WORKING PAPER NO. 176

MACROECONOMICS OF INTERNATIONAL PRICE DISCRIMINATION

BY GIANCARLO CORSETTI AND LUCA DEDOLA

September 2002
WORKING PAPER NO. 176

MACROECONOMICS OF INTERNATIONAL PRICE DISCRIMINATION¹

BY GIANCARLO CORSETTI²
AND LUCA DEDOLA³

September 2002

¹ We thank Pierpaolo Benigno, Don Davis, Charles Engel, Raffaele Giordano, Pierre Gourinchas, Dale Henderson, Sylvain Leduc, Patrizia Pagano, Paolo Pesenti, Chris Sims and seminar participants at the Bank of Italy, Columbia University, Insead, the International Monetary Fund, the International Research Forum on Monetary Policy at the European Central Bank, the 2002 NBER Summer Institute, the New York Fed, New York University, Princeton University, Yale University, and the 2001 meeting of the SED for helpful comments. Corsetti’s work on this paper is part of a research network on “The Analysis of International Capital Markets: Understanding Europe’s Role in the Global Economy”, funded by the European Commission under the Research Training Network Programme (Contract No. HPRN-CT-1999-00067). The views expressed here are those of the authors, and do not necessarily reflect the position of the Bank of Italy, the European Central Bank or any other institution with which the authors are affiliated.

² University of Rome III, Yale University and CEPR

³ Bank of Italy and Ente Enaudi
International Research Forum on Monetary Policy

The International Research Forum on Monetary Policy is a new series of conferences organised and sponsored by the European Central Bank, the Board of Governors of the Federal Reserve System, the Center for German and European Studies at Georgetown University and the Center for Financial Studies at Goethe University. The conferences aim to provide opportunities to discuss new research on theoretical and empirical aspects of monetary policy that are relevant from a global perspective.

The Forum will meet regularly at locations alternating between the euro area and the United States.

The first conference in this new series took place at the European Central Bank on 5 and 6 July 2002. The issues, numbers 174 to 181 of the European Central Bank Working Paper Series contain the papers presented on that occasion. The programme of the conference and the invited discussions can be accessed at http://www.ecb.int/events/conf/intforum/.

© European Central Bank, 2002

Address Kaisersstrasse 29
D-60311 Frankfurt am Main
Germany
Postal address Postfach 16 03 19
D-60066 Frankfurt am Main
Germany
Telephone +49 69 1344 0
Internet http://www.ecb.int
Fax +49 69 1344 6000
Telex 411 144 ecb d

All rights reserved.
Reproduction for educational and non-commercial purposes is permitted provided that the source is acknowledged.
The views expressed in this paper do not necessarily reflect those of the European Central Bank.

ISSN 1561-0810
Contents

Abstract 4

Non-technical summary 5

1 Introduction 7

2 The model 11

3 Pricing-to-market and incomplete pass-through 17
   3.1 Firms’ optimization and optimal price discrimination 17
   3.2 Optimal pricing in the presence of arbitrage between retail and wholesale markets 19
   3.3 Exchange rate pass-through 21

4 Equilibrium characterization 22
   4.1 Equilibrium conditions 22
   4.2 Price elasticities of export demand and equilibrium multiplicity 25

5 Exchange rates and relative prices under financial autarky 28
   5.1 Exchange rate volatility 28
   5.2 Terms of trade 30
   5.3 The real exchange rate 32

6 Equilibrium under alternative asset structures 33

7 Market segmentation and national welfare 39
   7.1 Distribution services and risk sharing 39
   7.2 Monetary policy and the natural rate of employment 41

8 Competitive goods markets and gains from cooperation 42

9 Conclusions 46

Appendix 49

References 54

European Central Bank working paper series 60
Abstract

This paper builds a baseline two-country model of real and monetary transmission under optimal international price discrimination. Distributing traded goods to consumers requires nontradables; because of distributive trade, the price elasticity of export demand depends on the exchange rate. Profit-maximizing monopolistic firms drive a wedge between wholesale and retail prices across countries. This entails possibly large deviations from the law of one price and incomplete pass-through on import prices. Yet, consistent with expenditure-switching effects, a nominal depreciation generally worsens the terms of trade. Moreover, the exchange rate and the terms of trade can be more volatile than fundamentals. For plausible ranges of the distribution margin, there can be multiple steady states, whereas large differences in nominal and real exchange rates across equilibria translate into small differences in consumption, employment and the price level. Finally, we show that with competitive goods markets international policy cooperation is redundant even under financial autarky.

JEL classification: F3, F4

Keywords: exchange rate pass-through, wholesale and retail services, nominal rigidities, optimal cyclical monetary policy, international cooperation.
Non-technical summary

Cross-border relative price adjustments are at the core of the conventional wisdom on the international transmission, exemplified by the enduring contributions of Friedman [1953] and Mundell [1960]. Yet, cross-border price differentials are large and persistent, and that the law of one price fails to hold for most types of goods and services. Most important, prices seem to respond only mildly, if at all, to changes in the nominal exchange rate. To the extent that incomplete pass-through is due to destination-specific markup adjustment, this is evidence of market segmentation and pricing-to-market (henceforth PTM) by firms.

If firms optimally insulate local prices from exchange rate movements, the exchange rate may have a lesser impact on the relative prices of domestic and foreign goods, in relation to what is implied by the received view. Then, to what extent do exchange rate movements induce desirable changes in the price of foreign relative to domestic goods? What are the overall implications of endogenous PTM for the international transmission? Does international market segmentation create policy spillovers which could motivate the need for international cooperation in policy design?

To address these issues, we build a general-equilibrium two-country model with nominal wage rigidities and monopolistic competition in production, in which upstream firms with monopoly power optimally charge different prices to competitive retailers situated in different locations. What makes the elasticity of demand differ across markets is the need for distribution services whose production is intensive in local inputs. We show that this way of modelling vertical relationships among firms located in different markets turns out to be quite effective in narrowing the gap between open macro theory and key stylized facts of the international economy, and leads to a number of novel results.

In our model, deviations from the law of one price at both wholesale and retail levels derive endogenously from optimal pricing by monopolistic firms. Second, since the price elasticity of export demand is non-linear in the exchange rate, optimal international price discrimination leads to incomplete exchange rate pass-through. Third, despite low pass-through, a nominal depreciation can worsen the terms of
trade — consistent with the received wisdom about the international transmission and the possibility of expenditure switching effects. By the same token, nominal and real exchange rates are positively correlated in equilibrium. Fourth, the exchange rate tends to be more volatile than fundamentals. Fifth, there can be multiple equilibria, raising the possibility of exchange rate volatility driven by sunspots. For a realistic range of the distribution margin, there is one steady state in which the nominal and real exchange rate are equal to the level dictated by PPP, and two steady states in which there are large deviations from PPP. However, because of low equilibrium pass-through, large differences in the level of the nominal and real exchange rates across equilibria translate into very small differences in consumption, employment and price levels.

Including wholesale and retail services, marketing and advertisement, and local transportation, distributive trade accounts for an important share of the retail price of consumption goods: for the US, the average distributive margin is as high as 40 percent (see Burstein, Neves and Rebelo [2000]). In recent years, a number of contributions have included distributive trade in open macro models in order to account for the very low exchange rate pass-through at the consumer price level. Yet, this literature has not analyzed market segmentation resulting from the vertical interaction among monopolistic producers and retailers, nor its implications for the international transmission and policy design. This is precisely the goal of this paper.

Our open economy model with endogenous price discrimination builds upon the analytical framework of Corsetti and Pesenti [2001a,b] and Obstfeld and Rogoff [1995, 2000a]. The specification of consumption preferences in these models is such that terms of trade movements in response to worldwide shocks can be sufficient to generate optimal risk-sharing: introducing Arrow-Debreu securities would not change the equilibrium allocation. We show that this is no longer the case when we allow for distributive trade. For such reason, the predictions and policy implications of our model are quite different from those of the above contributions and the literature drawing on them. Specifically, monetary policy rules supporting the flex-price allocation may not be optimal from a welfare perspective.
1 Introduction

One of the most striking stylized facts of the international economy is the magnitude of cross-border price differentials. First, even in regions with the longest track record of free trade, the empirical evidence shows that the law of one price fails to hold for most types of goods and services.\(^1\) Although this law fails to hold also within national boundaries, the deviations from it are much more dramatic at the international level — leading some researcher to talk about a specific ‘border effect’ (i.e. the effect of switching currency across jurisdictions) on prices of tradables.\(^2\) Second, prices seem to respond only mildly, if at all, to changes in the nominal exchange rate. Exchange rate pass-through, quite low for consumer prices, is far from complete also for international prices.\(^3\) To the extent that incomplete pass-through is due to destination-specific markup adjustment by firms with market power, this is evidence of market segmentation. As firms ‘price-to-market’ (henceforth PTM), buyers across national markets face systematically different prices for otherwise identical goods.\(^4\)

While a deeper understanding of these facts is an exciting challenge to microeconomic research, the evidence of large international price discrepancies raises important macroeconomic issues. Namely, cross-border relative price adjustments are at the core of the conventional wisdom on the international transmission, exemplified by the enduring contributions of Friedman [1953] and Mundell [1960]. If firms optimally insulate local prices from exchange rate movements, the exchange rate may have a lesser impact on the relative prices of domestic and foreign goods, in relation

\(^1\)See Rogoff [1996] for an excellent survey on the evidence on the failure of the law of one price. In his analysis of US exchange rate movements, using both consumers and producers price indices, Engel [1999] finds that a great deal of the amount of deviations from PPP are due to a failure of the law of one price for internationally traded goods.

\(^2\)Engel and Rogers [1996] find that nominal currency movements appear to be important determinants of international price discrepancies.

\(^3\)According to the evidence surveyed by Goldberg and Knetter [1997], 1/2 is the median fraction by which exporters to the US offset a dollar appreciation by lowering their export prices.

\(^4\)Krugman [1987] labeled the phenomenon of exchange rate induced price discrimination ‘pricing-to-market’. Overall, the average degree of PTM found by Marston’s [1990] classic study of Japanese industries is in the neighborhood of 50 percent. Similar findings are in Knetter [1989, 1993] and Gagnon and Knetter [1995].
to what is implied by the received view. For instance, Krugman [1989] notes that,
in the presence of deviations from the law of one price and incomplete pass-through,
large swings in the exchange rate may bring about only negligible changes in the
equilibrium allocation. But what are the overall implications of endogenous PTM
for the international transmission? To what extent do exchange rate movements
induce desirable changes in the price of foreign relative to domestic goods? Does
international market segmentation create policy spillovers which could motivate the
need for international cooperation in policy design?

To address these issues, we build a general-equilibrium two-country model with
nominal wage rigidities and monopolistic competition in production, in which up-
stream firms with monopoly power optimally charge different prices to competitive
retailers situated in different locations. What makes the elasticity of demand differ
across markets is the need for distribution services whose production is intensive
in local inputs. We show that this way of modelling vertical relationships among
firms located in different markets turns out to be quite effective in narrowing the
gap between open macro theory and key stylized facts of the international economy,
and leads to a number of novel results.

In our model, deviations from the law of one price at both wholesale and retail
levels derive endogenously from optimal pricing by monopolistic firms. Second, since
the price elasticity of export demand is non-linear in the exchange rate, optimal
international price discrimination leads to incomplete exchange rate pass-through.
Third, despite low pass-through, a nominal depreciation can worsen the terms of
trade — consistent with the received wisdom about the international transmission
and the possibility of expenditure switching effects. By the same token, nominal and
real exchange rates are positively correlated in equilibrium. Fourth, the exchange
rate tends to be more volatile than fundamentals. Fifth, there can be multiple
equilibria, raising the possibility of exchange rate volatility driven by sunspots.

Specifically, we show that for a realistic range of the distribution margin there
is one steady state in which the nominal and real exchange rate are equal to the
level dictated by PPP, and two steady states in which there are large deviations
from PPP. However, because of low equilibrium pass-through, large differences in
the level of the nominal and real exchange rates across equilibria translate into very small differences in consumption, employment and price levels.

Distribution services are traditionally invoked as a key reason for the failure of relative PPP.\(^5\) Dornbusch [1989], for instance, suggests that these services may account for his finding that the price of an identical consumption basket is higher in high-income economies than in low-income ones. A common objection against this view is that the role of these services is too small to be empirically relevant, one reason being that transportation costs associated with international trade are quite contained. However, including wholesale and retail services, marketing and advertisement, and local transportation, distributive trade actually accounts for an important share of the retail price of consumption goods: for the US, the average distributive margin is as high as 40 percent (see Burstein, Neves and Rebelo [2000]).

Strong evidence of the importance of distribution services in accounting for international price discrimination is presented by Goldberg and Verboven [2001], based on comprehensive and detailed data for automobile prices sold in five European countries. According to their estimates, a 1 per cent change in the nominal exchange rate induces a 0.46 per cent adjustment in the export prices in exporter currency (i.e., equivalent to a 0.46 pass-through coefficient). Of this, between 0.37 and 0.39 per cent can be attributed to a change in the local components of marginal costs (i.e., incurred in the destination country).\(^6\)

In recent years, a number of contributions have included distributive trade in open macro models in order to account for the very low exchange rate pass-through at the consumer price level.\(^7\) Yet, this literature has not analyzed market segmentation resulting from the vertical interaction among monopolistic producers and retailers,

---

\(^5\)Recent literature has explored the role of barriers to trade and transportation costs, without however linking them explicitly to international price discrimination. See Obstfeld and Rogoff [2001] on the role of transportation costs in explaining major puzzles in international finance, and the evidence in Parsley and Wei [2000] on transportation costs and the border effect.

\(^6\)Goldberg and Verboven [2001] gauge that local costs are up to 35 per cent of the value of a car, mainly due to distribution services provided by local dealers.

\(^7\)Erxeg and Levin [1995], McCulmaun and Nelson [1999], and Burstein, Neves and Rebelo [2000] assume that distribution requires local inputs, focusing on the case of perfect competition in the goods market.
nor its implications for the international transmission and policy design. This is precisely the goal of this paper.

By modelling vertical relationships between firms located in different national markets, we pursue a different approach relative to recent contributions that view market segmentation as a direct implication of price rigidities. In this literature, foreign exporters preset consumer prices in local currency.\(^8\) One problem with this class of models is that, while accounting for price differences across markets and the border effect, they imply that an exchange rate depreciation should improve a country’s terms of trade. Obstfeld and Rogoff [2000a] however find evidence at odds with such prediction: in the data, exchange rate depreciations tend to be associated with a worsening of the terms of trade, consistent with the traditional wisdom.

Our open economy model with endogenous price discrimination builds upon the analytical framework of Corsetti and Pesenti [2001a,b] and Obstfeld and Rogoff [1995, 2000a]. The specification of consumption preferences in these models is such that terms of trade movements in response to worldwide shocks can be sufficient to generate optimal risk-sharing: introducing Arrow-Debreu securities would not change the equilibrium allocation. We show that this is no longer the case when we allow for distributive trade: no equilibrium with trade in international bonds can lead to optimal risk sharing — not even when nominal rigidities and monopoly power distortions are removed. For such reason, the predictions and policy implications of our model are quite different from those of the above contributions and the literature drawing on them. Specifically, monetary policy rules supporting the flex-price allocation may not be optimal from a welfare perspective.

The paper is organized as follows. Section 2 presents the model. Section 3 discusses optimal pricing by monopolistic firms facing country-specific demand elasticities. Section 4 analyzes the novel features of the equilibrium with endogenously

\(^8\)An incomplete list of papers assuming local currency prices includes Betts and Devereux [2000], Chari, Kehoe and McGrattan [2000], Devereux and Engel [2000], and Kollman [1997], among others (see Lane [2001] and Engel [2002] for a survey of the literature, and Corsetti and Pesenti [2001b] for a generalization of this approach). Early contributions simply assume that foreign exporters quoted prices in local currency. Recent work by Bacchetta and van Wincoop [2000], Corsetti and Pesenti [2002] and Devereux and Engel [2001] analyzes the problem of producers who can choose whether presetting prices in domestic currency only, or in both domestic and foreign currencies.
segmented markets, highlighting the result of steady state multiplicity. Section 5 and 6 study exchange rate determination and derive a set of predictions of the model that are consistent with the main stylized facts on international prices. Section 7 characterizes the command optimum allocation and discusses the scope of monetary policy in this model. Section 8 looks at the limiting case of no monopoly power in the goods market.

2 The model

Goods, inputs and market structure The world economy consists of two countries of equal size, denoted H and F. Each country is specialized in one type of tradable goods, produced in a number of varieties or brands defined over a continuum of unit mass. Brands of tradable goods are indexed by $h \in [0, 1]$ in the Home country and $f \in [0, 1]$ in the Foreign country. In addition, each country produces an array of differentiated nontradable goods, indexed by $n \in [0, 1]$. Nontraded goods are either consumed, or used to make intermediate tradable goods $h$ and $f$ available to domestic consumers.\(^9\)

Firms producing tradable and nontraded goods are monopolistic supplier of one brand of goods only. These firms employ differentiated domestic labor inputs in a continuum of unit mass. Each worker occupies a point in this continuum, and acts as a monopolistic supplier of a differentiated type of labor input to all firms in the domestic economy. Households/workers are indexed by $j \in [0, 1]$ in the Home country and $j^* \in [0, 1]$ in the Foreign country. Firms operating in the distribution sector, instead, are assumed to operate under perfect competition.\(^10\) They buy tradable goods, and distribute them to consumers using nontraded goods as the only input in production.

\(^9\)For notational simplicity, we ignore distribution costs incurred in the non-traded good market, as these can be accounted for by varying the level of productivity in the nontraded sector.

\(^10\)Due to this assumption, we note from the start that the equilibrium allocation studied below would be identical in a vertically integrated economy, where exporters with monopoly power own local retailers.
Technology  Let $Y(h)$ denote total output of a differentiated tradable good $h$, and 
$L(h, j)$ the demand for labor input of type $j$ by the producer of good $h$. By the 
same token, $Y(n)$ denotes total production of a differentiated nontradable good $n$, 
and $L(n, j)$ the corresponding demand for labor input $j$. The production function 
of the Home traded and non traded goods are, respectively:

$$Y_t(h) = Z_{H,t} \left[ \int_0^1 L_t(h, j) \frac{\phi}{\phi - 1} \, dj \right]^{\frac{\phi}{\phi - 1}}$$

$$Y_t(n) = Z_{N,t} \left[ \int_0^1 L_t(n, j) \frac{\phi}{\phi - 1} \, dj \right]^{\frac{\phi}{\phi - 1}}$$

where $\phi$ is the elasticity of substitution among labor inputs, which is the same across 
sectors, and $Z$ denotes stochastic productivity parameters, which are sector-specific. 
Similar expressions hold for firms in the Foreign country, whereas the elasticity of 
substitution is also $\phi$, but the productivity shocks are not necessarily symmetric.

Our specification of the distribution sector is in the spirit of Tirole’s ([1995], 
page 175) matter-of-fact remark that “production and retailing are complements, 
and consumers often consume them in fixed proportions”. As in Erceg and Levin 
[1995] and Burstein, Neves and Rebelo [2000], we thus assume that bringing one unit 
of traded goods to consumers requires $\eta$ units of a basket of differentiated nontraded 
goods\(^{11}\)

$$\eta = \left[ \int_0^1 \eta(n) \frac{\phi}{\phi - 1} \, dn \right]^{\frac{\phi}{\phi - 1}}.$$  \hspace{1cm} (2)

We note here that the Dixit-Stiglitz index above also applies to the consumption 
of differentiated nontraded goods, specified in the next subsection. In equilibrium, 
then, the basket of nontraded goods required to distribute tradable goods to con-
sumers will have the same composition of the basket of nontradable goods consumed 
by the representative domestic household.\(^{12}\)

\(^{11}\)In a more general setting, the key assumption is that the consumer elasticity of substitution 
between tradables and nontrades be higher than the retailer elasticity of substitution between 
tradables and nontrades (see Burstein, Neves and Rebelo [2000]).

\(^{12}\)For simplicity, we do not distinguish between nontradable consumption goods that directly enter 
the agents’ utility and nontraded distribution services, that are jointly consumed with goods. This 
distinction may however be important in more empirically oriented studies (e.g., see MacDonald 
and Ricci [2001]).
Preferences. Home agent’s lifetime expected utility $U$ is defined as:

$$U_t(j) = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U \left[ C_t(j), \frac{M_t(j)}{P_t}, \ell_t(j), \Omega_t \right]$$

(3)

where $\beta < 1$ is the discount rate and the instantaneous utility $U$ is a positive function of a consumption index $C(j)$ – to be defined below – and real balances $M_t(j)/P_t$, while is a negative function of labor effort $\ell_t(j)$. Instantaneous utility is state dependent, as indexed by the vector of random variables $\Omega_t$. We assume the following additively-separable parameterization:

$$U \left[ C_t(j), \frac{M_t(j)}{P_t}, \ell_t(j) \right] = \ln C_t(j) + \frac{\chi_t}{1-\varepsilon} \left( \frac{M_t(j)}{P_t} \right)^{1-\varepsilon} - \kappa_t \ell_t(j)$$

(4)

We allow for velocity shocks in the form of a stochastically varying utility of real balances, and shocks to the disutility of labor. Thus we have $\Omega_t = \{\chi_t, \kappa_t\}$. Foreign agents’ preferences are:

$$U^*_t(j^*) = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ \ln C^*_t(j^*) + \frac{\chi^*_t}{1-\varepsilon} \left( \frac{M^*_t(j^*)}{P^*_t} \right)^{1-\varepsilon} - \kappa^*_t \ell^*_t(j^*) \right]$$

(5)

where the discount rate $\beta$ is the same as in the Home country, but shocks to velocity $\chi^*_t$ and the disutility of labor $\kappa^*_t$ (hence the natural rate of unemployment) can be different across countries.

Households consume all types of (domestically-produced) nontraded goods, and both types of traded goods. So, $C_t(n,j)$ is consumption of brand $n$ of Home nontraded good by agent $j$ at time $t$; $C_t(h,j)$ and $C_t(f,j)$ are the same agent’s consumption of Home brand $h$ and Foreign brand $f$. For each type of good, we assume that one brand is an imperfect substitute to all other brands, with constant elasticity of substitution $\theta > 1$. Consumption of Home and Foreign goods by Home agent $j$ is defined as:

$$C_{N,t}(j) \equiv \left[ \int_0^1 C_t(n,j) \frac{\theta-1}{\theta} d\theta \right]^{\frac{1}{\theta-1}},$$

$$C_{H,t}(j) \equiv \left[ \int_0^1 C_t(h,j) \frac{\theta-1}{\theta} dh \right]^{\frac{1}{\theta-1}},$$

$$C_{F,t}(j) \equiv \left[ \int_0^1 C_t(f,j) \frac{\theta-1}{\theta} df \right]^{\frac{1}{\theta-1}},$$

Consumption of different goods by Foreign agent $j^*$ — $C^*_t(n,j^*)$, $C^*_t(h,j^*)$ and $C^*_t(f,j^*)$ — and the corresponding consumption indexes — $C^*_{N,t}(j^*)$, $C^*_{H,t}(j^*)$ and $C^*_{F,t}(j^*)$ — are similarly defined.
The utility from consumption of tradable goods of individuals $j$ and $j^*$ is:

$$C_{T,t}(j) = 2C_{H,t}(j)^{1/2}C_{F,t}(j)^{1/2}, \quad C_{T,t}(j^*) = 2C_{H,t}(j^*)^{1/2}C_{F,t}(j^*)^{1/2} \quad (6)$$

Note that, consistent with the assumption that each country specializes in the production of a single type of traded good, the elasticity of substitution between goods produced in different countries (set equal to one) is below the elasticity of substitution among goods produced in one country ($\theta \geq 1$). The full consumption basket in each country is

$$C_t(j) = \frac{C_{T,t}(j)^\gamma C_{N,t}(j)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}, \quad C_t^*(j^*) = \frac{C_{T,t}(j^*)^\gamma C_{N,t}(j^*)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \quad 0 < \gamma < 1 \quad (7)$$

including both traded and non traded goods with weight $\gamma$ and $1-\gamma$, that are the same in every country.

**Price indices** Let $p_t(h)$ and $p_t^*(h)$ denote the retail price of brands $h$ expressed in the Home and foreign currency, respectively. The utility-based consumption price indexes are\(^{13}\)

$$P_{H,t} = \left[ \int_0^1 [p_t(h)]^{1-\theta} dh \right]^{\frac{1}{1-\theta}}, \quad P_{H,t}^* = \left[ \int_0^1 [p_t^*(h)]^{1-\theta} dh \right]^{\frac{1}{1-\theta}}. \quad (8)$$

The price indexes $P_{N,t}$, $P_{N,t}^*$, $P_{F,t}$ and $P_{F,t}^*$, are analogously defined. By the same token, we hereafter write the utility-based price indexes of tradable:

$$P_{T,t} = P_{H,t}^{1/2}P_{F,t}^{1/2}, \quad P_{t}^* = \left( P_{H,t}^* \right)^{1/2} \left( P_{F,t}^* \right)^{1/2} \quad (9)$$

and the utility-based CPI:

$$P_t = P_{T,t}^{1-\gamma}, \quad P_t^* = \left( P_{T,t}^* \right)^{1-\gamma}. \quad (10)$$

\(^{13}\)The price index is the solution to the following Home goods’s expenditure minimization problem for the Home consumer $j$

$$\min \int_0^1 p_{H,t} C_t(h,j) dh \quad s.t. \quad C_{H,t} = 1$$
Households’ budget constraints and asset markets  Home agents hold Home currency $M_t$, two international bonds, $B_{H,t}$ and $B_{F,t}$, respectively denominated in Home and Foreign currency, and a well-diversified portfolio of domestic firm. They earn labor income $W_t \ell$ and pay non-distortionary (lump-sum) net taxes $T_t$, denominated in Home currency. The individual flow budget constraint for agent $j$ in the Home country is:  

$$M_t(j) + B_{H,t+1}(j) + \mathcal{E}_t B_{F,t+1}(j) \leq M_{t-1}(j) + (1 + \delta_t) B_{H,t}(j)$$  

$$+ (1 + i_t^s) \mathcal{E}_t B_{F,t}(j) + \int_0^1 \Pi(h,j) dh + \int_0^1 \Pi(n,j) dn +$$  

$$W_t(j) \ell_t(j) - T_t(j) - P_{H,t} C_{H,t}(j) - P_{F,t} C_{F,t}(j) - P_{N,t} C_{N,t}(j)$$

where $\mathcal{E}_t$ is the nominal exchange rate, expressed as Home currency per unit of Foreign currency; $i_t$ and $i_t^s$ are the nominal yields in Home and Foreign currency, paid at the beginning of period $t$ but known at time $t - 1$; and $\int \Pi(h,j) dh + \int \Pi(n,j) dn$ is total dividend revenue from equities, from all firms $h$ and $n$ in the economy. A similar expression holds for the representative individual $j^s$ in the Foreign country.

The international bonds are assumed to be in zero net-supply. Thus we have  

$$\int_0^1 B_{H,t}(j) dj + \int_0^1 B^*_{H,t}(j^s) dj^s = 0,$$  

$$\int_0^1 B_{F,t}(j) dj + \int_0^1 B^*_{F,t}(j^s) dj^s = 0$$

so that in the aggregate $B_{H,t} = -B^*_{H,t}$ and $B_{F,t} = -B^*_{F,t}$.

Government budget constraint and policy instruments  The government budget constraint in the Home country is:  

$$\int_0^1 [M_t(j) - M_{t-1}(j)] dj + \int_0^1 T_t(j) dj = 0$$

whereas we abstract from government spending, and seigniorage revenue is rebated to households in a lump-sum fashion.\footnote{The notation conventions follow Obstfeld and Rogoff [1996, ch.10]. Specifically, $M_t(j)$ denotes agent $j$’s nominal balances accumulated during period $t$ and carried over into period $t + 1$, while $B_{H,t}(j)$ and $B_{F,t}(j)$ denote agent $j$’s bonds accumulated during period $t - 1$ and carried over into period $t$. The model could be extended to encompass government spending and public debt. A fiscal shock, modeled as random fluctuations in government spending, is isomorphic to productivity shocks (see Corsetti and Pesenti [2001a]).}
In order to characterize monetary policy, it is convenient to define a variable $\mu_t$ such that

$$
\frac{1}{\mu_t} = \beta(1 + i_{t+1})E_t \left( \frac{1}{\mu_{t+1}} \right)
$$

(14)

Given the time path of $\mu$, there is a corresponding sequence of Home nominal interest rates. Without loss of generality, we assume that the government affects the stock of Home monetary assets by controlling the short-term rate $i_{t+1}$: Home monetary easing at time $t$ leading to a lower $i_{t+1}$ is associated with a higher $\mu_t$.\(^6\) Similar considerations hold for the Foreign country, where the government is assumed to control the interest rate $\mu^*_t$.

**Household optimization and wage setting** The Home agent $j$ chooses a consumption plan, a portfolio plan and a wage rate, so as to maximize (4) subject to the budget constraint (11) and total labor demand (87). An analogous optimization problem is solved by Foreign agent $j^*$. The full characterization of these problems is presented in the Appendix. Hereafter, we discuss the choice of an optimal wage rate in the presence of nominal rigidities.

In our baseline model, we allow for nominal rigidities by assuming that workers and firms agree on the nominal wage rate one period in advance.\(^7\) As in Obstfeld and Rogoff [2000a], the first order condition for the optimal wage contract yields

$$
W_t(j) = \frac{\phi}{\phi - 1} \frac{E_{t-1} \left( \kappa_t \ell_t(j) \right)}{E_{t-1} \left( \ell_t(j) / P_t C_t(j) \right)}
$$

(15)

With a competitive labor market, the nominal wage rate equates the disutility of labor to the marginal utility of consumption of an additional unit of nominal revenue. Because of workers monopoly power, the wage rate is set with a markup $\phi / (\phi - 1)$ over the expected utility cost of labor effort, expressed in units of domestic currency.

\(^6\)With logarithmic utility, in equilibrium $\mu_t$ is equal to nominal spending $P_t C_t$.

\(^7\)Christiano, Eichenbaum and Evans [2001] and Smets and Wouters [2001] are recent structural models providing convincing evidence that price stickiness induced by wage stickiness is an important determinant of persistent macroeconomic fluctuations. Here we abstract from issues in inflation dynamics that could be analyzed, for instance, by assuming Calvo-style adjustment of prices or wages. See Kollman [1997] and Chari et al. [2000] among others.
Having set the wage rate optimally, workers stand ready to provide any amount of labor to firms at the ongoing rate, as long as the real wage is above the marginal disutility of labor. We restrict the size of shocks in such a way that this will always be the case.

3 Pricing-to-market and incomplete pass-through

3.1 Firms’ optimization and optimal price discrimination

International price discrimination is a key feature of the international economy captured by our model. In what follows we show that, even if the elasticity of substitution $\theta$ among different brands of type $h$ ($f$) good is both constant and equal in the two countries, the need for distribution services intensive in local nontraded goods implies that the elasticity of demand for the $h$ ($f$) brand at wholesale level will generally be different across markets. Firms will thus want to charge different prices at Home and in the Foreign country.\(^{18}\)

Consider first the optimal pricing problem faced by firms producing nontraded goods for the Home market. The demand for their product is

$$C(n) + \eta(n) = [p_h(n)]^{-\theta} F_{N,t}^W [C_{H,t} + \eta(C_{H,t} + C_{F,t})]$$

(16)

It is easy to see that their optimal price will result from charging a constant markup over unit labor cost:

$$p_h(n) = P_{N,t} = \frac{\theta}{\theta - 1} \frac{W_t}{Z_{N,t}}$$

(17)

Note that nominal wage rigidities do not translate into price rigidities: the nontraded good prices $p_h(n)$ in fact move inversely with productivity in the sector. Yet, $p_h(n)$ does not respond to contemporaneous monetary shocks. An analogous expression characterizes $P_{N,t}^e$.

A notable feature of our specification is that, because of distribution costs, there is a wedge between the producer price and the consumer price of each good. Let $\pi_t(h)$ denote the price of brands $h$ expressed in the Home currency, at producer level.

\(^{18}\)Bergin and Feenstra [2001] assume a class of preferences that exhibit a non-constant demand elasticity in order to study the quantitative impact of PTM on real exchange rate persistence.
As we assume a competitive distribution sector, the consumer price of good $h$ is simply

$$p_t(h) = p^*(h) + \eta P^*_t$$  \hspace{1cm} (18)

To derive “pricing to market” by firms producing tradables, we proceed in two steps. In this subsection, we solve the unconstrained optimal pricing problem of the Home representative firm. In the next subsection we will study the implications for optimal pricing of the possibility of arbitrage across retail and wholesale markets in different countries. In the unconstrained case, the Home representative firm in the tradable sector solves the following profit maximization problem

$$\max \tilde{p}(h), \tilde{p}^*(h) \quad [\tilde{p}(h)C_t(h) + \xi_t \tilde{p}_t^*(h)C_t^*(h)] - \frac{W_t}{Z_{H,t}} [C_t(h) + C_t^*(h)]$$ \hspace{1cm} (19)

where

$$C_t(h) = \left( \frac{P_{H,t}}{\tilde{p}(h) + \eta P^*_t} \right)^{-\theta} C_{H,t}, \quad C_t^*(h) = \left( \frac{P^*_H}{\tilde{p}_t^*(h) + \eta P^*_t} \right)^{-\theta} C_{H,t}^*$$ \hspace{1cm} (20)

Making use of (17), the optimal wholesale prices $\tilde{p}(h)$ and $\tilde{p}^*(h)$ are:

$$\tilde{p}_t(h) = \frac{\theta}{\theta - 1} \left( 1 + \frac{\eta}{\theta - 1} \frac{Z_{H,t}}{Z_{N,t}} \right) \frac{W_t}{Z_{H,t}} \left( 1 + \frac{\eta \xi_t W^*_t}{\eta W_t} \frac{Z_{H,t}}{Z_{N,t}} \right) \frac{W_t}{Z_{H,t}}$$ \hspace{1cm} (21)

$$\xi_t \tilde{p}_t^*(h) = \frac{\theta}{\theta - 1} \left( 1 + \frac{\eta}{\theta - 1} \frac{Z_{H,t}}{Z_{N,t}} \right) \frac{W_t}{Z_{H,t}} \left( 1 + \frac{\eta \xi_t W^*_t}{\eta W_t} \frac{Z_{H,t}}{Z_{N,t}} \right) \frac{W_t}{Z_{H,t}}$$ \hspace{1cm} (22)

Different from the case of nontraded goods, markups in both markets have a state-contingent component (in brackets in the above expression), that varies as a function of productivity shocks. In addition to productivity shocks, the markup in the foreign market also responds to monetary innovations and relative wages.

With $\eta > 0$, the elasticity of the demand for the Home goods will generally be different at Home and abroad, reflecting any asymmetry in relative productivity

\footnote{It is easy to show that allowing for monopoly power in the retail sector would imply double marginalization at consumer prices. However, as retailers face a constant-elasticity demand, their markup over marginal cost would be constant. Therefore, the double marginalization would influence only the expressions for the level of consumer prices, while leaving all our main results substantially unaffected.}
and/or relative wages. As the monopolistic firm takes into account the implications of distributive trade on the demand elasticity for its product, it finds it optimal to charge different wholesale prices to firms distributing in the Home and in the Foreign market. In general, the optimal (desired) prices of intermediate tradable goods will not obey the law of one price ($\tilde{p}_h(h) \neq \mathcal{E}_t \tilde{p}^*_h(h)$).²⁰

Specifically, in the export market, the price elasticity of the demand for the good $h$ is a non-linear function of the exchange rate:

$$
- \frac{\partial C^*_t(h)}{\partial \tilde{p}_t(h)} \frac{\tilde{p}_t(h)}{C^*_t(h)} = \theta \frac{\tilde{p}_t(h)}{\tilde{p}^*_t(h) + \eta P^*_N} = \theta \frac{1}{1 + \frac{\eta \mathcal{E}_t W^*_t}{W^*_t Z^*_N}}
$$

This expression shows that a relatively appreciated Home exchange rate (a low $\mathcal{E}_t$) corresponds to a relatively large price elasticity of the demand for export.²¹ Conversely, in the Home market, the price elasticity of the demand for the good $h$ only depends on relative productivity levels across sectors:

$$
- \frac{\partial C_t(h)}{\partial \tilde{p}_t(h)} \frac{\tilde{p}_t(h)}{C_t(h)} = \theta \frac{\tilde{p}_t(h)}{\tilde{p}(h) + \eta P_N} = \theta \frac{1}{1 + \frac{\eta \mathcal{E}_t W^*_t}{W^*_t Z^*_N}}
$$

Note that in either market the price elasticity is a monotonic function of the distribution margin, defined as the share of distributive trade in the consumer price of the good $h$ — i.e. $\eta P^*_N/\tilde{p}^*_h(h)$ and $\eta P_N/\tilde{p}(h)$ in the export and the domestic market, respectively. The higher the distribution share, the lower the price elasticity. Analogous considerations apply to optimal pricing by Foreign firms in the Home and the Foreign markets.

### 3.2 Optimal pricing in the presence of arbitrage between retail and wholesale markets

While charging (21) and (22) would maximize firms profits, arbitrage in the good markets may prevent optimal price discrimination between domestic and foreign

²⁰Note also that in our specification the marginal cost of producing the good $h$ is constant, regardless of destination.

²¹Observe that the above price elasticity converges to $\theta > 1$ for extreme rates of Home currency appreciation (a very small $\mathcal{E}_t$), while converges to $1$ for extreme rates of Home currency depreciation (a very large $\mathcal{E}_t$).

dealers.\textsuperscript{22} We now analyze optimal pricing allowing for the possibility that retailers arbitrage across markets. Consider the consumer price of the good \(h\) in both markets, calculated adding the distribution costs (\(\eta P_N\) and \(\eta^* P_N^*\)) to the optimal producer prices above:

\[
p_t(h) = \frac{\theta}{\theta - 1} \left( 1 + \eta \frac{\theta}{\theta - 1} \frac{Z_{H,t}}{Z_{N,t}} \right) \frac{W_i}{Z_{H,t}}
\]  

(25)

\[
\mathcal{E}_t p_t^*(h) = \frac{\theta}{\theta - 1} \left( 1 + \eta \frac{\theta}{\theta - 1} \frac{\mathcal{E}_t W_i^*}{W_i} \frac{Z_{H,t}}{Z_{N,t}} \right) \frac{W_i}{Z_{H,t}}
\]  

(26)

If the representative Home firm set the wholesale price in the Foreign country above the consumer price of its own good in the Home country, firms distributing the good \(h\) in the Foreign country would find it profitable to buy it from Home retailers rather than in the wholesale market.\textsuperscript{23} But this implies that optimal price discrimination is possible only as long as the following no-arbitrage conditions are verified:

\[
\mathcal{E}_t p_t^*(h) = \mathcal{E}_t \left( p_t^*(h) + \eta P_N^* \right) \geq \bar{p}_t(h)
\]

\[
p_t(h) = \bar{p}_t(h) + \eta P_N^* \geq \mathcal{E}_t p_t^*(h)
\]

Using optimal prices, these conditions can be synthetically written as

\[
\frac{1}{\theta} \leq \frac{\mathcal{E}_t W_i^* Z_{N,t}}{W_i Z_{H,t}^*} \leq \theta
\]  

(27)

According to this expression, for given relative nominal marginal costs in the non-traded goods sector, a large depreciation of the nominal exchange rate could reduce the Home consumer price of \(h\) in Foreign currency below the optimal export price \(\overline{p}_t^*(h)\) — violating the second inequality above. In that case, arbitrage in the goods market would force the domestic price and the foreign wholesale price to be set

\textsuperscript{22}We are assuming that markets can be segmented along national lines. However, in our model this assumption can be easily justified with a system of selective and exclusive distribution, in which the manufacturer can choose his/her dealers and restrain them from reselling to anyone but end-users. For instance, Regulation 123/85 of the European Commission has allowed these kind of practices in the European Union to some extent (see Goldberg and Verboven [2001] for the implications of the Regulation in the European car market).

\textsuperscript{23}It is easy to verify that it can never be profitable to buy the \(h\) good at the Home (Foreign) retail price, and sell it at the Foreign (Home) retail price after paying distribution costs. There is no arbitrage opportunity across retail markets.
equal to each other: \( p_t(h) = \mathcal{E}_t \bar{p}^*_t(h) \). By the same token, a large appreciation of the exchange rate could reduce the foreign retail price of \( h \) in the Home currency below the wholesale price at Home. In this case, ruling out arbitrage requires firms to set \( \bar{p}_t(h) = \mathcal{E}_t \bar{p}^*_t(h) \).

Leaving to the Appendix the characterization of prices when shocks are such that the no-arbitrage condition is binding, we provide an intuitive account of our main results. Suppose that to rule out arbitrage Home firms must set: \( p_t(h) = \mathcal{E}_t \bar{p}^*_t(h) \). Relative to the optimal prices (21) and (22), Home firms will now raise \( \bar{p}_t(h) \) above (21) while lowering \( \bar{p}^*_t(h) \) below (22). As the two prices cannot be set independently, the drop in the markup in the foreign market is partly offset by a higher markup at home.\(^{24}\)

### 3.3 Exchange rate pass-through

In addition to explaining deviations from the law of one price, distribution costs also have important implications for the optimal degree of exchange rate pass-through. Assume at first that the no-arbitrage condition (27) is satisfied as a strict inequality. *Other things equal*, the change of producer prices in response to exchange rate movements is:

\[
- \frac{\partial \bar{p}^*_t(h)}{\partial \mathcal{E}_t} = \frac{1}{\mathcal{E}_t} \frac{1 + \eta Z_{H,t} \mathcal{E}_t W_t}{\theta - 1 Z_{S,t} W_t} < 1
\]

i.e. given wages and productivity shocks, the degree of exchange rate pass-through on import prices is less than 100 per cent. At consumer price level, it is even lower, as the denominator of the above expression is augmented by \( \eta P_N^* \), that is, the second term in the denominator is multiplied by \( \theta > 1 \).

Note that the right-hand side of the above expression is just the inverse of the state-contingent component of the markup charged by Home firms in the export market. Clearly, anything that increases the Home-export markup will dampen the effect of the exchange rate on Home-export prices. So, a relatively appreciated

\(^{24}\)Even when the no-arbitrage condition is binding, wholesale prices will still be different in the Home and Foreign markets: when distribution services are required, the law of one price never holds.
exchange rate (a low $E_t$), a low monopoly power (a high $\theta$), or a small need for
distribution services (a low $\eta$) all imply a relatively small markup charged in the
export market, and a correspondingly high degree of exchange rate pass-through.

Allowing for arbitrage between the retail and the wholesale markets cannot but
reinforce our results. As discussed in the previous section, when the exchange rate
depreciation is large enough, Home firms will set the foreign wholesale price equal
to the domestic consumer price. To the extent shocks cause Home firms to raise
the Home price of their goods, the degree of pass-through will clearly be even lower
than implicit in (22).

Clearly, the above expression will not hold in general in equilibrium. From
our analysis below, if follows that it will hold in the presence of nominal shocks ,
as well as shocks to productivity in the Home nontradables and Foreign tradables
sectors. For a general specification of the joint distribution of all shocks, exchange
rate pass-through cannot be captured in such a simple way — although it will still
be incomplete.

Our framework provides a framework to contrast partial- and general-equilibrium
results. For instance, nominal shocks have real effects only because of nominal
rigidities. Without wage contracts, nominal shocks would not alter relative wages
(the exchange rate and the wage rate would move in opposite direction), and pass-
through would be 100 per cent. Even if all prices and wages were fully flexible,
however, our model predicts that price differentials would still reflect productivity
differentials across sectors, as well as wage differentials arising from country-specific
shocks. In other words, firms’s willingness to engage in price discrimination across
markets is not dependent on the presence of nominal rigidities.

4 Equilibrium characterization

4.1 Equilibrium conditions

In this section we characterize the equilibrium of our economy and study the features
of deterministic steady states. We focus on symmetric equilibria within a country
(though not necessarily symmetric across countries): we hereafter drop the indexes
The world equilibrium is characterized as follows. Given the stochastic processes driving monetary stances ($\mu_t$ and $\mu^*_t$) and the shocks to productivity and preferences (all the $Z$’s, $\chi_t, \chi^*_t, \kappa_t$, and $\kappa^*_t$), and given the initial holdings of bonds ($B_{H,0}$ and $B_{F,0}$) and money ($M_0$ and $M^*_0$), for $t \geq 0$ the equilibrium is a set of processes for the nominal exchange rate $E_t$, the Home allocations and prices ($l_t, C_{H,t}, C_{F,t}, C_{N,t}, M_{t+1}, B_{H,t+1}, B_{F,t+1}, \overline{P}_{H,t}, \overline{P}_{F,t}, P_{N,t}$ and $W_t$) and their Foreign counterparts ($l^*_t, C^*_{H,t}, C^*_{F,t}, C^*_{N,t}, M^*_{t+1}, B^*_{H,t+1}, B^*_F,t+1, \overline{P}^*_{H,t}, \overline{P}^*_{F,t}, P^*_{N,t}$ and $W^*_t$) that (a) satisfy the Home and Foreign consumers’ optimality conditions, (b) maximize firms profits, (c) satisfy the market clearing conditions for each asset and each good, in all the markets where it is traded, and (d) satisfy the resource constraints.

For convenience, Table 1 presents a subset of equilibrium conditions that completely characterize the model. In the table, $\bar{A}_{t+1}$ denotes Home non-monetary wealth at the beginning of time $t+1$, that is

$$\bar{A}_{t+1} \equiv (1 + i_{t+1}) B_{H,t+1} + (1 + i^*_t) E_{t+1} B_{F,t+1};$$

(29)

$\overline{A}$ is similarly defined. Equations (32) state the risk-adjusted uncovered interest parity condition, where we have used the fact that in equilibrium $\mu_t = P_t C_t$ and $\mu^*_t = P^*_t C^*_t$. Note that in the absence of uncertainty, the uncovered interest parity conditions become:

$$\frac{E_t}{E_{t+1}} = \frac{\mu_t}{\mu_{t+1}} \left( \frac{\mu^*_t}{\mu^*_{t+1}} \right)^{-1} = \frac{1 + \bar{i}_{t+1}}{1 + \bar{i}^*_{t+1}}$$

Finally, equation (33) is the bond market clearing, while (1) is the current account (see the Appendix for a derivation).

Now, for given wages, the nominal exchange rate and external borrowing are simultaneously determined by (32), (1) and (33). Employment and consumption in both countries (corresponding to the expressions (35), (37), (36) and (38)) can be derived as a function of exogenous shocks, wages and the nominal exchange rate. In each country, the domestic nominal wage rate is preset given the joint distribution of employment, $\kappa$ (or $\kappa^*$) and domestic monetary shocks.
Table 1: Equilibrium

\[
W_t = \frac{\phi}{\phi - 1} \frac{E_{t-1} \kappa_t \ell_t}{E_{t-1} \left( \ell_t \mu_t \right)}
\]

\[
W_t^* = \frac{\phi}{\phi - 1} \frac{E_{t-1} \kappa_t^* \ell_t^*}{E_{t-1} \left( \ell_t^* \mu_t^* \right)}
\]

\[
E_t = \frac{\mu_t E_t}{\mu_t^*} \left( \frac{\mathcal{E}_{t+1}}{\mu_{t+1}} \right) = \frac{\mu_t}{\mu_t^*} \frac{E_t}{E_t^*} \left( \frac{\mathcal{E}_{t+1}}{\mu_{t+1}} \right)
\]

\[
\mathcal{A}_t = -\mathcal{E}_t \mathcal{A}_t
\]

\[
E_t \left\{ \frac{\beta \mu_t}{\mu_{t+1}} \mathcal{A}_{t+1} \right\} = \mathcal{A}_t - 2 \left[ \frac{1 + \frac{\eta}{\sigma - 1} \frac{Z_{F,t}}{Z_{N,t}} - \mathcal{E}_t \frac{\mu_t}{\mu_{t+1}} \frac{Z_{N,t}}{Z_{F,t}}}{1 + \frac{\eta}{\sigma - 1} \frac{Z_{N,t}}{Z_{F,t}}} \right]
\]

\[
\ell_t = \frac{\theta - 1}{\sigma} \frac{\mu_t}{W_t} \left[ 1 + \frac{\ell_t^i / \mu_t}{\sigma - 1} \frac{\mathcal{E}_t^i}{W_t} \frac{Z_{N,t}}{Z_{F,t}} \right]
\]

\[
\ell_t^i = \frac{\theta - 1}{\sigma} \frac{\mu_t^*}{W_t^*} \left[ 1 + \frac{\ell_t^i / \mu_t^*}{\sigma - 1} \frac{\mathcal{E}_t^i}{W_t^*} \frac{Z_{N,t}}{Z_{F,t}} \right]
\]

\[
C_t = \frac{\theta - 1}{\bar{\theta}} \frac{\mu_t}{W_t} \left( \frac{Z_{N,t}}{Z_{H,t}} - \frac{\eta}{\bar{\theta} - 1} \right) - 2 \left( \frac{Z_{N,t}}{Z_{H,t}} + \frac{\eta}{\bar{\theta} - 1} \right)^{-2} - 2 \left( \frac{Z_{N,t}}{Z_{H,t}} \mathcal{E}_t W_t^* \bar{\theta} - 1 \right)^{-2}
\]

\[
C_t^* = \frac{\theta - 1}{\bar{\theta}} \frac{\mu_t^*}{W_t^*} \left( \frac{Z_{N,t}^*}{Z_{F,t}} + \frac{\eta}{\bar{\theta} - 1} \right)^{-2} - 2 \left( \frac{Z_{N,t}^*}{Z_{F,t}} W_t^* \mathcal{E}_t \bar{\theta} - 1 \right)^{-2}
\]
4.2 Price elasticities of export demand and equilibrium multiplicity

Consider a deterministic steady state in which external wealth is zero, i.e. $B_H = B_T = 0$, so that the trade balance is zero, and all exogenous variables are set equal to constant values, symmetric across countries. By combining (32), (1) and (33) we can solve the balanced trade condition for the exchange rate:

$$E = \frac{\mu}{\mu^*} \left( 1 - \frac{\eta}{Z_N W} + \frac{\eta \theta}{\theta - 1} \right)$$  \hspace{1cm} (39)

Observe that the exchange rate is a non-linear function of relative monetary policy stance $\mu/\mu^*$ and relative productivity.

The following proposition characterizes the solution to the exchange rate equation in a non-stochastic steady state, and establishes that our economy can have either one or three steady states.

**Proposition 1** If $\frac{\mu^*}{\mu} = 1$, $\frac{Z_V}{Z_N} = \frac{Z_H}{Z_N} = \frac{Z_T}{Z_N}$, and $\frac{\kappa}{\kappa^*} = 1$, there always exists a steady state in which $\frac{W}{W^*} = \frac{\kappa \mu}{\kappa^* \mu^*} = 1$, the nominal exchange rate $E$ is equal to 1, and the consumption and labor allocation as determined by equations (35) and (37) are equal across countries. Moreover, when the exchange-rate elasticity of imports expenditure is larger than the exchange-rate elasticity of exports expenditure in Home currency, both evaluated at the symmetric equilibrium with $E = 1$

$$\left[ \frac{\partial (\tilde{P}_V C_V)}{\partial E} \frac{\partial E}{\tilde{P}_V C_V} \right]_{E=1} \geq \left[ \frac{\partial (\tilde{E}_H C_H)}{\partial E} \frac{\partial E}{\tilde{E}_H C_H} \right]_{E=1},$$  \hspace{1cm} (40)

there will be two more steady state equilibria in which $\frac{W}{W^*} = 1$, but the nominal exchange rate is equal to $E_h$ and $E_l$, respectively, where $E_h > 1$, and $0 < E_l < 1$. The consumption and labor allocation, as determined by equations (35) and (37), will differ across countries.

A sketch of the proof is as follows.\(^{25}\) The equilibrium condition (39) can be

\(^{25}\)To save space, we have not included a formal proof, that is however available upon request. In terms of parameters and exogenous productivities levels, condition (40) implies that (for $0 < \theta < \infty$):

$$(\theta - 1) \left( \frac{(\theta - 3) - \sqrt{(\theta - 3)^2 - 4\theta \eta Z_N}}{2\theta} \right) \leq \frac{Z_N}{Z_V} \leq (\theta - 1) \left( \frac{(\theta - 3) + \sqrt{(\theta - 3)^2 - 4\theta \eta Z_N}}{2\theta} \right).$$
written as

\[
\left[ 1 + \frac{\eta}{\theta - 1} \frac{W}{\theta} \frac{Z_N^*}{Z_N} \right] \mu = \left[ 1 + \frac{\eta}{\theta - 1} \frac{\mathcal{E} W^*}{\theta} \frac{Z_N^*}{Z_N} \right] \mathcal{E} \mu^*, \tag{41}
\]

which, by (23) and the analogous expression for the Home demand for Foreign goods, is a function of price elasticities of Home imports and exports:

\[
\left[ -\frac{\partial C_F}{\partial P_F} \frac{\bar{P}_F}{C_F} \right] \mu = \left[ -\frac{\partial C_H^*}{\partial P^*_H} \frac{\bar{P}^*_H}{C^*_H} \right] \mathcal{E} \mu^*.	ag{42}
\]

The balanced trade equilibrium condition is clearly violated when the exchange rate is either strongly appreciated (\(\mathcal{E}\) is very low) or strongly depreciated (\(\mathcal{E}\) is very high). In the first case, the left-hand side of the above expression will be larger than the right-hand side, corresponding to a Home trade deficit. In the second case, the opposite will be true and the Home trade balance will be in surplus.

Now, reminiscent of the Marshall-Lerner condition, when expression (40) holds, a small depreciation around a perfectly symmetric equilibrium with \(\mathcal{E} = 1\) will locally increase the value of imports of the Home country above the value of its exports. But we have seen above that, for a Home exchange rate that is depreciated enough, the value of Home exports will always be larger than the value of imports. Hence, there will be some \(\mathcal{E}_h > 1\) such that the balanced trade equilibrium condition is satisfied. By symmetry, this must also be true at some \(\mathcal{E}_l < 1\). \(^{26}\)

Key to our multiplicity is that markets are incomplete and the exchange-rate elasticity of export and import demand are non-linear in the exchange rate. We have shown above that the price elasticity of demand in the foreign market is monotonically related to the size of the distribution margin. For instance, the equilibrium is unique when \(\eta\) is sufficiently low — that is, the distribution margin in the consumer price is sufficiently small, so that the foreign demand for the Home good is sufficiently elastic. It follows that the condition for multiplicity of equilibria cannot be verified when \(\eta = 0\).

\(^{26}\)Starting from our specification with \(\eta = 0\) (that is, looking at the class of models similar to Obstfeld and Rogoff [2000a]), it can be shown that multiplicity cannot be obtained by simply modifying preferences, i.e. by increasing the share of nontradables in utility and making the elasticity of substitution between tradables and nontradables different from 1.
The specific features of the asymmetric equilibria shed light on the far-reaching implications of international price discrimination. In these equilibria, each country has better terms of trade and can therefore afford a larger volume of imports, enjoying higher consumption. Consumer prices are below their level in the perfect symmetric equilibrium, but because of imperfect pass-through and distributive trade, the difference reflects only a small fraction of the large difference in the nominal exchange rate. Across equilibria, consumption and employment differ slightly, compared to the magnitude of differences in the real exchange rate.

These features are best illustrated by means of a simple numerical example. We exactly solve for the model steady state by assuming that labor is 4 times as productive in the tradable sector relative to the nontradable sector – namely, we set \( \frac{Z_T}{Z_N} = 4 \); the values of \( \eta \) and \( \theta \) are set equal to 1/2 and 10, such that markups vary between 25 and 50 percent, and the distribution margin is between 50 and 60 percent across sectors — a number that is not far from available estimates for the US and OECD countries. Tradables and nontradables are given the same weight in consumption, i.e. \( \gamma = .5 \). Relative to the perfectly symmetric equilibrium, there is a second equilibrium in which the Home nominal and real exchange rates are 41 and 36 per cent stronger, respectively, while the Home terms of trade is 28 per cent lower. Yet, because of distributive trade, aggregate consumption and employment are relatively unaffected — they are 3 and 1 per cent higher relative to their values in the symmetric equilibrium, respectively. Despite the extremely large differences in the exchange rate and relative wages, Home CPI is only 3 per cent lower!

It is worth stressing that the proposition above also establishes the possibility of multiple temporary equilibria in a version of our model under financial autarky, but without nominal rigidities — so that (30) and (31) in table 1 are replaced by the expressions below:

\[
W_t = \frac{\phi}{\phi - 1} \kappa_t \mu_t, \quad W_t^* = \frac{\phi}{\phi - 1} \kappa_t^* \mu_t^*.
\]

In such an economy (39) yields the value of the exchange rate at each point in time.

Further research is clearly needed to check the robustness of multiplicity to changes in the specification of the model, and verify whether condition (40) or its analog in an economy with bond, is quantitatively reasonable (see Corsetti, Dedola
and Leduc (2002)). Yet, the interest of this result is apparent. In the presence of multiple equilibria, the same fundamentals could correspond to different allocations and nominal and real exchange rates. Thus, persistent deviations from PPP could arise from sunspots coordinating agents expectations on one of the possible equilibria. Moreover, monetary or real shocks could move the economy from a unique equilibrium, to a region of multiplicity, and vice versa. Finally, there are relevant policy implications. Limiting exchange rate movements — whether or not coordinated at the international level — may rule out multiplicity altogether.

5 Exchange rates and relative prices under financial autarky

5.1 Exchange rate volatility

In order to understand how the exchange rate and the terms of trade respond to shocks in our economy, we find it convenient to start our analysis by focusing on a decentralized equilibrium under the assumption of financial autarky. As we will see below, most results under financial autarky are qualitatively identical in our economy with international trade in bonds.\footnote{In the case of permanent real shocks, uncontingent nominal bonds are actually useless to hedge risk, and the response of the economy is the same as under financial autarky.} With no trade in assets, however, our model becomes relatively more tractable, so that we can simplify exposition. In addition, the static version of our model below is directly comparable to many recent contributions in open macro that, by assumption or endogenously, rule out current account dynamics (see Corsetti and Pesenti [2001a], Heathcote and Perri [2000] and Obstfeld and Rogoff [2000b]).

In what follows, we first derive the exchange rate response to nominal and real shocks, showing under what conditions the nominal exchange rate will be more volatile than the underlying shocks to fundamentals. In the next subsection, we will show that, notwithstanding the low pass-through, exchange rate movements are associated with changes in relative prices: nominal depreciations tend to worsen the terms of trade. We then analyze the volatility of these variables in equilibrium.
Finally, we will show that nominal and real exchange rates be positively correlated in equilibrium. Throughout our analysis, we will restrict parameter values as to ensure a unique equilibrium.\footnote{Relative to the parameters’ values assumed in our numerical example in the previous section, a unique equilibrium can be obtained by lowering slightly the distribution margins, e.g. setting $\eta = 1/3$ instead of $\eta = 1/2$.}

Under no trade in financial assets, the exchange rate is determined by the balanced-trade condition, whose solution thus coincides with the exchange rate in a deterministic steady state (39). Linearizing the counterpart of (39) around a symmetric equilibrium with $Z_H = Z_F = Z_T$ and equal wage levels across countries we obtain:

$$
\tilde{E}_t = \tilde{\mu}_t - \tilde{\mu}_t^* + \frac{Z_T}{Z_N} \frac{2(\tilde{E}_t + \tilde{W}_t^* - \tilde{W}_t) + \tilde{Z}_{N,t} + \tilde{Z}_{H,L,t} - \tilde{Z}_{N,L,t} - \tilde{Z}_{F,L,t}}{(1 + \rho^* \frac{Z_N}{Z_T}) (1 + \rho^* \frac{Z_N}{Z_T})}
$$

Consider first the response of the exchange rate to permanent shocks to monetary policy, in the form of an unexpected innovation to the ratio of $\mu$ to $\mu^*$. It is easy to see that, in the long run, the nominal exchange rate moves one-to-one with relative monetary stances. As all prices are ex-ante flexible, money is neutral: relative wages do not respond to anticipated monetary innovations, and the term $\tilde{E}_t + \tilde{W}_t^* - \tilde{W}_t$ in the expression above disappears. In the short run, instead, (given nominal wages) the impact on the nominal exchange rate can be written as:

$$
\tilde{E}_t = \frac{(1 + \rho^* \frac{Z_N}{Z_T}) (1 + \rho^* \frac{Z_N}{Z_T})}{(1 + \rho^* \frac{Z_N}{Z_T}) (1 + \rho^* \frac{Z_N}{Z_T}) - 2\eta \frac{Z_N}{Z_T}} (\tilde{\mu}_t - \tilde{\mu}_t^*)
$$

Holding the condition for uniqueness of equilibrium, the coefficient multiplying the relative monetary shock in the above equation is always positive and larger than one. On impact, a permanent monetary expansion depreciates the nominal exchange rate by more than its long-run value, generating expectations of appreciation in the future.\footnote{Note that the above result is not directly comparable with Dornbusch’s overshooting. Without trade in financial assets the uncovered interest parity does not hold. Moreover, by definition of $\mu$, permanent shocks to this variable affect neither Home, nor Foreign interest rates.}

In response to real productivity shocks, the exchange rate jumps to its new
equilibrium value on impact:

\[
\hat{E}_t = \left(1 + \frac{\eta}{\beta - \eta} \frac{Z_{F,t}}{Z_{H,t}}\right) \left(1 + \frac{\beta}{\beta - \eta} \frac{Z_{H,t}}{Z_{F,t}}\right) - 2\eta \frac{Z_{F,t}}{Z_{H,t}} \left[Z_{N,t} + \hat{Z}_{H,t} - \hat{Z}_{F,t} - \hat{Z}_{F,t}^2\right]
\]

(45)

The response to shocks to the traded and the nontraded sector is symmetric: on impact, the nominal exchange rate depreciates with any domestic productivity shock, while appreciates with any Foreign shock. Intuitively, a positive shock to productivity in the Home tradable sector reduces the wholesale price of the Home goods in the Foreign market. Although the retail price also falls, increasing Foreign demand, the value of export drops: for a given value of Home imports, balanced trade in equilibrium requires a depreciation of the exchange rate.

By the same token, a positive productivity shock to the Home nontraded sector reduces unit distribution costs in the Home market, raising the price elasticity of Home demand for imports: import prices tend to fall. However, because of falling distribution costs, retail prices fall by more, boosting Home import demand. Then, the exchange rate must depreciate to ensure a zero trade balance. Note that the size of exchange rate movements in response to productivity shocks is amplified when \(\eta\) and \(\theta\) are relatively high.\(^{30}\)

5.2 Terms of trade

Consider now the link between nominal exchange rate movements and the terms of trade. The latter can be written as the product of two ratios: the first is the ratio between the markup charged by Foreign producers to Home retailers and the markup charged by Home producers to Foreign retailers. The second is the ratio between marginal costs of tradables in the two countries. When (27) holds as a strict inequality – i.e. the no-arbitrage condition is satisfied, we have:

\[
\frac{\hat{F}_{F,t}}{\hat{E}_{F,t}^* W_{F,t}} = \frac{1 + \frac{\eta}{\theta - 1} \frac{Z_{F,t}}{Z_{F,t}} W_{F,t}}{1 + \frac{\eta}{\theta - 1} \frac{Z_{F,t}}{Z_{F,t}} W_{F,t}} \frac{\hat{E}_{F,t}^* W_{F,t}}{\hat{E}_{H,t} W_{H,t}} \left[\frac{Z_{F,t}}{Z_{H,t}} \frac{W_{F,t}}{W_{H,t}}\right]
\]

(46)

\(^{30}\)When the Marshall-Lerner conditions are locally violated, instead, the response of the exchange rate to nominal and real shocks may change sign even for equilibria that are in a neighborhood of the symmetric steady state. Clearly, for parameters’ values close to those that make the sign switch, the exchange rate becomes extremely volatile.
Note that the expression above is the same under perfect competition in the goods market \((\theta \rightarrow \infty)\) as with no distribution services \((\eta = 0)\).\(^{31}\) The effect of \(\eta\) on the elasticity of foreign demand for Home products is relevant to pricing only when Home firms have monopoly power.

Clearly, for a nominal depreciation to worsen the terms of trade, the impact of exchange rate movements on relative marginal costs must be larger than the relative markup adjustment by foreign and domestic firms. Linearizing the above equation around a symmetric steady state, we obtain

\[
\frac{\ddot{P}_V}{\hat{E} \ddot{P}_H} = \left(1 - \frac{\eta}{\theta - 1}\right) \left(\tilde{E}_t - \tilde{W}_t^* + \tilde{W}_t\right) + \left(\tilde{Z}_{H,t} - \tilde{Z}_{H,t}^*\right) - \frac{\eta}{\theta - 1} \left(\tilde{Z}_{N,t} - \tilde{Z}_{N,t}^*\right)
\]

\[\text{1 + \frac{\eta}{\theta - 1}}\]

(47)

where, conditional on the shock, \(\tilde{E}_t\) is given by either (44) or (45). Under mild conditions on the degree of monopoly power and distribution margins, such that \(\eta < \theta - 1\), the coefficient of \(\tilde{E}_t\) in the above expression is positive. Hence, monetary shocks induce a positive correlation between the terms of trade and the exchange rate.

The correlation between the terms of trade and the nominal exchange rate is also positive in the presence of real shocks to productivity in the Home tradable sector. As these shocks unambiguously depreciate the nominal exchange rate, they worsen the terms of trade both directly (second term on the right-hand side of the expression above) and through their effect on \(\hat{E}\). For this reason, it is possible that shocks to the tradeable sector cause the terms of trade to be more volatile than \(\hat{E}\).

Conversely, the correlation between \(\hat{E}\) and the terms of trade induced by shocks to productivity to the Home nontradable sector is not necessarily positive. While unambiguously depreciating the nominal exchange rate, these shock also have a positive effect on the terms of trade. This is because, by reducing the cost of distributive trade in the Home market, they raise the price elasticity of the Home demand for Foreign products. A higher price elasticity tends to lower the optimal price charged by Foreign wholesalers. While this effect is small for reasonable values

\(^{31}\)However, \(\eta = 0\) implies a rather different exchange rate determination: \(\hat{E}\) becomes a function of relative monetary stances only. Hence, the indirect effect of the exchange rate on the terms of trade is also different.
of \( \eta/(\theta - 1) \), its presence implies that the volatility of the terms of trade in response to shocks to nontradables tends to be lower than the volatility of \( \mathcal{E} \).

### 5.3 The real exchange rate

By using the definition of the real exchange rate, we can write

\[
\frac{\mathcal{E}_t P^s_t}{P_t} = \left( \frac{\mathcal{E}_t P^s_{t,t}}{P_{t,t}} \right) \left( \frac{P^s_{t,t}}{P^a_{t,t}} \right) = \left( \frac{\mathcal{E}_t P^s_{t,t}}{P_{t,t}} \right) \left( \frac{P^s_{N,t}}{P^a_{t,t}} \right)^{1-\gamma}
\]  

(48)

In our model, movements in the real exchange rate are due to both movements in the relative price of traded goods across countries, and to movements of the relative price of tradables in terms of nontrades. This is in sharp contrast with models adopting a similar specification, but not allowing for distributive trade. By setting \( \eta = 0 \), in fact, there would be no deviation from the law of one price. The first term on the right-hand side of the above definition would be constant, and the variability of the real exchange rate would only depend on the variability of the relative price of nontrades within each country — a prediction that is inconsistent with the findings for the US real exchange rate in Engel [1999].

Consider now the response of the real exchange rate to nominal shocks. Linearizing the real exchange rate around a symmetric steady state, we obtain:

\[
\left( \frac{\mathcal{E}_t P^s_t}{P_t} \right) = \left[ 1 - \gamma + \frac{\theta}{\theta - 1} \frac{Z_t}{Z_s} \right] \mathcal{E}_t, \tag{49}
\]

where \( \tilde{\mathcal{E}}_t \) is given by (44). Since \( 0 < \gamma < 1 \), this expression shows that monetary shocks always move nominal and real exchange rates in the same direction. Thus, an unexpected monetary expansion at home will bring about both a nominal and a real depreciation. However, since the coefficient of \( \tilde{\mathcal{E}}_t \) in the above expression is less than one, the real exchange rate will move by less.

With shocks to traded and nontraded productivity, we have, respectively:

\[
\left( \frac{\mathcal{E}_t P^s_t}{P_t} \right) = \left[ 1 - \frac{\gamma}{1 + \frac{\theta}{\theta - 1} \frac{Z_t}{Z_s}} \right] \mathcal{E}_t
\]

\[
\left( \frac{\mathcal{E}_t P^s_t}{P_t} \right) = \left[ 1 - \frac{\gamma}{1 + \frac{\theta}{\theta - 1} \frac{Z_t}{Z_s}} \right] \left( 1 + \mathcal{E}_t \right)
\]  

(50)
where $\hat{\pi}_t^e$ is given by (45). We have seen above that, regardless of the sector in which they occur, domestic productivity shocks always depreciate the nominal exchange rate. It is then apparent from the above expressions that they also depreciate the real exchange rate. Observe that the real depreciation will be attenuated in the case of shocks to $Z_{H,t}$, relative to the case of shocks to $Z_{N,t}$.

Together, the above results imply that the real exchange rate and relative consumption be negatively correlated in the presence of nominal disturbances and shocks to tradable goods production. To see this, note that in our model the real exchange rate can also be written as:

$$\frac{\hat{E}_t P^*_t}{P_t} = \frac{\hat{\pi}_t^e}{\hat{\pi}_t} \frac{C_t}{C_t^*}$$

(51)

We have shown above that, in response to shocks to $Z_{H_t}$, the real exchange rate moves less than the nominal one, that is $\frac{\hat{E}_t P^*_t}{P_t} < \hat{\pi}_t^e$ (while $\hat{\pi}_t = \hat{\pi}_t^e = 0$). Then, it must be the case that the ratio of Home to Foreign consumption $\frac{C_t}{C_t^*}$ comoves negatively with $\frac{\hat{E}_t P^*_t}{P_t}$. Key to this result is the high variability of the terms of trade implied by our setup with distribution costs — we further explore this issue in related work addressing the Backus and Smith puzzle (see Corsetti, Dedola and Leduc [2002]).

6 Equilibrium under alternative asset structures

In this section, we analyze our model under alternative specifications of the asset markets. We carry out numerical exercises assuming financial autarky, trade in

---

32 As first pointed out by Backus and Smith [1993], in the data there is virtually no correlation between relative consumption levels and real exchange rates. Clearly, this empirical regularity cannot be explained by models allowing for trade frictions but relying on complete asset markets: in such a setup, efficient risk-sharing would imply higher consumption levels for countries that experience a fall in the real price of consumption. Relative consumption levels across countries would then be perfectly correlated with the real exchange rate. What is less obvious is that the ‘Backus-Smith anomaly’ would not be explained by simply combining nontradability with incomplete markets frictions. Important examples are provided by Obstfeld and Rogoff [2001] and Chari, Kehoe and McGrattan [2000]. According to the numerical results in the latter contributions, limiting international asset trade to bonds only does not seem to matter for the correlation between relative consumption and the real exchange rate — that remains high and positive as in their complete market specification.
uncontingent nominal bonds, and trade in nominal claims contingent on real and monetary shocks — i.e., claims to receiving one unit of currency in each state of nature. We study the response of our economy to unexpected nominal and real shocks, defined as a 1 per cent deviation from their initial steady state values, allowing for both permanent shocks and temporary shocks (that last only one period). When trade in assets is limited to nominal uncontingent bonds, it is well known that the effects of shocks on the wealth distribution across countries will generate an endogenous dynamics (see Obstfeld and Rogoff [1996], Chapter 10). Thus, we solve for the equilibrium path assuming that after the shock realization the economy evolves under perfect foresight to its new steady state, characterized by a different world distribution of wealth. When shocks are permanent, this adjustment is obviously immediate.

As discussed above, multiple equilibria are possible in our model depending on the size of the distribution margin. It turns out that we can rule out multiplicity by choosing parameters’ values corresponding to a realistic magnitude of the distribution margin — around 50 percent. Relative to our first numerical exercise in Section 4, we can therefore ensure a unique equilibrium by lowering the value of \( \eta \) from .5 to .28, while keeping all other parameters’ values unchanged.

The results of our exercises are shown in Table 2, that reports the percentage changes in the nominal (NER) and real exchange rate (RER), the terms of trade (TOT), real GDP (at constant steady-state prices), employment and consumption for the Home country under different asset trading arrangements. Observe that, qualitatively, the equilibrium response to shocks in the economies with trade in bonds are in line with our analysis in the previous section. Relative to the economy with no asset trade, however, there are apparent quantitative differences. Notably, there is a large drop in the exchange rate volatility between financial autarky and the bond economy (volatility is even lower with trade in contingent bonds). What explains such differences? In general, a positive productivity shock in the Home country tends to worsen the terms of trade, reflecting greater scarcity of the Foreign traded good. In the bond-only economy, markets for insurance against country-specific shocks do not exist. But by running a current account deficit, the Home country can export less and import more, thus limiting the relative supply of Home
goods in the international market, and containing the fall in its terms of trade. In financial autarky, instead, all trade has to be *quid pro quo*. Relative to the bond economy, the Home country must export more and import less. But this, in turn, requires larger movements in the terms of trade and in nominal and real exchange rates. With this intuitive explanation in mind, we now turn to discussing our exercises in detail.

**Nominal and real exchange rate** With incomplete assets markets, positive nominal and real shocks result in a nominal exchange rate depreciation. Consider first the response of the nominal exchange rate to monetary shocks (first two columns in Table 2). With incomplete markets, the NER is more volatile than the underlying shock. A 1 percent increase in $\mu$ — equivalent to a 1 percentage point drop in the nominal interest rate — depreciates $E$ by 1.3 per cent in the bond economy, and by a striking 8.8 per cent under financial autarky.\(^\text{33}\) The intuition provided by (44) for the case of financial autarky carries over in our bond-only economy — although the effect is subdued due to the possibility of international borrowing and lending.

With complete markets, instead, the exchange rate and $\mu$ move in proportion as

$$E_t = \frac{\mu_t}{\mu_t^*}. \quad (52)$$

We should note here that the above solution also characterizes the exchange rate in our incomplete-market economy in the limiting case $\eta \to 0$ — when our specification becomes similar to Corsetti Pesenti [2001a], Obstfeld and Rogoff [2000a] and Devereux and Engel [2000]. With constant demand elasticities (no distributive trade), nominal shocks cannot produce any excess volatility. Furthermore, the exchange rate responds to real shocks in the economy only through endogenous changes in the relative monetary policy stance $\mu_t / \mu_t^*$.\(^\text{33}\)

\(^{33}\)In all our exercises, following a permanent shock to $\mu$, the exchange rate displays no endogenous dynamics. It immediately jumps to the new depreciated level, and stays there. Recall that a permanent shock to $\mu$ implies constant domestic and foreign interest rates. A temporary increase in $\mu$ instead decreases the domestic interest rate for one period. By the uncovered interest parity, the exchange rate is expected to *appreciate* in the future, although it will be *depreciated* with respect to the initial steady state level.
Table 2: Impact Responses of Selected Variables to Nominal and Real Shocks under Alternative Asset Markets Structures  
(Percentage deviations from steady-state values)

<table>
<thead>
<tr>
<th></th>
<th>Monetary Shock</th>
<th>Shock to Tradable</th>
<th>Shock to Nontradable</th>
<th>Economy-wide Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Temporary</td>
<td>Permanent</td>
<td>Temporary</td>
<td>Permanent</td>
</tr>
<tr>
<td><strong>NER</strong></td>
<td>8.8%</td>
<td>8.8%</td>
<td>3.9%</td>
<td>3.9%</td>
</tr>
<tr>
<td><strong>RER</strong></td>
<td>6.8%</td>
<td>6.8%</td>
<td>3.0%</td>
<td>3.0%</td>
</tr>
<tr>
<td><strong>Financial Autarky</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TOT</strong></td>
<td>6.8%</td>
<td>6.8%</td>
<td>3.9%</td>
<td>3.9%</td>
</tr>
<tr>
<td><strong>GDP</strong></td>
<td>0.9%</td>
<td>0.9%</td>
<td>0.2%</td>
<td>0.2%</td>
</tr>
<tr>
<td><strong>Employment</strong></td>
<td>0.9%</td>
<td>0.9%</td>
<td>-0.1%</td>
<td>-0.1%</td>
</tr>
<tr>
<td><strong>Consumption</strong></td>
<td>0.02%</td>
<td>0.02%</td>
<td>-0.3%</td>
<td>-0.3%</td>
</tr>
<tr>
<td><strong>NER</strong></td>
<td>1.3%</td>
<td>1.3%</td>
<td>0.2%</td>
<td>3.9%</td>
</tr>
<tr>
<td><strong>RER</strong></td>
<td>1.0%</td>
<td>1.0%</td>
<td>0.1%</td>
<td>3.0%</td>
</tr>
<tr>
<td><strong>Bond economy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TOT</strong></td>
<td>1.0%</td>
<td>1.0%</td>
<td>1.0%</td>
<td>3.9%</td>
</tr>
<tr>
<td><strong>GDP</strong></td>
<td>0.9%</td>
<td>0.9%</td>
<td>0.2%</td>
<td>0.2%</td>
</tr>
<tr>
<td><strong>Employment</strong></td>
<td>0.9%</td>
<td>0.9%</td>
<td>-0.1%</td>
<td>-0.1%</td>
</tr>
<tr>
<td><strong>Consumption</strong></td>
<td>0.8%</td>
<td>0.8%</td>
<td>0.1%</td>
<td>-0.3%</td>
</tr>
<tr>
<td><strong>NER</strong></td>
<td>1.0%</td>
<td>1.0%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td><strong>RER</strong></td>
<td>0.8%</td>
<td>0.8%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td><strong>Complete Markets</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TOT</strong></td>
<td>0.8%</td>
<td>0.8%</td>
<td>0.9%</td>
<td>0.9%</td>
</tr>
<tr>
<td><strong>GDP</strong></td>
<td>0.9%</td>
<td>0.9%</td>
<td>0.2%</td>
<td>0.2%</td>
</tr>
<tr>
<td><strong>Employment</strong></td>
<td>0.9%</td>
<td>0.9%</td>
<td>-0.1%</td>
<td>-0.1%</td>
</tr>
<tr>
<td><strong>Consumption</strong></td>
<td>0.9%</td>
<td>0.9%</td>
<td>0.1%</td>
<td>0.1%</td>
</tr>
</tbody>
</table>
Without trade in contingent bonds and \( \eta > 0 \), real shocks to productivity depreciate NER independently of monetary policy (third through sixth column in the table). The magnitude of the effects on NER of innovations to \( Z_H \) and \( Z_N \) is comparable (slightly lower in the case of shocks to \( Z_N \)). In the bond economy, temporary productivity shocks have a much smaller impact than permanent shocks (.2 percent vs. 3.9 percent), against which uncontingent bonds provide no insurance mechanism.

Observe that the real exchange rate is positively correlated with the nominal exchange rate (so that it always depreciates in response to positive nominal and real shocks). The table also shows that RER is less volatile than NER in response to nominal shocks and productivity shocks in the tradable goods sector. It is as volatile or more volatile in response to shocks to the nontradable goods sector.

Finally, recall that, in the bond economy, the long-run value of the real exchange rate will be a function of the accumulation of foreign assets and liabilities in response to temporary productivity shocks and nominal shocks (permanent productivity shocks do not cause a current account imbalance). These long-run movements will however tend to be quite small relative to the initial steady state. With temporary shocks, the real exchange rate will therefore overshoot in the short run its long-run value.

**Terms of trade** In all our exercises, the terms of trade is positively correlated with the nominal exchange rate (TOT and NER move in the same direction). As shown by our table, a nominal depreciation always worsens the relative price of Home exports, regardless of the nature of the shock – raising the possibility of expenditure switching effects from exchange rate movements.

Conditional on the nature of the underlying disturbance, however, the terms of trade can be more or less volatile than the nominal (and real) exchange rate. The table confirms this result for the economy in financial autarky, extending it to the case of trade in nominal bonds. In our economy, exporters adjust their optimal prices and markups, passing only a fraction of the movements in exchange rates onto import prices. In financial autarky, for instance, monetary shocks cause the

\[ ^{34} \text{In light of our discussion in the previous section, this is so because the ratio } \eta/(\theta - 1) \text{ is small in our parameterization.} \]
TOT to worsen by 6.8 percent, 2 percentage points less than the depreciation of the NER. In the bond economy, the TOT fall by 30 percent less than the NER, moving approximately in proportion with the nominal shocks.

Shocks in the nontradable sector have a minor effect on the terms of trade. Conversely, shocks to the tradable sector raise the volatility of the terms of trade above that of the nominal (and real) exchange rate. Combining both shocks — i.e. allowing for an aggregate macro shock to domestic productivity — relative volatility depends on how persistent the shock is (see the last two columns of Table 2). Permanent innovations cause the nominal exchange rate to move more than the real exchange rate and the terms of trade. Temporary shocks cause the nominal exchange rate to move by less.\textsuperscript{35}

**Consumption, labor and GDP** Across most exercises, exchange rate movements are several times larger than the average changes in consumption and output. For instance, in response to permanent shocks to productivity, the nominal and real exchange rate are between 5 and 20 times more volatile than real output, which, in turn, is in general more volatile than consumption. So, the model may be consistent with a high volatility of the real exchange rate relative to real variables.

By the same token, domestic nominal price movements tend to be quite contained with respect to those in international nominal prices, due to the low degree of pass-through that characterizes the model. As domestic CPI is equal to $\mu/C$, it is easy to gauge its response to shocks from that of real consumption. Changes in the domestic price level are in general a great deal smaller than changes in the nominal exchange rate, particularly so in the face of nominal shocks: in this case, the CPI response is ten times smaller than that of the nominal exchange rate.

\textsuperscript{35}Depending on the specification of shocks, our model can be consistent with a key stylized fact for the OECD countries, that the exchange rate is more volatile than the terms of trade.
7 Market segmentation and national welfare

7.1 Distribution services and risk sharing

In this section, we characterize the command optimum allocation and discuss the implications of distribution services for the risk-sharing mechanism in the model.\footnote{In all our welfare exercises, following Obstfeld and Rogoff [2000a,b], we assume that liquidity services from money are negligible in utility terms, i.e. $\chi \to 0$.}

Consider the welfare allocation that a planner would choose by maximizing a weighted average of representative consumer’s utility in each country, subject to the resource constraint (including distribution requirements). Assuming equal welfare weights across countries, it is easy to show that the command optimum allocation should satisfy

\[
\tilde{q}_t = \frac{1}{\kappa_t} \left\{ 1 + \frac{\gamma}{2} \left[ \left( \frac{1}{1 + \frac{\kappa_t^* Z_{N,t}}{\kappa_t Z_{H,t}}} \right) + \frac{\eta}{\kappa_t Z_{F,t}} \right] - 1 \right\} \tag{53}
\]

\[
\tilde{C}_t = \frac{Z_{N,t}}{\kappa_t} \left( \frac{1}{\eta + \frac{Z_{N,t}}{Z_{H,t}}} \right) \left( \frac{1}{\eta + \frac{\kappa_t^* Z_{N,t}}{\kappa_t Z_{F,t}}} \right)
\]

where a tilde (\(\tilde{}\)) denotes first-best quantities. In the first best allocation, Home consumption and employment generally respond to both domestic and foreign shocks. Specifically, Home consumption is increasing in Home productivity shocks as well as in $Z_{F,t}$, but it is insulated from shocks to Foreign nontradables. Home employment, instead, is decreasing in response to positive Home productivity shocks, while increasing in response to Foreign productivity shocks. Moreover, in the employment equation, the expression in curly brackets can be either larger or smaller than unity. For given $Z$’s, Home employment will either increase or decrease depending on whether $\kappa$ turns out to be small or large relative to $\kappa^*$ — reflecting a differential in the disutility of work effort.

With full risk sharing, shocks to Home $Z$’s are unambiguously welfare-improving for the Home representative agent (who consume more and work less as a result). A productivity shocks to Foreign nontradables, instead, will reduce the welfare of
the Home representative agent (who will work more for a given consumption). The international welfare effect of a productivity improvement in the Foreign tradable sector is ambiguous: Home agents end up working but also consuming more.

Notably, with no distribution services (i.e. \( \eta = 0 \) — see Obstfeld and Rogoff [2000a]), the above expressions become

\[
\tilde{\ell}_t(\eta = 0) = \frac{1}{\kappa_t} \quad \text{and} \quad \tilde{C}_t(\eta = 0) = \frac{Z_{N,t}}{\kappa_t} \left( \frac{Z_{H,t}}{Z_{N,t}} \right)^{\frac{\gamma}{\gamma - 1}} \left( \frac{\kappa_t}{\kappa_t^*} \frac{Z_{F,t}}{Z_{N,t}} \right)^{\frac{1}{\gamma - 1}}. \tag{54}
\]

Home employment is now completely insulated from Foreign shocks, while Home consumption is still a function of the same domestic and Foreign disturbances. The presence of distribution services changes the risk-sharing mechanisms in our model, relative to previous literature relying on similar specifications of consumption preferences and asset markets incompleteness.

To shed light on this important issue, consider a world economy with flexible prices and wages and no monopoly power in either the labor or the goods markets (i.e., both \( \phi \) and \( \theta \) are arbitrarily large). The only friction is due to incomplete asset markets: for the sake of simplicity, we assume that there is no trade in financial assets. Using our previous results, it is easy to see that the competitive equilibrium allocation\(^{37}\) is

\[
l_t = \frac{1}{\kappa_t} \quad \text{and} \quad C_t = \frac{Z_{N,t}}{\kappa_t} \left( \frac{1}{\eta + \frac{Z_{N,t}}{Z_{H,t}}} \right)^{\frac{\gamma}{\gamma - 1}} \left( \frac{\kappa_t}{\kappa_t^*} \frac{Z_{H,t}}{Z_{N,t}} - \eta \frac{Z_{F,t}}{Z_{N,t}} \frac{Z_{N,t}}{Z_{H,t}} \right)^{\frac{1}{\gamma - 1}}. \tag{55}
\]

Comparing these expressions with the planner allocations (53) and (54), it is apparent that a decentralized competitive allocation coincides with the first best either if all shocks are global, or if \( \eta = 0 \). In our setup without distribution services, even if there is no trade in financial assets, terms of trade movements in response to worldwide shocks are sufficient to generate optimal risk sharing (according to the benchmark given by (54)). An essential ingredient for this result is a unit elasticity of consumption preferences for Home and Foreign traded goods — see Cole and Obstfeld [1991], Corsetti and Pesenti [2001a] and Obstfeld and Rogoff [2000a].

In the general case with \( \eta > 0 \), however, terms of trade movements are insufficient to generate optimal risk sharing. Compare once again the allocations (53) and

\(^{37}\)See also Section 8 below.
(55). First, competitive prices do not provide the right incentive for domestic employment to respond to Home and Foreign shocks to productivity and preferences. In particular, competitive goods markets do not generate the negative cross-country correlation of effort called for by optimal risk sharing in the face of all shocks. Second, terms of trade movements do not insulate domestic consumption from productivity shocks to nontradables in the other country.

Furthermore, in equilibrium price movements may adversely propagate some shocks. For instance, according to the first-best allocation (53) an improvement in Home tradables productivity $Z_{H,t}$ should unambiguously increase Home consumption. In the competitive equilibrium, instead, for intermediate ranges of $\eta$, a positive shock to $Z_{H,t}$ may entail a drop in Home consumption due to worsening Home terms of trade. An important conclusion is that, even disregarding firms’ market power, the presence of distribution services drives the market and the planner allocations apart under restrictions to international assets trading.

7.2 Monetary policy and the natural rate of employment

Despite nominal wage contracts, in our model there exists a simple monetary rule that can sustain a flex-price, flex-wage allocation. Consider the following monetary rules under preset wages:

$$
\mu_t = \frac{\Gamma_t}{K_t}, \quad \mu_t^* = \frac{\Gamma_t^*}{K_t^*}
$$

(56)

where $\Gamma_t$ and $\Gamma_t^*$ are deterministic function of time (therefore known at time $t - 1$), scaling the level of wages in the economy. Note that, according to the above rules, the domestic monetary stance is tighter in those states of the world when the disutility of working is higher. Moreover, the above rule is completely “inward-looking”, in the sense that monetary authorities only react to domestic shocks to the labor market.

Proposition 2 In a decentralized equilibrium, the monetary policy rule (56) supports the flexible-wage, flexible-price allocation.

As monetary policy in either country completely stabilizes the marginal disutility of working, according to (30) and (31) workers will find it optimal to set a nominal wage equal to $\Gamma_t \phi / (\phi - 1)$ and $\Gamma_t^* \phi / (\phi - 1)$. This is exactly the flexible wage (43)
that would result when (56) is implemented. It follows that the above monetary rules support the flexible wage allocation.

The Proposition above establishes that market segmentation due to distributive trade does not prevent the possibility that monetary policy rules sustain the allocation without nominal rigidities. As shown by Corsetti and Pesenti [2001b], instead, this is impossible when market segmentation and incomplete pass-through can be attributed to local currency pricing (even partial local currency pricing) by firms.

In the presence of market segmentation, however, the results in the previous section suggest that this policy will not be optimal.\footnote{The literature on optimal monetary policy in the open economy under nominal rigidities has been growing very fast in the last few years. Besides the papers by Corsetti and Pesenti [2001b] and Obstfeld and Rogoff [2000a,b] already mentioned, an incomplete list includes Benigno and Benigno [2000], Clarida, Galí and Gertler [2001], Devereux and Engel [2000], Galí and Monacelli [2000], Sutherland [2001], Svensson [2000], Tille [2002] and Walsh [1999]. Excellent surveys of these issues in closed economy are contained in Galíñoxed [2000] and Woodford [2000].} Even disregarding the complex issues raised by the possibility of multiplicity of equilibria — therefore assuming that the equilibrium is unique — a benevolent government may be able to do better than simply replicating an equilibrium without nominal rigidities. The monetary authority can optimize over the trade-offs between different kinds of distortions, due to international price discrimination, the lack of risk sharing, and nominal rigidities in the labor market. In other words, the monetary authority will try to balance the different distortions arising from market segmentation and incomplete markets, rather than completely redressing that induced by wage stickiness.

8 Competitive goods markets and gains from cooperation

A crucial departure of our model from related contributions allowing for distributive trade is the analysis of the interaction of nontraded retail services and firms’ market power. Many of our results — most notably, imperfect pass-through at the import price level — derive exactly from this feature of our specification. It is nonetheless instructive to analyze our model in the limiting case in which different brands of
Table 3: The solution with competitive good markets in financial autarky

\[ W_t = \frac{\phi}{\phi - 1} E_{t-1} \kappa_t \mu_t \]  \hspace{1cm} (57)

\[ W_t^* = \frac{\phi}{\phi - 1} E_{t-1} \kappa_t^* \mu_t^* \]  \hspace{1cm} (58)

\[ \mathcal{E}_t = \frac{\mu_t - \eta \mu_t^*}{E_{t-1} \kappa_t \mu_t} \left( \frac{Z_{\text{F},t}}{Z_{\text{H},t}} \right) \]  \hspace{1cm} (59)

\[ \ell_t = \frac{\phi - 1}{\phi} \frac{\mu_t}{E_{t-1} \kappa_t \mu_t} \]  \hspace{1cm} (60)

\[ \ell_t^* = \frac{\phi - 1}{\phi} \frac{\mu_t^*}{E_{t-1} \kappa_t^* \mu_t^*} \]  \hspace{1cm} (61)

\[ C_t = \frac{\phi - 1}{\phi} \frac{\mu_t Z_{\text{H},t}}{E_{t-1} \kappa_t \mu_t} \left( \frac{Z_{\text{H},t}}{Z_{\text{F},t}} + \eta \right)^{-\frac{1}{2}} \left[ \frac{E_{t-1} \kappa_t \mu_t}{\mu_t} \left( \frac{Z_{\text{F},t}}{Z_{\text{H},t}} - \frac{Z_{\text{N},t}}{Z_{\text{N},t}} \right) \right]^{-\frac{1}{2}} \]  \hspace{1cm} (62)

\[ C_t^* = \frac{\phi - 1}{\phi} \frac{\mu_t^* Z_{\text{N},t}}{E_{t-1} \kappa_t^* \mu_t^*} \left( \frac{Z_{\text{N},t}}{Z_{\text{F},t}} + \eta \right)^{-\frac{1}{2}} \left[ \frac{E_{t-1} \kappa_t \mu_t}{\mu_t} \left( \frac{Z_{\text{F},t}}{Z_{\text{H},t}} - \frac{Z_{\text{N},t}}{Z_{\text{N},t}} \right) \right]^{-\frac{1}{2}} \]  \hspace{1cm} (63)
tradable goods are perfect substitutes ($\theta \to \infty$), so that, with competitive goods markets, firms are no longer able to segment markets across national borders. The law of one price holds at the producer price level (this is apparent when evaluating expression (28) as $\theta/(\theta - 1)$ approaches unity). When prices are measured net of distribution costs, it also holds at the consumer price level.

With perfectly competitive goods markets, the solution of the model considerably simplifies. For the case of financial autarky, we are actually able to solve the model in closed form. The allocation under perfect competition in the goods market and without trade in assets is shown in Table 3. Under complete pass-through, exchange rate determination in (59) is consistent with the law of one price. The exchange rate is a linear function of monetary policy stances as well as of productivity shocks — in general depreciating with a positive shock to Home tradable production. The levels of Home and Foreign employment, (60) and (61), only depend on the domestic monetary stance. Conversely, domestic consumption responds to policy shocks anywhere in the world economy. A Foreign monetary expansion that appreciates the exchange rate (in the domestic currency) improves the Home terms of trade and raises Home consumption. In this case a positive shock to $\mu^*_k$ triggers an expenditure switching effect.

The equilibrium is always unique. Yet, in a neighborhood of a symmetric equilibrium, a small nominal depreciation can either improve or worsen the trade balance, depending on the relative size of distributive trade: the condition (40) is satisfied only when the distribution margin is larger than 50 per cent, that is, $\eta \frac{Z_T}{Z_N} > 1$. Observe that, relative to the benchmark of competitive goods markets, allowing for imperfect competition may either increase or decrease the volatility of the nominal and real exchange rates, depending on whether the condition (40) — the opposite of Marshall-Lerner — holds. This can be seen by evaluating (44) and (49) for $\theta \to \infty$, when $\eta \frac{Z_T}{Z_N}$ is close to one.

The following proposition states that, with competitive goods markets, monetary

---

30 The case of perfect competition in a small open economy without nominal rigidities is analyzed by Burstein, Neves and Rebelo [2000].

40 It is interesting to note the different pattern when firms have monopoly power. In this case, labor depends on both domestic and foreign monetary shocks.
authorities can do no better than implementing (56) — even though under financial
autarky the resulting allocation does not coincide with the command optimum (53).
Most crucially, international cooperation is completely superfluous.

**Proposition 3** Without trade in assets, if \( \frac{\theta}{\theta - 1} = 1 \), the monetary policy rule
\( \mu_t = \Gamma_t / \kappa_t \) — as specified in (56) — is optimal in both the world Nash equilibrium
and under cooperation (independently of the welfare weights attributed to the Home
and Foreign country).

To prove this proposition, we note that the first order conditions of the policy
problem in a world Nash equilibrium include the following equation:

\[
\frac{1}{\mu_t} \left( \frac{\kappa_t}{E_{t-1} (\kappa_t \mu_t)} \right) + \gamma \frac{\kappa_t \mu_t - E_{t-1} (\kappa_t \mu_t)}{2 \mu_t} \left( \frac{\kappa_t \mu_t - E_{t-1} (\kappa_t \mu_t)}{\mu_t^2} \frac{Z_{S,t}}{Z_{S,t} \mu_t} \right) = 0 \quad (64)
\]

It is easy to check that the monetary rule \( \mu_t = \Gamma_t / \kappa_t \) is a solution to the above
equation for any Foreign monetary policy stance \( \mu_t^* \).41 To prove the second part of
the proposition, consider the problem of a benevolent centralized planner, setting
\( \mu_t \) and \( \mu_t^* \) to maximize a weighted average of the expected utility of the Home and
Foreign agents, according to arbitrary weights \( \Omega \) and \( 1 - \Omega \):

\[
\max_{\mu_t, \mu_t^*} E_{t-1} \left[ \Omega \left( \log C_t - \kappa_t l_t \right) + (1 - \Omega) \left( \log C_t^* - \kappa_t^* l_t^* \right) \right] \quad (65)
\]

subject to (62) and (63). The first order condition for \( \mu_t \) is

\[
\Omega \left[ \frac{1}{\mu_t} - \frac{\kappa_t}{E_{t-1} (\kappa_t \mu_t)} + \gamma \frac{\kappa_t \mu_t - E_{t-1} (\kappa_t \mu_t)}{2 \mu_t} \right] + \]

\[
(1 - \Omega) \gamma \left[ \frac{1}{\mu_t} - \frac{\kappa_t}{E_{t-1} (\kappa_t \mu_t)} - \eta Z_{F,t} \frac{\kappa_t \mu_t - E_{t-1} (\kappa_t \mu_t)}{\mu_t^2} \right] = 0 \quad (66)
\]

It is apparent that setting \( \mu_t \) according to (56) solves the above equation regardless
of the value of \( \mu_t^* \) and \( \Omega \).

Intuitively, in the absence of firms’ market power, the monetary authority in
each country can achieve a constrained-Pareto efficient allocation by redressing the

\[41\] The above policy would also be optimal in an economy with staggered wage contracts, provided
that the policy makers choose a constant \( \Gamma \) preventing a nominal drift in wages. A constant \( \Gamma \)
however may not be optimal when money yields non-negligible utility services.
distortion from wage stickiness. They cannot, however, address the distortion due to monopoly power in the labor market, nor they can engineer enough risk sharing, as required to achieve the command optimum allocation.\footnote{Using (56), employment and consumption under the optimal rules can be easily obtained from (62) and (60). Because of workers’ market power, they are lower than employment and consumption in (55) by the fraction $\frac{\phi - 1}{\phi}$.}

Notably, competitive goods markets make international policy cooperation completely redundant even under financial autarky, which means that there are no monetary policy spillovers in equilibrium. This example extends a key result in Obstfeld and Rogoff [2000b] to an environment with low pass-through to consumer prices and less than optimal risk-sharing.

9 Conclusions

In the last few years, a vast and growing literature in international macroeconomics has been devoted to the study of macroeconomic interdependence using general-equilibrium models with nominal rigidities at either price or wage level. This literature has broken new ground in the analysis of stabilization policies in open-economy, boosting our understanding of the international transmission of both policy and real shocks. Yet, its theoretical development has long been hindered by the lack of explicit modeling of endogenous price discrimination and pass-through. The model in this paper is a step in this direction, bringing open macro theory closer in line with those key stylized facts of the international economy related to large discrepancies in international prices.

Relative to the existing literature, this paper opens a new perspective to model the macroeconomy, and raises new issues. Several contributions have recently stressed the importance of placing international price discrimination centerstage in the analysis of open economies. In this paper we have shown that, among the alternative ways to do it, modelling vertical relationships among firms located in different markets is a realistic and promising strategy. For instance, we show that, due to the presence of downstream retailers, upstream firms with monopoly power may face different demand elasticities in national markets even under symmetric, constant elasticity
preferences across countries. Thus, they will optimally charge different prices to
domestic and foreign dealers — within the limits dictated by the possibility of inter-
national arbitrage between wholesale and retail good markets. As a consequence, the
law of one price fails to hold at both producer and consumer levels, independently of
nominal rigidities. Second, as firms optimally adjust markups in the face of demand
fluctuations, the response of prices to exchange rate movements is muted at both
consumer and producer levels. Exchange rate pass-through is not complete. Third,
our model yields that an exchange rate depreciation generally worsens the terms of
trade. Thus, despite low pass-through the international transmission of monetary
shock can result into expenditure switching effects. Fourth, our specification can si-
multaneously account for the high volatility of the exchange rate relative to output,
and the Backus Smith anomaly — an issue further analyzed by Corsetti, Dedola
and Leduc (2002). To our knowledge, this has never been accomplished before.

Finally, we have shown that there could be multiple equilibria in which large dif-
fferences in the nominal and real exchange rate are associated with small difference
in the real allocation and the price level. The possibility of multiplicity is an im-
portant issue that requires further research at both empirical and theoretical level,
raising a number of questions in the design of stabilization policies at the national
and international level.

Key to our approach is that distributive trade requires local inputs, that is, that
there are cost linkages among firms across national boundaries. It is worth stressing
that cost linkages are not exclusively due to distributive trade. Realistically, local
inputs can be employed in some final stage of manufacturing of the final product
at local level, combined with traded intermediate goods. Encompassing both dis-
tributive trade and manufacturing at local level, the share of the consumer prices
that can be attributed to the cost of local inputs may actually become quite high,
reinforcing many of the novel results of our analysis.

The model in this paper has been purposely kept simple by means of convenient
assumptions. For instance, the elasticity of substitution among individual goods
(brands) is the same in all sectors, the elasticity of substitution among types of
good is set equal to one, and there is no difference between nontraded goods and
distribution services. Thus an important task for future work would be to appro-
appropriately relax these assumptions in order to confront the model more directly with
the data.

Most crucially, we have assumed the most basic vertical structure: an upstream
monopolist sells the good it produces to a perfectly competitive downstream firm,
the retailer. In this case, without distortionary taxation at national level, vertical
integration would be completely neutral as regards the allocation. An important task
for future research would be to generalize our setup to richer interactions between
upstream and downstream firms, bringing more realism – and more microeconomics
– into the construction of open macro models.
A Appendix

The problem of the Home representative consumer  The first order conditions of the Home consumer’s problem with respect to $C_{H,t}(j)$, $C_{F,t}(j)$, $C_{N,t}(j)$, $B_{H,t+1}(j)$, $B_{F,t+1}$ and $M_t(j)$ are, respectively:

$$\frac{\gamma}{C_{H,t}(j)} = 2\lambda_t(j) P_{H,t} \quad (67)$$

$$\frac{\gamma}{C_{F,t}(j)} = 2\lambda_t(j) P_{F,t} \quad (68)$$

$$\frac{1 - \gamma}{C_{N,t}(j)} = \lambda_t(j) P_{N,t} \quad (69)$$

$$\lambda_t(j) = \beta E_t \lambda_{t+1}(j) (1 + \dot{u}_{t+1}) \quad (70)$$

$$E_t \lambda_t(j) = \beta E_t \lambda_{t+1}(j) (1 + \dot{u}_{t+1}) \quad (71)$$

$$\lambda_t(j) = \beta E_t \lambda_{t+1}(j) + \lambda_t \left( \frac{M_t(j)}{P_t} \right) - \varepsilon \frac{-\varepsilon}{P_t} \quad (72)$$

From these conditions, it is easy to see that, at the optimum, the individual demand for Home, Foreign and nontraded consumption goods are a constant share of total consumption expenditure:

$$P_t C_t(j) = \frac{2}{\gamma} P_{H,t} C_{H,t}(j) = \frac{2}{\gamma} P_{F,t} C_{F,t}(j) = \frac{1}{1 - \gamma} P_{N,t} C_{N,t}(j). \quad (73)$$

Using these expressions, it is easy to verify that

$$P_t C_t(j) = P_{H,t} C_{H,t}(j) + P_{F,t} C_{F,t}(j) + P_{N,t} C_{N,t}(j). \quad (74)$$

The intertemporal allocation of consumption is determined according to the Euler equation:

$$\frac{1}{C_t(j)} = \beta (1 + \dot{u}_{t+1}) E_t \left( \frac{P_t}{P_{t+1} C_{t+1}(j)} \right) \quad (75)$$

Finally, condition (72) can be written as the money demand function:

$$\left( \frac{M_t(j)}{P_t} \right)^\varepsilon = \chi_t \frac{1 + \dot{u}_{t+1}}{\dot{u}_{t+1}} C_t(j) \quad (76)$$
Define the variable $Q_{t,t+1}(j)$ as:

$$Q_{t,t+1}(j) = \frac{\beta P_t C_t(j)}{P_{t+1} C_{t+1}(j)}$$  \hfill (77)$$

which is agent $j$’s stochastic discount rate. Comparing (77) with (70) and (71), we obtain:

$$E_t Q_{t,t+1}(j) = \frac{1}{1 + i_{t+1}}, \quad E_t \left[ Q_{t,t+1}(j) \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right] = \frac{1}{1 + i_{t+1}^*}$$  \hfill (78)$$

It follows that the risk-adjusted uncovered interest parity linking domestic and foreign nominal interest rates is:

$$\frac{1 + i_{t+1}}{1 + i_{t+1}^*} = E_t \left( \frac{\mathcal{E}_{t+1}}{P_{t+1} C_{t+1}(j)} \right) \left[ E_t \left( \frac{\mathcal{E}_t}{P_t C_t(j)} \right) \right]^{-1}$$  \hfill (79)$$

Note that, in the absence of uncertainty the previous condition collapses to the familiar expression $1 + i_{t+1} = (1 + i_{t+1}^*) \mathcal{E}_{t+1} / \mathcal{E}_t$.

Using (78) we can write:

$$M_t(j) + B_{H,t+1}(j) = \frac{i_{t+1} M_t(j)}{1 + i_{t+1}} + E_t \left[ Q_{t,t+1}(j) \left( M_t(j) + (1 + i_{t+1}) B_{H,t+1}(j) \right) \right]$$  \hfill (80)$$

and:

$$\mathcal{E}_t B_{F,t+1}(j) = E_t \left[ Q_{t,t+1}(j)(1 + i_{t+1}^*) \mathcal{E}_{t+1} B_{F,t+1}(j) \right]$$  \hfill (81)$$

It follows that the flow budget constraint (11) is:

$$\frac{i_{t+1} M_t(j)}{1 + i_{t+1}} + E_t \left[ Q_{t,t+1}(j) A_{t+1}(j) \right] \leq A_t(j) + R_t(j) - T_t(j) - P_t C_t(j)$$  \hfill (82)$$

where $A_{t+1}$ is wealth (net assets) at the beginning of period $t+1$, defined as:

$$A_{t+1}(j) = M_t(j) + (1 + i_{t+1}) B_{H,t+1}(j) + (1 + i_{t+1}^*) \mathcal{E}_{t+1} B_{F,t+1}(j).$$  \hfill (83)$$

Optimization implies that households exhaust their intertemporal budget constraint: the flow budget constraint hold as equality and the transversality condition below is satisfied:

$$\lim_{N \to \infty} E_t \left[ Q_{t,N}(j) A_N(j) \right] = 0$$  \hfill (84)$$

where $Q_{t,N} = \prod_{s=t+1}^{N} Q_{s-1,s}$. If an interior solution exists (as is the case given our parameterization), the resource constraint holds as equality as well. Similar results characterize the optimization problem of Foreign agent $j^*$.  

50
**Wage setting** Consider now the problem of choosing an optimal nominal wage rate one period in advance. Let $W(j)$ denote the nominal wage of worker $j$, and define the Home country wage index $W$ as

$$W_t = \left[ \int_0^1 W_i(j)^{1-\phi} dj \right]^{1/\phi}$$  \hspace{1cm} (85)

The (constant-elasticity) demand for labor input $j$ by the firm $n$ can be expressed as follows

$$L_t(n, j) = \left[ \frac{W_t(j)}{W_t} \right]^{-\phi} \frac{Y_t(n)}{Z_{N,t}}$$  \hspace{1cm} (86)

Using the fact that $\phi$ is the same across sector, the total demand for the labor input supplied by $j$ to all domestic firms is

$$L_t(i, j) = \left[ \frac{W_t(j)}{W_t} \right]^{-\phi} \left[ \int_{h=0}^1 \frac{Y_t(h)}{Z_{H,t}} dh + \int_{n=0}^1 \frac{Y_t(n)}{Z_{N,t}} dn \right]$$  \hspace{1cm} (87)

Workers are assumed to be monopolistic suppliers of a particular type of labor; thus, they take into account the above demand schedule when fixing the nominal wage rate. We posit that the number of workers is large enough so that they ignore the impact of their own pricing decision on the aggregate the aggregate wage index. The first order condition for this problem is the expression (15) in the main text.

**Firms’ pricing and the no-arbitrage conditions** This Appendix characterizes optimal price setting when shocks are such that the no-arbitrage condition is binding. The desired prices by the representative Home firm is the solution to the following profit maximization problem

$$\text{Max}_{\mathbf{p}(h), \mathbf{p}^*(h)} \left[ \bar{p}_t(h) C_t(h) + \bar{E}_t \bar{p}^*_t(h) C^*_t(h) \right] - \frac{W_t}{Z_{H,t}} \left[ C_t(h) + C^*_t(h) \right]$$

s.t.

$$C_t(h) + C^*_t(h) = \left[ \bar{p}_t(h) + \eta P_{N,t} \right]^{-\theta} \bar{p}^*_t(h) + \left[ \bar{p}_t(h) + \eta P^*_t(h) \right]^{-\theta} \left( \bar{p}^*_t \right)^\theta C^*_t$$

$$\bar{p}_t(h) + \eta P_{N,t} - E_t \bar{p}^*_t(h) \geq 0$$

$$E_t \left( \bar{p}^*_t(h) + \eta P^*_t \right) - \bar{p}_t(h) \geq 0$$
The relevant FOC’s for \( \pi(h) \) and \( \pi^x(h) \) (including the complementary slackness conditions and dropping time subscripts) are, respectively:

\[
[\pi(h) + \eta P_N]^{-\theta} P_H^\theta C_H \left( 1 - \theta \frac{\pi(h)}{\pi(h) + \eta P_N} + \theta \frac{W}{Z_H} \right) + \xi_1 = \xi_2
\]

\[
[\pi^x(h) + \eta P_N^x]^{-\theta} (P_H^x)^\theta C_H^x \left( 1 - \theta \frac{\pi^x(h)}{\pi^x(h) + \eta P_N^x} + \theta \frac{W}{Z_H} \right) + \xi_2 = \xi_1
\]

\[
\xi_1 \left[ \pi(h) + \eta P_N - \mathcal{E} \pi^x(h) \right] = 0, \quad \pi(h) + \eta P_N - \mathcal{E} \pi^x(h) \geq 0, \quad \xi_1 \geq 0
\]

\[
\xi_2 \left[ \mathcal{E} (\pi^x(h) + \eta P_N^x) - \pi(h) \right] = 0, \quad \mathcal{E} (\pi^x(h) + \eta P_N^x) - \pi(h) \geq 0, \quad \xi_2 \geq 0
\]

The optimal prices discussed in the main text are derived for the case in which \( \xi_1 = \xi_2 = 0 \). An important implication of the these prices is that, if the Home monopolist can freely discriminate across national markets, the same is true of the Foreign one. Before proceeding further, note that, if \( \xi_1 \geq 0 \), and \( \xi_2 \geq 0 \), then it must be that \( \eta \, (P_N + \mathcal{E} P_N^x) = 0 \), which can be true only for \( \eta = 0 \). Obviously, as long as distribution costs are strictly positive, the two constraints cannot be binding at the same time.

Thus, we have to characterize optimal price-setting when either condition is binding. Without loss of generality set \( \xi_1 \geq 0 \) and \( \xi_2 = 0 \), i.e. \( \pi(h) + \eta P_N = \mathcal{E} \pi^x(h) \).

The relevant FOC’s for \( \pi(h) \) and complementary slackness conditions are:

\[
\frac{P_H^\theta C_H \left( 1 - \theta \frac{\pi(h)}{\pi(h) + \eta P_N} + \theta \frac{W}{Z_H} \right)}{[\pi(h) + \eta P_N]^\theta} = -\xi_1 \leq 0,
\]

\[
\frac{(\mathcal{E} P_H^x)^\theta C_H^x \left( 1 - \theta \frac{\pi^x(h) + \eta P_N^x}{\pi^x(h) + \eta (P_N + \mathcal{E} P_N^x)} + \theta \frac{W}{Z_H} \right)}{[\pi^x(h) + \eta (P_N + \mathcal{E} P_N^x)]^\theta} = \xi_1 \geq 0
\]

It follows that, under symmetry, the optimal price \( T_H \) should satisfy

\[
\left( 1 + \frac{\eta}{\theta - 1} \frac{Z_H}{Z_N} \right) \frac{W}{Z_H} < T_H < \left( 1 - \frac{\eta}{\theta - 1} \frac{Z_H}{Z_N} \right) \frac{W}{Z_H} + \frac{\eta}{\theta - 1} \frac{\mathcal{E} W^x}{Z_N},
\]

while solving the following quadratic equation:

\[
C_H \frac{\theta \left( 1 + \frac{\eta}{\theta - 1} \frac{Z_H}{Z_N} \right) \frac{W}{Z_H} - (\theta - 1) T_H}{T_H + \eta \frac{W}{Z_N}} + C_H \frac{\theta \left( \frac{1}{Z_H} - \frac{\eta}{\theta - 1} \frac{1}{Z_N} \right) W + \frac{\eta}{\theta - 1} \frac{\mathcal{E} W^x}{Z_N} - (\theta - 1) T_H}{T_H + \eta \frac{W}{Z_N} + \frac{\mathcal{E} W^x}{Z_N}} = 0.
\]

The optimality condition for \( T_F \) mirrors the above expression.
The current account  We focus on an equilibrium in which domestic agents are symmetric within a country (although there could be asymmetries across countries). Aggregating the individual budget constraints and using the government budget constraint we obtain an expression for the Home current account:

$$E_t \left\{ Q_{t+1} A_{t+1} \right\} = \overline{A}_t + R_t - P_t C_t$$

(88)

where $\overline{A}$ is defined as wealth net of money balances, or

$$\overline{A}_{t+1} = A_{t+1} - M_t.$$

(89)

Now, $R_t$ is defined as:

$$R_t \equiv \left( \bar{P}_{H,t} + \eta P_{N,t} \right) C_{H,t} + P_{N,t} C_{N,t} + \mathcal{E}_t \bar{P}_{H,t}^a C_{H,t}^a + \eta P_{N,t} C_{F,t}$$

$$= P_t C_t + \mathcal{E}_t \bar{P}_{H,t}^a C_{H,t}^a + \eta P_{N,t} \frac{P_{F,t}}{P_{F,t}} C_{F,t} - \eta \mathcal{E}_t \bar{P}_{H,t}^a C_{H,t}^a$$

$$= P_t C_t - \frac{\gamma}{2} P_t C_t + \frac{\gamma}{2} \mathcal{E}_t \bar{P}_{t}^a C_{t}^a + \eta \frac{P_{N,t}}{P_{F,t}} P_t C_t - \frac{\gamma}{2} \mathcal{E}_t \bar{P}_{H,t}^a C_{H,t}^a$$

$$= P_t C_t \left( 1 - \frac{\gamma}{2} + \frac{\gamma}{2} \frac{P_{N,t}}{P_{F,t}} \right) + \frac{\gamma}{2} \mathcal{E}_t \bar{P}_{t}^a C_{t}^a \left( 1 - \eta \frac{P_{N,t}}{P_{F,t}} \right)$$

Thus the Home current account becomes

$$E_t \left\{ Q_{t+1} A_{t+1} \right\} = \overline{A}_t - \frac{\gamma}{2} \mu_t \left( 1 - \eta \frac{P_{N,t}}{P_{F,t}} \right) + \frac{\gamma}{2} \mathcal{E}_t \mu_t^a \left( 1 - \eta \frac{P_{N,t}}{P_{F,t}} \right)$$

Clearly, in equilibrium $\overline{A}_t = -\mathcal{E}_t \overline{A}_t$, for all $t$.  

ECB • Working Paper No 176 • September 2002  53
References


[51] Walsh, Carl E. [1999]. “Monetary Policy Trade-Offs in the Open Economy,”
mimeo, UC Santa Cruz.

European Central Bank working paper series

For a complete list of Working Papers published by the ECB, please visit the ECB’s website (http://www.ecb.int).


1.23 “Analysing and combining multiple credit assessments of financial institutions” by E. Tabakis and A. Vinci, February 2002.


1.26 “Optimal contracts in a dynamic costly state verification model” by C. Monnet and E. Quintin, February 2002.


1.28 “Openness, imperfect exchange rate pass-through and monetary policy” by F. Smets and R. Wouters, February 2002.


131 “Measurement bias in the HICP: what do we know, and what do we need to know?” by M. A. Wynne and D. Rodriguez-Palenzuela, March 2002.


133 “Can confidence indicators be useful to predict short term real GDP growth?” by A. Mourougane and M. Roma, March 2002.


135 “The optimal mix of taxes on money, consumption and income” by F. De Fiore and P. Teles, April 2002.

136 “Retail bank interest rate pass-through: the new evidence at the euro area level” by G. de Bondt, April 2002.

137 “Equilibrium bidding in the eurosystem’s open market operations” by U. Bindseil, April 2002.


140 “Price setting and the steady-state effects of inflation” by M. Casares, May 2002.


146 “Competition and stability – what’s special about banking?”, by E. Carletti and P. Hartmann, May 2002.


“G-7 inflation forecasts” by F. Canova, June 2002.


“Euro area production function and potential output: a supply side system approach” by A. Willman, June 2002.

“The euro bloc, the dollar bloc and the yen bloc: how much monetary policy independence can exchange rate flexibility buy in an interdependent world?” by M. Fratzscher, June 2002.

“Youth unemployment in the OECD: demographic shifts, labour market institutions, and macroeconomic shocks” by J. F. Jimeno and D. Rodriguez-Palenzuela, June 2002.


“Quantifying Embodied Technological Change” by P. Sakellaris and D. J. Wilson, July 2002.

“Optimal public money” by C. Monnet, July 2002.

“Model uncertainty and the equilibrium value of the real effective euro exchange rate” by C. Detken, A. Dieppe, J. Henry, C. Marin and F. Smets, July 2002.


“Euro area corporate debt securities market: first empirical evidence” by G. de Bondt, August 2002.
165 "The industry effects of monetary policy in the euro area" by G. Peersman and F. Smets, August 2002.

166 "Monetary and fiscal policy interactions in a micro-founded model of a monetary union" by R. M.W.J. Beetsma and H. Jensen, August 2002.

167 "Identifying the effects of monetary policy shocks on exchange rates using high frequency data" by J. Faust, J.H. Rogers, E. Swanson and J.H. Wright, August 2002.


169 "Modeling model uncertainty" by A. Onatski and N. Williams, August 2002.

170 "What measure of inflation should a central bank target?" by G. Mankiw and R. Reis, August 2002.

171 "An estimated stochastic dynamic general equilibrium model of the euro area" by F. Smets and R. Wouters, August 2002.

172 " Constructing quality-adjusted price indices: a comparison of hedonic and discrete choice models" by N. Jonker, September 2002.

173 "Openness and equilibrium determinacy under interest rate rules" by F. de Fiore and Z. Liu, September 2002.

174 "International monetary policy coordination and financial market integration" by A. Sutherland, September 2002.

175 "Monetary policy and the financial accelerator in a monetary union" by S. Gilchrist, J.O. Hairault and H. Kempf, September 2002.