EUROPEAN CENTRAL BANK

EUROSYSTEM

WORKING PAPER SERIES

NO 1481 / OCTOBER 2012

CLEARING, COUNTERPARTY RISK AND AGGREGATE RISK

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In 2012 all ECB publications feature a motif taken from the €50 banknote

Acknowledgements
We would like to thank Pierre-Olivier Gourinchas (editor), Ayhan Kose (co-editor), Stijn Claessens (guest editor), Ana Fostel (IMF ARC discussant), Iman van Lelyveld (DNB discussant), Christophe Perignon and two anonymous referees for helpful comments on earlier drafts. This paper was prepared for the Twelfth Jacques Polak Annual Research Conference on Monetary and Macroprudential Policies held November 10-11, 2011 in Washington, D.C. Many thanks for insightful comments to participants at the IMF Twelfth Jacques Polak Annual Research Conference, the Fifth Financial Risk International Forum in Paris, the Financial Infrastructures Conference (Tilburg University/Dutch National Bank) and the European Central Bank. The views expressed do not necessarily reflect those of the European Central Bank or the Eurosystem. Biais gratefully aknowledges the support of the European Research Council, Grant 295484, Trading and Post-trading.

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ISSN 1725-2806 (online)

Abstract

We study the optimal design of clearing systems. We analyze how counterparty risk should be allocated, whether traders should be fully insured against that risk, and how moral hazard affects the optimal allocation of risk. The main advantage of centralized clearing, as opposed to no or decentralized clearing, is the mutualization of risk. While mutualization fully insures idiosyncratic risk, it cannot provide insurance against aggregate risk. When the latter is significant, it is efficient that protection buyers exert effort to find robust counterparties, whose low default risk makes it possible for the clearing system to withstand aggregate shocks. When this effort is unobservable, incentive compatibility requires that protection buyers retain some exposure to counterparty risk even with centralized clearing.

JEL classification: G22, G28, D82

Keywords: Risk-sharing; Moral hazard; Optimal contracting; Counterparty risk; Central Clearing Counterparty; Mutualization; Aggregate and idiosyncratic risk

Non-Technical Summary

Would the development of Centralised Clearing Platforms (hereafter CCPs) improve the working of markets? In September 2009, the G20 leaders, followed by the Dodd-Frank Wall Street Reform & Consumer Protection Act, and then the European Commission, answered yes. They proposed that all standardised OTC derivatives contracts be centrally cleared.

How, and to what extent, can clearing improve the allocation of risk and how should it be designed? Should it be decentralized or centralised? Should it provide full insurance against counterparty default? Is it likely to decrease or increase risk-taking? Is clearing enough to cope with counterparty risk, or should it be complemented by other risk-mitigation tools? We take an optimal contracting approach to analyse these issues and offer policy implications.

We analyse an environment in which protection buyers hold risky assets, which are exposed to both aggregate risk and idiosyncratic risk. To hedge that risk, protection buyers contract with limited-liability protection sellers. Protection sellers are able to provide insurance against protection buyers' risk but they can default on their contractual obligations with positive probability, thus generating counterparty risk for protection buyers.

In line with the classic theory of insurance, our analysis identifies three ways in which counterparty risk can be mitigated. First, resources can be deposited in safe assets and used to make promised payments in case of counterparty default. This is comparable to self-insurance, whereby an agent saves to insure against future negative shocks. However, safe deposits entail opportunity costs of foregoing more productive investments. Second, agents can search for counterparties with low default risk. This is comparable to self-protection, whereby an agent exerts effort to reduce default probabilities. Such search effort is costly as it entails time and resources spent on, e.g., due diligence. Third, trading parties can mutualize their risk.

We show that the main advantage of centralised clearing is the mutualisation of idiosyncratic risk. Mutualisation allows economising on the opportunity costs of safe deposits, as well as on the costs of finding creditworthy counterparties. Hence, whenever the protection buyers' risk has an idiosyncratic component, centralised clearing improves on no or decentralized clearing in our model.

While mutualisation can insure against idiosyncratic components of buyers' risk, it cannot provide insurance against aggregate risk. When the latter is significant, it is efficient that protection buyers exert effort to find robust counterparties, whose low default risk makes it possible for the clearing system to withstand aggregate shocks. We show that when this effort is privately observable, leading to moral hazard concerns, incentive compatibility requires that protection buyers retain some exposure to counterparty risk, even with centralised clearing. Hence, under moral hazard and aggregate risk, the CCP should be designed to maintain proper incentives of members to search for solid counterparties.

We discuss a number of policy issues. Regarding the question of whether participation in the CCP should be mandatory, we argue that if all clearing is initially decentralised, it may be difficult to change expectations and coordinate on the Pareto dominant equilibrium with centralised clearing. In that case, making centralised clearing mandatory is Pareto improving. Regarding the governance and regulation of CCPs, we show that in our set-up, a CCP which is mutually owned by its users implements the optimal clearing mechanism. By contrast, a for-profit shareholder-owned CCP may not achieve the same level of efficiency and may thus have to be regulated.

1 Introduction

Counterparty risk is the risk to each party of a contract that his or her counterparty will not live up to its contractual obligations. As vividly illustrated by the failure of Lehman Brothers and the near failures of AIG and Bear Stearns, counterparty risk is a real issue for investors. These episodes underscore that institutions should monitor the risk of their counterparties and strive to contract with creditworthy ones.

Clearing entities, and in particular Centralized Clearing Platforms (hereafter CCPs), can offer insurance against counterparty risk. The clearing entity interposes between the two parties. If one of them is unable to meet its obligations to the other, the clearing entity makes the payment on behalf of the defaulting party. Would the use of CCPs make markets safer? In September 2009, the G20 leaders, followed by the Dodd-Frank Wall Street Reform & Consumer Protection Act, and then the European Commission, answered yes. They proposed that all standardized OTC derivatives contracts be centrally cleared.[1]

Was this the right move? More generally, how, and to what extent, can clearing improve the allocation of risk, and how should it be designed? Should it be decentralized or centralized? Should it provide full insurance against counterparty default? Is it likely to decrease or increase risk exposures? Is clearing enough to cope with counterparty risk, or should it be complemented by other risk-mitigation tools? We take an optimal contracting approach to analyze these issues and offer policy implications.

We consider a simple model in which a continuum of risk-averse agents who hold risky assets (protection buyers) faces a continuum of risk-neutral limited-liability agents (protection sellers). For example, protection buyers can be financial institutions holding a portfolio of loans and seeking insurance against the default of these loans. For simplicity we assume that the asset held by each protection buyer can take on only two values, high and low, with equal probability. The protection sellers offer to insure the protection buyers against the risk of a low value of this asset.[2] The problem is that protection sellers themselves may default. This creates counterparty risk for protection buyers and reduces the extent to which they can hedge their own risk. At some cost, protection buyers can exert effort to search for good

[1] The G20 meeting in September 2009 chose December 2012 as the deadline for this change. It is not clear this deadline will be met.

[2] To achieve this, the two parties can trade a Credit Default Swap, or possibly another derivative, such as a forward contract. Our optimal contracting approach enables us to consider general contract structures, without restricting attention to particular instruments.

counterparties with low default risk ("due diligence"). When deciding whether to do so, protection buyers trade off the benefits of better insurance (granted by good counterparties) and the cost of effort. If protection buyers are sufficiently risk-averse, or the search cost is low enough, then it is optimal to exert effort to find a creditworthy counterparty.

Even when they exert effort, protection buyers remain exposed to some counterparty risk since the default probability of the protection seller is always strictly positive. In this context, how can a clearing entity offer insurance against counterparty risk and improve welfare? To clarify the economic drivers underlying this issue, we distinguish three cases. First, the risk exposures of the protection buyers are independent, and their search effort is observable and contractible (no moral hazard). Second, the risk exposure of the protection buyers has an aggregate component, but there is no moral hazard. Third, the risk exposure has an aggregate component and there is moral hazard. The optimal design as well as the usefulness of clearing arrangements vary across these three cases.

Consider the first case (no aggregate risk, no moral hazard). With decentralized clearing, there are clearing agents interposing between each pair of protection buyer and protection seller. For a fee, these clearing agents can insure protection buyers against the default of their counterparty. A clearing agent chooses a portfolio of liquid, low-return assets (cash) and illiquid, high-return assets. To be able to pay protection buyers, clearing agents must set aside liquid assets. This has an opportunity cost since the return on liquid assets is lower than on illiquid ones. Because of this cost, it is optimal to insure only partially against counterparty risk. Since they are not fully insured against counterparty risk, it is optimal for sufficiently risk-averse protection buyers to exert effort and search for creditworthy protection sellers.

With centralized clearing, the CCP interposes between all protection buyers and all protection sellers. Hence, the total insurance payment by the CCP is the sum of all the individual payments to protection buyers. Since the individual risks are independent, the law of large numbers applies and the sum of all payments is deterministic. Correspondingly, the fees levied by the CCP are exactly equal to the amount of insurance needed and it is no longer necessary to set aside liquid assets. Thus, the *first* benefit of mutualization via a CCP is that it avoids the opportunity cost of holding liquid assets. Since this cost is not incurred, full insurance against counterparty risk is optimal. This is the *second* benefit of mutualization. Also, with mutualization, the protection buyers effectively insure each other. Hence, they are not affected by the default of protection sellers. Consequently there is no need to search for good counterparties. Avoiding the search cost is the *third* benefit of mutualization.

Consider now the second case (aggregate risk, no moral hazard).[3] To model aggregate risk, we assume there are two equiprobable macro-states, referred to as good and bad. In the good state, the probability that each individual protection buyer's asset value is high is greater than one half. In the bad state, it is lower than one half.[4] Conditional on the realization of the macro-state, the values of the protection buyers' assets are i.i.d. Hence, the aggregate value of the protection buyers' assets is larger in the good state than in the bad state. While mutualization among protection buyers continues to be useful, it cannot provide insurance against the aggregate risk. The protection sellers become valuable again, even with centralized clearing, because the resources they bring to the table are useful to insure protection buyers against aggregate risk. Correspondingly, the effort to search for good counterparties is also valuable. If protection buyers are sufficiently risk-averse, the optimal contract involves i) effort to locate good counterparties, and ii) full insurance thanks to the mutualization of idiosyncratic risk and transfers from protection sellers in the bad macro-state.

Finally, consider the third and most intricate case (aggregate risk and moral hazard). In this case, the CCP cannot observe whether protection buyers exert search effort to find creditworthy protection sellers or not. Should the CCP continue to promise full insurance against counterparty risk as in the second case above? If it does, then protection buyers have no incentive to incur the cost associated with the search for creditworthy counterparties. Consequently, the average amount of resources brought to the table by protection sellers would be small. Their default rate would be high in the bad macro-state and the CCP would have to pay a lot of insurance. This liability could exceed the resources of the CCP, and push it into bankruptcy. To avoid this, the CCP should not offer full insurance against counterparty risk when there is moral hazard. This risk exposure, while suboptimal in the first-best, is needed in the second-best to maintain the incentives of protection buyers to

[3]In the first case, we analyzed why centralized clearing dominated decentralized clearing. For brevity, in the second and third cases we only consider centralized clearing. This is without loss of generality. In our optimal contracting framework, the optimal centralized mechanism dominates decentralized clearing by construction.

[4]Ex–ante, the probability that the value of the asset is good is exactly one half on average. The model with aggregate risk nests the model without aggregate risk as a particular case.

exert search effort.

In sum, our analysis yields the following implications. Centralized clearing is superior to decentralized clearing, since it enables the mutualization of risk. Policy makers are therefore right to promote centralized clearing. They should, however, keep in mind the limitations of centralized clearing and endeavor to mitigate their adverse consquences. In particular, while the mutualization delivered by centralized clearing reduces the exposure to *idiosyncratic* risk, it does not reduce the exposure to *aggregate* risk. Minimizing that exposure requires exerting effort to find creditworthy counterparties, robust to macro-shocks, and also attracting diverse counterparties, whose default risks are not too correlated. Our analysis also underscores that centralizing clearing can reduce both the social value and the private incentives to exert the search effort. While improving the allocation of counterparty risk, the centralization of clearing might therefore increase the aggregate counterparty default rate. Finally, under the plausible assumption that the effort to find creditworthy counterparties is unobservable, there is a moral hazard problem and the CCP must be designed to maintain the incentives of protection buyers. This precludes full insurance against counterparty default. The incentive constraint is especially important when aggregate risk is significant. In particular, when *aggregate* risk is large, incentive compatibility requires that protection buyers retain some exposure to the *idiosyncratic* component of risk.

Our analysis contributes to the micro-prudential and the macro-prudential study of clearing mechanisms. Micro-prudential analyses focus on one financial institution, studying its regulation, e.g., to avoid excessive risk-taking. Macro-prudential analyses consider a population of financial institutions and focus on the equilibrium interactions between these institutions, as well as on aggregate outcomes generated by these interactions. All of these features are present in our analysis. This is because, by construction, CCPs raise macro-prudential issues since they clear the trades of a population of financial institutions. Furthermore, our analysis emphasizes the interaction between the design of CCPs and the presence of aggregate risk. It underscores that when aggregate risk is significant, CCPs are useful but should not provide full insurance against counterparty risk, lest this would jeopardize the incentives of market participants to search for creditworthy counterparties.

The next section presents the institutional background. Section 3 reviews the literature. Section 4 presents the model. Section 5 analyzes the case with no aggregate risk and no moral hazard. Section 6 turns to the case with aggregate risk and no moral hazard. Section

7 examines the situation in which there is both aggregate risk and moral hazard. Section 8 offers a discussion. Section 9 concludes. Proofs not given in the text are in the online appendix.

2 Institutional background

Definition of clearing: After a transaction is agreed upon, it needs to be implemented. This typically involves the following actions:

- Determining the positions of the different counterparties (how many securities or contracts have been bought and sold and by whom, how much money should they receive or pay). This is the narrow sense of the word "clearing."
- Transferring securities or assets (to custodians, which are financial warehouses) and settling payments. This activity is referred to as "settlement."
- Reporting to regulators, calling margins and deposits, netting.
- Handling counterparty failures.

Understood in a broad sense, clearing refers to this whole process. The market-wide system used for clearing operations is often referred to as the "market infrastructure."

The basic mechanism of clearing and counterparty risk: Clearing in spot markets differs somewhat from its counterpart in derivative markets. First consider the case in which A and B agree on a spot trade: B buys an asset (stock, bond, commodity) from A, against the payment of price P. The clearing entity receives the asset from A and transfers it to B (or his custodian or storage facility). The clearing entity also receives the payment of P dollars from B and transfers it to the account of A.[5]

Derivative markets are more complicated because contracts are typically written over a longer maturity and are often contingent on certain events. Consider for example the case of a CDS. A sells protection to B against the default of a given bond. Before the maturity of the contract, as long as the underlying bond does not default, B must pay an insurance premium to A. Just like the payment of the price for the purchase of an asset, this payment

[5]In practice, this process might involve additional intermediaries, such as the brokers of A and B. For simplicity, these are not discussed here.

can take place via the clearing agent. If the underlying bond defaults before the maturity of the contract, A must pay the face value of the bond to B, while B must transfer the bond to A. Thus, the clearing entity receives the bond from B and transfers it to A, and receives the cash payment of the face value from A and transfers this to the account of B.

Clearing entities also typically provide insurance against the default of trading counterparties. For example, in the CDS trade described above, if the underlying bond defaults *and* A is bankrupt, then the clearing entity can provide the insurance instead of A: In this case, it is the clearing entity that receives the bond and pays cash to B. Such insurance is more significant in derivative markets than in spot markets: other things equal, the risk of default of one of the counterparties is greater over the long maturity of derivative contracts than during the few days or hours it takes to clear and settle a spot trade. To meet the default costs, the clearing entity must have capital and reserves.

Bilateral versus centralized clearing: The clearing process can be bilateral and operated in a decentralized manner. In this case the trade between A and B is cleared by a "clearing broker" or "prime broker." If on the same day there is a trade between two other institutions, C & D, it can be cleared by a different broker. In contrast, with Central Counterparty Clearing (hereafter CCC) the clearing process for several trades (between A & B as well as between C & D) is realized within a single entity, referred to as the Central Clearing Platform (hereafter CCP). In this centralized clearing system, the CCP takes on the counterparty risk of all the trades. This implies that the CCP can be exposed to a large amount of counterparty default risk. To cope with such risk, the CCP needs relatively large capital and reserves. Such reserves can be built up by levying a fee on the brokers using its services (possibly contingent on activity levels). The CCP can also issue equity capital subscribed to by the brokers and financial institutions using its services. To the extent that the counterparty loss on a given trade is paid for by the capital and reserves of the CCP provided by all the members of the CCP, centralizing clearing leads to the mutualization of counterparty default risk.

CCC has been the prevailing model for futures and stock exchanges. A polar case is the Deutsche Börse, where the trading platform and the clearing platform are vertically integrated. In contrast, decentralized clearing is most frequent when trades are conducted in OTC markets.[6] Up to now, a large fraction of the Credit Default Swaps market has

[6] For a recent paper analyzing whether centralized clearing of OTC transactions can improve welfare and

been OTC and cleared in a decentralized way. Note, however, that trading mechanisms and clearing mechanisms are distinct. Thus, it is possible to have OTC trading and CCC. In that case, the search for counterparties and the determination of the terms of trade is decentralized, while the two parties who struck a deal clear the trade in a CCP.[7]

3 Literature

Similarly to the present paper, Stephens and Thompson (2011) study the case in which protection sellers can default.[8] They assume protection sellers are privately informed about their type.[9] They analyze the risk of contracting with a bad protection seller. When they extend their analysis to centralized clearing, they show that it can lead to an inefficient increase in that risk. Pirrong (2011) also warns that centralized clearing could lead to an increase in counterparty default and notes that "with asymmetric information, it is not necessarily the case that the formation of a CCP is efficient." Similarly, Koeppl (2012), who, like us, emphasizes the mutualization benefits of CCPs, shows that they can "upset market discipline."

While we also find that centralized clearing can increase counterparty default, we show, in contrast to Stephens and Thompson (2011), Pirrong (2009), and Koeppl (2012), that the optimal CCP is welfare improving relative to bilateral clearing.[10] This difference in conclusions stems from a difference in approaches. Instead of considering features of the CCP that are exogenously given, we take an optimal contracting approach to study the design of the optimal clearing mechanism. By construction the resulting CCP is Pareto optimal (subject to information, resource and technology constraints.) From a normative viewpoint, our contribution is thus to identify the conditions and the design under which centralized clearing brings about efficiency gains.

Acharya and Bisin (2010) and Leitner (2012) also offer insights into the optimal design of centralized clearing. As noted by Acharya and Bisin (2010), no protection buyer can control

economize on settlement costs, see Koeppl, Monnet and Temzelides (2011).

[7] This can be the case, e.g., for swap deals struck on the OTC market, and then cleared through LCH.Clearnet or SwapClear. In that case, the original swap is transformed into two deals: between the swap buyer and the CCP, and between the CCP and the swap seller.

[8] Thompson (2010) and Biais, Heider, Hoerova (2010) also study inefficiencies associated with the default of protection sellers, but they do not model CCPs.

[9] Thus, they consider an adverse–selection model, which contrasts with our moral–hazard setup.

[10] In our analysis, the CCP leads to an increase in counterparty risk relative to the decentralized clearing case, because better risk-sharing undermines incentives to search for creditworthy counterparties.

the trades of his counterparty with other investors in OTC markets. So when a protection seller contracts with an additional protection buyer, this exerts a negative externality on other protection buyers. It increases counterparty risk and generates inefficiencies in equilibrium (similar to the inefficiencies arising in the non–exclusive contracting model of Parlour and Rajan, 2001). Acharya and Bisin (2010) show how centralized clearing can eliminate such inefficiencies by implementing price schedules that penalize the creation of counterparty risk. Furthermore, Leitner (2012) shows how, within a central mechanism, position limits prevent agents from entering into excessive contracts. Our focus is different. In Acharya and Bisin (2010) and Leitner (2012), the benefit of centralized clearing is that it enables to control the risk exposure of protection sellers. In our analysis, the benefit of centralized clearing is the mutualization of counterparty default risk. Also, while centralized clearing makes excessive risk positions observable in Acharya and Bisin (2010) or elicitable in Leitner (2012), the benefit of centralized clearing in our analysis applies even when the effort to search for creditworthy counterparties is observable.

Our analysis is related to Koeppl and Monnet (2010), who also consider the mutualization benefit of CCPs. But the market frictions they analyze differ from ours. They consider a bargaining process that gives rise to inefficiencies and a setting where institutions privately conduct trades, which they must be incentivized to reveal. Our focus is on optimal contracts, attaining information constrained Pareto optimality, and on trades that are observable and contractible. Unlike Koeppl and Monnet (2010), we assume that protection buyers must exert effort to screen and monitor counterparties (and we also consider the case in which this effort is unobservable). Our conclusion that, to preserve incentives, protection buyers should not be fully insured against counterparty risk is the opposite of what Koeppl and Monnet (2010) conclude. Another related paper is Carapella and Mills (2012). Focusing on information acquisition incentives, they show that by providing counterparty risk insurance (and multi-lateral netting), CCPs reduce counterparties' incentives to acquire information about centrally cleared securities, making such securities less information sensitive and more liquid. They consider asymmetric information about the value of the underlying asset, while we study asymmetric information about the effort of the protection buyer to find creditworthy counterparties.

Our analysis of the mutualization benefits of CCPs also echoes the analysis of the netting benefits of CCPs by Duffie and Zhu (2011). Taking deposit constraints in different systems as given, they study which system is more economical in terms of collateral requirements. This is motivated by their observation that collateral deposits are costly. The objective of their analysis is the *netting efficiency* of the system. In contrast, while we also take into account the cost of deposits, we endogenize the deposits requested, and the objective in our analysis is the *risk–sharing efficiency* of the system. While the risk aversion of the agents and their incentive compatibility constraints play an important role in our analysis, they are absent from Duffie and Zhu (2011).[11]

Finally, our analysis is also related to Diamond and Dybvig (1983) and Hellwig (1994). In Diamond and Dybvig (1983), a continuum of households can experience early or late consumption needs. Because consumption needs are i.i.d. across households, uncertainty about these idiosyncratic shocks washes out in the aggregate by the law of large numbers. Competitive banks implement the Pareto–optimal mechanism when consumption needs are observable, as they can fully mutualize the idiosyncratic liquidity risk of households. Hellwig (1994) studies aggregate interest rate risk in a Diamond and Dybvig-like set-up. Efficient risk–sharing requires that households bear some of this aggregate risk, even when consumption needs are publicly observable. When such needs are privately observed, incentive compatibility constraints give an additional reason for households to be exposed to aggregate risk. Our analysis also considers risk-sharing in an environment with both idiosyncratic and aggregate shocks, and under incentive compatibility constraints, but there are substantial differences in the objects of analysis in Hellwig (1994) and in our paper: banks versus CCP, value of assets versus consumption needs, liquidity risk versus counterparty risk, adverse selection versus moral hazard.

4 The model

There are five dates, $t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ and 1. A unit mass continuum of risk-averse protection buyers faces a large population of risk-neutral protection sellers.[12] The discount rate of all market participants is normalized to one and the risk-free rate to 0.

At time $t = 0$, each protection seller i is endowed with one unit of assets-in-place, which

[11] We consider only one market, whereas Duffie and Zhu (2009) analyze netting efficiency in a multi–market setting. They point out that when traders intervene in different markets, having separate CCPs in different markets can lead to excessive collateral deposits.

[12] Concavity of the objective function of the protection buyer can reflect institutional, financial or regulatory constraints, such as leverage constraints or risk-weighted capital requirements.

returns $\tilde{R}_i$ at $t = 1$. The protection sellers are heterogeneous. Some of them are solid, creditworthy institutions, which we hereafter refer to as "good". They generate $\tilde{R}_i = R > 1$ with probability p and 0 otherwise. Others are more fragile, less creditworthy institutions, hereafter referred to as "bad". They generate $\tilde{R}_i = R$ with probability $p - \delta$ only.[13] When protection seller i is good (resp. bad), we denote this by $\xi_i = 1$ (resp. $\xi_i = 0$), and correspondingly the probability of default of protection seller i is denoted by $1 - p(\xi_i)$. The protection sellers' positions are completely illiquid, i.e., their liquidation value before time 1 is zero. All protection sellers are risk-neutral, have no initial endowment apart from the illiquid asset generating R or 0, and have limited liability.

At time $t = 0$, each protection buyer j is endowed with an asset whose random final value $\tilde{\theta}_j$ realizes at time $t = 1$. We assume that $\tilde{\theta}_j$ can take on two values: $\bar{\theta}$ with probability $\frac{1}{2}$ and $\underline{\theta}$ otherwise. The asset owned by the protection buyer can be thought of as a loan portfolio, and $\underline{\theta}$ can be interpreted as occurring when the loans are only partially repaid. We assume that $R > \bar{\theta} - \underline{\theta} = \Delta\theta$, which implies that when protection sellers do not default, they can fully insure protection buyers against their risk $\tilde{\theta}$. We also assume that all exogenous random variables are independent.

Because the protection buyers are risk–averse while the protection sellers are risk–neutral, there are potential gains from trade. But to reap these gains from trade, protection buyers must contact protection sellers. At time $t = \frac{1}{4}$, protection buyer j can choose to exert effort ($e_j = 1$) and devote resources to finding a good counterparty. This involves searching for counterparties, screening them, and checking their risk exposure and financial solidity.[14] Denote the corresponding cost by B. Matches occur at time $t = \frac{1}{2}$. When exerting effort, a protection buyer finds a good protection seller with probability one. Alternatively, protection buyer j may choose not to exert costly effort ($e_j = 0$). In this case, he finds a bad protection seller with probability one.[15] The preferences of the protection buyers are quasi–linear, that

[13] Stephens and Thompson (2011) also analyze a model with "good" and "bad" protection sellers. But they define good and bad types differently. In Stephens and Thompson (2011), good protection sellers invest the insurance premium in liquid but low return assets, which can be pledged to pay the insurance, while bad protection sellers invest the premium in illiquid high return assets, which cannot be pledged to pay the insurance. In contrast, in the present paper, bad protection sellers have both a higher probability of default and a lower rate of return than good protection sellers. Furthermore, in Stephens and Thompson (2011) the presence of the two types of protection sellers is associated with an adverse selection problem, while in the present paper it corresponds to a moral hazard problem.

[14] This can be interpreted in terms of due diligence.

[15] A richer model of the search process would have probabilities strictly between 0 and 1 to find good counterparties. The probabilities would reflect the number of available good protection sellers. While our 0–1 specification is more stylized, and hence more tractable, both specifications yield the same qualitative

is, there exists a concave utility function u such that the utility of protection buyer j with consumption x is $u(x) - e_j B$.

Ex–ante, the types of protection sellers are unobservable. At time $t = \frac{1}{2}$, however, protection buyer j observes the type of protection seller i with whom he is matched.

At time $\frac{3}{4}$, aggregate macroeconomic uncertainty is resolved. But for simplicity and clarity, we assume until Section 6 that there is no aggregate risk and hence we postpone the exposition of how we model this risk until then. We relax this assumption in Section 6 and study how the introduction of aggregate risk alters the economics of clearing and risk-sharing in our framework.

Our model starts at time $t = 0$, when a market infrastructure is put in place. We consider three possibilities: bilateral trade between a protection buyer and a protection seller (no clearing), trilateral trade with a clearing agent (decentralized clearing), or multilateral contracting with a CCP (centralized clearing). Figure 1 summarizes the sequence of events.

For simplicity, but without affecting the results qualitatively, we assume that the protection buyer has all the bargaining power. Thus, contracts are designed to maximize the protection buyers' expected utility, subject to the participation, feasibility and incentive constraints spelled out below.

At time 1, the realizations of $\tilde{R}_i$, $\tilde{\theta}_j$ and $\tilde{\gamma}$ are observable and contractible. Until Section 7, we assume for simplicity and clarity that the effort e_j of protection buyers is observable and contractible. Thus, the optimal clearing arrangement we characterize until Section 7 implements the first–best, subject to the limited liability and search constraints. In Section 7, we introduce moral hazard and analyze the information-constrained optimal clearing arrangement that implements the second-best.

5 Idiosyncratic risk and observable effort

In this section, we study optimal risk-sharing contracts when protection buyers' efforts are observable and they are only exposed to idiosyncratic risk. We first characterize the optimal bilateral contract between a protection buyer and a protection seller without a clearing agent. We then consider trilateral contracting between the two parties and a single clearing agent. We conclude the section with the analysis of the optimal multilateral contract with a CCP.

insights.

5.1 Bilateral contracting without clearing agent

The contract, offered at time $t = \frac{1}{2}$ once a match has been made, spells out the transfer τ from the protection seller to the protection buyer.[16] When τ is positive, the protection seller pays the protection buyer. When τ is negative, the protection buyer pays the protection seller. The transfer τ is contingent on the value of the assets of the protection buyer $(\tilde{\theta}_j)$ and the protection seller $(\tilde{R}_i)$. Figure 2 depicts the bilateral relations, and transfers τ, in the market without clearing. Only two (representative) pairs of protection–buyers and protection–sellers are depicted, but the same structure applies to all matches. The protection seller has limited liability, hence τ is such that

$$R_i \geq \tau(\theta_j, R_i), \quad \forall(\theta_j, R_i). \tag{1}$$

First, consider the case in which the protection buyer chooses to exert the search effort $(e_j = 1)$ and hence he is matched with a good protection seller $(\xi_i = 1)$. The transfer τ maximizes his expected utility subject to the limited liability and the participation constraint of the protection seller. Thus

$$\max_{\tau} \; E[u(\tilde{\theta} + \tau(\tilde{\theta}, \tilde{R}_i))|\xi_i = 1] - B, \tag{2}$$

subject to (1) and the participation constraint:

$$0 \geq E[\tau(\tilde{\theta}, \tilde{R}_i)|\xi_i = 1]. \tag{3}$$

The solution to this optimization problem is given in Proposition 1.

Proposition 1 *In the bilateral contract with effort,*

$$\bar{\theta} + \tau(\bar{\theta}, R) = \bar{\theta} + \tau(\bar{\theta}, 0) = \underline{\theta} + \tau(\underline{\theta}, R), \; \tau(\underline{\theta}, R) = \frac{\Delta\theta}{1+p} \quad \textit{and} \quad \tau(\underline{\theta}, 0) = 0.$$

Proposition 1 states that there is full risk-sharing in the optimal bilateral contract with effort as long as the protection seller does not default. The protection buyer is exposed to counterparty risk in state $(\underline{\theta}, 0)$, which occurs with probability $\frac{1}{2}(1-p)$.

[16] An important setting for which our analysis is relevant is the CDS market. In our optimal contracting approach, however, instead of specifying payoffs matching the features of a given type of contract, we allow for general transfers. For complex distributions such generality could prove untractable. For the simple distributions we assume it does not.

Second, consider the case in which the protection buyer does not exert effort. The optimal contract solves

$$\max_{\tau} \; E[u\left(\tilde{\theta} + \tau(\tilde{\theta}, \tilde{R}_i)\right) |\xi_i = 0], \tag{4}$$

subject to (1) and

$$0 \geq E[\tau(\tilde{\theta}, \tilde{R}_i)|\xi_i = 0]. \tag{5}$$

Proposition 2 presents the solution to this optimization problem.

Proposition 2 *In the bilateral contract without effort,*

$$\bar{\theta} + \tau(\bar{\theta}, R) = \bar{\theta} + \tau(\bar{\theta}, 0) = \underline{\theta} + \tau(\underline{\theta}, R), \; \tau(\underline{\theta}, R) = \frac{\Delta\theta}{1 + p - \delta} \quad \text{and} \quad \tau(\underline{\theta}, 0) = 0.$$

As in Proposition 1, there is full risk-sharing as long as the protection seller does not default. But in Proposition 2 the protection buyer receives a higher transfer in state $\underline{\theta}$ when the protection seller does not default, and he pays less to the protection seller in state $\bar{\theta}$. The protection seller is willing to accept these terms (apparently more attractive to the protection buyer), because the probability that he will actually pay the protection buyer is lower (while the probability that he will be paid remains the same). Indeed, without effort, the probability of counterparty default is higher, and equal to $\frac{1}{2}(1 - p + \delta)$. Combining Proposition 1 and Proposition 2 we obtain the condition under which, without clearing, search effort is optimal.

Proposition 3 *Without clearing, the protection buyer prefers to exert the search effort if and only if u is concave enough and B is low enough.*

5.2 Decentralized clearing

Now we turn to the case in which, in addition to the protection buyer and the protection seller, there is a clearing agent. The contract designed at time $t = \frac{1}{2}$ now involves, in addition to τ, the transfer τ^C from the clearing agent to the protection buyer. When τ^C is positive the clearing agent pays the protection buyer, while when τ^C is negative the protection buyer pays the clearing agent. Figure 3 depicts the trilateral relations, and transfers τ and τ^C in the market with decentralized clearing. Again, only two (representative) pairs of protection–buyers and protection–sellers are depicted, but the same structure applies to all matches.

The clearing agent is risk-neutral, has limited liability and is endowed with c units of an asset with a per-unit return at time $t = 1$ of $\rho > 1$. The asset is illiquid, however, and cannot be used to make insurance payments at time $t = 1$. To be able to pay the protection seller at $t = 1$, the clearing agent must liquidate a fraction α of his asset at time $t = 0$ and invest it in a liquid asset. The opportunity cost of doing so is that the return on the liquid asset is lower than that on the illiquid asset.[17] For simplicity we assume the liquid asset only earns the risk–free return, and we normalize the risk–free rate of return to 0.

Since the protection seller faces no such opportunity cost, he will continue to insure the protection buyer against the $\tilde{\theta}$ risk, as long as he is not in default and the clearing agent will draw from his safe deposit to pay the protection buyer only when $\theta_j = \underline{\theta}$ and $R_i = 0$. The limited liability condition of the clearing agent implies that

$$\alpha c \geq \tau^C(\underline{\theta}, 0), \tag{6}$$

while his participation constraint is

$$\rho c \leq \alpha c + (1-\alpha)\,\rho c - E(\tau^C|\xi_i). \tag{7}$$

In the two propositions below, we characterize the optimal contract with effort. It solves

$$\max_{\alpha,\tau,\tau^C} \; E[u(\theta + \tau + \tau^C)|\xi = 1] - B \tag{8}$$

subject to the limited liability and participation constraints of the protection seller, (1) and (3), and the clearing agent (6) and (7), respectively.

Proposition 4 *When there is no moral hazard and no aggregate risk, in the contract with effort, the clearing agent invests $\alpha c = \tau^C(\underline{\theta}, 0)$ in the safe asset. As long as the protection seller does not default, there is full risk-sharing, i.e.,*

$$\bar{\theta} + \tau(\bar{\theta}, R) + \tau^C(\bar{\theta}, R) = \bar{\theta} + \tau(\bar{\theta}, 0) + \tau^C(\bar{\theta}, 0) = \underline{\theta} + \tau(\underline{\theta}, R) + \tau^C(\underline{\theta}, R),$$

but the protection buyer is not fully insured against counterparty risk, i.e.,

$$\underline{\theta} + \tau^C(\underline{\theta}, 0) < \underline{\theta} + \tau(\underline{\theta}, R) + \tau^C(\underline{\theta}, R).$$

[17] This aspect of our model is in line with Thompson (2010). In contrast with Thompson's assumption that the portfolio choice of the protection seller is private information, we assume the liquid holdings of the clearing agent are observable. Thus, in our analysis, in contrast with Thompson (2010), there is no moral–hazard problem associated with safe liquid holdings.

Insurance against counterparty risk is only partial since its provision entails the opportunity cost $\rho - 1$. To minimize this cost, the fraction of funds invested in the safe asset is just equal to the amount to be paid to the protection buyer in case the protection seller defaults. As in the case without clearing, there is full risk-sharing as long as the protection seller does not default, and hence:

$$\tau(\underline{\theta}, R) = \frac{\Delta\theta}{1+p}. \tag{9}$$

The binding participation constraint of the clearing agent can be written as:[18]

$$(\rho - 1)\tau^C(\underline{\theta}, 0) = -[\frac{1+p}{2}\tau^C(\underline{\theta}, R) + \frac{1-p}{2}\tau^C(\underline{\theta}, 0)], \tag{10}$$

i.e., the expected net transfer from the protection buyer to the clearing agent (i.e., the right–hand–side of (10)) covers the latter's opportunity cost of liquidating a fraction of his initial assets to hold liquid assets to pay insurance against counterparty default (i.e., the left–hand–side of (10)).

A measure of the insurance against counterparty risk is the ratio of the marginal utilities of the protection buyer in state $\underline{\theta}$ when the protection seller defaults and when he does not default:

$$\frac{u'(\underline{\theta} + \tau^C(\underline{\theta}, 0))}{u'\left(\underline{\theta} + \tau(\underline{\theta}, R) + \tau^C(\underline{\theta}, R)\right)} > 1. \tag{11}$$

The ratio is greater than one since there is only partial insurance against counterparty risk (Proposition 4). The greater the insurance provided by the clearing agent, the lower this ratio, and with full insurance the ratio goes down to 1. Substituting the transfers $\tau(\underline{\theta}, R)$ and $\tau^C(\underline{\theta}, R)$ from (9) and (10), the ratio in (11) rewrites as a function φ of the transfer $\tau^C(\underline{\theta}, 0)$ paid by the clearing agent in state $\underline{\theta}$ when the protection seller defaults (and of exogenous parameters):

$$\varphi\left(\tau^C(\underline{\theta}, 0)\right) \equiv \frac{u'(\underline{\theta} + \tau^C(\underline{\theta}, 0))}{u'\left(\underline{\theta} + \frac{\Delta\theta}{1+p} - \frac{1-p+2(\rho-1)}{1+p}\tau^C(\underline{\theta}, 0)\right)}.$$

Higher insurance against counterparty default (larger $\tau^C(\underline{\theta}, 0)$) reduces φ, lowering the ratio of marginal utilities closer to one. The following proposition characterizes the optimal degree of counterparty risk insurance in the contract with effort.

[18] For details, see the proof of Proposition 4.

Proposition 5 *When there is no moral hazard and no aggregate risk, in the contract with effort, if* $\varphi(0) \leq 1 + \frac{2(\rho-1)}{1-p}$*, then the clearing agent is not used and* $\alpha^* = 0$*. Otherwise, if*

$$\varphi(c) > 1 + \frac{2(\rho - 1)}{1 - p}, \tag{12}$$

then $\alpha^* = 1$*, while if (12) does not hold, then* $\alpha^* \in (0, 1)$ *and* $\tau^C(\underline{\theta}, 0)$ *is given by*

$$\varphi\left(\tau^C(\underline{\theta}, 0)\right) = 1 + \frac{2(\rho - 1)}{1 - p}. \tag{13}$$

The optimal degree of insurance against counterparty risk balances its benefits (left-hand side of (13)) with the opportunity cost of holding cash reserves (right-hand side of (13)). When the opportunity cost of cash reserves $(\rho - 1)$ is very high, there is little insurance. As the probability of counterparty default $(1 - p)$ rises, insurance increases.

The optimal contract without effort solves

$$\max_{\alpha,\tau,\tau^C} E[u(\theta + \tau + \tau^C)|\xi = 0] \tag{14}$$

subject to the limited liability and participation constraints of the protection seller and the clearing agent. As in the effort case, i) there is full risk-sharing as long as the protection seller does not default, and ii) the contract provides only partial insurance against the default of the protection seller.[19] Again, one can define a measure of the amount of insurance against counterparty risk as the ratio of the marginal utilities across the states $(\underline{\theta}, 0)$ and $(\underline{\theta}, R)$:

$$\phi\left(\tau^C(\underline{\theta}, 0)\right) \equiv \frac{u'(\underline{\theta} + \tau^C(\underline{\theta}, 0))}{u'\left(\underline{\theta} + \frac{\Delta\theta}{1+p-\delta} - \frac{1-p+\delta+2(\rho-1)}{1+p-\delta}\tau^C(\underline{\theta}, 0)\right)}.$$

We then have the following counterpart of Proposition 5.

Proposition 6 *When there is no moral hazard and no aggregate risk, in the contract without effort, if* $\phi(0) \leq 1 + \frac{2(\rho-1)}{1-p+\delta}$*, then the clearing agent is not used and* $\alpha^* = 0$*. Otherwise, if*

$$\phi(c) > 1 + \frac{2(\rho - 1)}{1 - p + \delta}, \tag{15}$$

then $\alpha^* = 1$*, while if (15) does not hold, then* $\alpha^* \in (0, 1)$ *and* $\tau^C(\underline{\theta}, 0)$ *is given by*

$$\phi\left(\tau^C(\underline{\theta}, 0)\right) = 1 + \frac{2(\rho - 1)}{1 - p + \delta}. \tag{16}$$

[19] For the sake of brevity, we do not state these results formally. Their derivation is exactly as in Proposition 4. One only replaces p with $(p - \delta)$, so $\tau(\underline{\theta}, R)$, for example, is now given by $\tau(\underline{\theta}, R) = \frac{\Delta\theta}{1+(p-\delta)}$.

In the contract without effort, the clearing agent provides more insurance to the protection buyer compared to the contract with effort. This is in line with the observation that the amount put in the safe asset is increasing in the probability of counterparty default. But, since insurance is costly, it remains partial. And since the amount paid to the protection buyer in case of counterparty default is higher when there is no effort, so is the amount paid to the clearing agent in state $\bar{\theta}$ to compensate him for insuring against counterparty risk. The protection buyer prefers to exert search effort if and only if

$$\frac{1+p}{2}u\left(\underline{\theta}+\tau(\underline{\theta},R|\xi_j=1)+\tau^C(\underline{\theta},R|\xi_j=1)\right)+\frac{1-p}{2}u(\underline{\theta}+\tau^C(\underline{\theta},0|\xi_j=1))-B\geq$$
$$\frac{1+p-\delta}{2}u\left(\underline{\theta}+\tau(\underline{\theta},R|\xi_j=0)+\tau^C(\underline{\theta},R|\xi_j=0)\right)+\frac{1-p+\delta}{2}u(\underline{\theta}+\tau^C(\underline{\theta},0|\xi_j=0)),$$

which holds if the cost of effort (B) is low and the increase in default risk due to lack of effort (δ) is high.

5.3 Multilateral contracting with a CCP

Now consider the case in which there is a CCP interposing between all the pairs of protection buyers and sellers. We maintain our assumption that effort is observable, so the CCP can request it without facing incentive constraints.[20]

At time $t=0$, a benevolent central planner designing the contracts (including the features of the CCP) maximizes the ex–ante expected utility of a representative protection buyer, subject to limited liability, participation and budget constraints. To compare with the case of decentralized clearing of the previous section, we assume that the CCP is endowed with c units of an asset with a per-unit return at time $t=1$ of $\rho>1$. As in the case of decentralized clearing, it is without loss of generality or efficiency to focus on contracts where the CCP pays $\tau^C(\underline{\theta},0)\geq 0$ to the protection buyer when state $(\underline{\theta},0)$ occurs and where it pays $\tau^C(\bar{\theta},R)=\tau^C(\bar{\theta},0)=\tau^C(\underline{\theta},R)\leq 0$ in the other states. That is, the CCP receives a participation fee paid by its members for the CCP's insurance against the default of protection sellers. Figure 4 depicts the multilateral relations, and transfers τ and τ^C in the market with centralized clearing. Only 6 (representative) pairs of protection–buyers and protection–sellers are depicted, but the same structure applies to all matches.

[20] Since the link between the search effort of the protection buyer and the type of the protection seller he is matched with is one to one, it makes no difference whether the contract is contingent on effort or on protection seller type.

The aggregate net transfer from the CCP at time $t = 1$ is

$$\int_{i=0}^{1} \{\mathbf{1}(\theta_i = \underline{\theta})\mathbf{1}(R_i = 0)\tau^C(\underline{\theta}, 0) + [1 - \mathbf{1}(\theta_i = \underline{\theta})\mathbf{1}(R_i = 0)]\tau^C(\bar{\theta}, R)\}di, \tag{17}$$

where $\mathbf{1}$ denotes the indicator variable and i indexes the protection buyer-protection seller pairs. Because the $\tilde{\theta}_i$ and $\tilde{R}_i$ are independent, the aggregate mass of defaults is deterministic by the law of large numbers and (17) is equal to

$$\frac{1 - p(\xi)}{2}\tau^C(\underline{\theta}, 0) + \frac{1 + p(\xi)}{2}\tau^C(\bar{\theta}, R).$$

The participation constraint of the CCP is

$$\alpha c\,(\rho - 1) \leq -\frac{1 - p(\xi)}{2}\tau^C(\underline{\theta}, 0) - \frac{1 + p(\xi)}{2}\tau^C(\bar{\theta}, R). \tag{18}$$

Note that increasing α tightens the participation constraint. Holding liquid assets carries an opportunity cost for which the CCP must be compensated. As in the single clearing agent case, the participation constraint is binding and the following result is immediate.

Proposition 7 *When there is no moral hazard and no aggregate risk, the optimal CCP does not liquidate any of its assets ($\alpha = 0$) and provides full insurance against counterparty risk.*

The intuition of Proposition 7 is the following. When there is no aggregate risk, by mutualizing all the contracts, the CCP can collect an aggregate amount of fees that exactly covers the aggregate amount of insurance payments. Thanks to mutualization, the CCP, unlike individual clearing agents, does not need to hold liquid assets and therefore avoids the corresponding opportunity cost. Since there is no opportunity cost of providing insurance against counterparty risk, the CCP can fully insure protection buyers, regardless of whether they exert search effort or not. This leads to the next proposition.

Proposition 8 *When there is no moral hazard and no aggregate risk, it is optimal for protection buyers not to exert search effort.*

When all risk is idiosyncratic, the CCP can mutualize all risk and fully insure against counterparty risk. Consequently, when there is no aggregate risk, protection buyers are fully insured. Creditworthy protection sellers are no longer needed and protection buyers no longer need to search for them.[21] In this context, while the CCP achieves the first-best when there is no aggregate risk, it increases the occurrence of counterparty default. Yet, this increase in (aggregate) default is not a symptom of a market failure.

With only idiosyncratic risk and mutualization by the CCP, protection sellers actually no longer need to be part of the risk-sharing transaction. Risk-mutualization among protection buyers is enough. This is not the case when there is aggregate risk, a situation to which we turn next. In that case, mutualization is not enough to deliver full insurance, protection sellers play an important role and protection buyers' effort can be optimal, as it increases the total amount of resources available to insure against aggregate risk.

6 Aggregate risk and observable effort

6.1 Aggregate risk

We now study the optimal contract with a CCP when there is aggregate risk, while maintaining in this section the assumption that there is no moral hazard. To extend our analysis to aggregate risk, we extend our set-up as follows. There are two macro-states in the economy, high and low, each occurring with equal probability. In the high state, the probability of the protection buyer's high asset value $\bar{\theta}$ increases to $\frac{1}{2}+\gamma$, while the probability of the low asset value $\underline{\theta}$ decreases to $\frac{1}{2}-\gamma$. In the low state, the opposite happens. The probability of $\bar{\theta}$ becomes $\frac{1}{2}-\gamma$, while the probability of $\underline{\theta}$ becomes $\frac{1}{2}+\gamma$. Conditional on the macro-state, the realizations of the $\tilde{\theta}_j$ are i.i.d. Note that ex-ante, the two values $\underline{\theta}$ and $\bar{\theta}$ are equiprobable, as in previous sections. But ex-post, after observing the macro-state at time $t=\frac{3}{4}$, one of them becomes more likely. When $\gamma=0$ there is no aggregate risk and protection buyers are exposed only to idiosyncratic risk. At the other extreme, when $\gamma=\frac{1}{2}$, there is only aggregate risk. The values of all the protection buyers' assets are perfectly correlated.

[21] This underscores that, in spite of formal similarities, the setup we consider is very different from that analyzed by Holmström and Tirole (1998). In Holmström and Tirole (1998), at cost B, the agent can reduce the probability of low output. Thus, effort increases the average output in the economy. Even with risk-neutrality this can be valuable, if B is low enough. In the present model, at cost B, the agent can find a protection seller with low probability of default. But, effort does not increase the average output in the economy: the overall output of the protection sellers is exogenous and unaffected by the protection buyers' efforts. Therefore, the effort of the protection buyers can be useful only if it increases their risk-sharing ability. However, with mutualization and no aggregate risk, full risk-sharing can be achieved even if the protection sellers are bad. Hence, effort is not optimal.

We now consider the intermediate case $\gamma \in (0, \frac{1}{2})$, i.e., there is combination of aggregate and idiosyncratic risk. We also assume that

$$pR > \frac{\Delta\theta}{2} > (p - \delta) R, \tag{19}$$

that is, when all protection buyers exert effort, the aggregate amount of resources of good protection sellers (pR) is sufficient to fully insure protection buyers (whose exposure is $\frac{\Delta\theta}{2}$), while this may not be feasible when protection buyers do not exert the search effort and only bad protection sellers are used (whose aggregate resources are $(p - \delta) R$). To focus on the effect of aggregate risk in the simplest possible set-up, we also assume that the CCP is not endowed initially with any asset, i.e., $c = 0$.

The macro-state occurs at time $\frac{3}{4}$, after the market infrastructure is in place, contracts have been designed and search effort has been exerted. The realization of the macro-state is publicly observable and contractible. Thus, the transfers are now contingent on the macro-state, $\tau(\theta, R, \gamma)$. To reduce the burden of notation we denote by $\left(\bar{\tau}, \bar{\tau}^C\right)$ the transfers of the protection seller and of the CCP in the high macro–state and by $\left(\underline{\tau}, \underline{\tau}^C\right)$ the transfers in the low macro–state. As before, contracts are designed at time $t = 0$ to maximize the expected utility of a representative protection buyer.

6.2 Optimal clearing and contracting with effort

Consider first the case in which protection buyers exert search effort. The optimal contract sets $(\underline{\tau}, \underline{\tau}^C, \bar{\tau}, \bar{\tau}^C)$ to maximize

$$\frac{1}{2}E[u(\theta + \bar{\tau} + \bar{\tau}^C)|\xi_i = 1, \text{high macro–state}] + \frac{1}{2}E[u(\theta + \underline{\tau} + \underline{\tau}^C)|\xi_i = 1, \text{low macro–state}] - B, \tag{20}$$

subject to the participation constraint of protection sellers

$$0 \geq \frac{1}{2}E[\bar{\tau}|\xi_i = 1, \text{high macro–state}] + \frac{1}{2}E[\underline{\tau}|\xi_i = 1, \text{low macro–state}], \tag{21}$$

and their limited liability constraints and the budget constraint of the CCP. The solution is given in the following proposition:

Proposition 9 *With aggregate risk but no moral hazard, the optimal contract with centralized clearing and search effort provides full insurance to protection buyers, whose expected* *utility is*

$$u\left(E\left[\tilde{\theta}\right]\right) - B. \tag{22}$$

When protection sellers do not default, the transfers to protection buyers are

$$\underline{\tau}(\underline{\theta}, R) = \frac{1}{\frac{1+p}{2} - \gamma(1-p)}\frac{\Delta\theta}{2} > \bar{\tau}(\underline{\theta}, R) = \frac{1}{\frac{1+p}{2} + \gamma(1-p)}\frac{\Delta\theta}{2}, \tag{23}$$

while in case of counterparty default, the transfer from the CCP to protection buyers is

$$\underline{\tau}^C(\underline{\theta}, 0) = \bar{\tau}^C(\underline{\theta}, 0) = \frac{\Delta\theta}{2}. \tag{24}$$

Note that the transfers from protection sellers to protection buyers are feasible. The transfer $\underline{\tau}(\underline{\theta}, R)$ reaches its maximum when $\gamma = \frac{1}{2}$ (only aggregate risk) and the feasibility constraint

$$R \geq \underline{\tau}(\underline{\theta}, R) \tag{25}$$

holds by condition (19).

Since the aggregate resources of protection sellers are sufficient to provide full insurance, the expected utility of a protection buyer exerting search effort is the same with or without aggregate risk. Competitive and risk-neutral protection sellers are willing to provide insurance as long as they break even on average. Hence, insurance comes at no cost to protection buyers, except for the search cost B. As in the case without aggregate risk, the CCP can provide optimal risk-sharing. But unlike the case without aggregate risk, protection buyers must now search for creditworthy protection sellers. Their resources are needed to insure against aggregate risk.

6.3 Optimal clearing and contracting without effort

We now consider the contract without effort. The optimal contract sets $(\underline{\tau}, \underline{\tau}^C, \bar{\tau}, \bar{\tau}^C)$ to maximize

$$\frac{1}{2}E[u(\theta+\bar{\tau}+\bar{\tau}^C)|\xi_i = 0, \text{high macro–state}] + \frac{1}{2}E[u(\theta+\underline{\tau}+\underline{\tau}^C)|\xi_i = 0, \text{low macro–state}], \tag{26}$$

subject to the participation constraint of the protection seller

$$0 \geq \frac{1}{2}E[\bar{\tau}(\tilde{\theta}, \tilde{R}_i)|\xi_i = 0, \text{high macro–state}] + \frac{1}{2}E[\underline{\tau}(\tilde{\theta}, \tilde{R}_i)|\xi_i = 0, \text{low macro–state}],$$

and their limited liability constraints and the budget constraint of the CCP. The solution is given by the following proposition.

Proposition 10 *Consider the case with aggregate risk but no moral hazard. If*

$$R \geq \frac{1}{\frac{1+p-\delta}{2} - \gamma\left(1-p+\delta\right)} \frac{\Delta\theta}{2} \tag{27}$$

then the optimal contract with centralized clearing and no search effort provides full insurance to protection buyers, whose expected utility is

$$u\left(E\left[\tilde{\theta}\right]\right). \tag{28}$$

When protection sellers do not default, the transfers to protection buyers are

$$\underline{\tau}(\underline{\theta}, R) = \frac{1}{\frac{1+p-\delta}{2} - \gamma(1-p+\delta)} \frac{\Delta\theta}{2} > \bar{\tau}(\underline{\theta}, R) = \frac{1}{\frac{1+p-\delta}{2} + \gamma(1-p+\delta)} \frac{\Delta\theta}{2}, \tag{29}$$

while in case of counterparty default, the transfer from the CCP to protection buyers is

$$\bar{\tau}^C(\underline{\theta}, 0) = \underline{\tau}^C(\underline{\theta}, 0) = \frac{\Delta\theta}{2}. \tag{30}$$

If (27) does not hold, then protection buyers are fully insured in the high macro-state but not in the low one so that

$$\bar{\tau}^C(\underline{\theta}, 0) > \underline{\tau}^C(\underline{\theta}, 0). \tag{31}$$

The contract without search effort mirrors the contract with search effort (Proposition 9) except that the probability of seller default is higher ($1 - p + \delta$ instead of $1 - p$). The main difference is, however, that the aggregate resources of protection sellers may not be enough to fully insure protection buyers. Condition (27) stems from substituting the transfer $\underline{\tau}(\underline{\theta}, R)$ from (29) into the feasibility constraint (25). The right-hand-side of (27) increase in the amount of aggregate risk γ. Higher aggregate risk requires a larger total transfer from protection sellers to protection buyers in the low macro–state. Hence, there exists a threshold level of aggregate risk above which full insurance is no longer feasible,

$$\gamma^* = \frac{1 + p - \delta - \frac{\Delta\theta}{R}}{2\left(1-p+\delta\right)} < \frac{1}{2}. \tag{32}$$

When aggregate risk is large, in the sense that $\gamma > \gamma^*$, and protection buyers do not exert search effort, then the aggregate amount of resources in the low macro-state is insufficient to provide full risk-sharing even with a CCP, and (31) applies. Hence, when $\gamma > \gamma^*$, protection buyers prefer to exert search effort if they are sufficiently risk averse and the search cost is low (u concave and B low enough).

When aggregate risk is small, $\gamma < \gamma^*$, the aggregate resources of bad protection sellers are enough to provide full insurance. Protection buyers do not need to incur the search cost B and obtain expected utility (28), which is greater than the expected utility in (22). In that case, searching for creditworthy counterparties is not optimal (as in Proposition 8).[22]

When protection buyers decide ex-ante not to exert effort, one might wonder why they do not search for good protection sellers ex–post when the low macro-state occurs? This is because, when the low macro-state occurs, it is too late to share the aggregate risk. The effort to search for creditworthy counterparties enhances the risk-sharing capacity, but this can only be done ex–ante, before the resolution of uncertainty.

To illustrate our analysis, consider the case in which there is only aggregate risk, i.e., $\gamma = \frac{1}{2}$. In that case condition (27) is equivalent to $(p-\delta)\, R \geq \frac{\Delta\theta}{2}$ which cannot hold due to (19). Thus, full risk-sharing is not feasible without exerting search effort. The following proposition describes protection buyers' decision to search for good protection sellers.

Proposition 11 *With aggregate risk only and no moral hazard, protection buyers prefer to exert the search effort if and only if*

$$u\left(E\left[\tilde{\theta}\right]\right) - B \geq \frac{1}{2} u\left(\bar{\theta} - (p-\delta)\, R\right) + \frac{1}{2} u\left(\underline{\theta} + (p-\delta)\, R\right). \tag{33}$$

The left–hand–side of (33) is the expected utility of fully hedged protection buyers when they exert effort. The right–hand–side is their expected utility without effort. When the low macro–state occurs, the value of the assets of all protection buyers is $\underline{\theta}$. The aggregate amount of resources of (bad) protection sellers is $(p-\delta)\, R$. The entire amount is transferred to protection buyers when $\underline{\theta}$ occurs to partially insure them. To ensure the participation of the protection sellers, the protection buyers pay them $(p-\delta)\, R$ when $\bar{\theta}$ occurs.

7 Aggregate risk and moral hazard

We now turn to the unobservable effort case. As before, the contract is designed at time $t = 0$ to maximize the ex–ante expected utility of a representative protection buyer, but now, in addition to participation, budget and limited liability constraints, the optimal contract also has to satisfy the protection buyer's incentive compatibility constraint.

[22] Note that, if $\gamma = 0$, i.e., there is no aggregate risk, condition (27) is equivalent to $R \geq \frac{\Delta\theta}{1+p-\delta}$ which holds under (19).

Consider first the case in which effort is requested from the protection buyer. The expected utilities of the protection buyer under effort and no effort can be formally written as in (20) and (26), respectively. Note, however, that under moral hazard, the transfers $\left(\tau, \tau^C\right)$ cannot be contingent on effort, i.e., the same contract must be implemented, whether the agent exerts effort or not. Expressing the expected utilities explicitly (using Proposition 4 and its proof) and comparing them, effort is incentive compatible, i.e., the benefit from finding a solid counterparty is larger than the search cost, when

$$\frac{1}{2}\left(\frac{1}{2}-\gamma\right)\delta\left[u(\underline{\theta}+\bar{\tau}(\underline{\theta},R)+\bar{\tau}^C(\underline{\theta},R))-u(\underline{\theta}+\bar{\tau}^C(\underline{\theta},0)\right]+$$
$$\frac{1}{2}\left(\frac{1}{2}+\gamma\right)\delta\left[u(\underline{\theta}+\underline{\tau}(\underline{\theta},R)+\underline{\tau}^C(\underline{\theta},R))-u(\underline{\theta}+\underline{\tau}^C(\underline{\theta},0))\right]\geq B. \tag{34}$$

The first term on the left–hand side is the probability that the high macro–state occurs, times the probability of $\underline{\theta}$ in the high macro–state, times the increase in the probability of counterparty default due to the lack of effort. This first term multiplies the difference between the protection buyer's utility when his counterparty does not default and when it does, in the good macro–state. The next terms on the left–hand side are similar, but for the low macro–state. Hence, the incentive compatibility of search effort hinges on the exposure of the protection buyer to counterparty risk.[23]

Suppose the transfers are as in Proposition 9. That is, the protection buyer obtains full insurance against counterparty default. In that case condition (34) does not hold. Thus, protection buyers have no incentive to search for creditworthy counterparties. Consequently, for such transfers, there is a high (and socially suboptimal) default rate. Hence, we state the following lemma:

Lemma 1 *When protection buyers obtain full insurance against counterparty risk, they do not exert unobservable effort to search for creditworthy counterparties. Hence, to induce such effort, centralized clearing must be accompanied with only partial insurance against counterparty risk.*

[23] The central planner could use a revelation game to elicit truthful messages from the protection buyer and the protection seller about the latter's type. For simplicity we rule this out. To micro-found this restriction one could allow for collusion between the protection buyer and the protection seller (as in Laffont and Martimort, 2000), and assume it is costly for the protection seller to be revealed bad to the central planner (e.g. due to higher regulation cost and compliance burden or loss of reputation). This would preclude costless revelation. If the revelation cost is large enough, it becomes efficient for the central planner to impose a condition similar to (34), precluding full insurance against counterparty risk.

The next proposition solves for the second-best contract with search effort. The classic trade-off between incentives and insurance arises and incentive-compatibility requires that protection buyers remain exposed to some risk.

Proposition 12 *With aggregate risk and unobservable search effort, the optimal contract that induces effort no longer offers full insurance to protection buyers and we have*

$$\begin{aligned}
\underline{\theta} + \bar{\tau}(\underline{\theta}, R) + \bar{\tau}^C(\underline{\theta}, R) &= \underline{\theta} + \underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\underline{\theta}, R) \\
&> \bar{\theta} + \bar{\tau}(\bar{\theta}, R) + \bar{\tau}^C(\bar{\theta}, R) = \bar{\theta} + \underline{\tau}(\bar{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R) \\
&> \underline{\theta} + \bar{\tau}^C(\underline{\theta}, 0) = \underline{\theta} + \underline{\tau}^C(\underline{\theta}, 0)
\end{aligned}$$

The optimal transfers are given by:

$$\frac{1}{2}\delta\left[u(\underline{\theta} + \underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\underline{\theta}, R)) - u(\underline{\theta} + \underline{\tau}^C(\underline{\theta}, 0))\right] = B,$$

$$\underline{\tau}(\bar{\theta}, R) = -p\underline{\tau}(\underline{\theta}, R) + \frac{2\gamma(1-p)}{\frac{1}{2}(1+p) - \gamma(1-p)}\underline{\tau}^C(\underline{\theta}, 0),$$

$$\frac{u'(\underline{\theta} + \underline{\tau}^C(\underline{\theta}, 0))}{u'(\underline{\theta} + \underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\underline{\theta}, R))} = \frac{(1-p)\,u'(\bar{\theta} + \underline{\tau}(\bar{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R))}{u'(\underline{\theta} + \underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\underline{\theta}, R)) - pu'(\bar{\theta} + \underline{\tau}(\bar{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R))},$$

$$\bar{\tau}^C(\bar{\theta}, R) = \frac{-\left(\frac{1}{2} - \gamma\right)(1-p)}{\frac{1}{2}(1+p) + \gamma(1-p)}\underline{\tau}^C(\underline{\theta}, 0) \text{ and } \underline{\tau}^C(\bar{\theta}, R) = \frac{-\left(\frac{1}{2} + \gamma\right)(1-p)}{\frac{1}{2}(1+p) - \gamma(1-p)}\underline{\tau}^C(\underline{\theta}, 0).$$

In the first-best, protection buyers obtain full insurance when they exert search effort. With moral hazard, this is no longer the case. Lemma 1 states that protection buyers must be exposed to counterparty risk to ensure that they exert search effort: There must be a wedge between $\underline{\theta} + \tau(\underline{\theta}, R) + \tau^C(\underline{\theta}, R)$ and $\underline{\theta} + \tau^C(\underline{\theta}, 0)$. Such a wedge can be obtained either by raising $\underline{\theta} + \tau(\underline{\theta}, R) + \tau^C(\underline{\theta}, R)$ or by lowering $\underline{\theta} + \tau^C(\underline{\theta}, 0)$. The latter is more costly since a protection buyer's utility is minimal when $\underline{\theta}$ is realized and the protection seller defaults. To ensure the participation of protection sellers, the increase in $\tau(\underline{\theta}, R)$ must be compensated by a decrease in $\tau(\bar{\theta}, R)$. Hence, protection buyers are no longer insured against their θ-risk, $\underline{\theta} + \tau(\underline{\theta}, R) + \tau^C(\underline{\theta}, R) > \bar{\theta} + \tau(\bar{\theta}, R) + \tau^C(\bar{\theta}, R)$. They are, however, fully insured against aggregate risk. Incentive compatibility requires exposure to counterparty but not aggregate risk, since effort affects the former but not the latter. Moreover, under effort, the aggregate resources of protection sellers are sufficient to absorb the macro-shock.

The optimal contract without effort is as in Proposition 10. Protection buyers prefer the contract with effort if and only if their expected utility with effort is higher than without effort. Since their expected utility in the optimal contract with unobservable effort is necessarily lower than in the first-best, while their expected utility without effort is unaffected by the observability of effort, we have the following proposition:

Proposition 13 *The set of parameters for which the optimal contract requests effort is smaller when effort is unobservable (moral hazard) than when it is observable.*

The proposition states that moral hazard reduces the equilibrium provision of search effort. It therefore reduces the creditworthiness of protection sellers and increases the occurrence of counterparty default in equilibrium.

8 Discussion

8.1 Should participation in the CCP be mandatory?

In the above analysis with decentralized clearing, clearing agents must set aside funds to provide some insurance against counterparty risk. With centralized clearing, this is not necessary as the insurance against counterparty risk is funded by contributions from the CCP's members. If one institution anticipates that all the others will participate in the CCP, then it is optimal for that institution to also participate and benefit from mutualization. But if each institution anticipates that no one else will participate in the CCP, then it is optimal for each of them to opt for decentralized clearing instead. Indeed, when no (or only few) institution(s) participate in the CCP, there is no scope for mutualization. Thus, there are two pure strategy Nash equilibria in the game where institutions have to decide ex–ante whether to opt for centralized or decentralized clearing: one equilibrium in which all institutions opt for decentralized clearing and the other in which they all opt for centralized clearing. If all clearing is initially decentralized, then it may be difficult to change expectations and coordinate on the (Pareto dominant) equilibrium with centralized clearing. In that case, making centralized clearing mandatory is Pareto improving.

8.2 OTC trading of heterogeneous contracts and exchange trading of standardized contracts

The model presented above should be thought of as a model of OTC trading of relatively homogeneous contracts. As in OTC markets, transactions between protection buyers and protection sellers are bilateral deals. Since by assumption all $\tilde{\theta}$ are identically distributed, the risk underlying all transactions is homogeneous.[24] In the market we consider, both decentralized and centralized clearing can be used, but the latter Pareto dominates the former. How would the analysis and results change in two alternative settings: i) OTC trading of heterogeneous contracts, and ii) exchange trading of standardized contracts?[25] These two cases raise different challenges.

One way to think of OTC trading of heterogeneous contracts in our setup is to consider the case in which the θ-risk is not identically distributed across protection buyers. Different protection buyers then have different risk profiles. If these were observable, then different protection buyers should hold different contracts (τ, τ^C) to accommodate their heterogeneity. Such a fine-tuning of contracts can be complicated. If a protection buyer's risk profile was private information, then additional incentive compatibility conditions should be imposed, which could severely constrain the set of feasible allocations. Because of these constraints, the mutualization benefits of centralized clearing could be difficult to reap in OTC markets for heterogeneous contracts. This conclusion is in line with Koeppl and Monnet (2010), who show that with customized contracts, fungibility is limited, which reduces the scope for insurance through mutualization.

On the other hand, to model trading of highly standardized contracts (as on exchanges), consider the case in which the θ-risk is perfectly correlated across protection buyers. The θ-risk becomes aggregate and cannot be mutualized. Yet, centralized clearing remains efficient, because the counterparty risk can be mutualized. To reap the benefits of such mutualization, the CCP should strive to attract a diverse population of protection sellers, e.g., from different states, countries or industries.

Another important difference between the trading process of our model and the one prevailing on derivative exchanges is anonymity. In our model, as in OTC markets, the deals are bilateral. Thus, each party observes the identity of its counterparty and can use the knowledge of the counterparty's risk profile in the contract. In contrast, trading on exchanges is often anonymous. Such anonymity may preclude searching, screening and monitoring of solid counterparties. Our analysis shows that this can increase systemic risk. Markets where counterparty risk is an important issue, in particular the markets for derivatives with their

[24] In contrast, counterparty risk could be heterogeneous when good and bad protection sellers coexist. With such coexistence, the features of clearing arrangements should be adjusted to fit the counterparty risk of each protection seller.

[25] Exchange trading of heterogeneous products would be unrealistic.

long contract maturities, should not be anonymous. Instead, institutions should be able (and encouraged) to screen and monitor the creditworthiness of their counterparties in these markets.

8.3 Governance

The analysis above spells out the optimal design of the CCP, maximizing the expected utility of protection buyers, subject to the participation, incentive and feasibility constraints of the different parties. In practice, who would perform and implement this design, and thus set up a socially optimal CCP? CCPs are, in a sense, utilities, providing services to financial institutions. Such utilities are often structured as cooperatives, or mutuals, whose members are both owners and users. Consider a mutual CCP, whose members would be the protection buyers. Its objective and constraints would be those analyzed above, and it would therefore implement the optimal CCP we characterized. By contrast, consider a for-profit, shareholder-owned CCP. Its objective, the maximization of profit, would not necessarily coincide with the maximization of social welfare. We have shown above that, in the presence of moral hazard, the CCP should expose its members to some counterparty risk to maintain their incentives. Suppose that the for-profit CCP did not do that and offered full insurance. Then, if the protection buyers believed the CCP would indeed deliver full insurance, they would not exert search effort and would be willing to pay large fees for this full insurance. In the good macro-state, the CCP would use part of these fees to pay insurance, and the remaining part would be profits. In the bad macro-state, the rate of counterparty failure would be large and the CCP would go bankrupt. To the extent that a large population of protection buyers operates within the CCP, such a bankruptcy would be a systemic event. The government would have to step in and bail out protection buyers, thus confirming their initial expectation that full insurance is being provided. Thus, CCPs managed as for-profit organizations may be prone to gambling and generating systemic risk. Hence, they should be regulated. In particular, their capital should be large enough to absorb counterparty defaults so that government bail outs are not needed. Also, their risk exposure should be monitored and their ability to withstand aggregate shocks be tested.

9 Conclusion

In line with the classic theory of insurance, our analysis identifies three ways in which counterparty risk can be mitigated. First, resources can be deposited in safe assets and used to make promised payments in case of counterparty default. This is comparable to self-insurance, whereby an agent saves to insure against future negative shocks (see Ehrlich and Becker, 1972). Second, traders can exert effort to find creditworthy counterparties, whose default risk is low. This is comparable to self-protection, whereby an agent exerts effort to reduce damage probabilities (again as in Ehrlich and Becker, 1972). Third, trading parties can mutualize their risk.

We show that an appropriately designed centralized clearing mechanism enables trading parties to benefit from the mutualization of (the idiosyncratic component of) risk. Centralized clearing therefore dominates no-clearing and decentralized clearing. But we also warn that such an arrangement has limitations. First, mutualization can only deal with idiosyncratic risk. It leaves trading parties exposed to aggregate risk. Dealing with aggregate risk can require that agents search for solid, creditworthy counterparties. Second, mutualization can weaken the incentives of the trading parties to incur the unobservable effort necessary to find these solid counterparties. To cope with this moral hazard problem, an incentive compatible clearing mechanism must be put in place. Such a mechanism requires that protection buyers remain exposed to some counterparty risk in order to preserve their incentives to search for solid counterparties. We thus uncover a tradeoff between the ability of the system to withstand aggregate shocks (which requires that incentives be maintained), and the extent to which idiosyncratic risk can be mutualized in a CCP.

Two limitations of our model are that we assume the CCP i) maximizes social welfare and ii) controls counterparty risk *indirectly* via the incentives of protection buyers. In further research, it would be important to analyze how the governance, regulation and competitive position of CCPs affect their willingness and ability to put in place optimal clearing systems and to *directly* monitor the creditworthiness of their members. Such monitoring would involve running stress–tests and controlling the capitalization and risk–exposure of CCP members.

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Appendix

Proof of Proposition 1 The optimal contract with effort solves

$$\max_{\tau} \frac{1}{2}pu\left(\bar{\theta}+\tau(\bar{\theta},R)\right)+\frac{1}{2}(1-p)\,u\left(\bar{\theta}+\tau(\bar{\theta},0)\right)+\frac{1}{2}pu\left(\underline{\theta}+\tau(\underline{\theta},R)\right)+\frac{1}{2}(1-p)\,u(\underline{\theta}+\tau(\underline{\theta},0))-B$$

subject to the participation constraint of the good protection seller

$$0\geq\frac{1}{2}p\tau(\bar{\theta},R)+\frac{1}{2}(1-p)\,\tau(\bar{\theta},0)+\frac{1}{2}p\tau(\underline{\theta},R)+\frac{1}{2}(1-p)\,\tau(\underline{\theta},0). \tag{35}$$

and the limited liability constraints in (1).

The first-order conditions yield:

$$\begin{aligned} u'\left(\bar{\theta}+\tau(\bar{\theta},R)\right) &= \lambda+\mu_1, \\ u'\left(\bar{\theta}+\tau(\bar{\theta},0)\right) &= \lambda+\mu_2, \\ u'\left(\underline{\theta}+\tau(\underline{\theta},R)\right) &= \lambda+\mu_3, \\ u'\left(\underline{\theta}+\tau(\underline{\theta},0)\right) &= \lambda+\mu_4, \end{aligned}$$

where λ is the Lagrange multiplier on the participation constraint and μ_1 to μ_4 are the Lagrange multipliers on the limited liability constraints, respectively.

We proceed in four steps. First, we show that $\lambda>0$ and the participation constraint must bind. Suppose not. Then, $\lambda=0$ and the first-order conditions imply that all the limited liability constraints bind (all μ's must be strictly positive since the marginal utilities are strictly positive). Using the now implied transfers in the participation constraint yields $0\geq pR$, which is a contradiction since $pR>0$.

Second, we show that at least one limited liability constraint has to bind. Suppose not. Then, all μ's are equal to zero and the marginal utilities are equalized across all states so that $\tau(\bar{\theta},R)=\tau(\bar{\theta},0)$, $\tau(\underline{\theta},R)=\tau(\underline{\theta},0)$ and $\tau(\underline{\theta},0)=\Delta\theta+\tau(\bar{\theta},0)$. Since $\tau(\underline{\theta},0)<0$ (the limited liability constraint is slack), we get that $\Delta\theta+\tau(\bar{\theta},0)<0$ implying that $\tau(\bar{\theta},R)=\tau(\bar{\theta},0)<0$. Since $\tau(\underline{\theta},R)=\tau(\underline{\theta},0)<0$, we get $\tau(\bar{\theta},R)+\tau(\underline{\theta},R)<0$.

From the binding participation constraint we have

$$\tau(\bar{\theta},R)+\tau(\underline{\theta},R)=-\frac{1-p}{p}\left[\tau(\bar{\theta},0)+\tau(\underline{\theta},0))\right]>0,$$

which contradicts $\tau(\bar{\theta},R)+\tau(\underline{\theta},R)<0$.

Third, we show that the limited liability constraint for $\tau(\underline{\theta},0)$ must bind. Suppose not and $0>\tau(\underline{\theta},0)$. Then it would be possible to increase $\tau(\underline{\theta},0)$ by ε and decrease $\tau(\bar{\theta},R)$ by

ε. This keeps the participation constraint binding and yields a higher utility to the buyer (due to concavity of u and $\underline{\theta} < \bar{\theta}$). But this means that $0 > \tau(\underline{\theta}, 0)$ cannot be optimal.

Finally, we show that the other limited liability constraints are slack. We derive the optimal allocation under this assumption and then show that the limited liability constraints are indeed slack. Since the marginal utilities are equalized across the three states in which the limited constraints do not bind, we have $\tau(\bar{\theta}, R) = \tau(\bar{\theta}, 0)$ and $\tau(\underline{\theta}, R) = \Delta\theta + \tau(\bar{\theta}, R)$. Substituting into the binding participation constraint, we obtain $\tau(\bar{\theta}, 0) = -\frac{p}{1+p}\Delta\theta < 0$ and hence $\tau(\underline{\theta}, R) = \frac{\Delta\theta}{1+p} < \Delta\theta < R$. Note that the limited liability constraints are indeed slack in this allocation. QED

Proof of Proposition 2 For brevity this proof is omitted. It goes along the same lines as the proof of Proposition 1.

Proof of Proposition 3 The expected utility of the protection buyer under effort is higher than his expected utility under no effort if and only if

$$E[u\left(\tilde{\theta} + \tau(\tilde{\theta}, \tilde{R}_i)\right) |\xi_i = 1] - B \geq E[u\left(\tilde{\theta} + \tau(\tilde{\theta}, \tilde{R}_i)\right) |\xi_i = 0].$$

Substituting for the transfers τ from Propositions 1 and 2, respectively, we get

$$(1+p)\, u\left(\underline{\theta} + \frac{\Delta\theta}{1+p}\right) - 2B \geq (1+p-\delta)\, u\left(\underline{\theta} + \frac{\Delta\theta}{1+p-\delta}\right) + \delta u(\underline{\theta})$$

or, after collecting terms and re-arranging,

$$\frac{u\left(\underline{\theta} + \frac{\Delta\theta}{1+p-\delta}\right) - u(\underline{\theta})}{\frac{\Delta\theta}{1+p-\delta}} - \frac{u\left(\underline{\theta} + \frac{\Delta\theta}{1+p-\delta}\right) - u\left(\underline{\theta} + \frac{\Delta\theta}{1+p}\right)}{\frac{\Delta\theta}{1+p-\delta}\frac{\delta}{1+p}} \geq \frac{2B}{\frac{\delta\Delta\theta}{1+p-\delta}}$$

Note that the first term is the slope of a line between $u(\underline{\theta})$ and $u\left(\underline{\theta} + \frac{\Delta\theta}{1+p-\delta}\right)$, while the second term is the slope of a line between $u\left(\underline{\theta} + \frac{\Delta\theta}{1+p}\right)$ and $u\left(\underline{\theta} + \frac{\Delta\theta}{1+p-\delta}\right)$. QED

Proof of Proposition 4 Let μ and μ^C denote the Lagrange multipliers on the protection seller's and clearing agent's participation constraints, respectively. Let μ_0 and μ_1 denote the Lagrange multipliers on the feasibility constraints $0 \leq \alpha \leq 1$ and let μ_2 be the Lagrange multiplier on the feasibility constraint $\alpha c \geq \tau^C(\underline{\theta}, 0)$.

Since the only role of the clearing agent will be to insure against counterparty risk, we can assume without loss of generality that the transfers with the clearing agent are the same whenever the protection seller does not default, $\tau^C(\bar{\theta}, R) = \tau^C(\underline{\theta}, R) = \tau^C(\bar{\theta}, 0) \equiv \hat{\tau}^C$. Moreover, the structure of the contracting problem between the protection buyer and the protection seller, and its solution, are similar to those in the proof of Proposition 1. For ease of exposition, we therefore ignore the limited liability constraints of the protection seller, except the one in state $(\underline{\theta}, 0)$, which will bind, i.e., $\tau(\underline{\theta}, 0) = 0$. The intuition is the following. It would be suboptimal for a protection buyer to pay a bankrupt protection seller in a state when the protection buyer actually wants to *receive* a payment, because $\tilde{\theta}$ takes on its low value, $\underline{\theta}$. Also, as before, the payment from the protection buyer to the protection seller does not depend on the seller's asset return when the buyer's asset as a high value, $\tau(\bar{\theta}, R) = \tau(\bar{\theta}, 0) \equiv \hat{\tau}$. The protection seller is risk-neutral but the protection buyer is risk-averse. Hence, the optimal contract gives the buyer the same consumption in states $\left(\bar{\theta}, R\right)$ and $(\bar{\theta}, 0)$.

The first-order conditions with respect to $\hat{\tau}$, $\tau(\underline{\theta}, R)$, $\hat{\tau}^C$, $\tau^C(\underline{\theta}, 0)$ and α yield

$$
\begin{aligned}
u'(\bar{\theta} + \hat{\tau} + \hat{\tau}^C) &= \mu, && (36)\\
u'(\underline{\theta} + \tau(\underline{\theta}, R) + \hat{\tau}^C) &= \mu, && (37)\\
u'(\bar{\theta} + \tau(\bar{\theta}, 0) + \hat{\tau}^C) &= \mu^C, && (38)\\
(1-p)\, u'(\underline{\theta} + \tau^C(\underline{\theta}, 0)) &= \mu^C (1-p) + 2\mu_2, && (39)\\
\mu^C (\rho - 1)\, c &= \mu_0 - \mu_1 + \mu_2 c, && (40)
\end{aligned}
$$

respectively. By (36) and (37), the participation constraint of the protection seller binds, i.e., $\hat{\tau} = p\tau(\underline{\theta}, R)$, and there is full risk-sharing as long as the protection seller does not default, with $\tau(\underline{\theta}, R) = \frac{\Delta\theta}{1+p}$.

By (38), $\mu^C > 0$ and the participation constraint of the clearing agent binds. By (36) and (38), we have that $\mu = \mu^C$. Substituting into (39), we get

$$u'(\underline{\theta} + \tau^C(\underline{\theta}, 0)) = \mu + \frac{2\mu_2}{1-p},$$

which, combined with (37), yields

$$\underline{\theta} + \tau^C(\underline{\theta}, 0) < \underline{\theta} + \tau(\underline{\theta}, R) + \hat{\tau}^C.$$

Hence, the clearing agent only provides a partial insurance against counterparty default.

Moreover, $\alpha > 0$ must hold since otherwise the clearing agent cannot provide any insurance against counterparty risk (as $\tau^C(\underline{\theta}, 0) = 0$ in this case). Hence, $\mu_0 = 0$. It follows from

(40) that $\mu_2 > 0$ must hold and $\alpha = \frac{\tau^C(\underline{\theta},0)}{c}$. Since the participation constraint of the clearing agent binds, we have that $\hat{\tau}^C = -\tau^C(\underline{\theta},0)\frac{1-p+2(\rho-1)}{1+p}$. QED

Proof of Proposition 5 Using (39) and (40), we can write:

$$\varphi\left(\tau^C\right) \equiv \frac{u'(\underline{\theta}+\tau^C(\underline{\theta},0))}{u'\left(\underline{\theta}+\frac{\Delta\theta}{1+p}-\tau^C(\underline{\theta},0)\frac{1-p+2(\rho-1)}{1+p}\right)} = 1+\frac{2\left(\rho-1\right)}{1-p}+\frac{2\mu_1}{\left(1-p\right)cu'(\underline{\theta}+\frac{\Delta\theta}{1+p}+\tau^C(\underline{\theta},R))} \tag{41}$$

If $\varphi\left(\tau^C(\underline{\theta},0)\right) > 1+\frac{2(\rho-1)}{1-p}$ at $\tau^C(\underline{\theta},0) = c$ (which is the maximum $\tau^C(\underline{\theta},0)$ as $\alpha = 1$), then $\varphi\left(\tau^C(\underline{\theta},0)\right) > 1+\frac{2(\rho-1)}{1-p}$ for all smaller $\tau^C(\underline{\theta},0)$ and hence $\mu_1 > 0$ and $\alpha = 1$. Then, $\tau^C(\underline{\theta},0) = c$. If $\varphi\left(0\right) \leq 1+\frac{2(\rho-1)}{1-p}$, then (41) cannot hold for any $\tau^C(\underline{\theta},0) > 0$ and the clearing agent is not used. Otherwise, $\tau^C(\underline{\theta},0)$ is given by $\varphi\left(\tau^C(\underline{\theta},0)\right) = 1+\frac{2(\rho-1)}{1-p}$. QED

Proof of Proposition 6 For brevity this proof is omitted. It goes along the same lines as the proofs of Propositions 4 and 5.

Proof of Proposition 8 Consider first the optimal contract with effort. Written explicitly, the objective function (8) is

$$\frac{1}{2}pu\left(\bar{\theta}+\tau(\bar{\theta},R)+\tau^C(\bar{\theta},R)\right)+\frac{1}{2}(1-p)u\left(\bar{\theta}+\tau(\bar{\theta},0)+\tau^C(\bar{\theta},0)\right)+$$
$$\frac{1}{2}pu\left(\underline{\theta}+\tau(\underline{\theta},R)+\tau^C(\underline{\theta},R)\right)+\frac{1}{2}\left(1-p\right)u\left(\underline{\theta}+\tau(\underline{\theta},0)+\tau^C(\underline{\theta},0)\right).$$

Using $\tau(\bar{\theta},R) = \tau(\bar{\theta},0), \tau^C(\bar{\theta},R) = \tau^C(\bar{\theta},0) = \tau^C(\underline{\theta},R), \tau(\underline{\theta},0) = 0$ (see the proof of Proposition 4), the objective function becomes

$$\frac{1}{2}u\left(\bar{\theta}+\tau(\bar{\theta},R)+\tau^C(\bar{\theta},R)\right)+\frac{1}{2}pu\left(\underline{\theta}+\tau(\underline{\theta},R)+\tau^C(\bar{\theta},R)\right)-\frac{1}{2}\left(1-p\right)u\left(\underline{\theta}+\tau^C(\underline{\theta},0)\right) \tag{42}$$

Using Proposition 7 (binding participation constraint for CCP and $\alpha = 0$), we have

$$\tau^C(\bar{\theta},R) = -\frac{(1-p)}{(1+p)}\tau^C(\underline{\theta},0). \tag{43}$$

Using (43) to substitute for $\tau^C(\bar{\theta},R)$ in (42) and taking the first-order condition with respect to $\tau^C(\underline{\theta},0)$ yields

$$-\left[u'\left(\bar{\theta}+\tau(\bar{\theta},R)-\frac{(1-p)}{(1+p)}\tau^C(\underline{\theta},0)\right)+pu'\left(\underline{\theta}+\tau(\underline{\theta},R)-\frac{(1-p)}{(1+p)}\tau^C(\underline{\theta},0)\right)\right]$$
$$=(1+p)u'\left(\underline{\theta}+\tau^C(\underline{\theta},0)\right) \tag{44}$$

As in the previous subsections, there is full risk-sharing as long as the protection seller does not default. Hence,

$$u'\left(\bar{\theta}+\tau(\bar{\theta},R)-\frac{(1-p)}{(1+p)}\tau^{C}(\underline{\theta},0)\right)=u'\left(\underline{\theta}+\tau(\underline{\theta},R)-\frac{(1-p)}{(1+p)}\tau^{C}(\underline{\theta},0)\right).$$

Using (43) to substitute back for $\tau^C(\bar{\theta},R)$, equation (44) simplifies to

$$-u\acute{}\left(\underline{\theta}+\tau(\underline{\theta},R)+\tau^{C}(\bar{\theta},R)\right)=u\acute{}\left(\underline{\theta}+\tau^{C}(\underline{\theta},0)\right)$$

implying $-\tau^C(\bar{\theta},R)=\tau^C(\underline{\theta},0)+\tau(\underline{\theta},R)$. Using full risk-sharing across states and the binding participation constraint of the protection seller, we get $\tau(\underline{\theta},R)=\frac{\Delta\theta}{1+p}$ and $\tau^C(\underline{\theta},0)=\frac{\Delta\theta}{2}$. The expected utility under effort is thus

$$EU(\xi_i=1)=u\left(\underline{\theta}+\frac{\Delta\theta}{2}\right)-B=u\left(E\left[\tilde{\theta}\right]\right)-B. \tag{45}$$

Following the same steps for the optimal contract without effort yields $\tau(\underline{\theta},R)=\frac{\Delta\theta}{1+p-\delta}$ and $\tau^C(\underline{\theta},0)=\frac{\Delta\theta}{2}$. The expected utility without effort is thus

$$EU(\xi_i=0)=u\left(\underline{\theta}+\frac{\Delta\theta}{2}\right)=u\left(E\left[\tilde{\theta}\right]\right). \tag{46}$$

Comparing expected utility of the protection buyer under effort and without effort, we have

$$EU(\xi_i=1)=u\left(E\left[\tilde{\theta}\right]\right)-B<u\left(E\left[\tilde{\theta}\right]\right)=EU(\xi_i=0).$$

QED

Proof of Proposition 9 Let μ denote the Lagrange multiplier on the protection seller's participation constraint. First-order conditions with respect to $\bar{\tau}(\bar{\theta},R)$ $(=\bar{\tau}(\bar{\theta},0))$, $\bar{\tau}(\underline{\theta},R)$, $\bar{\tau}^C(\underline{\theta},0)$, $\underline{\tau}(\bar{\theta},R)$ $(=\underline{\tau}(\bar{\theta},0))$, $\underline{\tau}(\underline{\theta},R)$ and $\underline{\tau}^C(\underline{\theta},0)$ yield

$$u\acute{}(\bar{\theta}+\bar{\tau}(\bar{\theta},R)+\bar{\tau}^{C}(\bar{\theta},R))=\mu \tag{47}$$

$$u\acute{}(\underline{\theta}+\bar{\tau}(\underline{\theta},R)+\bar{\tau}^{C}(\bar{\theta},R))=\mu \tag{48}$$

$$\frac{\left(\frac{1}{2}+\gamma\right)u\acute{}(\bar{\theta}+\bar{\tau}(\bar{\theta},R)+\bar{\tau}^{C}(\bar{\theta},R))}{\frac{1}{2}(1+p)+\gamma(1-p)}+\frac{\left(\frac{1}{2}-\gamma\right)pu\acute{}(\underline{\theta}+\bar{\tau}(\underline{\theta},R)+\bar{\tau}^{C}(\bar{\theta},R))}{\frac{1}{2}(1+p)+\gamma(1-p)}=u\acute{}(\underline{\theta}+\bar{\tau}^{C}(\underline{\theta},0)) \tag{49}$$

$$u\acute{}(\bar{\theta}+\underline{\tau}(\bar{\theta},R)+\underline{\tau}^{C}(\bar{\theta},R))=\mu \tag{50}$$

$$u\acute{}(\underline{\theta}+\underline{\tau}(\underline{\theta},R)+\underline{\tau}^{C}(\bar{\theta},R))=\mu \tag{51}$$

$$\frac{\left(\frac{1}{2}+\gamma\right)u\acute{}(\bar{\theta}+\underline{\tau}(\bar{\theta},R)+\underline{\tau}^{C}(\bar{\theta},R))}{\frac{1}{2}(1+p)+\gamma(1-p)}+\frac{\left(\frac{1}{2}-\gamma\right)pu\acute{}(\underline{\theta}+\underline{\tau}(\underline{\theta},R)+\underline{\tau}^{C}(\bar{\theta},R))}{\frac{1}{2}(1+p)+\gamma(1-p)}=u\acute{}(\underline{\theta}+\underline{\tau}^{C}(\underline{\theta},0)) \tag{52}$$

By (47), the participation constraint of the protection seller binds. Using (47) and (48) in (49), and simplifying yields:

$$u'(\underline{\theta} + \bar{\tau}^C(\underline{\theta}, 0)) = \mu$$

Similarly, using (50) and (51) in (52), and simplifying yields

$$u'(\underline{\theta} + \underline{\tau}^C(\underline{\theta}, 0)) = \mu \tag{53}$$

As before, the CCP will collect the fees such that they exactly cover the insurance payments in each state,

$$\left(\frac{1}{2} - \gamma\right)(1-p)\,\bar{\tau}^C(\underline{\theta}, 0) = -\left[\frac{1}{2} + \gamma + \left(\frac{1}{2} - \gamma\right)p\right]\bar{\tau}^C(\bar{\theta}, R)$$

in the high state and

$$\left(\frac{1}{2} + \gamma\right)(1-p)\,\underline{\tau}^C(\underline{\theta}, 0) = -\left[\frac{1}{2} - \gamma + \left(\frac{1}{2} + \gamma\right)p\right]\underline{\tau}^C(\bar{\theta}, R)$$

in the low state. Hence, we have that

$$\bar{\tau}^C(\bar{\theta}, R) = -\frac{\left(\frac{1}{2} - \gamma\right)(1-p)\,\bar{\tau}^C(\underline{\theta}, 0)}{\frac{1}{2}(1+p) + \gamma(1-p)} \text{ and } \underline{\tau}^C(\bar{\theta}, R) = -\frac{\left(\frac{1}{2} + \gamma\right)(1-p)\,\underline{\tau}^C(\underline{\theta}, 0)}{\frac{1}{2}(1+p) - \gamma(1-p)} \tag{54}$$

Using full risk-sharing across states, the binding participation constraint of the protection seller and (54), we get

$$\bar{\tau}^C(\underline{\theta}, 0) = \underline{\tau}^C(\underline{\theta}, 0) = \frac{\Delta\theta}{2}$$

Furthermore,

$$\bar{\tau}(\underline{\theta}, R) = \frac{1}{\frac{1}{2}(1+p) + \gamma(1-p)}\bar{\tau}^C(\underline{\theta}, 0),\ \underline{\tau}(\underline{\theta}, R) = \frac{1}{\frac{1}{2}(1+p) - \gamma(1-p)}\underline{\tau}^C(\underline{\theta}, 0),$$
$$\bar{\tau}(\bar{\theta}, R) = -\Delta\theta + \bar{\tau}(\underline{\theta}, R),\ \underline{\tau}(\bar{\theta}, R) = -\Delta\theta + \underline{\tau}(\underline{\theta}, R)$$

and $\bar{\tau}^C(\bar{\theta}, R)$, $\underline{\tau}^C(\bar{\theta}, R)$ are given by (54).

QED

Proof of Proposition 10 For brevity, we only sketch the proof. First, suppose that the feasibility constraint (25) does not bind. Then the proof is identical to the one of Proposition 9 once we replace p with $(p - \delta)$.

When the feasibility constraint binds,

$$R < \frac{1}{\frac{1}{2}(1+p-\delta) - \gamma(1-p+\delta)}\frac{\Delta\theta}{2},$$

then we denote the Lagrange multiplier on the feasibility constraint by ψ. The first-order condition (51) in the case without search effort becomes

$$u'(\underline{\theta} + \underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R)) = \mu + \frac{\psi}{\left(\frac{1}{2} + \gamma\right)(p - \delta)}.$$

Following the same steps as in the proof of Proposition 9 now yields

$$u'(\underline{\theta} + \bar{\tau}^C(\underline{\theta}, 0)) = \mu$$

but

$$u'(\underline{\theta} + \underline{\tau}^C(\underline{\theta}, 0)) > \mu,$$

and hence $\bar{\tau}^C(\underline{\theta}, 0) > \underline{\tau}^C(\underline{\theta}, 0)$.

QED

Proof of Proposition 11 For the special case of $\gamma = \frac{1}{2}$, the first-order conditions of (26) (the case without effort) with respect to $\bar{\tau}(\bar{\theta}, R)$, $\underline{\tau}(\underline{\theta}, R)$ and $\underline{\tau}^C(\underline{\theta}, 0)$ (the other three variables drop out of the objective function and the constraints) yield

$$u'(\underline{\theta} + \underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R)) = \mu + \frac{\psi}{p - \delta} = u'(\underline{\theta} + \underline{\tau}^C(\underline{\theta}, 0)),$$

where λ and ψ are the Lagrange multipliers on the participation and the feasibility constraint, respectively. Hence, $\underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R) = \underline{\tau}^C(\underline{\theta}, 0)$ and, using $\underline{\tau}(\underline{\theta}, R) = R$ and $\underline{\tau}^C(\bar{\theta}, R)$ from the CCP's resource constraints (see also 54),

$$\underline{\tau}^C(\bar{\theta}, R) = -\frac{\frac{1}{2}(1 - p + \delta)\,\underline{\tau}^C(\underline{\theta}, 0)}{2(p - \delta)} \tag{55}$$

we have

$$\underline{\tau}^C(\underline{\theta}, 0) = (p - \delta)\,R$$

Using the binding participation constraint of the protection seller, we have

$$\bar{\tau}(\bar{\theta}, R) = -(p - \delta)\,R.$$

QED

Proof of Proposition 12 Let μ and ν denote the Lagrange multipliers on the protection seller's participation constraint and on the protection buyer's incentive constraint, respectively. The first-order conditions with respect to $\bar{\tau}(\bar{\theta}, R)$, $\bar{\tau}(\underline{\theta}, R)$, $\bar{\tau}^C(\underline{\theta}, 0)$, $\underline{\tau}(\bar{\theta}, R)$, $\underline{\tau}(\underline{\theta}, R)$ and $\underline{\tau}^C(\underline{\theta}, 0)$ are:

$$u'(\bar{\theta} + \bar{\tau}(\bar{\theta}, R) + \bar{\tau}^C(\bar{\theta}, R)) = \mu \quad (56)$$

$$p\,(1+\nu)\,u'(\underline{\theta} + \bar{\tau}(\underline{\theta}, R) + \bar{\tau}^C(\bar{\theta}, R)) = p\mu \quad (57)$$

$$\frac{(1-p)\left[\left(\frac{1}{2}+\gamma\right)u'(\bar{\theta} + \bar{\tau}(\bar{\theta}, R) + \bar{\tau}^C(\bar{\theta}, R)) + p\,(1+\upsilon)\left(\frac{1}{2}-\gamma\right)u'(\underline{\theta} + \bar{\tau}(\underline{\theta}, R) + \bar{\tau}^C(\bar{\theta}, R))\right]}{\left[\frac{1}{2}(1+p)+\gamma\,(1-p)\right]\left[1-p\,(1+\nu)\right]}$$
$$= u'(\underline{\theta} + \bar{\tau}^C(\underline{\theta}, 0)) \quad (58)$$

$$u'(\bar{\theta} + \underline{\tau}(\bar{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R)) = \mu \quad (59)$$

$$p\,(1+\nu)\,u'(\underline{\theta} + \underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R)) = p\mu \quad (60)$$

$$\frac{(1-p)\left[\left(\frac{1}{2}-\gamma\right)u'(\bar{\theta} + \underline{\tau}(\bar{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R)) + p\,(1+\nu)\left(\frac{1}{2}+\gamma\right)u'(\underline{\theta} + \underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R))\right]}{\left[\frac{1}{2}(1+p)-\gamma\,(1-p)\right]\left[1-p\,(1+\nu)\right]}$$
$$= u'(\underline{\theta} + \underline{\tau}^C(\underline{\theta}, 0)) \quad (61)$$

By (56), $\mu > 0$ and hence the participation constraint of the protection seller binds. Using (56) and (57) in (58), we get:

$$u'(\underline{\theta} + \bar{\tau}^C(\underline{\theta}, 0)) = \frac{(1-p)\,\mu}{1-p\,(1+\nu)} \quad (62)$$

Similarly, using (59) and (60) in (61), we get:

$$u'(\underline{\theta} + \underline{\tau}^C(\underline{\theta}, 0)) = \frac{(1-p)\,\mu}{1-p\,(1+\nu)} \quad (63)$$

It follows that $\bar{\tau}^C(\underline{\theta}, 0) = \underline{\tau}^C(\underline{\theta}, 0)$. By (56) and (59), $\bar{\tau}(\bar{\theta}, R) + \bar{\tau}^C(\bar{\theta}, R) = \underline{\tau}(\bar{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R)$. By (57) and (60), $\bar{\tau}(\underline{\theta}, R) + \bar{\tau}^C(\bar{\theta}, R) = \underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R)$. Using these equalities and (54) to substitute in the binding participation constraint, we get

$$-\underline{\tau}(\bar{\theta}, R) = p\underline{\tau}(\underline{\theta}, R) - \frac{2\gamma\,(1-p)\,\underline{\tau}^C(\underline{\theta}, 0)}{\frac{1}{2}(1+p)-\gamma\,(1-p)}$$

which yields $\underline{\tau}(\bar{\theta}, R)$ as a function of $\underline{\tau}(\underline{\theta}, R)$ and $\underline{\tau}^C(\underline{\theta}, 0)$.

Furthermore, it must be that $\nu > 0$ in the optimum so that the incentive constraint binds. Suppose not, i.e., $\nu = 0$. Then, marginal utilities are equalized across all states, implying that $\bar{\tau}(\underline{\theta}, R) + \bar{\tau}^C(\bar{\theta}, R) = \bar{\tau}^C(\underline{\theta}, 0)$ and $\underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R) = \underline{\tau}^C(\underline{\theta}, 0)$. But then, the incentive constraint cannot hold as $B > 0$. A contradiction. Since the incentive constraint binds, we have, using $\bar{\tau}(\underline{\theta}, R) + \bar{\tau}^C(\bar{\theta}, R) = \underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R)$ and $\bar{\tau}^C(\underline{\theta}, 0) = \underline{\tau}^C(\underline{\theta}, 0)$,

$$u(\underline{\theta} + \underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R)) - u(\underline{\theta} + \underline{\tau}^C(\underline{\theta}, 0)) = \frac{2B}{\delta}$$

This equation yields $\underline{\tau}(\underline{\theta}, R)$ as a function of $\underline{\tau}^C(\underline{\theta}, 0)$.

Next, note that by (59) and (60),

$$\frac{u'(\bar{\theta} + \underline{\tau}(\bar{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R))}{u'(\underline{\theta} + \underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R))} = 1 + \nu > 1$$

since $\nu > 0$. It follows that $\underline{\theta} + \underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R) > \bar{\theta} + \underline{\tau}(\bar{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R)$. Furthermore, by (63), we have

$$u'(\underline{\theta} + \underline{\tau}^C(\underline{\theta}, 0)) = \frac{(1-p)}{1 - p(1+\nu)} u'(\bar{\theta} + \underline{\tau}(\bar{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R))$$

Since $1 + \nu > 1$, we have that $\bar{\theta} + \underline{\tau}(\bar{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R) > \underline{\theta} + \underline{\tau}^C(\underline{\theta}, 0)$. The optimal transfer $\underline{\tau}^C(\underline{\theta}, 0)$ is given by:

$$u'(\underline{\theta} + \underline{\tau}^C(\underline{\theta}, 0)) = \frac{(1-p)\, u'(\bar{\theta} + \underline{\tau}(\bar{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R)) u'(\underline{\theta} + \underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R))}{u'(\underline{\theta} + \underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R)) - p u'(\bar{\theta} + \underline{\tau}(\bar{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R))}$$

In sum, protection buyer's utilities are no longer equalized, even when the protection seller does not default, and

$$\begin{aligned} \underline{\theta} + \underline{\tau}(\underline{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R) = \underline{\theta} + \bar{\tau}(\underline{\theta}, R) + \bar{\tau}^C(\bar{\theta}, R) > \bar{\theta} + \underline{\tau}(\bar{\theta}, R) + \underline{\tau}^C(\bar{\theta}, R) \\ = \bar{\theta} + \bar{\tau}(\bar{\theta}, R) + \bar{\tau}^C(\bar{\theta}, R) > \underline{\theta} + \underline{\tau}^C(\underline{\theta}, 0) = \underline{\theta} + \bar{\tau}^C(\underline{\theta}, 0) \end{aligned}$$

holds in the optimum.

QED

Figure 1: Sequence of events

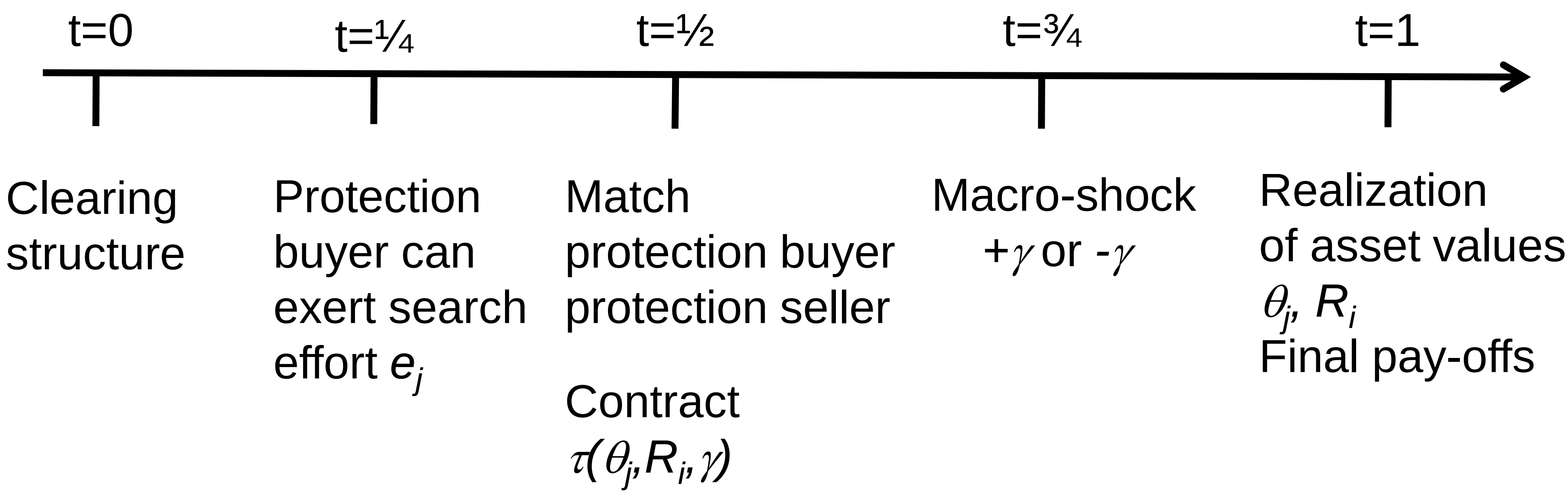

Figure 2: No clearing

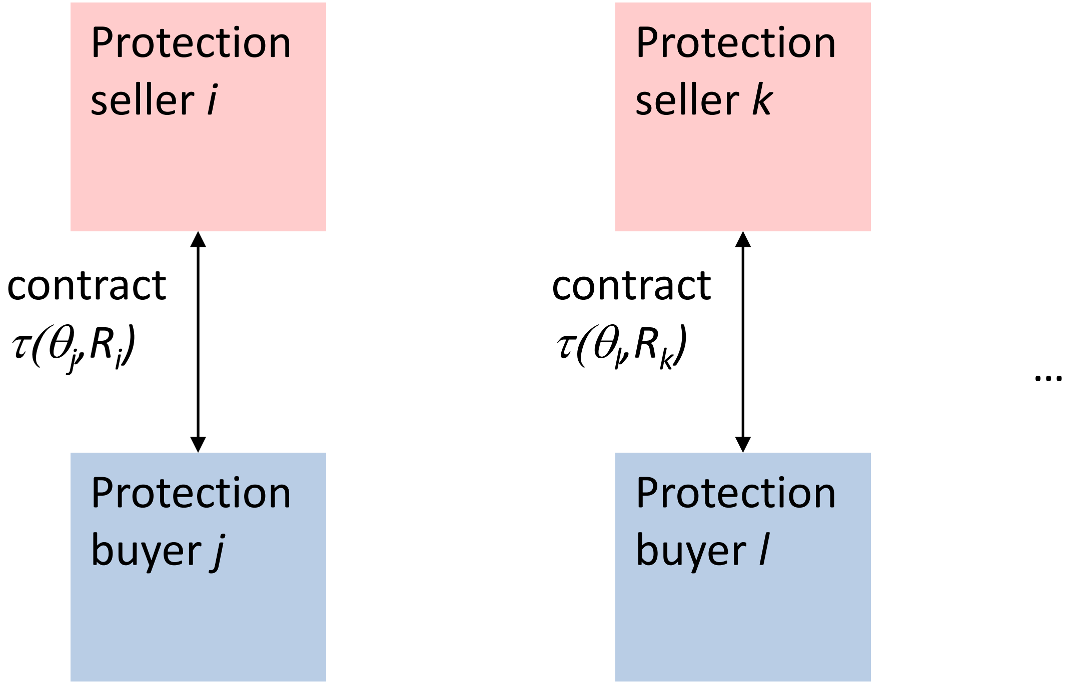

Figure 3: Decentralized clearing

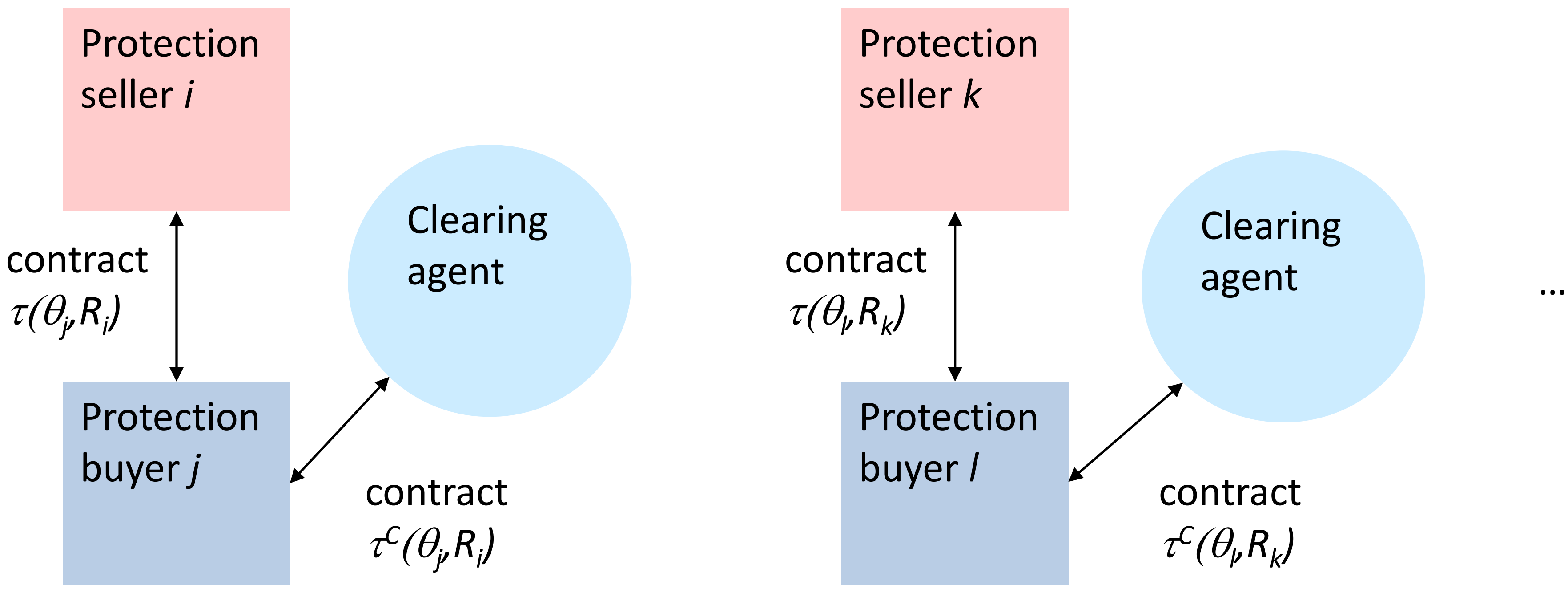

Figure 4: Centralized clearing