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EMPLOYMENT, HOURS AND OPTIMAL MONETARY POLICY

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Abstract

We characterize optimal monetary policy in a New Keynesian search-and-matching model where multiple-worker firms satisfy demand in the short run by adjusting hours per worker. Imperfect product market competition and search frictions reduce steady state hours per worker below the efficient level. Bargaining results in a convex 'wage curve' linking wages to hours. Since the steady-state real marginal wage is low, wages respond little to hours. As a result, firms overuse the hours margin at the expense of hiring, which makes hours too volatile. The Ramsey planner uses inflation as a instrument to dampen inefficient hours fluctuations.

JEL classification: E30, E50, E60

Keywords: employment, hours, wage curve, optimal monetary policy.

Non-Technical Summary

This paper shows that the availability of *two* labor input margins at the firm level, employment and hours per employee in a large firm setup, gives rise to novel optimal policy prescriptions regarding tax and monetary policies.

Our model has a couple of important and, as we argue, realistic features as compared with much of the existing literature. First, firms' price setting and hiring decisions are subject to costs and frictions. As in Barnichon (2010), Kuester (2010) and Thomas (2011), these two types of frictions are not artificially separated from each other, but affect the *same* firms. This is important, because a firm that has chosen a particular price will adjust its labor input to meet the demand it faces at that price. Second, firms can adjust only hours per worker to satisfy demand in the short run; they can change their workforce only with a lag in response to persistent changes in demand.

The steady state displays distortions along the two labor input margins. First, due to the combination of monopolistic competition in product markets and labor market frictions (wage bargaining coupled with a 'right-to-manage' choice of hours), hours per employee are too low. Second, as a result, the shadow value of the marginal worker is too low and therefore hiring is below the efficient level.

In a large firm, there is another (and well-known) effect on employment that goes in the opposite direction. Hiring shifts the burden of future production away from the intensive and towards the extensive margin. Hours per employee fall and, through intra-firm bargaining the wage paid to *all* workers falls, too. In isolation, this externality leads to overhiring. We demonstrate that, in a standard calibration, the first effect on employment dominates and steady state employment is too low.

We show that the optimal tax policy mix is a subsidy to private consumption (to raise production and hours per employee), combined with a firm revenue tax (to counter the overhiring result that would obtain due to the large-firm externality, see above).

In the absence of fiscal instruments, the steady state distortions lead to inefficient business cycle fluctuations. Our model features a 'wage curve'. The wage set through bargaining is a convex function of hours per worker; the real marginal wage increasing in hours. A low steady-state real marginal wage implies that the real wage and thus real marginal costs are not very sensitive to hours. As a consequence of this real wage rigidity, firms overuse the hours margin relative to the employment margin in response to shocks. The optimal monetary policy uses inflation as an instrument to dampen inefficient fluctuations in hours worked.

1 Introduction

New Keynesian models with labor market frictions have been extensively used for optimal monetary policy analysis.[1] Deviations from Walrasian wage setting affect inflation through the real marginal costs and thus the monetary transmission mechanism (see Walsh, 2005; Krause and Lubik, 2007a; and Krause et al., 2008). Papers inspecting optimal monetary policy in a search-and-matching model usually assume that hours per worker are constant and a firm is composed of a single worker, so he contributes to the firm's profit through its marginal productivity. In this paper, we characterize optimal policy in a large-firm setup with two margins of labor, namely employment and hours per employee. Two related forces affecting wages come up in this realistic framework. First, a firm can hire multiple workers and an additional worker is not productive straight away. Therefore, he contributes to profits by affecting the wage of *all other* workers, which results in a 'wage externality'. Second, a firm can adjust the two labor margins, which sheds light on the 'wage curve', i.e. the real wage is a function of the number of hours worked. A form of 'real wage rigidity' emerge – when wages respond little to hours – due to imperfect competition and labor market frictions. Unlike the typical search-and-matching model, firms can exploit it by overusing the hours margin when adjusting production. We argue that deviation from price stability might occur in this setup. With its lever on inflation and real marginal costs, the Ramsey planner can affect the real wage and thus the firm's hours decision.

Once we allow for firms to have many workers, we move away from the standard one-worker-firm model of Mortensen and Pissarides (1994) and towards a large-firm model. Many studies considering labor market frictions in monetary economies focus on one of two labor input margins, either the extensive margin (employment) or the intensive margin (hours). Ohanian and Raffo (2012), however, stress the importance of accounting for both the extensive and the intensive labor input margin. In our model, employment is predetermined,

[1] See Thomas (2008), Faia (2009), Blanchard and Galí (2010), Ravenna and Walsh (2012), for instance.

i.e. it takes one period for a newly hired worker to become productive. When faced with a shock, firms cannot expand the extensive margin of labor on impact, but instead adjust hours in a 'right-to-manage' fashion. We believe that predetermined employment is a reasonable assumption since VAR evidence suggests that, on impact, employment and unemployment respond little (if at all) to demand shocks (see Monacelli et al., 2010, Brueckner and Pappa, 2012).

The right-to-manage feature by which firms choose hours worked unilaterally, coupled with wage bargaining, results in a 'wage curve', i.e. the real wage becomes a function of the number of hours worked. Kuester (2010) shows that a model with such a 'wage channel' performs well at matching impulse responses in US data. In the presence of the wage curve, instantaneous hiring vs. predetermined employment is not an innocuous model choice. With instantaneous hiring as in Sunakawa (2013), the marginal worker generates profits by contributing directly to production. With predetermined employment, an additional worker contributes to current profits by reducing the wage payments to all existing workers, creating a 'wage externality'. An additional worker reduces the number of hours needed to satisfy future demand as firms shift production from the intensive to the extensive labor margin. Through the wage curve, the reduction in hours results in a reduction in the real wage.

We assume that producers are monopolistically competitive firms that face price rigidities as well as search-and-matching frictions in the labor market (see Barnichon, 2010, Kuester, 2010 and Thomas, 2011). Ebell and Haefke (2009) justify the large-firm assumption under monopolistic competition by arguing that a firm's size is related to its market power. While the separation of these two frictions in a 'producer-retailer structure', as in Trigari (2006), is a useful device for many research questions, we argue that optimal policy prescriptions are rather sensitive to this assumption.

Our contribution is to show that our large-firm model combining the two labor margins gives rise to optimal monetary and fiscal policies. We highlight that both the intensive and the extensive labor margins are distorted in the competitive allocation. Hours per employee are inefficiently low because of the combination of imperfect competition in product markets and labor market frictions (wage bargaining coupled with a 'right-to-manage' choice of hours). Hiring is inefficient for two reasons. All things equal, the wage externality implies that firms tend to over-hire in order to benefit from the reduction in wages (see Stole and Zwiebel, 1996). This effect is, however, dominated by the fact that each employee works too few hours, implying that the steady-state value of an additional worker is lower than is efficient. As a result, both hours and employment are suboptimally low at the steady state. We show how a combination of a tax on firm revenues, a consumption subsidy and a compensating transfer to unemployed home production workers makes the steady state efficient. Under an optimal tax policy mix, fluctuations in real marginal costs due to price setting frictions represent the only cyclical distortions. Therefore, strict inflation targeting is optimal and implements the efficient allocation. However, when tax instruments are unavailable and the steady state is distorted, the Ramsey optimal policy deviates from price stability.

Labor inputs distortions at the steady state result in inefficient cyclical fluctuations in response to shocks. The wage per worker, set through bilateral bargaining, is an increasing and convex function of hours worked, as captured by the wage curve. The slope of the wage curve (i.e. the real marginal wage) is too low at the steady state due to the hours distortion described above. This implies that, in response to shocks, wages do not rise much for a given increase in hours worked. Firms exploit this endogenous 'real wage rigidity' by overusing the hours margin, and potentially underusing the employment margin, when responding to changes in demand or technology. Price rigidities give the Ramsey planner a tool to influence the real marginal wage and thus the tradeoff between the intensive and the extensive labor margin. Inflation is used as a countercyclical policy instrument (with respect to hours worked) to dampen inefficient fluctuations in hours. We show that the magnitude of the deviation of price stability depends on the size of the hours distortion, which in turn is driven by the bargaining power of workers, the disutility cost of hours and the return to hours in production (see also Barnichon, 2012).

Several authors have analyzed the implications of labor search frictions and price rigidities for optimal policy. Ravenna and Walsh (2011) study optimal monetary policy in a linear-quadratic framework, while Sala et al. (2008) use an ad-hoc loss function. These authors do not distinguish between the two labor margins. Thomas (2008) shows that imperfect wage adjustment creates inefficient hiring and leads to optimal deviations from price stability. Blanchard and Galí (2010) study the effect of real wage rigidities on the inflation-unemployment trade-off. However, they all restrict their attention to the case of an efficient steady state. Faia (2009) shows that deviations from the efficient steady state, through the Hosios condition, imply that optimal monetary policy does not fully stabilize prices. Sunakawa (2013) extends this analysis by assuming that hours are chosen in a right-to-manage fashion. Importantly, unlike that author, employment is predetermined in our model. Finally, Ravenna and Walsh (2012) characterize optimal tax policies in a model where price rigidities and labor search frictions affect different sectors. We provide policy recommendations in a setting where firms, with two labor input margins, face *both* labor search frictions and price adjustment costs. Another strand of the literature has explored various aspects of the large-firm setup, but has so far not provided an optimal monetary policy analysis.[2] Our paper aims to fill this gap.

The remainder of the paper is structured as follows. Section 2 presents the model. Section 3 describes the distortions at the steady state and the optimal taxation results. In Section 4, we analyze optimal monetary policy. Section 5 concludes.

[2] Cahuc and Wasmer (2001), Cahuc et al. (2008), Mortensen (2009) have shown that a large-firm model combined with search-matching frictions generates inefficiencies in the competitive allocation. However, they do not consider hours, such that the only labor market distortion is over-hiring by firms. Beugnot and Tidball (2010) incorporate price setting in a large-firm model, where both hiring and pricing decisions are in the same sector and the aggregate production function features increasing returns to scale. Here, we distinguish between two labor margins and allow for increasing returns to hours only.

2 Model

Our model features search-and-matching frictions in the labor market and bilateral wage bargaining à la Mortensen and Pissarides (1994).[3] We adopt the large-firm version of this model where firms employ many workers, see Chapter 2 in Pissarides (2000). We allow for variable hours per worker such that labor input can be adjusted along two margins, the extensive margin (employment) and the intensive one (hours per employee). Firms operate under monopolistic competition and face quadratic price adjustment costs à la Rotemberg (1982). Labor search frictions and goods market imperfections affect the same firms, as in Barnichon (2010), Kuester (2010) and Thomas (2011).[4] Employment is predetermined and firms adjust hours unilaterally to satisfy demand in the short run, as in the 'right-to-manage' model of e.g. Trigari (2006).

2.1 Households

In the representative household or family, a fraction n_t of members are employed in the market economy and receive the real wage w_{it} from each firm i for providing hours of work h_{it}. Each employed family member works for all firms on the unit interval. The remaining $1 - n_t$ family members are unemployed; they are instead engaged in home production. The family maximizes lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - n_t \int_0^1 \frac{\lambda_h h_{it}^{1+\sigma_h}}{1+\sigma_h} di \right], \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, C_t denotes consumption, $\lambda_h > 0$ captures the weight on hours in labor disutility, while $\sigma_h \geq 0$ determines the curvature of labor disutility. There exists an insurance technology guaranteeing complete consumption risk sharing between

[3] The online appendix contains detailed model derivations.

[4] Many New Keynesian models with labor market search, e.g. Faia (2009) and Ravenna and Walsh (2012), instead separate these two frictions in what is known as a 'producer-retailer structure'.

family members, such that C_t denotes consumption enjoyed by a member as well as overall family consumption. As in Ravenna and Walsh (2012), consumption consists of final goods sold in the market and home-produced goods, i.e $C_t = C_t^m + (1 - n_t) b$ where b is the productivity of workers in home production.

The family maximizes lifetime utility (1) subject to the sequence of budget constraints

$$(1 + \tau^c) C_t^m + \frac{B_t}{R_t P_t} = n_t \int_0^1 w_{it} (h_{it}) \, di + \frac{B_{t-1}}{P_t} + D_t + (1 - n_t) T^b + T_t, \tag{2}$$

where τ^c is a tax on consumption, P_t is the price level, B_t are one-period nominal bonds that cost $1/R_t$ units of currency in t and pay a safe return of one currency unit in period $t+1$. Consumption expenditure C_t^m and bond purchases B_t are financed through wage income by employed members, interest income on bond holdings, real profits D_t, lump sum transfers T_t, and lump sum transfers to the unemployed T^b. Rewriting the household budget constraint in terms of total consumption gives

$$(1 + \tau^c) C_t + \frac{B_t}{R_t P_t} = n_t \int_0^1 w_{it} (h_{it}) \, di + (1 - n_t) b^c + \frac{B_{t-1}}{P_t} + D_t + T_t, \tag{3}$$

where an unemployed worker produces $(1 + \tau^c) b$ units of market consumption goods and receives the lump sum transfer T^b, i.e. $b^c = (1 + \tau^c) b + T^b$. So far, we have described the representative family. Given that all families are identical in equilibrium and their mass is normalized to unity, C_t represents household consumption as well as economy-wide consumption. The first order conditions for consumption and bonds imply $1 = R_t E_t \{\beta_{t,t+1} / \Pi_{t+1}\}$, where $\beta_{t-1,t} = \beta \frac{C_{t-1}}{C_t}$ is the stochastic discount factor and $\Pi_t = P_t / P_{t-1}$ is the gross inflation rate.

2.2 Labor Market Search and Matching

Firms post vacancies and unemployed workers search for jobs. Let $\mathcal{M}_t = \mathcal{M}_0 u_t^{\eta} v_t^{1-\eta}$ denote the number of successful matches. The matching technology is a Cobb-Douglas function of the unemployment rate $u_t = 1 - n_t$ and the aggregate number of vacancies $v_t = \int_0^1 v_{it} di$, where $\eta \in (0, 1)$ is the elasticity of the number of matches to unemployment and $\mathcal{M}_0 > 0$ is a scale parameter. The probability of a vacancy being filled next period is $q_t = \mathcal{M}_t / v_t = \mathcal{M}_0 \theta_t^{-\eta}$, where the ratio of vacancies to unemployed workers $\theta_t = v_t / u_t$ is a measure of labor market tightness. The job finding rate is $p_t = \mathcal{M}_t / u_t = q_t \theta_t$. A constant fraction λ of matches are destroyed each period, such that

$$n_{it+1} = (1 - \lambda) n_{it} + q_t v_{it} \tag{4}$$

describes the evolution of employment at firm i. Notice that current hires become productive only in the next period, making employment predetermined.

2.3 Wage Determination

Firms bargain with each worker bilaterally over the real wage w_{it} and split the joint surplus according to their respective bargaining weights γ and $(1 - \gamma)$. It can be shown that the bargaining wage satisfies[5]

$$w_{it} = \gamma \left(\frac{h_{it}}{\varphi} w'_{it} (h_{it}) + \kappa_v \theta_t \right) + (1 - \gamma) \left(\frac{\lambda_h h_{it}^{1+\sigma_h}}{1 + \sigma_h} \frac{1}{\Lambda_t} + b^c \right), \tag{5}$$

where κ_v is the per-period cost to the firm of posting a vacancy, φ is the elasticity of output to hours (defined below) and $\Lambda_t = 1/[(1 + \tau^c) C_t]$ is the Lagrange multiplier on the household budget constraint (2). An employed worker suffers the disutility $\lambda_h h_{it}^{1+\sigma_h}/(1 + \sigma_h)$ from working, which we divide by Λ_t to convert utils into consumption goods. His outside option

[5] See the online appendix.

is represented by b^c.

The firm's surplus from employing a marginal worker equals the latter's contribution to profits. As in Barnichon (2010) and Thomas (2011), a firm sets its price prior to hiring and wage bargaining. Once it has set a price, it adjusts hours unilaterally to satisfy demand at that price. Therefore, firm revenues are independent of n_{it} and the contribution of the marginal worker to firm profits is the marginal reduction in the wage bill, $(h_{it}/\varphi)w'_{it}(h_{it})$ with $w'_{it}(h_{it}) \equiv \frac{\partial w_{it}}{\partial h_{it}}$ defining the real marginal wage, and not his marginal revenue product as in the standard search-and-matching model. The decrease in costs due to an additional hire - through lower average hours and lower wages paid to all workers - is what we call the 'wage externality' and it is discussed in detail below.

By the method of undetermined coefficients, we find the following solution to (5),

$$w_{it}(h_{it}) = \gamma\kappa_v\theta_t + (1-\gamma)\, b^c + \varkappa \frac{\lambda_h h_{it}^{1+\sigma_h}}{1+\sigma_h}(1+\tau^c)\, C_t, \tag{6}$$

where we define

$$\varkappa \equiv \frac{1-\gamma}{1-\gamma\frac{1+\sigma_h}{\varphi}}. \tag{7}$$

The derivative of (6) is the real marginal wage,

$$w'_{it}(h_{it}) = \varkappa \cdot mrs_{it}. \tag{8}$$

where mrs_t denotes the marginal rate of substitution between consumption and leisure,

$$mrs_{it} = \lambda_h h_{it}^{\sigma_h}(1+\tau^c)\, C_t. \tag{9}$$

We impose $1-\gamma(1+\sigma_h)/\varphi > 0$ in (7).[6] This implies that $\varkappa > 0$, such that the real marginal wage under bargaining (8) is positively related to hours worked. Furthermore,

[6]When computing the steady state numerically, we verify that this condition is satisfied.

under the usual assumption that $\frac{1+\sigma_h}{\varphi} > 1$, we have that $\varkappa > 1$.[7] If wages were set in a Walrasian labor market, we would have $\varkappa = 1$ instead.[8] The parameter $\varkappa$ captures the distortion imposed by the wage bargaining process. Ceteris paribus, a greater $\varkappa$ implies that raising hours worked results in a larger increase in the equilibrium wage. Given $\frac{1+\sigma_h}{\varphi} > 1$, the slope of the wage curve is increasing in the bargaining power of workers, γ. For a given bargaining power of workers γ, the parameter $\varkappa$ is increasing in the curvature of the disutility of hours worked $1 + \sigma_h$, and decreasing in the returns to hours in production, φ. The wage curve is steeper, the higher is the utility cost of hours $(1 + \sigma_h)$ relative to the degree of returns to hours, φ.

2.4 Production

Final output Y_t is an aggregate of intermediate goods Y_{it} bundled according to the function $Y_t = (\int_0^1 Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di)^{\frac{\varepsilon}{\varepsilon-1}}$, where ε is the elasticity of substitution between the individual goods varieties. Given a price P_{it} for each variety i, the corresponding demand function is given by $Y_{it}^d = (P_{it}/P_t)^{-\varepsilon} Y_t$.

Firms indexed by $i \in (0, 1)$ use labor to produce intermediate goods under monopolistic competition. Output of an individual firm Y_{it} is produced according to the following production function,

$$Y_{it} = A_t n_{it} h_{it}^{\varphi}, \tag{10}$$

where A_t is a technology index common to all firms. The parameter φ measures the short-run returns to hours or the elasticity of output to hours. Production is thus linear in employment and (potentially) non-linear in hours per worker h_{it}. The firm sets a price at the beginning of the period and commits to satisfying demand at that price. Taking into account the firm's

[7] We do not consider as empirically relevant the case where $\frac{1+\sigma_h}{\varphi} < 1$.

[8] The efficient wage is derived in the online appendix.

production technology (10), its demand constraint is

$$\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t = A_t n_{it} h_{it}^{\varphi}. \tag{11}$$

Since employment is predetermined, the firm adjusts hours worked in the short run in order to produce the amount of output demanded. More formally, the firm chooses a price P_{it}, hours worked h_{it}, vacancies v_{it}, and next period's employment n_{it+1}, to maximize the present discounted stream of future profits,

$$E_0 \sum_{t=0}^{\infty} \beta_{0,t} \left[\left(1-\tau^f\right) \frac{P_{it}}{P_t} Y_{it}^d - w_{it}\left(h_{it}\right) n_{it} - \kappa_v v_{it} - \frac{\kappa_p}{2} \left(\frac{P_{it}}{P_{it-1}} - 1\right)^2 Y_t \right], \tag{12}$$

subject to the law of motion for employment (4), the equilibrium wage (6), and the demand constraint (11). In the objective function (12), τ^f is a tax on firm revenues and $\beta_{0,t} = \beta_{0,1}\beta_{1,2}\cdots\beta_{t-1,t}$. Firm revenues are taxed if $\tau^f > 0$ and subsidized if $\tau^f < 0$. Following Rotemberg (1982), price changes are subject to quadratic adjustment costs scaled by the parameter $\kappa_p \geq 0$. Substituting demand into the firm's objective function (12), we can write the firm's optimization problem as a Lagrangian problem,

$$\begin{aligned} \max_{\{h_{it}, v_{it}, n_{it+1}, P_{it}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta_{0,t} \{ \left(1-\tau^f\right) \left(\tfrac{P_{it}}{P_t}\right)^{1-\varepsilon} Y_t - w_{it}\left(h_{it}\right) n_{it} - \kappa_v v_{it} - \frac{\kappa_p}{2} \left(\tfrac{P_{it}}{P_{it-1}} - 1\right)^2 Y_t \\ - s_{it} [\left(\tfrac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t - A_t n_{it} h_{it}^{\varphi}] - \varphi_{nt} \left[n_{it+1} - (1-\lambda) n_{it} - q_t v_{it} \right] \}, \end{aligned}$$

where s_{it} and φ_{nt} are the Lagrange multipliers on the demand constraint and on the firm's employment dynamics, respectively. Since all firms choose the same price, hours, vacancies and future employment level in equilibrium, we drop the i-subscript from here on.

Hours Worked Notice that a worker's marginal product per hour, defined as $mph_t \equiv \frac{\partial(Y_t/n_t)}{\partial h_t}$, is

$$mph_t = \varphi A_t h_t^{\varphi-1}. \tag{13}$$

If $\varphi > 1$, we have short run increasing returns to hours, implying $\frac{\partial mph_t}{\partial h_t} > 0$. This means that increasing hours by 1% raises output per worker by more than 1%. As argued by Oi (1962) and Solow (1964), increasing returns to hours can be rationalized through unobserved variations in factor utilization, such as work intensity, or effort. See also Barnichon (2010, 2012) and Galí and van Rens (2014).

The first order condition for hours worked states that the Lagrange multiplier on the demand constraint s_t equals the real marginal wage divided by the marginal product of hours,

$$s_t = \frac{w_t'(h_t)}{mph_t}. \tag{14}$$

Equation (14) describes the firm's real marginal costs, i.e. the change in the wage bill for a unit increase in output. Since employment is predetermined, firms increase production to satisfy demand by increasing hours worked. Using more hours has two effects on real marginal costs. On one hand, it increases the real marginal wage $w_t'(h_t)$, or the cost of one additional worker-hour, provided that $\varkappa > 0$ in (8), and therefore s_t. On the other hand, when $\varphi \neq 1$, the marginal productivity of hours also varies with hours worked. Under the standard assumption of decreasing returns to hours ($\varphi < 1$), mph_t falls with hours, raising real marginal costs. Under increasing returns to hours ($\varphi > 1$), the marginal product of hours mph_t instead rises with hours, which reduces s_t. Then real marginal costs respond less positively to a rise in hours worked.

The real marginal cost can be seen as the cost of using the hours margin, rather than the employment margin, to adjust output in response to a persistent shock. It is increasing in the slope of the wage curve, $w_t'(h_t)$, and decreasing in the marginal product of hours, mph_t.

Vacancy Posting The first order conditions for vacancies and next period's employment together imply

$$\frac{\kappa_v}{q_t} = E_t\left\{\beta_{t,t+1}\left[\chi_{t+1} + (1-\lambda)\frac{\kappa_v}{q_{t+1}}\right]\right\}, \tag{15}$$

where χ_t is the shadow value of the marginal worker. A firm posts vacancies until the cost of hiring a worker equals the expected discounted future benefits from this extra worker. The costs of hiring a worker are given by the vacancy posting costs, κ_v, multiplied by the average duration of a vacancy, $1/q_t$. The benefits of hiring a worker are his shadow value, χ_{it}, plus the vacancy posting costs saved in case the employment relationship continues.

In the one-worker-firm framework with instantaneous hiring, the shadow value of a marginal worker, χ_t, corresponds to his marginal productivity net of his wage. In our setup with large firms and predetermined employment, the marginal worker reduces future hours worked of all the firm's employees by shifting production from the intensive to the extensive margin. The reduction in hours in turn reduces the wage determined in the bargaining process. Formally, the shadow value of a marginal worker captures the reduction in the wage bill induced by an additional hire,

$$\chi_t = -\frac{\partial w_t\left(h_t\right) n_t}{\partial n_t} = -w_t\left(h_t\right) + \frac{h_t}{\varphi} w_t'\left(h_t\right). \tag{16}$$

On one hand, hiring an additional worker costs the firm w_t. On the other, it allows the firm to reduce the number of hours, and through (6) the wage payments to, all other workers.

The degree of returns to hours has a direct and an indirect effect on the shadow value. First, if the degree of returns to hours φ is high, a given reduction in hours reduces output by a larger amount. Then hiring an additional worker and reducing hours is less attractive and the shadow value is lower. Second, there is an indirect effect through the wage externality. If the degree of returns to hours φ is high relative to the utility cost of hours $(1+\sigma_h)$, the wage curve is rather flat and raising hours has a smaller effect on the equilibrium wage. Then the shadow value of the marginal worker is lower and hiring is discouraged.

Substituting out the real marginal wage in (16), the shadow value can be expressed as

$$\chi_t = -w_t\left(h_t\right) + \frac{h_t}{\varphi} s_t mph_t. \tag{17}$$

Price Setting The first order condition for prices yields the New Keynesian Phillips Curve,

$$\kappa_p \Pi_t (\Pi_t - 1) = \varepsilon s_t - \left(1 - \tau^f\right)(\varepsilon - 1) + \kappa_p E_t \left\{ \beta_{t,t+1} \Pi_{t+1} (\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \right\}. \quad (18)$$

Our specification of price rigidities follows Faia (2009) and Sunakawa (2013), but differs from Barnichon (2010), Kuester (2010) and Thomas (2011), who adopt Calvo (1983) price staggering. The price set by a firm determines the shadow value of the marginal worker, and thus its hiring decision. In the Calvo setup, sticky-price firms choose a different employment level than flexible-price firms. This firm-specificity of labor alters the slope of the New Keynesian Phillips Curve. For simplicity, we opt for the Rotemberg scheme, which delivers the standard New Keynesian Phillips Curve slope.[9]

2.5 Equilibrium

The government budget constraint equates current income (bond issues and tax revenues) with current expenditure (government consumption, lump-sum transfers, and maturing government bonds),

$$\frac{B_t}{R_t P_t} + \tau^c C_t^m + \tau^f Y_t = G_t + T_t + (1 - n_t) T^b + \frac{B_{t-1}}{P_t}. \quad (19)$$

The costs of posting vacancies and adjusting prices are passed on to households in the form of lower dividends. Aggregate (after-tax) profits are

$$D_t = \left(1 - \tau^f\right) Y_t - w_t n_t - \kappa_v v_t - \frac{\kappa_p}{2} (\Pi_t - 1)^2 Y_t. \quad (20)$$

Combining the aggregated household budget constraint with the government budget constraint (19) and the aggregate profit equation (20), we obtain the aggregate accounting

[9] In addition, the Rotemberg price setting scheme allows us to write down the model in non-linear form, which we need to derive the Ramsey first order conditions later in the paper.

identity,

$$Y_t + (1 - n_t)\, b^c = C_t + G_t + \kappa_v v_t + \frac{\kappa_p}{2} (\Pi_t - 1)^2 Y_t. \tag{21}$$

The model is closed with a description of monetary policy.

2.6 Model Summary

We condense the decision rules of households and firms into three equilibrium conditions determining hours (h_t), vacancies (v_t) and real marginal costs (s_t),

$$s_t = \varkappa \frac{\lambda_h h_t^{\sigma_h} (1 + \tau^c) C_t}{\varphi A_t h_t^{\varphi - 1}}, \tag{22}$$

$$\begin{aligned} & \tfrac{\kappa_v}{\mathcal{M}_0}(\tfrac{v_t}{1-n_t})^{\eta} - (1 - \lambda)\, \beta E_t \left\{ \tfrac{C_t}{C_{t+1}} \tfrac{\kappa_v}{\mathcal{M}_0}(\tfrac{v_{t+1}}{1-n_{t+1}})^{\eta} \right\} \\ = \; & \beta E_t \left\{ \tfrac{C_t}{C_{t+1}} [-\gamma \kappa_v \tfrac{v_{t+1}}{1-n_{t+1}} - (1 - \gamma)\, b^c + (1 - \tfrac{\varphi}{1+\sigma_h}) A_{t+1} h_{t+1}^{\varphi} s_{t+1}] \right\}, \end{aligned} \tag{23}$$

$$\kappa_p (\Pi_t - 1)\, \Pi_t + (1 - \tau^f)(\varepsilon - 1) - \varepsilon s_t = \kappa_p \beta E_t \left\{ \tfrac{C_t}{C_{t+1}} (\Pi_{t+1} - 1)\, \Pi_{t+1} \tfrac{A_{t+1} n_{t+1} h_{t+1}^{\varphi}}{A_t n_t h_t^{\varphi}} \right\}. \tag{24}$$

The technological constraints determining, respectively, the number of workers (n_{t+1}) and consumption (C_t) are given by the evolution of employment and the resource constraint,

$$n_{t+1} = (1 - \lambda)\, n_t + \mathcal{M}_0 (1 - n_t)^{\eta} v_t^{1-\eta}, \tag{25}$$

$$(1 - \tfrac{\kappa_p}{2}(\Pi_t - 1)^2) A_t n_t h_t^{\varphi} + (1 - n_t)\, b^c = C_t + G_t + \kappa_v v_t. \tag{26}$$

Finally, monetary policy pins down a path for inflation (Π_t). We are now ready to provide a formal definition of equilibrium.

Definition 1 *A competitive equilibrium is a set of allocations* $\{h_t, v_t, n_{t+1}, C_t\}_{t=0}^{\infty}$*, prices* $\{s_t\}_{t=0}^{\infty}$*, tax policies* $\{\tau^f, \tau^c, T^b\}$ *and monetary policy* $\{\Pi_t\}_{t=0}^{\infty}$*, such that, given an initial* *employment level n_0, households maximize utility, firms maximize profits, and all markets clear.*

2.7 Calibration

We calibrate the model parameters as follows.[10] The discount factor in household preferences is set to $\beta = 0.99$, implying a steady-state annualized real interest rate of 4%. The steady-state output level Y is normalized to unity. Steady-state technology is then set to obtain an unemployment rate of 9.6% in the steady state, which corresponds to the average unemployment rate in the Euro Area between 1999 and 2013. The resulting value is $A = 1.30$. Following Barnichon (2010), we set σ_h, the curvature of labor disutility in hours, equal to 2. The household's weight on labor disutility λ_h is calibrated such that hours equal 0.9 in steady state.[11] Following Christoffel et al. (2009), we set the probability of filling a job, q, to 0.7, the job separation rate, λ, to 0.03 and the vacancy posting costs, κ_v to 0.058. From these parameters, we can deduce the probability of finding a job, p, equals 0.28, the degree of labor market tightness, θ, equals 0.40 which both correspond to Christoffel et al.'s (2009) calibration. The productivity in home production, b, is 0.74. We set the bargaining power of workers, γ, to 0.4. The standard Hosios (1990) condition is satisfied, such that the elasticity of matches to unemployment, η, equals the bargaining power of workers. We assume increasing returns to hours by setting $\varphi = 1.5$, as in Barnichon (2010).[12] The substitution elasticity between intermediate goods is set to $\varepsilon = 6$, yielding a net price markup of 20%, and the price adjustment cost, κ_p, is set to 20, as in Faia (2009) and Sunakawa (2013). The share of government spending in total market output in steady state is roughly one fifth, $G = 0.21$ as measured in Euro Area data. In our benchmark calibration, tax rates are set to zero, $\tau^f = \tau^c = 0$.

[10] The online appendix describes the model's steady state which can be written recursively.

[11] This gives a value of $\lambda_h = 0.74$.

[12] We impose this value such that, for our calibration of σ_h and γ, the slope of the wage curve, $\varkappa$ in (7), is larger than one. Notice that setting $\gamma = 0.5$ would yield an infinite slope.

3 Steady-State Distortions and Optimal Tax Policy

In this section, we first derive the efficient allocation. Second, we characterize the steady-state distortions to employment and to the number of hours that arise in the competitive equilibrium. Third, we derive the optimal tax policy mix that removes these distortions. Finally, we analyze the effects of the steady-state distortions on the model dynamics when prices are flexible.

3.1 Efficient Allocation

The social planner problem is to maximize household utility subject to the evolution of aggregate employment, which we regard as a technological constraint, and the resource constraint.

Definition 2 *An efficient allocation is a set of paths* $\{h_t, v_t, n_{t+1}, C_t\}_{t=0}^{\infty}$ *which maximizes utility (1), subject to the employment dynamics constraint and the resource constraint,*

$$n_{t+1} = (1-\lambda)\, n_t + \mathcal{M}_0\,(1-n_t)^{\eta}\, v_t^{1-\eta}, \tag{27}$$

$$A_t n_t h_t^{\varphi} + (1-n_t)\, b = C_t + G_t + \kappa_v v_t. \tag{28}$$

The efficient allocation is characterized by two conditions determining hours and employment. First, it can be shown that the efficient hours choice satisfies

$$1 = \frac{\lambda_h h_t^{\sigma_h} C_t}{\varphi A_t h_t^{\varphi-1}}. \tag{29}$$

Equation (29) states that the utility cost of providing one additional hour of work must equal its marginal benefit captured by the marginal product of hours. Second, the efficient choice of employment satisfies

$$\frac{\kappa_v}{\mathcal{M}_0}(\frac{v_t}{1-n_t})^\eta - (1-\lambda)\,\beta E_t\{\frac{C_t}{C_{t+1}}\frac{\kappa_v}{\mathcal{M}_0}(\frac{v_{t+1}}{1-n_{t+1}})^\eta\} \tag{30}$$
$$= \beta E_t\left\{\frac{C_t}{C_{t+1}}\left[-\eta\kappa_v\frac{v_{t+1}}{1-n_{t+1}} - (1-\eta)\,b + (1-\eta)\,(1-\frac{\varphi}{1+\sigma_h})A_{t+1}h_{t+1}^{\varphi}\right]\right\}.$$

The left hand side of (30) is the net hiring cost, while the right hand side is the efficient shadow value of a marginal worker.

3.2 Steady-State Distortions

We show that the competitive steady state is distorted by comparing the decentralized decision rules concerning the choice of hours and employment with the respective efficiency conditions.

Hours Margin Comparing the hours choice in the competitive equilibrium (22) with the efficiency condition (29), we can state the following result.

Result 1 *A distortion in the decentralized intensive labor margin arises if*

$$\frac{s}{\varkappa} \neq 1+\tau^c. \tag{31}$$

Following Galí et al. (2007), we characterize the distortion in the number of hours worked in terms of a wedge between the marginal rate of substitution and the marginal product of hours. This wedge or ‘inefficiency gap’ is driven by a wage markup and a price markup, representing inefficiencies in labor markets and in product markets, respectively.

First, in the steady state, the real marginal wage (8) is related to the marginal rate of substitution as follows

$$w'(h) = \mu_w mrs, \tag{32}$$

where $\mu_w = \varkappa$ can be viewed as a wage markup. In a typical calibration we have $\frac{1+\sigma_h}{\varphi} > 1$ and therefore $\varkappa > 1$, such that the real marginal wage under bargaining is larger than the marginal rate of substitution. This means that wages rise faster with hours than in the efficient case. Recall that $\varkappa = 1$ if wages were set in a Walrasian labor market. The parameter $\varkappa$ thus captures the distortion imposed by the bargaining process.

Second, using the firm's first order condition for hours (14), we can relate the real marginal wage to the marginal product of hours as follows,

$$w'(h) = \frac{mph}{\mu_p}, \tag{33}$$

where $\mu_p = 1/s$ represents a price markup. Setting (32) and (33) equal to eliminate $w'(h)$ and defining the steady-state inefficiency gap as $gap = \frac{mrs}{mph}$, we obtain

$$gap = (\mu_w \mu_p)^{-1} = \frac{s}{\varkappa}. \tag{34}$$

In the absence of taxes ($\tau^f = \tau^c = 0$), Galí et al (2007)'s inefficiency gap (34) corresponds to the hours distortion (31) in our model. More specifically, we have $gap = 1/(\varkappa\mu) < 1$. The inefficiency in the choice of hours comes from two sources.

First, because of monopolistic competition in goods markets captured by the markup μ, which reduces real marginal costs below unity ($s < 1$), output and thus hours per worker are too low.

Second, wages are not set as in a Walrasian labor market but are instead chosen through bargaining. Suppose that wages are set such that the demand for hours by the firm equals the supply of hours by the household. If the household could choose hours optimally, it would set h_t to maximize utility (1) subject to the budget constraint (3). The associated first order condition is that the real marginal wage equals the marginal rate of substitution between leisure and consumption, $w_t'(h_t) = mrs_t$.

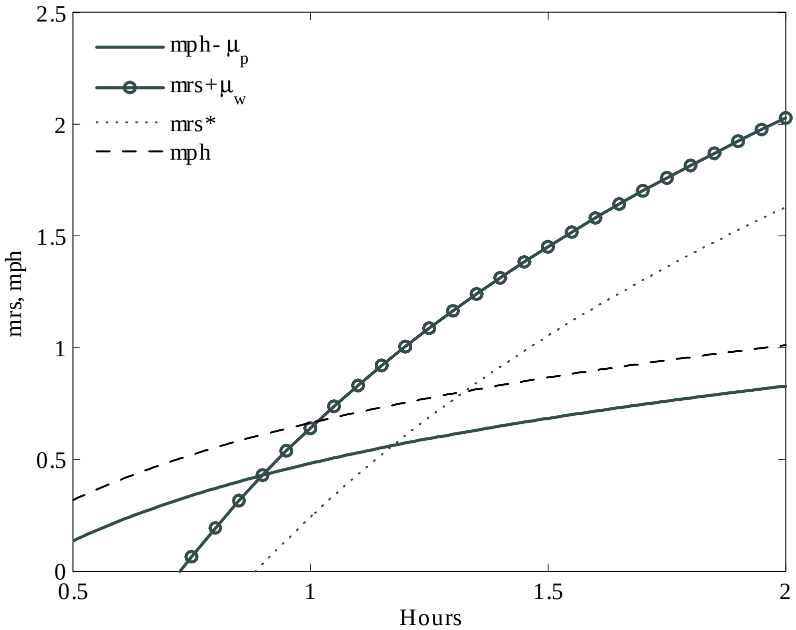
The wedge introduced by wage bargaining is greater than unity ($\varkappa > 1$). For a given product market distortion $\mu > 1$, it is theoretically possible that hours are efficient ($s/\varkappa = 1$), or even too high ($s/\varkappa > 1$). This is the case only if $(1 + \sigma_h)/\varphi < 1$ such that $\varkappa < 1$, which happens if labor disutility does not rise strongly with hours, such that σ_h is close to zero, and there are strongly increasing returns to hours, such that φ is much above 1. We do not consider this case here.

Figure 1 displays the (log) real marginal wage as a function of hours by using (32) and (33), setting taxes to zero ($\tau^c = \tau^f = 0$). The competitive equilibrium allocation for hours worked is at the intersection of the two curves. Notice that in this partial equilibrium exercise, we plot the marginal rate of substitution (9) as a function of hours, keeping consumption constant at the *competitive* level, C. Figure 1 also plots mrs^* and mph as a function of hours worked. We keep consumption constant at the *efficient* level, C^*, in mrs^*. The efficient number of hours worked can be read off from the intersection of the two latter curves.

The figure shows two results. First, hours worked are lower than in the efficient allocation. This is what we call the 'hours distortion'. Second, the real marginal wage is lower than it would be in a Walrasian labor market. How do our results differ from Galí et al. (2007)? First, since we consider two labor input margins, hours and employment, the hours distortion in (34) is not the only inefficiency. We analyze the employment distortion in more detail below. Second, since in our model the marginal product of hours is increasing in hours worked, the mph-curve is positively sloped in Figure 1. The real wage is therefore unambiguously too low in the competitive equilibrium.

Trigari (2006) and Sunakawa (2013) show that the 'right-to-manage' assumption, by which firms choose hours unilaterally, results in a wedge between the marginal rate of substitution and the marginal product of labor. In an alternative setup where both wages and hours are determined through Nash bargaining, this wedge is removed, such that hours are

Figure 1: **Labor Market Allocation: Hours**



Note: The curves depict the (log) real marginal wage, the (log) marginal product of labor and the (log) marginal rate of substitution as a function of hours. The continuous line displays the real marginal wage implied by the firm's first order condition for hours (33). The dashed line displays the marginal product of hours (13). The line with circles displays the real marginal wage determined through bargaining (32). The dotted line displays the marginal rate of substitution (9) when $\tau^c = 0$ and consumption is efficient.

set efficiently. This is what Trigari (2006) calls 'efficient bargaining'. She considers predetermined employment, but assumes a producer-retailer structure where price rigidities and hiring frictions are located in different sectors. Sunakawa (2013) considers firms that face both price rigidities and hiring frictions, but assumes instantaneous hiring. In our model, both features are present: price rigidities and hiring frictions affect the same firms *and* employment is predetermined. Therefore, we cannot use efficient bargaining as a benchmark, because a firm that has set a price needs to be able to adjust total hours in order to satisfy demand at the chosen price. Since employment is predetermined, the only labor margin adjustable in the short run is the number of hours per worker.

Employment Margin As in Pissarides (2000) and Krause and Lubik (2007b), we highlight the distortions associated with vacancy posting decisions. To do so, we derive two steady-state equations in unemployment (u) and vacancies (v): the Beveridge Curve and the job creation condition for both the competitive and the efficient allocation.

Under symmetry, the law of motion for employment (4) is $n_t = (1-\lambda)\, n_{t-1} + q_t v_t$, which in steady state becomes $n = qv/\lambda$ or

$$v = \left(\frac{(1-u)\,\lambda}{\mathcal{M}_0 u} \right)^{\frac{1}{1-\eta}} u, \tag{35}$$

after substituting out n and q. The Beveridge Curve (35) traces out the number of vacancies v as a function of unemployment u, for a given matching efficiency $\mathcal{M}_0$ and separation rate λ. Since the efficient allocation is characterized by the same law of motion for employment (4), the Beveridge Curve holds in the competitive and in the efficient allocation.

The competitive job creation condition is derived by combining the vacancy posting condition (15) with the shadow value (17) at the steady state. After several substitutions, we obtain

$$\left(\frac{v}{u}\right)^{\eta} = \frac{1}{\Omega} \left[-\gamma \kappa_v \frac{v}{u} - (1-\gamma)\, b^c + \left(1 - \frac{\varphi}{1+\sigma_h}\right) s \frac{Y}{n} \right], \tag{36}$$

where $\Omega = [1-(1-\lambda)\,\beta]\kappa_v / (\beta \mathcal{M}_0)$. In the efficient allocation, the steady-state job creation condition is given by

$$\left(\frac{v}{u}\right)^{\eta} = \frac{1}{\Omega} \left[-\eta \kappa_v \frac{v}{u} - (1-\eta)\, b + \left(1 - \frac{\varphi}{1+\sigma_h}\right) (1-\eta) \frac{Y^*}{n^*} \right]. \tag{37}$$

Comparing the job creation condition (36) with its efficient counterpart (37), we can see that there are three channels through which unemployment affects the number of vacancies.

The first channel is related to the effect of unemployment on vacancy duration. This is captured by the first term on the right hand side of the job creation condition. For a given matching efficiency $\mathcal{M}_0$, labor market tightness falls when unemployment rises, which lowers the duration of a vacancy q^{-1} and encourages hiring. A distortion arises if $\gamma \neq \eta$. To restore efficiency, the well-known Hosios (1990) condition needs to hold, which requires that the workers' bargaining power γ must equal the elasticity of vacancy duration to the number of vacancies, η. When this elasticity is high, a firm that posts a vacancy greatly increases vacancy duration for all other firms, creating a congestion effect. This effect is offset if workers have a lot of bargaining power, which discourages firms from posting vacancies. In contrast to the standard search-and-matching model, however, the Hosios condition is not enough to guarantee efficient vacancy posting in this model.

The second channel is a deviation of the worker's outside option b^c from the efficient value b, which we recall represents the productivity in home production. A distortion arises if $b^c \neq b$. Any consumption tax or subsidy ($\tau^c \neq 0$), distorts the choice of market production relative to home production, and hence the worker's outside option, since b^c is no longer equal to b in this case.[13]

The third channel is related to the combination of monopolistic competition and search frictions in our large-firm setup. This is captured by the last term on the right hand side of the job creation condition. A distortion arises if

$$s\frac{Y}{n} \neq (1-\eta)\,\frac{Y^*}{n^*}. \tag{38}$$

The inequality captures two opposing effects on hiring. First, because of the hours distortion described above, output per worker is suboptimally low in the competitive equilibrium, $Y/n < Y^*/n^*$. Since hours per worker are too low, an additional worker is less productive and therefore less valuable to the firm, reducing the firm's incentives to hire. Therefore, employment is reduced below its efficient level.

Second, in a typical calibration, steady-state real marginal costs are greater than the elasticity of matches to vacancies, $s > 1-\eta$. Recall that real marginal costs (14) represent

[13] This distortion is also present in the model of home production described by Ravenna and Walsh (2012).

the cost of adjusting hours, relative to employment, in order to accommodate changes in demand. Real marginal costs are high if $w'(h)$ is large, i.e. the wage curve is steep. In that case, an additional worker and a corresponding reduction in the number of hours per worker allows for a large wage cut for all other workers. This boosts the firm's incentive to hire.

Intuitively, when the firm hires a new worker, its other workers have to work fewer hours to produce a given amount of output. The bargained wage falls and this raises the shadow value of a worker and hence the number of vacancies posted. Thus, firms have an incentive to over-employ workers in order to reduce their bargaining position within the firm (see Stole and Zwiebel, 1996). As shown by Ebell and Haefke (2009), this over-hiring externality is reinforced when the degree of competition is low (the price markup μ is high such that $s = 1/\mu$ is close to 1). The following result summarizes this discussion.

Result 2 *A distortion in the extensive labor margin arises when the shadow value of a worker is too low or too high. Two forces work in opposite directions. On the one hand, the monopolistic competition distortion depresses the number of hours per worker, h, and therefore productivity (output per worker, Y/n),*

$$\frac{Y}{n} < \left(\frac{Y}{n}\right)^*, \tag{39}$$

resulting in too low employment. On the other hand, if steady-state real marginal costs are greater than the match elasticity to vacancies,

$$s > (1 - \eta), \tag{40}$$

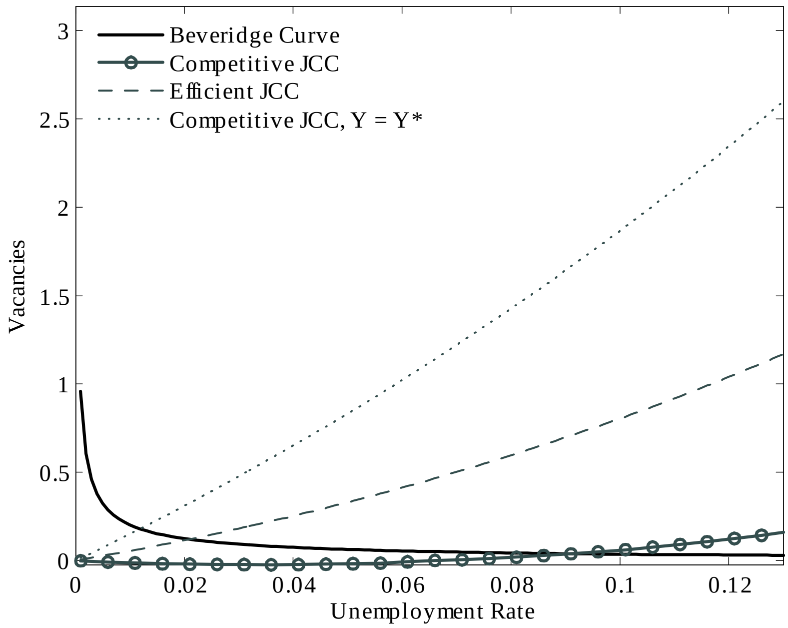
employment is too high in equilibrium.

Figure 2 depicts the Beverige curve, as well as the competitive and the efficient job creation condition (JCC).[14] We keep the steady-state output levels in the competitive and

[14] Notice that the competitive Beveridge curve is identical to the efficient one.

in efficient allocation, Y and Y^* respectively, constant in this partial equilibrium exercise, and we use the relation $n = 1 - u$.

Figure 2: **Labor Market Allocation: Employment**



Each curve depicts the number of vacancies as a function of the unemployment rate. The continuous line displays the Beveridge Curve (35). The line with circles displays the competitive job creation condition (36). The dashed line displays the efficient job creation condition (37). The dotted line displays the competitive job creation condition (36) when output is efficient, $Y = Y^*$.

Under the standard Hosios condition $\gamma = \eta$ and with all tax rates set to zero ($\tau^f = \tau^c = T^b = 0$), the difference between the competitive and efficient JCC stems entirely from the inequality (38). The Beveridge curve is the downward sloping curve in (u, v)-space. A higher number of vacancies is associated with a higher level of employment (and hence lower unemployment). The JCC is upward-sloping; as unemployment rises, the shadow value of a worker rises overall and the number of vacancies increases. The equilibrium is located at the intersection of the Beveridge curve with the relevant JCC.

The figure shows that the efficient JCC is flatter than the competitive JCC. The competitive equilibrium is thus characterized by too much unemployment. What if hours are large enough such that output in the competitive economy is at the efficient level? The corresponding JCC is depicted as the steepest upward-sloping curve in Figure 2. The intersection with the Beveridge Curve is at a point where unemployment is below the efficient level. The remaining distortion in the shadow value is given by the inequality $s > 1 - \eta$, leading to the over-hiring distortion described above.

3.3 Optimal Tax Policy

Under flexible prices, the efficient allocation can be implemented through an appropriate tax policy mix. More precisely, the policy maker needs to choose the fiscal instruments which remove the hours and employment distortions at the steady state.

Definition 3 *An optimal tax policy is a set $\{\tau^f, \tau^c, T^b\}$, such that the zero-inflation steady state in the competitive equilibrium coincides with the efficient steady state.*

As described above, the intensive margin of labor is distorted when the inefficiency gap is not equal to the gross consumption tax, see (31). In addition, there are three potential sources of distortion on the extensive margin of labor which can be shown by comparing the decentralized JCC (36) with the efficient JCC (37). In the following, we first assume that the standard Hosios condition is satisfied ($\gamma = \eta$), which allows us to derive a simple expression for an optimal mix of revenue taxes, consumption subsidies and transfers to the unemployed, that jointly correct for inefficiencies in vacancy posting and hours. Second, we relax the Hosios assumption and derive the optimal tax policy mix for the general case.

Special Case We assume that the standard Hosios condition is satisfied ($\gamma = \eta$). In this special case, we can show analytically that the remaining distortions in hours and employment can be removed with our tax instruments. To derive the optimal revenue and consumption tax rates, we replace $s = \left(1 - \tau^f\right)/\mu$, $Y/n = Ah^\varphi$ and $Y^*/n^* = Ah^{*\varphi}$ in (31) and (38) to obtain:

$$\frac{1-\tau^{f}}{\mu} h^{\varphi} = (1-\eta)\, h^{*\varphi}, \tag{41}$$

$$\frac{1-\tau^{f}}{\mu \varkappa} = 1+\tau^{c}. \tag{42}$$

The optimal fiscal policy mix is given by the tax rates τ^{f*}, τ^{c*} and T^{b*} which jointly satisfy the two efficiency conditions (41) and (42), as well as $b = b^{c}$, such that $h = h^{*}$ and $n = n^{*}$.

Result 3 *Under the standard Hosios condition $\gamma = \eta$, the optimal tax policy mix is given by*

$$1-\tau^{f*} = \mu\,(1-\eta)\,, \tag{43}$$

$$\tau^{c*} = -\gamma \frac{1+\sigma_h}{\varphi}, \tag{44}$$

$$T^{b*} = -\tau^{c*} b. \tag{45}$$

First, we focus on the *extensive* labor margin as described in the first efficiency condition (41). Given an optimal consumption tax such that hours worked are efficient ($h = h^{*}$), efficiency in vacancy posting is restored with an appropriate revenue tax (43). The optimal revenue tax depends on the price markup μ and on the elasticity of matches to vacancies, $1-\eta$. Recall that the large-firm setup - in isolation - features an over-hiring externality when condition (40) is satisfied. Firms employ too many workers in order to reduce hours per worker and thus the wage bill through (6). Overhiring in turn generates congestion effects by reducing the probability of other firms to find a worker. Therefore, the optimal revenue tax to be imposed on a monopolistic firm equals the gross markup adjusted for the congestion externalities it creates.[15] Equation (43) shows that, if there are no matching frictions and therefore no congestion externalities, such that $\eta = 0$, we have the standard result from the New Keynesian model prescribing an optimal revenue *subsidy* equal to the

[15] Felbermayr et al. (2012) show that the over-hiring externality can also be corrected by increasing unemployment benefits.

gross markup. If instead $\mu(1-\eta) < 1$, as is the case in a calibrated search-and-matching model, firm revenues are instead *taxed* at the optimum, $\tau^{f*} > 0$.

Second, we turn to the *intensive* labor margin as described in the second efficiency condition (42). Given an optimal revenue tax such that employment is efficient ($n = n^*$), an appropriate consumption tax can correct the inefficiency in hours worked, such that $h = h^*$. Imposing $\tau^f = \tau^{f*}$ (43) in (42), the optimal consumption tax that removes the hours distortion simplifies to (44). The optimal consumption tax is negative: at the optimum, market consumption should be subsidized. Recall that the hours distortion is driven by the gap between the real marginal cost, s, and the slope of the wage curve, $\varkappa$, as shown in (31). The latter corresponds to the deviation from Walrasian wage setting and is driven by $(1+\sigma_h)/\varphi$ and γ, see (7). A high relative disutility cost of hours, $(1+\sigma_h)/\varphi$, or a high worker bargaining power, γ, shift the real marginal wage curve up for any given number of hours worked, see Figure 1. The farther the real marginal wage is from the marginal rate of substitution, the greater is the required consumption subsidy, which shifts the marginal rate of substution down, see (9).

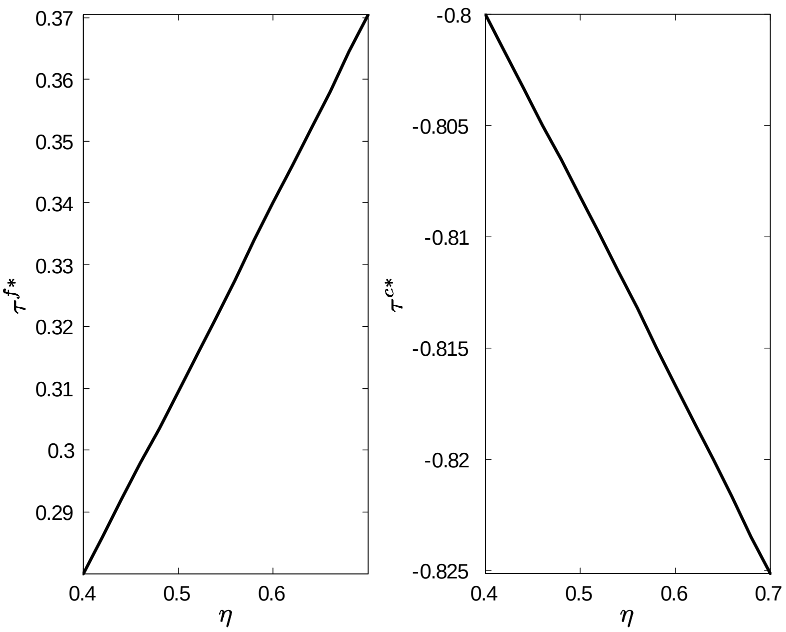
Third, we choose an appropriate lump sum transfer to the unemployed such that we can abstract from the choice between market and home production. To remove the distortionary effect of the consumption tax on vacancy posting, we allow for transfers to unemployed workers $T^{b*} = -\tau^{c*}b$, such that $b^c = b$ (see the definition of b^c in the household budget constraint (3)).

General Case In the general case where the standard Hosios condition does not hold ($\gamma \neq \eta$), the optimal tax policy mix has no neat analytical form but instead depends on the entire model's steady state and therefore has to be derived numerically.[16] We continue to assume that unemployed workers receive lump-sum transfers T^{b*}, such that $b = b^c$.

[16] First, the optimal revenue tax is such that employment in the competitive steady state equals its efficient level, $n = n^*$. Second, the optimal consumption tax removes the hours distortion by satisfying the efficiency condition (42).

Figure 3 displays the optimal tax policy mix as a function of the elasticity of vacancy duration to the number of vacancies, η. The other model parameters are set to their benchmark values, in particular, the workers' bargaining power is set to $\gamma = 0.4$.

Figure 3: **Optimal Taxation and Congestion Effects**



The left panel shows the optimal tax on firm revenues, τ^{f*}, as a function of the elasticity of matches to unemployment, η. The right panel shows the optimal consumption tax, τ^{c*}, as a function of η.

For a given price markup, the higher the congestion effects (large η), the larger is the required tax on firm revenues. Large tax revenues lower steady-state real marginal costs $s = \frac{1-\tau^f}{\mu}$ and hence the gap between s and $1-\eta$. At the same time, for a given bargaining friction $\varkappa$, lowering steady-state real marginal costs s through $(1-\tau^f)$ increases the inefficiency gap $\frac{\varkappa}{s}$, implying that consumption has to be subsidized by more. Therefore, the higher the elasticity η, the greater are both the optimal revenue tax and the optimal consumption subsidy.

3.4 Effect on Model Dynamics

We now analyze how the steady-state distortions affect the variables' *dynamics* in a flexible-price model by comparing impulse responses in the competitive model with the efficient ones. Taxes are set to zero, $\tau^c = \tau^f = T^b = 0$. Two shocks are considered, a technology shock (A_t) and a government spending shock (G_t). The transmission channels of these shocks can be better understood by examining the log-linearized equations driving the two labor margins. In the competitive allocation, the hours and hiring decisions, (22) and (23), are given in linearized form by:

$$\hat{s}_t + \hat{A}_t + [\varphi - (1+\sigma_h)]\hat{h}_t = \hat{C}_t, \tag{46}$$

$$\frac{\eta}{\beta}\hat{\theta}_t = -\hat{r}_t + \varpi \frac{Ah^{\varphi}}{1/q} s E_t\{\hat{A}_{t+1} + \varphi\hat{h}_{t+1} + \hat{s}_{t+1}\} + [(1-\lambda)\,\eta - \gamma p]\, E_t\{\hat{\theta}_{t+1}\}, \tag{47}$$

where $\hat{r}_t = -E_t\{\hat{\beta}_{t,t+1}\}$ is the real interest rate and we define $\varpi \equiv \frac{1-\beta(1-\lambda)}{\beta}\kappa_v(1 - \frac{\varphi}{1+\sigma_h})$. A flexible-price version of our model is characterized by constant real marginal costs $s_t = 1/\mu$, such that $\hat{s}_t = 0$. As emphasized by Monacelli et al. (2010), there are two channels at work in hiring decisions.[17] The first is the real interest rate channel: any shock which increases the real interest rate $\hat{r}_t$, e.g. a public spending expansion, reduces the shadow value of an additional worker and, in turn, discourages hiring. The second channel is the marginal value of employment channel which is captured by the second term in (47): the marginal value of a worker depends on his contribution to the wage bill through the reduction in hours per worker.[18]

In the efficient allocation, the hours and vacancy posting decisions (29) and (30) are given in linearized form by:

$$\hat{A}_t + [\varphi - (1+\sigma_h)]\hat{h}_t = \hat{C}_t, \tag{48}$$

[17] Since we do not consider investment in this model, the capital accumulation channel of Monacelli et al. (2010) is absent here.

[18] We use a slightly different term than Monacelli et al. (2010), who call this the 'marginal value of work' channel, because in our model, there are two margins of work: employment and hours.

$$\frac{\eta}{\beta}\hat{\theta}_t = -\hat{r}_t + \varpi \frac{Ah^{*\varphi}}{1/q^*}(1-\eta) E_t\{\hat{A}_{t+1} + \varphi \hat{h}_{t+1}\} + [(1-\lambda)\eta - \eta p] E_t\{\hat{\theta}_{t+1}\}. \qquad (49)$$

The hours decision is identical in the efficient and in the decentralized allocation under flexible prices, see (46) and (48). Comparing the linearized hiring condition in the competitive equilibrium (47) with its efficient counterpart (49), we notice two differences.

First, price stickiness induces inefficient fluctuations in employment through variations in real marginal costs, $\hat{s}_t$. Suppose that after an expansionary demand shock, prices do not adjust upwards in the same proportion. Then real marginal costs rise. From (47), we see that the shadow value of a worker rises, because it becomes more expensive to expand hours in order to satisfy the higher demand. This effect vanishes under flexible prices where real marginal costs are constant, $\hat{s}_t = 0$.

Second, to the extent that $\frac{Y/n}{1/q}s$ differs from $\frac{Y^*/n^*}{1/q^*}(1-\eta)$, there are inefficient employment fluctuations even under flexible prices, owing to the steady-state distortions explained above. If the former elasticity, $\frac{Y/n}{1/q}s$, is higher than its efficient counterpart, $\frac{Y^*/n^*}{1/q^*}(1-\eta)$, hiring responds too strongly to the marginal value of a worker and therefore to hours worked. Under which conditions does the competitive allocation feature employment inefficiencies? As explained above, if distortions (31) and (38) are removed, the gap between the efficient and the competitive steady state disappears, which in turn makes the dynamics identical. However, the bargaining process implies that there is a wedge between the real marginal cost ($s < 1$) and the parameter reflecting wage bargaining ($\varkappa > 1$).

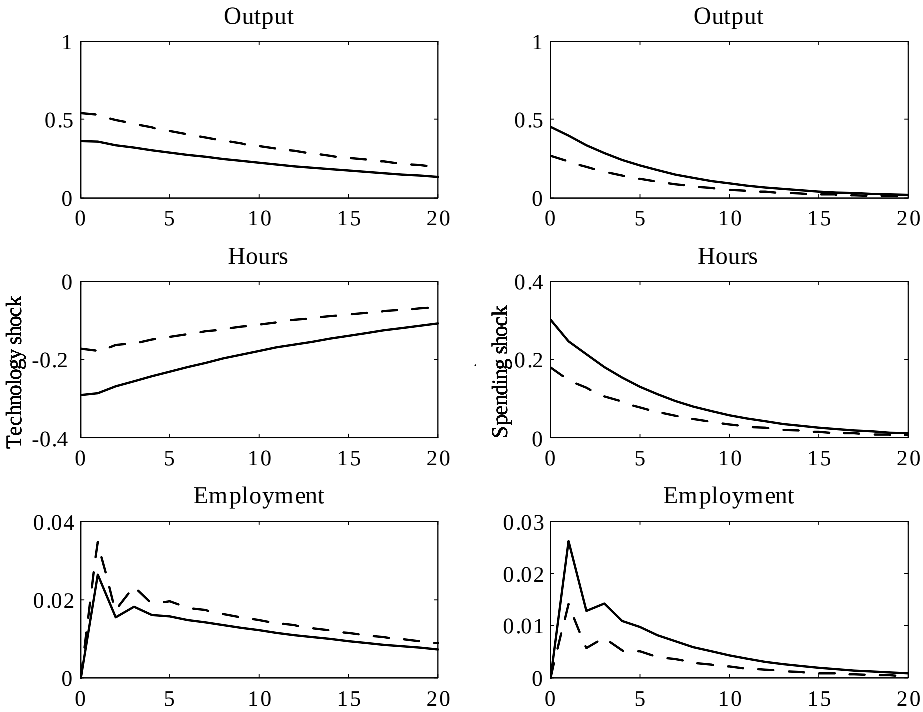
Figure 4 compares the impulse responses of output, hours and unemployment in response to technology and government spending shocks in the competitive flexible-price equilibrium and in the efficient allocation. Technology and government spending follow autoregressive processes in logs,

$$\hat{A}_t = (1-\rho_a)\ln A + \rho_a \hat{A}_{t-1} + \varepsilon_t^a, \quad \varepsilon_t^a \sim N(0, \sigma_a), \qquad (50)$$

$$\hat{G}_t = \left(1-\rho_g\right)\ln G + \rho_g \hat{G}_{t-1} + \varepsilon_t^g, \quad \varepsilon_t^g \sim N\left(0, \sigma_g\right), \tag{51}$$

where $\hat{A}_t = \ln(A_t/A)$ and $\hat{G}_t = \ln(G_t/G)$. We calibrate $\rho_a = \rho_g = 0.95$ and $\sigma_a = \sigma_g = 0.008$, as in Faia (2009). The model is the same as before, except that we replace the price setting condition (18) with its flexible-price counterpart, $s_t = 1/\mu$.

Figure 4: **Competitive and Efficient Dynamics**



The left (right) panel shows the impulse response functions to a positive technology shock (government spending shock, resp.). The continuous lines correspond to the dynamics in the competitive allocation (in the absence of taxes and under flexible prices, i.e. $s_t = 1/\mu$). The dashed lines correspond to the dynamics in the efficient allocation. Employment is expressed in percentage-point deviations from the steady state. Output and hours are in percent deviations from the steady state.

A government spending expansion ($\hat{G}_t$) implies an increase in expected future taxes which, through the wealth effect, discourages household consumption and makes workers supply more labor. Hours rise by more than is efficient. This is because the steady-state real marginal wage is too low, see Figure 1, such that a rise in hours raises the wage and therefore firms' production costs only by a small amount. Firms exploit this by expanding hours worked by a large amount. As described above, the shadow value of a worker is too sensitive to hours worked in the competitive allocation, such that firms post too many vacancies and employment rises too much in response to the spending expansion. Consequently, both the extensive and the intensive margins are too volatile compared to the efficient allocation.

An improvement in technology ($\hat{A}_t$) implies that workers have to work fewer hours to produce a given amount of output. The marginal value of a worker increases (driven by $\hat{A}_t$), although the rise in productivity is dampened by the reduction in hours. Figure 4 shows that hours drop by more in the competitive allocation than in the efficient allocation. This reduces the shadow value and hiring by firms relative to the efficient case, see (47). In addition, hiring responds by more to hours worked in the competitive allocation than in the efficient allocation. It follows that employment increases by less in the competitive equilibrium than in the efficient allocation.

We show in the next section that the gap between the efficient and the competitive allocation gives room for optimal deviations from price stability.

4 Optimal Monetary Policy

In the following, we characterize optimal monetary policy when prices are sticky. To this end, we compute the paths that the Ramsey policy maker chooses for the model variables in order to maximize household utility, subject to the decision rules of households and firms. A formal definition of the Ramsey policy is given next.

Definition 4 *The Ramsey optimal policy is a set of plans for the control variables* $\{h_t, v_t, n_{t+1}, C_t, s_t, \Pi_t\}_{t=0}^{\infty}$ *that, for a given initial employment level* n_0*, maximizes household utility (1) subject to the implementability conditions (22)-(26).*

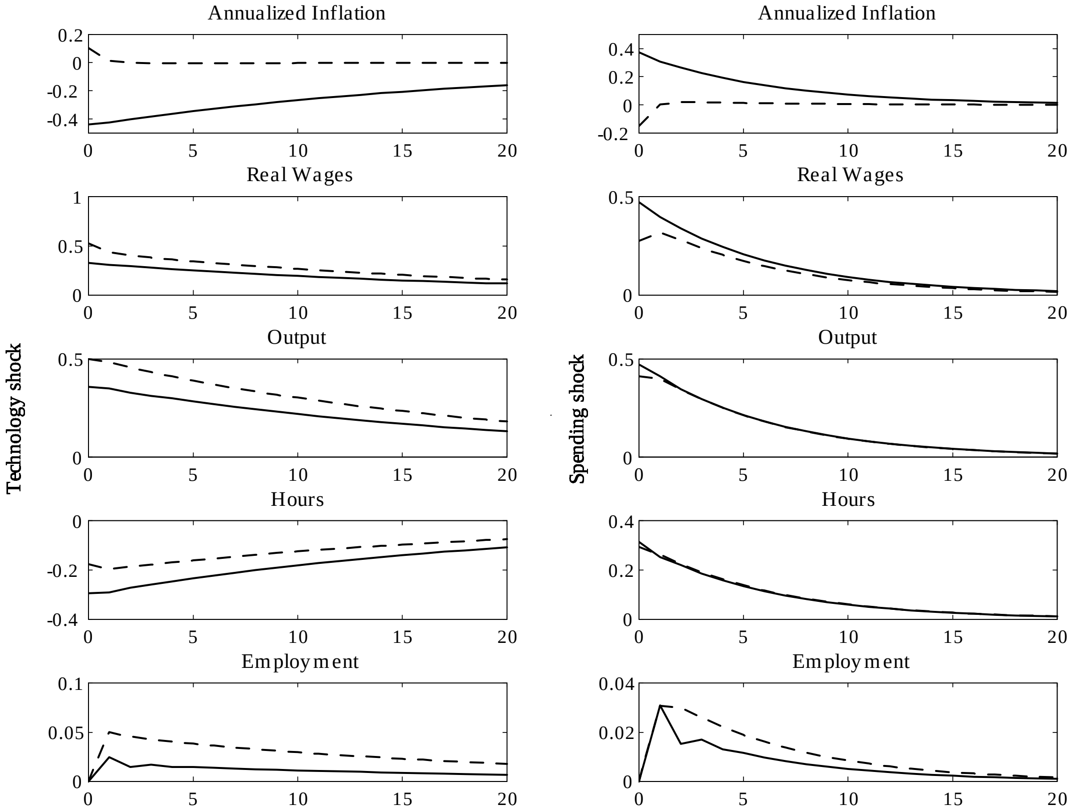
Thomas (2008) shows that in the absence of wage markup fluctuations, e.g. in the form of nominal or real wage rigidities, strict inflation targeting is optimal when the steady state is efficient. Since real marginal costs are the only time-varying wedges around an efficient steady state, the optimal monetary policy stabilizes real marginal costs over the cycle. This is true also in our model. Under the optimal tax policy $(\tau^{f*}, \tau^{c*}, T^{b*})$, price stability replicates the efficient allocation. However, if taxes are unavailable and the steady state is distorted, price stability is no longer optimal.

We investigate how the optimal policy is affected by steady-state distortions (31) and (38). To this end, we derive the first order conditions of the Ramsey problem and linearize them around the steady state. First, we study the optimal dynamics in response to technology shocks and government spending shocks. Second, we simulate the model under the Ramsey policy and compute the optimal inflation volatility.

The impulse response functions of inflation, real wages, output, hours and employment, under the optimal Ramsey policy and in the competitive allocation, are shown in Figure 5. The left and the right panels show the responses to a technology shock and to a spending shock, respectively.

The figure shows that under the optimal policy, inflation is countercyclical with respect to hours worked. After a spending expansion, real marginal costs (and therefore inflation) need to *fall* in order to compensate for the inefficiently large rise in hours, see (47). In contrast, a positive technology shock generates an inefficiently large reduction in hours, which has to be offset by a *rise* in inflation. In the next paragraph, we show that the deviation from price stability depends on the size of the two distortions. This finding differs from Sunakawa (2013) who shows that the Ramsey policy does not deviate from price stability in a search-and-matching model with right-to-manage and instantaneous hiring. With instantaneous hiring, a worker's shadow value depends on his marginal productivity. Here, since employment is predetermined, a worker's shadow value depends instead on the reduction in hours and in the wage paid to all other workers through the bargaining distortion (31). As shown above, hours decrease too much in response to a technology shock at the cost of a small variation in employment. This inefficiency is corrected by modifying real marginal costs and therefore

Figure 5: **Competitive and Ramsey Optimal Policy Dynamics**



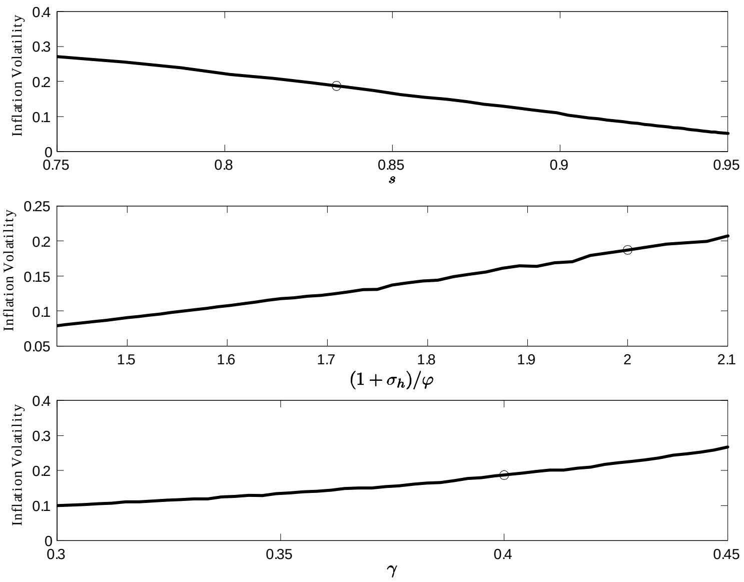
The left (right) panel shows the impulse response functions to a positive technology shock (government spending shock). The continuous lines correspond to the dynamics in a model without taxes under the Taylor-type rule $R_t/R = (\pi_t/\pi)^{1.5}$. The dashed lines correspond to the dynamics in the Ramsey allocation. Annualized inflation and employment is expressed in percentage-point deviations from the steady state. All other responses are in percent deviations from the steady state.

inflation.

We now investigate in more detail what the main drivers of the optimal inflation volatility are. As shown in (31) and (38), the steady-state distortions are reflected by two gaps. First, the gap between real marginal costs s and the intra-firm bargaining parameter $\varkappa$. Second, the gap between real marginal costs s and the elasticity of matches to vacancies, $1-\eta$. We first consider the optimal inflation volatility as a function of s.[19] In practice, we compute the volatility for a grid of values for ε, the elasticity of substitution between goods varieties. The real marginal cost is $s = (\varepsilon - 1)/\varepsilon$. Consider Figure 6.

[19]The volatility is given by the standard deviation of annualized inflation under the Ramsey policy.

Figure 6: **Optimal Inflation Volatility**



The optimal inflation volatility is computed as the standard deviation of annualized inflation (in percent). The upper panel displays inflation volatility as a function of the real marginal cost, s. The middle panel displays inflation volatility as a function of the net disutility cost of hours, $(1 + \sigma_h)/\varphi$. The lower panel displays inflation volatility as a function of the workers' bargaining power, γ.

The upper panel in Figure 6 shows that optimal inflation volatility is 0.18% in our benchmark calibration.[20] A high volatility goes hand in hand with low real marginal costs (i.e. a high price markup, μ). All things equal, higher real marginal costs diminish the steady-state hours distortion as s gets closer to $\varkappa$, see (34).[21] At the same time, the steady-state employment distortion is worsened as s deviates more from $1-\eta$, see (40). The deviation from price stability therefore depends on the relative size of these two distortions. We can shut down the hours distortion by imposing an optimal consumption subsidy $\tau^c = \tau^{c*}$. Since the only distortion left is the employment distortion, we find that volatility of optimal inflation is reduced to 0.09% (not shown).

We now have a closer look at the hours distortion, given by (31), which depends on the gap between the real marginal cost, s, and the intra-firm bargaining parameter, $\varkappa$. As mentioned above, a large value of $\varkappa$ implies that the wage curve is steep in hours, see (8).

[20] The relative inflation volatility (σ_π/σ_y) is 0.10.

[21] To understand this result, recall that in our benchmark calibration. $s = 0.83$, $\varkappa = 3$ and $1 - \eta = 0.6$.

It follows from distortion (31) that the inefficiency gap is large since the level of hours is too low compared to the Walrasian labor market allocation (i.e. $\varkappa = 1$). To confirm this intuition, the middle and lower panels in Figure 6 display the volatility of optimal inflation as a function of $(1 + \sigma_h)/\varphi$ and γ, respectively, the two drivers of the intra-firm bargaining parameter $\varkappa$.[22] Recall that $\varkappa$ is a positive function of $(1 + \sigma_h)/\varphi$. If the disutility cost of hours is high compared to the returns to hours in production, we are further away from a Walrasian labor market and the steady-state distortion resulting from bargaining is larger. Similarly, parameter $\varkappa$ is a positive function of the workers' bargaining power γ. Intuitively, real wages in steady state deviate more from the Walrasian allocation when workers have greater bargaining power. From Figure 1 we see that the real marginal wage is too low, such that wages respond less to hours. Firms exploit this insensitivity of wages by overusing the hours margin in response to shocks, at the expense of the employment margin. Hours become too volatile over the business cycle. This in turn generates optimal inflation volatility. The deviation from price stability depends on parameters affecting the trade-off between the intensive and the extensive margins of labor: the larger is the deviation from Walrasian wage setting, $\varkappa$, the greater is the optimal deviation from price stability.

This result differs from Sunakawa (2013) who shows that, in the absence of real wage rigidities, price stability is optimal in a right-to-manage model with instantaneous hiring. The reason is that in that setup, real wages and hours replicate the Walrasian allocation. We show that the intra-firm bargaining process combined with predetermined employment generates optimal deviations from price stability by generating hours and real wages that differ from the Walrasian allocation.

[22] In practical terms, we vary φ, taking $(1 + \sigma_h)$ as given.

5 Conclusion

We study optimal policy in New Keynesian search-and-matching model where firms can adjust their workforce as well as hours per worker. Firms operate under monopolistic competition; they set a price and commit to satisfying demand at that price. Since employment is predetermined, firms adjust hours per worker in a 'right-to-manage' fashion in order to satisfy demand in the short run. The right-to-manage assumption, combined with wage bargaining, results in a convex 'wage curve', such that the real marginal wage is an increasing function of hours per worker. In a large-firm model, this wage curve generates an externality since a change in the number of hours per worker affects the wage of *all other workers*. We show that product market imperfections and labor market frictions combine to reduce steady-state output and hours below their efficient levels. Since the real marginal wage is too low at the steady state, wages respond little to hours worked. Firms exploit this real wage rigidity by overusing the hours margin when adjusting their production level in response to shocks. As a result, hours are too volatile along the business cycle. A policy of strict inflation targeting is suboptimal in this environment. Inflation can be used to affect the real wage set through bargaining and dampen inefficient fluctuations in hours.

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1 Model Description

Our model features hours and employment as inputs into production, (possibly) increasing returns to hours, multiple-workers firms, search frictions in the labor market and quadratic price adjustment costs. A hat denotes the log deviation of a certain variable from its steady state level.

1.1 Households

There is a unit mass of households. A fraction n_t of workers are employed in the market economy and receive the wage w_{it} from firm $i \in (0,1)$ for providing hours h_{it}. A fraction $1-n_t$ of workers are unemployed; they are instead engaged in home production with productivity b. The representative household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - n_t \int_0^1 g(h_{it})\, di \right], \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, C_t is household consumption and $g(h_{it})$ denotes individual labor disutility of working h_t hours at firm i to those n_t household members that are employed. Each employed household member works for all firms on the unit interval; therefore, we sum labor disutility across all firms. As in Ravenna and Walsh (2012), consumption consists of final goods sold in the market and home-produced goods,

$$C_t = C_t^m + (1-n_t)\, b.$$

There exists an insurance technology guaranteeing complete consumption risk sharing between household members, such that C_t denotes consumption enjoyed by a member as well as overall household consumption. Labor disutility is given by

$$g(h_{it}) = \frac{\lambda_h h_{it}^{1+\sigma_h}}{1+\sigma_h}, \tag{2}$$

where $\lambda_h > 0$ captures the weight on hours in labor disutility, while $\sigma_h \geq 0$ determines the degree of increasing marginal disutility of hours. The household maximizes utility (1) subject to a sequence of budget constraints,

$$(1+\tau^c)\, C_t^m + \frac{B_t}{R_t P_t} = n_t \int_0^1 w_{it}(h_{it})\, di + \frac{B_{t-1}}{P_t} + D_t + (1-n_t)\, T^b + T_t, \tag{3}$$

where B_t are one-period nominal bonds that cost $1/R_t$ units of currency in t and pay a safe return of one currency unit in period $t+1$. Consumption expenditure C_t^m and bond purchases B_t are financed through wage income by employed members, where w_t is the real wage per worker, interest income on bond holdings, real profits D_t, unemployment benefits $(1-n_t)\, T^b$ from the government and lump sum transfers T_t. Consumption expenses are taxed at rate τ^c (subsidized if $\tau^c < 0$). Rewriting the household budget constraint in terms of total consumption gives

$$(1+\tau^c)\, C_t + \frac{B_t}{R_t P_t} = n_t \int_0^1 w_{it}(h_{it})\, di + (1-n_t)(1+\tau^c)\, b + \frac{B_{t-1}}{P_t} + D_t + (1-n_t)\, T^b + T_t. \tag{4}$$

So far, we have described the representative household. Given that all households are identical in equilibrium and the mass of households is normalized to unity, C_t is household consumption as well as economy-wide consumption. Writing the Lagrangian conveniently as

$$\max_{\{C_t, B_t\}_{t=0}^{\infty}} \mathcal{L}_t = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - n_t \int_0^1 g(h_{it})\, di - \Lambda_t \left[(1+\tau^c)\, C_t + \frac{B_t}{R_t P_t} - \ldots - \frac{B_{t-1}}{P_t} \right] \right\}.$$

We derive the first order conditions for consumption C_t and bonds B_t,

$$\Lambda_t = \frac{1}{(1+\tau^c)\,C_t}, \tag{5}$$

$$\frac{\Lambda_t}{P_t} = \beta R_t E_t \left\{ \frac{\Lambda_{t+1}}{P_{t+1}} \right\}, \tag{6}$$

Combining the first order conditions (5) and (6), we derive the Euler equation for consumption,

$$1 = R_t E_t \left\{ \frac{\beta_{t,t+1}}{\Pi_{t+1}} \right\}, \tag{7}$$

where $\beta_{t-1,t} = \beta \frac{\Lambda_t}{\Lambda_{t-1}}$ is the growth of the marginal utility of consumption between $t-1$ and t, and $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate. Combining the last expression with the first order condition for consumption, the stochastic discount factor becomes

$$\beta_{t-1,t} = \beta \frac{C_{t-1}}{C_t}. \tag{8}$$

Hours are not chosen by the household but are instead set by the firm in a right-to-manage (RTM) fashion (see Section 1.4).

1.2 Final Goods Firms

Final output Y_t is an aggregate of intermediate goods Y_{it} bundled according to function

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{9}$$

where ε is the elasticity of substitution between the individual varieties. Given a price P_{it} for each variety i, final good firms choose optimally the inputs Y_{it} to minimize total expenditure $\int_0^1 P_{it} Y_{it} di$ subject to the CES aggregator function (9). This yields the following demand functions,

$$Y_{it}^d = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t, \tag{10}$$

where P_t is the price of the final good. We interpret P_t as the consumer price index.

1.3 Labor Market Search and Matching

Firms post vacancies and unemployed workers search for jobs. Let $\mathcal{M}_t$ denote the number of successful matches. The matching technology is assumed to be a Cobb-Douglas function of the unemployment rate $u_t = 1 - n_t$ and the aggregate number of vacancies $v_t = \int_0^1 v_{it} di$,

$$\mathcal{M}_t = \mathcal{M}_0 u_t^{\eta} v_t^{1-\eta},$$

where $\eta \in (0,1)$ and $\mathcal{M}_0 > 0$. The probability of a vacancy being filled next period q_t equals the number of matches divided by the number of vacancies posted,

$$q_t = \frac{\mathcal{M}_t}{v_t} = \mathcal{M}_0 \theta_t^{-\eta}, \tag{11}$$

where the ratio of vacancies to unemployed workers,

$$\theta_t \equiv \frac{v_t}{u_t}, \tag{12}$$

is a measure of labor market tightness. The job finding rate equals the number of matches divided by the number of unemployed,

$$p_t = \frac{\mathcal{M}_t}{u_t} = q_t \theta_t \tag{13}$$

An alternative expression for the job finding rate is the probability of filling a vacancy multiplied by the degree of labor market tightness. A constant fraction λ of matches are destroyed each period, such that employment at firm i evolves as

$$n_{it+1} = (1-\lambda) n_{it} + q_t v_{it}. \tag{14}$$

1.4 Intermediate Goods Firms

1.4.1 Production Function

Intermediate firms produce differentiated goods under monopolistic competition, are located on the unit interval and are indexed by $i \in [0,1]$.[1] Output of an individual firm Y_{it} is produced according to the following production function

$$Y_{it} = A_t n_{it} f(h_{it}), \tag{15}$$

where A_t is a technology index common to all firms, n_{it} is employment in firm i, and the function $f(.)$ allows for decreasing or increasing returns to hours in production. Production is thus linear in employment and (potentially) non-linear in hours per worker h_{it}. We specify the function $f(.)$ as h_{it}^{φ}, such that (15) becomes

$$Y_{it} = A_t n_{it} h_{it}^{\varphi}. \tag{16}$$

A worker's marginal product per hour, defined as $mph_{it} = \frac{\partial(Y_{it}/n_{it})}{\partial h_{it}}$, is

$$mph_{it}(h_{it}) = \varphi A_t h_{it}^{\varphi-1}. \tag{17}$$

The parameter φ measures the short run returns to hours (elasticity of output to hours), $\frac{\partial Y_{it}/Y_{it}}{\partial h_{it}/h_{it}} = \varphi$. If $\varphi > 1$, we have short run increasing returns to hours. This means that increasing hours by 1% raises output by more than 1%. In response to an expansionary demand shock, firms increase hours such that measured productivity (output per hour) increases. Note also that if $1 < \varphi < 2$, the marginal product per hour (17) is increasing and concave in hours.

1.4.2 Profit Maximization

Once the firm has set its price, it is demand-constrained and has to produce the amount of output demanded at that price. The firm faces a demand function given by (10) and has production function technology given by (16). It therefore faces the following demand constraint

$$\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t = A_t n_{it} h_{it}^{\varphi}. \tag{18}$$

[1] Ultimately, we may drop the subscript i, since all firms are symmetric in this model.

Since employment is predetermined, a firm cannot raise output by increasing n_{it}. Faced with higher demand, the firm adjusts hours to satisfy demand in the short run. Formally, firms choose the number of hours worked h_{it}, the number of vacancies v_{it} to post, the number of workers n_{it+1} to hire, and which price P_{it} to set, so as to maximize the present discounted stream of future profits,

$$E_0 \sum_{t=0}^{\infty} \beta_{0,t} \left(\left(1-\tau^f\right) \frac{P_{it}}{P_t} Y_{it}^d - w_{it}\left(h_{it}\right) n_{it} - \kappa_v v_{it} - pac_{it} \right), \tag{19}$$

where τ^f is a tax on firm revenues, κ_v is the cost of posting a vacancy (common to all firms), v_{it} is the number of vacancies posted by the i'th firm, pac_{it} are price adjustment costs to be specified below and $\beta_{0,t}$ is the stochastic discount factor, defined recursively as $\beta_{0,t} = \beta_{0,1}\beta_{1,2}\dots\beta_{t-1,t}$. Firm revenues are taxed if $\tau^f > 0$ and subsidized if $\tau^f < 0$. Firms maximize (19) subject to the law of motion for employment at firm i (14), the demand constraint (18), and price adjustment costs

$$n_{it+1} = (1-\lambda) n_{it} + q_t v_{it},$$

$$\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t = A_t n_{it} h_{it}^{\varphi},$$

$$pac_{it} = \frac{\kappa_p}{2}\left(\frac{P_{it}}{P_{it-1}} - 1\right)^2 Y_t.$$

Substituting demand (10) into the firm's objective function (19), we can write the firm's optimization problem as a Lagrangian problem,

$$\begin{aligned} \max_{\{h_{it},v_{it},n_{it+1},P_{it}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta_{0,t} \Bigg\{ & \left(1-\tau^f\right)\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} Y_t - w_{it}\left(h_{it}\right) n_{it} - \kappa_v v_{it} - \frac{\kappa_p}{2}\left(\frac{P_{it}}{P_{it-1}} - 1\right)^2 Y_t \\ & - s_{it}\left[\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t - A_t n_{it} h_{it}^{\varphi}\right] \\ & - \varphi_{nt}\left[n_{it+1} - (1-\lambda) n_{it} - q_t v_{it}\right]\Bigg\}, \end{aligned}$$

where s_{it} and φ_{nt} are the Lagrange multipliers on the demand constraint and on the firm's employment dynamics, respectively. The multiplier on the demand constraint, s_{it}, represents firm i's real marginal costs.

Hours Worked The first order condition for hours worked is $0 = \beta_{0,t}\{-n_{it} w_{it}'\left(h_{it}\right) + \varphi s_{it} A_t n_{it} h_{it}^{\varphi-1}\}$, or expressed differently and using (17),

$$s_{it} = \frac{w_{it}'\left(h_{it}\right)}{mph_{it}\left(h_{it}\right)}. \tag{20}$$

According to (20), real marginal costs are equal to the ratio of the real marginal wage and the marginal product of hours per worker. Since employment is predetermined, the firm needs to raise hours per worker in order to increase production. This comes at a marginal cost of $w_{it}'\left(h_{it}\right)$ per worker. See also Thomas (2011).

Job Creation Condition The first order condition for vacancies $\{v_{it}\}_{t=0}^{\infty}$ is $0 = \beta_{0,t}\{-\kappa_v + \varphi_{nt}q_t\}$, such that we can express the Lagrange multiplier on the employment law of motion φ_{nt} as

$$\varphi_{nt} = \frac{\kappa_v}{q_t}. \tag{21}$$

The first order condition for employment $\{n_{it+1}\}_{t=0}^{\infty}$ is

$$0 = -\beta_{0,t}\varphi_{nt} + E_t\left\{\beta_{0,t+1}\left[s_{it+1}A_{t+1}h_{it+1}^{\varphi} - w_{it+1}\left(h_{it+1}\right) + (1-\lambda)\varphi_{nt+1}\right]\right\},$$

Dividing by $\beta_{0,t}$, using the relation $\beta_{t,t+1} = \frac{\beta_{0,t+1}}{\beta_{0,t}}$ and rearranging, we obtain

$$\varphi_{nt} = E_t\left\{\beta_{t,t+1}\left[s_{it+1}A_{t+1}h_{it+1}^{\varphi} - w_{it+1}\left(h_{it+1}\right) + (1-\lambda)\varphi_{nt+1}\right]\right\}. \tag{22}$$

Finally, using (21) to substitute out the Lagrange multiplier φ_{nt}, the first order condition for employment becomes

$$\frac{\kappa_v}{q_t} = E_t\left\{\beta_{t,t+1}\left[s_{it+1}A_{t+1}h_{it+1}^{\varphi} - w_{it+1}\left(h_{it+1}\right) + (1-\lambda)\frac{\kappa_v}{q_{t+1}}\right]\right\}. \tag{23}$$

A firm posts vacancies until the cost of hiring a worker equals the expected discounted future benefits from an extra worker. The costs of hiring a worker are given by the vacancy posting costs divided by the probability of filling a vacancy, equivalent to vacancy posting costs multiplied by the average duration of a vacancy, $1/q_t$. The first two terms on the right hand side of (23) correspond to expected shadow value of a worker, i.e. the expected value of an additional worker (see Section 1.6 for details). Since we do not assume instantaneous hiring, the vacancy posting condition captures the expected shadow value of a worker *tomorrow*, rather than the *current* shadow value.

Price Setting We assume quadratic price adjustment costs à la Rotemberg (1982). The parameter κ_p captures the size of price adjustment costs. Iterating pac_{it} one period, we see that pac_{it+1} also depends on P_{it},

$$pac_{it+1} = \frac{\kappa_p}{2}\left(\frac{P_{it+1}}{P_{it}} - 1\right)^2 Y_{t+1}.$$

The derivatives of pac_{it} and pac_{it+1} with respect to the firm price P_{it} are, respectively,

$$\frac{\partial pac_{it}}{\partial P_{it}} = \kappa_p \frac{1}{P_{it-1}}\left(\frac{P_{it}}{P_{it-1}} - 1\right)Y_t \tag{24}$$

$$\frac{\partial pac_{it+1}}{\partial P_{it}} = -\kappa_p \frac{P_{it+1}}{P_{it}^2}\left(\frac{P_{it+1}}{P_{it}} - 1\right)Y_{t+1}. \tag{25}$$

The first order condition for prices is

$$0 = \beta_{0,t}\left[\left(1-\tau^f\right)(1-\varepsilon) + \varepsilon s_{it}\frac{P_t}{P_{it}}\right]\frac{Y_t}{P_t}\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} - \beta_{0,t}\frac{\partial pac_{it}}{\partial P_{it}} - E_t\left\{\beta_{0,t+1}\frac{\partial pac_{it+1}}{\partial P_{it}}\right\}.$$

Dividing by $\beta_{0,t}$ and plugging in the derivatives (24) and (25), we have

$$\begin{aligned} 0 \;=\; & \left[\left(1-\tau^f\right)(1-\varepsilon) + \varepsilon\frac{P_t}{P_{it}}s_{it}\right]\frac{Y_t}{P_t}\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} \\ & -\kappa_p\frac{1}{P_{it-1}}\left(\frac{P_{it}}{P_{it-1}} - 1\right)Y_t + \kappa_p E_t\left\{\frac{\beta_{0,t+1}}{\beta_{0,t}}\frac{P_{it+1}}{P_{it}^2}\left(\frac{P_{it+1}}{P_{it}} - 1\right)Y_{t+1}\right\}. \end{aligned}$$

Dividing by Y_t/P_{it}, collecting terms and rewriting the stochastic discount factor as $\beta_{t,t+1}$, this becomes

$$0 = \left[-\left(1-\tau^f\right)(\varepsilon-1) + \varepsilon \frac{P_t}{P_{it}} s_{it} \right] \left(\frac{P_{it}}{P_t} \right)^{1-\varepsilon} + \kappa_p \Upsilon_{it},$$

where

$$\Upsilon_{it} = \frac{P_{it}}{P_{it-1}} \left(\frac{P_{it}}{P_{it-1}} - 1 \right) - E_t \left\{ \beta_{t,t+1} \frac{P_{it+1}}{P_{it}} \left(\frac{P_{it+1}}{P_{it}} - 1 \right) \frac{Y_{t+1}}{Y_t} \right\}.$$

Thus, the optimal price satisfies

$$\frac{P_{it}}{P_t} = \frac{\varepsilon s_{it}}{\left(1-\tau^f\right)(\varepsilon-1) + \frac{\kappa_p \Upsilon_{it}}{(P_{it}/P_t)^{1-\varepsilon}}},$$

Imposing symmetry ($P_{it} = P_t$, $s_{it} = s_t$, and $Y_{it} = Y_t$), this simplifies to

$$\kappa_p \Pi_t (\Pi_t - 1) = \varepsilon s_t - \left(1-\tau^f\right)(\varepsilon-1) + \kappa_p E_t \left\{ \beta_{t,t+1} \Pi_{t+1} (\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \right\}. \quad (26)$$

1.5 Wage Determination

In our model, wages are determined through Nash bargaining. To understand how bargaining affects wages, we first derive the efficient wage setting (Walrasian wages) and we compare it with bargained wages.

1.5.1 Efficient Wage Setting

Walrarian wages are set such that the demand for hours the firm equals the supply of hours by the household. If the household can choose hours optimally, it will set h_{it} to maximize utility (1) subject to the budget constraint (4). The Lagrangian problem is

$$\max_{\{h_{it}\}_{t=0}^{\infty}} \mathcal{L}_t = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - n_t \int_0^1 g(h_{it})\, di - \Lambda_t \left[(1+\tau^c) C_t \ldots - n_t \int_0^1 w_{it}(h_{it})\, di \right] \right\},$$

and the associated first order condition states that the real marginal wage must equal the marginal rate of substitution between hours and consumption,

$$w'_{it}(h_{it}) = \frac{g'(h_{it})}{\Lambda_t} = mrs_{it}. \quad (27)$$

Plugging in the marginal utility of consumption Λ_t, the real marginal wage becomes $w'_{it}(h_{it}) = \lambda_h h_{it}^{\sigma_h} (1+\tau^c) C_t$.

1.5.2 Wage Bargaining

In the model, workers and firms bargain bilaterally over the real wage w_{it} and split the surplus according to their respective bargaining weight given by γ and $(1-\gamma)$, respectively.

Firm The value of firm i in period t is

$$V_i^f(w_{it}) = (1-\tau^f)\frac{P_{it}}{P_t}Y_{it}^d - w_{it}(h_{it})n_{it} - \kappa_v v_{it} - \frac{\kappa_p}{2}\left(\frac{P_{it}}{P_{it-1}}-1\right)^2 Y_t + E_t\left\{\beta_{t,t+1}V_i^f(w_{it+1})\right\}. \quad (28)$$

For the firm, the surplus from employing a marginal worker is computed by maximizing (28) with respect to n_{it}, subject to the demand constraint $(\frac{P_{it}}{P_t})^{-\varepsilon}Y_t = A_t n_{it} h_{it}^{\varphi}$. The surplus from employing a marginal worker, defined as $S_i^f(w_{it}) \equiv \frac{\partial V_i^f(w_{it})}{\partial n_{it}}$, is given by

$$S_i^f(w_{it}) = s_{it}A_t h_{it}^{\varphi} - w_{it}(h_{it}) + (1-\lambda)E_t\left\{\beta_{t,t+1}S_i^f(w_{it+1})\right\}, \quad (29)$$

A vacancy is filled with probability q_t and remains open otherwise. The per-period cost of posting a vacancy is κ_v. The value of posting a vacancy (in terms of consumption) is

$$V_i^v(w_{it}) = -\kappa_v + E_t\left\{\beta_{t,t+1}\left[q_t S_i^f(w_{it+1}) + (1-q_t)V_i^v(w_{it+1})\right]\right\}.$$

The firm posts vacancies as long as the value of a vacancy is greater than zero. In equilibrium, $V_i^v(w_{it}) = 0$ and so the vacancy posting condition is

$$\frac{\kappa_v}{q_t} = E_t\left\{\beta_{t,t+1}S_i^f(w_{it+1})\right\}. \quad (30)$$

Worker Denote the value of being employed by the i^{th} firm $W_i(w_{it})$ and the value of being unemployed U_t. In period t, an employed worker receives the wage w_{it} and suffers the disutility $g(h_{it})$ given by (2). In the next period, he is either still employed by firm i with probability $1-\lambda$, in which case he has an expected value of $E_t\left\{\beta_{t,t+1}W_i(w_{it+1})\right\}$, or the employment relation is dissolved with probability λ, then his expected value is $E_t\left\{\beta_{t,t+1}U_{t+1}\right\}$. The worker's asset value of being matched to firm i is

$$W_i(w_{it}) = w_{it} - \frac{g(h_{it})}{\Lambda_t} + E_t\left\{\beta_{t,t+1}\left[(1-\lambda)W_i(w_{it+1}) + \lambda U_{t+1}\right]\right\}, \quad (31)$$

where we divide labor disutility $g(h_{it})$ by the marginal utility of consumption Λ_t to convert utils into consumption units. The value of being unemployed U_t is in turn given by

$$U_t = b^c + E_t\left\{\beta_{t,t+1}\left[\theta_t q_t \int_0^1 \frac{v_{jt}}{v_t}W_j(w_{jt+1})\,dj + (1-\theta_t q_t)U_{t+1}\right]\right\}, \quad (32)$$

where $b^c = (1+\tau^c)b + T^b$. An unemployed worker produces $(1+\tau^c)b$ units of market consumption goods in period t and receives transfer T^b from the government. In the next period, he faces a probability $\frac{v_{jt}}{u_t}q_t$ of finding a new job with firm j and a probability $1-\theta_t q_t$ of remaining unemployed.

Defining the worker's surplus as $S_{it}^w(w_{it}) = W_i(w_{it}) - U_t$, we can subtract (32) from (31) to write

$$S_{it}^w(w_{it}) = w_{it} - \frac{g(h_{it})}{\Lambda_t} - b^c + E_t\left\{\beta_{t,t+1}\left[(1-\lambda)S_{it+1}^w - \int_0^1 \theta_t q_t \frac{v_{jt}}{v_t}S_{jt+1}^w dj\right]\right\}. \quad (33)$$

Wage Bargaining Under Nash bargaining, the equilibrium wage satisfies

$$w_{it} = \arg\max_{w_{it}} \left(S_{it}^{w}\left(w_{it}\right)\right)^{\gamma}\left(S_{i}^{f}\left(w_{it}\right)\right)^{1-\gamma},$$

such that the surplus-sharing rule implies

$$S_{it}^{w}\left(w_{it}\right) = \frac{\gamma}{1-\gamma} S_{i}^{f}\left(w_{it}\right). \tag{34}$$

Using the surplus-sharing rule (34) to replace S_{it}^{w}, S_{it+1}^{w} and S_{jt+1}^{w}, this becomes

$$\frac{\gamma}{1-\gamma} S_{i}^{f}\left(w_{it}\right) = w_{it} - \frac{g\left(h_{it}\right)}{\Lambda_t} - b^{c} + \frac{\gamma}{1-\gamma} E_t\left\{\beta_{t,t+1}\left[\left(1-\lambda\right) S_{i}^{f}\left(w_{it+1}\right) - \int_0^1 \theta_t q_t \frac{v_{jt}}{v_t} S_{i}^{f}\left(w_{it+1}\right) dj\right]\right\}.$$

Substituting out $S_{i}^{f}\left(w_{it}\right)$ using the firm's surplus (29), we obtain

$$\begin{aligned} & \frac{\gamma}{1-\gamma}\left[s_{it} A_t h_{it}^{\varphi} - w_{it}\left(h_{it}\right) + \left(1-\lambda\right) E_t\left\{\beta_{t,t+1} S_{i}^{f}\left(w_{it+1}\right)\right\}\right] \\ = \; & w_{it}\left(h_{it}\right) - \frac{g\left(h_{it}\right)}{\Lambda_t} - b^{c} + \frac{\gamma}{1-\gamma} E_t\left\{\beta_{t,t+1}\left(\left(1-\lambda\right) S_{i}^{f}\left(w_{it+1}\right) - \int_0^1 \theta_t q_t \frac{v_{jt}}{v_t} S_{i}^{f}\left(w_{it+1}\right) dj\right)\right\}. \end{aligned}$$

Finally, using the vacancy-posting rule (30) to replace $S_{i}^{f}\left(w_{it+1}\right)$ and $S_{i}^{f}\left(w_{it+1}\right)$, the equilibrium wage satisfies

$$\frac{\gamma}{1-\gamma}\left(s_{it} A_t h_{it}^{\varphi} - w_{it}\left(h_{it}\right) + \left(1-\lambda\right)\frac{\kappa_v}{q_t}\right) = w_{it} - \frac{g\left(h_{it}\right)}{\Lambda_t} - b^{c} + \frac{\gamma}{1-\gamma}\frac{\kappa_v}{q_t}\left(1-\lambda-\theta_t q_t\right),$$

or, after rearranging,

$$w_{it}\left(h_{it}\right) = \gamma\left(s_{it} A_t h_{it}^{\varphi} + \kappa_v \theta_t\right) + \left(1-\gamma\right)\left(\frac{g\left(h_{it}\right)}{\Lambda_t} + b^{c}\right).$$

Substituting out s_{it} using (20) gives

$$w_{it}\left(h_{it}\right) = \gamma\left(\frac{1}{\varphi} h_{it} w_{it}'\left(h_{it}\right) + \kappa_v \theta_t\right) + \left(1-\gamma\right)\left(\frac{g\left(h_{it}\right)}{\Lambda_t} + b^{c}\right). \tag{35}$$

Equilibrium Wage Rearranging (35), and replacing $g\left(h_{it}\right)$ using (2) and Λ_t using the first order condition for consumption (5), we obtain

$$w_{it}\left(h_{it}\right) = \gamma \kappa_v \theta_t + \left(1-\gamma\right) b^{c} + \frac{\gamma}{\varphi} h_{it} w_{it}'\left(h_{it}\right) + \left(1-\gamma\right)\frac{g\left(h_{it}\right)}{\Lambda_t}. \tag{36}$$

Using the method of undetermined coefficients, we guess that the solution to (36) takes the form

$$w_{it}\left(h_{it}\right) = \gamma \kappa_v \theta_t + \left(1-\gamma\right) b^{c} + \varkappa \frac{g\left(h_{it}\right)}{\Lambda_t}, \tag{37}$$

where

$$\varkappa \equiv \frac{1-\gamma}{1-\gamma\frac{1+\sigma_h}{\varphi}}. \tag{38}$$

Under bargaining, the real marginal wage is

$$w'_{it}(h_{it}) = \varkappa \frac{g'(h_{it})}{\Lambda_t} = \mu_{wt} \cdot mrs_{it}(h_{it}), \tag{39}$$

where $\mu_{wt} = \varkappa$ represents a wage markup. Alternatively, using (20), we can relate the real marginal wage to the marginal product of hours as follows,

$$w'_{it}(h_{it}) = \frac{mph_{it}(h_{it})}{\mu_{pt}}. \tag{40}$$

where $\mu_{pt} = 1/s_{it}$ represents a price markup. See Gali, Gertler and Lopez-Salido (2007). To ensure that the real marginal wage is positive, such that the wage is increasing in hours worked, we impose the following parameter restriction:

$$1 - \gamma \frac{1+\sigma_h}{\varphi} > 0, \tag{41}$$

such that $\varkappa > 0$. Furthermore, in a typical calibration we have $\frac{1+\sigma_h}{\varphi} > 1$ and therefore $\varkappa > 1$, such that the real marginal wage under bargaining lies above the marginal rate of substitution. This means that, under Nash bargaining, wages rise faster with hours than is efficient. Comparing the efficient real marginal wage (27) with the real marginal wage under bargaining (39), we note that the parameter $\varkappa$ encapsulates the distortion imposed by the bargaining process. Given a certain value for the curvature of labor disutility σ_h, the bargaining distortion is increasing both in the degree of returns to hours φ and in the bargaining power of workers γ.

For future reference, note that the parameter governing the curvature of the labor disutility function, σ_h, is also the elasticity of the real marginal wage to hours, i.e.

$$\sigma_h = h_{it} \frac{w''_{it}(h_{it})}{w'_{it}(h_{it})}. \tag{42}$$

Rearranging (35), and replacing $g(h_{it})$ using (2) and Λ_t using the first order condition for consumption (5), we obtain an alternative expression for the equilibrium wage,

$$w_{it}(h_{it}) = \gamma \kappa_v \theta_t + (1-\gamma) b^c + \varkappa \frac{\lambda_h h_{it}^{1+\sigma_h}}{1+\sigma_h} (1+\tau^c) C_t.$$

1.6 Worker Shadow Value

We define the total reduction in the wage bill $w_{it}(h_{it}) n_{it}$ induced by an additional worker, the shadow value of a marginal worker, as

$$\chi_{it} \equiv -\frac{\partial w_{it}(h_{it}(n_{it})) n_{it}}{\partial n_{it}}. \tag{43}$$

The wage is a function of hours, $w_{it}(h_{it})$, as shown in (37) above. In turn, hours are a function of the number of workers, $h_{it}(n_{it})$. To see this, we rewrite the production function (16) in terms of a labor requirement (in terms of hours) for a given level of output Y_{it}, i.e.

$$h_{it} = Y_{it}^{1/\varphi} (A_t n_{it})^{-1/\varphi}.$$

We can derive the derivative of hours to employment as

$$\frac{\partial h_{it}}{\partial n_{it}} = -\frac{1}{\varphi}\frac{h_{it}}{n_{it}},$$

For a given amount of output, hiring an additional worker thus allows the firm to reduce the number of hours of all other workers. The firm effectively reduces the intensive margin and raises the extensive margin of production. The shadow value of a worker becomes

$$\chi_{it} = -w_{it}(h_{it}) - w'_{it}(h_{it})\frac{\partial h_{it}}{\partial n_{it}}n_{it} = -w_{it}(h_{it}) + w'_{it}(h_{it})\frac{h_{it}}{\varphi}. \tag{44}$$

The shadow value of the marginal worker has two components. The first is the wage payment going to the worker, the second represents the reduction in the wage bill due to the additional hire. Hiring an extra worker allows the firm to lower hours per worker for all its workers and, through the wage curve (37), to lower the wage of all its workers. Substituting out the derivative $w'_{it}(h_{it})$ from (20) yields

$$\chi_{it} = -w_{it}(h_{it}) + \frac{h_{it}}{\varphi}s_{it}mph_{it}. \tag{45}$$

Finally, using the bargaining wage (37), we can express the shadow value as

$$\chi_{it} = -\gamma\kappa_v\theta_t - (1-\gamma)b^c + \varkappa\left(\frac{1+\sigma_h}{\varphi} - 1\right)\frac{g(h_{it})}{\Lambda_t}.$$

Notice that we can rewrite the vacancy posting condition (23) as

$$\frac{\kappa_v}{q_t} = E_t\left\{\beta_{t,t+1}\left[\chi_{it+1} + (1-\lambda)\frac{\kappa_v}{q_{t+1}}\right]\right\}. \tag{46}$$

Let's analyze how the shadow value is related to hours worked. Differentiating (44) yields

$$\chi'_{it}(h_{it}) = -w'_{it}(h_{it}) + \frac{1}{\varphi}\left(w'_{it}(h_{it}) + w''_{it}(h_{it})h_{it}\right).$$

Rearranging, we get

$$\chi'_{it}(h_{it}) = w'_{it}(h_{it})\left(-1 + \frac{1}{\varphi}\left(1 + \frac{w''_{it}(h_{it})}{w'_{it}(h_{it})}h_{it}\right)\right).$$

Using again the elasticity of the wage to hours in (42), this becomes

$$\chi'_{it}(h_{it}) = w'_{it}(h_{it})\left(\frac{1+\sigma_h}{\varphi} - 1\right).$$

If condition (41) is satisfied and $\frac{1+\sigma_h}{\varphi} > 1$, the worker's marginal value increases with hours per worker. When computing the steady state, we verify that these conditions are satisfied.

1.7 Real Marginal Costs

Alternatively to (20), we can express real marginal costs in terms of the worker's shadow value by rearranging (45),

$$s_{it} = \frac{w_{it}(h_{it})}{A_t h_{it}^{\varphi}} + \frac{\chi_{it}}{A_t h_{it}^{\varphi}}. \tag{47}$$

In (47), both the wage and the shadow value are divided by *endogenous* labor productivity, i.e. the marginal product of labor in terms of employment,

$$\frac{\partial Y_{it}}{\partial n_{it}} = A_t h_{it}^{\varphi}.$$

Since the production function is linear in the number of workers, the marginal product of employment is equal to the average product (output per worker).

The first component of real marginal costs in (47) represents unit labor costs, $\frac{w_{it} n_{it}}{Y_{it}}$. Let's compare expression (47) for real marginal costs with the standard New Keynesian model,

$$s_{it} = \frac{w_{it} n_{it}}{Y_{it}} \quad \text{and} \quad s_{it} = \underbrace{\frac{w_{it} n_{it}}{Y_{it}}}_{\text{unit labor costs}} + \frac{n_{it}}{Y_{it}} \underbrace{\left[\frac{\kappa_v}{q_t} - E_t \left\{ \beta_{t,t+1} (1-\lambda) \frac{\kappa_v}{q_{t+1}} \right\} \right]}_{\chi_t \,=\, \text{net hiring costs}}.$$

In the New Keynesian model, real marginal costs are given unit labor costs. In the search-and-matching framework, real marginal costs have an additional component: net hiring costs. Net hiring costs are today's hiring costs (vacancy posting costs times the duration of a vacancy) less the saving of tomorrow's expected hiring costs in case the employment relationship continues. Under the assumption of instantaneous hiring, this term represents the current shadow value of a worker. See Krause and Lubik (2007) and Faia (2009).

Replacing the derivative $w'_{it}(h_{it})$ in the first order condition for hours (20) using (39), and rearranging, real marginal costs become

$$s_{it} = \varkappa \frac{g'(h_{it})/\Lambda_t}{\varphi A_t h_{it}^{\varphi-1}} = \varkappa \frac{mrs_{it}}{mph_{it}} \tag{48}$$

Using the real marginal cost expression (48), we may rewrite the shadow value (45) as

$$\chi_{it} = -\gamma \kappa_v \theta_t - (1-\gamma) b^c + \left(1 - \frac{\varphi}{1+\sigma_h}\right) A_t h_{it}^{\varphi} s_{it}, \tag{49}$$

and the bargaining wage (37) becomes

$$w_{it} = \gamma \kappa_v \theta_t + (1-\gamma) b^c + \frac{\varphi}{1+\sigma_h} A_t h_{it}^{\varphi} s_{it}. \tag{50}$$

Under efficient wage setting, real marginal costs would instead be given by the marginal rate of substitution divided by the marginal product of hours,

$$s_{it} = \frac{g'(h_{it})/\Lambda_t}{\varphi A_t h_{it}^{\varphi-1}} = \frac{mrs_{it}}{mph_{it}}. \tag{51}$$

1.8 Government

The government budget constraint equates current income (bond issues and tax revenues) with current expenditure (government spending, lump-sum transfers, and maturing government bonds),

$$\frac{B_t}{R_t P_t} + \tau^c C_t^m + \tau^f Y_t = G_t + (1 - n_t) T^b + T_t + \frac{B_{t-1}}{P_t}. \tag{52}$$

1.9 Aggregate Accounting

Aggregating the budget constraint (3) across households yields

$$(1 + \tau^c) C_t^m + \frac{B_t}{R_t P_t} = w_t n_t + \frac{B_{t-1}}{P_t} + D_t + (1 - n_t) T^b + T_t. \tag{53}$$

We assume that the costs of posting vacancies are distributed to households; they are included in firm profits D_t. Price adjustment costs are also subsumed in D_t, such that aggregate (after-tax) profits are

$$D_t = \left(1 - \tau^f\right) Y_t - w_t n_t - \kappa_v v_t - \frac{\kappa_p}{2} (\Pi_t - 1)^2 Y_t. \tag{54}$$

We combine the aggregate household budget constraint (53) with the government budget constraint (52) and the aggregate profit equation (54) to obtain the aggregate accounting identity,

$$Y_t + (1 - n_t) b = C_t + G_t + \kappa_v v_t + \frac{\kappa_p}{2} (\Pi_t - 1)^2 Y_t. \tag{55}$$

1.10 Exogenous Shocks

Technology and government spending follow autoregressive processes in logs,

$$\log(A_t) = (1 - \rho_a) \log(A) + \rho_a \log(A_{t-1}) + \varepsilon_t^a, \quad \varepsilon_t^a \sim N(0, \sigma_a),$$

$$\log(G_t) = (1 - \rho_g) \log(G) + \rho_g \log(G_{t-1}) + \varepsilon_t^g, \quad \varepsilon_t^g \sim N(0, \sigma_g).$$

2 Steady State

The following equations summarize the zero-inflation ($\Pi = 1$) steady state.

Unemployment:

$$u = 1 - n \tag{56}$$

Number of matches:

$$\mathcal{M} = \mathcal{M}_0 u^\eta v^{1-\eta} \tag{57}$$

Job finding rate:

$$p = \frac{\mathcal{M}}{u} \tag{58}$$

Vacancy filling rate:

$$q = \frac{\mathcal{M}}{v} \tag{59}$$

Labor market tightness

$$\theta = \frac{v}{u} \tag{60}$$

Production function:

$$Y = Anh^{\varphi} \tag{61}$$

Vacancy posting:

$$\frac{\kappa_v}{q} = \frac{\beta}{1-\beta(1-\lambda)}\chi \tag{62}$$

Worker's shadow value:

$$\chi = -\gamma\kappa_v\theta - (1-\gamma)\,b^c + \left(1 - \frac{\varphi}{1+\sigma_h}\right) Ah^{\varphi}s \tag{63}$$

Price setting:

$$\varepsilon s = \left(1-\tau^f\right)(\varepsilon - 1) \tag{64}$$

Real marginal costs:

$$s = \varkappa \frac{\lambda_h h^{\sigma_h}(1+\tau^c)\,C}{\varphi A h^{\varphi-1}} \tag{65}$$

where $\varkappa$ is a positive constant determined by $\varkappa \equiv (1-\gamma)/\left(1-\gamma\frac{1+\sigma_h}{\varphi}\right)$.

Aggregate accounting:

$$Y + bu = C + G + \kappa_v v \tag{66}$$

Finally, the bargaining wage w can be computed residually using

$$w = \gamma\kappa_v\theta + (1-\gamma)\,b^c + \frac{\varphi}{1+\sigma_h}Ah^{\varphi}s. \tag{67}$$

We will see in Section 6.1 that we set $T^b = -\tau^c b$.

3 Summary

3.1 Recursive Steady State

We can rewrite the model's steady state recursively such that

$$\begin{aligned}
A &= \frac{Y}{nh^{\varphi}}, \\
s &= \left(1-\tau^f\right)\frac{(\varepsilon-1)}{\varepsilon}, \\
\kappa_v &= \frac{c_v Y}{\theta(1-n)}, \\
\chi &= \frac{\kappa_v}{q}\left[\frac{1-\beta(1-\lambda)}{\beta}\right], \\
b &= \frac{1}{(1-\gamma)}\left[\left(1-\frac{\varphi}{1+\sigma_h}\right)Ah^{\varphi}s - \chi - \gamma\kappa_v\theta\right], \qquad \text{under the assumption } T^b = -\tau^c b \\
C &= Y - G - \kappa_v v + (1-n)\,b, \\
\lambda_h &= (1+\sigma_h)\left[\frac{1-\gamma\frac{1+\sigma_h}{\varphi}}{1-\gamma}\frac{\varphi}{1+\sigma_h}\frac{As}{h^{1+\sigma_h-\varphi}(1+\tau^c)\,C}\right].
\end{aligned}$$

3.2 Nonlinear Model

Endogenous variables: $\beta_{t-1,t}$, h_t, θ_t, χ_t, n_t, s_t, Π_t, Y_t, C_t, R_t, v_t, u_t, w_t, A_t, G_t. Exogenous variables: ε_t^a, ε_t^g. We replace n_{t+1} with n_t and n_t with n_{t-1} to be consistent with the timing convention in Dynare.

$$Y_t = A_t n_{t-1} h_t^{\varphi} \tag{68}$$

$$1 = R_t E_t \left\{ \frac{\beta_{t,t+1}}{\Pi_{t+1}} \right\} \tag{69}$$

$$\beta_{t-1,t} = \beta \frac{C_{t-1}}{C_t} \tag{70}$$

$$\frac{\kappa_v}{\mathcal{M}_0} \theta_t^{\eta} = E_t \left\{ \beta_{t,t+1} \left[\chi_{t+1} + (1-\lambda) \frac{\kappa_v}{\mathcal{M}_0} \theta_{t+1}^{\eta} \right] \right\} \tag{71}$$

$$\chi_t = -\gamma \kappa_v \theta_t - (1-\gamma) b^c + \left(1 - \frac{\varphi}{1+\sigma_h} \right) A_t h_t^{\varphi} s_t \tag{72}$$

$$n_t = (1-\lambda) n_{t-1} + \mathcal{M}_0 \theta_t^{1-\eta} (1 - n_{t-1}) \tag{73}$$

$$\kappa_p \Pi_t (\Pi_t - 1) = \varepsilon s_t - \left(1 - \tau^f\right) (\varepsilon - 1) + \kappa_p E_t \left\{ \beta_{t,t+1} \Pi_{t+1} (\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \right\} \tag{74}$$

$$s_t = \varkappa \frac{\lambda_h h_t^{\sigma_h} (1+\tau^c) C_t}{\varphi A_t h_t^{\varphi - 1}} \tag{75}$$

$$Y_t + (1 - n_{t-1}) b = C_t + G_t + \kappa_v v_t + \frac{\kappa_p}{2} (\Pi_t - 1)^2 Y_t \tag{76}$$

$$\theta_t = \frac{v_t}{1 - n_{t-1}} \tag{77}$$

$$u_t = 1 - n_{t-1} \tag{78}$$

$$w_t = \gamma \kappa_v \theta_t + (1-\gamma) b^c + \frac{\varphi}{1+\sigma_h} A_t h_t^{\varphi} s_t \tag{79}$$

3.3 Linearized Model

Endogenous variables: $\hat{\beta}_{t-1,t}$, $\hat{h}_t$, $\hat{\theta}_t$, $\hat{\chi}_t$, $\hat{n}_t$, $\hat{s}_t$, $\hat{\Pi}_t$, $\hat{Y}_t$, $\hat{C}_t$, $\hat{R}_t$, $\hat{v}_t$, $\hat{u}_t$, $\hat{w}_t$, $\hat{A}_t$, $\hat{G}_t$. Exogenous variables: ε_t^a, ε_t^g.

$$\hat{Y}_t = \hat{A}_t + \hat{n}_{t-1} + \varphi \hat{h}_t \tag{80}$$

$$0 = \hat{R}_t + E_t \{ \hat{\beta}_{t,t+1} - \hat{\Pi}_{t+1} \} \tag{81}$$

$$\hat{\beta}_{t-1,t} = -(\hat{C}_t - \hat{C}_{t-1}) \tag{82}$$

$$\eta \hat{\theta}_t = E_t \left\{ \beta \hat{\beta}_{t,t+1} + [1 - \beta (1-\lambda)] \hat{\chi}_{t+1} + (1-\lambda) \eta \beta \hat{\theta}_{t+1} \right\} \tag{83}$$

$$\chi \hat{\chi}_t = -\gamma \kappa_v \theta \hat{\theta}_t + \left(1 - \frac{\varphi}{1+\sigma_h} \right) A h^{\varphi} s \left(\hat{A}_t + \varphi \hat{h}_t + \hat{s}_t \right) \tag{84}$$

$$\hat{n}_t = (1 - \lambda - q\theta) \hat{n}_{t-1} + \frac{1-n}{n} q\theta (1-\eta) \hat{\theta}_t \tag{85}$$

$$\hat{\Pi}_t = \frac{\varepsilon s}{\kappa_p} \hat{s}_t + \beta E_t \{ \hat{\Pi}_{t+1} \} \tag{86}$$

$$\hat{s}_t = (1 + \sigma_h - \varphi)\,\hat{h}_t + \hat{C}_t - \hat{A}_t \tag{87}$$
$$Y\hat{Y}_t - bn\hat{n}_{t-1} = C\hat{C}_t + G\hat{G}_t + \kappa_v v\hat{v}_t \tag{88}$$
$$\hat{\theta}_t = \hat{v}_t + \frac{n}{1-n}\hat{n}_{t-1} \tag{89}$$
$$\hat{u}_t = -\frac{n}{1-n}\hat{n}_{t-1} \tag{90}$$
$$w\hat{w}_t = \gamma\kappa_v\theta\hat{\theta}_t + \frac{\varphi}{1+\sigma_h}Ah^{\varphi}s\left(\hat{A}_t + \hat{h}_t + \hat{s}_t\right) \tag{91}$$

4 Efficient Allocation

In this section, we consider the planner problem of choosing consumption, employment, vacancies and hours in the absence of price setting frictions.

4.1 Planner Optimization Program

The planner problem reads

$$\max_{\{C_t,n_{t+1},v_t,h_t\}} E_0 \sum_{t=0}^{\infty} \beta^t [\ln C_t - n_t \frac{\lambda_h h_t^{1+\sigma_h}}{1+\sigma_h}],$$

subject to the employment dynamics equation and the resource constraint,

$$n_{t+1} = (1-\lambda)\,n_t + \mathcal{M}_0\,(1-n_t)^{\eta}\,v_t^{1-\eta}, \tag{92}$$
$$A_t n_t h_t^{\varphi} + (1-n_t)\,b = C_t + G_t + \kappa_v v_t. \tag{93}$$

The optimal policy problem can be specified as

$$\max_{\{C_t,n_{t+1},v_t,h_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \ln C_t - n_t \frac{\lambda_h h_t^{1+\sigma_h}}{1+\sigma_h} + \varphi_{n,t}[n_{t+1} - (1-\lambda)\,n_t - \mathcal{M}_0\,(1-n_t)^{\eta}\,v_t^{1-\eta}]$$
$$+ \varphi_{C,t}\left[A_t n_t h_t^{\varphi} + (1-n_t)\,b - C_t - G_t - \kappa_v v_t\right]\Big\}.$$

The first order conditions for C_t, n_{t+1}, v_t and h_t are, respectively,

$$0 = \frac{1}{C_t} - \varphi_{C,t},$$

$$0 = \varphi_{n,t} + \beta E_t\{-\tfrac{\lambda_h h_{t+1}^{1+\sigma_h}}{1+\sigma_h} + \varphi_{n,t+1}[-(1-\lambda) + \mathcal{M}_0\eta\,(1-n_{t+1})^{\eta-1}\,v_{t+1}^{1-\eta}] + \varphi_{C,t+1}\left(A_{t+1}h_{t+1}^{\varphi} - b\right)\},$$
$$0 = -\varphi_{n,t}\mathcal{M}_0\,(1-\eta)\,(1-n_t)^{\eta}\,v_t^{-\eta} - \varphi_{C,t}\kappa_v,$$
$$0 = -\lambda_h h_t^{\sigma_h} n_t + \varphi\varphi_{C,t}A_t n_t h_t^{\varphi-1}.$$

Dividing by β^t, replacing $\frac{v_t}{1-n_t}$ with θ_t, and rearranging, the first order conditions become

$$\varphi_{C,t} = \frac{1}{C_t}, \tag{94}$$

$$-\varphi_{n,t} = \beta E_t\{-\tfrac{\lambda_h h_{t+1}^{1+\sigma_h}}{1+\sigma_h} + \varphi_{n,t+1}[-(1-\lambda) + \mathcal{M}_0\eta\theta_{t+1}^{1-\eta}] + \varphi_{C,t+1}A_{t+1}h_{t+1}^{\varphi} - \varphi_{C,t+1}b\}, \tag{95}$$
$$-\varphi_{n,t} = \varphi_{C,t}\frac{\kappa_v}{\mathcal{M}_0\,(1-\eta)}\theta_t^{\eta}, \tag{96}$$
$$\varphi_{C,t}\varphi A_t h_t^{\varphi-1} = \lambda_h h_t^{\sigma_h}. \tag{97}$$

4.2 Efficient Choice of Hours

The optimal choice of hours is obtained by combining (94) and (97),

$$\varphi A_t h_t^{\varphi-1} = \lambda_h h_t^{\sigma_h} C_t, \tag{98}$$

4.3 Efficient Job Creation Condition

Plugging (97) into (95) to eliminate $\varphi_{C,t+1}A_{t+1}h_{t+1}^{\varphi}$, we obtain

$$-\varphi_{n,t} = \beta E_t\{(\tfrac{1+\sigma_h}{\varphi}-1)\tfrac{\lambda_h h_{t+1}^{1+\sigma_h}}{1+\sigma_h} - \varphi_{n,t+1}[(1-\lambda)-\mathcal{M}_0\eta\theta_{t+1}^{1-\eta}] - \varphi_{C,t+1}b\}.$$

Substituting out the Lagrange multipliers on employment dynamics, $-\varphi_{n,t}$ and $-\varphi_{n,t+1}$, using (96), we obtain

$$\varphi_{C,t}\tfrac{\kappa_v}{\mathcal{M}_0(1-\eta)}\theta_t^{\eta} = \beta E_t\{(\tfrac{1+\sigma_h}{\varphi}-1)\tfrac{\lambda_h h_{t+1}^{1+\sigma_h}}{1+\sigma_h} - \tfrac{\kappa_v}{\mathcal{M}_0(1-\eta)}\varphi_{C,t+1}\theta_{t+1}^{\eta}[(1-\lambda)-\mathcal{M}_0\eta\theta_{t+1}^{1-\eta}] - \varphi_{C,t+1}b\}.$$

Dividing by $\varphi_{C,t}$ and using (11) to replace $\frac{\theta_t^{\eta}}{\mathcal{M}_0}$, the vacancy posting condition becomes

$$\tfrac{\kappa_v}{q_t(1-\eta)} = \beta E_t\{\tfrac{\varphi_{C,t+1}}{\varphi_{C,t}}[(\tfrac{1+\sigma_h}{\varphi}-1)\tfrac{\lambda_h h_{t+1}^{1+\sigma_h}}{1+\sigma_h}\frac{1}{\varphi_{C,t+1}} + \tfrac{\kappa_v}{q_{t+1}(1-\eta)}[(1-\lambda)-q_{t+1}\eta\theta_{t+1}] - b]\}.$$

Multiplying by $(1-\eta)$ and rearranging, we obtain

$$\frac{\kappa_v}{q_t} = \beta E_t\{\tfrac{\varphi_{C,t+1}}{\varphi_{C,t}}[(1-\eta)(\tfrac{1+\sigma_h}{\varphi}-1)\tfrac{\lambda_h h_{t+1}^{1+\sigma_h}}{1+\sigma_h}\frac{1}{\varphi_{C,t+1}} + (1-\lambda)\frac{\kappa_v}{q_{t+1}} - \eta\kappa_v\theta_{t+1} - (1-\eta)b]\}.$$

Replacing $\frac{\varphi_{C,t+1}}{\varphi_{C,t}}$ with $\beta_{t,t+1}$, and $1/\varphi_{C,t+1}$ with C_{t+1}, we obtain

$$\frac{\kappa_v}{q_t} = E_t\{\beta_{t,t+1}[-\eta\kappa_v\theta_{t+1} - (1-\eta)b + (1-\eta)(\tfrac{1+\sigma_h}{\varphi}-1)\tfrac{\lambda_h h_{t+1}^{1+\sigma_h}}{1+\sigma_h}C_{t+1} + (1-\lambda)\frac{\kappa_v}{q_{t+1}}]\}.$$

Thus, the job creation condition in the efficient allocation is

$$\frac{\kappa_v}{q_t} = E_t\{\beta_{t,t+1}[\chi_{t+1} + (1-\lambda)\frac{\kappa_v}{q_{t+1}}]\}. \tag{99}$$

where

$$\chi_t = -\eta\kappa_v\theta_t - (1-\eta)b + (1-\eta)\left(\frac{1+\sigma_h}{\varphi}-1\right)\frac{\lambda_h h_t^{1+\sigma_h}}{1+\sigma_h}C_t$$

is the efficient shadow value of a worker. Using (98), the shadow value can be expressed more simply as

$$\chi_t = -\eta\kappa_v\theta_t - (1-\eta)b + (1-\eta)\left(1-\frac{\varphi}{1+\sigma_h}\right)A_t h_t^{\varphi}. \tag{100}$$

4.4 Steady State

The 12 parameters u, $\mathcal{M}$, p, q, θ, n, Y, v, χ, h, s, C are determined by the following 12 equations in the efficient steady state.

$$\begin{aligned}
0 &= u-(1-n)\\
0 &= \mathcal{M}-\mathcal{M}_0 u^{\eta} v^{1-\eta}\\
0 &= p-\frac{\mathcal{M}}{u}\\
0 &= q-\frac{\mathcal{M}}{v}\\
0 &= \theta-\frac{v}{u}\\
0 &= n-\frac{qv}{\lambda}\\
0 &= Y-Anh^{\varphi}\\
0 &= \frac{\kappa_v}{q}-\frac{\beta}{1-\beta(1-\lambda)}\chi\\
0 &= \chi+\eta\kappa_v\theta+(1-\eta)\,b-\left(1-\frac{\varphi}{1+\sigma_h}\right)(1-\eta)\,Ah^{\varphi}\\
0 &= \varphi Ah^{\varphi-1}-\lambda_h h^{\sigma_h} C\\
0 &= s-1\\
0 &= Y+ub-C-G-\kappa_v v
\end{aligned}$$

where $\varkappa \equiv \frac{1-\gamma}{1-\gamma\frac{1+\sigma_h}{\varphi}}$.

5 Steady State Distortions

In this section, we compare the efficient allocation with the competitive one. Because our model has two labor margins, hours and employment, we have to distinguish between two labor wedges, one at the intensive margin and one at the extensive margin. As shown here, the competitive and the efficient allocations differ only by two equations, i.e. the hours and the vacancy decisions.

5.1 Hours Margin

We show how the optimal choice of hours in the competitive equilibrium differs from the efficient one. Following Gali et al (2007), we define the 'inefficiency gap' as the ratio of the marginal rate of substitution between leisure and consumption, to the marginal product of labor, where labor is measured in terms of *hours per worker*,

$$gap_t \equiv \frac{mrs_t}{mph_t} = \frac{g'(h_t)/\Lambda_t}{\varphi A_t h_t^{\varphi-1}}. \tag{101}$$

We can replace $g'(h_t)$ and Λ_t by their expressions in (2) and (5), to obtain

$$gap_t = \frac{\lambda_h h_t^{\sigma_h}(1+\tau^c)C_t}{\varphi A_t h_t^{\varphi-1}}. \tag{102}$$

One can easily see from (20) and (39) that the inefficiency gap is a function of the real marginal cost,

$$gap_t = \frac{s_t}{\varkappa}. \tag{103}$$

If the firm has all the bargaining power ($\gamma = 0$) such that $\varkappa = 1$, the inefficiency gap coincides with the real marginal cost.

Considering the efficient allocation, we can deduce from (98) that inefficiency gap in the efficient allocation is given by the gross tax on consumption,

$$gap_t^* = 1 + \tau^c. \tag{104}$$

5.2 Employment Margin

The job creation condition in the efficient allocation (99) differ from the one in the competitive market (46) only through the expression of the shadow value of a worker. In the competitive allocation, the shadow value is given by (49):

$$\chi_t = -\gamma\kappa_v\theta_t - (1-\gamma)\, b^c + \left(1 - \frac{\varphi}{1+\sigma_h}\right) A_t h_t^{\varphi} s_t. \tag{105}$$

In the efficient allocation, the shadow value is given by (100):

$$\chi_t = -\eta\kappa_v\theta_t - (1-\eta)\, b + (1-\eta)\left(1 - \frac{\varphi}{1+\sigma_h}\right) A_t h_t^{\varphi}. \tag{106}$$

As in Krause and Lubik 2007, we now derive two *steady state* equations in unemployment (u) and vacancies (v). The two equations are the employment dynamics relation (the Beveridge curve) and the job creation condition. We derive the two steady state equations for the competitive equilibrium and for the efficient allocation. The Beveridge curve is the same in the two allocations. We analyze the tradeoff between n and h. Below, we set the matching efficiency $\mathcal{M}_0$ and compute the steady state vacancy filling rate as $q = \mathcal{M}_0 (1-n)^{\eta} v^{-\eta}$.

5.2.1 Beveridge Curve

Under symmetry, the law of motion for employment (14) is $n_t = (1-\lambda)\, n_{t-1} + q_t v_t$, which in steady state becomes $n = \frac{qv}{\lambda}$ or:

$$v = \frac{n\lambda}{q}.$$

Substituting out $n = 1-u$ and $q = \frac{\mathcal{M}}{v} = \frac{\mathcal{M}_0 u}{v}\left(\frac{v}{u}\right)^{1-\eta}$, we get

$$v = \left(\frac{(1-u)\,\lambda}{\mathcal{M}_0 u}\right)^{\frac{1}{1-\eta}} u. \tag{107}$$

The law of motion for employment is an equation in four unknowns: n, λ, q and v. So, we can fix the vacancy filling rate q and the separation rate λ and compute vacancies v for any given employment level n. Alternatively, we can view the same equation as a Beveridge Curve, see (107), which is a relation in v, $\mathcal{M}_0$, λ and u. Using (107), we can fix the matching efficiency $\mathcal{M}_0$ and the separation rate λ and then trace out the number of vacancies v as a function of unemployment u.

We can show that v is a downward-sloping function of u,

$$\begin{aligned}
\frac{\partial v}{\partial u} &= -\left(\frac{\lambda}{\mathcal{M}_0}\right)^{\frac{1}{1-\eta}} \frac{1}{1-\eta}\left(\frac{1}{u}-1\right)^{\frac{1}{1-\eta}-1} \frac{1}{u^2}u + \left(\frac{(1-u)\lambda}{\mathcal{M}_0 u}\right)^{\frac{1}{1-\eta}}, \\
&= -\frac{1}{1-\eta}\left(\frac{(1-u)\lambda}{\mathcal{M}_0 u}\right)^{\frac{1}{1-\eta}}\left(\frac{1-u}{u}\right)^{-1}\frac{1}{u} + \left(\frac{(1-u)\lambda}{\mathcal{M}_0 u}\right)^{\frac{1}{1-\eta}}, \\
&= \left(1-\frac{1}{1-\eta}\frac{1}{1-u}\right)\left(\frac{(1-u)\lambda}{\mathcal{M}_0 u}\right)^{\frac{1}{1-\eta}} < 0.
\end{aligned}$$

The Beveridge Curve is downward-sloping: in steady state, a higher number of vacancies is associated with a higher level of employment (and hence lower unemployment).

5.2.2 Job Creation Condition

In the competitive equilibrium, we can express the steady state job creation condition by combining the vacancy posting condition (62) with the shadow value (63):

$$\frac{\kappa_v}{q} = \frac{\beta}{1-\beta(1-\lambda)}\left[-\gamma\kappa_v\theta - (1-\gamma)b^c + \left(1-\frac{\varphi}{1+\sigma_h}\right)Ah^{\varphi}s\right]. \tag{108}$$

Substituting out the vacancy filling rate q using $q = \mathcal{M}_0 u^{\eta} v^{-\eta}$ and labor market tightness $\theta = \frac{v}{u}$, this becomes

$$\frac{\kappa_v}{\mathcal{M}_0 u^{\eta} v^{-\eta}} = \frac{\beta}{1-\beta(1-\lambda)}\left[-\gamma\kappa_v\frac{v}{u} - (1-\gamma)b^c + \left(1-\frac{\varphi}{1+\sigma_h}\right)Ah^{\varphi}s\right].$$

Substituting out the real marginal cost s using the price setting equation (**??**) and the production function (61) to replace Ah^{φ}, we can write

$$\frac{\kappa_v}{\mathcal{M}_0}\left(\frac{v}{u}\right)^{\eta} = \frac{\beta}{1-\beta(1-\lambda)}\left[-\gamma\kappa_v\frac{v}{u} - (1-\gamma)b^c + \left(1-\frac{\varphi}{1+\sigma_h}\right)\frac{Y}{1-u}\frac{1-\tau^f}{\mu}\right]. \tag{109}$$

What determines the slope of the job creation condition (109)? There are three (partial equilibrium) effects of higher unemployment on the number of vacancies. First, for a given matching efficiency $\mathcal{M}_0$, the vacancy filling rate q increases when unemployment rises, which lowers during of a vacancy $\frac{1}{q}$ and encourages hiring. Second, higher unemployment lowers labor market tightness θ, which has a dampening effect on the bargained wage and thereby boosts hiring. Third, the firm can influence the number of hours per worker, and therefore wage bargaining, through its hiring decision. More specifically, when the firm hires a new worker, all other workers have to work fewer hours to produce a given amount of output. The bargained wage falls and this has the effect of raising the shadow value of a worker and hence the number of vacancies posted. A firm that hires a worker is creating an externality by reducing the bargained wage for other firms as well, raising their incentive to hire.

In the efficient allocation, the steady state job creation condition is

$$\frac{\kappa_v}{q} = \frac{\beta}{1-\beta(1-\lambda)}\left[-\eta\kappa_v\theta - (1-\eta)b + (1-\eta)\left(1-\frac{\varphi}{1+\sigma_h}\right)Ah^{\varphi}\right]$$

Substituting out the vacancy filling rate q using $q = \mathcal{M}_0 u^{\eta} v^{-\eta}$, and tightness $\theta = \frac{v}{u}$, and replacing Ah^{φ} with Y/n, this becomes

$$\frac{\kappa_v}{\mathcal{M}_0}\left(\frac{v}{u}\right)^{\eta} = \frac{\beta}{1-\beta(1-\lambda)}\left[-\eta\kappa_v\frac{v}{u} - (1-\eta)b + (1-\eta)\left(1-\frac{\varphi}{1+\sigma_h}\right)\frac{Y}{1-u}\right] \tag{110}$$

5.3 Employment Dynamics

Let's study in more detail the linearized hiring condition in the competitive allocation and in the efficient allocation. We combine the linearized vacancy posting condition (83) and the shadow value (84) to obtain,

$$\frac{\eta}{\beta}\hat{\theta}_t = -\hat{r}_t + \varpi\frac{Ah^{\varphi}}{\chi}sE_t\{\hat{A}_{t+1} + \varphi\hat{h}_{t+1} + \hat{s}_{t+1}\} + [(1-\lambda)\eta - \gamma p]\, E_t\{\hat{\theta}_{t+1}\}, \tag{111}$$

where the term $\hat{r}_t = -E_t\{\hat{\beta}_{t,t+1}\} = \hat{R}_t - E_t\{\hat{\Pi}_{t+1}\}$ is the real interest rate and we define

$$\varpi = \frac{1-\beta(1-\lambda)}{\beta}\left(1 - \frac{\varphi}{1+\sigma_h}\right).$$

As in Monacelli, Perotti and Trigari (2010), we can use (111) to analyze how hiring responds to government spending shocks. There are two channels at work.[2] The first channel is the *real interest rate channel*, which is the same as in Monacelli et al (2010). A rise in government spending leads to higher taxes, which tightens the household budget constraint and leads to a rise shadow value of wealth, Λ_t. As a result, the real interest rate $\hat{r}_t$ increases. This reduces the shadow value of an additional worker and thus discourages hiring. The second channel is the *marginal value of employment channel*. We use a slightly different term than Monacelli et al (2010), who call this the 'marginal value of work' channel, because in our model, there are two margins of work: employment and hours.

In the efficient allocation, vacancy posting is also given by (83). Then, linearizing the efficient shadow value (100) and combining it with (83), we obtain the efficient hiring condition,

$$\frac{\eta}{\beta}\hat{\theta}_t = -\hat{r}_t + \varpi\frac{Ah^{*\varphi}}{\chi^*}(1-\eta)\, E_t\{\hat{A}_{t+1} + \varphi\hat{h}_{t+1}\} + [(1-\lambda)\eta - \eta p]\, E_t\{\hat{\theta}_{t+1}\}. \tag{112}$$

Comparing the linearized hiring condition in the competitive equilibrium (111) with its efficient counterpart (112), we notice two differences.

First, price stickiness induces inefficient fluctuations in employment through variations in the real marginal cost, $\hat{s}_t$. Suppose that after a government spending expansion, prices do not adjust upwards in the same proportion. Then real marginal costs rise. From (111) we see that the shadow value of a worker rises, because it becomes more expensive to expand hours in order to satisfy the higher demand. This effect vanishes under flexible prices because real marginal costs are constant at $s = \left(1-\tau^f\right)/\mu$, and so $\hat{s}_t = 0$.

Second, the coefficient on the marginal value of work is ϖ multiplied by $\frac{Ah^{\varphi}}{\chi}s$ in the competitive allocation and $\frac{Ah^{*\varphi}}{\chi^*}(1-\eta)$ in the efficient allocation. Therefore, to the extent that $\frac{Y/n}{\chi}s$ differs from $\frac{Y^*/n^*}{\chi^*}(1-\eta)$, there are inefficient employment fluctuations even under flexible prices, owing to the steady state distortions explained above. If the ratio of the output per worker to the shadow value, $\frac{Y/n}{\chi}$, is higher than is efficient, hiring responds too strongly to shocks. We saw that, in isolation, there is overhiring in the case where the steady state real marginal cost exceeds the elasticity of the matching function to vacancies, $s > (1-\eta)$. The above analysis shows that the same condition makes employment respond too much to shocks.

[2] Since we do not consider investment in this model, Monacelli et al's (2010) capital accumulation channel is absent here.

6 Optimal Policy

We investigate the optimal policy which allows to correct the discrepancy between the efficient and the competitive allocation.

6.1 Optimal Steady State Policy

We then characterize the tax policies that make the two steady states equal to each other. In the competitive equilibrium, the choice of hours (103) and the shadow value of a worker (105) are given at the steady state by

$$\chi = -\gamma\kappa_v\theta - (1-\gamma)\,b^c + \left(1 - \frac{\varphi}{1+\sigma_h}\right) Ah^{\varphi}\frac{1-\tau^f}{\mu}, \tag{113}$$

$$gap = \frac{1-\tau^f}{\varkappa\mu}. \tag{114}$$

where s has been replaced with its expression (**??**): $s = \frac{1-\tau^f}{\mu}$.

In the efficient steady state, the choice of hours (104) and the shadow value of a worker (106) are

$$\chi = -\eta\kappa_v\theta - (1-\eta)\,b + (1-\eta)\left(1 - \frac{\varphi}{1+\sigma_h}\right) Ah^{\varphi}, \tag{115}$$

$$gap = 1 + \tau^c. \tag{116}$$

First, equalizing the inefficiency gap (114) with its efficient counterpart (116), we can express the optimal consumption tax as:

$$1 + \tau^{c*} = \frac{1-\tau^f}{\varkappa\mu}. \tag{117}$$

All else equal, the consumption tax τ^{c*} is increasing in the returns to hours parameter φ.

Second, we compare the shadow value in the decentralized economy (113) with its efficient counterpart (115). A consumption tax or subsidy ($\tau^c \neq 1$), distorts the choice of market production relative to home production, and hence the worker's outside option. To remove this effect, we assume that transfers to unemployed workers is such that

$$T^{b*} = -\tau^c b.$$

We can see from (113) and (115) that the Hosios condition is not sufficient to remove the inefficiencies in vacancies. Assuming $\gamma = \eta$, the term in hours is identical only if we impose a constant revenue subsidy equal to

$$1 - \tau^{f*} = \mu\,(1-\eta)\,. \tag{118}$$

Without revenue taxes, real marginal costs in the decentralized flexible-price allocation are constant and equal to the inverse markup. Thus, efficiency requires that the inverse markup be aligned with the weight on vacancies in the matching function. When $\eta = 0$, we have the standard result from the New Keynesian model stating that the optimal revenue subsidy equals the markup μ.

Notice that in the special case where the Hosios condition holds ($\gamma = \eta$) and the optimal revenue tax (118) is imposed, the optimal consumption tax simplifies to

$$\tau^{c*} = -\gamma\frac{1+\sigma_h}{\varphi}.$$

6.2 Optimal Cyclical Policy

6.2.1 Implementability Conditions

We condense the optimality conditions of households and firms into two implementability conditions given by the (modified) vacancy posting and price setting equations. Plugging the worker's shadow value in χ_{t+1} (72) into the vacancy posting condition (71) yields the competitive (i.e. decentralized) evolution of labor market tightness,

$$\frac{\kappa_v}{\mathcal{M}_0}\theta_t^\eta = E_t\{\beta_{t,t+1}[-\gamma\kappa_v\theta_{t+1} - (1-\gamma)\, b^c + (1-\tfrac{\varphi}{1+\sigma_h})A_{t+1}h_{t+1}^\varphi s_{t+1} + (1-\lambda)\frac{\kappa_v}{\mathcal{M}_0}\theta_{t+1}^\eta]\}$$

We replace θ_t with $(1-n_t)^{-1} v_t$ and $\beta_{t,t+1}$ with $\beta\frac{C_t}{C_{t+1}}$ and rearrange the equation to obtain

$$\begin{aligned} & \frac{\kappa_v}{\mathcal{M}_0}(1-n_t)^{-\eta} v_t^\eta C_t^{-1} - \beta E_t\{(1-\lambda)\frac{\kappa_v}{\mathcal{M}_0}(1-n_{t+1})^{-\eta} v_{t+1}^\eta C_{t+1}^{-1}\} \\ = \; & \beta E_t\{[-\gamma\kappa_v(1-n_{t+1})^{-1} v_{t+1} - (1-\gamma)\, b + (1-\tfrac{\varphi}{1+\sigma_h})A_{t+1}h_{t+1}^\varphi s_{t+1}]C_{t+1}^{-1}\}. \end{aligned} \qquad (119)$$

Rearranging the price setting equation (26), we have

$$\left[\varepsilon s_t - \left(1-\tau^f\right)(\varepsilon-1) - \kappa_p(\Pi_t-1)\Pi_t\right]Y_t = -\kappa_p E_t\left\{\beta_{t,t+1}(\Pi_{t+1}-1)\Pi_{t+1}Y_{t+1}\right\}$$

We again replace Y_t with $A_t n_t h_t^\varphi$ and $\beta_{t,t+1}$ with $\beta\frac{C_t}{C_{t+1}}$ to obtain

$$\left[\kappa_p(\Pi_t-1)\Pi_t + \left(1-\tau^f\right)(\varepsilon-1) - \varepsilon s_t\right]A_t n_t h_t^\varphi C_t^{-1} = \kappa_p\beta E_t\left\{(\Pi_{t+1}-1)\Pi_{t+1}A_{t+1}n_{t+1}h_{t+1}^\varphi C_{t+1}^{-1}\right\}. \qquad (120)$$

Real marginal costs represent the third implementability constraint for the Ramsey planner,

$$s_t = \varkappa\frac{\lambda_h h_t^{\sigma_h} C_t}{\varphi A_t h_t^{\varphi-1}}. \qquad (121)$$

In addition, the Ramsey planner must respect the evolution of employment and the resource constraint. Replacing q_t with $\mathcal{M}_0\theta_t^{-\eta} = \mathcal{M}_0[v_t/(1-n_t)]^{-\eta}$ in the equation determining employment dynamics (73), we get

$$n_{t+1} = (1-\lambda)\, n_t + \mathcal{M}_0(1-n_t)^\eta v_t^{1-\eta} \qquad (122)$$

The resource constraint reads

$$\left(1-\frac{\kappa_p}{2}(\Pi_t-1)^2\right)A_t n_t h_t^\varphi + (1-n_t)\, b^c = C_t + G_t + \kappa_v v_t, \qquad (123)$$

where we have plugged in the production function to substitute for Y_t.

6.2.2 Ramsey Problem

Definition: Let $\{\lambda_{1,t},\lambda_{2,t},\lambda_{3,t},\lambda_{4,t},\lambda_{5,t}\}_{t=0}^\infty$ denote sequences of Lagrange multipliers on the constraints (119) to (123), respectively. For given stochastic processes $\{A_t,G_t\}_{t=0}^\infty$ and for a given n_0, plans for the control variables $\{C_t,n_t,v_t,h_t,s_t,\Pi_t\}_{t=0}^\infty$ and for the co-state variables $\{\lambda_{1,t},\lambda_{2,t},\lambda_{3,t},\lambda_{4,t},\lambda_{5,t}\}_{t=0}^\infty$ represent a first best constrained allocation if they solve the following optimization problem. Note that we adopt the Dynare timing convention here, i.e. we lag all the n_t's.

$$\min_{\{\lambda_{1,t},\lambda_{2,t},\lambda_{3,t},\lambda_{4,t},\lambda_{5,t}\}_{t=0}^\infty} \max_{\{C_t,n_t,v_t,h_t,s_t,\Pi_t\}_{t=0}^\infty} \mathcal{L},$$

where

$$
\begin{aligned}
\mathcal{L} \quad = \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t + n_{t-1} \frac{\lambda_h h_t^{1+\sigma_h}}{1+\sigma_h} \right. \\
& + \lambda_{1,t} \frac{\kappa_v}{\mathcal{M}_0} [(1-n_{t-1})^{-\eta} v_t^{\eta} C_t^{-1} - \beta E_t \{(1-\lambda)(1-n_{t+1})^{-\eta} v_{t+1}^{\eta} C_{t+1}^{-1}\}] \\
& - \lambda_{1,t} \beta E_t \{[-\gamma \kappa_v (1-n_{t+1})^{-1} v_{t+1} - (1-\gamma) b + (1 - \tfrac{\varphi}{1+\sigma_h}) A_{t+1} h_{t+1}^{\varphi} s_{t+1}] C_{t+1}^{-1}\} \\
& + \lambda_{2,t} \left[\kappa_p (\Pi_t - 1) \Pi_t + \left(1-\tau^f\right)(\varepsilon - 1) - \varepsilon s_t \right] A_t n_t h_t^{\varphi} C_t^{-1} \\
& - \lambda_{2,t} \beta E_t \left\{ (\Pi_{t+1} - 1) \Pi_{t+1} A_{t+1} n_t h_{t+1}^{\varphi} C_{t+1}^{-1} \right\} \\
& + \lambda_{5,t} [s_t - \frac{\varkappa \lambda_h}{\varphi} h_t^{1+\sigma_h - \varphi} C_t A_t^{-1}] \\
& + \lambda_{3,t} [n_t - (1-\lambda) n_{t-1} - \mathcal{M}_0 (1-n_{t-1})^{\eta} v_t^{1-\eta}] \\
& \left. + \lambda_{4,t} [(1 - \tfrac{\kappa_p}{2} (\Pi_t - 1)^2) A_t n_{t-1} h_t^{\varphi} + (1-n_{t-1}) b - C_t - G_t - \kappa_v v_t] \right\}
\end{aligned}
$$

Noticing that the first two constraints have forward-looking components, we rewrite the problem in a recursive way as proposed by Marcet and Marimon (2011), such that $\vartheta_{1,t} = \lambda_{1,t-1}$ and $\vartheta_{2,t} = \lambda_{2,t-1}$. We impose the additional initial conditions $\vartheta_{1,0} = \vartheta_{2,0} = 0$.

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