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TIME-TO-BUILD APPROACH IN A STICKY PRICE, STICKY WAGE OPTIMIZING MONETARY MODEL

BY MIGUEL CASARES

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Abstract

One of the most significant characteristics of optimizing models is that the behavioral equations involved are typically forward looking, i.e., agents are concerned about the future rather than the past. This creates difficulties when modelling some of the business-cycle patterns widely observed in modern economies. For example, it is not easy to obtain the delay in the response of the rate of inflation to a monetary shock. This paper shows that an optimizing monetary model with endogenous capital, sticky prices, sticky wages, and adjustment costs of investment, can replicate a lag in the maximum response of both output and inflation to an interest rate shock when taking into account a time-to-build requirement for investment projects.

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Non-technical summary

This paper explores the implications of considering the time requirement for installing capital goods in the production processes of the economy. The optimal level of capital must be decided in advance in order to leave time for building and installation. The implications of this assumption are discussed in the paper, mainly by examining the responses of macroeconomic variables to an unexpected rise in the nominal interest rate. The empirical evidence has shown that if the interest rate unexpectedly rises, investment, output, and inflation all gradually fall, reporting a maximum value of their fall with certain time delay. This type of outcome is obtained in our model when featuring rigidities in price and nominal wage setting, adjustment costs of installing capital, and types of capital with a sufficiently long time requirement for installation.
1 Introduction

Over the past decade numerous examples of optimizing monetary models featuring nominal rigidities have made a strong impact in the literature. Both their theoretical appeal as microfounded models highly applicable in policy analysis, and their ability to explain short-run effects of monetary policy have contributed to their popularity among researchers. However, their dependence on a forward-looking decision making-process makes it quite difficult for these models to capture some of the business cycle features observed in the data. In particular, models are not very successful in replicating co-movements between the nominal interest rate and output, the existence of a liquidity effect on monetary expansions, the slightly procyclical behavior of the real wage, or the delay in the response of output and inflation to a monetary shock.

The purpose of this paper relates precisely to this last well-documented fact, namely, to derive a model that will help to reveal why responses of both output and inflation to monetary stimulus, far from being immediate, delay several quarters before reaching their maximum impact. This phenomenon has been widely investigated in recent papers using optimizing models featuring frictions in price-setting, wage-setting, or both. A representative list of these should include Chari, Kehoe, and McGrattan (2000), Erceg, Henderson, and Levin (2000), and Christiano, Eichenbaum, and Evans (2001).

Following Kydland and Prescott (1982), the procedure is to introduce time-to-build requirements for investment projects in a model with endogenous capital. The capital stock decided today will not be utilized in the production process until several periods from today due to the building and installation time needed. As a result, there is greater inertia in capital and investment behavior so that they take longer to respond to monetary shocks. This delay will have some impact on productivity and costs. Since prices are typically set by looking at marginal costs, time-to-build may also affect pricing decisions in a way similar to that observed in the data: prices (and inflation) respond more slowly to monetary innovations.1

Other basic features of the model are sticky prices and sticky wages on the supply side, and, habit persistence in consumption decisions, adjustment costs of investment, and transactions-facilitating money demand on the demand side.

The rest of the paper is divided into five sections. Section 2 describes the model. The equations governing the dynamic behavior of capital accumulation and the rate of inflation are derived

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1 A recent work by Edge (2000) has shown that the time-to-build assumption also helps capturing the liquidity effect in response to a positive money-growth shock. She used an optimizing monetary model with price stickiness and flexible wages. However, she did not report any inflation lag to the monetary shock. We presume this result to be due to the absence of a time-to-build constraint imposed on the demand for capital entering the production function.
in section 3, so as to examine the implications of the time-to-build approach. Calibration of the parameters is carried out in section 4, while section 5 describes the consequences of taking the time-to-build approach in the responses of the model to a monetary policy shock in a benchmark calibrated economy and in several other variants of this. Finally, section 6 summarizes the conclusions.

2 The model

The economy consists of a continuum of households indexed by \( i \in [0,1] \) who are also producers. They all share a set of preferences, transactions technology, production technology, the same capital accumulation restrictions, and the same rigidities when setting prices and nominal wages.

Household preferences

The following constant relative risk aversion (CRRA) utility function ranks household preferences over period \( t \)

\[
U(c_t, c_{t-1}, l_t) = (1 - \sigma)^{-1} \left[ \frac{c_t}{c_{t-1}} \right]^{1-\sigma} + \Upsilon(1 - \gamma)^{-1}l_t^{1-\gamma},
\]

where \( \sigma, \Upsilon, \gamma > 0 \), and \( 0 \leq \zeta \leq 1 \). Utility depends on current consumption \( c_t \), previous period consumption \( c_{t-1} \), and leisure time \( l_t \). The habit formation element can be ruled out by setting \( \zeta = 0.0 \). Consumption units are bundles of many differentiated goods aggregated as in Dixit and Stiglitz (1977)

\[
c_t = \left[ \int_0^1 [c_t(i)]^{(\theta_P - 1)/\theta_P} \, di \right]^{\theta_P/(\theta_P - 1)} \quad \text{with } \theta_P > 1,
\]

where \( c_t(i) \) is the consumption of the \( i^{th} \) single good.

Transactions technology

Households produce a single good and consume multiple goods. They use monetary services to facilitate their transactions and save some transaction costs. The transactions technology is represented by a function that indicates the level of transaction costs, \( h_t \), which depend positively on the number of consumption bundles, \( c_t \), and negatively on the amount of real money balances, \( m_t \), held at the exchange

\[
h_t = h(c_t, m_t) = \begin{cases} 
0 & \text{if } c_t = 0 \\
 h_0 + c_t h_1 \left[ \frac{c_t}{m_t} \right]^{h_2/(1-h_2)} & \text{if } c_t > 0 
\end{cases}
\]

(2)
with \( h_0, h_1 > 0 \), and \( 0 \leq h_2 \leq 1 \). The transactions-facilitating role of money is shown by the negative signs of the partial derivative \( h_m < 0 \), and the cross derivative \( h_{cm} < 0 \). Transaction cost figures are given in consumption-bundle units.

**Capital accumulation, time-to-build, and adjustment costs of investment**

Households save part of their production to be transformed into capital to be used for future production. Following the seminal paper by Kydland and Prescott (1982), it is assumed that installing capital goods takes time; the so-called time-to-build requirement. Thus, there is a gap between the point when the decision is taken for the demand for capital and the point when it is actually used in the production process; this gap is known as the time-to-build period. Following on from Kydland and Prescott’s setup, it is assumed that types of capital with different time-to-build periods can simultaneously be used as production factors. Hence, there exist \( J \) different types of capital ranging from the one-period time-to-build type to the \( J \)-period time-to-build type.

The existence of multiple types of capital brings about situations in which many investment projects are running simultaneously. In turn, notation becomes somewhat complicated. Generalizing Kydland and Prescott’s definition for the multiple capital case, let \( s_{pt}(j) \) denote the net increase in the stock of \( j \)-type capital that, over period \( t \), is still \( p \) periods from completion:

\[
s_{pt}(j) = k_{t+p}(j) - (1 - \delta)k_{t+p-1}(j), \quad \text{with } p = 1, \ldots, j,
\]

and where \( \delta \) is the depreciation rate.

In period \( t \), the choice variables on the \( J \) varieties of capital are \( k_{t+1}(1), k_{t+2}(2), \ldots, k_{t+J}(J) \) in order to determine the capital accumulation processes \( s_{1t}(1), s_{2t}(2), \ldots, s_{Jt}(J) \) respectively. In other words, the time-to-build requirement makes producers decide on the \( j \)-type capital accumulation \( j \) periods in advance.

Investment over period \( t \) in the \( j \)-type capital, \( x_t(j) \), is defined as the average figure

\[
x_t(j) = (1/j) \sum_{p=1}^{j} s_{pt}(j),
\]

which implies that resources for investment projects are allocated evenly throughout their time-to-build period, as assumed in the calibration preferred by Kydland and Prescott. Then, total investment \( x_t \) will be the sum of all the \( J \) varieties of capital

\[
x_t = \sum_{j=1}^{J} x_t(j). \tag{3}
\]

A second restriction regarding capital decisions is that some adjustment costs are involved in installing the new units of capital. Particularly, it is assumed here that these adjustment costs are
affected positively by the increase in the stock of capital to be fully installed in the next period and negatively by the current stock of capital. The functional form chosen here is linear homogeneous on these two variables, as recommended by Hayashi (1982). Hence, the amount of adjustment costs in period \( t \) for the installation of the \( j \)-type capital \( A_t(j) \) is given by the following function

\[
A_t(j) = A(k_{t+1}(j), k_t(j)) = \Theta_1 \frac{k_{t+1}(j) - (1-\delta)k_t(j)^{1+\Theta_2}}{[k_t(j)]^{\Theta_2}}, \quad \text{for } j = 1, \ldots, J
\]

with \( \Theta_1, \Theta_2 > 0 \). Adjustment costs are measured in consumption-bundle units. The total amount of adjustment costs is

\[
A_t = \sum_{j=1}^{J} A_t(j). \tag{4}
\]

Production technology

The amount of output produced in period \( t \), \( y_t \), is obtained by employing the \( J \)-type varieties of capital and the demand for labor \( n_t^d \) within the following technology

\[
y_t = f(k_t(1), \ldots, k_t(J), n_t^d) = \left[ \sum_{j=1}^{J} \Phi_j [k_t(j)]^{\alpha/\upsilon} \right]^{\upsilon/\alpha} \left[ n_t^d \right]^{1-\alpha} \tag{5}
\]

with \( 0 < \alpha < 1, \upsilon > \alpha, 0 < \Phi_j < 1, \) and \( \sum_{j=1}^{J} \Phi_j = 1 \). This production function exhibits constant returns to scale on the differentiated capital goods and on labor. Let \( K_t \) denote the capital bundle obtained in period \( t \) from the CES aggregator of the production technology as

\[
K_t = \left[ \sum_{j=1}^{J} \Phi_j [k_t(j)]^{\alpha/\upsilon} \right]^{\upsilon/\alpha},
\]

which enables the production function (5) to be expressed in a convenient Cobb-Douglas style

\[
y_t = \tilde{f}(K_t, n_t^d) = [K_t]^\alpha \left[ n_t^d \right]^{1-\alpha}. \tag{5'}
\]

Price and wage setting

Households sell their production in a monopolistic competition market. Then, the amount produced \( y_t(i) \) by the \( i \)-th household for period \( t \) will depend on the selling price \( P_t(i) \), the aggregate price level \( P_t^A \), and aggregate output \( y_t^A \), as determined by the Dixit-Stiglitz demand equation\(^2\)

\[
y_t(i) = \left[ \frac{P_t(i)}{P_t^A} \right]^{-\theta_P} y_t^A, \tag{6}
\]

\(^2\)It is implicitly assumed that households can also produce the aggregate output good (by using the Dixit-Stiglitz output aggregator technology defined in the text). If so, the profit-maximizing criterion leads to the demand equation (6), and the zero-profit condition leads to the Dixit-Stiglitz price level definition.
in which \( \theta_P \) is the elasticity of substitution between differentiated goods, \( P_t^A = \left[ \int_0^1 P_t(i)^{1-\theta_P} di \right]^{1/(1-\theta_P)} \) is the aggregate Dixit-Stiglitz price level, and \( y_t^A = \left[ \int_0^1 [y_t(i)]^{(\theta_P-1)/\theta_P} di \right]^{(\theta_P-1)/\theta_P} \) is the Dixit-Stiglitz aggregate output taking the same aggregation scheme as for consumption goods.

According to Calvo (1983), households can set their price optimally only with a probability of \( 1 - \eta_P \). With a probability of \( \eta_P \), their selling price will be automatically raised by the steady state rate of inflation.\(^3\) In such case, households will adjust the price in accordance with the rule \( P_t = (1 + \pi)P_{t-1} \) with \( \pi \) as the steady state rate of inflation. In turn, the Dixit-Stiglitz aggregate price level can be reformulated as follows

\[
P_t^A = \left[ (1 - \eta_P) [P_t(i)]^{1-\theta_P} + \eta_P(1 + \pi)^{1-\theta_P} [P_{t-1}^A]^{1-\theta_P} \right]^{1/(1-\theta_P)},
\]

(7)

where \( P_t(i) \) is the optimal selling price during period \( t \).

In the labor market, each household supplies differentiated labor services in monopolistic competition. Since capital is predetermined, labor demand is the amount needed to produce the level of output given by the market demand eq. (6). The units of labor demand entering the production function are bundles of differentiated labor services. A CES technology aggregates the labor services supplied in order to meet the labor demand. Hence, the \( z \)-th household labor demand is \( n_t^d(z) = \left[ \int_0^1 [n_t^d(i, z)]^{(\theta_W-1)/\theta_W} di \right]^{\theta_W/(\theta_W-1)} \) with \( \theta_W > 1 \), and \( n_t^d(i, z) \) is the amount of labor supplied by the \( i \)-th household to the production process of the \( z \)-th household.

Just as in the goods market, the individual nominal wage \( W_t(i) \) set by the \( i \)-th household determines the amount of labor supplied by this household to any other \( z \)-th household through the Dixit-Stiglitz demand condition\(^4\)

\[
n_t^d(i, z) = \left[ \frac{W_t(i)}{W_t^A} \right]^{-\theta_W} n_t^d(z),
\]

where \( \theta_W \) is the elasticity of substitution between differentiated labor units, and \( W_t^A \) is the aggregate nominal wage defined as \( W_t^A = \left[ \int_0^1 [W_t(i)]^{1-\theta_W} di \right]^{1/(1-\theta_W)} \). Summing over \( z \) in order to compute the quantity of labor supplied by the \( i \)-th household to all the households, we have

\[
n_t^d(i) = \left[ \frac{W_t(i)}{W_t^A} \right]^{-\theta_W} n_t^A,
\]

(8)

---

\(^3\)This departs from the original Calvo assumption of maintaining prices unchanged. The price adjusting rule at hand was selected because it is consistent with optimal pricing in steady state. Examples of models using such non-optimal pricing scheme are Yun (1996), and Erceg et al. (2000).

\(^4\)This condition can be derived from the labor-related cost minimizing program for the differentiated labor services. Furthermore, the aggregate nominal wage defined below is obtained assuming that neither profits nor losses are made in the process.
where $n_t^A = \int_0^1 n_t^d(z) dz$ is aggregate labor. In accordance to this result, the time constraint of the representative $i$-th household can be written as follows

$$\left[ \frac{W_t(i)}{W_t^A} \right]^{-\theta_W} n_t^A + t_t = 1,$$

with the total time normalized to 1 unit of time. Nominal wage setting also incorporates rigidities à la Calvo. Thus, the household can set the nominal wage optimally only under some fixed probability $1 - \eta_W$. Otherwise, with a probability $\eta_W$, the household will raise last period nominal wage by the steady state rate of inflation, $W_t = (1 + \pi)W_{t-1}$. Consequently, the Dixit-Stiglitz aggregate nominal wage level can also be expressed as

$$W_t^A = \left[ (1 - \eta_W) [W_t(i)]^{1-\theta_W} + \eta_W (1 + \pi)^{1-\theta_W} [W_{t-1}^A]^{1-\theta_W} \right]^{1/(1-\theta_W)},$$

where $W_t(i)$ is the optimal nominal wage in period $t$.

**Government and household budget constraints**

The government gives lump-sum transfers to households that are financed by either increasing money balances or bonds. Let us denote $g_t$ as the government transfers per household in consumption-bundle terms, $b_{t+1}$ as the amount of government bonds purchased in period $t$ and to be reimbursed in period $t + 1$ also in consumption-bundle terms per household, and $r_t$ as the real interest rate that government bonds will yield from $t$ to $t + 1$. Thus, the government’s budget constraint in consumption-bundle units per household is

$$g_t = m_t - (1 + \pi_t)^{-1}m_{t-1} + (1 + r_t)^{-1}b_{t+1} - b_t.$$  

where $\pi_t$ is the rate of inflation during period $t$ defined as $\pi_t = (P_t^A/P_{t-1}^A) - 1$.

Let us now turn to the household budget constraint. The $i$-th household’s budget constraint expressed in consumption-bundle units can be expressed as follows

$$\frac{P_t(i)}{P_t^A} y_t(i) + \frac{W_t(i)}{P_t^A} n_t^d(i) + g_t =$$

$$C_t + x_t(i) + A_t(i) + \frac{W_t^A}{P_t} n_t^d(i) + m_t - (1 + \pi_t)^{-1}m_{t-1} + (1 + r_t)^{-1}b_{t+1}(i) - b_t(i) + h_t.$$ 

As shown in (11), there are three sources to raise income: output sales, labor income, and government transfers. Income is spent on consumption, investment, adjustment costs of investment, labor demand payments, increasing real money holdings, on net purchases of government bonds, and on transaction costs.

Both selling price and nominal wage vary across households depending on whether they were optimally set or not. As a consequence, other variables are also different among households. The
selling price, \( P_t(i) \), will directly determine individual output, \( y_t(i) \), given by the market demand equation (6), and specific labor demand, \( n^d_t(i) \), as needed to produce \( y_t(i) \). The selling price will also affect capital and investment decisions through its impact on expected future prices and output. As a result, both \( x_t(i) \) and \( A_t(i) \) are household specific. The nominal wage, \( W_t(i) \), will determine labor supply, \( n^d_t(i) \), given the demand equation (8). Purchases of government bonds, \( b_{t+1}(i) \), are also specific to each household. An increase (decrease) of government bonds can be viewed as a surplus (deficit) that households register as a consequence of their decisions on the selling price and nominal wage.

By contrast, behavior across households is symmetric in choices regarding consumption and money demand. Their optimizing behavior on allocating differentiated consumption goods leads to the same level of consumption expenditure \( \int_0^1 P_t(i)c_t(i)di \) which for all of them is equal to \( P_tA_t c_t \). Thus, households will consume \( c_t \) consumption-bundle units. The demand for real money balances \( m_t \) also evolves symmetrically since it depends on \( c_t \) and the nominal interest rate as determined by the optimizing conditions. In turn, transaction costs \( h_t \) also move identically across households. Finally, the amount of lump-sum transfers received from the government, \( g_t \), is assumed to be the same for all households.

The overall resources constraint

A two-step derivation of the overall resources constraint will be carried out. The first step will be to determine the relationship between the aggregate output of the economy in consumption-bundle units and the Dixit-Stiglitz output aggregator. Next, that result will be used to express aggregate output in terms of production factors and equate this to aggregate demand.

The economy’s aggregate output during period \( t \) in consumption-bundle units \( y^*_t \) is given by

\[
y^*_t = \int_0^1 \frac{P_t(i)y_t(i)}{P_t^A} di,
\]

where, by inserting the individual good demand equation (6) in place of \( y_t(i) \), it yields

\[
y^*_t = \int_0^1 \frac{P_t(i)}{P_t^A} \left[ \frac{P_t(i)}{P_t^A} \right]^{-\theta_P} y_t^A di = \int_0^1 \left[ \frac{P_t(i)}{P_t^A} \right]^{1-\theta_P} y_t^A di.
\]

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5 Either the surplus or deficit obtained may be conceptually interpreted in the same way as profits or losses in a production economy constituted by optimizing firms.

6 The optimal demand function for the differentiated consumption good is \( c_t(i) = \left[ \frac{P_t(i)}{P_t^A} \right]^{-\theta_P} c_t \). Taking into account both this demand function and the Dixit-Stiglitz aggregate price level \( P_t^A = \left[ \int_0^1 [P_t(i)]^{1-\theta_P} di \right]^{1/(1-\theta_P)} \), nominal consumption expenditures are \( \int_0^1 P_t(i)c_t(i)di = \int_0^1 P_t(i) \left[ \frac{P_t(i)}{P_t^A} \right]^{-\theta_P} c_t di = \left[ P_t^A \right]^{1-\theta_P} c_t \int_0^1 P_t(i)^1-\theta_P di = \left[ P_t^A \right]^{1-\theta_P} c_t \left[ P_t^A \right]^{1-\theta_P} di = P_tA_t c_t \).
and removing both \( P_t^A \) and \( y_t^A \) from the integral

\[
y_t^* = \left[ P_t^A \right]^{\theta P - 1} y_t^A \int_0^1 [P_t(i)]^{1 - \theta P} di.
\]

It is now possible to use the Dixit-Stiglitz price level definition \( P_t^A = \left[ \int_0^1 [P_t(i)]^{1 - \theta P} di \right]^{1/(1 - \theta P)} \) to demonstrate that the aggregate output level \( y_t^* \) is exactly equal to the Dixit-Stiglitz output aggregator \( y_t^A \):

\[
y_t^* = \left[ P_t^A \right]^{\theta P - 1} y_t^A \left[ P_t^A \right]^{1 - \theta P} = y_t^A.
\]

By bringing in the Dixit-Stiglitz output aggregator definition \( y_t^A = \left[ \int_0^1 [y_t(i)]^{(\theta P - 1)/\theta P} di \right]^{\theta P/(\theta P - 1)} \), and using the Cobb-Douglas production function (5'), it emerges that

\[
y_t^* = \left[ \int_0^1 \left[ K_t(i) \right]^\alpha n_t^d(i)^{1 - \alpha} \right]^{(\theta P - 1)/\theta P} \theta P/(\theta P - 1) = c_t + x_t^A + A_t^A + h_t,
\]

In the labor market, aggregate labor income \( \int_0^1 W_t(i) n_t^d(i) di \) is equal to the aggregate amount of labor payments \( \int_0^1 W_t^A n_t^d(i) di \). It turns out that these both are equal to \( W_t^A n_t^A \). Using this result, assuming that the government budget constraint is held, and inserting the definition of \( y_t^* \) obtained above, the overall resources constraint which is the total sum of the budget constraint for all the households becomes

\[
\left[ \int_0^1 \left[ K_t(i) \right]^\alpha n_t^d(i)^{1 - \alpha} \right]^{(\theta P - 1)/\theta P} \theta P/(\theta P - 1) = c_t + x_t^A + A_t^A + h_t,
\]

where \( x_t^A = \int_0^1 x_t(i) di \), and \( A_t^A = \int_0^1 A_t(i) di \).

3 The capital accumulation and inflation equations

The economic behavior of households is determined by solving their optimizing program which involves maximizing the sum of current and expected discounted future utility values subject to the sequence of budget constraints, time constraints, and market demand constraints. The optimizing program is described in the appendix to this paper.\(^7\) In this section we will examine the resulting behavioral equations governing changes in capital accumulation and inflation since the introduction of a time-to-build requirement gives rise to relevant effects on their dynamic behavior.

\(^7\)In an economy with producers maximizing profits as separate entities, the optimizing program would be equivalent assuming that households lend their capital stock to producers in return for a rental rate. The time-to-build requirement must also be considered influencing producer’s demand for capital.
The capital accumulation equation

Assuming a maximum time-to-build requirement of $J$ periods, the capital accumulation decisions made in period $t$ are the values of $k_{t+1}(1), k_{t+2}(2), \ldots, k_{t+J}(J)$. As representative of any of these, the optimal value of $k_{t+j}(j)$ is obtained through its first order condition, $(k_{t+j}^{\text{loc}}(j))$, derived in the appendix:\footnote{For the sake of simplicity, we use the notation $g_{xt} = \frac{\partial g(x)}{\partial x_t}$ to represent the partial derivative during period $t$ of any $g(.)$ function with respect to $x_t.$}

$$(1/j)E_t \sum_{p=1}^{j} \left[ \beta^{p-1} \lambda_{t+p-1} - (1 - \delta) \beta^p \lambda_{t+p} \right] = 

\begin{align*}
\beta^{j}E_t \left[ \xi_{t+j} f_{k_{t+j}(j)}^{t+j} \right] - \beta^{j-1}E_t \left[ \lambda_{t+j-1} A_{k_{t+j}(j)}^{t+j-1} \right] - \beta^{j}E_t \left[ \lambda_{t+j} A_{k_{t+j}(j)}^{t+j} \right], \quad (k_{t+j}^{\text{loc}}(j))
\end{align*}

where $\beta$ is the household’s intertemporal discount factor, $\lambda_{t+j}$ is the Lagrange multiplier of the budget constraint in period $t + j$, and $\xi_{t+j}$ is the Lagrange multiplier of the market demand constraint in period $t + j$. Let us define $\psi_{t+j} = w_{t+j}^{A} / n_{t+j}^{foc}$ as the real marginal cost over period $t + j$. The labor demand first order condition for period $t + j$ implies that the relationship between $\xi_{t+j}$ and $\lambda_{t+j}$ is

$$
\xi_{t+j} = \lambda_{t+j} \psi_{t+j},
$$

(13)

When the value obtained in (13) is inserted into the first order condition $(k_{t+j}^{\text{loc}}(j))$, this gives

$$(1/j)E_t \sum_{p=1}^{j} \left[ \beta^{p-1} \lambda_{t+p-1} - (1 - \delta) \beta^p \lambda_{t+p} \right] = 

\begin{align*}
\beta^{j}E_t \left[ \lambda_{t+j} \psi_{t+j} f_{k_{t+j}(j)}^{t+j} \right] - \beta^{j-1}E_t \left[ \lambda_{t+j-1} A_{k_{t+j}(j)}^{t+j-1} \right] - \beta^{j}E_t \left[ \lambda_{t+j} A_{k_{t+j}(j)}^{t+j} \right].
\end{align*}

(14)

The first order conditions regarding government bond purchases imply that

$$
\beta^p E_t \lambda_{t+p} = E_t \left[ \lambda_t \prod_{k=0}^{p-1} (1 + r_{t+k})^{-1} \right].
$$

(15)

Hence, when all the Lagrange multipliers in (14) are substituted for their value relative to $\lambda_t$ implied by (15), it yields after some algebra

$$(1/j)E_t \left[ \lambda_t (1 - \delta) \lambda_t \prod_{k=0}^{j-1} (1 + r_{t+k})^{-1} + \delta \lambda_t \sum_{p=1}^{j-1} \prod_{k=0}^{p-1} (1 + r_{t+k})^{-1} \right] = 

E_t \left[ \lambda_t \prod_{k=0}^{j-1} (1 + r_{t+k})^{-1} \psi_{t+j} f_{k_{t+j}(j)}^{t+j} \right] - E_t \left[ \lambda_t \prod_{k=0}^{j-2} (1 + r_{t+k})^{-1} A_{k_{t+j}(j)}^{t+j-1} \right] - E_t \left[ \lambda_t \prod_{k=0}^{j-1} (1 + r_{t+k})^{-1} A_{k_{t+j}(j)}^{t+j} \right]. \quad (14)$$
When both sides are divided by $E_t \left[ \lambda_t \prod_{k=0}^{j-1} (1 + r_{t+k})^{-1} \right]$, this gives

$$(1/j)\left( 1 - \frac{1}{1 + \delta} \right)(1 + \delta) = \delta A_t^{-1} E_t \left[ \psi_{t+j} f_{t+k}^{1+j} \right] - \delta E_t \left[ (1 + r_{t+j-1})A_{k_{t+j}(j)}^{1+j-1} \right] - E_t \left[ A_{k_{t+j}(j)}^{1+j} \right].$$  

(16)

Loglinearizing (16) around the steady-state solution, and neglecting products of close-to-zero numbers results in following expression, which governs dynamic behavior in capital accumulation

$$(1/j)E_t \sum_{p=0}^{j-1} r_{t+p} = E_t \left[ \psi f_k (\hat{\psi}_{t+j} + \hat{f}_{k_{t+j}(j)}^{1+j}) \right] - \delta - (1 + r)A_{k_{t+j}(j)}^{1+j-1} - A_k^{1+j} \hat{A}_{k_{t+j}(j)},$$  

(17)

where variables without time subscripts denote steady-state figures and "hat" variables represent percent deviations from steady state.9 Interpretation of (17) is quite straightforward: expected marginal cost equal to expected marginal return. On the right hand side, we see the expected return on $k_{t+j}(j)$. This is what remains of the net marginal increase in production due to the additional capital raised $\psi f_k (\hat{\psi}_{t+j} + \hat{f}_{k_{t+j}(j)}^{1+j}) - \delta$ once the necessary adjustment costs $-(1 + r)A_{k_{t+j}(j)}^{1+j-1} - A_k^{1+j} \hat{A}_{k_{t+j}(j)}$ have been deducted. The appearance of the real marginal cost $\hat{\psi}_{t+j}$ in (17) is a result of the prevailing monopolistic competition setup. Thus, a rise in the real marginal cost means more costly labor, and more capital accumulation through a substitution effect. Likewise, higher expected marginal productivity of capital $\hat{f}_{k_{t+j}(j)}$ will bring about more capital accumulation to exploit its marginal return. With respect to marginal adjustment costs, $A_{k_{t+j}(j)}^{1+j-1}$ has a positive sign and represents the marginal costs of having $k_{t+j}(j)$ installed, and $\hat{A}_{k_{t+j}(j)}^{1+j}$ has a negative sign, since it saves some adjustment costs for the next period.

On the left hand side there is an opportunity cost of $k_{t+j}(j)$ measured by the expected average of real interest rates that the financial asset (government bond) is yielding during the time-to-build required for $k_{t+j}(j)$. Noteworthily, the introduction of the time-to-build requirement makes a stream of interest rates matter on the optimal capital decision. As there are $J$ different types of capital, total investment will be influenced by $r_t, E_r r_{t+1}, ..., E_r r_{t+j}$. As a result, investment will more closely depend on a medium-term real interest rate definition rather than the short-term value that comes in when there is no time-to-build.10

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9 Regarding the derivatives of the adjustment costs function in steady state appearing in (17), $A_k^*$ is its steady-state derivative in any period with respect to the stock of capital of the previous period, whereas $A_k$ is its steady state derivative in any period with respect to the stock of capital of that same period.

10 If there were no time-to-build restriction in the optimizing program, the first order equation governing capital accumulation would imply that $r_t = \psi f_k (\hat{\psi}_t + \hat{f}_k) - \delta - (1 + r)A_k^{*} \hat{A}_k^{1-1} - A_k \hat{A}_k$. 

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The inflation equation

We now depart from the first order condition applicable to that fraction of households \((1 - \eta_P)\) who were able to optimize when setting their selling price. For a representative \(i\)-th household, the selling price first order condition \((P_{t, \eta}^{foc})\) derived in the appendix implies that

\[
P_t(i) = \theta_P \frac{E_t \left[ \sum_{i=0}^{\infty} \beta^i \eta_P (1 + \pi)^{-i} \xi_{t+i} \left( P_{t+i}^A \right)^{\theta_P} \xi_{t+i} \right]}{\theta_P - 1} E_t \left[ \sum_{i=0}^{\infty} \beta^i \eta_P (1 + \pi)^{-i} \lambda_{t+i} \left( P_{t+i}^A \right)^{\theta_P-1} \xi_{t+i} \right].
\]

(18)

For convenience, first order condition (13) can be used to omit the Lagrange multipliers \(\xi_{t+i}\), and first order condition (15) to omit the Lagrange multipliers \(\lambda_{t+i}\). The resulting expression can be rearranged and log-linearized to obtain

\[
\log P_t(i) = \beta \eta_P E_t \log P_{t+1} + (1 - \beta \eta_P) \log P_t^A + (1 - \beta \eta_P) \tilde{\psi}_t(i),
\]

(19)

where \(\tilde{\psi}_t(i) = \tilde{\beta} - \tilde{\beta}_{\eta_P}(i)\) is the percent change from steady state in the real marginal cost for households who can set the price optimally. As shown in (19), the price set by these households \(P_t(i)\) depends positively on the expected future evolution of both the aggregate price level and the real marginal cost.

The aggregate price level equation (7) introduced in the previous section in log-linear terms yields

\[
\log P_t^A = (1 - \eta_P) \log P_t(i) + \eta_P \log P_{t-1}^A.
\]

(20)

By combining equations (19) and (20), defining inflation as \(\pi_t = \log P_t^A - \log P_{t-1}^A\), and neglecting the constant term, it is possible to describe the inflation dynamics of the model as

\[
\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \beta \eta_P)(1 - \eta_P)}{\eta_P} \tilde{\psi}_t(i).
\]

(21)

The equation obtained is the so-called New Keynesian Phillips curve widely used in the literature (see Yun (1996), King and Wolman (1996), or Goodfriend and King (1997)). Current inflation depends with a positive sign on next period’s inflation and real marginal costs under optimal price setting. The influence of future marginal costs on current inflation is almost as significant as that of current marginal costs, since the discount factor \(\beta\) is typically very close to one after calibration.

In the New Keynesian Phillips curve (21), the real marginal cost \(\tilde{\psi}_t(i)\) refers to that which is computed when the selling price is set optimally. It is common in the literature for the real marginal cost to be identical among producers, thus leading to the same individual and aggregate real marginal costs. However, this result depends on a number of assumptions that are not satisfied.
in our time-to-build model. Since capital is predetermined in our setup, the labor-capital ratio depends upon the pricing conditions of each producer as does the real marginal cost. Therefore, if prices are sticky, real marginal costs will differ among producers.

A second consequence of using the time-to-build approach is that the real marginal cost is positively affected by the level of output produced. Since current capital cannot be adjusted, more labor will be hired as more output is produced. Then, labor marginal productivity will fall and real marginal costs will rise. Real marginal costs rise as output increases.

With no symmetry in real marginal costs, some algebra is necessary to express the inflation equation depending on the aggregate real marginal cost \( \psi_t^A \). Thus, we define \( \psi_t = \hat{w}_t^A - \hat{P}_{n_t^A} \) where marginal productivity is computed with respect to aggregate labor of the economy \( n_t^A \). The task now is to find the relationship between \( \psi_t(i) \) and \( \psi_t^A \), in order to insert it into the New Keynesian Phillips curve (21).

The Dixit-Stiglitz demand function (6) can be expressed in log-linear terms as follows

\[
\bar{y}_t = -\theta_P (\log P_t(i) - \log P_t^A) + \hat{y}_t^A. \tag{6'}
\]

Taking into consideration that both \( \bar{y}_t(i) \) and \( \hat{y}_t^A \) are obtained by using Cobb-Douglas production technology, \( \alpha \) it yields

\[
\alpha \hat{K}_t(i) + (1 - \alpha) \hat{n}_t^d(i) = -\theta_P (\log P_t(i) - \log P_t^A) + \alpha \hat{K}_t^A + (1 - \alpha) \hat{n}_t^A,
\]

where \( \hat{K}_t^A \) are percent deviations from steady state of the aggregate capital bundle \( \hat{K}_t^A = \int_0^1 K_t(i) \, di \).

The previous expression can be rearranged in order to be solved for \( \hat{n}_t^d(i) \)

\[
\hat{n}_t^d(i) = \frac{\theta_P}{1 - \alpha} \left( \log P_t(i) - \log P_t^A \right) - \frac{\alpha}{1 - \alpha} \left( \hat{K}_t(i) - \hat{K}_t^A \right) + \hat{n}_t^A. \tag{22}
\]

By subtracting (22) from (6'), we get

\[
\hat{P}_{n_t^d(i)} = \frac{\alpha \theta_P}{1 - \alpha} \left( \log P_t(i) - \log P_t^A \right) + \frac{\alpha}{1 - \alpha} \left( \hat{K}_t(i) - \hat{K}_t^A \right) + \hat{P}_{n_t^A}. \tag{23}
\]

The loglinearized aggregate price level equation (20) implies that \( \log P_t(i) - \log P_t^A = \eta_P (1 - \eta_P)^{-1} \pi_t \). When this result is inserted into (23), it yields

\[
\hat{P}_{n_t^d(i)} = \frac{\alpha \theta_P \eta_P}{(1 - \alpha) (1 - \eta_P)} \pi_t + \frac{\alpha}{1 - \alpha} \left( \hat{K}_t(i) - \hat{K}_t^A \right) + \hat{P}_{n_t^A}.
\]

\[^{11}\text{In an optimizing monopolistic competition model with endogenous investment and price stickiness à la Calvo, the real marginal cost is identical across producers when the following three conditions are satisfied: i) capital installation coincide in time with capital demand, ii) the adjustment costs of investment are considered outside the producer’s optimizing program, and iii) production technology exhibits constant returns to scale. Assumptions i) and ii) are not satisfied in our model. If i), ii), and iii) were satisfied, the real marginal costs would be independent of both the selling price and the amount of output produced. See Yun (1996), and Christiano et al. (2001) for two representative examples.}

\[^{12}\text{Loglinearizing the overall resources constraint (12), and using } y_t^* = y_t^A \text{ result in } \hat{y}_t^A = \alpha \hat{K}_t^A + (1 - \alpha) \hat{n}_t^A.\]
Finally, by substituting \( \tilde{f}_t^\text{it} \) from the above equation into the definition \( \tilde{\psi}_t^\text{it} = \tilde{w}_t^A - \tilde{f}_t^\text{it} \), we obtain

\[
\tilde{\psi}_t^\text{it} = \tilde{\psi}_t^A - \frac{\alpha \theta \rho \eta_P}{(1 - \alpha)(1 - \eta_P)} \pi_t - \frac{\alpha}{1 - \alpha} \left( \tilde{K}_t^\text{it} - \tilde{K}_t^A \right). \tag{24}
\]

With falling inflation, the real marginal cost under optimal pricing \( \tilde{\psi}_t^\text{it} \) is greater than the aggregate real marginal cost \( \tilde{\psi}_t^A \). We now can substitute (24) in the New Keynesian Phillips curve (21) so as to present the inflation dynamic equation

\[
\pi_t = \phi_1 E_t \pi_{t+1} + \phi_2 \tilde{\psi}_t^A + \phi_3 \left( \tilde{K}_t^\text{it} - \tilde{K}_t^A \right), \tag{25}
\]

where \( \phi_1 = \frac{\beta}{(1 + \frac{(1 - \beta \eta_p)(1 - \eta_P)}{(1 - \alpha)(1 - \beta \eta_p)(1 - \eta_P)})} \), \( \phi_2 = \frac{(1 - \beta \eta_p)(1 - \eta_P)}{\eta_P (1 + \frac{(1 - \beta \eta_p)(1 - \eta_P)}{(1 - \alpha)(1 - \beta \eta_p)(1 - \eta_P)})} \), and \( \phi_3 = \frac{(1 - \beta \eta_p)(1 - \eta_P) \alpha}{\eta_P (1 - \alpha)(1 + \frac{(1 - \beta \eta_p)(1 - \eta_P)}{(1 - \alpha)(1 - \beta \eta_p)(1 - \eta_P)})} \). Inflation depends positively on expected next period’s inflation, \( E_t \pi_{t+1} \), and on current aggregate real marginal costs, \( \tilde{\psi}_t^A \), and negatively on the current capital bundle gap, \( \tilde{K}_t^\text{it} - \tilde{K}_t^A \). Note that the discount rate attached to \( E_t \pi_{t+1} \) is lower in (25) than in (21), thus implying that future aggregate real marginal costs have less influence on inflation than the marginal costs under optimal prices which appear in (21). Moreover, the coefficient that shows the impact of real marginal costs on inflation is also lower in (25) than in (21).

### 4 Calibration

The parameters of the model are calibrated to suit the impulse response functions analysis conducted below. A Taylor-type monetary policy rule with the nominal interest rate as the instrument will be shocked in the impulse-response analysis. Accordingly, some of the parameters of the model were calibrated relying on empirical works regarding the effects of interest rate shocks. The required evidence was specifically found in Bernanke and Mihov (1995), Christiano, Eichenbaum, and Evans (1996, 1998), and Rotemberg and Woodford (1997). In an attempt to generalize the results of these authors, we would say that an annualized 1% interest rate shock has a negative impact on both output and inflation, characterized by a delay of between two and six quarters in their maximal responses. Moreover, the impact on inflation appears to be quantitatively low (about one tenth of the shock) whereas output drops by about 0.5% of its value. These findings are used below to calibrate \( \zeta, \sigma, \) and \( \Theta_2 \). In addition, standard figures generally accepted in optimizing dynamic models are adopted for the calibration of the remaining parameters.

In the utility function specification (1), the habit formation term is set at \( \zeta = 0.9 \), in order to induce a three-quarter lag in the maximal response of consumption to an interest rate shock. The consumption relative risk aversion is \( \sigma = 4.0 \) so that the size of responses from consumption and output to an interest rate shock is realistic (numbers reported below). The leisure relative risk
aversion coefficient is set at $\gamma = 8.0$, leading to quite low elasticity of labor supply with respect to the real wage (+0.25), which is consistent with results based on the microeconomic evidence reported by Pencavel (1986). As for the scale parameter $\Upsilon$, this is set to imply that one third of total time is devoted to work in aggregate magnitudes. Finally, the household’s intertemporal discount factor is $\beta = 0.995$, assuming a rate of intertemporal preference of 0.5% per quarter, that is 2% per year. 

In the transaction costs function (2), the value assigned to the constant term $h_0$ will imply that transaction costs take 1% of output in steady state. The figure actually selected will depend on the number of types of capital in the model. The scale parameter $h_1$ is set to give a steady state ratio of real money over consumption, $m/c$, equal to 1.5. This criterion leads to set $h_1 = 0.026$. As for the real money share parameter in the transactions technology, it is set at $h_2 = 0.85$ which brings about an elasticity of the nominal interest rate in the money demand function equal to $-0.15$. 

In the adjustment costs function (4), the scale parameter $\Theta_1 = 419.8$ implies that, in steady state, adjustment costs of investment are equal to 6% of total investment, i.e., nearly 1.5% of output. As for the elasticity parameter, $\Theta_2$, it is set $\Theta_2 = 2.4$ so that in the sticky-price, sticky-wage calibrated model with $J = 4$, the peak response of investment to an interest rate shock is 6 times that of consumption and 2.6 times that of output. The capital depreciation rate is $\delta = 0.025$ for all types of capital. 

In the production function (5), the $J$ different types of capital have identical weights, $\Phi_j = J^{-1}$ for all $j$ belonging to $[1,...,J]$. The capital bundle share parameter in the production function $(5')$ is $\alpha = 0.36$. In addition, the coefficient $\nu$ is set at $\nu = 10.0$ so that the elasticity of substitution between differentiated capital goods is -0.96, close to its ceiling of -1.0. 

Following the empirical results by Basu and Fernald (1994), and Basu (1996), the elasticity of substitution between differentiated consumption goods is $\theta_P = 10.0$, implying a 10% mark-up of price over marginal cost in steady state. The elasticity of substitution between labor services is $\theta_W = 4.0$, as suggested by Griffin (1992, 1996). Neither prices nor nominal wages can be adjusted optimally with a probability equal to $\eta_P = \eta_W = 0.75$, as in Erceg et al. (2000). Subsequently, both prices and wages are set optimally on average once a year.14

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13 Based on equipment, structures, and residential investment data, Edge (2000) estimates these weights for the US economy in a time-to-build model with six different types of capital.

14 Christiano et al. (2001) estimate the average length of optimal price and wage contracts in the US. They report that prices are set every 6 months, whereas wages last on average nearly a year. Smets and Wouters (2001) also estimated these parameters in the Euro area and found optimal prices to last an average of two and a half years, and optimal wages one year.
As for the coefficients in the inflation equation (25), calibration of parameters $\beta, \theta_P, \eta_P,$ and $\alpha$ leads to the following figures: $\phi_1 = 0.41, \phi_2 = 0.0348,$ and $\phi_3 = -0.0196$.

The last feature of the model to be calibrated is the value assigned to $J$, the number of types of capital classified according to their time-to-build requirement. Since this point is crucial in the paper, we will view four different possibilities: $J = 1, J = 2, J = 4,$ and $J = 8$. As a result, we have the model ranging from a case in which there is only a simple type of capital with a one-quarter time-to-build requirement, to one in which there are eight different types of capital requiring from one quarter to eight quarters to be fully installed. The next section deals with the implications of having different time-to-build properties on this benchmark calibrated model and on several of its variants.

5 What does time-to-build do?

As mentioned above, the consequences of considering the time-to-build approach in the model will be investigated by observing how the variables of the model respond to a nominal interest rate shock. Thus, we need to define the monetary policy that is in place. Monetary authorities are assumed to follow a monetary policy rule on the nominal interest rate instrument.\textsuperscript{15} Specifically, the following Taylor-type rule with interest rate smoothing is implemented

\begin{equation}
R_t - R = (1 - \mu_3)\mu_1E_{t-1}\left[\pi_{t+1} - \pi\right] + (1 - \mu_3)\mu_2E_{t-1}\tilde{y}_t + \mu_3(R_{t-1} - R) + \varepsilon_t, \tag{26}
\end{equation}

where $\tilde{y}_t$ is the output gap, and $\varepsilon_t$ is a white-noise nominal interest rate shock. The output gap is defined as the percentage difference between current output and potential output. Potential output is calculated in the model as the amount of output that would be produced if all the prices and wages were fully flexible to adjust optimally. Expected values of inflation deviations and the output gap are considered when applying the rule since the actual values are assumed not to be available at the time of the decision. The coefficients chosen for the monetary policy rule are $\mu_1 = 1.5, \mu_2 = 0.5, \text{ and } \mu_3 = 0.85$.

A one-unit positive shock in (26) is considered to imply a one-unit unexpected rise in the nominal interest rate at the time of the shock. Attention is focused on the responses of the following variables of the model plotted in Figures 1-5: nominal interest rate ($R$), the capital bundle ($k$), investment ($x$), consumption bundle ($c$), output ($y$), the capital bundle gap ($kgap$), real wage ($w$), marginal productivity of labor ($fn$), the real marginal cost ($\psi$), and the rate of inflation ($\pi$). All the variables represent aggregate magnitudes of the economy except for the capital bundle gap

\textsuperscript{15} Alternatively, the monetary policy rule could be formulated with respect to a money-growth instrument. This alternative was examined and results in the impulse response functions were found to be very similar.
which is a log-difference between the capital bundle under optimal price setting and the aggregate capital bundle. The numbers reported are percent deviations from steady state except for the nominal interest rate and the inflation rate which are level deviations from steady state values. The impulse response functions are plotted on a 3-year time horizon, from quarter 0 to quarter 12. The monetary policy shock occurs in quarter 1.

The impact of the time-to-build approach on the business cycle properties of the model will be analyzed by comparing cases $J = 1$, $J = 2$, $J = 4$, and $J = 8$.

**Benchmark economy**

Figure 1 shows the results for the benchmark model with sticky prices ($\eta_P = 0.75$), sticky wages ($\eta_P = 0.75$), consumption habit formation, and adjustment costs of investment. Table 1 and Table 2 contain the capital bundle, investment, consumption, output, and inflation responses when $J = 1$ and $J = 8$, respectively. As displayed in Figure 1 and reported in Tables 1 and 2, the introduction of longer time-to-build types of capital leads to significant changes in the overall picture. To begin with, the capital bundle responses are smoother with longer time-to-build types of capital. Subsequently, falls in investment are less pronounced and more gradual with some delay reported in the maximum fall.

Consumption response barely changes in a longer time-to-build economy. In turn, the effects of the time-to-build requirement on output response are a result of its impact on investment decisions. Declines in output, like those in investment, are more gradual and less striking in economies with longer time-to-build types of capital. For example, output reports a maximum fall of 3.36% at the time of the shock when $J = 1$, and slightly above 2% two quarters after the shock when $J = 4$. If $J = 8$, the maximum fall of output is 1.68% of its value and occurs at both two and three quarters after the shock. In turn, maximum output response to an interest rate shock registers a lag only when the model incorporates types of capital with moderately long time-to-build requirements. In such a case, the output response follows a u-shape pattern, consistent with the prediction suggested by the empirical evidence.

Wage stickiness in the labor market brings about a small and very slow fall in the real wage which does not seem to be significantly affected by the time-to-build conditions prevailing in the economy.

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16 These output responses may appear unrealistically strong. However, since the model is specified in quarterly observations an interest rate shock should be multiplied by four to be expressed in conventional annualized terms. Therefore, responses of output to a one-unit annualized interest rate shock are equivalent to those reported divided by four. Output would therefore decline by around 0.5% when $J = 4$ or $J = 8$, which is consistent with the empirical evidence provided by Christiano, Eichenbaum, and Evans (1996, 1998), and Rotemberg and Woodford (1997).

17 Since the shock was contractionary, u-shape responses take the inverse form of the hump-shape patterns typically found after an expansionary shock.
The effect of a nominal interest rate shock on inflation resembles that of the real marginal costs (see Figure 1). The capital bundle gap has little impact on inflation. In log-linear magnitudes, the real marginal cost is the difference between the real wage and the marginal productivity of labor. Of the two, it is the latter that most closely determine the response in the real marginal cost and ultimately the response in inflation. The response in the marginal productivity of labor becomes smoothed and significantly delayed in the presence of longer time-to-build capital. The real marginal cost follows pretty much the same pattern but in the opposite direction, falling more gradually when the economy features longer time-to-build capital. As a result, we also obtain a u-shape response in inflation to the nominal interest rate shock similar to that displayed by output and investment (see Table 2 and Figure 1). Interestingly, the purely forward-looking inflation equation (26) replicates a lag in the response of inflation to an interest rate shock when a sufficiently long time-to-build requirement is considered.

In short, output and inflation were found to display a u-shape response to an interest rate shock in the benchmark calibrated model, when there are types of capital with a time-to-build requirement exceeding a certain length.

Flexible-price economy \((\eta_P = 0.0)\)

Flexible prices lead to symmetric behavior across households in terms of output, demand for labor, and demand for capital. As a result, the capital bundle gap is always zero. Figure 2 shows the impulse response functions to an interest rate shock. Similar responses to the sticky-price case are reported on the demand side of the economy. Figure 2 also shows that falls in both investment and output are more gradual and less drastic when longer time-to-build types of capital are present in the economy.

By contrast, the supply side shows very different patterns from the ones it displays in the sticky-price benchmark economy. Optimal price setting behavior across all households leads to a flat real marginal cost: all the producers set prices so as to equate marginal productivity of labor and the real wage, \(\tilde{w}_{t+i} = \tilde{f}_{n_{t+i}}\). Subsequently, a substantial drop in prices is needed to lift the real wage up to the level of rising marginal productivity of labor. In turn, two unrealistic features are found in the supply-side responses. First, the real wage becomes strongly countercyclical. Second, the impact of the nominal interest rate shock on inflation is immediate and quantitatively very substantial. These two findings are robust to any time-to-build specification. Thus, a longer time-to-build capital in the economy only makes the real wage responses less pronounced and slower smaller to peak, while remaining clearly countercyclical. As for inflation, its response is
somewhat diminished and more persistent with longer time-to-build capital but remains relatively strong with no u-shape pattern reported.

**Flexible-wage economy** ($\eta_W = 0.0$)

Another variant of the model emerges from when fully flexible wages are assumed. In other words, all households can optimize when setting the nominal wage that they will receive for their labor supply. Figure 3 displays the impulse response functions obtained in this economy.

As Figure 3 shows, the demand sector of the economy (consumption, investment, and output) is not substantially altered when wages are flexible. In the event of a nominal interest rate shock, they all report similar falls to those that occur in the benchmark economy. Moreover, the impact of longer time-to-build requirements seems to be the same as in the benchmark economy: both investment and output move towards a u-shape response when longer time-to-build types of capital are considered.

The story is on the supply side, however, quite different. Now that all the nominal wages are flexible to adjust optimally, they move down dramatically. After a contractionary shock, households wish to work more as consumption falls and leisure time increases. Consequently, they set a lower nominal wage. With fully flexible nominal wages, the real wage increases so unrealistically as to become highly procyclical, dropping by 400% when $J = 1$. In a longer time-to-build economy, declines are not as steep and (by 130% when $J = 8$) but they last longer. These severe falls in the real wage govern responses in real marginal costs and inflation. Hence, both the real marginal cost and inflation also report very deep falls. No delay is found to occur in the maximum response of inflation even when $J = 8$.

**No adjustment-cost economy** ($\Theta_1 = 0.0$)

As Figure 4 shows the absence of adjustment costs of investment gives rise to an incredibly large responses to an nominal interest rate shock. The most relevant of these is the investment response, which registers a dramatic fall at the time of the shock. Investment plunges by more than 4000% if $J = 1$ and by nearly 900% if $J = 8$! These huge falls are partially transmitted to the output responses which also report very high figures.

The consequences of such dramatic falls in investment and output spread to bring about unrealistic responses on the supply side of the economy. As labor demand falls with output, the marginal productivity of labor soars leading to a steep decline in the real marginal cost. As a result, inflation falls severely at the time of the shock. The introduction of longer time-to-build

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18 This same result has been reported by Casares and McCallum (2000).
capital types has a moderating effect on all the responses reported but figures remain extremely high.

No consumption habit formation economy ($\zeta = 0.0$)

Finally, the last variant of the benchmark economy is the economy without habit formation in consumption preference. Figure 5 contains the impulse response functions to an interest rate shock. As expected, the most significant change is to be seen in consumption response which is now somewhat greater and peaked at the time of the shock under any time-to-build specification. This response is persistent over time and still noticeable 12 quarters after the shock.

As the consumption lag disappears, the delay in output response relies only on the delay in investment. Therefore, the u-shape pattern of output is no longer as marked as it was in the benchmark economy that featured consumption habit formation.

All the variables on the supply side respond very much as they did in the benchmark economy. Thus, the real marginal cost and the rate of inflation are affected in the same way by the time-to-build requirement. The effect of longer time-to-build types of capital is to reduce and slow down falls in both variables.

6 Conclusions

This paper has explored some business cycle implications of including a time-to-build requirement in a sticky-price-sticky-wage optimizing monetary model with endogenous investment and adjustment costs. The setup described in the seminal paper by Kydland and Prescott (1982) was extended so as to include in the production technology different types of capital that varied in their time-to-build requirement.

As installing capital goods takes time, the capital accumulation decision is made in advance and capital cannot be adjusted to produce current output. In turn, the dynamic behavior of capital and inflation are affected by the time-to-build requirement. It has been demonstrated, for example, how optimal capital depends not only on the current real interest rate but also on the expected real interest rates during the time-to-build period. In addition, the time-to-build requirement in a sticky-price economy leads to asymmetry of the real marginal cost across producers, which has a significant impact on the dynamics of inflation.

The effects of an unexpected rise in the interest rate using a Taylor-type monetary policy rule were analyzed under several time-to-build specifications. It was found that the introduction of longer time-to-build types of capital has relevant implications: the responses of both output and inflation are smoother and delay in reaching their peak. In other words, the model replicates
u-shape output and inflation responses to an interest rate shock when including types of capital ranging from the one-quarter time-to-build capital to the eight-quarter time-to-build capital.

The process originating the u-shape responses in output and inflation begins with the slower reaction of investment as capital takes longer to complete installation. In such a case, output also responds more slowly since investment is one of its components. On the supply side, the fall in labor is also delayed by the lag in output response. The presence of sticky wages smooths the real wage and leaves the real marginal cost mostly determined by the marginal productivity of labor. Consequently, the delay in labor response gives rise to delays in both the marginal productivity of labor and the real marginal costs responses. Ultimately, inflation reports a lag in reaching its peak response as optimal prices are set by looking at real marginal costs.

However, these results are not robust to three variants of the model: flexible prices, flexible wages, and no adjustment costs of investment. If either prices or nominal wages are fully flexible, the real marginal cost and inflation responses are much greater and observed at the time of the shock, even when time-to-build requirements are in place. In a version of the model without adjustment costs of investment the responses of all the variables to an interest rate shock are simply unrealistically high.
APPENDIX. The household optimizing program.

The optimizing program of a representative household consists of maximizing the sum of current and expected discounted future utility values subject to the sequence of budget constraints, time constraints and market demand constraints:

$$\text{Max} \sum_{t=0}^{\infty} \beta^t U (c_{t+i}, c_{t-1+i}, l_{t+i})$$

subject to:

$$E_t \left[ \frac{(1+\pi)^P_t}{E_{t+i}} \right] 1 - \theta_p \frac{y_{t+i}}{W_{t+i}} + \left[ \frac{(1+\pi)^P_t}{W_{t+i}} \right] 1 - \theta W_{t+i} n_{t+i}^A + g_{t+i} +$$

$$- \alpha_{t+i} - (1/j) \sum_{j=1}^{J} \sum_{p=1}^{J} (k_{t+p+i}(j) - (1-\delta)k_{t+p-1+i}(j)) - \sum_{j=1}^{J} A(k_{t+i+1}(j), k_{t+i}(j)) +$$

$$- u_{t+i}^A n_{t+i}^d - m_{t+i} + (1 + \pi_{t+i})^{-1} m_{t-1+i} - (1 + r_{t+i})^{-1} b_{t+1+i} + b_{t+i} - h(c_{t+i}, m_{t+i}) = 0,$$

$$E_t \left[ \frac{(1+\pi)^P_t}{W_{t+i}} \right] - \theta W_{t+i} n_{t+i}^A + l_{t+i} - 1 = 0,$$

$$E_t \left[ f(k_{t+i+1}(1), ..., k_{t+i}(J), n_{t+i}^d) - \frac{(1+\pi)^P_t}{W_{t+i}} \right] - \theta W_{t+i} y_{t+i} = 0,$$

for $i = 0, 1, 2, ..., \infty$, and $j = 1, ..., J$. The solution in period $t$ comprises the three constraints for $i = 0$, and the following first order conditions related to each choice variable

$$U_{t+i}^l + \beta U_{t+i}^{l+1} - \lambda_t \left( 1 + h_{t+i}^l \right) = 0,$$

$$U_{t+i}^l + \varphi_t = 0,$$

$$\lambda_t \left( 1 + h_{t+i}^l \right) + \beta E_t \left[ A_{t+1} + 1 \right] = 0,$$

$$\lambda_t (1 + r_t)^{-1} - \beta E_t \lambda_{t+i+1} = 0,$$

$$\lambda_t (1 + r_t)^{-1} + \beta E_t \lambda_{t+i+1} = 0,$$

where $\lambda_{t+i}$, $\varphi_{t+i}$, and $\xi_{t+i}$ are respectively the Lagrange multipliers of the budget, time, and market demand constraints in any period $t + i$. If both the selling price $P_t$ and the nominal wage $W_t$ can also be optimally set by the household, their respective first order conditions are
\[
E_t \sum_{i=0}^{\infty} \left[ (1-\theta_P)\beta^i \eta_P^i \left( \lambda_{t+i} \left[ \frac{(1+\pi)^i P_t}{P_{t+i}} \right] -\theta_P \left( \frac{(1+\pi)^i y^*_{{P,t+i}}}{y^*_{{P,t+i}}} \right) \right) \right] + (P_t^{foc})
\]

\[
E_t \sum_{i=0}^{\infty} \left[ \theta_P \beta^i \eta_P^i \left( \xi_{t+i} \left[ \frac{(1+\pi)^i P_t}{P_{t+i}} \right] -\theta_P^{-1} \left( \frac{(1+\pi)^i y^*_{{P,t+i}}}{y^*_{{P,t+i}}} \right) \right) \right] = 0,
\]

\[
E_t \sum_{i=0}^{\infty} \left[ (1-\theta_W)\beta^i \eta_W^i \left( \lambda_{t+i} \left[ \frac{(1+\pi)^i W_t}{W_{t+i}} \right] -\theta_W \left( \frac{(1+\pi)^i y^*_{{W,t+i}}}{y^*_{{W,t+i}}} \right) \right) \right] - (W_t^{foc})
\]

\[
E_t \sum_{i=0}^{\infty} \left[ \theta_W \beta^i \eta_W^i \left( \varphi_{t+i} \left[ \frac{(1+\pi)^i W_t}{W_{t+i}} \right] -\theta_W^{-1} \left( \frac{(1+\pi)^i y^*_{{W,t+i}}}{y^*_{{W,t+i}}} \right) \right) \right] = 0,
\]

References


Table 1. Benchmark economy.
Time-to-build capital takes 1 quarter ($J = 1$).

Responses to a unit monetary policy shock.

<table>
<thead>
<tr>
<th>Quarters after the shock</th>
<th>Capital bundle</th>
<th>Investment</th>
<th>Consumption</th>
<th>Output</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-10.12</td>
<td>-0.51</td>
<td>-3.36</td>
<td>-0.129</td>
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<td>-1.33</td>
<td>-0.088</td>
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<tr>
<td>6</td>
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<td>-1.73</td>
<td>-0.81</td>
<td>-1.08</td>
<td>-0.080</td>
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</tbody>
</table>

Table 2. Benchmark economy.
Time-to-build capital takes from 1 to 8 quarters ($J = 8$).

Responses to a unit monetary policy shock.

<table>
<thead>
<tr>
<th>Quarters after the shock</th>
<th>Capital bundle</th>
<th>Investment</th>
<th>Consumption</th>
<th>Output</th>
<th>Inflation</th>
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<td>-1.99</td>
<td>-0.78</td>
<td>-1.17</td>
<td>-0.079</td>
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</tbody>
</table>
Figure 1. Benchmark economy.

Time-to-build capital takes from $1$ to $J$ quarters.

Impulse response functions to a unit monetary policy shock.
Figure 2. Flexible-price economy ($\eta_P = 0.0$).
Time-to-build capital takes from 1 to $J$ quarters.
Impulse response functions to a unit monetary policy shock.
Figure 3. Flexible-wage economy ($\eta_W = 0.0$).

Time-to-build capital takes from 1 to $J$ quarters.

Impulse response functions to a unit monetary policy shock.
Figure 4. No adjustment-cost economy (Θ_1 = 0.0).
Time-to-build capital takes from 1 to J quarters.
Impulse response functions to a unit monetary policy shock.
Figure 5. No consumption habit formation economy ($\zeta = 0.0$).
Time-to-build capital takes from 1 to $J$ quarters.
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