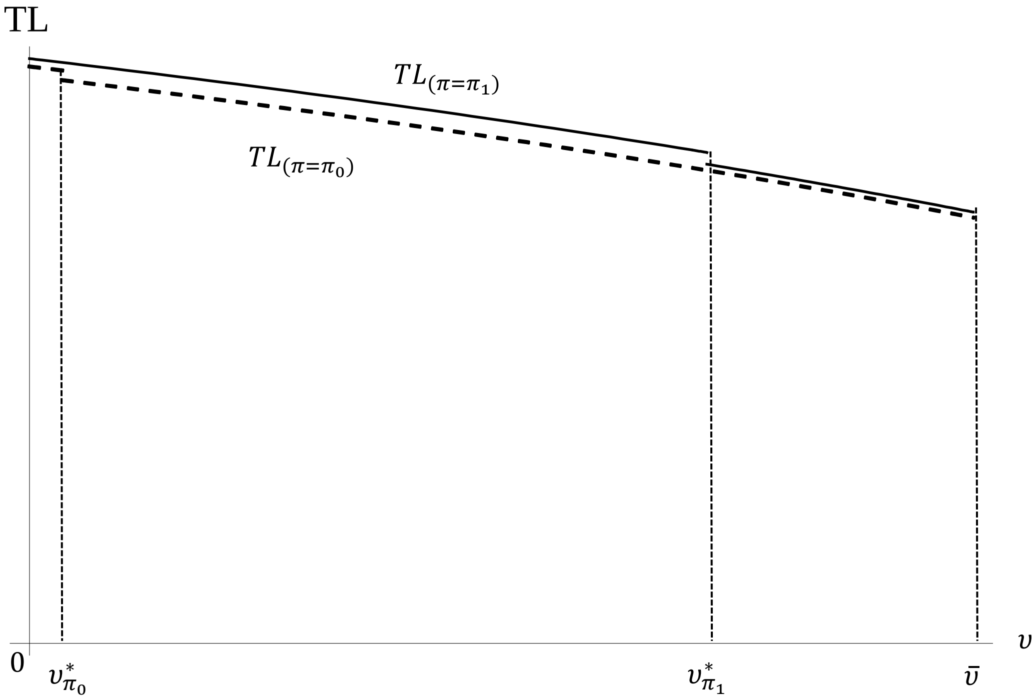

Figure 5.a: Credit availability as a function of the degree of competition and the level of aggregate liquidity risk when $\overline{\pi} < \underline{\pi}$. The figure plots the supply of loans in the equilibrium with default (i.e., ρ<1) and without default (i.e., ρ=1) for different levels of aggregate liquidity risk π (π_0 , π_1) with $\pi_0 < \pi_1$ as a function of the degree of competition υ. The dashed line corresponds to the case when $\pi = \pi_0$, while the solid one represents the case when $\pi = \pi_1$. Irrespective of the value of π, the total amount of loans granted by banks decreases with competition. At $\upsilon = \upsilon^*$, where the equilibrium with and without default coexist, the figure shows that credit availability is higher in the equilibrium with default than in the one without default both when $\pi = \pi_0$ and when $\pi = \pi_1$. The figure is drawn for the following set of parameters: $V = 1.5, \pi_0 = 0.6, \pi_1 = 0.8, \lambda_L = 0.8, \lambda_H = 0.81$, which imply $\overline{\pi} = 0.502 < \underline{\pi} = 0594$.

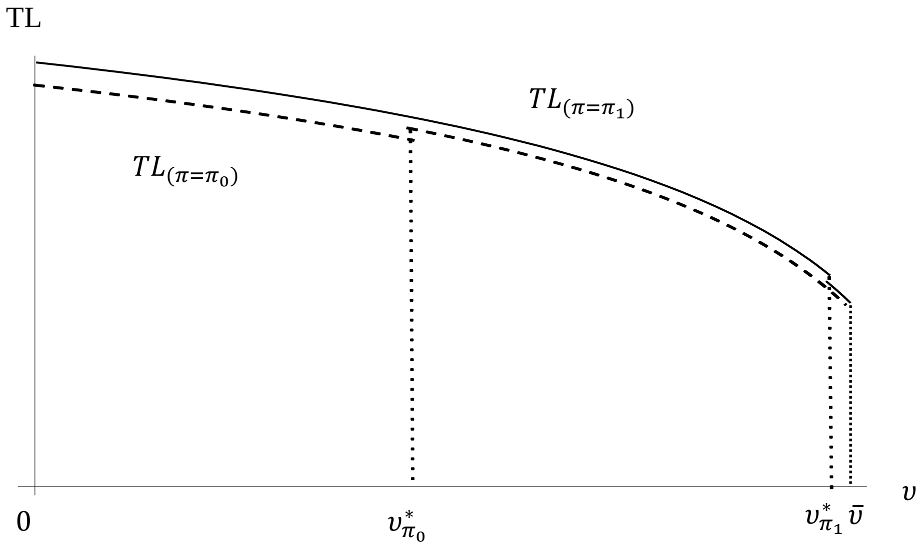

Figure 5.b: Credit availability as a function of the degree of competition and the level of aggregate liquidity risk when $\overline{\pi} > \underline{\pi}$. The figure plots the supply of loans in the equilibrium with default (i.e., ρ<1) and without default (i.e., ρ=1) for different levels of aggregate liquidity risk π (π_0 , π_1) with $\pi_0 < \pi_1$ as a function of the degree of competition υ. The dashed line corresponds to the case when $\pi = \pi_0$, while the solid one represents the case when $\pi = \pi_1$. Irrespective of the value of π, the total amount of loans granted by banks decreases with competition. At $\upsilon = \upsilon^*$, where the equilibrium with and without default coexist, the figure shows that credit availability is higher in the equilibrium with default than in the one without default. when $\pi = \pi_1$, while the opposite is true when $\pi = \pi_0$. The figure is drawn for the following set of parameters: $V = 6, \pi_0 = 0.43, \pi_1 = 0.8, \lambda_L = 0.8, \lambda_H = 0.81$, which imply $\overline{\pi} = 0.474 > \underline{\pi} = 0.4277$.

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