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DOES A LEVERAGE RATIO REQUIREMENT INCREASE BANK STABILITY?

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MACROPRUDENTIAL RESEARCH NETWORK

Macroprudential Research Network
This paper presents research conducted within the Macroprudential Research Network (MaRs). The network is composed of economists from the European System of Central Banks (ESCB), i.e. the national central banks of the 27 European Union (EU) Member States and the European Central Bank. The objective of MaRs is to develop core conceptual frameworks, models and/or tools supporting macro-prudential supervision in the EU.

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MaRs is chaired by Philipp Hartmann (ECB). Paolo Angelini (Banca d'Italia), Laurent Clerc (Banque de France), Carsten Detken (ECB), Simone Manganelli (ECB) and Katerina Šmídková (Czech National Bank) are workstream coordinators. Javier Suarez (Center for Monetary and Financial Studies) and Hans Degryse (Katholieke Universiteit Leuven and Tilburg University) act as external consultants. Fiorella De Fiore (ECB) and Kalin Nikolov (ECB) share responsibility for the MaRs Secretariat.

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The paper is released in order to make the research of MaRs generally available, in preliminary form, to encourage comments and suggestions prior to final publication. The views expressed in the paper are the ones of the author(s) and do not necessarily reflect those of the ECB or of the ESCB.

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Abstract

Basel III has introduced a non-risk-weighted leverage ratio requirement (LRR) which complements the internal ratings based (IRB) capital requirements. It provides a backstop against model risk which arises if some loans get incorrectly rated and become toxic. We study the effects of the LRR on lending strategies and its implications for banks' stability. We show that the LRR might induce banks with low-risk lending strategies to diversify their portfolios into high-risk loans until the LRR is no longer the binding capital constraint on them. If the LRR is lower than the average bank's IRB requirement, the aggregate capital costs of banks do not increase. However, because the diversification makes banks' portfolios more alike the banking sector as a whole may become more exposed to model risk in each loan category. This may undermine banking sector stability. On balance, our calibrated model motivates a significantly higher LRR than the current one.

Keywords: Bank regulation, Basel III, capital requirements, credit risk, leverage ratio

JEL-codes: D41, D82, G14, G21, G28

Non-technical summary

In the aftermath of the global financial crisis, the international rules harmonizing banks' capital requirements have gone through a major overhaul (the so called Basel III accord). To supplement the previous risk-weighted capital requirements, a non-risk-weighted leverage ratio requirement has been added as a new element. The Basel III recommendation is that a bank's capital to assets ratio (including off balance sheet items) must be at least 3%. A central motivation to the leverage ratio requirement (LRR) is to provide a more robust capital buffer against the "model" risk that banks and regulators may get the risk-weights wrong, as happened before the crisis.

We analyze a model of a competitive banking sector in which the risk-weighted capital requirements, which are based on the bank's internal customer credit ratings, have been supplemented with a LRR. Because banks view (equity) capital as the relatively costly form of financing their lending, the LRR is punitive to banks specializing in low-risk loan customers which may require less risk-weighted capital than the 3% required by the LRR. Therefore, in the new equilibrium in bank loan market, formerly low-risk lending banks would add some high-risk loans in their portfolio while banks with formerly high-risk lending strategies would assume some of the low-risk loans. As a result, bank portfolios would become more similar with one another, and hence more correlated. The overall cost of equity in the banking sector does not change, if the overall supply of loans of each kind is not changed, and also the lending rates are not affected much by the adjustment process. However, if a model risk materializes in some loan category, meaning that loan default rates turn out to be much higher than

the expectation according to which loans were priced and regulatory risk-weighted were set, the problem could now affect a larger number of banks. This is a consequence of the more similar portfolios. Hence, banking sector stability as a whole could be undermined by the introduction of the LRR, if the materialized model risk is big enough.

An illustrative calibrated version of our model has the policy implication that a significant increase of the current 3% LRR recommendation, up to the point where the LRR matches the average bank's risk-weighted capital requirement in the economy (more than 6% in our calibration), could have positive stability effects without compromising lending costs. This is because a higher LRR would simply provide a more sufficient buffer against even very big model risk while banks could (almost) costlessly adjust to it by changing their loan portfolios. As the lower LRR would in any case result in much of the portfolio adjustment, it is better to let this happen with the higher LRR, with higher stability gains.

1 Introduction

The new Basel III framework contains a *leverage ratio requirement* (LRR) which has been added to supplement risk-based, internal ratings based (IRB) minimum capital requirements on banks, introduced already in Basel II. According to the current LRR calibration, banks must have a minimum of three percent of capital of non-risk-weighted total assets, including off-balance sheet items (see Basel Committee on Banking Supervision, 2011, p. 61).[1]

The Basel Committee on Banking Supervision (2009, pp. 2-3) argues that a LRR would "help contain the build up of excessive leverage in the banking system, introduce additional safeguards against attempts to game the risk based requirements, and help address model risk". The "gaming" of the requirements might include not just dubious practices, such as giving unrealistically low internal ratings to loans in order to reduce capital requirements, but also legitimate forms of regulatory capital arbitrage.[2] By providing an all-encompassing "floor" to capital requirements the LRR reduces incentives to such manoeuvres.[3]

In this paper we discuss the introduction of the LRR into the Basel framework, focusing on the third of Basel Committee's motivations to introduce it, i.e., the possibility that there is model risk embedded in the IRB capital requirements. These requirements are, in both Basel II and the revised Basel III framework, based on an asymptotic single risk-factor model by Vasicek (2002), and if the model is correct, bank capital suffices for covering the unexpected losses with a 99.9% probability.[4] To

[1] It might be more logical to talk about a capital to assets ratio requirement or an inverse of a leverage ratio requirement. For simplicity, however, we henceforth use the term leverage ratio requirement keeping in mind that in actuality it is imposed in terms of a minimum capital to assets ratio.

[2] E.g., banks have shifted loan risks from the banking book to the trading book or to off-balance sheet items, often with the help of securitizations, coupled with too optimistic rating agency ratings. Before Basel III such manoeuvres have effectively resulted in lower risk-based capital requirements (see e.g. Acharya et al., 2013).

[3] Cf. Blum (2008) who shows that a LRR reduces the moral hazard which is associated with internal ratings based requirements because it reduces the profit that may be obtained by giving unrealistically low internal ratings to loans.

[4] There have been no major changes in the risk-weighting system of the IRB capital requirements

keep things simple, we shall consider a competitive banking sector with loans of two types, called low-risk and high-risk loans, whose risks are determined by the the Vasicek model. Such a setting has previously been used by Repullo and Suarez (2004) to study the allocational and welfare effects of the Basel II requirements.

As Repullo and Suarez (2004) have shown, when the IRB requirements are the only capital requirements in the model, banks have an incentive to specialize in either low-risk or high-risk lending. This is because - as banks have the obligation to use not just their capital but also their interest income for covering the losses from the defaulting loans - there is a positive probability that one of two specialized financial institutions fails and the other one does not.[5] In this case the owners of a "mixed portfolio" bank would have to use income from high-risk loans for covering losses from low-risk loans or vice versa. Hence, in order to take full advantage of limited liability, banks prefer to specialize. We view the specialized banking market, as described by Repullo and Suarez (2004), as a simplified representation of a real world banking sector where some banks have a portfolio which is sufficiently risky so that the LRR is irrelevant for them, while for other banks the LRR turns out to be a binding constraint.[6] We generalize Repullo and Suarez (2004)'s specialization result to our

when moving from Basel II to Basel III; the risk-weights are determined by the same function of the default probability of loans in both frameworks (Basel Committee on Banking Supervision, 2006, p. 64, and Basel Committee on Banking Supervision, 2011, p. 39). See Basel Committee on Banking Supervision (2005) for an intuitive explanation of the way in which the Vasicek (2002) model is applied within the Basel II and Basel III frameworks.

[5] The equilibrium interest income of banks depends, not just on loan risks, but also on the market structure of the banking sector. Hence, if one wanted to construct capital requirements which would make the actual bank failure rates - rather than just the probability that bank capital suffices for unexpected losses - identical for all banks, one would have to make the capital requirements depend on the market structure of the banking sector. Repullo and Suarez (2004) discuss such capital requirements in the context of a perfectly competitive banking sector. However, they also show that the capital requirements which harmonize bank safety do not maximize welfare.

[6] As detailed bank portfolio data is not publicly available, it is difficult to judge how often banks actually are affected by the additional LRR. However, differences in banks' general risk profiles are nevertheless evident, indicating some degree of specialization. For instance, the share of net loans to customers in relation to trading assets (often seen as riskier business) may vary greatly (for a sample of leading European banks, see Liikanen 2012, Table A3.2). There is also evidence that banks may focus on either corporate or retail loans, the former of which are normally seen as riskier. For instance, in a sample of the largest Nordic banks, we find variation in the ratio of corporate and commercial loans to residential mortgage loans in the range of 50 to 150 percent.

setting with the LRR by showing that some banks will hold a fixed ratio of low-risk and high-risk loans while the other banks hold only either low-risk loans or high-risk loans, depending on the type of equilibrium.

The key insight from these results is that banks can adapt to a relatively low LRR without a significant impact on loan interest rates, by simply reshuffling loans among themselves. In particular, banks previously specialized in low-risk loans, facing the LRR which would otherwise raise their funding costs, can maintain their (zero-) profitability by adding some high-risk loans to their portfolios. Banks previously specialized in high-risk loans will adopt some low-risk loans so that consequently, low-risk loans will be held by a larger number of banks and there will be fewer banks specializing only on high-risk loans. In the absence of model risk this will increase both welfare and bank stability. Such an adjustment through reshuffling of low-risk and high-risk loans works for LRRs which are lower than or equal to the average risk-based capital requirement of all loans in the banking sector.[7] For higher LRRs, both the reshuffling strategy of banks and loan interest rates would have to adjust and the aggregate amount of bank capital would have to increase. In this case, the welfare benefit from increased bank stability would have to be weighed against the welfare loss from increased capital costs of banks.

When discussing model risk we assume that economic agents (regulators, banks, and loan customers) base their actions on common estimates of the loan default probabilities, and we define model risk as the possibility that the common estimates might turn out to be false. More specifically, we assume that some bank loans turn out to have much higher default probabilities than expected by any agent in the economy; i.e., that they unexpectedly turn toxic. Our approach can be motivated by the model of Gennaioli et al. (2012), in which a bias which is called "local thinking" may make economic agents neglect some rare risks. Empirical examples of such shocks to default

[7] Hence, the critique by e.g. some low-risk lending institutions in Europe against the LRR that the LRR will unjustly raise their cost of capital is not necessarily generally justified.

probabilities may be provided by the US subprime crisis and the European sovereign debt crisis.

We find that the reshuffling of loans induced by a leverage ratio requirement may be a double-edged sword because it causes both a positive diversification effect and a negative contamination effect. If, e.g., loans that have been taken for "low-risk loans" turn out to be riskier than the high-risk loans and the LRR lies within the range in which it does not force banks to increase their aggregate amount of capital, then the"reshuffling" has the consequence that each bank which originally specialized on "low-risk loans" now holds also high-risk loans. High-risk loans are now relatively less risky, but still subject to a higher capital requirement. Hence the diversification tends to make banks originally specialised on low-risk loans more stable. On the other hand, the reshuffling also has the consequence that the number of banks which have some "low-risk loans" in their portfolios (and are contaminated by them) grows larger. The policy implication suggested by our numerical results is that welfare is likely to be increased if the LRR is set at the highest level at which the banking sector can still adapt to it by reshuffling of loans, without having to adjust loan interest rates much. In our model with two loan risk categories, this LRR level equals the average IRB capital requirement of all the loans in the banking sector.

The rest of the paper is organized as follows. In Section 2 we recapitulate the main features of the Basel II IRB framework and present a generalized framework in which banks are also subject to a LRR. In Section 3 we discuss the two kinds of equilibria that the generalized model may have. In Section 4 we present a welfare function for our model, to be used in making some policy suggestions on the basis of our numerical results. In Section 5 we present the calibrated version of our model with which we analyze loan interest rates and bank stability both in the absence and in the presence of model risk in Section 6. Section 7 concludes.

2. Model

We assume that there are two kinds of firms, called low-risk (L) and high-risk (H) firms, a competitive banking sector, and a government which regulates banks. There are just two periods, $T = 0$ and $T = 1$ (see Figure 1). At time $T = 0$ each bank first collects capital from its owners and deposits from depositors, and lends them out to the firms as loans of size 1.

The low-risk and high-risk firms invest in low-risk and high-risk projects, respectively. The projects are of size 1, and their only source of funding are the bank loans. The number of firms of type η ($\eta = L, H$) is assumed to be a constant, denoted by n_η. Hence, we implicitly assume that the demand for loans is inelastic and independent of interest rates.[8] We refer to the loans to the two kinds of firms as low-risk and high-risk loans, and the interest rate on a loan of type η ($\eta = L, H$) is denoted by r_η.

[Figure 1]

The results of the investments projects are realized in period $T = 1$. The project chosen by each firm can either succeed or fail. A successful project of type η ($\eta = L, H$) produces $1 + a_\eta$, of which the bank receives $1 + r_\eta$, but if a project is unsuccesful, it produces only $1 - \lambda$. In this case the loan defaults and the bank receives $1 - \lambda$. Hence, λ expresses the loss given default of the bank. In the next step, the deposits are withdrawn and the bank dissolves. The bank will fail if and only if its assets do not suffice for covering the deposits. In this case the deposits will be covered partly by a deposit insurance provided by the government. This implies that the deposits earn

[8]Like in any model with the assumption of inelastic demand, which we adopt for simplicity, inelasticity can be given a variety of interpretations. For example, we may think of the firms as small firms run by entrepreneurs who are competent only in running a low-risk or a high-risk firm. In this case the number of firms may be thought of as being determined by the differing opportunity costs that being an enterpreneur has for different individuals, e.g by the different salaries w that the entrepreneurs could earn elsewhere in the economy. The case of a constant number of firms can then be viewed as a case in which the opportunity cost w is so low for all the considered entrepreneurs that an increase in the interest rate does not reduce their number. With inelastic loan demand, firms make positive profits in our model. We have also extended our model to the case of elastic loan demand, but the main results stay the same. The extended results are available from the authors' upon request.

a riskless interest rate, which is normalized to zero. We also normalize the deposit insurance premium to zero.[9] Discussion on the cost that deposit insurance causes to the government is postponed to section 4 where we study the welfare implications of our model.

Each bank has a large well-diversified portfolio, and the size of each bank is normalized to 1. We denote the portfolio of a bank by α which is defined as the share of high-risk loans among all its loans.

By assumption, bank capital has an expected cost δ over the riskless interest rate of zero.[10] The government imposes two types of capital requirements on banks. First, banks are subject to what constitutes the counterpart of the requirements of the Basel IRB framework. The Basel II documents draw a distinction between the expected and the unexpected losses of a bank. According to them, banks are "expected in general to cover their (e)xpected (l)osses on an ongoing basis, e.g. by provisions and write-offs", so that capital will be needed only for absorbing the unexpected losses.[11] We denote the default probability of a loan of type η ($\eta = L, H$) by $\bar{p}_\eta$, so that the expected loss from each type η loan equals $\lambda\bar{p}_\eta$, whereas a bank's average unexpected losses from such loans are expressed by the (positive or negative) difference between the actual average losses from loans of type η and the value $\lambda\bar{p}_\eta$.

The amount of capital that the IRB approach requires for each loan of size 1 is a function $b(\bar{p})$ of the default probability $\bar{p}$ of the loan. We shall shortly return to the definition of and the motivation behind the function b. We assume that the regulator requires the bank to have the amount of capital $b(\bar{p}_\eta)$ for each loan of type η ($\eta = L, H$), and also to provision the funds $\lambda\bar{p}_\eta$ for the expected losses. In the simplified world of our two-period model, the only legitimate source for the provisions

[9] A small enough flat positive deposit insurance premium would not change our results. We have also analyzed the effect of imposing a risk-sensitive actuarially fair deposit insurance premium on banks, and found that the results of our equilibrium analysis to be presented in Section 3 below still hold.

[10] For the standard justifications of this assumption see e.g. Repullo and Suarez (2004), p. 501.

[11] Basel Committee on Banking Supervision (2005), p. 7.

are the funds of the bank owners, implying that they are quite analogous with bank capital.[12] The capital that the owners are required to provide altogether (in the form of provisions and equity) for each loan of type η ($\eta = L, H$) is given by

$$k_\eta = \lambda \bar{p}_\eta + b\left(\bar{p}_\eta\right) \tag{1}$$

This means that if the bank has the portfolio α, the Basel II type requirement imposed on the bank owners states that the total capital that they must provide satisfies the condition

$$k \geq k_{B2}\left(\alpha\right) \tag{2}$$

where

$$k_{B2}\left(\alpha\right) = \left(1 - \alpha\right) k_L + \alpha k_H \tag{3}$$

Secondly, the bank is also subject to a LRR which states that the bank must have the amount k_{lev} of capital per loan. Together, the two requirements amount up to the statement that the amount of capital k that the bank owners provide must satisfy

$$k \geq \kappa\left(\alpha\right) \tag{4}$$

when

$$\kappa\left(\alpha\right) = \max\left\{\left(1-\alpha\right) k_L + \alpha k_H, k_{lev}\right\} = \begin{cases} k_{lev},\ \alpha \leq \alpha_{lev} \\ k_{B2}\left(\alpha\right),\ \alpha > \alpha_{lev}. \end{cases} \tag{5}$$

Here α_{lev} is the portfolio for which both risk-based constraint and the LRR constraint are binding, and it is given by

$$\alpha_{lev} = \frac{k_{lev} - k_L}{k_H - k_L} \tag{6}$$

[12] Cf. Repullo-Suarez (2004, p. 502) who also view the general loan provisions as a form of capital and, accordingly, simplify their presentation by leaving out provisions and the term $\lambda \bar{p}$ from the definition (7) of the Basel II capital requirement.

[Figure 2]

Figure 2 shows the requirement (4) as a function of the portfolio α (solid line) and contrasts it with the corresponding risk-based requirement (broken line), which does not include a leverage ratio requirement. When $\alpha < \alpha_{lev}$, the LRR is the binding constraint and the Basel III requirement is stronger than the corresponding risk-based requirement, but when $\alpha > \alpha_{lev}$, the risk-based requirement is binding.

According to Basel Committee for Banking Supervision (2006, p. 64), the Basel II IRB capital requirement for a loan with default probability $\bar{p}$ and maturity $M = 1$ is

$$b(\bar{p}) = \lambda\Phi\left(\frac{\Phi^{-1}(\bar{p}) + \sqrt{\rho}\Phi^{-1}(\theta)}{\sqrt{1-\rho}}\right) - \lambda\bar{p} \tag{7}$$

where Φ is the cumulative distribution function of the standardized normal distribution, θ is a parameter which has been given the value $\theta = 0.999$ in the Basel IRB framework, λ is the loss given default of the loan, and (assuming that the firm-size adjustment does not apply) the correlation parameter ρ is given by

$$\rho = 0.12\left(2 - \frac{1 - e^{-50\bar{p}}}{1 - e^{-50}}\right) \tag{8}$$

As explained in Basel Committee on Banking Supervision (2005), the formula (7) is motivated by the single risk factor model elaborated in Vasicek (2002). This model applies to a case in which the portfolio of the bank is well-diversified, i.e. in which the bank has granted a very large number of small loans. In this case the capital requirement of the form (7) suffices to cover the unexpected loan losses from the bank's correlated loan portfolio with probability θ.[13]

We follow the notation of Repullo-Suarez (2004) in our presentation of the Vasicek model. The success probability of the project of a firm i is driven by the random

[13] Vasicek (2002), formula (3) on p. 160; observe change in notation.

variable x_i which is defined by

$$x_i = \mu_i + \sqrt{\rho}z + \sqrt{1-\rho}\varepsilon_i \tag{9}$$

The project of the firm i fails and its loan defaults if $x_i > 0$. Here $z \sim N(0,1)$ is the systematic risk factor, and the random variables $\varepsilon_i \sim N(0,1)$ are independent of each other and of z. The value of μ_i is equal to the constant μ_L for low-risk loans, and with the constant μ_H for the high-risk loans. It is easy to see that the unconditional default probability $\overline{p}_\eta$ of the loans of type η $(\eta = L, H)$ is given by

$$\overline{p}_\eta = \Phi\left(\mu_\eta\right) \tag{10}$$

We next consider the default probabilities of the loans after the systematic risk factor z has been realized. Given z, the default probability $p_\eta(z)$ of a loan i of type η $(\eta = L, H)$ is the probability that

$\mu_\eta + \sqrt{\rho}z + \sqrt{1-\rho}\varepsilon_i > 0,$

which is given by

$$p_\eta(z) = P\left(\varepsilon_i > -\frac{\mu_\eta + \sqrt{\rho}z}{\sqrt{1-\rho}}\right) = \Phi\left(\frac{\mu_\eta + \sqrt{\rho}z}{\sqrt{1-\rho}}\right) \tag{11}$$

Since the bank portfolio is fully diversified by assumption, for each given z the actual loan losses of the bank are $\lambda p_\eta(z)$. The unexpected loan losses are given by $\lambda p_\eta(z) - \lambda\bar{p}_\eta$, which is an increasing function of z. These results make it easy to grasp the motivation behind the capital requirement (7): comparing (7), (10), and (11) we observe that

$b(\bar{p}_\eta) = \lambda p_\eta\left(\Phi^{-1}(\theta)\right) - \lambda\bar{p}_\eta$

and that the unexpected losses from the loans of each type η will not exceed the capital requirement $b(\bar{p}_\eta)$ if and only if $z \leq \Phi^{-1}(\theta)$. Clearly, the probability of this event is θ, i.e. 0.999.

The *net worth* of a bank is the difference between the repayments it receives from its loans and, as we have normalized the deposit interest rate to zero, its amount of deposits. The amount of deposits of the bank which holds the amount k of capital is $1-k$. Hence, if the bank has the portfolio α, its net worth is

$$\begin{aligned}\pi\left(k,\alpha,r_L,r_H;z\right) &= (1-\alpha)\left((1-p_L(z))(1+r_L)+p_L(z)(1-\lambda)\right)\\ &\quad +\alpha\left((1-p_H(z))(1+r_H)+p_H(z)(1-\lambda)\right)-(1-k)\\ &= k+(1-\alpha)\left(r_L-p_L(z)(\lambda+r_L)\right)+\alpha\left(r_H-p_H(z)(\lambda+r_H)\right)\end{aligned} \tag{12}$$

The actual *final payoff* to the owners of the bank equals the bank's net worth, as long as the net worth is positive, and zero otherwise. At time $T=0$ the *net value* of the bank equals the difference of the investment that the bank owners make and the discounted value of the expected final payoff at time $T=1$. As we have normalized the size of the bank to one, and as the cost of bank capital is δ, the net value of the bank is

$$V\left(k,\alpha,r_L,r_H\right)=-k+\frac{1}{1+\delta}\Pi\left(k,\alpha,r_L,r_H\right) \tag{13}$$

where

$$\Pi\left(k,\alpha,r_L,r_H\right)=\int_{-\infty}^{\widehat{z}_\alpha}\pi\left(k,\alpha,r_L,r_H;z\right)d\Phi(z) \tag{14}$$

and $\widehat{z}_\alpha$ is the value of z for which the integrand becomes zero. Intuitively, if $z>\widehat{z}_\alpha$, the liabilities of the bank are larger than its assets. In this case the bank will fail and be of a zero net worth (rather than of a negative net worth) to its owners.

It is easy to see that when the net value of a bank is given by (13), it is optimal for banks to choose the minimum amount of capital $\kappa(\alpha)$ which is allowed by the capital requirement (cf. Repullo and Suarez, 2004, p. 502). Since the banking sector is competitive, in equilibrium the net value of each bank must be zero, i.e. for each α that some banks choose it must be the case that

$$V\left(\kappa(\alpha),\alpha,r_L,r_H\right)=0 \tag{15}$$

The equilibrium conditions of our model may now be formulated by stating that (15) is valid for all the portfolios α that some banks choose, that the net value is not positive for any portfolios, and that the loan demand and supply are identical for both kinds of loans.

3. Equilibria

As Repullo and Suarez (2004, p. 503) point out, banks would have in the current setting an incentive to specialize to either low-risk or high-risk loans under almost any additive capital requirement if they were not subject also to the LRR. To see why this is the case, we first observe that the probability that a bank with some portfolio α fails is distinct from, and smaller than, the probability that the amount of capital specified by the Basel II IRB framework does not suffice for its intended purpose (i.e. for covering the unexpected losses of the bank). As we just saw, the latter will be the case if $z > \Phi^{-1}(\theta)$ and the probability of this event is for all values of α by construction $1-\theta = 0.001$. On the other hand, the bank with the portfolio α will become insolvent and fail if $z > z_\alpha$ where z_α is the value of z for which the net worth of the bank defined by (12) is zero, and the value of z for which this is the case depends not just on the capital requirement but also on α, r_L, and r_H. Intuitively, a bank does not always fail when its capital is insufficient for covering its unexpected losses, because in this case the bank is obliged to use also its interest income from its non-defaulting loans for paying the losses from its defaulting loans, and the probability with which the losses can be covered in this manner depends both on the interest rates and on the portfolio of the bank. Hence z_α will be larger than $\Phi^{-1}(\theta)$ and, as a rule different for different values of α.[14]

[14] It is easy to see from (1), (7), (10), (11), and (12) that if the interest rates were set to zero, i.e. if $r_L = r_H = 0$, and k was given the value which corresponds to the Basel II IRB framework, i.e. $k = (1-\alpha)k_L + \alpha k_H$, the value of $\widehat{z}_\alpha$ would be $\widehat{z}_\alpha = \Phi^{-1}(\theta)$ for all values of α. In this case the probability with which a bank with portfolio α fails would be $1-\theta$ independently of α, as the intuitive motivation behind the Basel II requirements suggests. We also observe that the value $\widehat{z}_\alpha$ is an increasing function of the interest rates and must hence be larger than $\Phi^{-1}(\theta)$ when the interest

In particular, considering a specialized low-risk loan and a specialized high-risk loan bank, for which $\alpha = 0$ and $\alpha = 1$, respectively, we conclude that $\widehat{z}_0 \neq \widehat{z}_1$, implying that there are values of z between $\widehat{z}_0$ and $\widehat{z}_1$for which one of the specialized banks fails and the other one does not.[15] In these cases the final payoff from two specialized banks will be larger than the final payoff of a single mixed-portfolio bank. To see why this is the case, we put $k = (1-\alpha)\,k_L + \alpha k_H$ and conclude from (12) that

$$\pi\left(k, \alpha, r_L, r_H; z\right) = (1-\alpha)\,\pi\left(k_L, 0, r_L, r_H; z\right) + \alpha\pi\left(k_H, 1, r_L, r_H; z\right) \tag{16}$$

This result states that the net worth a bank with portfolio α (which appers on the left-hand side) is a linear function of α when the capital requirement is the additive Basel II requirement. The final payoff from a bank equals its net worth when the net worth is positive and zero otherwise, and we may conclude from (16) that final payoffs satisfy

$$\begin{aligned} &\max\left\{\pi\left(k, \alpha, r_L, r_H; z\right), 0\right\} \\ \leq\, &(1-\alpha)\max\left\{\pi\left(k_L, 0, r_L, r_H; z\right), 0\right\} + \alpha\max\left\{\pi\left(k_H, 1, r_L, r_H; z\right), 0\right\} \end{aligned} \tag{17}$$

Here the first (the second) term on the right-hand size is the final payoff from a specialized low-risk (high-risk) bank which is subject to the capital requirement k_L (k_H) and which has the size $1-\alpha$ (the size α), while the left-hand side represents the final payoff from a mixed-portfolio bank which has the same loans. It is clear this

rates are positive. Hence, in this case the probability of bank failure is smaller than the probability that $z > \Phi^{-1}(\theta)$, i.e. $1-\theta$.

[15]As a matter of fact, Repullo and Suarez (2004, pp. 507-508) have shown that under the Basel IRB requirements $\widehat{z}_1 > \widehat{z}_0$, when the model is calibrated realistically. This means that specialized high-risk loan banks will have smaller failure rates than specialized low-risk loan banks. In order to understand this seemingly counter-intuitive result, it is useful to first recapitulate that as long as the systematic risk factor z satisfies $z < \Phi^{-1}(\theta)$, bank capital suffices (in case of both types of banks) for covering the unexpected losses. Comparing a specialized high-risk loan and a specialized low-risk loan bank, and thinking of z increasing beyond the border line value $z = \Phi^{-1}(\theta)$, we observe that in the case of high-risk loan bank the number of loan defaults increases with z at a greater pace, and that the high-risk loan bank has a larger interest income from each of its non-defaulting loans. Intuitively, Repullo and Suarez's result shows that the latter effect is stronger than the former one, and that the larger interest income more than compensates for the larger number of defaulting loans.

inequality will be strict if and only if one of the net worths on the right-hand side is positive and the other one is negative, i.e. if z is between $\widehat{z}_0$ and $\widehat{z}_1$. Intuitively, in this case one of the specialized banks fails but the other one does not, and a mixed-portfolio bank must use the income from its high-risk loans for paying the losses from its low-risk loans or vice versa.

Taking expectations with respect to z in (17) and using (13) and (14), we now conclude that

$$V(k, \alpha, r_L, r_H) < (1-\alpha) V(k_L, 0, r_L, r_H) + \alpha V(k_H, 0, r_L, r_H)$$

Intuitively, the net value of the mixed-portfolio bank slightly smaller than the aggregate value of a low-risk loan and a high-risk loan bank with the same loans, so that the owners of the mixed-portfolio bank have an incentive to split their bank into two separate (high-risk and low-risk) financial institutions.

Theorem 1 constitutes the analogy of this specialization result in the more general setting of ours. This theorem, which is concerned with the combination of IRB requirements and a LRR, is less intuitive than its earlier counterpart, but the logic of its proof is analogous: it is based on the fact that as long as the capital requirement is a linear function of α, the benefit obtained from specialization is either zero or positive, and the fact that the capital requirement (5) is linear α in each of the regions $[0, \alpha_{lev}]$ and $[\alpha_{lev}, 1]$ when these regions are considered separately.

Theorem 1. If there is an equilibrium in which the low-risk and high-risk interest rates are r_L and r_H, there is an equilibrium with the same interest rates in which each bank has one of the portfolios $\alpha = 0$, $\alpha = \alpha_{lev}$ and $\alpha = 1$.

Because of Theorem 1 we consider in this section only banks with portfolios $\alpha = 0$, $\alpha = \alpha_{lev}$, and $\alpha = 1$ when we determine the equilibrium interest rates of the model.[16] In what follows we shall refer to these three kinds of banks as the low-risk

[16] Our Theorem 5 (to be found in the Appendix) specifies constraints which must be valid if the model has equilibria with other bank portfolios. For our purposes, the essential feature of Theorem 5 is that its constraints are equality constraints, and the parameter values that satisfy them may accordingly be viewed as exceptions. See also footnote 32 below.

loan banks, the mixed-portfolio banks, and the high-risk loan banks, respectively. Still assuming that banks are of unit size, we shall denote the number of the three kinds of banks by m_L, m_M, and m_H. In equilibrium all existing banks must be of zero net value (i.e., those of the portfolios $\alpha = 0, \alpha_{lev}, 1$ that some banks have must satisfy (15)) and the demand and supply for loans must be identical. Remembering that n_η denotes the demand for loans of type η ($\eta = L, H$), the latter of these conditions may be formulated as

$$\begin{cases} m_L + (1 - \alpha_{lev})\, m_M = n_L \\ \alpha_{lev} m_M + m_H = n_H \end{cases} \tag{18}$$

It turns out that the equilibria of the current model may be classified into two types, A and B. To understand them intuitively, we contrast them with the Basel II equilibrium which obtains in the absence of the LRR. In this equilibrium there are just low-risk loan banks and high-risk loan banks, and the interest rates r_L and r_H are determined by (15) with $\alpha = 0, \kappa(\alpha) = k_L$ and with $\alpha = 1, \kappa(\alpha) = k_H$.

It is clear from (12), (13), and (14) that the net value of a bank which has specialized in low-risk loans is independent of the high-risk interest rate and vice versa. Accordingly, we denote the net value of a specialized low-risk (high-risk) loan bank by $V_L(k, r_L)$ ($V_H(k, r_H)$) when k is the capital requirement which applies to low-risk (high-risk) loans banks. We denote the interest rates which are valid in the Basel II equilibrium by $\bar{r}_L$ and $\bar{r}_H$. These interest rates are determined by the conditions

$$\begin{cases} V_L(k_L, \bar{r}_L) = 0 \\ V_H(k_H, \bar{r}_H) = 0 \end{cases} \tag{19}$$

Intuitively, we may think that the Basel II equilibrium has emerged in a setting in which there is a large number of potential bankers who specialize in either low-risk or high-risk lending and enter the market until their profits (which are measured by the net values of the banks) have sunk to zero. Viewing the Basel II equilibrium in this manner, we observe that the introduction of a LRR does not affect the business model of the high-risk loan bankers in any obvious way. However, the behaviour of the low-risk loan bankers has to change if a leverage ratio requirement $k_{lev} > k_L$ is introduced, since after its introduction the amount of capital of a specialized low-risk loan bank may no longer be just k_L per loan.

There are two obvious ways in which a low-risk loan banker might adapt to the LRR. Firstly, he could choose to increase the amount of capital up to the value k_{lev}. We shall denote the low-risk interest rate which obtains in an equilibrium in which some bankers follow this strategy by $r_{BL}(k_{lev})$. Clearly, $r_{BL}(k_{lev})$ must be determined by

$$V_L(k_{lev}, r_{BL}(k_{lev})) = 0 \tag{20}$$

Alternatively, the low-risk loan banker might reshuffle loans with some of the high-risk loan bankers in such a way that their banks become mixed-portfolio banks and that the capital required by the LRR is no longer larger than the capital required by the Basel II requirements.

We shall denote the interest rate which applies to low-risk loans when there are both mixed-portfolio banks and specialized high-risk loan banks in the market by $r_{AL}(k_{lev})$. This interest rate is determined by (15), which now receives the form

$$V(k_{lev}, \alpha_{lev}, r_{AL}(k_{lev}), \bar{r}_H) = 0 \tag{21}$$

In the calibrated version of our model it has turned out that

$$r_{AL}(k_{lev}) < r_{BL}(k_{lev}) \tag{22}$$

for all values of k_{lev} between k_L and k_H. The reasons for the validity of (22) may be understood intuitively as follows. The interest rate $r_{BL}(k_{lev})$ is larger than the interest rate in the absence of the leverage ratio requirement, $\bar{r}_L$, because an increase in the interest rate must compensate for the costs that the extra capital demanded by the LRR causes for the specialized low-risk loan bank. Also the interest rate $r_{AL}\left(k_{lev}\right)$ is larger than $\bar{r}_L$, but this is because of a much subtler effect: as explained in the begining of this section, a mixed-portfolio bank has a smaller expected net worth than two specialized financial institutions with same loans. Given that in the calibrated version of the model bank failure probabilities are small, the increase in the low-risk loan interest rate which is needed to compensate for this effect is much smaller than the difference between $\bar{r}_L$ and $r_{BL}\left(k_{lev}\right)$.

Hence, as long as there are specialized high-risk loan banks on the market, the low-risk loans will be held by mixed-portfolio banks rather than by low-risk loan banks. This conclusion may be formulated rigorously as the following theorem.

Theorem 2. If

$$0 < \alpha_{lev} \leq \frac{n_H}{n_L + n_H} \tag{23}$$

there is an equilibrium (called equilibrium of type A) in which all banks are either specialized high-risk loan banks or mixed-portfolio banks with the portfolio α_{lev}. In this equilibrium the interest rates have the values $r_L = r_{AL}\left(k_{lev}\right)$ and $r_H = \bar{r}_H$ that are determined by

$$\begin{cases} V\left(k_{lev}, \alpha_{lev}, r_{AL}\left(k_{lev}\right), \bar{r}_H\right) = 0 \\ V_H\left(k_H, \bar{r}_H\right) = 0 \end{cases}$$

and the number of the banks is given by

$$\begin{cases} m_L = 0 \\ m_M = n_L / \left(1 - \alpha_{lev}\right) \\ m_H = n_H - \left(\alpha_{lev} / \left(1 - \alpha_{lev}\right)\right) n_L \end{cases} \tag{24}$$

According to (6), α_{lev} is an increasing function of the LRR k_{lev}, and α_{lev} receives the values from 0 to 1 as k_{lev} receives the values from k_L to k_H. Hence, the inequality (23) is valid and an equilibrium of type A exists whenever k_{lev} is sufficiently close to the capital requirement for low-risk loans, k_L. This inequality has a simple intuitive interpretation. The right-most term of (23) is the share of high-risk loans within loan demand, and the condition (23) states that this share is larger than or equal to the share of high-risk loans in the portfolios of the mixed-portfolio banks. This statement must, obviously, be valid if the only banks that there are on the market in addition to the mixed-portfolio banks are banks which specialize in high-risk loans.

If α_{lev} has the largest value allowed by the condition (23), the mixed-portfolio banks will supply all the high-risk loans in the market, and if α_{lev} is even larger, an equilibrium of type A is no longer possible. In this case the model turns out to have an equilibrium of another kind, which we call an equlibrium of type B and which will be characterized by Theorem 3 below. To understand the nature of the type B equilibrium intuitively, we first recapitulate that specialized low-risk banks are possible in the presence of a LRR if the low-risk interest rate increases to the value $r_{BL}(k_{lev})$ which suffices to compensate for the extra costs that the LRR causes. This shift is irrelevant for the profitability of a high-risk loan bank, but it tends to make the business model of a mixed portfolio bank more attractive. Intuitively, the high-risk loan bankers reshuffle loans with some of the low-risk loan bankers so that their banks become mixed-portfolio banks. Given that the banking sector is competitive and that the low-risk interest rate is fixed by the condition (20), the increased profitability of the mixed-portfolio banks shows up as a decreased high-risk loan interest rate. Accordingly, in a type B equilibrium the low-risk interest rate is higher but the high-risk interest rate is lower than in the Basel II equilibrium.

Theorem 3. If

$$\frac{n_H}{n_L + n_H} \leq \alpha_{lev} \leq 1 \tag{25}$$

there is an equilibrium (called equilibrium of type B) in which all banks are either specialized low-risk loan banks or mixed-portfolio banks with the portfolio α_{lev}. In this equilibrium the interest rates have the values $r_L = r_{BL}(k_{lev})$ and $r_H = r_{BH}(k_{lev})$

that are determined by

$$\begin{cases} V_L\left(k_{lev}, r_{BL}\left(k_{lev}\right)\right) = 0 \\ V\left(k_{lev}, \alpha_{lev}, r_{BL}\left(k_{lev}\right), r_{BH}\left(k_{lev}\right)\right) = 0 \end{cases}$$

and the number of the banks is given by

$$\begin{cases} m_L = n_L - \left(\left(1 - \alpha_{lev}\right)/\alpha_{lev}\right) n_H \\ m_M = n_H/\alpha_{lev} \\ m_H = 0 \end{cases} \tag{26}$$

We may conclude from (18) also that there can be an equilibrium in which there are only mixed-portfolio banks only if $\alpha_{lev} = \frac{n_H}{n_L+n_H}$, i.e. if α_{lev} lies on the borderline of the regions in which the equilibria of types A and B are possible. The Theorems 1-3 suffice to fix the equilibrium interest rates for all other values of α_{lev}.[17]

4. Welfare function

Following Repullo-Suarez (2004, p. 511), we measure welfare by the sum of the expected profits of the entrepreneurs and the payoff of the goverment. The latter term represents the (direct and indirect) social costs of bank failure.[18] We use the welfare function for both a conventional welfare analysis, in which we evaluate the extent to which the introduction of a LRR increases or reduces welfare, and for an analysis of the effects of the LRR in the presence of model risk.

A succesful entrepreneur of type η ($\eta = L, H$) earns $a_\eta - r_\eta$ (i.e., the difference

[17] See also Remarks 2 and 3 and Theorem 5 in the Appendix. The interest rates will not be fixed uniquely by the equilibrium conditions when α_{lev} has the "borderline value" $\alpha_{lev} = n_H/\left(n_L + n_H\right)$ for which the mixed-portfolio banks can satisfy the whole loan demand. For this value of α_{lev} it can only be concluded that the low-risk and high-risk interest rates r_L and r_H must satisfy the condition

$$V\left(k_{lev}, \alpha_{lev}, r_L, r_H\right) = 0$$

and that the two interest rates must be between the values that they would have in the equilibria of types A and B.

[18] Our welfare function does not include a term which represents the expected profits of banks, because their expected profits are zero in equilibrium, and we do not consider the welfare of depositors, which must be constant in the presence of deposit insurance.

of the revenue $1+a_\eta$ and the payment to the bank, $1+r_\eta$) if her project succeeds and nothing if the project fails. Hence, the expected profit of an entrepreneur of type η is given by

$$u_\eta = (1-\bar{p}_\eta)(a_\eta - r_\eta) \tag{27}$$

and the aggregate expected profits of the entrepreneurs amount up to

$$U = n_L u_L + n_H u_H = n_L (1-\bar{p}_L)(a_L - r_L) + n_H (1-\bar{p}_H)(a_H - r_H) \tag{28}$$

In our model, the payoff of the government is given by the sum of three terms, which represents the social costs of failure of the banks of each kind. The social costs of bank failure are given by the aggregate

$$G = m_L G_0 + m_M G_{\alpha_{lev}} + m_H G_1 \tag{29}$$

where

$$G_\alpha = E \min\{\pi(\kappa(\alpha), \alpha, r_L, r_H; z), 0\} - s(1-\Phi(\widehat{z}_\alpha)) \tag{30}$$

represents the expected social costs of the failure of a single bank with the portfolio α. In (30) the first term represents the expected value of direct costs of bank failure, i.e. the liabilities that the considered bank imposes on the deposit insurance system.[19] The latter term represents the indirect negative welfare effects that bank failures have on the economy, e.g. via disruptions in the supply of credit and savings opportunities. The multiplier s is assumed to be a constant in it.

Our welfare function will be the sum of the aggregate profits and the aggregate social costs which are due to all banks. This sum is given by

$$W = U + G \tag{31}$$

[19] These costs are, of course, 0 when the bank does not fail, and when it does, they are given by the (negative) net worth $\pi(k, \alpha, r_L, r_H, z)$ of the bank.

It turns out that our welfare function may be put in an intuitive form which does not contain an explicit reference to the interest rates r_L and r_H or to the profits of the firms (cf. Repullo-Suarez, 2004, p. 511). In what follows we use the more intuitive notations $\widehat{z}_L$, $\widehat{z}_M$, and $\widehat{z}_H$ for the $\widehat{z}$ values $\widehat{z}_0$, $\widehat{z}_{\alpha_{lev}}$, and $\widehat{z}_1$ that correspond to low-risk loan banks, mixed portfolio banks, and high-risk loan banks, respectively.

Theorem 4. The welfare function W equals

$$W = n_L\left(\left(1-\bar{p}_L\right)a_L - \bar{p}_L\lambda\right) + n_H\left(\left(1-\bar{p}_H\right)a_H - \bar{p}_H\lambda\right) - \delta K - sD$$

where K is the aggregate amount of bank capital and

$$D = m_L\left(1-\Phi\left(\widehat{z}_L\right)\right) + m_M\left(1-\Phi\left(\widehat{z}_M\right)\right) + m_H\left(1-\Phi\left(\widehat{z}_H\right)\right)$$

is the expected number of bank failures.

In Theorem 4 the first two terms of the formula for W are independent of capital requirements,[20] and we may now conclude that the optimization problem of the government consists of choosing the capital requirement so that the sum

$$\delta K + sD$$

is minimized. Since in our model there are three parameters (i.e., k_L, k_{lev}, and k_H) which a social planner is free to choose, a standard welfare analysis of our model would consist in finding the values of k_L, k_{lev}, and k_H which minimize $\delta K + sD$. However, we find that a welfare analysis of this kind would have little relevance for the study of actual economies. The actual IRB requirements of Basel II are not optimal in the sense of producing the maximal value for the welfare function W among all possible capital requirements k_L and k_H,[21] and we may view them as having been determined

[20] In the above expression of the welfare function W, the expected social value of each firm of type η ($\eta = L, H$) is represented by

$$\left(1-\bar{p}_\eta\right)a_\eta - \bar{p}_\eta\lambda$$

and here the first term may be viewed as the expected social value from the success of the firm (since a_η is the difference of the revenue from a succesful project, $1 + a_\eta$, and the investment needed for it, 1). The latter term represents the expected social costs from the failure of the firm (since λ is the loss given default of the bank).

[21] Cf. Repullo and Suarez (2004, pp. 511-3). Repullo and Suarez consider also "corrected" Basel II capital requirements, which yield precisely the same failure rate for specialized low-risk loan and high-risk loan banks. As they point out, also these requirements are not optimal in the sense of maximizing the considered welfare function for some fixed, given value of s (cf. Table 3 in ibid., p. 512).

by an optimization procedure which is outside the scope of our model. Accordingly, we shall investigate the problem of choosing the LRR k_{lev} optimally in an economy which has some fixed (not necessarily socially optimal) values of the risk-based capital requirements k_L and k_H.

Theorems 2 and 3 yield simple expressions for the aggregate amount of bank capital K.

Remark 1. The aggregate amount of bank capital K has the value

$K_A = n_L k_L + n_H k_H$

in an equilibrium of type A, and the value

$K_C = (n_L + n_H)\, k_{lev}$

in an equilibrium of type B.

Remark 1 shows that although the amount of capital of the mixed-portfolio banks is an increasing function of the LRR in the "small LRR" region in which the equilibrium A is possible, the decrease in the number of high-risk loan banks which corresponds to the increase of the LRR (and which is due to the fact that the mixed-portfolio banks take over an increasing part of the high-risk loans market as the LRR increases) suffices to compensate for this effect so that the aggregate amount of bank capital stays constant. Hence, the optimal value of k_{lev} in this region is simply the value which minimizes D, the expected number of defaulting banks. However, when an equilibrium of type B obtains, an increase in k_{lev} will have the negative effect of increasing capital costs, which must be weighted against its possible positive role in decreasing bank failures.

5. Calibration

We now present a calibrated version of our model. We have followed Repullo-Suarez (2009, p. 16) in giving the loss given default parameter the value $\lambda = 0.45$ and the cost of capital δ the value $\delta = 0.04$.[22] The rest of the parameter values are based on the data in Table I.[23]

[Table I]

We have taken the investment grade loans (loans of the categories from AAA to BBB) to constitute the counterpart of low-risk loans in our model and correspondingly, we have viewed the non-investment grade loans as the counterpart of high-risk loans.[24] When the total number of loans is normalized to 1, one may readily calculate from Table I both the demand for loans of each category, i.e. n_L and n_H, and also the average default probabilities of the loans of each category, which we denote by $\bar{p}_{L,E}$ and $\bar{p}_{H,E}$ (where 'E' stands for 'estimated'). The values $\mu_{L,E}$ and $\mu_{H,E}$ of μ which correspond to low-risk and high-low loans are determined by $\bar{p}_{L,E}$ and $\bar{p}_{H,E}$ in accordance with (10), and the values $\bar{p}_{L,E}$ and $\bar{p}_{H,E}$ also determine the correlation parameters ρ_L and ρ_H and the capital requirements k_L and k_H of our model in accordance with (1), (7), and (8). The resulting values are shown in Table II.

[Table II]

6. Model risk and bank portfolio choice

We wish to analyze the consequences of actions based on the calibrated model both when it is correct and when it turns out to be false. More rigorously, we assume that the agents act in accordance with the calibrated model during period $T = 0$, so that the portfolios of banks and the interest rates correspond to the Nash equilibium of the

[22] The size of the parameter δ has been recently actively discussed; see e.g. Hanson et al. (2011). The estimates they refer to suggest that the 4% assumption may be somewhat high but still reasonable.

[23] Table I reproduces the data on the shares of the loans with different ratings in the portfolio of an average quality bank, which has been presented in Gordy (2000), p. 132, Table I. The data on estimated default probabilities which appears in Table I is from Gupton et al. (1997), p. 76, Table 6.11.

[24] Such an aggregation is motivated by the observation from Table I that the default probabilities of the investment grade loans are quite close to one another. There is more variation between the non-investment grades but as a whole, default probabilities are broadly speaking "polarized" between the investment grade and non-investment grade groups.

calibrated model. However, we also assume that the loan defaults and banks failures might have probabilities which differ from the ones in the calibrated model.[25] To keep things simple, we assume that if the model is incorrect, the correct model has the structure described in Section 2 but corresponds to values of μ_L and μ_H which are different from the values $\mu_{L,E}$ and $\mu_{H,E}$ in the calibrated model.

6.1. Leverage ratio requirement, loan portfolios and interest rates

We begin by considering the effects that the LRR has on the loan portfolio choices of banks. These choices are not affected by the kind of model risk we consider. Figure 3 depicts the low-risk and the high-risk interest rates as functions of the LRR. In region A, the interest rates are almost constant. The LRR which has been introduced as a part of the Basel III reform corresponds to the value $k_{lev} = 0.03$ in Figure 3. This value has been indicated with a dashed vertical line in the figure. Since the value $k_{lev} = 0.03$ lies in region A, our model predicts that the LRR of the Basel III framework can have only quite small effects on the interest rates, provided that the banks are free to include high-risk loans in their portfolio.

[Figure 3]

It has sometimes been suggested that a leverage ratio type of requirement should be much higher than the 3% requirement of the Basel III framework. Figure 3 illustrates also the effects of such a scenario: an increased LRR (beyond approximately 6%) might lead to a considerable decrease in high-risk interest rates and to a considerable increase in low-risk interest rates. This will be the case when the requirement is so high that the banks which own low-risk loans are not in the position to cope with it by including high-risk loans in their portfolio.

[Figure 4]

[25] This situation may be thought of as a result of Knightian uncertainty (i.e. uncertainty which does not allow the agents to associate models with Bayesian priors). We may think that, not having prior probabilities for models, regulators, banks, and the banks' customers might be forced to base their actions on a model which can turn out to be false.

Figure 4 shows the number of the banks of each kind as a function of the LRR. The curve $m = m_L(k_{lev})$ depicts the number of the specialized low-risk loan banks, and the curve $m = m_H(k_{lev})$ depicts the number of the specialized high-risk loan banks. As the figure illustrates, for the low values of the LRR (in the region A) there will be only high-risk loan banks and mixed-portfolio banks on the market, and for the high values of the LRR (in region B) there are only low-risk loan banks and mixed-portfolio banks on the market. At the border line of the two regions both (23) and (25) are valid with equality, and the model shows a multiplicity of equilibria.

The portfolio α_{lev} of each mixed-portfolio bank is determined by the LRR k_{lev} in accordance with (6). Since we have assumed the demand for loans to be inelastic and normalized the size of the banks to 1, the number of the banks is independent of k_{lev}. This value is indicated by the horizontal line $m = 1$ in Figure 3. The space between this straight line and the curve $m = m_H(k_{lev})$ (in region A) or $m = m_L(k_{lev})$ (in region B) indicates the number $m_M(k_{lev})$ of the mixed-portfolio banks with portfolio α_{lev}.

The above results are valid independently of the existence of the kind of model risk that we wish to consider (i.e. of whether the assumed values $\mu_{L,E}$ and $\mu_{H,E}$ correspond to the actual default probabilities of loans), but bank failure probabilities will, of course, depend on the possibility of model risk. Beginning with the case in which the values $\mu_{L,E}$ and $\mu_{H,E}$ are correct, we recapitulate that in this case the capital requirements of the Basel II IRB regime suffice to cover the unexpected loan losses with the probability 99.9%. However, the probability of bank failure is below 0.1% under these requirements, because the banks use also their interest income from the non-defaulting loans for covering their loan losses. A high-risk loan bank differs from a low-risk loan bank in so far that a larger fraction of its loans default, and in so far that the interest income for each non-defaulting loan is larger. In the calibrated version, the aggregate effect turns out to be that under Basel II the failure probability of high-risk loan banks is smaller than the failure probability of low-risk loan banks, i.e.[26]

$1-\Phi\left(\widehat{z}_H\right) < 1-\Phi\left(\widehat{z}_L\right)$

The mixed portfolio banks have a failure probability which is between these values, and (since the Basel II requirement is a binding constraint for them), an increase in the share of the high-risk loans in their portfolios will decrease their failure probabilities. In other words,

$1-\Phi\left(\widehat{z}_H\right) < 1-\Phi\left(\widehat{z}_M\right) < 1-\Phi\left(\widehat{z}_L\right)$

and $1-\Phi\left(\widehat{z}_M\right)$ decreases towards $1-\Phi\left(\widehat{z}_H\right)$ as the share of high-risk loans increases in the portfolio of the mixed-portfolio bank.

In region A, the expected number of bank failures D (defined in Theorem 4) receives the form

$D = m_M\left(1-\Phi\left(\widehat{z}_M\right)\right) + m_H\left(1-\Phi\left(\widehat{z}_H\right)\right)$

In this region an increase in the LRR will increase the number of the riskier mixed portfolio banks (i.e. it will increase m_M and decrease m_H), which tends to increase the number of bank failures, but it also decreases the failure probability of each mixed portfolio bank when considered separately (i.e. it decreases $1-\Phi\left(\widehat{z}_M\right)$).

[Figure 5]

As Figure 5 illustrates, the result of these opposing effects turns out to be that the expected number of bank failures decreases as a function of the LRR k_{lev}. The expected number of bank failures is a decreasing function of k_{lev} also in the region B, because - in accordance with Remark 1 - in this region the total amount of capital of the banks increases as a function of k_{lev}.

According to Theorem 4 our welfare function W may be expressed in a form in which the effects of the capital requirement policy shows up only in the last two terms

$-\delta K - sD$

The amount of bank capital K stays constant in region A according to Remark 1, and one may immediately conclude from Figure 5 that in this region welfare increases as

[26] This result has earlier been presented in Repullo and Suarez (2004, p. 57, Table 1).

a function of k_{lev}. In region B, an increase in k_{lev} will not just decrease the number of bank failures, but also increase the aggregate costs of bank capital. In this region the weights δ and s will determine whether a raise in k_{lev}will increase or decrease welfare.

6.2. Model risk and bank stability

We now turn to the analysis of model risk and assume that the default probabilities of the loans of either category are distinct from the ones that appear in the calibrated model of Table II. As the values of the default probabilities correspond to the parameter μ via (10), our assumption may also be formulated by stating that all agents believe that the values μ_L or μ_H have the values $\mu_{L,E}$ or $\mu_{H,E}$ that appear in Table II, whereas, in reality, either the actual value of μ_L (which we denote by $\mu_{L,a}$ for clarity) satisfies $\mu_{L,a} > \mu_{L,E}$ or the actual value of μ_H (which we denote by $\mu_{H,a}$) satisfies $\mu_{H,a} > \mu_{H,E}$.[27] Our analysis of the former possibility is, in an obvious way, motivated by the subprime crisis and the recent developments during the European sovereign debt crisis.[28]

We have calculated the value of the expected number of bank failures as a function of p_{La} and of p_{Ha} for four different leverage ratio requirement regimes. The results are shown in Figures 6 and 7.

[Figure 6]

Figure 6 shows the expected number of bank failures when the default probability p_{La} varies from the value $\bar{p}_L = 0.0523\%$, which we have used in our calibration, up to 30%. The curve $D = D_0(p_{La})$ shows the expected number of failing banks in the absence of the leverage ratio requirement, i.e. under the Basel II regime, and the

[27] As we stated above, we are assuming that the model of Section 2 is correct, although being used with an incorrect calibration. In particular, we are assuming that the correlation parameters ρ_L and ρ_H are determined by the actual (rather than the estimated) values of μ_L and μ_H via (8)and (10).

[28] As we discussed in the introduction, our way of modelling the unanticipated model risk can be also motivated by Gennaioli et al. (2011) who argue that a bias which they call "local thinking" may lead to neglecting rare risks. This combined with investors' preference for safe assets may have contributed to the emergence of seemingly low-risk subprime loan based assets. In other words, their theory may explain why 1) what we have called model risk can be unanticipated, and 2) why such model risk is particularly relevant in the case of (seemingly) low-risk assets.

curve $D = D_1(p_{La})$ shows the expected number of bank failures under the leverage ratio requirement which has been included in the Basel III framework, i.e. $k_{lev} = 0.03$. As it is seen from Figure 6, if the default probability of low-risk loans is larger than the banks and regulators believe it to be, but not too much (below ca. 17.1%), the introduction of a leverage ratio requirement of the size $k_{lev} = 0.03$ will decrease the expected number of bank failures and increase welfare. However, the opposite is the case when p_{La} is very large.[29]

This result can be understood intuitively by remembering that in region A the leverage ratio requirement affects the number of bank failures in two ways. First, an increase in the leverage ratio requirement will increase both the amount of capital and the number of high-risk loans in the portfolios of the mixed-portfolio banks, and this diversification effect makes them safer. The model error tends to strengthen this effect, given that now the loans which are called "low-risk loans" are, as a matter of fact, quite risky. Secondly, an increase in the leverage ratio requirement increases also the volume of the loans that are held by the mixed-portfolio banks (and also the number of the mixed-portfolio banks, given that their size has been normalized to one). This contamination effect tends to increase the number of bank failures. Figure 6 shows that the first of these effects dominates the latter one for not too large values of the actual default probability p_{La} whereas the latter effect dominates the first one when p_{La} is sufficiently large.[30]

Since in region A a leverage ratio requirement does not increase the aggregate bank capital in the banking sector (which is 0.0582 in region A), adding a leverage

[29] Note that the threshold value of the actual default probability, 17.1%, is, although high, not necessarily unreasonable given the experience from the subprime crisis. For instance, "S&P now expects the default rate on subprime loans issued in 2005, 2006, and 2007 to be 11 percent, 30 percent, and 49 percent, respectively." (7.6.2009 in The Truth About Mortgage.com)

[30] When putting these numerical results into perspective, however, one should note that at the point where Basel III is worse for bank stability than Basel II, the absolute number of bank failures is already so high that any marginal increase or decrease in that number would probably not alter the fact that the banking sector is in deep trouble. Hence, while these results are quite interesting in a qualitative sense, policy conclusions might have to be based on further analysis with models which can be calibrated in a more realistic manner.

ratio requirement of $k_{lev} = 0.03$ increases welfare whenever the curve $D = D_1(p_{La})$ is below the curve $D = D_0(p_{La})$ in Figure 6. The curve $D = D_2(p_{La})$ corresponds to the largest leverage ratio requirement, $k_{lev} = 0.0582$, which may be implemented in the economy without increasing the amount of aggregate bank capital. This is the value which separates the regions A and B.[31] As Figure 6 shows, an increase of the leverage ratio requirement from $k_{lev} = 0.03$ up to the limit of the regions A and B will decrease the expected number of bank failures and increase welfare for all the considered values of p_{La}. Hence, the results from the calibrated model suggest that when the model risk is associated with low-risk loans, it would make sense to increase the LRR up to the point where the LRR equals the aggregate amount of risk-based capital requirements in the banking sector.

Finally, the curve $D = D_3(p_{La})$ shows the expected number of bank failures for a leverage ratio requirement of the size $k_{lev} = k_H = 0.112$. This limiting case has identical effects with a flat-rate (Basel I type) capital requirement of size 11.2%, and it corresponds to a considerable further decrease in the bank failure probability. However, the welfare comparisons between this regime and three other considered regimes will depend also on the relative weight that is given to the extra capital that is needed for implementing the leverage ratio requirement.

[Figure 7]

It is also interesting to study the model risks that are associated with high-risk loans. Analogously with Figure 6, Figure 7 shows the expected number of bank failures as a function of the actual default probability p_{Ha} of high-risk loans when p_{Ha} varies from the value $\bar{p}_H = 3.78\%$ to 30%. The curves D_0, D_1, D_2, and D_3 correspond to the same capital requirement regimes as the corresponding curves of Figure 6. The curves $D = D_0(p_{Ha})$ (which corresponds to Basel II regime) and $D = D_3(p_{Ha})$ (which corresponds to a flat-rate capital requirement k_H) are indistinguishable in Figure 7,

[31] The expected number of bank failures changes discontinuously at the border line of the regions A and B, and, more rigorously, the curve $D = D_2(p_{La})$ represents the limit of the number of bank failures when k_{lev} approaches the border line value from the left.

because their difference stems only from the different capital requirements for low-risk loan banks, and the failure rates of these banks are in the considered case quite small in comparison with the failure rates of high-risk loan banks.

As Figure 7 shows, if the actual value of the default probability p_{Ha} is not close to $\bar{p}_H$ (if p_{Ha} is larger than ca. 4.79%), the leverage ratio requirement 3% of the Basel III framework, which corresponds to the curve $D = D_1(p_{Ha})$, will tend to increase bank failures (in comparison with Basel II), and the "borderline" leverage ratio requirement 0.0582, which corresponds to the curve $D = D_2(p_{Ha})$, will tend to increase bank failures even more.

To understand this result intuitively, one should keep in mind that when p_{Ha} is large, in region A an increase of k_{lev} decreases the number of the specialized high-risk loan banks, which is a positive diversifying effect that tends to decrease the number of bank failures, but it also increases the riskiness of each mixed-portfolio bank (because it makes the mixed-portfolio banks include more high-risk loans in their portfolios). The latter effect is made stronger by the fact that the mixed-portfolio banks have, in addition to their high-risk loans, only loans with a low interest rate and a low capital requirement in their portfolios. As Figure 7 indicates, for the larger values of p_{Ha} the negative contaminating effect exceeds the positive diversifying effect from the decrease in the number of specialized high-risk loan banks.

It should be noted that the positive welfare effect of a 3% LRR in the absence of model risk, which is illustrated by Figure 5, was based on a rather specific feature of the Basel II framework, i.e. the fact that under the Basel II regime the banks that specialize in high-risk loans have a smaller failure probability than the banks that specialize in low-risk loans. It is natural to ask whether our results concerning model risk are in a similar way specific for the Basel IRB formula (7). We have run simulations in which the capital requirements k_L and k_H are risk-based in the sense that $k_L < k_H$, but have not been chosen in accordance with (7), and it has turned out that the qualitative effects concerning model risk have remained valid for other risk-based capital requirement regimes.[32]

7. Concluding remarks

We have studied the credit allocation and bank stability effects of introducing a leverage ratio requirement (LRR) on top of risk-based capital requirements, as in Basel III. We considered a specialized banking sector in which banks with high-risk loan portfolios were meant to represent banks which are not directly affected by the LRR (as their capital requirement would, in any case, be above the LRR), while banks with low-risk loan portfolios represented banks for which the LRR is a binding constraint. In our setting, the specialization of banks stems from limited liability, but the basic logic of our analysis would remain valid also in a model in which banks specialize for some other reason (e.g., bank-specific skills or informational advantages in screening loan customers in a certain customer segment).

We showed that if the LRR is below the average risk-based capital requirement in the banking sector, then both low-risk and high-risk loan rates and volumes remain essentially unchanged. This is because the LRR will be a binding capital constraint only on banks specializing in low-risk lending, so the banking sector can adapt by more banks granting both low-risk and high-risk loans. For LRRs above the aforementioned threshold, low-risk lending rates would significantly increase and high-risk lending

[32] In the analysis of this section we did not consider the possibility of a multiplicity of equilibria, but simply assumed that the equilibrium is either of type A or of type B. According to Theorem 5 (in the Appendix), the model will show a multiplicity of equilibria in region A if $\widehat{z}_0 = \widehat{z}_1$ (i.e. if the failure rates of low-risk and high-risk loan banks are identical) but not otherwise. Further, in region B there can be at most one value of LRR for which a multiplicity of equilibria occurs.

As we saw above, in the reasonably calibrated versions of the model $\widehat{z}_1 > \widehat{z}_0$, implying that the model can show a multiplicity of equilibria for at most a single, exceptional value of the LRR (see also the discussion in footnote 14 above). If it were the case that $\widehat{z}_0 = \widehat{z}_1$, the model would have many equilibria in the whole region in which equilibrium A is possible, but one could still state that the introduction of a LRR belonging to the region A would reduce their multiplicity, forcing all banks to have a fixed minimum proportion of high-risk loans in their portfolios. Hence, the LRR would also in this case tend to make banks more similar, hence increasing both the contamination effect and the diversification effect that we considered above.

rates would fall. Bank failures would decrease because of the increased amount of bank capital. However, in the presence of model risk, modelled as an unanticipated shock to the default probability of loans, a relatively low like the current 3% LRR might even reduce bank stability, counter to regulatory intentions. If the model risk is associated with low-risk loans, bank stability would be reduced if the model risk were severe. This is because for a sufficiently high model risk the beneficial effect from spreading (the seemingly) low-risk loans to a larger number of banks is dominated by the effect of contaminating a larger number of banks by the assets turned "toxic". If the unanticipated model risk concerns high-risk loans' default probability, then the current moderate LRR (almost) always increases bank failures as a result of the contamination effect.

A welfare implication of our analysis when the model risk concerns low-risk loans is that the LRR could be increased up to the point where the LRR equals the average risk-based capital requirement in the banking sector. Up to that point the additional cost of capital to the sector would be insignificant because banks can adapt to the higher LRR simply by reshuffling loans among themselves. However, the advantage would be that bank stability would improve (unless an unanticipated shock to the low-risk loans' default probability is extremely high).

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Appendix. Proofs of theorems

We begin the presentation of the proofs by introducing a new notation. The value $\widehat{z}_\alpha$, which appears in (14), depends in addition to α implicitly also on k, r_L, and r_H. In what follows we shall denote it by $\widehat{z}_\alpha(k, r_L, r_H)$ for the sake of clarity; i.e., for any given combination of α, k, r_L, and r_H we shall denote by $\widehat{z}_\alpha(k, r_L, r_H)$ the value of z for which

$$\pi(k, \alpha, r_L, r_H; z) = 0$$

In the proofs that follow we shall repeatedly appeal to the following lemma.

Lemma 1. Assume that the capital requirement $\kappa(\alpha)$ is a linear function of α in the interval $[\alpha_1, \alpha_2]$, assume that the interest rates r_L and r_H are fixed, and let α have some value in (α_1, α_2).

(a) The value $\widehat{z}_\alpha(\kappa(\alpha), r_L, r_H)$ is strictly between $\widehat{z}_{\alpha_1}(\kappa(\alpha_1), r_L, r_H)$ and $\widehat{z}_{\alpha_2}(\kappa(\alpha_2), r_L, r_H)$ if and only if

$$\widehat{z}_{\alpha_1}(\kappa(\alpha_1), r_L, r_H) \neq \widehat{z}_{\alpha_2}(\kappa(\alpha_2), r_L, r_H) \tag{32}$$

Otherwise, $\widehat{z}_\alpha(\kappa(\alpha), r_L, r_H) = \widehat{z}_{\alpha_1}(\kappa(\alpha_1), r_L, r_H) = \widehat{z}_{\alpha_2}(\kappa(\alpha_2), r_L, r_H)$.

(b) The net value of a bank with the portfolio α satisfies

$$\begin{aligned} & V(\kappa(\alpha), \alpha, r_L, r_H) \\ & \leq \tfrac{\alpha_2-\alpha}{\alpha_2-\alpha_1} V(\kappa(\alpha_1), \alpha_1, r_L, r_H) + \tfrac{\alpha-\alpha_1}{\alpha_2-\alpha_1} V(\kappa(\alpha_2), \alpha_2, r_L, r_H) \end{aligned} \tag{33}$$

The condition (33) is valid with strict inequality if and only if (32) is valid.

Proof. By linearity, the capital requirement $\kappa(\alpha)$ may be expressed in the form

$$\kappa(\alpha) = \frac{\alpha_2 - \alpha}{\alpha_2 - \alpha_1}\kappa(\alpha_1) + \frac{\alpha - \alpha_1}{\alpha_2 - \alpha_1}\kappa(\alpha_2) \tag{34}$$

Now (12) implies that

$$\begin{aligned}&\pi\left(\kappa\left(\alpha\right),\alpha,r_L,r_H;z\right)\\&=\tfrac{\alpha_2-\alpha}{\alpha_2-\alpha_1}\pi\left(\kappa\left(\alpha_1\right),\alpha_1,r_L,r_H;z\right)+\tfrac{\alpha-\alpha_1}{\alpha_2-\alpha_1}\pi\left(\kappa\left(\alpha_2\right),\alpha_2,r_L,r_H;z\right)\end{aligned}\tag{35}$$

It is clear that if (32) is *not* valid, the right-hand side of this equality is zero for

$$z=\widehat{z}_{\alpha_1}\left(\kappa\left(\alpha_1\right),r_L,r_H\right)=\widehat{z}_{\alpha_2}\left(\kappa\left(\alpha_2\right),r_L,r_H\right)$$

and, hence, in this case $\widehat{z}_{\alpha}\left(\kappa\left(\alpha\right),r_L,r_H\right)$, $\widehat{z}_{\alpha_1}\left(\kappa\left(\alpha_1\right),r_L,r_H\right)$, and $\widehat{z}_{\alpha_2}\left(\kappa\left(\alpha_2\right),r_L,r_H\right)$ are all equal. If, on the other hand, (32) is valid, the value of z for which

$$\pi\left(\kappa\left(\alpha\right),\alpha,r_L,r_H;z\right)=0$$

must according to (35) be such that one of the two values $\pi\left(\kappa\left(\alpha_1\right),\alpha_1,r_L,r_H;z\right)$ and $\pi\left(\kappa\left(\alpha_2\right),\alpha_2,r_L,r_H;z\right)$ is positive and the other one negative, i.e. the value $\widehat{z}_{\alpha}\left(\kappa\left(\alpha\right),r_L,r_H\right)$ must be strictly between $\widehat{z}_{\alpha_1}\left(\kappa\left(\alpha_1\right),r_L,r_H\right)$ and $\widehat{z}_{\alpha_2}\left(\kappa\left(\alpha_2\right),r_L,r_H\right)$.

(b) Define $\underline{z}$ and $\bar{z}$ by

$$\underline{z}=\min\left\{\widehat{z}_{\alpha_1}\left(\kappa\left(\alpha_1\right),r_L,r_H\right),\widehat{z}_{\alpha_2}\left(\kappa\left(\alpha_2\right),r_L,r_H\right)\right\}$$

and

$$\bar{z}=\max\left\{\widehat{z}_{\alpha_1}\left(\kappa\left(\alpha_1\right),r_L,r_H\right),\widehat{z}_{\alpha_2}\left(\kappa\left(\alpha_2\right),r_L,r_H\right)\right\}$$

We conclude from (35) that

$$\begin{aligned}&\max\left\{\pi\left(\kappa\left(\alpha\right),\alpha,r_L,r_H;z\right),0\right\}\\&\leq\tfrac{\alpha_2-\alpha}{\alpha_2-\alpha_1}\max\left\{\pi\left(\kappa\left(\alpha_1\right),\alpha_1,r_L,r_H;z\right),0\right\}+\tfrac{\alpha-\alpha_1}{\alpha_2-\alpha_1}\max\left\{\pi\left(\kappa\left(\alpha_2\right),\alpha_2,r_L,r_H;z\right),0\right\}\end{aligned}\tag{36}$$

and that this result is valid with strict inequality if and only one of the values $\pi\left(\kappa\left(\alpha_1\right),\alpha_1,r_L,r_H;z\right)$ and $\pi\left(\kappa\left(\alpha_2\right),\alpha_2,r_L,r_H;z\right)$ is positive and the other one negative, i.e. if $\underline{z}<z<\bar{z}$.

Let E denote expectation with respect to the random variable z, which follows a standardized normal distribution. One may conclude from (14) and(36) that

$$\begin{aligned}\Pi\left(\kappa\left(\alpha\right),\alpha,r_L,r_H\right)&=E\left(\max\left\{\pi\left(\kappa\left(\alpha\right),\alpha,r_L,r_H;z\right),0\right\}\right)\\&\leq\tfrac{\alpha_2-\alpha}{\alpha_2-\alpha_1}E\left(\max\left\{\pi\left(\kappa\left(\alpha_1\right),\alpha_1,r_L,r_H;z\right),0\right\}\right)\end{aligned}$$

$$+\tfrac{\alpha-\alpha_1}{\alpha_2-\alpha_1}\left(E\max\left\{\pi\left(\kappa\left(\alpha_2\right),\alpha_2,r_L,r_H;z\right),0\right\}\right)$$
$$=\tfrac{\alpha_2-\alpha}{\alpha_2-\alpha_1}\Pi\left(\kappa\left(\alpha_1\right),\alpha_1,r_L,r_H\right)+\tfrac{\alpha-\alpha_1}{\alpha_2-\alpha_1}\Pi\left(\kappa\left(\alpha_2\right),\alpha_2,r_L,r_H\right)$$

and that this result is valid with strict inequality if and only if there are values of z for which $\underline{z}< z < \bar{z}$, i.e. if (32) is valid.

Finally, combining this result with (13) and (34), it follows that

$$V\left(\kappa\left(\alpha\right),\alpha,r_L,r_H\right)=-\kappa\left(\alpha\right)+\tfrac{1}{1+\delta}\Pi\left(\kappa\left(\alpha\right),\alpha,r_L,r_H\right)$$
$$\leq\tfrac{\alpha_2-\alpha}{\alpha_2-\alpha_1}\left(-\kappa\left(\alpha_1\right)+\tfrac{1}{1+\delta}\Pi\left(\kappa\left(\alpha_1\right),\alpha_1,r_L,r_H\right)\right)$$
$$+\tfrac{\alpha-\alpha_1}{\alpha_2-\alpha_1}\left(-\kappa\left(\alpha_2\right)+\tfrac{1}{1+\delta}\Pi\left(\kappa\left(\alpha_2\right),\alpha_2,r_L,r_H\right)\right)$$
$$=\tfrac{\alpha_2-\alpha}{\alpha_2-\alpha_1}V\left(\kappa\left(\alpha_1\right),\alpha_1,r_L,r_H\right)+\tfrac{\alpha-\alpha_1}{\alpha_2-\alpha_1}V\left(\kappa\left(\alpha_2\right),\alpha_2,r_L,r_H\right)$$

and that also this inequality is strict if and only if (32) is valid.

Proof of Theorem 1. Suppose that in an equilibrium there are banks with the portfolio α, $0<\alpha<\alpha_{lev}$. Let n_α be the number of these banks. Given that the considered situation is an equilibrium, the interest rates r_L and r_H must be such that

$$V\left(k_{lev},\alpha,r_L,r_H\right)=0.$$

We now apply Lemma 1(b) with $\alpha_1=0$, $\alpha_2=\alpha_{lev}$ and with κ being given by (5), and we conclude that $V\left(k_{lev},\alpha,r_L,r_H\right)$ is smaller than or equal to a weighted average of $V\left(k_{lev},0,r_L,r_H\right)$ and $V\left(k_{lev},\alpha_{lev},r_L,r_H\right)$. However, since the value of no portfolio can yield a positive net value to a bank in equilibrium, this can only be the case if

$$V\left(k_{lev},0,r_L,r_H\right)=V\left(k_{lev},\alpha_{lev},r_L,r_H\right)=0$$

Hence, also a situation in which n_α banks with portfolio α are replaced with low-risk banks and mixed-portfolio banks with the same loans (i.e., with $\left(1-\left(\alpha/\alpha_{lev}\right)\right)n_\alpha$ low-risk banks and $\left(\alpha/\alpha_{lev}\right)n_\alpha$ mixed-portfolio banks) must be an equilibrium of the model.

Similarly, suppose that there are n_α banks with the portfolio α, $\alpha_{lev}<\alpha<1$, in equilibrium, and denote the equilibrium interest rates by r_L and r_H. Again, it must be the case that

$$V\left(\kappa\left(\alpha\right),\alpha,r_L,r_H\right)=0$$

and this time we may conclude, applying Lemma 1(b) with $\alpha_1 = \alpha_{lev}$ and $\alpha_2 = 1$, that

$$V(k_{lev}, \alpha_{lev}, r_L, r_H) = V(k_H, 1, r_L, r_H) = 0.$$

Hence, also a situation in which n_α banks with portfolio α are replaced with mixed-portfolio banks and high-risk loan banks with the same loans (i.e., with $((1-\alpha)/(1-\alpha_{lev}))\, n_\alpha$ mixed-portfolio banks and $(1-(1-\alpha)/(1-\alpha_{lev}))\, n_\alpha$ high-risk banks) must be an equilibrium of the model.

Proof of Theorem 2. It follows directly from the definitions of the interest rates $r_{AL}(k_{lev})$ and $\bar{r}_H$ that the net value of a mixed-portfolio bank and a specialized high-risk loan bank are zero. One may further conclude from (20) and (22) that the net value of a specialized low-risk loan bank must be negative for these interest rates. Now Lemma 1(b) implies that the net value of a bank with any portfolio α must be non-positive. Hence, a situation with these interest rates is an equilibrium if the supply and demand of loans are equal, i.e. if (18) is valid. However, (18) follows from (24), and the assumption (23) guarantees that the values that (24) specifies for the number of the three kinds of banks are all non-negative.

Proof of Theorem 3. The conditions which characterize the interest rates $r_{BL}(k_{lev})$ and $r_{BH}(k_{lev})$ in this theorem imply immediately that specialized low-risk banks and mixed-portfolio banks are of a net value zero. We may now conclude that from (21) and (22) that

$$V(k_{lev}, \alpha_{lev}, r_{BL}(k_{lev}), \bar{r}_H) > V(k_{lev}, \alpha_{lev}, r_{AL}(k_{lev}), \bar{r}_H) = 0$$

and in a second step from this result and the condition

$$V(k_{lev}, \alpha_{lev}, r_{BL}(k_{lev}), r_{BH}(k_{lev})) = 0$$

that

$$r_{BH}(k_{lev}) < \bar{r}_H$$

Hence, for the considered interst rates a specialized high-risk loan bank would be of a negative net value. Now Lemma 1(b) implies that the net value of a bank with any portfolio α must be non-positive. Again, we conclude that the considered situation is an equilibrium if the supply and demand of loans are equal, i.e. if (18) is valid. This time (18) follows from (26), and the assumption (25) guarantees that the values that (26) specifies for the number of the three kinds of banks are all non-negative.

Proof of Theorem 4. Letting E represent expectation with respect to z and remembering (14), we conclude that for each portfolio α

$$\begin{aligned} &E\left(\min\left\{\pi\left(k,\alpha,r_L,r_H,z\right),0\right\}\right) \\ &\quad = E\left(\pi\left(k,\alpha,r_L,r_H,z\right)\right) - E\left(\max\left\{\pi\left(k,\alpha,r_L,r_H,z\right),0\right\}\right) \\ &\quad = E\left(\pi\left(k,\alpha,r_L,r_H,z\right)\right) - \Pi\left(k,\alpha,r_L,r_H\right) \end{aligned}$$

The definition (13) and the equilibrium condition $V\left(k,\alpha,r_L,r_H\right) = 0$ imply that the latter term equals $(1+\delta)\,k$. The first term can be evaluated using (12) and remembering that, by definition, $E\left(p_L\right) = \bar{p}_L$ and $E\left(p_H\right) = \bar{p}_H$. Putting these results together, it follows that, for each portfolio α,

$$\begin{aligned} &E\left(\min\left\{\pi\left(k,\alpha,r_L,r_H,z\right),0\right\}\right) \\ &\quad = (1-\alpha)\left(r_L - \bar{p}_L\left(\lambda + r_L\right)\right) + \alpha\left(r_H - \bar{p}_H\left(\lambda + r_H\right)\right) - \delta k \end{aligned}$$

When this result is combined with (18), (29), and (30), one may conclude with some elementary algebra that

$$G = n_L\left(r_L - \bar{p}_L\left(\lambda + r_L\right)\right) + n_H\left(r_H - \bar{p}_H\left(\lambda + r_H\right)\right) - \delta K - sD$$

where

$$D = m_L\left(1 - \Phi\left(\widehat{z}_L\right)\right) + m_M\left(1 - \Phi\left(\widehat{z}_M\right)\right) + m_H\left(1 - \Phi\left(\widehat{z}_H\right)\right)$$

is the expected number of bank failures and K is the aggregate amount of capital of the banks. Finally, together with our definition (28) of U, our latter formula for G implies that the welfare $W = U + G$ may also be expressed in the form

$$W = n_L\left(\left(1-\bar{p}_L\right)a_L - \bar{p}_L\lambda\right) + n_H\left(\left(1-\overline{p}_H\right)a_H - \bar{p}_H\lambda\right) - \delta K - sD$$

which is the result that was to be proved.

Proof of Remark 1. Since the amount of capital of a mixed-portfolio bank and of a high-risk loan bank are k_{lev} and k_H, respectively, one may conclude from (24) and

(6) that the total amount of bank capital in an equilibrium of type A is

$$\begin{aligned} K_A &= m_M k_{lev} + m_H k_H = \tfrac{n_L}{1-\alpha_{lev}} k_{lev} + \left(n_H - \tfrac{\alpha_{lev}}{1-\alpha_{lev}} n_L\right) k_H \\ &= \left(\tfrac{k_H - k_L}{k_H - k_{lev}} n_L\right) k_{lev} + n_H k_H - \tfrac{k_{lev} - k_L}{k_H - k_{lev}} n_L k_H \\ &= \tfrac{k_L (k_H - k_{lev})}{k_H - k_{lev}} n_L + n_H k_H = n_L k_L + n_H k_H \end{aligned}$$

Similarly, since the amount of capital of a low-risk loan bank and of a mixed-portfolio bank both equal k_{lev}, one may conclude from (26) that the total amount of bank capital in an equilibrium of type B is

$$\begin{aligned} K_C &= m_L k_{lev} + m_M k_{lev} = (m_L + m_H) k_{lev} \\ &= \left(n_L - \tfrac{1-\alpha_{lev}}{\alpha_{lev}} n_H + \tfrac{n_H}{\alpha_{lev}}\right) k_{lev} \\ &= (n_L + n_H) k_{lev} \end{aligned}$$

Above we saw that the model will have a multiplicity of equilibria at the border line of the regions A and B. Theorem 5, which is not included in the body of this paper, characterizes the other cases in which a multiplicity of equilibria is possible. The proof of theorem appeals to two observations, which we formulate as Remarks 2 and 3.

Remark 2. Specialized low-risk loan banks and specialized high-risk loan banks cannot co-exist in equilibrium in the presence of a LRR.

Proof of Remark 2. Suppose that there are both specialized low-risk loan banks and specialized high-risk loan banks on the market. Now the high-risk loan interest rate must have the value r_H which is determined by the condition

$$V_H (k_H, r_H) = 0$$

i.e. it must be the case that $r_H = \bar{r}_H$. On the other hand, the low-risk loan interest rate must have the value r_L which is determined by the condition

$$V_L (k_{lev}, r_L) = 0,$$

i.e. it must be the case that $r_L = r_{BL} (k_{lev})$. However, now one may conclude from (21) and (22) that

$$V (k_{lev}, \alpha_{lev}, r_{BL} (k_{lev}), \bar{r}_H) > V (k_{lev}, \alpha_{lev}, r_{AL} (k_{lev}), \bar{r}_H) = 0$$

i.e. that a mixed-porftolio bank has a positive value. Hence, the considered situation is not an equilibrium.

Remark 3. An equilibrium of type A and an equilibrium of type B can coexist for the same value of α_{lev} only if

$\alpha_{lev} = \frac{n_H}{n_L+n_H}$

Proof of Remark 3. Assume first that

$\alpha_{lev} < \frac{n_H}{n_L+n_H}$

We wish to prove that an equilibrium of type B does not exist. If it does, in it all banks have one of the portfolios $\alpha = 0, \alpha_{lev}$ so that (18) is valid with $m_H = 0$. However, plugging $m_H = 0$ and

$\alpha_{lev} < \frac{n_H}{n_L+n_H}$

into (18), it follows that

$m_L = n_L - \frac{1-\alpha_{lev}}{\alpha_{lev}} n_H = (n_L + n_H) - \frac{n_H}{\alpha_{lev}} < (n_L + n_H) - (n_L + n_H) = 0$

i.e. that the number of specialized low-risk loan banks is negative. This, however, is possible, and we may conclude that a type B equilibrium does not exist.

Assume next that

$\alpha_{lev} > \frac{n_H}{n_L+n_H}$

We wish to prove that an equilibrium of type A does not exist. If it does, all banks have in it one of the portfolios $\alpha = \alpha_{lev}, 1$ and the demand and supply of loans must satisfy (18) with $m_L = 0$. Plugging the assumptions $m_L = 0$ and

$\alpha_{lev} > \frac{n_H}{n_L+n_H}$

into (18), we conclude that

$m_H = n_H - \frac{\alpha_{lev}}{1-\alpha_{lev}} n_L = \frac{n_L+n_H}{1-\alpha_{lev}} \left(\frac{n_H}{n_L+n_H} - \alpha_{lev} \right) < 0$

i.e. that the number of specialized high-risk loan banks is negative. However, this is impossible, and we may conclude that the type A equilibrium does not exist.

Theorem 5.

(a) Suppose that the LRR k_{lev} lies in the region in which

$0 < \alpha_{lev} < \frac{n_H}{n_L + n_H}$

The model has other equilibria besides the type A equilibrium if and only if

$\widehat{z}_0 (k_L, \bar{r}_L, \bar{r}_H) = \widehat{z}_1 (k_H, \bar{r}_L, \bar{r}_H)$

(i.e. if and only if the mixed-portfolio banks and the high-risk loan banks have precisely the same failure rate in the absence of the LRR).

(b) Suppose that the LRR k_{lev} lies in the region in which

$\frac{n_H}{n_L + n_H} < \alpha_{lev} < 1$

and let r_{BL} and r_{BH} denote the interest rates that correspond to the type B equilibrium for the given k_{lev}. There are other equilibria in addition to the type B equilibrium if and only if

$\widehat{z}_0 (k_{lev}, r_{BL}, r_{BH}) = \widehat{z}_{\alpha_{lev}} (k_{lev}, r_{BL}, r_{BH})$

(i.e. if the low-risk loan banks and the mixed-portfolio banks have precisely the same failure rate under the considered leverage ratio requirement).

Proof of Theorem 5.

In this proof we shall apply Lemma 1 not just to the considered Basel III type requirement, but also to the Basel II capital requirement, which we shall denote by κ_{B2}. The Basel II requirement is given by

$\kappa_{B2} (\alpha) = (1 - \alpha) k_L + \alpha k_H$

and it is linear in α in the whole interval $[0, 1]$. For the sake of clarity, we shall denote the Basel III requirement of the form (5) which is under consideration (and which depends on α_{lev}) by $\kappa_{B3} (\alpha)$.

(a) Assume first that

$\widehat{z}_0 (k_L, \bar{r}_L, \bar{r}_H) = \widehat{z}_1 (k_H, \bar{r}_L, \bar{r}_H)$.

Since is linear in α in the whole interval $[0, 1]$, we may apply Lemma 1 to κ_{B2}, to the Basel II interest rates $r_L = \bar{r}_L$ that $r_H = \bar{r}_H$, and to $\alpha_1 = 0$, $\alpha = \alpha_{lev}$, and $\alpha_2 = 1$. In this way we conclude that

$$\widehat{z}_{\alpha_{lev}}\left(k_{lev},\bar{r}_L,\bar{r}_H\right)=\widehat{z}_1\left(k_H,\bar{r}_L,\bar{r}_H\right) \tag{37}$$

and that

$$V\left(k_{lev},\alpha_{lev},\bar{r}_L,\bar{r}_H\right)$$
$$=\left(1-\alpha_{lev}\right)V\left(k_L,0,\bar{r}_L,\bar{r}_H\right)+\alpha_{lev}V\left(k_H,1,\bar{r}_L,\bar{r}_H\right)=0.$$

According to the definition (21) of $r_{AL}\left(k_{lev}\right)$, this result implies that $r_{AL}\left(k_{lev}\right)=\bar{r}_L$. In a next step we apply Lemma 1(b) to κ_{B3} with $r_L=\bar{r}_L$, $r_H=\bar{r}_H$, $\alpha_1=\alpha_{lev}$, $\alpha_2=1$. Remembering (37), we conclude that

$$V\left(\kappa_{B2}\left(\alpha\right),\alpha,\bar{r}_L,\bar{r}_H\right)=0$$

for any portfolio α for which $\alpha_{lev}<\alpha<1$. Hence, the model has a multiplicity of equilibria in which banks choose portfolios between α_{lev} and 1. E.g., also the symmetrical situation in which the interest rates have their Basel II values $r_L=\bar{r}_L$, $r_H=\bar{r}_H$ and all banks choose the portfolio

$$\alpha=\frac{n_H}{n_L+n_H}>\alpha_{lev}$$

corresponds to an equilibrium of the model.

Conversely, assume that the model has an equilibrium **E** which is distinct from the type A equilibrium characterized by Theorem 2. We may conclude from Theorem 1 and Remark 2 that in this equilibrium the interest rates are identical with those of either an equilibrium of type A or an equilibrium of type B. However, Theorem 2 and Remark 3 imply that an equilibrium of type B does not exist, and the interest rates are identical with those of an equilibrium of type A, i.e. with $r_L=r_{AL}\left(k_{lev}\right)$ and $r_H=\bar{r}_H$. Given that the model has the equilibrium **E** besides the type A equilibrium, there must be some value $\bar{\alpha}\neq\alpha_{lev},1$, for which

$$V\left(\kappa_{B3}\left(\bar{\alpha}\right),\bar{\alpha},r_{AL}\left(k_{lev}\right),\bar{r}_H\right)=0 \tag{38}$$

If $\alpha<\alpha_{lev}$, we may conclude from (20)-(22) and Lemma 1(b) (by putting $\alpha_1=0$, $\alpha_2=\alpha_{lev}$, and $\kappa=\kappa_{B3}$) that

$$V\left(k_{lev},\alpha_{lev},r_{AL}\left(k_{lev}\right),\bar{r}_{H}\right)=0$$
$$\leq\tfrac{\alpha_{lev}-\alpha}{\alpha_{lev}}V_{L}\left(k_{lev},r_{AL}\left(k_{lev}\right)\right)+\tfrac{\alpha}{\alpha_{lev}}V\left(k_{lev},\alpha_{lev},r_{AL}\left(k_{lev}\right),\bar{r}_{H}\right)$$
$$<\tfrac{\alpha_{lev}-\alpha}{\alpha_{lev}}V_{L}\left(k_{lev},r_{BL}\right)=0$$

Hence, we may conclude that the value of $\bar{\alpha}\neq\alpha_{lev},1$ which satisfies (38) must belong to the interval $(\alpha_{lev},1)$. In this case (33) is valid with equality for $\kappa=\kappa_{B3}$, $\alpha_1=\alpha_{lev}$, $\alpha=\bar{\alpha}$, and $\alpha_2=1$, and we may conclude from Lemma 1(b) that

$$\widehat{z}_{\alpha_{lev}}\left(k_{lev},r_{AL}\left(k_{lev}\right),\bar{r}_{H}\right)=\widehat{z}_{1}\left(k_{H},r_{AL}\left(k_{lev}\right),\bar{r}_{H}\right)\tag{39}$$

In a next step, we apply Lemma 1 to the Basel II capital requirement $\kappa=\kappa_{B2}$ with $\alpha_1=0$, $\alpha=\alpha_{lev}$, and $\alpha_2=1$ and conclude from Lemma 1(a) and (39) that

$$\widehat{z}_{0}\left(k_{L},r_{AL}\left(k_{lev}\right),\bar{r}_{H}\right)=\widehat{z}_{1}\left(k_{H},r_{AL}\left(k_{lev}\right),\bar{r}_{H}\right)\tag{40}$$

and from Lemma 1(b) that

$$0=V\left(k_{lev},\alpha_{lev},r_{AL}\left(k_{lev}\right),\bar{r}_{H}\right)$$
$$=\left(1-\alpha\right)V_{L}\left(k_{L},r_{AL}\left(k_{lev}\right)\right)+\alpha V_{H}\left(k_{H},\bar{r}_{H}\right)$$

implying that

$$V_{L}\left(k_{L},r_{AL}\left(k_{lev}\right),\bar{r}_{H}\right)=0$$

However, according to the definition of $\bar{r}_L$ this means that

$$r_{AL}\left(k_{lev}\right)=\bar{r}_{L}$$

Combining the last result with (40), it follows that

$$\widehat{z}_{0}\left(k_{L},\bar{r}_{L},\bar{r}_{H}\right)=\widehat{z}_{1}\left(k_{H},\bar{r}_{L},\bar{r}_{H}\right)$$

This completes the proof of part (a).

(b) Assume first that

$$\widehat{z}_{0}\left(k_{lev},r_{BL},r_{BH}\right)=\widehat{z}_{\alpha_{lev}}\left(k_{lev},r_{BL},r_{BH}\right)$$

We apply Lemma 1(b) with $\alpha_1=0$, $\alpha_2=\alpha_{lev}$, and $\kappa=\kappa_{B3}$ (which is identical with constant function k_{lev} in the interval $[0,\alpha_{lev}]$) with and conclude that

$$V(k_{lev}, \alpha, r_{BL}, r_{BH}) = 0 \tag{41}$$

is valid for all values of α in the interval $[0, \alpha_{lev}]$. Hence, the model has other equilibria besides the equilibrum of type B; e.g. the symmetric situation in which the interest rates are $r_L = r_{BL}$ and $r_L = r_{BH}$ and all banks choose the portfolio

$\alpha = \frac{n_H}{n_L + n_H} < \alpha_{lev}$

is an equilibrium of the model.

Assume now conversely that the model has an equilibrium E which is distinct from the type B equilibrium of Theorem 3. Again, we conclude from Theorem 1 and Remark 2 that in this equilibrium the interest rates are identical with those of either a type A equilibrium or a type B equilibrium. However, Theorem 3 and Remark 3 imply that a type A equilibrium does not exist, and that the interest rates of equilibrium **E** are identical with those of a type B equilibrium, in which the interest rates have the values $r_L = r_{BL}$ and $r_L = r_{BH}$.

Given that there are many equilibria, there must be some value $\bar{\alpha} \neq 0, \alpha_{lev}$ of α which satisfies (41). As our next step we conclude from (22) and the equations that define $r_{AL}(k_{lev})$ and $r_{BH}(k_{lev})$ in Theorems 2 and 3, i.e.

$V(k_{lev}, \alpha_{lev}, r_{AL}(k_{lev}), \bar{r}_H) = 0$

and

$V(k_{lev}, \alpha_{lev}, r_{BL}, r_{BH}) = 0$

that

$r_{BH} < \bar{r}_H$

and, further, that

$V_H(k_H, r_{BH}) < V(k_H, \bar{r}_H) = 0.$

We may now apply Lemma 1(b) with $\kappa = \kappa_{B3}$ with $\alpha_1 = \alpha_{lev}$, and $\alpha_2 = 1$, and conclude that whenever $\alpha > \alpha_{lev}$,

$V(\kappa_{B3}(\alpha), \alpha, r_{BL}, r_{BH}) < 0$

Hence, the value $\bar{\alpha} \neq 0, \alpha_{lev}$ of α which satisfies (41) must be between 0 and α_{lev}. However, if this is the case, the condition (33) must be valid with equality for $\alpha_1 = 0$, $\alpha = \bar{\alpha}$, $\alpha_2 = \alpha_{lev}$, and $\kappa = \kappa_{B3}$, and we may conclude from Lemma 1(b) that

$$\widehat{z}_0 \left(k_{lev}, r_{BL}, r_{BH}\right) = \widehat{z}_{\alpha_{lev}} \left(k_{lev}, r_{BL}, r_{BH}\right).$$

This completes our proof.

TABLES

Table I. Data on the shares of loans of different categories among all granted loans and on their default probabilities.

Loan Category	Share in Portfolio (%)	Default Probability (%)
AAA	2.9	0.02
AA	5.0	0.02
A	13.4	0.03
BBB	31.2	0.07
BB	32.4	1.32
B	11.1	5.58
CCC	4.0	18.6

Table II. The parameter values of the calibrated version of the model.

Parameter	Explanation	Value
δ	Cost of equity (equity premium)	0.04
λ	Loss given default	0.45
$\bar{n}_L$	Demand for low-risk loans	0.525
$\bar{n}_H$	Demand for high-risk loans	0.475
$\bar{p}_L$	Default probability for low-risk loans	0.0523%
$\bar{p}_H$	Default probability for high-risk loans	3.77%
$\mu_{L,E}$	Parameter characterizing the assumed low-risk loan default probability distribution	-3.278
$\mu_{H,E}$	Parameter characterizing the assumed high-risk loan default probability distribution	-1.778
ρ_L	Correlation parameter for low-risk loans	0.237
ρ_H	Correlation parameter for high-risk loans	0.138
k_L	Basel II capital requirement for low-risk loans	0.00951
k_H	Basel II capital requirement for high-risk loans	0.112