MODEL UNCERTAINTY AND THE EQUILIBRIUM VALUE OF THE REAL EFFECTIVE EURO EXCHANGE RATE

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Abstract: On the basis of historical data aggregated over the period 1973 to 2000, we estimated four different equilibrium exchange rate models for the synthetic euro. Using the same data set, variable definitions and sample period offers the possibility to assess the uncertainty surrounding such equilibrium levels, both from a statistical and a theoretical perspective. We employed reduced form co-integration models, a structural VAR, a Natrex model (estimated in structural form) and the ECB’s small-sized euro area wide macro-econometric model. In this order the approaches feature an increasing degree of ‘structure’, in the sense of the constraints based on economic theory embedded in the econometric models that were estimated. The results confirm the high likelihood for the euro having been undervalued in Q4 2000, while stressing the significant empirical and theoretical uncertainty with respect to the equilibrium exchange rate level.

Keywords: equilibrium exchange rate, euro, model uncertainty.

JEL classification numbers: F31, F32.
**Non technical summary:**

Understanding what drives real exchange rate developments is important for monetary policy-makers, as the assessment of the macroeconomic impact of exchange rate changes will typically depend on the source of the shock that drives them. In particular, it is important, although not easy to assess to what extent exchange rates are driven by various economic fundamentals. A number of alternative econometric time series methods are available for estimating the determinants of the real effective exchange rate of a currency. Unfortunately, due to the short period of its existence, these methods can not be readily applied to the case of the effective euro exchange rate.

In this paper we explore what can be learnt from analysing the historical behaviour of a synthetic euro, which consists of a weighted average of the currencies of countries that currently comprise the euro area (excluding Greece). In our framework, the rest of the world is proxied by the four most important trading partners of the euro area, the United Kingdom, the United States, Japan and Switzerland. Both the exchange rate of the synthetic euro and the economic variables in the rest of the world are obtained by taking a trade-weighted average of the variables in each of these four countries.

The objective of the paper is to assess both the determinants of the level of the effective equilibrium exchange rate of the synthetic euro and the uncertainty surrounding it. Such uncertainty may arise from two different sources, namely random shocks to the data and lack of robustness across methods. To deal with these two types of uncertainty a suite of models characterised by an increasing degree of ‘structure’ is employed, using the very same data set, variable definitions and sample periods across methods so that results could be more consistently compared across methods than would otherwise be the case.

We employ four different types of models. First, a simple analysis is carried out on various measures of the real effective exchange rate to test for the long-run purchasing power parity (PPP) theory, on the basis of stationarity tests. Following the rejection of PPP, a richer analysis is then conducted, attempting to link the long-run real effective exchange rate to a number of candidate explanatory variables (such as productivity, the relative price of tradable and non-tradable goods, interest rates, consumption to GDP ratio, etc.) without imposing any theoretical structure on the specifications. The best model of this kind, links the exchange rate to interest rate differentials and the relative price of non-traded vs. traded goods.
Second, a times series (VAR) model is estimated, comprising, in its simplest form, relative output, the real exchange rate and relative prices. Restrictions on the long-run properties of the model allow to decompose the real exchange rate into various permanent and temporary components. Results are not fully conclusive in the sense that not much of the permanent component of the euro exchange rate can be linked to shocks affecting other variables, even when more variables were included.

Third, a small scale model, i.e. the NATREX framework, is used. Three structural equations for consumption, investment and net trade are estimated and simulated along with equations for the accumulation of capital stock and net foreign assets. Solving the resulting system provides an estimate for the equilibrium exchange rate of the synthetic euro consistent with this model as well as some views on the sensitivity of the latter to a number of key parameters for both the euro area and the rest of the world. The estimates seem to track relatively well the synthetic euro’s historical path, with however quite some sensitivity with regard to alternative assumptions.

Finally, a small-sized macro-econometric model – namely the euro area-wide model developed at the ECB comprising 15 behavioural equations – is simulated under a range of alternative assumptions. This allows to quantify the sensitivity of the long-run real exchange rate of the euro to a variety of assumptions on e.g. the current state of the economy in terms of its deviation from equilibrium. We also document its response to demand and supply shocks. This model gives information mostly on the steady-state value of the exchange rate, which in some cases is reached only after a substantial number of years.

In sum, the general result seems to be, in line with what is usually found for other currencies, that the estimates for the equilibrium exchange rate are model-dependent and surrounded by some non-negligible uncertainty. Irrespective of the model considered, however, the real exchange rate of the euro at the end of 2000 is found to be below its (model-dependent) equilibrium value. On the other hand, the degree of undervaluation differs widely across models - from 5 % to 27% - and is also not significant for all models. Regardless, our results could also be interpreted as a sign that the various models used were simply not capable of explaining the path followed over the recent years by the real exchange rate of the euro, so that alternative explanatory variables and models should be considered for that purpose.
1. Introduction

Understanding what drives real exchange rate developments is important for monetary policy-makers, as the assessment of the macroeconomic impact of exchange rate changes will typically depend on the source of the shock that drives them.\(^2\) In particular, it is important, although not easy, to assess to what extent exchange rates are driven by various economic fundamentals. A number of alternative econometric time series methods are available for estimating the determinants of the real effective exchange rate of a currency. Unfortunately, due to the short period of its existence, these methods can not be readily applied to the case of the effective euro exchange rate.

In this paper we explore what can be learnt from analysing the historical behaviour of a synthetic euro, which consists of a weighted average of the currencies of eleven countries that currently comprise the euro area (excluding Greece). We apply four standard approaches to the case of the synthetic euro, in a framework where the rest of the world is proxied by the four most important trading partners of the euro area, the United Kingdom, the United States, Japan and Switzerland. Both the exchange rate of the synthetic euro and the economic variables in the rest of the world are obtained by taking a trade-weighted average of the variables in each of these four countries.

The objective of the paper is to assess both the determinants of the level of the effective equilibrium exchange rate of the synthetic euro and the uncertainty surrounding it. Such uncertainty may arise from two different sources, namely stochastic components in the data and lack of robustness across methods. Therefore a suite of models which are characterised by an increasing degree of ‘structure’ is employed. Moreover, using the very same data set, variable definitions and sample periods across methods, allows to focus exclusively on stochastic and model uncertainty.

First, in Section 2, a reduced-form time-series approach is taken. A stationarity analysis is carried out on various measures of the real effective exchange rate to test for the long-run purchasing power parity theory. Following the rejection of the latter, a multivariate cointegration analysis is then conducted, attempting to link the long-run real effective exchange rate to a number of candidate explanatory

\(^2\) In this context, it should be noted that the exchange rate is just one of the many indicators the ECB looks at in the context of the second pillar of its stability-oriented monetary policy strategy.
variables (such as productivity, the relative price of tradable and non-tradable goods, interest rates, consumption to GDP ratio, etc.) without imposing any theoretical structure on the specifications.

Second, a structural VAR is estimated, comprising, in its simplest form, relative output, the real exchange rate and relative prices (Section 3). Long-run identification restrictions allow to decompose the real exchange rate into various permanent and temporary components.

Third, in Section 4 the NATREX framework is used, in which case three structural equations for consumption, investment and net trade are estimated and simulated along with equations for the accumulation of capital stock and net foreign assets. Solving the resulting system provides an estimate for the equilibrium exchange rate of the synthetic euro consistent with this model as well as some views on the sensitivity of the latter to a number of key parameters for both the euro area and the rest of the world.

Finally, in Section 5 a small-sized macro-econometric model – namely the euro area-wide model developed at the ECB comprising 15 behavioural equations – is simulated under a range of alternative assumptions. This allows to assess the sensitivity of the steady-state value of the real exchange rate of the euro to a variety of assumptions, also illustrating the dynamics of the return of this indicator to some long-run equilibrium value implied by the model following e.g. a permanent public expenditure shock.

The results of applying those four methodologies to estimate the equilibrium real exchange rate of the synthetic euro are compared in the concluding Section. However, it is worth pointing out at the beginning that the notion of “equilibrium” is quite different in each of the methodologies used as well as the horizon considered in terms of when is this equilibrium value reached. This may partly drive differences in the estimated degree of over- or undervaluation across methodologies (in addition to those differences deriving from model and parameter uncertainty). In Section 2, the estimated equilibrium real exchange rate is based on an estimated long-run cointegrating relationship of the real exchange rate with its fundamentals such as the terms of trade and relative productivity differences. However, as the fundamentals themselves may not be at their equilibrium value, this can hardly be interpreted as an equilibrium from an economic point of view. Moreover, when a gap arises, it is not always clear which of the variables (the real exchange rate or the fundamentals) will adjust to correct the long-run equilibrium.

In the structural VAR analysis of Section 3, the equilibrium real exchange rate is defined as that part of
the movements in the real exchange rate that is driven by identified real structural shocks such as 
productivity, labour supply or government spending shocks that can have a long-run impact on the real 
exchange rate. This notion of equilibrium exchange rate mainly differs from the previous one in that the 
equilibrium real exchange rate also includes the short-term adjustment dynamics to such permanent 
structural shocks. Finally, in Sections 4 and 5 a more economic interpretation of the notion of equilibrium 
is maintained. In the Natex approach, a distinction is made between the medium run equilibrium 
exchange rate, which would prevail if the economy were in external and internal equilibrium (equivalent 
to the macro-economic balance approach), and the long-run equilibrium exchange rate, which would hold 
in a steady-state with a constant net foreign debt and capital stock. A similar concept is used in Section 5.

2. A cointegration reduced-form approach to PPP

The aim of this section is to possibly estimate, on the basis of cointegration techniques, a reduced-form 
model for the real effective exchange rate of the synthetic euro and to compute a corresponding 
confidence interval around the estimated ‘long-run’ value. First, a simple Purchasing Power Parity (PPP) 
model is employed, where a number of alternative measures of the real effective exchange rate are tested 
for stationarity, thereby assessing whether PPP holds in the long run (Section 2.1). Then, a series of 
candidate explanatory variables are discussed, referring broadly to theories which may explain persistent 
deviations from PPP. There is, however, no intention to directly test those theories following an approach 
similar to, e.g., Mc Donald [1996]. Cointegration tests are conducted, both on a pair-wise basis and on a 
multilateral basis, linking the deviation from PPP to alternative explanatory variables (Section 2.2).

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3 An important caveat in this respect is that the commonly used term ‘equilibrium’ in the context of cointegration can hardly be 
interpreted as such in theoretical terms. The concept here is purely statistical, to the extent that e.g. the derived equilibrium 
depends on values for the explanatory variables that may not be at their equilibrium value from an economic viewpoint. In 
sections 4 and 5, in contrast, the concept of equilibrium employed is more interpretable in economic terms, to the extent that 
the exchange rate is obtained there by solving systems of structural equations under various assumptions.
2.1 Trade vs. broadly based real effective exchange rates and PPP tests

A number of different measures are commonly used to compute real effective exchange rates. In this section, we review the available indicators, analyse their stationarity properties and, in the event they are not, check whether they are cointegrated. This analysis allows us to assess whether the PPP theory holds in the long run and will provide useful information in deciding which indicator to use subsequently.

In all cases, the real effective exchange rate of the synthetic euro is computed against an aggregate Rest of the World (RoW) comprising the United Kingdom, the United States, Japan and Switzerland (covering approximately 50% of the trade of the euro-area with the RoW). Exchange rates, and other variables, have been aggregated using trade-based weights. Data are quarterly, covering the period 1973q1 to 2000q4.

2.1.1 Trade based real exchange rate and PPP

A first type of standard real exchange rate indicators, i.e. trade-based indicators, are typically used to analyse the competitiveness of an economy or to assess the extent to which the law of one price, or PPP, holds for traded goods. One such measure is \( EPX = E \cdot \frac{P_X}{P_X^*} \) where \( E \) is the effective nominal exchange rate (an increase denoting an appreciation), \( P_X \) export prices, and \( * \) denotes the RoW variables.

If PPP holds, the euro area export price \( P_X \) should converge to the corresponding RoW price expressed in euros, i.e. \( \frac{P_X^*}{E} \). However, the non-stationarity of \( EPX \) cannot be rejected, even with a break in the mean of the series. Chart 2.1 provides some visual evidence of this. Although this measure of the real exchange rate is not trended, its level shifts over time with a marked trough before the mid-eighties. This pattern reflects the strong appreciation of the US dollar in the early 1980s and its subsequent reversal.

Beyond measuring deviations from PPP, an alternative way to interpret this trade-based real exchange rate indicator is to consider it as a proxy for the external terms of trade, i.e. \( \frac{P_X}{P_M} \), where \( P_M \) is the euro area import price. This will obviously be the case if \( \frac{P_X^*}{E} \) is close to \( P_M \). Cointegration tests indeed confirm that the two concepts are related in the long-run. However, short-run deviations between the two

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4 Tests carried out were standard Dickey-Fuller, Augmented Dickey Fuller, and Perron-Vogelsang unit root tests, with unknown break dates. In all cases, tests were systematically run with one and four lags.

5 The maximum value of the Perron statistic occurs for a break in the level of the \( EPX \) series in 1985q3, with a t-statistic at –2.7 for an AR(1) and –2.7 for an AR(4), hence pointing to non-stationarity even in the presence of a break. This break could be related to the accession of Spain and Portugal.
indices can be quite large, in particular because commodity prices do not enter the $EPY$ measure, while they affect developments in the terms of trade through their large and specific effect on import prices (as can be seen on Chart 2.2).\footnote{A further test, for cointegration between the euro area external and internal terms of trade, namely the ratio of GDP to consumption prices, leads to a rejection. Since $P_T / P_M = P_{P/Q} / P_C \Rightarrow P_{P/Q} / P_M \neq P_{P/Q} / P_C$, the relative price of non-traded goods in consumption ($P_C / P_M$) and production goods ($P_T / P_M$) differ. This finding is consistent with the intuition that traded goods in both consumption and imports comprise products that are neither exported nor produced in the euro area, such as oil.} Irrespective of the interpretation favoured, it seems that no clear notion of equilibrium value for the real exchange rate can be derived from this trade-based PPP approach.

\subsection*{2.1.2 Broadly based real exchange rates and PPP}

A second type of real exchange rate indicators and PPP tests is more broadly based in the sense that it makes use of price indices which also involve non-traded goods, such as the GDP deflator ($EPY = E . P_Q / P_Q^*$) or the consumption deflator ($EPC = E . P_C / P_C^*$). In terms of PPP, the results for these two additional indicators are quite similar. Even allowing for a break in mean, most likely to have occurred in 1980q1, both the consumption-based and the GDP-based indices are clearly I(1), so that no constant equilibrium value for the euro can be derived from these indicators.\footnote{The Perron statistic for $EPC$ and $EPY$ reaches its minimum for a break in 1980q1 at $-2.5$ for an AR(1) and $-2.5$ for an AR(4).}

Moreover, as Chart 2.3 shows, while the two series exhibit similar cyclical behaviour, their underlying trends differ. The ratio between $EPY$ and $EPC$ is readily interpreted, being by construction equal to the relative internal terms of trade between the euro area and the RoW: $EPY / EPC = (E . P_Q / P_Q^*) / (E . P_C / P_C^*) = (P_Q / P_C) / (P_Q^*/ P_C^*)$. Also this gap is non-stationary, reflecting the fact that shocks to external terms of trade can have persistently different effects across countries. After the mid-eighties, however, the gap between the two broadly based real exchange rates seems to have stabilised.\footnote{Some comparison is finally required between the two measures, trade and broadly based. Chart 2.1 shows the gap between the GDP and the export based indicator. The gap is, by construction, equal to the ratio between relative prices of non-traded goods in the two economies: $(E . P_Q / P_Q^* ) / (E . P_X / P_X^*) = (P_Q / P_X) / (P_Q^* / P_X^*)$. This ratio appears non-stationary, although the hypothesis of a stationary process with a shift in mean in 1980q1 can be accepted. This indicates that, in line with visual inspection, the gap}
between the two real exchange rates has stabilised, once the GDP based real exchange rate had
depreciated more than the trade based one.\footnote{Both relative internal and external terms of trade experience a major break at the same date, in 1985q3, but the hypothesis of non-stationarity can more clearly be rejected for the internal ones. Perron statistics are \(-5.4\) for both the AR(1) and AR(4) for the internal measure, whereas the corresponding figures are only \(-4.2\) and \(-3.2\) for the external one. As a result, only tests allowing for two breaks, in both 1980 and 1985, could make it possible to reject non-stationarity of EPY and EPC.}

In the light of this analysis, since PPP has experienced major level shifts in 1980 and 1985, it seems
appropriate to interpret the resulting pattern using additional explanatory elements, beyond the already
mentioned terms of trade. It seems also warranted to focus on a broadly based indicator, so as to avoid
limiting the analysis to traded goods; in any event the gap between the two types of measures appear
rather stable over time. As to the choice between consumption and GDP based indicators, the latter seems
more attractive to the extent that the traded good component involved is less affected by commodity
prices. Moreover, in the long run, the GDP based indicator behaves more in line with the trade based one.

### 2.2 A general reduced form model for the real exchange rate

In this section, a number of variables are discussed that may explain the non-stationary behaviour of the
real exchange rate, along lines very similar to the so-called Behavioural Equilibrium Exchange Rate
approach advocated by Clarke and Mc Donald [1999]. As both the various exchange rate measures and
each of the explanatory variables could be considered as I(1) processes, cointegration analysis will be
employed.

#### 2.2.1 The specification employed

The general model employed reads:

\[
\text{EPY} = \text{function (NTTD, Prod, Wage, SAV, RIR, CAX, NFX)}.
\]

All variables are in relative terms, euro area over RoW. The explanatory variables are discussed below.\footnote{The minimum Perron statistic for the relative $P_0/P_X$ series is \(-4.2\) for a break in 1980q2, for an AR(1) and \(-4.2\) for an
AR(4). Since the previous analysis suggested that EPY had presumably experienced breaks in both 1980q1 (like EPC) and
1985q3 (like EPY / EPC), it then may be said that EPY and EPX are co-breaking around 1985q3 – although no formal test
has been conducted for that hypothesis. Finally both break points envisaged coincide with marked upward and downward
shifts in the real price of oil, pointing to a specific role of such terms of trade shocks for the real exchange rate of the euro.}

\footnote{Using the Perron test, \text{PROD}, \text{WAGE} and \text{SAV} could be considered as I(0) with a level break whereas \text{CAX} and \text{NFX} were
found to be I(1) even allowing for a shift in the mean. The results for RIR were less clear-cut with ADF tests rejecting a unit
root. For the analysis all variables were assumed to be I(1).}
NTTD is the relative price of non-traded to traded goods. Various theories, as recalled in Mc Donald [1996], of which the most well-known is perhaps the Balassa-Samuelson model, suggest that higher productivity growth in the traded goods sector should lead to both a relative increase in the relative price of non-traded goods and an appreciation of the real exchange rate.\textsuperscript{11} However, as shown e.g. in Asea and Mendoza [1994], deviations from PPP may be explained only at the margin by such considerations.

PROD denotes productivity and is measured as the apparent productivity of labour, i.e. GDP per employment. Obviously, to the extent that higher productivity also leads to an increase in the relative price of non-traded goods, both measures could capture similar trends (as in Hsieh [1982]).

WAGE is the real wage measured as the nominal wage deflated by the GDP deflator. This variable can be considered as an alternative measure of the previous variable. With a Cobb-Douglas production function, the real wage will equal both the marginal and average productivity of labour.

SAV is social savings, i.e. GDP minus private consumption and government consumption as a ratio to GDP. One way to consider this variable is in the context of a stock-flow approach in which net foreign asset increases would be facilitated by higher savings, thereby leading to a real appreciation, as suggested in Clark et al. [1994]. According to e.g. Stein [1999, 2001], this variable can also be a proxy for the degree of time preference, so that a higher savings ratio should lead to an appreciation of the exchange rate – although in the short run, the reverse could be observed.

RIR is the long-term real interest rate. In steady state, the real interest rate differential should be positively correlated with changes in the real exchange rate. It should be noted that finding a non-stationary real interest rate differential could be due to the short sample used or to the existence of some time-varying risk premia, to the extent that relative inflation is I(0).

CAX is the current account as a ratio to GDP. In a stock-flow equilibrium there is clearly a dynamic link between savings, the current account and net foreign assets. These relations are the key element on which, e.g., the approach of Clark et al. [1994] is based.

\textsuperscript{11} This variable is proxied, as described in the previous section, by the ratio EPY/EPX, then capturing a level shift around 1980.
NFX is net foreign assets as a ratio to GDP. This variable is proxied by accumulating the current account and therefore neglects valuation effects. In econometric terms, this construction seems to have the inconvenience of introducing further unit roots in the underlying process.

2.2.2 Estimation results and equilibrium value

A first set of tests has been conducted to assess bilaterally the relation between the GDP based exchange rate and the candidate variables. A second round of tests has been run using a ‘system approach’ where more than one variable were included in the regression.\(^\text{12}\)

On the basis of OLS estimates, in the bivariate case, all variables have the expected positive signs, except social savings. Of course, since the variables employed are possibly I(1) processes, such correlation may be spurious. More rigorously, using cointegration techniques (FME of Phillips and Hansen [1990] and ECM-VAR à la Johansen [1991] with two variables), a borderline significantly positive correlation is found to remain only for productivity and wages.\(^\text{13}\)

In the multivariate VECM framework with more than two variables, all possible different combinations of the variables were tested. Results are reported only if either the Johansen Trace or Max tests were passed. In sum, most systems did not deliver any cointegration relations, and coefficients were hardly significant.

Under the hypothesis that \(\text{Rank} = 1\), two systems delivered coefficients in line with expectations: first, a system comprising \(\text{EPY}\), productivity, the relative price of non-traded goods, and the interest rate differential (coefficients were not all significant); second, a system consisting of the same variables without productivity. Using the AIC criteria, the ‘best’ regression was found to be with the relative non-traded prices and the interest rate differential. This yields the following cointegrating relationship:\(^\text{14}\)

\[
\text{EPY} = 4.48 + 1.76 \text{NTTD} + 0.07 \text{RIR} \\
(0.03) (0.70) (0.02)
\]

\(^{12}\) Two different samples are used, first, from 1973q1 on for Savings, Productivity, Wages, Relative price of non-traded goods, and the Interest rate differential; second, from 1977q1 on when Current account or Foreign asset variables are introduced.

\(^{13}\) The weak significance is confirmed by the fact that on visual inspection the gap between the estimated long-run value and the actual exchange rate is very persistent. Using \(\text{EPC}\) the results are even less favourable to priors, since only savings and the traded-non-traded prices are both significant and with positive sign. The other variables show a negative sign contrary to expectations.

\(^{14}\) Reported standard errors have been bootstrapped. Similar results are obtained for \(\text{EPC}\).
Under the hypothesis $Rank = 2$, as usual when more than one cointegration relation is found, interpretation problems were faced. Although a number of combinations were found to have rank 2, e.g. involving $EPY$ with Wage with the interest rate differential, in many cases signs were opposite to expectations or the variables were not significant providing a very low fit\textsuperscript{15}. Chart 2.4 plots the value of the real exchange rate implied by the above mentioned cointegration relation. The latter provides a relatively satisfactory fit of the historical values of the real exchange rate of the euro. The preferred model is consistent with a large impact of the interest rate differential (see e.g. Edison and Melick [1999]), at odds with equilibrium considerations. On the other hand, it also suggests a strong role for terms of trade in explaining level shifts in the real exchange rate (as already pointed out e.g. by Maeso Fernandez et al. [2001]). On this basis, the real exchange rate of the euro appears slightly under its long-run value over 1999 and 2000, being undervalued by around 10% in 2000q4. However, bootstrapped 80% confidence intervals at the end of the sample yield bounds of $\pm$ 8%, so that this undervaluation is only borderline significant.

\textsuperscript{15} This finding seems to be because RIR is stationary.
Chart 2.3: Real effective exchange rates, consumption and GDP prices

Chart 2.4: Real Exchange Rate function of relative terms of trade and interest rate differential
3. A structural VAR model of the synthetic euro.

In Section 2 we have analysed the time series properties of the various measures of the real effective exchange rate of the synthetic euro. This analysis has shown that in agreement with the literature, it is difficult to reject the unit root hypothesis on the basis of univariate tests. Following this finding, Section 2 has also examined the long run relationship of the real effective exchange rate of the synthetic euro with various potential fundamental determinants. In general, the findings are discouraging in the sense that no strong cointegrating relationship has been found with these fundamentals. This suggests that at least part of the non-stationary behaviour of the real exchange rate cannot be explained by stochastic trends in the observed fundamentals. In this Section we examine how much of the long-run behaviour of the real exchange rate is left unexplained. In addition, we also analyse how much of the short-run developments in the real exchange rate can be explained by underlying real factors, such as preferences and technology, or nominal factors such as asset market disturbances. In order to do so, in Section 3.1, we apply the structural VAR methodology used by Clarida and Gali [1994] (CG). Section 3.2 discusses an extension of the CG framework. Finally, Section 3.3 reports SVAR-based estimates of equilibrium real exchange rates.

3.1 The CG model

CG estimate a trivariate VAR model consisting of the change in relative output levels, the change in the real exchange rate and a relative inflation rate. In order to identify the three structural shocks (a permanent supply shock, a permanent real demand shock and a nominal shock) to this VAR system, CG use a long-run triangular identification scheme proposed by Blanchard and Quah [1989]. The nominal shocks are identified by assuming that such shocks do not affect real variables (i.e. the real exchange rate or relative output) in the long run. Both supply and real demand shocks are expected to influence the real exchange rate in the long run, but only supply shocks can affect relative output levels in the long run. These restrictions are based on the long-run behaviour of a modified version of the Mundell-Fleming-Dornbusch model, which exhibits a vertical long-run supply curve and long-run neutrality of money. As

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16 See, for example, the survey of the literature in McDonald and Stein [1999], p.24.
discussed in Section 5, such long-run behaviour also characterises the Fagan et al. [2001] area-wide model.

In order to check whether the identification is successful, it is worth recalling the expected sign patterns of the three shocks on output, the real and nominal exchange rate and the price level. A permanent real demand shock, such as a permanent increase in real government spending, should permanently appreciate the currency in nominal and real terms, increase the price level and boost output in the short run. A positive supply shock produces a permanent rise in output, a fall in prices and may lead to a nominal and real depreciation of the currency. If the positive supply shock increases domestic real wealth and consumers have a home bias in consumption, such a shock may be accompanied by an upward shift in the aggregate demand curve, which would tend to reduce the depreciation of the currency. Finally, a purely nominal shock will, with sticky prices, lead to both a nominal and real depreciation of the exchange rate. Such a shock is also likely to produce a rise in the price level and a, perhaps, transitory effect on output.

Chart 3.1 reports the impulse responses of relative output and prices, and the nominal and real effective exchange rate to each of the three structural shocks. The data set is the same as in Section 2. The output measure we use is real GDP, whereas the price index used is the GDP deflator. The VAR system contains four lags and is estimated over the full sample period 1970q1 to 2000q4.

The results are not fully convincing. First, a positive supply shock does lead to a significant permanent rise in output and a fall in prices. However, in contrast to expectations the response of the real exchange rate is insignificant, whereas the nominal exchange rate appreciates. As discussed above, an insignificant response of the real exchange rate may be due to the upward shift in demand following the positive supply shock and its effect on real wealth. The result that a supply shock leads to an insignificant appreciation of the real exchange rate was also found by Clarida and Gali [1994] and McDonald and Swagel [1998] who focus on various currency pairs (including the US-dollar-DM exchange rate) over a somewhat shorter estimation period.

Second, a real demand shock leads to both a real and nominal permanent appreciation of the exchange rate. However, in contrast to expectations, this shock has no significant effect on relative output and only
a very temporary positive effect on relative prices. These results are contrary to most of the literature that applies similar models.\textsuperscript{18} It is consistent, however, with the results for the US dollar-DM bilateral exchange rate reported in Weber [1998]. As in our case, he finds that the relative demand shock has no significant effect on output, which presumably it should have if it is to serve any purpose in representing a demand shock. Weber [1998] concludes that the real demand shock is a “catch-all” term which reflects what is left of real exchange rate movements that cannot be forecast from the other variables in the system.

**Chart 3.1. Impulse responses: Clarida-Gali model**

\textsuperscript{17} See, for example, Bayoumi and Eichengreen [1994] for a similar reasoning.

\textsuperscript{18} See, for example, the survey in McDonald and Stein [1999], p. 44-48.
Note: 90% confidence bands

Finally, the nominal shock leads to a significant depreciation of the nominal exchange rate and a corresponding rise in relative prices. It does have a significant temporary positive effect on output, but no effect on the real exchange rate. In light of the underlying structural interpretation, it is somewhat puzzling that in spite of the rapid pass-through into prices, the temporary effect on output is so strong.\(^{19}\)

Table 3.1 reports the forecast error variance decomposition of the main endogenous variables. As suggested by the discussion above, we find that at all horizons more than 88% (71%) of the variability in the real (nominal) exchange rate of the countries comprising the euro area is driven by so-called real demand shocks. At the same time, those shocks do not explain any significant portion of the variability in relative output and prices beyond the one year horizon. This “orthogonality” of the real exchange rate with respect to the relative economic conditions in the two areas, shows that the CG model estimated on synthetic euro area data is not very capable of accounting for changes in the real exchange rate of the synthetic euro.\(^{20}\)

### 3.2 An extension of the CG model

One reason for the failure of the CG model to explain changes in the real exchange rate may be that the model is too simple. In particular, the aggregate supply and demand shocks are likely to be a mixture of a large number of different shocks. If these have a different impact on output, prices and the exchange rate, their effects in the aggregate may cancel out. In this section we therefore extend the trivariate model to include three additional endogenous variables. In particular, we augment the basic CG model with relative employment, the difference in the ratio of government consumption over GDP and the long-term nominal interest rate differential to identify three additional structural shocks: a labour supply shock, a government consumption shock and an inflation expectations shock. Identification is again achieved

---

\(^{19}\) These results are basically unchanged when we shorten the sample period (1980q1-2000q4)

\(^{20}\) To check whether some of these negative results may be due to the aggregation bias that may result from lumping countries with historically very different monetary policy regimes together, we also estimated the basic CG model for Germany. The differences are striking. In this case, all the signs are as expected. In particular, a permanent real demand shock has significant positive effects on output, prices and the real and nominal exchange rate, while a nominal shock has a significantly negative effect on the real exchange rate. As a result almost half of the contemporaneous forecast errors in the real exchange rate can be accounted for by nominal shocks. Of course, in the medium run, real demand shocks still dominate real exchange rate developments.
through a Choleski decomposition of the long-run covariance matrix as in Blanchard and Quah [1989] and CG.

Following Shapiro and Watson [1988] and Weber [1997], relative employment is ordered first in order to identify a labour supply shock. This is motivated by the assumption that only permanent shocks to the labour supply can affect employment in the long run. Other supply shocks, like productivity shocks, can have a permanent effect on output, but not on employment.

One particular source of permanent real demand shocks are changes in real government consumption. To examine the contribution of such shocks to developments in the real exchange rate, we include the ratio of real government consumption to GDP. Following relative employment and output, this variable is ranked third. This reflects the identifying assumption that permanent changes in the ratio of government consumption to GDP can not have a permanent impact on real GDP or employment; i.e. in the long run increases in government consumption fully crowd out other components of spending. This is in contrast to Rogers [1999], who orders the government consumption ratio before relative output. It turns out that reversing this ordering does not affect the results in any significant manner.

As a final extension, we include the nominal long-term interest rate differential. The apparently non-stationary behaviour of this variable during the sample period may reflect various factors. One possible explanation may be a time-varying risk premium. An alternative hypothesis may be that the persistent changes in the interest rate differential reflect changes in relative inflation expectations.\(^{21}\) The implications of both hypotheses for the response of output are quite different. If long-term interest rates rise due to an increased risk premium, then this should have a negative and possibly permanent impact on output. In contrast, if long-term interest rates rise in anticipation of a booming economy and rising prices, then it could be associated with a positive, but temporary effect on output and a positive effect on prices. It turns out that empirically the second hypothesis turns out to be true. We therefore order the long-term interest rate differential fifth (just before relative prices) in the extended VAR system.

\(^{21}\) Note that such permanent changes in inflation expectations are not in line with the actual behaviour of relative inflation over the sample period. Recall from Section 2 that relative inflation appears to be stationary, whereas a unit root in the long-term interest rate differential can not be rejected. This is a clear violation of the Fisher hypothesis. An alternative hypothesis is that the rise in interest rates reflects a permanent positive shock to the productivity growth differential, but this is not consistent with the stationarity of the growth differential.
Chart 3.2: Impulse responses: extended Clarida-Gali model

Note: 90% confidence bands

Chart 3.2 reports the impulse response functions. As expected, both labour supply and productivity shocks lead to a permanent positive increase in output and a fall in relative prices (first two columns of Chart 3.2). Positive productivity shocks permanently increase labour productivity and temporarily reduce employment. In contrast to the CG model in Section 3.1, these shocks now have a significant positive effect on the real or nominal exchange rate. The labour supply shocks have no significant effect on labour productivity or the nominal and real exchange rate. This suggests that the appreciation of the real exchange rate following a positive supply shock in Section 3.1 is mostly due to productivity shocks.
The qualitative effect of an increase in government consumption on the other variables in the system is as expected (third column). A positive government consumption shock temporarily stimulates relative output and employment. It also leads to a rise in prices, an increase in inflation expectations as captured by the long-term interest rate differential and an appreciation of the nominal and real exchange rate. However, overall these impulse responses are not very significant. The effects of the real demand shock are very similar to those in the CG model. Finally, the two nominal shocks (fifth and sixth column) have very similar qualitative effects. An increase in nominal long-term interest rates is associated with a positive impact on output, employment and prices. In response, the nominal exchange rate depreciates, but not very significantly. The real exchange rate basically remains unchanged.

Table 3.1. reports the forecast error variance decomposition of the extended model. Compared to the CG model, both supply shocks (the labour supply and the productivity shocks) now explain a larger fraction of the five-year ahead forecast error variance of the real exchange rate. However, the overall qualitative conclusion does not change. Real demand shocks explain the bulk of the real exchange rate variance, but have only negligible effects on the other endogenous variables. In other words, there is still a disconnection between the real exchange rate and the fundamentals of the euro area economy.

**Table 3.1.**

Forecast Error Variance Decomposition at 5 year horizon

<table>
<thead>
<tr>
<th></th>
<th>Labour supply shock</th>
<th>Productivity shock</th>
<th>Government consumption shock</th>
<th>Real demand shock</th>
<th>Inflation expectations shock</th>
<th>Nominal shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPY</td>
<td>CG</td>
<td>- 11</td>
<td>-</td>
<td>89</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>ECG</td>
<td>8</td>
<td>6</td>
<td>9</td>
<td>78</td>
<td>0</td>
</tr>
<tr>
<td>EEN</td>
<td>CG</td>
<td>- 27</td>
<td>-</td>
<td>70</td>
<td>-</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>ECG</td>
<td>17</td>
<td>18</td>
<td>4</td>
<td>63</td>
<td>1</td>
</tr>
<tr>
<td>P₀</td>
<td>CG</td>
<td>- 63</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>ECG</td>
<td>37</td>
<td>19</td>
<td>8</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Q</td>
<td>CG</td>
<td>- 94</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>ECG</td>
<td>61</td>
<td>30</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

Notes: EPY is the real effective exchange rate; EEN is the nominal effective exchange rate; P₀ is the relative GDP deflator; Q is relative GDP. CG is the basic Clarida-Gali model of Section 3.1. ECG is the extended Clarida-Gali model of Section 3.2.
Chart 3.3: Equilibrium real exchange rate

Note: Black line is the actual real effective exchange rate (using GDP deflators); dashed lines are the estimate (and 80% confidence band) of the SVAR-based equilibrium real exchange rate.

3.3 SVAR-based estimates of the equilibrium real exchange rate

On the basis of the structural VARs estimated in Sections 3.1 and 3.2, one can define the equilibrium real exchange rate as the historical component of the real exchange rate driven by the identified real supply and demand shocks: in particular the labour supply, the productivity and the government consumption shock. We exclude the so-called real demand shock from this list because, as discussed above, this shock, while explaining the bulk of the movements in the real exchange rate, does not appear to be related to the underlying fundamentals of the economy. Graph 3.3 shows estimates of the equilibrium real exchange
rate based on both the CG and the extended CG model for two sample periods: the full sample period (1970-2001) and a shorter sample period starting in 1980. In each of those cases the actual real exchange rate (solid line) is undervalued compared to its SVAR-based equilibrium level at the end of the sample period (2000:Q4). The degree of undervaluation differs, however, from –5% to –12%. The confidence bands around the estimates are quite wide and only in the latter case (CG model estimated over the period 1980-2000) is the degree of undervaluation significant.

4. The Natrex approach

A further step towards increasing the structure underlying the estimated model is to estimate a number of behavioural relations as commonly found in standard structural macroeconometric models. In this Section we consider a small-scale model based on the Natrex approach (NATural Real EXchange rate), originally formulated by Stein [1994]. The approach tries to link the real exchange rate to a set of fundamental variables explaining savings, investment and the current account. Natrex is based on a rigorous modelling of the stock-flow interaction in a macro-economic growth model. A distinction is made between a medium run equilibrium where external and internal equilibrium prevails (equivalent to the macroeconomic balance approach) and the long-run equilibrium with a constant net foreign debt and the capital stock at its steady state level. In fact, Natrex’ contribution is in providing the trajectory of the medium term equilibrium to its long-run equilibrium.

To our knowledge this is the first truly structural estimation of the Natrex model, which has so far only been tested in its reduced form. 22

4.1 The Natrex model

Our model is defined by the following identity and behavioural equations.

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22 The reader is referred to Detken and Marin [2001] for details. Gandolfo and Felettiagh [1998] and Verrue and Colpaert [1999] come close to a structural estimation. But the former ignore the long-run capital and debt accumulation equations as well as the investment and consumption structure of their model, while the latter exclude the exchange rate from their structural system and estimate it separately via a reduced form.
National account identity:

\[ \frac{C}{Q} + \frac{I}{Q} + \frac{TB}{Q} = 1 - \frac{SCN}{Q} \]  

(1)

Three behavioural equations:

\[ \frac{I}{Q} = \alpha_1 + \alpha_2 \hat{A} - \alpha_3 K \frac{Q}{Q_{t-1}} - \alpha_4 r^L_{t-3} - \alpha_5 R_{t-4} \]  

(2)

\[ \frac{C}{Q} = \alpha_6 + \alpha_7 K \frac{Q}{Q_{t-1}} - \alpha_8 F \frac{Q}{Q} - \alpha_9 r^S + \alpha_{10} i^L \]  

(3)

\[ \frac{TB}{Q} = -\alpha_{11} - \alpha_{12} R - \alpha_{13} C \frac{Q}{Q_{t-4}} + \alpha_{14} \frac{C^*}{Q_{t-4}} + \alpha_{15} \text{tot} \]  

(4)

Real uncovered interest rate parity and Fisher equation:

\[ R - R^m = r - r^* \]  

(5)

\[ (i - i^*) = (\pi - \pi^*) + (r - r^*) \]  

(6)

Stock accumulation:

\[ F - F_{t-1} = -CA \]  

(7)

\[ K = (1 - \delta) K_{t-1} + \frac{I}{P_t} \]  

(8)

Steady State of stock variables:

\[ \frac{F}{Q} = -\left(1 + q\right) \frac{CA}{q} \]  

(9)

\[ \frac{K}{Q_{t-1}} = \frac{1 + \delta}{\delta + g} \frac{I}{P_0} \frac{P_t}{P_{t-1}} \]  

(10)

TB is the trade balance, I is gross (public plus private) investment (excluding inventories), C public plus private consumption, SCN the variation in stocks. Nominal and real interest rates are labelled i and r, respectively, and the superscripts L, S and * stand for long-term rates, short term rates and foreign variables. Q and Q_t are nominal and real GDP, K is the real capital stock. A is a global productivity
factor$^{23}$ where ^ is the notation for the annual growth rate. $F$ is net foreign debt as resulting from accumulated current account balances. The terms of trade are labelled $tot$ and are measured as the price of exports divided by the price of imports. $R$ is the real (deflated by GDP prices) effective exchange rate with respect to the four largest trading partners and with superscript $eq$ the equilibrium rate. A higher value for $R$ indicates an appreciation. Expected inflation is equal to actual inflation and denoted by $\pi$. The parameter $\delta$ is the quarterly rate of depreciation of the capital stock, $q$ is the nominal and $g$ the real quarterly GDP growth rate. $P_Q$ and $P_I$ depict the GDP and the investment price index, respectively. The estimated coefficients $\alpha_i$ to $\alpha_{15}$ are written in such way that they are all expected to be positive$^{24}$.

Equation (2) shows that investment depends positively on the growth rate of productivity, which is the annual change in the Solow residual of a global factor augmenting production function. The negative sign of the capital stock in the investment equation proxies for decreasing marginal productivity of capital and secures the convergence of the capital stock to a stable ratio to GDP. Note that not only a higher real long-term interest rate but also a stronger exchange rate crowds out domestic investment. The reason is that in a multi sector economy with tradable and non-tradable goods the relative price of different goods (proxied by $R$) will affect the marginal productivity of capital$^{25}$.

The capital stock ratio as well as the foreign debt ratio are proxies for wealth in the consumption function, equation (3). The real short-term interest rate negatively affects consumption, for given time preferences, which is the standard result from optimal control. The nominal long-term interest rate enters positively as it should depict exogenous consumption preferences related to the state of the business cycle.

Equation (4) describes the behaviour of the trade balance, which improves with a weaker real exchange rate and reacts in a standard way to (lagged) measures of domestic and foreign demand. The terms of trade variable captures a short-term (J-curve-type) price effect, as the trade balance improves when export prices rise relative to import prices.

$^{23}$ The exogenous productivity variable, $A$, is global factor augmenting, i.e. $Y = AK^\delta L^{1-\delta}$. $A$ is the residual of regressing $K$ and $L$ on $Y$ in log first differences.

$^{24}$ The exception is $\alpha_{11}$ on which sign we do not have any prior.

$^{25}$ For a detailed microfoundation of the behavioural equations see Marin-Martinez [2001].
Real uncovered interest parity and the Fisher equation(s), reveal that in equilibrium, first, domestic and foreign real interest rates will be equal and that, second, we assume perfect foresight expectations (equations (4) and (5)).

Equation (7) states that the change in the level of foreign debt is equal to the current account balance. The real capital stock will adjust due to depreciation and new investments as depicted in equation (8).

Equations (9) and (10) reveal the steady state foreign debt ratio and the steady state capital stock ratio to be functions of the current account and investment to GDP ratios, respectively.

Theoretically the most important exogenous factors within the Natrex concept are a thrift parameter, measuring the preference for consumption (here the nominal long-term interest rate in the consumption equation), and productivity (the growth rate of $A$ in the investment equation). The other exogenous variables are $\text{SCN/Q}$, $r^*$ (r for medium-run equilibrium), $C^*/Q^*$, tot, $\delta$, $P_Q$, $P_r$, $q$, $g$, as well as $K/Q$, and $F/Q$ for the medium-run equilibrium and net factor income.

The exogeneity of net factor income deserves some discussion. The current account is the sum of the trade balance and net factor income. If net factor income is $-i^*F_{t-1}$ then the steady state condition for the foreign debt ratio would be $F/Q = (1+q)/(i^*-q) \frac{TB}{Q}$. In a dynamically efficient economy the interest rate would exceed the nominal growth rate and thus the trade balance would be positively related to the steady state foreign debt ratio. Thus the higher the net foreign debt position and the associated net foreign interest outflow, the more positive the trade balance must be to secure a constant current account ratio to GDP in the steady state. In equation (9) the trade balance is negatively related to the steady state foreign debt ratio, due to the fact that net factor income ratio to GDP is treated as an exogenous variable. While letting net factor income endogenously adjust to changes in the net foreign asset position is theoretically the correct thing to do, we found this relationship in the euro area data with a wrong sign. The empirical failure of this relationship can be due to several reasons, like unaccounted valuation effects on outstanding assets, dynamically inefficient economies or simply aggregation effects of euro area country data. Two more recent arguments support our finding. First, Lane and Milesi-Ferretti [2001] investigate

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26 When approximating net factor income by $-i^*F_{t-1}$, it turns out to have the opposite trend than the true historical net factor income series.
the relationship between average net foreign asset (nfa) positions and net factor income (nfi). For the period 1983-98 they find that for all our four rest of the world countries as well some euro area countries (Germany, the Netherlands, Greece and Portugal) net factor income has a sign opposite to that of the prevailing net foreign asset position. The authors show that the reason is that for these countries the real rate of return on foreign assets exceeded the real return on foreign liabilities for debtor countries and vice versa. Second, the current account might rather be a poor variable to model changes in the net foreign debt position due to autonomous movements in foreign direct and portfolio investment flows. In light of these empirical findings and the lack of more specific data on asset and liability returns, we decided to neutralise the influence of the net foreign asset position on net factor income. While the unconventional negative cross-country correlation between net foreign assets and net factor income reported by Lane and Milesi-Ferretti points out the need for more detailed data, we are aware that in the long-run a positive time series correlation between net foreign assets and net factor income should nevertheless hold. The modelling decision taken here thus rather reflects the empirical characteristics of the sample period under investigation than a theoretical innovation.

4.2 Estimating the model

We estimate separate Vector Error Correction (VEC) models for the three behavioural equations (2), (3) and (4). All variables considered are integrated of order 1. The data set begins in Q1 1970 and ends in Q4 2000. The start of the estimation period for the three equations has been determined by the number of lags in the VEC model. The upper part of Table 4.1 shows that we found one co-integrating vector for each behavioural equation. The coefficients $\alpha_1$ to $\alpha_{15}$ are derived from the co-integrating vector of the respective ECM model. Note that Table 4.1 shows the co-integration vector. Thus to obtain the sign of the relationship between the (left hand side) variable normalised to one and any other variable in the

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28 Using different lags in the co-integration vector is admittedly not standard. But theoretically the estimated coefficients for the long-run relationship should not be influenced by any lag specification. In practice, the only additional restriction made is to exclude some lagged difference terms in the VEC model. The choice of lags is data driven although some lagged response of investment and the trade balance is also theoretically a reasonable thing to assume. The results without any lag structure are relatively similar.
Table 4.1 Johansen Cointegration Test and VEC Estimation

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<tr>
<th></th>
<th>I/Q Equation</th>
<th>C/Q Equation</th>
<th>TB/Q Equation</th>
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<tr>
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<td>(C,1 to 4)</td>
<td>(C,1 to 3,7,8)</td>
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<td>No trend in data</td>
<td>No trend in data</td>
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<td>Intercept in CE</td>
<td>Intercept in CE</td>
<td>Intercept in CE</td>
</tr>
<tr>
<td><strong>LR TEST</strong></td>
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<td>1 cointegrating vector</td>
<td>1 cointegrating vector</td>
</tr>
<tr>
<td></td>
<td>at 1%</td>
<td>at 5%</td>
<td>at 1%</td>
</tr>
</tbody>
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**INVESTMENT COINTEGRATING VECTOR**
(t-statistics in parentheses)

<table>
<thead>
<tr>
<th>I/Q</th>
<th>A</th>
<th>K/Qr (-1)</th>
<th>r^2 (-3)</th>
<th>R(-4)</th>
<th>C</th>
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<tbody>
<tr>
<td>1.000000</td>
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<td>0.036445</td>
<td>0.499764</td>
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<td>-0.738905</td>
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<td>(-2.35347)</td>
<td>(6.62560)</td>
<td>(2.31693)</td>
<td>(3.81333)</td>
<td>(-10.4659)</td>
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</table>

Error Correction

<table>
<thead>
<tr>
<th>I/Q</th>
<th>A</th>
<th>K/Qr</th>
<th>r^2</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
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<td>0.299676</td>
<td>1.663852</td>
<td>-0.004339</td>
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<tr>
<td>(-2.64449)</td>
<td>(4.54863)</td>
<td>(4.86766)</td>
<td>(-0.31505)</td>
<td>(-1.86406)</td>
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</table>

**CONSUMPTION COINTEGRATING VECTOR**
(t-statistics in parentheses)

<table>
<thead>
<tr>
<th>C/Q</th>
<th>K/Qr</th>
<th>F/Q</th>
<th>r^2</th>
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<th>C</th>
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<td>(-5.48967)</td>
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<td>(-4.43080)</td>
<td>(-5.71252)</td>
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Error Correction

<table>
<thead>
<tr>
<th>C/Q</th>
<th>K/Qr</th>
<th>F/Q</th>
<th>r^2</th>
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<th>C</th>
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<tr>
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<td>(-2.48498)</td>
<td>(1.19938)</td>
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**TRADE BALANCE COINTEGRATING VECTOR**
(t-statistics in parentheses)

<table>
<thead>
<tr>
<th>TB/Q</th>
<th>R</th>
<th>C/Q (-4)</th>
<th>C/Q* (-4)</th>
<th>tot (-4)</th>
<th>C</th>
</tr>
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<tbody>
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<td>0.394262</td>
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<td>(1.74955)</td>
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<td>(-3.77029)</td>
<td>(2.98893)</td>
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Error Correction

<table>
<thead>
<tr>
<th>TB/Q</th>
<th>R</th>
<th>C/Q</th>
<th>C/Q*</th>
<th>tot</th>
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<td>(-2.21943)</td>
<td>(1.39522)</td>
<td>(5.12105)</td>
</tr>
</tbody>
</table>

(*) C = constant in Var, integers are lags included
vector the depicted sign has to be reversed. All coefficients have the right sign and are significant at conventional significance levels (except domestic consumption in the trade balance equation, which is only significant at the 10% level).

In the row labelled “Error Correction Term” we show the coefficient and the t-statistics of the error correction term for the respective indicated VEC equation. Weak exogeneity for the right hand side variables of equations (2), (3) and (4) (as revealed by the insignificance of the error correction terms in the respective equation in the VEC specifications) can be attested for the real long-term interest rate and the exchange rate in the investment VEC, for all explanatory variables in the consumption VEC as well as for domestic consumption in the trade balance VEC. The error correction terms for the respective variables of interest, the endogenous variables in the three estimated equations, are all significant at the 5% level.

Given our parameter estimates the equilibrium is a stable node as both eigenvalues of the dynamic system in $K/Q$ and $F/Q$ are negative. A positive estimate for $\alpha_3$ and $\alpha_8$ is a sufficient condition for stability of the system. The former obviously ensures convergence of the capital stock to its steady state value, while the latter takes care that a higher foreign debt ratio reduces consumption, improves the trade balance and stabilises the foreign debt ratio.

### 4.3 The medium and long-run equilibrium

The main difference between the medium and the long-run is characterised by the adjustment of the capital and debt stocks. In the medium run both are given by their historical values, while in the long-run they are assumed to have converged to their steady state ratios to GDP. A second difference is the assumption of real uncovered interest parity for the long-run equilibrium, which means that the domestic interest rates are assumed to have converged to the foreign rates$^{29}$. Thus the medium-run equilibrium is

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$^{29}$ Convergence is assumed for all interest rates except the nominal long rate in the consumption equation, which is supposed to proxy for domestic consumption preferences. In addition, the long-run equilibrium uses the HP filter trend for the short and long-run real foreign interest rates.
obtained with equations (1) to (4). The long-run equilibrium uses the full model so that additionally equations (5), (6), (9) and (10) are applied.

A common example to explain the logic of the Natrex model is to consider an exogenous rise in (national) savings preferences. In the medium run the domestic currency would depreciate due to a lower domestic interest rate and the capital outflow associated with an excess of savings over investment. The lower interest rate as well as the depreciated exchange rate would also stimulate investment. At the same time the economy accumulates net foreign assets (reduces the stock of foreign debt), which increases wealth (also the capital stock is rising), increases consumption and thus diminishes the savings surplus until the stock of net foreign assets reaches its steady state. According to standard Natrex logic\(^{30}\), in the long-run equilibrium the reduced stock of net foreign debt comes along with a larger foreign investment income. Thus the trade balance will have to deteriorate in equilibrium in order to keep the current account balanced. The real exchange rate will thus have to appreciate beyond its initial level to produce the deteriorated trade balance. This difference in sign between the change of the medium and the long-run equilibrium exchange rate compared to the initial level vanishes when net factor income is kept constant as it is done here. In our case the reduction of the foreign debt level and the rising capital stock in the transition to the long-run equilibrium also raise consumption and deteriorate the trade balance from its medium run value. The associated exchange rate reaction in order to obtain a weakening trade balance is a real appreciation. However, the exchange rate appreciates only up to a point so that there remains a real depreciation compared to the initial level before the savings shock. We thus find an overshooting of the real exchange rate but no sign reversal of the exchange rate effect in the long-run\(^{31}\).

After having thus estimated the coefficients of our model we solved it for the medium and the long-run equilibrium exchange rates. In order to capture the requirement to evaluate exogenous variables at their equilibrium levels (e.g. inventory investment at full employment level) we use the respective trend variables after applying the HP filter\(^{32}\). Chart 4.1 depicts the medium-run and Chart 4.2 the long-run

\(^{30}\) See Stein [2001] for his most recent thinking on the Natrex model.

\(^{31}\) In Detken and Marin [2001] it is shown how the conditions for an overshooting of the exchange rate can be obtained, see their Annex on the long-run equilibrium. In general, it depends on a whole array of coefficient estimates.

\(^{32}\) For inventory investments and net factor income even a very strong filter using a lambda of 150000 instead of the usual 1600 has been employed in order to smooth the very high volatility in these two series.
equilibrium exchange rates, labelled MREQ and LREQ, respectively, while EERG is the actual real effective exchange rate.

**Chart 4.1 Medium-run equilibrium and actual real effective exchange rate of the euro**

![Chart 4.1](chart1.png)

**Chart 4.2 Long-run equilibrium and actual real effective exchange rate of the euro**

![Chart 4.2](chart2.png)

The first observation in both Charts is that the real effective exchange rate of the synthetic euro was overvalued in the second half of the seventies until the early eighties. Second, the depreciation in the early eighties would be an equilibrium reaction as was the subsequent appreciation in the second half of the eighties.

Although the equilibrium exchange rates depicted in Charts 4.1 and 4.2 are a complex result of the combined influence of our exogenous variables, the main driving factors of the general pattern seem to be
productivity growth and the terms of trade. Euro area productivity growth as derived here declined to half its previous rate between 1975 and 1983, then recovering swiftly until 1988. From 1989 on it fell from about 1.6% to around 1% p.a.. According to the estimated long-run equilibrium the synthetic euro had been undervalued for prolonged periods between 1983 and 1991 during which the euro equilibrium rate has been boosted by strong productivity growth.

This positive impact of productivity growth on the equilibrium exchange rate depends on the parameter estimates. A permanent and positive shock to productivity growth increases the investment to GDP ratio. The resulting capital inflow in the medium-term leads an exchange rate appreciation. In the long-run, the worsened current account increases the foreign debt ratio, while the capital stock starts rising to a higher steady state ratio, so that the resulting wealth effect on consumers is ambiguous. With our estimates, the fall in consumption is weaker than the increase in the investment ratio, so that a deteriorated trade balance is necessary to restore equilibrium in the goods market, hence the real appreciation. This long-run appreciation (with overshooting in the medium term) could theoretically be reversed in the event where the wealth effect of net foreign asset on consumption would be stronger. Then the results would be closer to the theoretical framework envisaged in Sections 3 and 5.

The most striking feature of the graphs though is the opening gap between the equilibrium and the actual exchange rate since 1997, interrupted only by a brief euro appreciation in the second half of 1998. At the end of 1998 our Natex estimates suggest that the euro had been undervalued by around 0.7% with respect to the medium-run and by 2.3% with respect to the long-run equilibrium. At the end of 2000, the undervaluation of the real effective euro with respect to the four largest trading partners amounted to 25.4% and 27.7% with respect to our medium and long-run equilibrium estimates, respectively. Using the simulation results of Detken and Marin (2001) the nearly 24% real depreciation of the real effective euro between end 1998 and end 2000 could only be ex-post rationalised by a permanent, annual expected negative productivity growth shock for the euro area (relative to the rest of the world) of about 5% with respect to the medium-run and of 10% with respect to the long-run equilibrium. This calculation suggests that other factors than serious productivity growth differential projections must have been at the root of the euro’s depreciation.
Overall, the variability of the estimated medium-run equilibrium is very high due to the volatility in the productivity growth rate, the terms of trade and domestic interest rates. The long-run equilibrium would feature the same degree of variability if real interest rates were not replaced by their (foreign) stochastic trends. On the positive side, this could be used to refute the claim that exchange rate models based on fundamentals are always at a loss in explaining actual exchange rates because fundamentals are not volatile enough. Furthermore, the basic pattern of the synthetic euro exchange rate has been traced by our version of a Natrex model. On the other hand one caveat is appropriate. The exact level of the resulting equilibrium rate is quite fragile to small changes in the behavioural equations (changes experimented with were within the scope allowed by the microfoundation of the fundamental relationships). The basic pattern would remain similar, but the degree of over- or undervaluation can vary significantly.

5. A structural macro-model approach

As can be seen from the previous section, the determinants of the equilibrium exchange rate very much depend on the structure of the model that is used to compute the implied equilibrium value. As a result, experiments with alternative models are clearly warranted. Another key element is the difficulties to expect when experimenting with a fully new framework. Going beyond the simple small-scale model employed in the previous section, and following work by the IMF using Multimod, in this section use is made of a relatively standard macromodel estimated on euro area-wide data, the AWM (see Fagan et al. [2001]). Steady state simulations over very long horizons are carried out to assess the responses to shocks of the steady state real effective exchange rate of the synthetic euro under different assumptions.\(^{33}\)

5.1 The underlying theoretical model for the steady state exchange rate

The theoretical model underlying the Fagan et al. [2001] AWM is similar to the one described in the previous section, although more detailed as regards the supply side and policy modelling. The model is consistent with a classical long-run equilibrium – i.e. with a long-run vertical Phillips curve where output

\(^{33}\) See Bayoumi et al. [1994] for similar exercises. In our case, such exercise have clearly an exploratory character to the extent that macroeconometrics of the euro area on the basis of aggregated historical series are still in infancy phase. Moreover results derived from past data cannot possibly take into account the currently changing structure of the euro area economy. In addition the model used is itself a provisional tool, subject to continuous development.
is supply-determined in the long-run. Sluggishness in prices and wages implies that in the short-run output is demand-determined. In the long run, the real interest rate determines the capital to output ratio, output being consistent with the production function and employment at its natural level, with productivity being labour-augmenting. Domestic demand in the AWM is also a function of the exchange rate – via the competitiveness term in the real trade equations – and of net foreign assets – via the wealth effect in consumption. Stock-flow adjustments and resulting steady state identities are also similar to those presented for the NATREX framework.

In comparison with the NATREX framework, a key difference is however that the AWM does not allow the real exchange rate to play a role in investment determination. In addition, adjustment to the resulting long-run equilibrium is ensured via a number of channels not at work in the NATREX framework, in particular fiscal and monetary policies. Fiscal policy pins down the public debt to GDP ratio, whereas a postulated interest rate rule adjusts interest rates to ensure that inflation reaches its long-run value\(^{34}\). The AWM comprises moreover a forward-looking UIP condition, in which the future change in the nominal effective exchange rate is equal to the interest rate differential vis-à-vis the RoW. Finally, an implicit hypothesis of balanced capital flows is made in the definition of the accumulation of net foreign assets, which is consistent with a zero deviation from UIP at equilibrium.

In steady state, as output equals potential, what matters for the determination of the exchange rate is the demand side of the model. For any given real interest rate, there is a single – and therefore equilibrium – real exchange rate consistent with output at its potential. The resulting equilibrium real exchange rate is a function of the other two variables characterising a given equilibrium, namely the stock of net foreign assets and the saving ratio. Both latter variables result from the adjustment of demand components to supply, involving jointly private consumption and trade, since otherwise investment and public consumption are respectively pinned down by the real interest rate and the fiscal policy rule.\(^{35}\)

\(^{34}\) The assumption is made for the sake of illustration only of a simple and standard Taylor rule. Some endogenous interest rate setting, even though basic it might be, is needed in such a framework to ensure convergence to the steady state.

\(^{35}\) Further details on how the model converges to its steady state are provided in Fagan et al. [2001].
5.2 Simulation results

Using this estimated model for the euro-area, a number of experiments can be conducted. First, a very long-horizon – 200 years – out-of-sample analysis has been carried out, with a view to computing the steady-state real exchange rate compatible with a zero trade balance in the long-run. This amounts to taking an approach similar to the desired equilibrium exchange rate (DEER) approach in Bayoumi et al. [1994]. Second, stochastic simulations are run in order to estimate the uncertainty band around the theoretical steady state level of the exchange rate. Third, variant simulations involving supply and demand shocks are employed to evaluate the response of the long-run steady state real exchange rate. Such variant simulations are not constrained ex ante to eventually deliver a zero trade balance ratio like in the baseline.

5.2.1 Steady state analysis: the construction of a reference path

In this section, long-run out-of-sample dynamic simulations are carried out so as to characterise the steady state. It is important to emphasise that the results will crucially depend on the type of assumptions retained for the various exogenous variables entering the model. As a result, any steady state simulation could always incorporate some normative component. The hypotheses used for projecting exogenous variables were a constant steady state inflation and real growth rate (see details in Fagan et al. [2001]). Also some additional assumptions were made regarding e.g. the composition of growth in terms of factor productivity and labour supply growth. Another key element is the calibration of the intercepts in the various behavioural equations, which is necessary to the extent that the estimated intercept may reflect in-sample mean adjustment that may not be relevant from a steady-state perspective (or would not take due account of the theoretically necessary cross-equation restrictions).36 Taking as an objective a zero trade balance in the long-run, the simulation leads to a real effective exchange rate which is 24% higher than in 2000q4.

36 As to the supply side, the calibration of the intercepts is imposed by a number of restrictions such as those needed to get a vertical Phillips curve or the long-run value investment to GDP ratio, given the real interest rate. Similarly both monetary and fiscal policy equations are assumed to be compatible with the given real interest rate and debt to GDP ratio, respectively. In general, other intercepts are set to their last observed value. On the demand side, the consumption residual has been left constant, taking another value would change the long-run saving ratio and the net foreign asset ratio to GDP. For trade volumes and prices, intercepts have been set so as to avoid taking a starting point with a specially high or low trade balance.
5.2.2 Stochastic simulations: the initial condition dependency

In order to assess the robustness of this assessment, stochastic simulations were performed (see the details on the experiment in Fagan et al. [2001]). 500 random shocks were generated for each econometric equation drawn from a Gaussian distribution of zero mean, assuming no correlation across equations and taking for the variance of each shock the estimated one, on the basis of the historical econometric residuals to each equation. Shocks were applied only to the first period, which aims at mimicking a situation in which the initial conditions "reasonably" depart from the steady state - assuming that the estimated model is the true model of the economy.

![Chart 5.1 Stochastic simulations: Exchange rate quantiles](chart.png)

NB: Each line denoted EERx gives at each point in time the deviation from the steady state corresponding to the x% quantile for the real effective exchange rate.

Chart 5.1 shows that the initial uncertainty on the real exchange rate is relatively limited, with ± 0.8% for the 80% confidence intervals. The corresponding figure for inflation would be even lower, ± 0.4%. The real exchange rate remains persistently affected by the initial shock, within a confidence band of ± 0.6% around the steady state, whereas inflation reaches steady state after about fifteen years. The nominal exchange rate is in turn affected by a ± 1.5% uncertainty.
Such initial condition dependency could be perceived as an illustration of the so-called hysteresis in the equilibrium exchange rate (cf. Bayoumi et al. [1994]).\textsuperscript{37} For any given initial conditions, the real exchange rate does not necessarily coincide ex ante with its uniquely corresponding equilibrium foreign asset ratio. Assume e.g. the latter is higher than the one consistent with a given initial exchange rate. This mismatch implies that the real exchange rate needs to appreciate, in order to generate a trade deficit to gradually compensate the initial excess stock of foreign assets. Given the simultaneity between the two variables, both the real exchange rate and the foreign assets will eventually differ from their initial value.

Additional illustrative experiments were made, assuming that factor income would be exogenous in GDP points, and therefore not directly related to net foreign assets. This has the expected effect of dampening significantly the above mentioned persistence in the equilibrium real exchange rate, the uncertainty being reduced to almost zero (0.05%) in the long run, while the corresponding figure for the nominal rate increases to 3%. In other words, the apparent multiplicity of equilibria for the asset ratio and the real exchange rate would almost vanish under such a simplified modelling of the current account, to the expense of an increase in the uncertainty surrounding the steady state value of the nominal exchange rate. The latter would then be entirely due to the price level indeterminacy introduced in the model by using a Taylor rule, i.e. an equation is consistent with a well pinned-down inflation but no precisely defined price level.

5.2.3 Variant simulations: responses to demand and supply shocks

Taking the above mentioned steady state as a baseline, simulations were carried out, so as to assess the impact on the equilibrium exchange rate of various shocks to supply or demand variables. Shocks are permanent, and expected effects of such shifts in either the demand or the supply curve are similar to those considered in Section 3. In all cases results are reported in a tabular format for the following variables: the real exchange rate, the nominal exchange rate, and the GDP deflator (see Table 5.1). Long-run effects on the trade balance ratio are less than 0.1 p.p. and have therefore not been reported. Ex post,

\textsuperscript{37} As could be expected, also the foreign asset to GDP ratio exhibits initial condition dependency. In the case of the AWM, the indeterminacy reflects the initial condition dependency of the equilibrium price level, to the extent that the closure rules as specified do not involve a price level target. In addition the empirical relation estimated in the AWM between foreign asset stocks and returns reflects a negative correlation between the two variables. See the previous section on this point too.
the resulting steady state appears compatible therefore with the envisaged DEER interpretation, in all of the reported simulations.\textsuperscript{38}

Table 5.1: Steady state effects of the various shocks on exchange rates and prices

<table>
<thead>
<tr>
<th></th>
<th>Effective exchange Rate, real</th>
<th>Effective Exchange Rate, nominal</th>
<th>GDP deflator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiscal shock</td>
<td>+3.3</td>
<td>+1.3</td>
<td>+2.0</td>
</tr>
<tr>
<td>World shock</td>
<td>+0.7</td>
<td>+0.6</td>
<td>+0.1</td>
</tr>
<tr>
<td>Labour supply</td>
<td>-0.7</td>
<td>-1.0</td>
<td>+0.3</td>
</tr>
<tr>
<td>Productivity</td>
<td>-1.2</td>
<td>-1.9</td>
<td>+0.6</td>
</tr>
</tbody>
</table>

NB: The effects of the shocks are given in percentage points, an increase in the exchange rates being an appreciation of the synthetic euro. The shocks are all permanent on the levels of the variables. They respectively correspond to the following: Fiscal shock: + 1 p.p. GDP on the level of public consumption, real; World shock: + 1% on the level of the RoW GDP, real; Labour supply: + 1% on the level of total labour force; Productivity: + 1% on the level of total factor productivity.

In the case of the demand shocks, be it domestic or foreign, the response is qualitatively similar across simulations, namely the exchange rate has to appreciate so as to ensure a reshuffling of demand compatible with the given supply constraint. The required appreciation is partially achieved through a nominal increase in the effective exchange rate, itself consistent with the initial increase in interest rates which is required to contain the additional inflation resulting from the demand shock. The necessary appreciation and the inflationary effects are less strong in the case of the external shock, sensibly enough.

In the case of the supply shocks, (total factor productivity and labour supply, respectively) the expected depreciation of the real exchange rate is found, although slightly lower than for the domestic demand shock. This is achieved, however, with very limited changes in the domestic price level, to the extent that

\textsuperscript{38} The saving ratio is only marginally affected by the various shocks, shifting by less than half a percentage point in all cases. The impact on the ratio of net foreign assets to GDP is more sizeable, around 1% in all cases, but highly dependent on the hypothesis made regarding the valuation effects of a change in the exchange rate on the foreign assets held by the euro area.
interest rates have a response overall similar to what is observed in the case of demand shocks. On the other hand, effects on foreign assets are similar in magnitude. Finally, the long-run impact on GDP is exactly 1% in the case of the labour supply shock, whereas the effect is higher following the total factor productivity shock, since supply reacts more than one-to-one to such a shock (1.7%). This also explains why the required depreciation is correspondingly higher in the case of the productivity shock. It should be noted that this latter shock leaves the capital to output ratio unaffected, since the real interest rate does not change. Only the labour to output ratio is permanently affected, which makes this simulation quite different from what would result from the model presented in the previous section.

In terms of the magnitude implied, somewhat large shocks would then be needed to explain a permanent 24% deviation from a given steady state value of the real exchange rate. Public consumption e.g. should be decreased by some 8% of GDP, or similarly, labour supply by around 33%, to yield such a large (and permanent) shift in the exchange rate. This may cast some doubts on the possibility for such shocks to fully explain developments in the real exchange rate of the euro experienced since its introduction.

6. Conclusions

On the basis of historical data aggregated over the period 1973 to 2000, we have experimented with a number of econometric models – in the most general sense of the term – with a view to estimating alternative concepts of the “equilibrium” level for the real exchange rate of the synthetic euro. Using a number of competing models with the same data set, variable definitions and sample period offers the possibility to assess the uncertainty surrounding such equilibrium levels, both from an empirical (different estimates) and a theoretical viewpoint (different specifications). In this exercise, the “Rest of the World” is proxied by the US, the UK, Japan and Switzerland, aggregated on the basis of trade weights.

A number of methods have been employed, with an increasing degree of ‘structure’, in the sense of the constraints based on economic theory embedded in the econometric models that were estimated. The following results are worth highlighting:

- A reduced-form cointegration approach in the vein of Mc Donald [1996] shows that there is evidence that the real exchange rate of the synthetic euro is significantly and positively correlated with the
relative prices of non-traded goods vs. traded goods and the real interest rate differential vis-à-vis the rest of the world. In such a model, the 80% confidence interval would amount to 8% around the resulting long-run statistical equilibrium value, so that the observed undervaluation in 2000q4, amounting to 10%, cannot be said to be overly significant.

- Estimating a structural VAR à la Clarida and Gali [1994] enabled us to examine the contribution of various permanent and temporary shocks to the dynamics of the real effective exchange rate. The results suggest that the main factor driving the permanent component in the real exchange rate are real demand shocks, whereas supply and nominal shocks have a much less significant effect. Depending on the model used, the undervaluation of the euro at the end of 2000 would range between 5 and 12%, but is not always significant. In any event, the fact that the real demand shocks explain only a small part of the variation in output and inflation, suggests that the Clarida-Gali model is not able to relate movements in the real exchange rate to their underlying fundamentals.

- One step further in the direction of discussing an equilibrium exchange rate was made by resorting to the NATREX framework, originally introduced by Stein [1994]. The structural form of a small-size dynamic model comprising behavioural equations for trade, consumption and investment has been estimated by means of separate VEC models. The equilibrium long-run real exchange rate is derived by solving the model with the capital stock and net foreign assets at their steady state ratios to GDP. The simulated equilibrium exchange rate tracks the actual exchange rate quite well. However, since 1997 a gap arises pointing to a considerable degree of undervaluation of the euro exchange rate, beyond 25% at the end of 2000.

- Finally an existing fully-fledged medium-size macro-econometric model for the euro-area has been employed – as suggested in the DEER approach (cf. Bayoumi et al. [1994]) – to evaluate the impact of stochastic and deterministic shocks on the steady state real exchange rate for the euro area. A reference steady state out-of-sample projection has been constructed on the basis of a number of normative assumptions such as the need to get a zero trade balance in the long-run. The resulting real exchange rate would be 24% higher than in 2000q4. Stochastic simulations around this baseline show that the 90% confidence band around the steady state level of the real exchange rate is around 1%
wide. In line with the underlying theoretical framework, supply shocks lead to a depreciation in real terms whereas the opposite occurs following demand shocks.

In sum, the general result seems to be, in line with what is usually found for other currencies, that the estimates for the equilibrium exchange rate are model-dependent and surrounded by some non-negligible uncertainty. Irrespective of the model considered, however, the real exchange rate of the euro at the end of 2000 is found to be below its (model-dependent) equilibrium value. On the other hand, the degree of undervaluation differs widely across models - from 5 % to 27% - and is also not significant for all models. Regardless, our results could also be interpreted as a sign that the various models used were simply not capable of explaining the path followed over the recent years by the real exchange rate of the euro, so that alternative explanatory variables and models should be considered for that purpose.

An additional interesting element, which could pave the way for further research, is that the estimated degree of undervaluation seems to increase with the structure imposed on the model and, in parallel, also with the (implicit) horizon considered in terms of the underlying long-run equilibrium. The latter ranges from a couple of years, for reduced-form models, up to fifteen years or more for the structural models employed. In all cases, the equilibrium value of the exchange rate is higher than its current value. Reduced-form models are designed to explain the exchange rate over relatively short-run horizons, so that, in the case at hand, the equilibrium implied by them could be interpreted as one step in the convergence of the exchange rate towards a higher and longer run equilibrium, which would in turn be consistent with the more structural models.
References


ANNEX 1: Overview of the Database

The database used is a mixture of data for the euro area and series for the Rest of the World (ROW). The euro area series mainly come from the Area-wide model (AWM) database. Detailed information on the construction of the latter can be found in Fagan et al [2001]. The ROW variables are proxied by an aggregation of the United Kingdom, the United States, Japan and Switzerland (representing approximately 50% of the trade of the euro-area with the ROW). All variables are quarterly from 1970q1 onwards (where possible). This annex briefly explains how the database was compiled.

1. Country data

The country series come from a variety of sources, mainly from the OECD National Accounts, the BIS and the IFS. All data used are seasonally adjusted, and where necessary, country data is seasonally adjusted before aggregating. Where quarterly series are not available, annual series are transformed into a quarterly frequency using a standard spline procedure[??]. Where one source does not provide data for a series from 1970q1, two series are combined to create a longer historical series. If the same definition is not available across countries, the closest series is used; for example the long-term interest rate series is for 10 year bonds (or of lesser maturity for some countries).

2. Aggregation method

Both for the AWM and the ROW datasets, the method of aggregation is the so-called “Index method” with a fixed weighting scheme, i.e. \( \ln X_T = \sum w_i \ln X_i \) where \( w \) is the weighting index. For the AWM data, the weights employed are GDP PPP weights for 1995. For the exchange rate of the synthetic euro and the ROW data, overall trade-weights for each of the four countries over the period 1995-96-97 are used.

3. Additional series

Once the basic series are aggregated, additional series are then derived. In particular, the deflators are computed as the ratio between nominal and real variables (consistently with the aggregation method) and the effective exchange is then deflated by the relevant series (e.g. GDP deflator). Other variables deflated by the GDP deflator include wages and interest rates.
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