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AN MVAR FRAMEWORK TO CAPTURE EXTREME EVENTS IN MACRO-PRUDENTIAL STRESS TESTS

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Abstract

Severe financial turbulences are driven by high impact and low probability events that are the characteristic hallmarks of systemic financial stress. These unlikely adverse events arise from the extreme tail of a probability distribution and are therefore very poorly captured by traditional econometric models that rely on the assumption of normality. In order to address the problem of extreme tail events, we adopt a mixture vector autoregressive (MVAR) model framework that allows for a multi-modal distribution of the residuals. A comparison between the respective results of a VAR and MVAR approach suggests that the mixture of distributions allows for a better assessment of the effect that adverse shocks have on counterparty credit risk, the real economy and banks' capital requirements. Consequently, we argue that the MVAR provides a more accurate assessment of risk since it captures the fat tail events often observed in time series of default probabilities.

JEL classification: C15, E44, G01, G21

Keywords: *stress testing, MVAR, tier 1 capital ratio, counterparty risk, Luxembourg banking sector*

Non-Technical Summary

The recent crisis period brought attention not only to the underlying deficiencies in banking sector risk management practices it also highlighted some fundamental inadequacies in financial sector regulatory frameworks. As a result, the true magnitude of the potential risks to financial stability at the onset of the crisis seems to have been significantly underestimated.

Perhaps one of the more important lessons of the recent financial crisis was the clear indication that the usefulness and applicability of commonly used statistical models remains limited in the presence of extreme events. Moreover, such models can break down dramatically when confronted with shocks that are several standard errors from the historical mean. Owing to their inability to capture extreme events, these models seem inappropriate for stress-testing exercises given authorities' focus on extreme but plausible scenarios. Our work, framed in the context of a stress testing model for bank counterparty credit risk, attempts to capture the effect of exogenous fat tail events on bank capitalization levels by using a Mixture Vector Autoregressive (MVAR) model. This class of model relies on a multimodal marginal distribution in order to capture the effect of highly unlikely events. The use of a mixture of distributions, as opposed to a single distribution, in the model makes it possible to intercept fat tail events that would be poorly, or perhaps not at all, captured by models using a univariate marginal distribution. In our view, the mixture of distributions allows us to better capture the effect that adverse shocks have on counterparty credit risk as well as the real economy.

In this study, we estimate and compare two different econometric models in order to calculate tier 1 capital ratios for both baseline and adverse scenarios under severe exogenous shocks. We apply the following methodology. First we estimate a standard VAR that incorporates the probability of default, euro area GDP growth in real terms, the real interest rate and a Luxembourg property price index in order to link the economic environment with an empirical measure of financial sector stability; in our case the probability of default. Given the nature of the VAR model specification, the econometric framework adopted in this work also allows for feedback between the probability of default of banking sector counterparties and the real economy. By contrast, the MVAR incorporates a bivariate marginal distribution which is able to accommodate "fat tails" and thus capture distant tail effects. It is the difference between the marginal distributions of the respective econometric models that enables our evaluation of the impact that fat tail events have on capitalization levels.

We argue that, compared to a VAR model, using the MVAR model to assess counterparty risk provides a more accurate representation of the true risk since the latter can capture the more extreme movements often observed in time series of default probabilities. According to the results in the paper, the VAR model is shown to consistently underestimate counterparty credit risk. In simulations that apply adverse macroeconomic shocks to the variables included in the econometric model, it is found that the minimum level of tier 1 capital required to withstand these shocks is actually underestimated by the VAR model. For shocks to euro area real GDP growth the magnitude of this underestimation equates to a difference in the tier 1 capital ratio of approximately 104 basis points. For some banks, this may represent a significant amount of capital.

Accounting for increased tail risk, in our view, provides a more accurate assessment of how shocks affect the real economy and, in turn, tier 1 capitalization ratios. From the policymaker's perspective, these results have some important implications. First, it suggests that the level of counterparty credit risk is being underestimated. A direct consequence of this result is that tier 1 capital levels extracted from reduced-form models based on univariate marginal distributions may actually be insufficient to withstand the magnitude of exogenous shocks currently being used to assess bank capitalization requirements. Furthermore, in the context of an early warning indicator, it could possibly provide a wrong signal to policymakers. In view of these results, and in order to facilitate the policy maker's decision process, it would seem appropriate to revisit the applicability of stress testing models that rely on univariate marginal distributions in the presence of large negative shocks.

Future work may attempt to compare the results from the MVAR to other non-linear tools. In particular, it would be worth comparing the mixture VAR to the structural VAR with non-normal residuals. This alternative framework also allows for innovations to follow a mixture of normal distributions but uses this feature to provide the identifying assumptions necessary to recover structural shocks to the system. There is another area of comparison we could pursue. The estimation technique used to determine the parameters of the MVAR model has some parallels with the class of Markov regime switching models. However, for the MVAR, the underlying process that governs the dynamics of the regime transition probabilities respects a different set of assumptions. Therefore, it would seem interesting to compare the performance and output of the MVAR model to a regime switching model. Such a study could provide us with additional information on which class of model is best suited to capture fat tail events when a stress testing exercise is specified in terms of a reduced-form model.

1. Introduction

The financial crisis that began in 2008 highlighted not only the poor risk-management practices implemented by the financial sector; it also illustrated weaknesses in financial regulatory and oversight frameworks. In particular, three major post-crisis lessons emerged. First, analysing financial stability requires a system-wide perspective rather than a strict micro-prudential approach. Second, there is an important link between macroeconomic conditions and financial stability that, prior to the crisis, was poorly understood and inadequately monitored. Third, statistical models of the linkages between the financial system and the real economy may break down in the face of extreme events. To address these three challenges, this paper applies a mixture vector autoregression (MVAR) in the context of a macro-economic stress test in order to illustrate the inadequacy of commonly employed VAR models. In forward-looking simulations, the MVAR model can provide multi-modal distributions for counterparty risk in the banking sector, reflecting the possible asymmetries and non-linearities that may manifest in the linkages between macroeconomic developments and financial stability.

The Financial Sector Assessment Programmes (FSAP) launched by the IMF and the World Bank in 1999 spawned a variety of stress-testing tools and numerous surveys have been published on this subject; for example see Sorge (2004), ECB (2006), Drehmann (2008) and Foglia (2009). However, in the period leading up to the recent financial crisis these FSAP analyses failed to identify significant instabilities (Alfaro and Drehmann (2009), Borio and Drehmann (2009)). Part of the problem is that many stress-testing tools focus on the impact of an adverse macro-economic scenario on the balance sheets of individual banks. In this context the micro-prudential perspective, that makes the assumption that macro-economic shocks are exogenous, needs to be complemented by a system-wide - or macro-prudential - perspective. Borio (2009, 2010) argues that although micro-prudential analysis may find that financial institutions are sound on an individual basis, in actual fact, the financial system as a whole may be unstable. For this reason, macro-prudential analysis considers the cross-sectional dimension in order to identify possible common (or correlated) exposures *across* financial institutions that are not apparent when assessing the diversification of portfolios *within* individual institutions. In addition, macro-prudential analysis considers the time dimension to identify possible procyclical patterns in aggregate financial developments that could amplify business cycle fluctuations.

This leads to the second lesson of the crisis, concerning the link between macroeconomic conditions and financial stability. A recent survey by the Basel Committee on Banking Supervision (2011) reveals that much economic analysis focuses on transmission channels from the real to the financial sector (e.g. the macro stress tests mentioned previously) or from the financial to the real sector (e.g. the strand of literature on the bank balance sheet channel, external finance premium and the financial accelerator). However, there have been few attempts to capture transmission in both directions. Since there is no common theoretical framework in which to analyse these linkages in general equilibrium, empirical studies often adopt a reduced-form approach to limit misspecification errors. In general, this involves extending standard macroeconomic vector autoregression (VAR) models to include a measure of financial stability, whether derived from financial market data or from bank balance sheets.

For example, Jacobson, Lindé and Roszbach (2005) included the aggregate default frequency from a panel of non-financial corporations as an exogenous variable in a macroeconomic VAR. Using a separate panel logit regression they showed that macroeconomic variables also explain default risk at the firm level. However, one limitation of their approach was that the financial stability measure - the aggregate default frequency - entered into the VAR as an exogenous variable. Huang, Zhou and Zhu (2009) use a similar micro-macro setup. However, they obtain bank-specific probabilities of default from market data on CDS trades for 12 major U.S. banks. In their case, the macro VAR only includes financial variables such as stock returns, stock price volatility and interest rates in addition to bank probability of default and correlation aggregated over their portfolio of banks. The latter variables are projected forward using bank-specific logit regressions.

Hoggarth, Sorensen and Zicchino (2005) adopt a more integrated approach, by including their financial stability measure directly as an endogenous variable in a macroeconomic VAR for the UK. As a proxy for financial stability, they use loan write-offs from the aggregate banking sector balance sheet. Marcucci and Quagliariello (2008) performed a similar exercise for Italy using the flow of non-performing loans in the banking sector (normalised by total loans). Filosa (2008) experimented with two other measures of financial stability from the Italian banking sector (the stock of non-performing loans or the interest margin), as well as two exogenous macroeconomic variables (euro area interest rate and euro exchange rate). Dovern, Meier and Vilsmeier (2010) estimated a macroeconomic VAR for Germany adding either loan write-offs or return on equity in the banking sector. To avoid possible non-stationarity, the previous studies usually estimated VARs in growth rates. However, Alves (2005) and Åsberg-Sommar and Shahnazarian (2009) allowed for possible cointegration among the endogenous variables, estimating a constrained VAR known as a vector error correction (VEC) model. Both these studies used market-based indicators of expected default frequency instead of bank data.

The third lesson of the crisis considered here is that most statistical models face limitations in the presence of extreme events. Drehmann, Patton and Sorensen (2006, 2007) note that the linear specification of standard VARs may provide an adequate approximation to the data generating process during periods of relative tranquility, but can break down dramatically when considering shocks that are several standard errors from the historical mean. This makes them inappropriate for stress-testing, given the focus on “extreme but plausible scenarios,” a point also illustrated by the examples in Misina and Tessier (2008). Drehmann et al. estimate a non-linear macroeconomic VAR for the UK based on a third-order Taylor expansion. Unlike for the linear VAR, they find that impulse responses can differ significantly depending on the size and sign of the macro-economic shock. To illustrate the consequences in a stress-test context, they feed these impulse responses into a separate probit model for the aggregate corporate liquidation rate. Puddu (2010) integrates the financial variable (in his case the share of non-performing loans) directly as an additional endogenous variable in a non-linear VAR estimated separately for ten different economies. Monnin and Jokipii (2010) also find evidence of non-linearity when they introduce dummy variables in their panel VAR corresponding to periods when the banking sector “distance to default” was either high or low by historical standards.

Fong and Wong (2008) argued that stress tests based on standard linear models will necessarily underestimate the probability of default conditional on an adverse macroeconomic shock. This is because linear models assume that default rates and macroeconomic shocks follow underlying distributions that are normal and therefore unimodal. Such an assumption cannot sufficiently discriminate between tranquil periods and the extreme events that are the focus of stress tests. A simple histogram of loan default rates in Hong Kong banks reveals a clearly bimodal distribution with a minor peak at higher default rates around the time of the Asian financial crisis. To capture this bimodality, Fong and Wong implement a class of mixture vector autoregressive (MVAR) models proposed by Fong, Li, Yau and Wong (2007). This model can be viewed as a mixture of two Gaussian VARs, producing a predictive distribution with different peaks associated with tranquil periods and extreme events. Compared to the standard linear VAR, Monte Carlo simulations with the MVAR suggest much higher credit losses for a given set of macroeconomic shocks.

In this paper, we use the MVAR framework to extend previous work by Rouabah and Theal (2010) evaluating aggregate credit risk for Luxembourg’s banking sector. We compare stress-test results based on a mixture of normals (MVAR) model to those obtained with a standard linear VAR. We also calculate Basel II tier 1 capital ratios under the MVAR framework and compare these to the values obtained from a standard linear VAR model.

This remainder of this paper is structured as follows. First, we briefly review our measure of counterparty risk for Luxembourg. Then we describe the MVAR model before providing a brief description of the VAR. Afterwards, we perform the estimation of the MVAR model. In section 6 we compare the predictive abilities (via their predictive distributions) of the two models. In the seventh section we evaluate the response of the MVAR model to exogenous macroeconomic shocks. We then simulate Tier 1 capital requirements in the presence of these shocks. Lastly we conclude.

2. Evaluating Counterparty Creditworthiness in Luxembourg

In order to estimate the probability of default of counterparties in the Luxembourg banking sector, an aggregate balance sheet was constructed using the ratio of provisions on loans to total loans over all sectors. This ratio was then used as a proxy for the aggregate probability of default, thereby providing a metric for assessing the vulnerability of the Luxembourg financial system to various adverse macroeconomic scenarios.

The historical probability of default series used here consists of quarterly observations over the period spanning from the first quarter of 1995 until the third quarter of 2011. We represent the probability of default using the notation p_t. Since p_t is a probability, and therefore lies in the fixed interval $[0,1]$, a logit transform was applied in order to map the probabilities into the real number space, $\Re$.

$$y_t = \ln\left(\frac{1-p_t}{p_t}\right) \tag{1}$$

The analytical form of the logit transform is presented in equation (1). Here, p_t is transformed such that y_t takes on values in the interval $-\infty < y_t < \infty$. After transformation, y_t and p_t are inversely related. This inverse relationship also applies to the first difference series of y_t. The macroeconomic framework for the VAR(p) model adopted in this work consists of a joint system of four linear equations for the transformed probability of default, euro area real GDP growth, the real interest rate (EURIBOR 3-month) and a Luxembourg-specific real property price index. The VAR model is specified in terms of first differences, except for euro area GDP which is included as a growth rate. Following a lag selection test based on the Bayesian information criteria (BIC), two lags are used in the VAR in order to capture any autocorrelation in the time series data. This specification also allows for feedback effects between the macroeconomic variables and the transformed probability of default series. Furthermore, the possibility of using one or two lags of the endogenous variable in each equation facilitates the persistence and transmission of exogenous shocks through the system.

3. The MVAR model: A tool to capture extreme events

Fong et al. (2007) developed the MVAR model as a multivariate extension of the mixture autoregression model in Wong and Li (2000). An $MVAR(n, K; p_k)$ model with K components for an observed n-dimensional vector Y_t takes the following form:

$$F(y_t|\Im_{t-1}) = \sum_{k=1}^{K} \alpha_k \Phi\left(\Omega_k^{-1/2}\left(Y_t - \Theta_{k0} - \Theta_{k1}Y_{t-1} - \Theta_{k2}Y_{t-2} - \ldots - \Theta_{k1p}Y_{t-p_k}\right)\right) \qquad (2)$$

Where y_t is the conditional expectation of Y_t, p_k is the autoregressive lag order of the k^{th} component, $\Im_{t-1}$ is the available information set up to time $t-1$, $\Phi(\cdot)$ is the cumulative distribution function of the multivariate Gaussian distribution, α_k is the mixing weight of the k^{th} component distribution, Θ_{k0} is an n-dimensional vector of constant coefficients and $\Theta_{k1},\ldots,\Theta_{kp_k}$ are the $n \times n$ autoregressive coefficient matrices of the k^{th} component distribution. Lastly, Ω_k is the $n \times n$ variance-covariance matrix of the k^{th} component distribution. One convenient characteristic of the MVAR is that individual components of the MVAR can be non-stationary while the entire MVAR model remains stationary.

It is possible to estimate the parameters of the MVAR using the expectation-maximization (EM) algorithm of Dempster et al. (1977). This assumes a vector of (generally) unobserved variables $Z_t = (Z_{t,1}, \ldots, Z_{t,K})^{\mathrm{T}}$ defined as:

$$Z_{t,i} = \begin{cases} 1 & if\ Y_t\ comes\ from\ the\ i^{th}\ component; \quad 1 \le i \le K, \\ 0 & otherwise \end{cases} \qquad (3)$$

Where the conditional expectation of the binary indicator $Z_{t,i}$ gives the probability that an observation originates (or does not originate) from the i^{th} component of the mixture.

As shown by Fong et al. (2007), the conditional log-likelihood function of the MVAR model can be written as follows:

$$l = \sum_{t=p+1}^{T} \left\{ \sum_{k=1}^{K} Z_{t,k} \log(\alpha_k) - \frac{1}{2} \sum_{k=1}^{K} Z_{t,k} \log|\Omega_k| - \frac{1}{2} \sum_{k=1}^{K} Z_{t,k} \left(e_{kt}^{\mathrm{T}} \Omega_k^{-1} e_{kt} \right) \right\} \tag{4}$$

Where the following variable definitions apply:

$$\begin{aligned} e_{kt} &= Y_t - \Theta_{k0} - \Theta_{k1} Y_{t-1} - \Theta_{k2} Y_{t-2} - \ldots - \Theta_{kp_k} Y_{t-p_k} \\ &= Y_t - \tilde{\Theta}_k X_{kt} \\ \tilde{\Theta}_k &= \left[\Theta_{k0}, \Theta_{k1}, \ldots, \Theta_{kp_k} \right] \\ X_{kt} &= \left(1, Y_{t-1}^{\mathrm{T}}, Y_{t-2}^{\mathrm{T}}, \ldots, Y_{t-p_k}^{\mathrm{T}} \right)^{\mathrm{T}} \end{aligned} \tag{5}$$

A number of model parameters need to be estimated. The parameter vector of the MVAR model is, in this case, $\Psi\left(\hat{\alpha}_k, \hat{\tilde{\Theta}}_k^{\mathrm{T}}, \hat{\Omega}_k \right)$. Here $\hat{\alpha}_k$ are the estimated mixing weights of the K component distributions, $\hat{\tilde{\Theta}}_k^{\mathrm{T}}$ are the estimated $n \times n$ autoregressive coefficient matrices and $\hat{\Omega}_k$ are estimates of the K $n \times n$ variance covariance matrices. As discussed in Fong et al. (2007), for the purpose of identification, it is assumed that $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_K \geq 0$ and $\sum_k \alpha_k = 1$. In the vector X_{kt} in equation (5), the first element (i.e. the 1) is a scalar quantity.

As shown in Fong et al. (2007), the equations for the expectation and maximization steps can be obtained as follows. In the expectation step, the missing data Z are replaced by their expectation conditional on the parameters $\tilde{\Theta}$ and on the observed data $Y_1 \ldots Y_T$. If the conditional expectation of the k^{th} component of Z_t is denoted $\tau_{t,k}$ then the expectation step is calculated according to equation (6). Another way to interpret the expression for $\tau_{t,k}$ is to think of it in terms of a Markov transition probability. However, in the context of the MVAR it is a transition probability that is independent of its past values. This differs from a first order Markov process where the value of a transition probability at time t is conditional on its past value at time $t-1$. The MVAR can be thought of as a special case of a regime switching model in which the transition probabilities are independent rather than conditional on past values.

Expectation Step:

$$\tau_{t,k} = \frac{\alpha_k |\Omega_k|^{-\frac{1}{2}} \exp\left(-\frac{1}{2} e_{kt}^{\mathrm{T}} \Omega_k^{-1} e_{kt} \right)}{\sum_{k=1}^{K} \alpha_k |\Omega_k|^{-\frac{1}{2}} \exp\left(-\frac{1}{2} e_{kt}^{\mathrm{T}} \Omega_k^{-1} e_{kt} \right)}, k = 1, \ldots, K \tag{6}$$

Following the expectation step, the maximization step can then be used to estimate the parameter vector $\tilde{\Theta}$.

The M-step equations are defined in Fong et al. (2007) as:

Maximization Step:

$$\begin{aligned}
\hat{\alpha}_k &= \frac{1}{T-p}\sum_{t=p+1}^{T}\tau_{t,k}\,, \\
\hat{\tilde{\Theta}}_k^{\mathrm{T}} &= \left(\sum_{t=p+1}^{T}\tau_{t,k}X_{tk}X_{tk}^{\mathrm{T}}\right)^{-1}\left(\sum_{t=p+1}^{T}\tau_{t,k}X_{tk}Y_t^{\mathrm{T}}\right), \\
\hat{\Omega}_k &= \frac{\sum_{t=p+1}^{T}\tau_{t,k}\hat{e}_{kt}\hat{e}_{kt}^{\mathrm{T}}}{\sum_{t=p+1}^{T}\tau_{t,k}}
\end{aligned} \tag{7}$$

where $k = 1,\ldots,K$. The model parameters are subsequently obtained by maximizing the log-likelihood function given in equation (4).

While the EM algorithm can be used to estimate the model parameters, in practice it is possible for the maximization routine to converge to a local rather than a global optimum or to encounter a fixed point at which it is no longer possible to increase the likelihood. Nevertheless, starting from an arbitrary point $\Phi^{(0)}$ in the parameter space, the algorithm almost always converges to a local maximum. In this sense, however, the algorithm cannot guarantee convergence to the global maximum in the presence of multiple maxima. For this reason we modified the simple EM estimation described in Fong et al. (2007) and included a variable neighbourhood search (VNS) routine. This may also help to mitigate the slow convergence of the EM algorithm in the presence of a large amount of incomplete information (see McLachlan and Krishnan (2008)).

4. Data and Estimation of the VAR model

The data consists of historical probabilities of default calculated on a quarterly basis over the period spanning the first quarter of 1995 until the third quarter of 2011 yielding a total of 67 observations. Figure 1 shows a kernel density estimation of the distribution of the logit transformed probabilities of default.

[Figure 1 about here]

Along with the probability of default, the model incorporates data on euro area real GDP growth, the real interest rate and the change in real property prices for a Luxembourg price index. The series for euro area real GDP growth effectively enters the model as a proxy for financial sector aggregate profitability while property prices and the real interest rate are used to capture balance sheet effects and changes in counterparty creditworthiness. This combination of variables allows for possible feedback effects between the probability of default series and the evolution of the macroeconomic variables. Since data on the aggregate default rates of Luxembourg banking sector counterparties was unavailable, it was necessary to construct the series of historical default probabilities. These default probabilities were calculated using a ratio of provisions on loans to total loans to all sectors. Subsequently, this ratio was used as an approximation for the aggregate sector probability of default. It is important to emphasize that although provisioning provides an estimate of the probability of default, loan loss provisions are, in effect, an imperfect approximation of default rates over the business cycle. More specifically, provisioning in some countries can be used in order to take advantage of tax deductions and thus provisioning may only partially reflect credit risk concerns and the true degree of loan impairments on a bank's balance sheet. It should also be mentioned that, in some cases, loan loss provisions themselves can be used in order to adhere to regulatory capital requirements. Luxembourg does not have a national credit registry so we use loan provisions as a proxy for default probabilities. These data are reported as part of the supervisory framework. In this context, the loan loss provision data represents banks' expectations about counterparty default and are not a direct measure of default probability. The provisioning data may also be sensitive to banks' internal risk management models and practices. Given the length of the series, we have controlled for some firms which have entered and exited the sample over time. Lastly, both of these data series are backward looking, so the results should be interpreted with caution.

Using a standard lag length test, the Hannan-Quinn criterion selects a VAR model with a lag length of 2. The corresponding VAR(2) model was estimated and the variable coefficients and their respective standard errors are reported in table 1.

[Table 1 about here]

The signs of the coefficients appear appropriate for the expected dependence of the probability of default on the individual macroeconomic variables. For example, positive increases in the growth rates of euro area real GDP result in an increase in the variable Δy_t. However, this effect acts with a lag which may suggest that balance sheet changes occur gradually over time, possibly due to frictions. This is because y_t is inversely related to the probability of default, and therefore a decrease in euro area real GDP growth results in a positive increase in the probability of default for counterparties in Luxembourg's banking sector. Furthermore, the magnitude of the euro area GDP coefficients suggests that this effect is important for Luxembourg. This is consistent with the interpretation that Luxembourg's economy is sensitive to the fundamentals of the euro area economy[4]. There is a similar effect for the change in the property price index and the positive coefficient on the second lag of the change in property prices variable implies that increases in property values reduce counterparty default risk over time. Increases in the real interest rate also demonstrate a possible effect on the creditworthiness of counterparties but, although the interest rate coefficient is negative, it is not statistically significant. This finding may not be unusual, as other authors have encountered similar results; see for example Virolainen (2004). Since neither of the lagged coefficients Δy_t was found to be significant, this suggests that there is no significant degree of autocorrelation in the probability of default series but the same cannot be said for the euro area GDP time series. Consequently, exogenous shocks will not be highly persistent and this will reduce the impact these shocks can have on the transformed default series. These findings suggest that the VAR(2) model can capture a dynamic response to an initial shock. Using Monte Carlo simulations as detailed in Rouabah and Theal (2010)[5], it is possible to simulate distributions of the counterparty probability of default with the MVAR model.

Post estimation analyses were conducted on the residuals of the VAR model. Figure 2 shows the individual kernel density plots of the VAR(2) model residuals.

[Figure 2 about here]

The distributions of the residuals display clear evidence of non-normality, particularly for the default probability and property price series. This strongly suggests that the underlying data generating process is non-normal. Lutkepohl's joint test for normality rejects the null

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[4] In fact, we have omitted Luxembourg real GDP growth from the VAR estimation. The reasons for this are twofold. The first is that banks in Luxembourg are primarily foreign branches and subsidiaries which tend to be more sensitive to growth in the euro area as a whole rather than domestic developments in Luxembourg. Secondly, the financial sector contributes approximately 20% to Luxembourg's GDP growth, which could result in a cointegrating relationship between GDP growth and the logit-transformed probability of default series.

[5] A detailed description of the SUR model specification and its estimation is provided in Rouabah and Theal (2010).

hypothesis that the residuals are multivariate normal at the 1% level of significance leading to the conclusion that a VAR model fails to capture the data generating process underlying the four variables. Consequently, the results of the statistical tests on the VAR(2) model would seem to justify the use of the MVAR model as an appropriate alternative that may better capture the underlying dynamics of the model variables.

5. Estimation of the MVAR Model

The estimation results and statistical tests seem to confirm the inadequacy of the VAR model and lead to the conclusion that the MVAR may be the more appropriate choice of model. In order to estimate the MVAR model parameters and avoid convergence to a poorly behaved local maximum, it was necessary to implement an expectation maximization with variable neighbourhood search (EMVNS) algorithm[6]. Four neighbourhood structures were used to perturb the model parameters. Given the parameter specification of the MVAR model, the EMVNS is used to search the following neighbourhoods: 1) perturbation of the distribution mixing weights, 2) perturbation of the intercept vector, 3) perturbation of the autoregressive coefficient matrices and, 4) perturbations of the variance-covariance matrices. Since the MVAR log-likelihood function has as its parameter set all the expressions in equation (7), the above specification for the neighbourhood structures enables a thorough search of the parameter space of the log-likelihood function and increases the chances that the EMVNS converges to the global maximum.

It is important to address a few important details that are inherent in the empirical implementation and estimation procedure. First, the diagonal values of the variance-covariance matrix may need to be regularized in order to ensure it remains positive definite. We therefore calculate the condition number of the covariance matrix and add a small, positive value to the main diagonal of the matrix if necessary. The magnitude of this number is close to the empirical precision of the computer on which the code is executed. Second, to ensure a computationally efficient estimation procedure, one of the stopping conditions monitors the amount of CPU time consumed by a given iteration in the maximization phase of the algorithm. If convergence is not achieved within a pre-specified CPU time limit, the current iteration is terminated and the algorithm proceeds to the next one. The same limit is

[6] The EMVNS algorithm is described in more detail in Appendix 1.

applied to the search of a given neighbourhood structure. In both cases, the time limit employed is approximately 20 seconds[7].

We use a two-distribution mixture so results may be interpreted in terms of two regimes; one for economic "good" times and the other for economic "bad" times. The entries for $\tau_{t,k}$, the conditional probability that a given observation originates from component $k = 1,2$ of the distribution, suggest that component one probabilities are larger during so-called "good" economic periods, and component two probabilities tend to be associated with periods of economic stress, although this classification is not entirely rigorous. The actual number of observations associated with tranquil times is 34 compared to 29 associated with times of turmoil. The absence of a clearer distinction between good and bad times may be in part attributed to the increased volatility and the relatively short length of the time series data used in this work.

Tables (2a) and (2b) provide the estimated MVAR coefficients and their respective standard errors for the two component distributions. The mixing weights of the distributions were estimated to be $\alpha_1 = 0.55672$ for the first component distribution and $\alpha_2 = 0.44328$ for the second component distribution. The standard errors reported for the MVAR coefficients were estimated using Louis' (1982) method[8]. Louis' method is based on the empirical evaluation of both the observed information in Y and missing information in Z. In order to obtain these matrices it is necessary to evaluate the second derivatives of the likelihood function[9]. In the case of the MVAR the second derivatives were derived analytically, although for more complicated likelihood functions it is conceivable that they may not exist. If closed-form expressions are not available, second derivatives may be approximated numerically, although at the cost of lower numerical precision. In any case, once the second derivatives are known, the standard errors can be extracted from the complete information matrix, $\mathbf{I}$, which is defined as the difference between the observed information and missing information matrices as given by equation (8).

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[7] At first glance, this limit may seem excessively large. However, the algorithm was implemented in MATLAB, an interpreter language, for which run-times tend to be slower than for compiled languages. An implementation using compiled binary code may substantially reduce this limit.

[8] The procedure and equations for the estimation of the standard errors were obtained from C. S. Wong in the form of a private communication of an unpublished manuscript.

[9] As these expressions are outside the scope of this work, they are not provided here.

$$\mathbf{I} = \mathbf{I_c} - \mathbf{I_m} = E\left(\frac{\partial^2 \ell}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}'} \middle| \boldsymbol{\theta}, \mathbf{Y} \right)_{\hat{\boldsymbol{\theta}}} - \operatorname{var}\left(\frac{\partial \ell}{\partial \boldsymbol{\theta}} \middle| \boldsymbol{\theta}, \mathbf{Y} \right)_{\hat{\boldsymbol{\theta}}} \quad (8)$$

The following variable definitions apply here: ℓ is the MVAR log-likelihood function, $\mathbf{Y}$ is the observed data and $\boldsymbol{\theta}$ contains the estimated model coefficients. $\mathbf{I_c}$ and $\mathbf{I_m}$ are the complete information and missing information matrices, respectively.

[Figure 3 about here]

Figure 3 shows the convergence of the likelihood function. Although the number of iterations required to achieve convergence is fairly low, the curve remains monotonic. The EM algorithm required 24 iterations to converge[10].

[Table (2a) about here]

[Table (2b) about here]

Tables (2a) and (2b) provide the estimated coefficients and standard errors for the MVAR(4,2;2,2) model. The tables are informative and provide some insight into the dynamics and feedback mechanisms between the macroeconomic environment and the logit-transformed and first differenced probability of default series, Δy_t, thereby linking the financial sector with the macroeconomic environment.

For the first component distribution (table 2a), the signs of the coefficients in the first column seem appropriate for the expected link between the macroeconomic environment and the creditworthiness of Luxembourg's banking sector counterparties. In fact, the results for the first component distribution correspond rather closely to the VAR(2) model results in terms of both magnitude and the significance of variables. Nevertheless there are some important differences. Unlike the VAR model, in the MVAR model's first component, there is no effect from lagged values of euro area GDP growth on the transformed default series. At the same time it captures the autocorrelation, the strong effect of euro area GDP on Δy_t suggests that profitability quickly affects counterparty credit risk. However, there is a strong and persistent effect from GDP growth originating from the second MVAR component. In addition, the second component of the MVAR also displays evidence of persistence in Δy_t, but it acts

[10] Convergence was defined as an improvement in the log-likelihood function between two successive iterations that is less than 1×10^{-6}.

with a lag, as shown by the negative and significant coefficient of Δy_{t-2} in the equation for Δy_t. This implies that shocks to the macroeconomic variables will pass from period to period with, albeit, a lagged effect.

The dynamics captured by the euro area GDP growth equation also underscore the important differences between the two econometric models. In the VAR model, there is a feedback loop between lagged values of the transformed default probability and euro area real GDP growth as well as a persistent effect given that the two lagged GDP coefficients are highly statistically significant. This result implies that euro area real GDP growth tends to increase when counterparty PDs recede. In the MVAR case, with respect to GDP for the euro area, there appears to be a feedback mechanism at work between property prices, GDP growth and counterparty creditworthiness. However, this mechanism is somewhat diminished, in the first component distribution where the lagged GDP coefficients are not statistically significant. Conversely, this effect is clearly present in the second distribution component. In addition, there is a persistence effect, as the coefficient of the lagged term in euro area GDP growth is statistically significant. Taken together, the two MVAR components encompass the feedback mechanism present in the VAR model; however it is distributed across components or "regimes". Additionally, in the second MVAR component distribution, there is a statistically significant effect on GDP growth stemming from the change in the property price index. The positive property price coefficient suggests that a growing real estate sector results in an improvement in macroeconomic conditions. This effect is not present in the VAR model.

The behaviour of the interest rate equation is markedly different in the MVAR estimation compared to the VAR model. In the VAR model, the only statistically significant effect on the interest rate comes from changes in the property price index. Increases in property prices in the previous period are associated with lower interest rates at time t. Conversely, there is also a small, but opposite, effect occurring at the second lagged value of the property price index. Specifically, a decline in property prices in the second period previous tends to decrease interest rates suggesting that favourable conditions two periods before lead to improved market conditions in the current period. Insofar as Luxembourg property prices are correlated with those in the euro area, this result could be interpreted as a general symptom of favourable economic periods. If shocks are applied to the property index, they will exhibit some persistence, but this will be mitigated to some extent as a result of the second lag coefficient. The MVAR model displays strikingly different mechanics for the interest rate equation. For both the first and second component of the MVAR, there is evidence of a feedback mechanism between the transformed probability of default and the change in the real rate. For the first component, increases in counterparty creditworthiness appear to increase interest rates (i.e. since Δy_t and the probability of default are inversely related[11]). The second MVAR component captures the opposite effect; when the probability of default decreases, interest rates increase. Thus, the MVAR seems to capture two different economic regimes, although the result for the first component seems difficult to explain in a macroeconomic context. Furthermore, since the coefficients on Δy_{t-1} and Δy_{t-2} are both positive and statistically significant, there will be a strong persistence in the effect of the transformed default rates on Δr_t. Additionally, positive euro area real GDP growth leads to increases in the real interest rate. Finally, for both the VAR and MVAR models, lagged values of the change in the property price index are negatively related to changes in the real interest rate. Interestingly, the intercept coefficient in the first component of the MVAR is equal to -25 basis points and is statistically significant. This may suggest the first MVAR component can be associated with a macroeconomic environment in which interest rates are declining perhaps as a result of looser monetary policy and/or low inflation although we do not pursue this any further in this work.

The VAR model equation for the change in the property price index also displays distinctly different characteristics from the MVAR equation. In the case of the VAR(2) equation, only the second lag term of the index change has a positive and statistically significant effect on Δp_t. From an economic perspective, this seems simplistic and unrealistic. Indeed, the expectation would be that interest rates would affect the property price index. However, under the MVAR framework, the dynamics of Δp_t are much more complex. For the first component distribution alone, there are five statistically significant coefficient estimates. First, there is a strong and persistent relationship between changes in the real interest rate and Δp_t whereby increases in the real rate of interest strongly and persistently decrease property prices. A similar effect is present in the second MVAR component but with the exception that only the lag-1 value of the change in interest rate is statistically significant and that the sign on the coefficient is positive rather than negative. The second MVAR component also reveals a feedback between counterparty creditworthiness and property prices. Lagged values of the transformed default probabilities persistently drive up the

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[11] Specifically, the PD measure *increases* since if Δy_t decreases, PD augments.

property index, or, put differently, when default probabilities decline, property prices increase suggesting that borrowers are at lower risk of default and the property market starts to grow. However, in the context of the second MVAR component this effect is reinforced by the feedback mechanism between Δy_{t-1}, Δy_{t-2} and the real interest rate in the equation for Δr_t. More specifically, as Δp_t increases, this increases Δy_t through the second lag coefficient. In response, the transformed default probabilities lead to increases in the real interest rate. These increases in the real rate then increase the property price index through their first lagged value. This could conceivably lead to a real estate bubble, but the feedback loop is mitigated to some extent by the first lagged value of Δp_t. There is also an opposing effect present in the first MVAR component where lagged values of the change in real interest rate result in declines in the property price index.

In comparison to the VAR(2) model, the MVAR(4,2;2,2) estimation captures a much richer set of dynamics between the macroeconomic variables used in the model. Taken together, these results can be interpreted as evidence in favour of the existence of a house price channel of monetary policy transmission in Luxembourg. These findings suggest that the real estate sector may be a relevant indicator of economic performance in Luxembourg. This is roughly consistent with Morhs (2010) who finds that there is a response of both credit and GDP to residential property price shocks in Luxembourg.

[Table 3 about here]

Table 3 shows the estimated residual covariance matrices for the two components. These matrices are symmetric. The values of the covariance estimates are considerably small, most being less than 1×10^{-5}. However, there are certain values in particular that are statistically significant. For the first component distribution all values on the diagonal of the matrix express significance at a high level. In addition the covariance between Δy_t and $\Delta\ln\left(GDP_t^{EUR}\right)$ is significant and equal to 1.0995×10^{-4} and the covariance between Δy_t and property prices is also significant and equal to 3.0574×10^{-5}. This result seems to confirm the link between the property markets and the creditworthiness of banking sector counterparties in Luxembourg.

The picture is different under the second component distribution. None of the diagonal values of the covariance matrix are statistically significant. However, almost all of the off-diagonal elements are with the exception of the covariance between Δy_t and $\Delta\ln\left(GDP_t^{EUR}\right)$. This seems to confirm the interpretation that the first and second MVAR components can be thought of in terms of different macroeconomic “regimes”. Nevertheless, it is important to understand that the MVAR model is not entirely equivalent to a regime switching model. In fact, it can be interpreted as a regime switching model in which the transition probabilities do not follow a Markov process[12], although these conditional probabilities do change from period to period as an outcome of the EM estimation procedure.

As a post-estimation analysis, it is possible to plot the equation by equation residuals for the MVAR model as was done for the VAR. The individual MVAR residual plots are depicted in figure 4.

[Figure 4 about here]

As can be seen in the figure, there are noticeable differences between the VAR and MVAR residuals. The residuals of the MVAR equations for euro area real GDP growth, changes in the real interest rate and changes in the property price index show a clear Gaussian distributional form. For the Δy_t residuals, there are some peaks evident in the tails of the distribution. Furthermore, there is a slight shift to the left in the case of the second distributional component. However, for all four residual series, the distributions are centred about zero. To eliminate the discrepancies between the residual distributions in the Δy_t equation case, it might be worth considering the inclusion of additional macroeconomic variables into the model. More complex financial linkages may be captured through the addition of a domestic stock market index series, or example. Nevertheless, it is important to maintain a balance between the complexity of the model and a parsimonious estimation. The addition of Luxembourg-specific variables might also serve to better capture the national dimensions of the underlying Δy_t process.

6. Comparison of Predictive Distributions

To demonstrate the ability of the MVAR model to capture extreme events, we can estimate the predictive distributions, $F\left(Z_t \middle| \Im_{t-1}\right)$, at different time periods in the data sample. In the accompanying figures, values on the x-axis represent logit transformed probabilities of default (PD), consequently, they take on values in the interval $-\infty \leq 0 \leq \infty$. Since the transformed values and the PDs are inversely related, increasingly negative values in the figure represent increasing values of the probability of default. What the figures show are

[12] That is, the transition probabilities are not conditional on the past and are, instead, independent.

the conditional densities for the next observation Y_{t+1} (specifically the next observation for the logit-transformed probability of default). The x-axis gives the values for the transformed PDs. The y-axis is a frequency. Thus the figures give the density for the next observation of the transformed PD, given the current observation.

Figure (5) shows the one-step ahead predictive distribution of the VAR and MVAR models for the period 2007Q2 before the crisis. On the other hand, figure (6) shows the associated predictive distribution of the MVAR and VAR models during Q1 of 2009. This was identified as a high point of stress during the recent financial crisis. In interpreting the figures, it is useful to recall that, under the logit transformation, a decrease in the transformed indicator translates to an increase in the actual probability of default of the counterparty.

[Figure 5 about here]

[Figure 6 about here]

The red arrows in the plots indicate the realized values of the change in the credit risk indicator (Δy_t) at time t. The blue arrow shows the out-of-sample value at time $t+1$, while the pink arrow indicates the value of Δy_t at time $t-1$. These quantities are 0.062 and -0.290, for the periods 2007Q2 and 2009Q1, respectively. In Figure 5, both the MVAR and VAR distributions are somewhat similar during the period before the crisis. Nevertheless, the MVAR distribution is shifted slightly to the left, indicating higher PDs on average, and its dispersion is larger that that for the corresponding VAR predictive distribution. This suggests that, under the MVAR model, negative values of the transformed default rates (correspondingly, increased PD values) are more likely than under the VAR model. This means that the VAR predictive distribution underestimates the risk of deterioration in counterparty creditworthiness during “normal” times. In Figure 6, there are some strong differences between the two distributions. In this case, we have calculated the distributions for a period during a high point in the crisis, 2009 Q1. The MVAR distribution exhibits clear bimodality with a strong peak centred at approximately -0.21. In this case, the mean of the VAR predictive distribution is located only at -0.03. The secondary peak of the MVAR model clearly illustrates the higher probability assigned to negative movements in the transformed default series; equivalently increases in counterparty probability of default. As figures (5) and (6) show, there are clear differences in the assessment of counterparty creditworthiness in the next period between the MVAR and VAR model predictive distributions.

7. The response of the MVAR Model to Exogenous Macroeconomic Shocks

Having estimated the models, it is now possible to subject the models to exogenous, pre-specified adverse macroeconomic shocks. This provides an empirical measure of how the probability of default of counterparties responds to exogenous shocks in the macroeconomic environment. To predict the response of the system, we can use a Monte Carlo simulation to generate both a baseline and a conditional adverse scenario for the probability of default. The baseline scenario is constructed by first drawing a random sample from a standard normal distribution. Through recursion of the respective VAR or MVAR model equations, it is therefore possible to generate simulated forward values of both the probability of default and the macroeconomic variables over some finite horizon period. The end result of this process is that a distribution of the probabilities of default can be constructed. The distribution thus generated can subsequently be considered as the baseline scenario.

The adverse scenario is constructed in a similar manner, except that at various periods throughout the simulation horizon exogenous shocks are applied to the individual macroeconomic variable equations. Consequently, conditional on the shocks, the distribution of the adverse scenario probability of default is governed by the dynamics of the macroeconomic variables in combination with the persistence of the shocks induced by the lagged specification of the model. This ability to generate two separate distributions for the probability of default allows for comparison of the estimated baseline and adverse scenarios when an artificial and exogenous shock is applied to a particular macroeconomic variable. The application of the exogenous shocks to the variables of the model allows us to analyze the sensitivity of the probability of default distribution to specific adverse macroeconomic developments. Under this type of deterministic approach, the response of the distribution can be evaluated for more complex macroeconomic scenarios. In any case, comparing the distributions provides information on the probable impact of macroeconomic shocks on the probability of default and can thus the procedure can be considered as a form of stress test. In order to perform the actual stress test, we must decide on some exceptional but plausible stressed scenarios. It is critical that the scenarios selected are neither too extreme nor too mild in their impact on the system because if the exogenous shocks are chosen inappropriately then the exercise will provide no relevant insight.

Three different stressed scenarios were employed with shocks being applied individually to the selected macroeconomic variables. The scenarios were chosen in order to focus on the various aspects of the transmission mechanism between the macroeconomic environment and the counterparty credit risk of the Luxembourg banking sector. The three specific scenarios include both domestic and EU level effects and are taken over a horizon of 10 quarters starting in 2011 Q3 and with the simulation ending in 2013 Q4. The scenarios are comprised of the following macroeconomic conditions:

1. A decrease in Euro area real GDP growth of magnitude -0.025 in the first quarter of 2012, followed by successive shocks of -0.028, 0.0 and 0.01 in the subsequent quarters
2. An increase in real interest rates of 100 basis points beginning in the first quarter of 2012 and a further increase of 100 basis points in 2012 Q3
3. A reduction in real property prices of magnitude 4% in 2012 Q1 and subsequent losses of 4% over the remaining quarters of 2012

Shocks of this magnitude represent particularly severe disturbances. It is important to note that if the shocks are too small, the test will provide no insight into the possible impact on the probability of default. Conversely, if the shocks are too large in magnitude, then the probability of such an event occurring would be too small and the testing exercise risks being uninformative. All shocks are applied on a quarter-to-quarter basis over the separate scenarios. For both the baseline and adverse scenarios we performed 5000 Monte Carlo simulations of the model and used the 5000 simulated probabilities of default in the last quarter of 2013 to construct the histograms. The actual simulation results for the four scenarios are displayed in figures 7 through 9.

[Figure 7 about here]

[Figure 8 about here]

[Figure 9 about here]

For all scenarios, the histograms exhibit a characteristic shift to the right of the stressed distribution, indicating that the average probability of default under the adverse scenario increases relative to the baseline scenario. An associated increase in the standard deviation is also observed along with increased weight in the tails of the distributions. For the shock to euro area real GDP growth, in the VAR case, the mean probability of default increases from approximately 1.09% to 1.70% under the adverse scenario. The corresponding change for the MVAR estimation is from 1.09% to 3.2%. For the remaining scenarios the increase is from 1.05% to 1.42% for the VAR and 1.24% to 1.59% for the MVAR under the real interest rate scenario. For the property price shocks, the VAR distribution increases from 0.9% to 1.27% while the MVAR increases from 1.17% to 2.02%. Tail probabilities under the stressed VAR scenario do not exceed their MVAR counterparts and no scenario displays probabilities of default in excess of approximately 8.14%. Despite the severity of the scenarios, the results for the selected adverse scenarios suggest that exogenous shocks to fundamental macroeconomic variables have a limited and somewhat mild effect on the average probability of default, except in the MVAR euro area real GDP growth and property price scenarios. For instance, the largest change in average counterparty PDs occurs for the MVAR under shocks to euro area GDP growth with a change of 2.11%. Under the VAR scenarios, the largest change between the adverse and baseline scenario also occurs under the GDP scenario, but the magnitude of the change is only 0.61%. The MVAR increase is more than 3.4 times larger than that observed for the VAR model.

8. Simulation and Calculation of Capital Requirements

The results of the Monte Carlo simulations can also be used to gain insight into the capitalization level of the entire Luxembourg banking sector. Using equations (9) and (10) for capital requirements for corporate exposures and Basel II tier 1 capital ratios, respectively, it is possible to calculate capital requirements under the adverse scenario.

$$k_c^* = \left(LGD \times N\left[\frac{G(PD)}{\sqrt{(1-R_c)}} + \left(\frac{R_c}{(1-R_c)} \right)^{\frac{1}{2}} \times G(0.999) \right] - PD \times LGD \right) \times \left(\frac{1}{1-1.5b} \right) \tag{9}$$

$$capital\ ratio = \frac{K + \Pi}{RWA - 12.5E^c\left(k_c - k_c^*\right)} \tag{10}$$

In equation (9), $G(PD)$ represents the inverse normal distribution with the probability of default, PD, as its argument. Here $N(\cdot)$ is the cumulative normal distribution, R_c denotes asset correlation and b is the maturity adjustment. The asterisk superscript on k denotes capital requirements under the stressed scenario. In equation (10), K denotes tier 1 capital, Π and RWA denote profit and risk weighted assets, respectively, and E^c represents corporate exposures. In equation (10) we do not specify a profit model.

This is an informative stress test in that it provides information on capitalization ratios under adverse macroeconomic conditions. To calculate the capital ratio, we use data on bank profitability, risk weighted assets, loans and the amount of tier 1 capital held by banks. As the entire sector is studied, it is important to stress these values represent average quantities. Throughout the analysis, the loss given default (LGD) is assumed to be 0.5, or 50%, and a maturity adjustment is used based on the Basel II regulations for risk-weighted assets for corporate, sovereign and bank exposures. The mean value of the probability of default values obtained from the Monte Carlo simulation is used during the calculation of the Basel II correlation and capital requirements.

[Figure 10 about here]

Figure 10 presents a bar chart showing the banking sector capital ratios under the four stressed scenarios in comparison to the baseline scenario. There are some noticeable differences between the capital requirements calculation for the VAR and MVAR models. Empirically the difference is 1.37%, suggesting that the VAR(2) model underestimates the required amount of capital in face of exogenous shocks to euro area real GDP growth. Similar, although less dramatic, results can be observed for the other variables. For the real interest rate the magnitude of the difference is 0.10% while for property prices the difference is approximately equal to 0.88%.

These differences have significant consequences from a regulatory perspective, suggesting that estimations of Tier 1 capital performed using univariate VAR models consistently underestimate the required amount of Tier 1 capital needed to withstand adverse macroeconomic shocks. The difference between the calculated values has its origins in the distributional assumptions underlying the VAR and MVAR models. In the context of the MVAR, the model is capturing a significant amount of the tail effects that, being based on the assumption of univariate normality, the VAR model does not capture. This is an important result.

One word of caution concerning the interpretation of the capital requirements calculations is necessary. As mentioned in Jones, Hilbers and Slack (2004), stress tests can "*provide information on how much could be lost under a given scenario, but not how much is likely to be lost"*. The results of a stress test then are a numerical estimate of sensitivity conditional on a given set of adverse macroeconomic conditions and allow us to understand the response or sensitivity of a financial system to various risk factors. In the absence of a formalized selection criterion for the adverse scenario, a series of assumptions and judgments must be made in determining the exceptionality and plausibility of the shocks. Naturally, this introduces a wide margin of error into the testing results and they must consequently be interpreted with care. This is true even more so if the data is aggregated over the entire sector rather than being at the level of an individual bank.

Conclusion

The main results of this study suggest that, compared to a framework with a unimodal distribution, using the MVAR model to assess counterparty risk provides a more accurate representation of the true risk by better capturing the more extreme movements observed in empirical measures of credit risk. According to the results in this paper, the VAR model consistently underestimates counterparty credit risk. In a simulation that applies adverse macroeconomic shocks to the econometric model, it is found that the level of Tier 1 capital required to withstand these shocks is underestimated by the VAR model. For shocks to euro area real GDP growth the magnitude of this underestimation is approximately 1.4% of Tier 1 capital. Financially, for some banks, this may represent a significant amount of capital. The underestimation of capital requirements in the case of the univariate model may demonstrate that there is an information gain provided by the MVAR model which is not present in the VAR framework. However, at this time there is no statistical test that we can apply to these results in order to empirically evaluate their significance.

One limitation of the MVAR is that it does not take into account endogenous actions by financial institutions or monetary authorities. While such an assumption may be valid in the short-term, in the long-run this is clearly unrealistic and an oversimplification. When stressed, financial institutions will readjust their balance sheets by selling distressed assets or rebalancing portfolios as part of their normal risk management activities. Additionally, central banks and governments will intervene during crisis either through monetary policy or more exceptional measures as was observed during the most recent period of instability. These effects, of considerable importance to the promotion of financial stability, are not captured by the types of econometric models employed in stress testing. Consequently, the actual response of the financial system to an exogenous shock may be quite different than the outcome predicted by a stress test.

Future work may attempt to compare the results from the MVAR to other non-linear tools. In particular, it would be worth comparing the mixture VAR to the structural VAR with non-normal residuals proposed by Lanne and Luetkepohl (2010). This alternative framework also allows for innovations to follow a mixture of normal distributions but uses this feature to provide the identifying assumptions necessary to recover structural shocks to the system. Maciejowska (2010) reports Monte Carlo results indicating that the EM algorithm (also used in this paper) outperforms alternatives in estimating this form of the Structural VAR. Another natural non-linear model to serve as a basis for comparisons would be the Markov-Switching VAR based on the work by Krolzig (1997) or by Sims and Zha (2006).

References

Alfaro, R. and M. Drehmann (2009) “Macro stress tests and crises: what can we learn?” Bank for International Settlements Quarterly Review, December.

Alves, I. (2005) “Sectoral fragility: factors and dynamics,” Bank for International Settlements Working Paper 22.

Åsberg Sommar, P. and H. Shahnazarian (2009) “Interdependencies between Expected Default Frequency and the Macro Economy,” *International Journal of Central Banking*, 5(3):83-110.

Basel Committee on Banking Supervision (2011) “The transmission channels between the financial and real sectors: a critical survey of the literature,” Bank for International Settlements Working Paper 18.

Bessadok, A., P. Hansen and A. Rebaï. (2009). “EM algorithm and Variable Neighborhood Search for fitting finite mixture model parameters,” *Proceedings of the International Multiconference on Computer Science and Information Technology*, Vol. 4, pp. 725-733.

Biernacki, C., G. Celeux and G. Govaert. (2003). “Choosing starting values for the EM algorithm for getting the highest likelihood in multivariate Gaussian mixture models,” *Computational Statistics & Data Analysis,* Vol. 41, Issues 3-4, pp. 561-575.

Borio, C. (2010). “Implementing a macroprudential framework: Blending boldness and realism”, *Bank for International Settlements*.

Borio, C. (2009). “Implementing the macroprudential approach to financial regulation and supervision”, *Financial Stability Review*, Banque de France, No. 13, pp. 31-41.

Borio, C. and M. Drehmann (2009) “Assessing the risk of banking crises – revisited,” Bank for International Settlements Quarterly Review, March, 29-46.

Dempster, A.P., N.M. Laird and D.B. Rubin. (1977). “Maximum Likelihood from Incomplete Data via the EM Algorithm,” *Journal of the Royal Statistical Society*. *Series B (Methodological)*, Vol. 39, No. 1. pp. 1-38.

Dovern, J., C.-P. Meier and J. Vilsmeier (2010) “How resilient is the German banking system to macroeconomic shocks?” *Journal of Banking and Finance*, 34:1839-1848.

Drehmann, M. (2008) “Stress tests: objectives, challenges and modelling choices,” Sveriges Riksbank Economic Review, June.

Drehmann, M., A.J. Patton and S. Sorensen (2006) “Corporate defaults and large macroeconomic shocks,” Bank of England, mimeo.

Drehmann, M., A.J. Patton and S. Sorensen (2007) “Non-linearities and Stress Testing,” in Risk Measurement and Systemic Risk, Frankfurt: European Central Bank, 283-307.

European Central Bank (2006) "Country-Level macro stress-testing practices," ECB Financial Stability Review, June, 147-154.

Filosa, R. (2008). "Stress testing of the stability of the Italian banking system: a VAR approach", *The IUP Journal of Monetary Economics*, 6(4):18-62 (also available as University of Modena Working Paper*)*.

Foglia, A. (2009) "Stress Testing Credit Risk: A Survey of Authorities' Approaches," International Journal of Central Banking, 5(3):9-45.

Fong, P.W., W.K. Li, C.W. Yau and C.S. Wong. (2007). "On a mixture vector autoregressive model", *The Canadian Journal of Statistics*, Vol. 35, No. 1, pp. 135-150.

Fong, T.P. and C.S. Wong. (2008). "Stress testing banks' credit risk using mixture vector autoregressive models," *Hong Kong Monetary Authority*, Working Paper 13/2008.

Hartigan, J. A., and M. A. Wong. (1979). "A K-Means Clustering Algorithm," *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, Vol. 28, No. 1, pp. 100-108.

Hoggarth, G., S. Sorensen and L. Zicchino. (2005). "Stress tests of UK banks using a VAR approach", *Bank of England Working Paper 282*, November.

Huang, X., H. Zhou, and H. Yhu. (2009). "A framework for assessing the systemic risk of major financial institutions", *Journal of Banking and Finance*, Vol. 33, pp. 2036-2049.

Jacobson, T., J. Lindé, K. Roszbach (2005) "Exploring interactions between real activity and the financial stance," *Journal of Financial Stability*, 1(3):308-341.

Jones, M. T., Hilbers, P, and Slack, G. (2004). "Stress Testing Financial Systems: What to Do When the Governor Calls", *IMF Working Paper* 04/127.

Krolzig, H.M. (1997**)** Markov-switching vector autoregressions: Modelling, statistical inference and application to business cycle analysis, Berlin: Springer

Lanne, M. and Lütkepohl, H. (2010). "Structural vector autoregressions with nonnormal residuals," *Journal of Business and Economic Statistics*, Vol. 28, Issue 1, pp. 159-168.

Louis, T. A. (1982). "Finding the observed information matrix when using the EM algorithm," *Journal of the Royal Statistical Society. Series B (Methodological)*, Vol. 44, No. 2, pp. 226-233.

Maciejowska, K. (2010). "Estimation methods comparison of SVAR model with the mixture of two normal distributions – Monte Carlo analysis," *EUI Working Papers*, European University Institute.

Marcucci, J. and M. Quagliariello (2008) "Is bank portfolio riskiness procyclical? Evidence from Italy using a vector autoregression," *Journal of International Financial Markets, Institutions and Money*, 18:46-63.

McLachlan, G. J. and T. Krishnan. (2008). “The EM Algorithm and Extensions,” *Second Edition*, John Wiley & Sons, Inc. Hoboken, New Jersey.

Misina, M. and D. Tessier (2008) “Non-linearities, model uncertainty and macro stress testing,” Bank of Canada Working Paper 2008-30.

Mladenović, N. and P. Hansen. (1997). “Variable Neighborhood Search,” *Computers & Operations Research*, Vol. 24, No. 11, pp. 1097-1100.

Monnin, P. and T. Jokipii (2010) “The Impact of Banking Sector Stability on the Real Economy,” Swiss National Bank Working Paper 2010-5.

Morhs, R. (2010). “Monetary policy transmission and macroeconomic dynamics in Luxembourg: Results from a VAR analysis,” *Banque central du Luxembourg*, Working Paper No. 49.

Puddu, S. (2010) “Real sector and banking system: real and feedback effects. A non-linear VAR approach,” mimeo University of Lausanne.

Rouabah, A. and J. Theal. (2010). “Stress Testing: The Impact of Shocks on the Capital Needs of the Luxembourg Banking Sector”, *Banque centrale du Luxembourg*, Working Paper No. 47.

Sims, C. and T. Zha (2006) “Were there regime switches in U.S. monetary policy?” American Economic Review, 96(1):54-81.

Sorge, M. (2004). “Stress-testing financial systems: an overview of current methodologies,” *BIS Working Papers*, No. 165.

Virolainen, K. (2004). “Macro stress testing with a macroeconomic credit risk model for Finland”, *Bank of Finland Discussion Papers*, No. 18.

Wong, C.S. and W.K. Li. (2000). “On a mixture autoregressive model,” *Journal of the Royal Statistical Society. Series B (Statistical Methodology)*, Vol. 62, No. 1, pp. 95-115.

Figure 1: Kernel Density Plot of the Logit Transformed Probability of Default Series

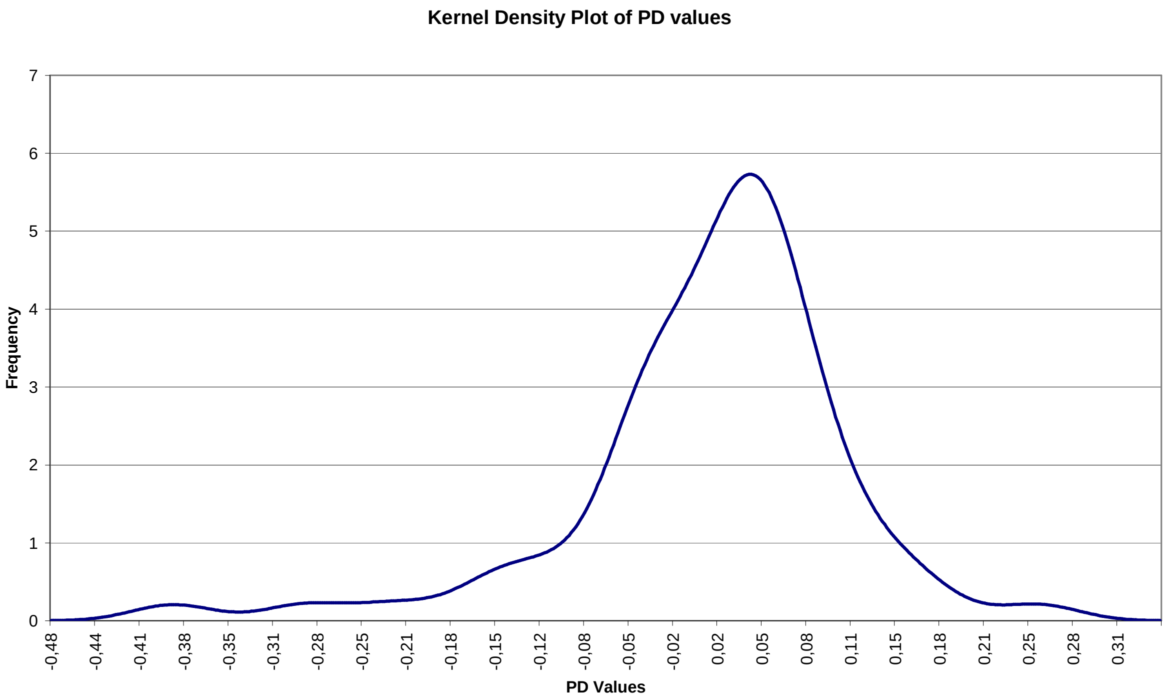


Figure 2: Kernel density plots of the residuals for VAR equation Δy_t

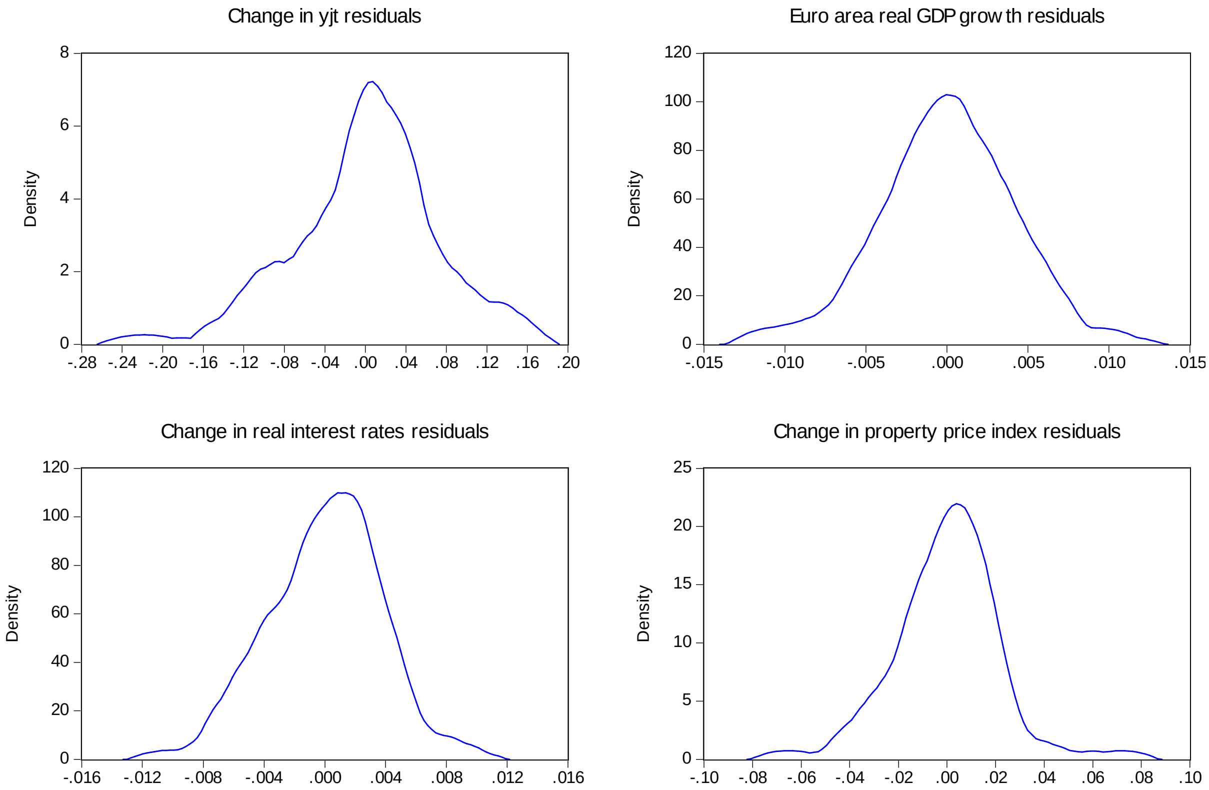


Figure 3: Kernel Density Plots of the Distribution of the MVAR Component 1 and Component 2 Residuals

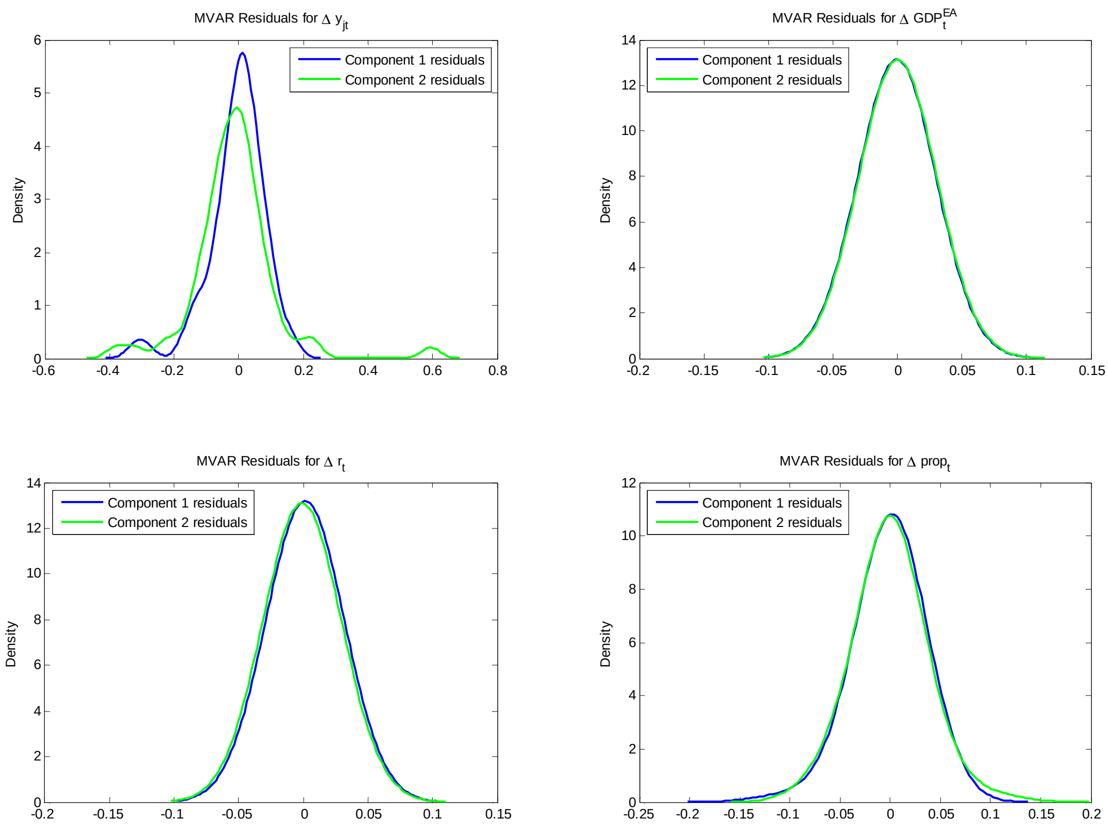


Figure 4: Plot of the Convergence Progress of the Log-likelihood Function of the MVAR(4,2;2,2) Model

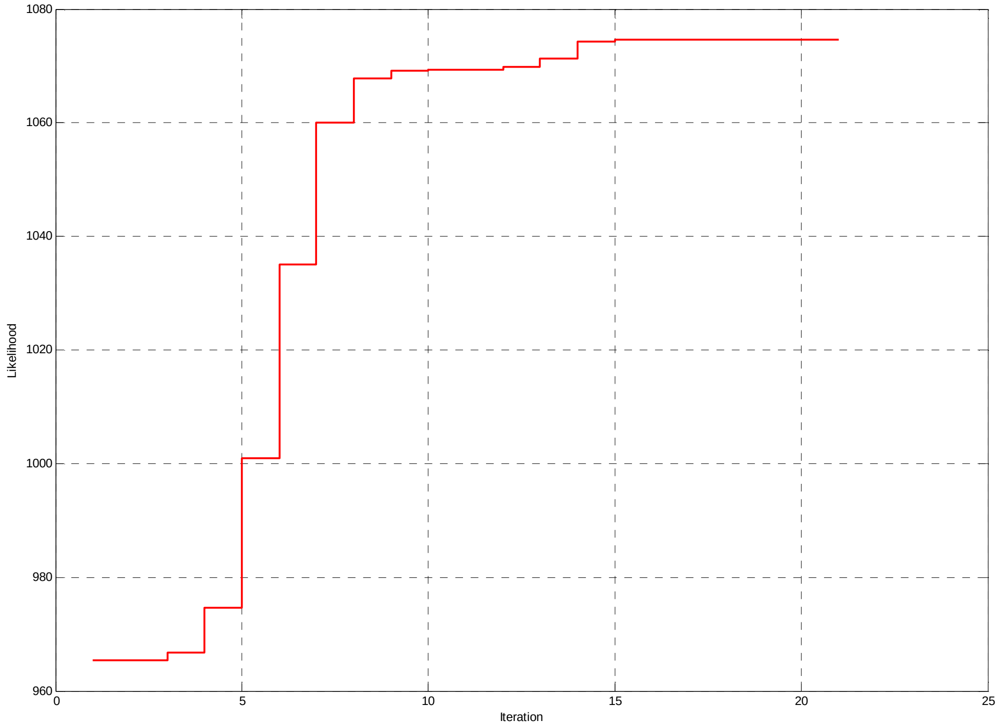


Figure 5: Predictive distributions at time $t = 49$ (2007Q2) for the MVAR and VAR models

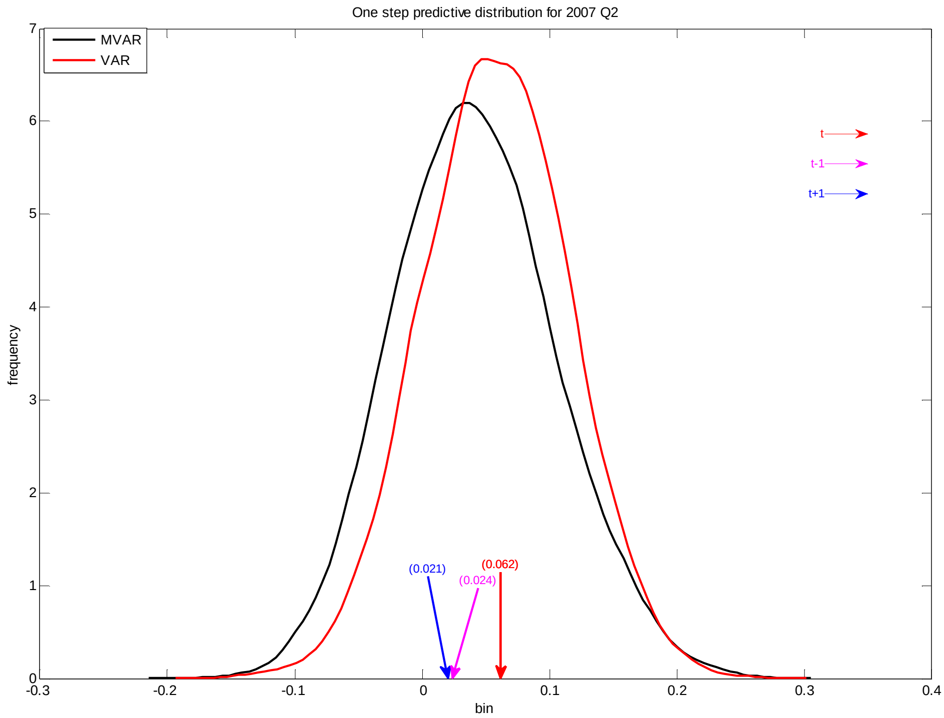


Figure 6: Predictive distributions at time $t = 56$ (2009Q1) for the MVAR and VAR models

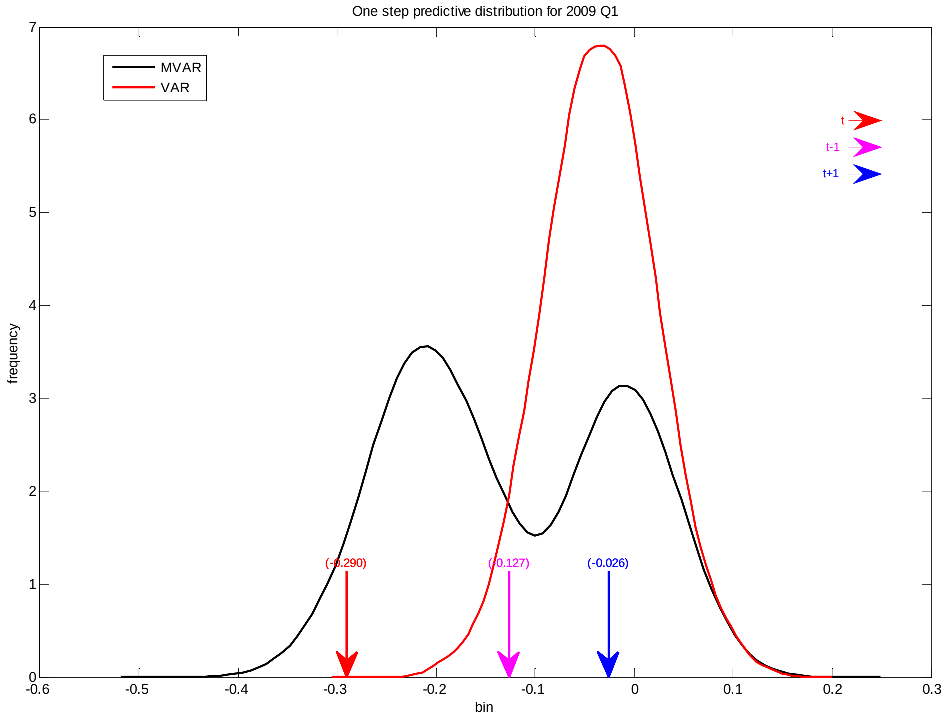
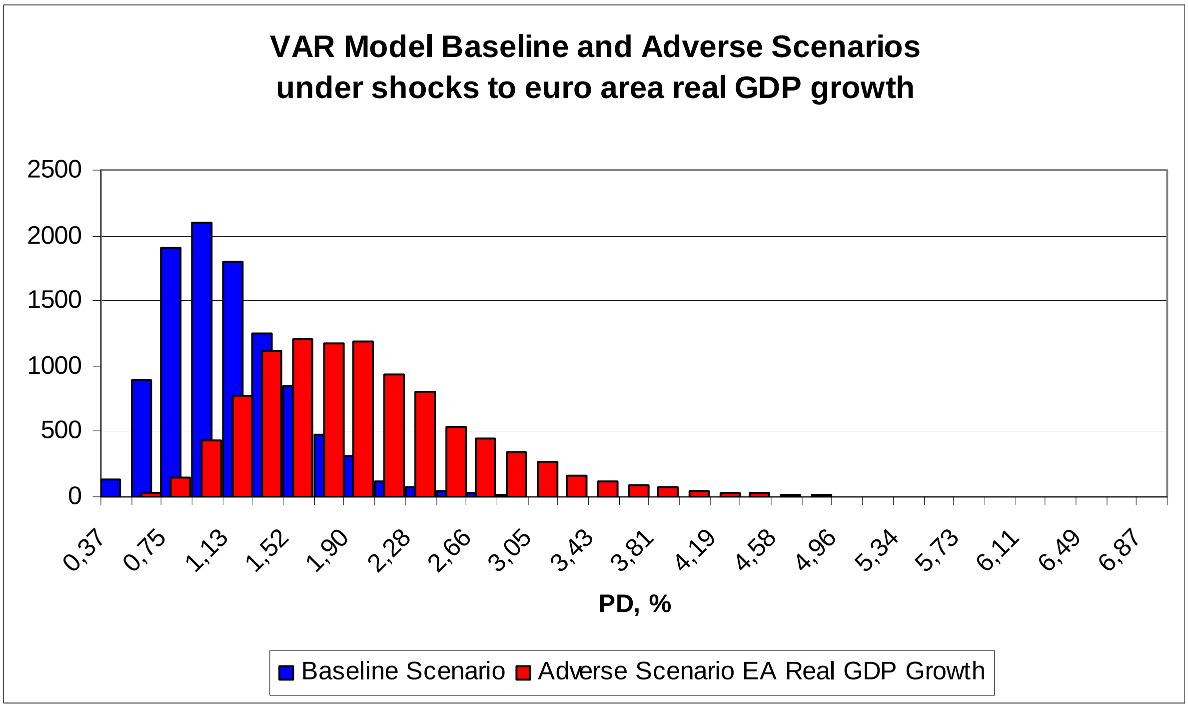


Figure 7: Distributions of Adverse and Baseline scenarios under adverse shocks to euro area real GDP growth



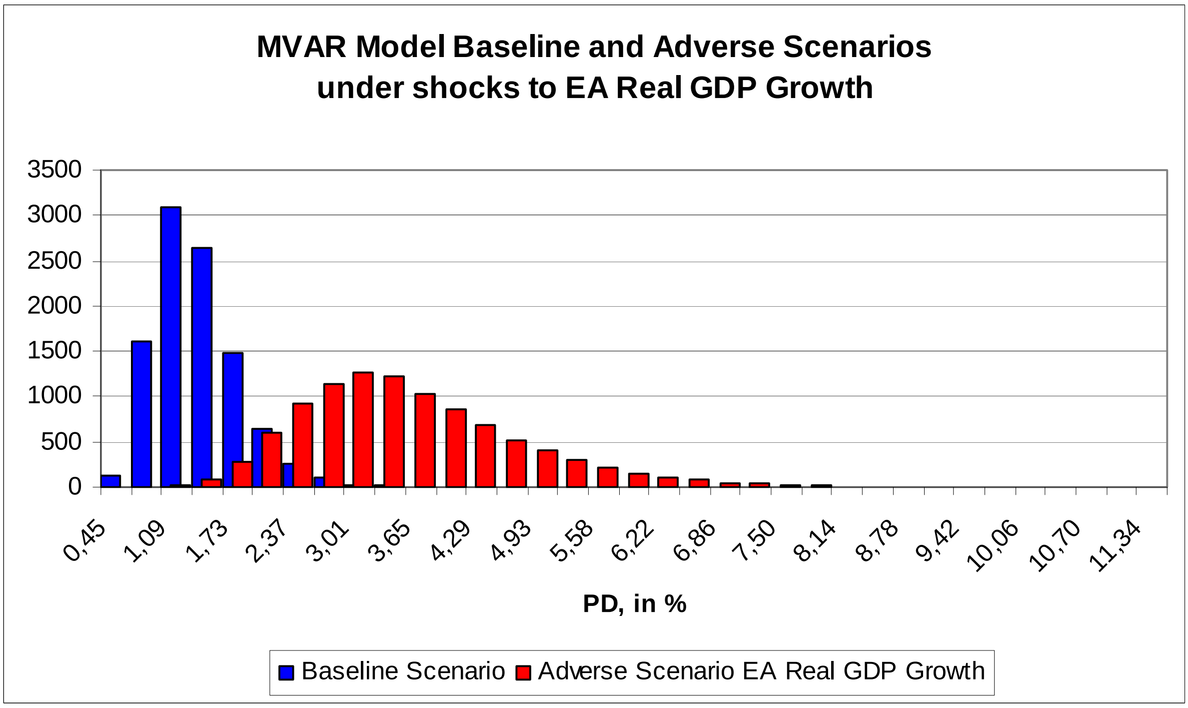
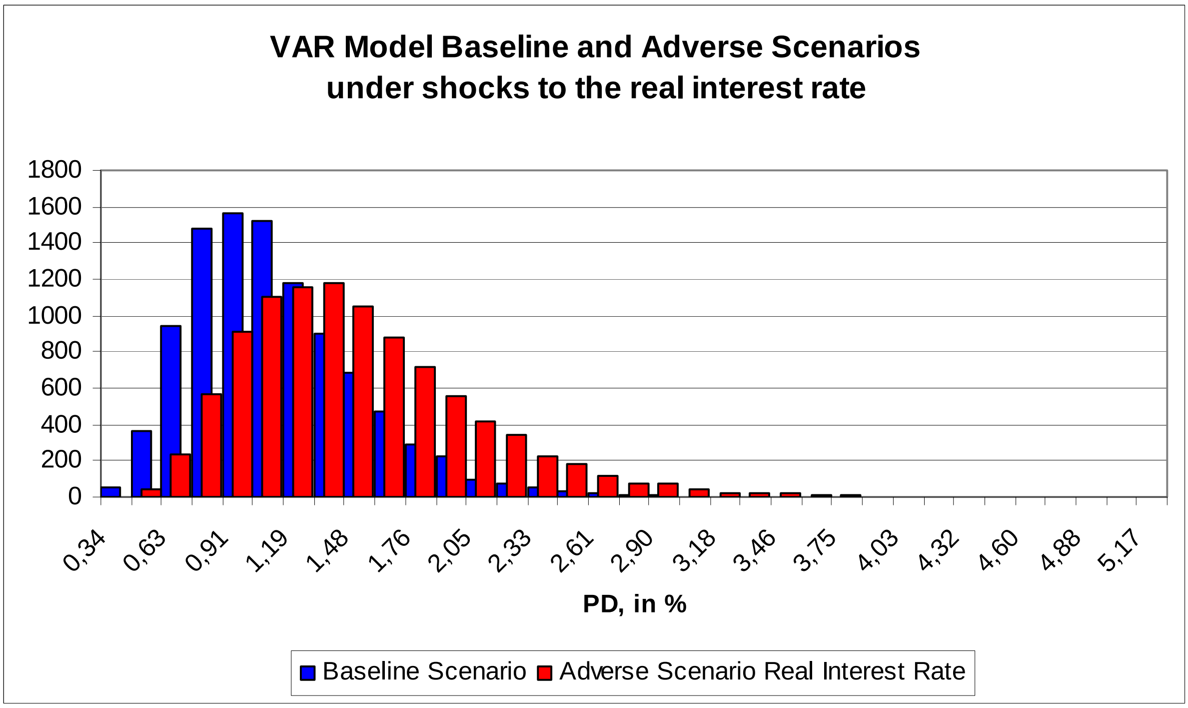


Figure 8: Distributions of Adverse and Baseline scenarios under adverse shocks to the real interest rate



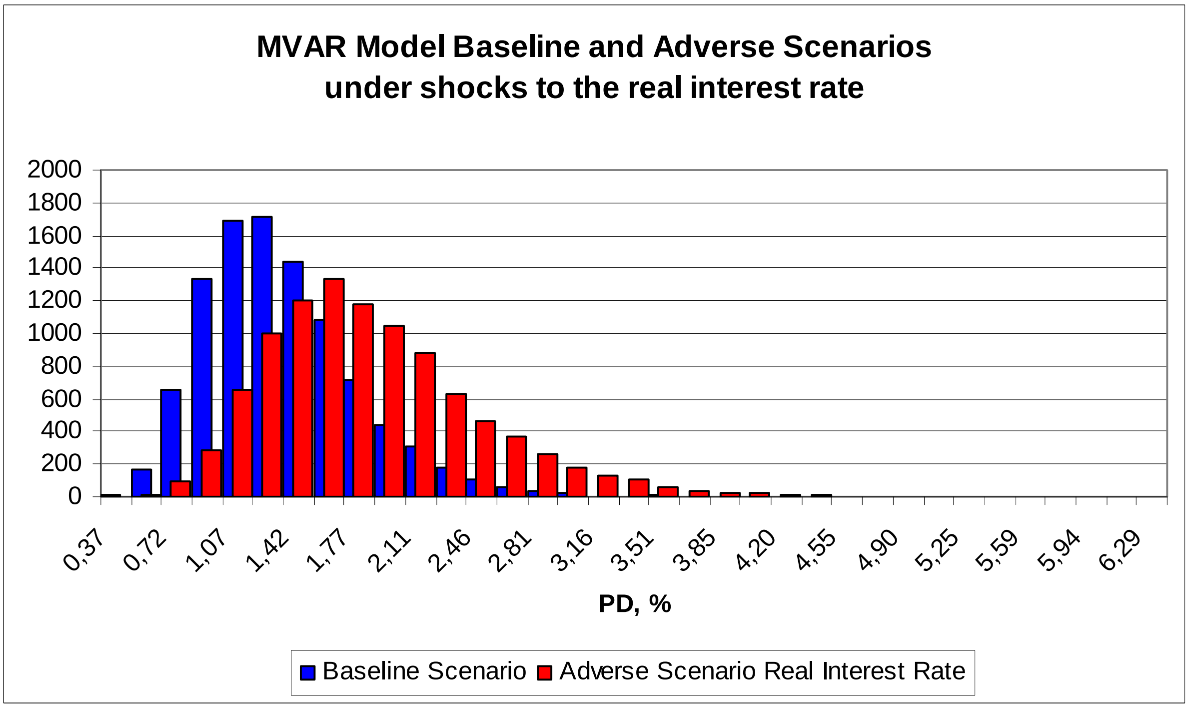
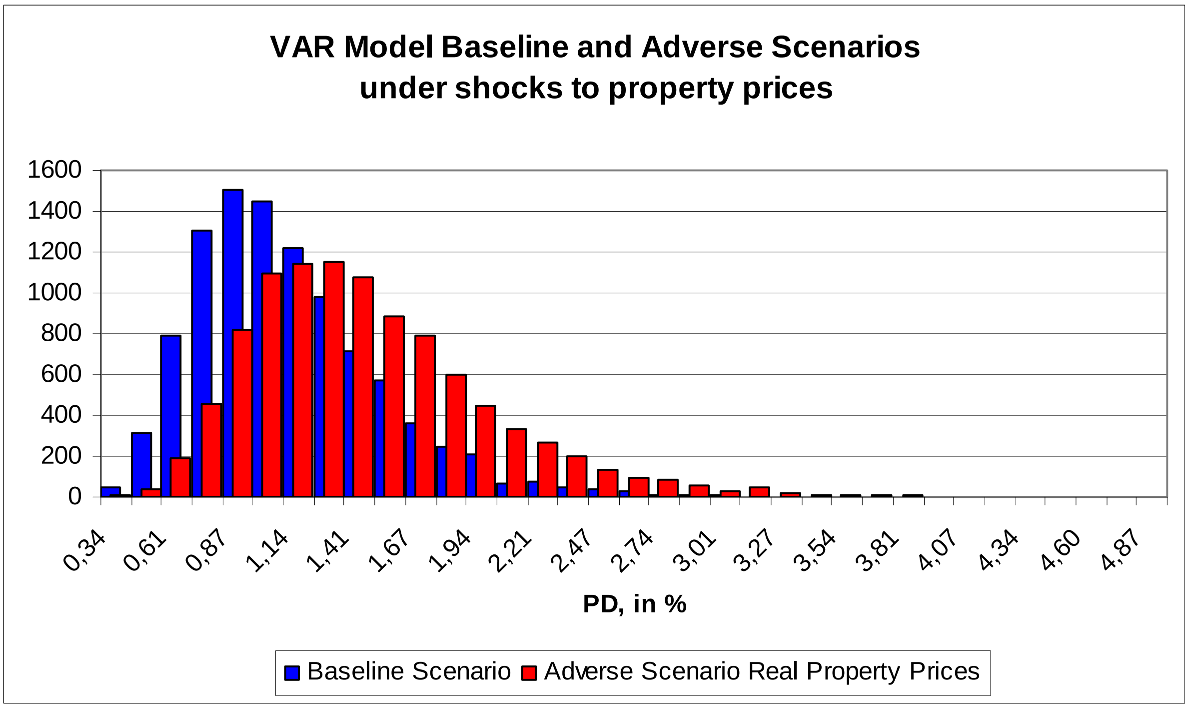


Figure 9: Distributions of Adverse and Baseline scenarios under adverse shocks to changes in Luxembourg's real property price index



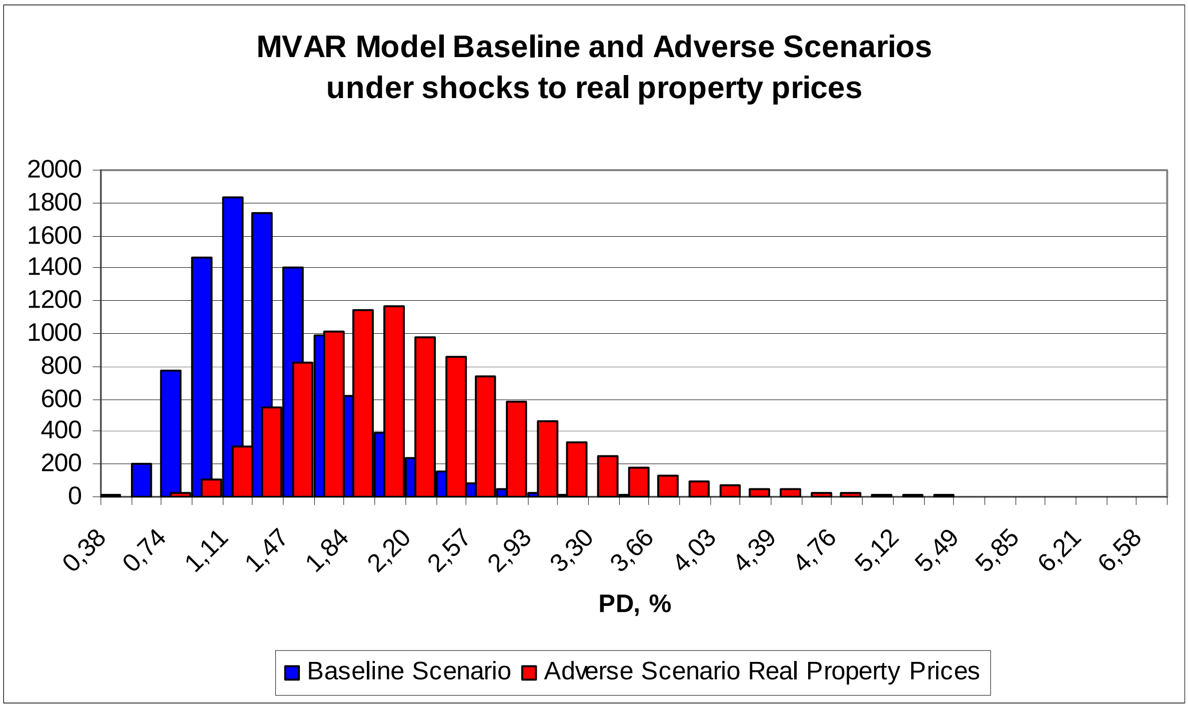


Figure 10: Differences in the Capital Requirements as Evaluated Under the VAR(2) and MVAR(4,2;2,2) Models

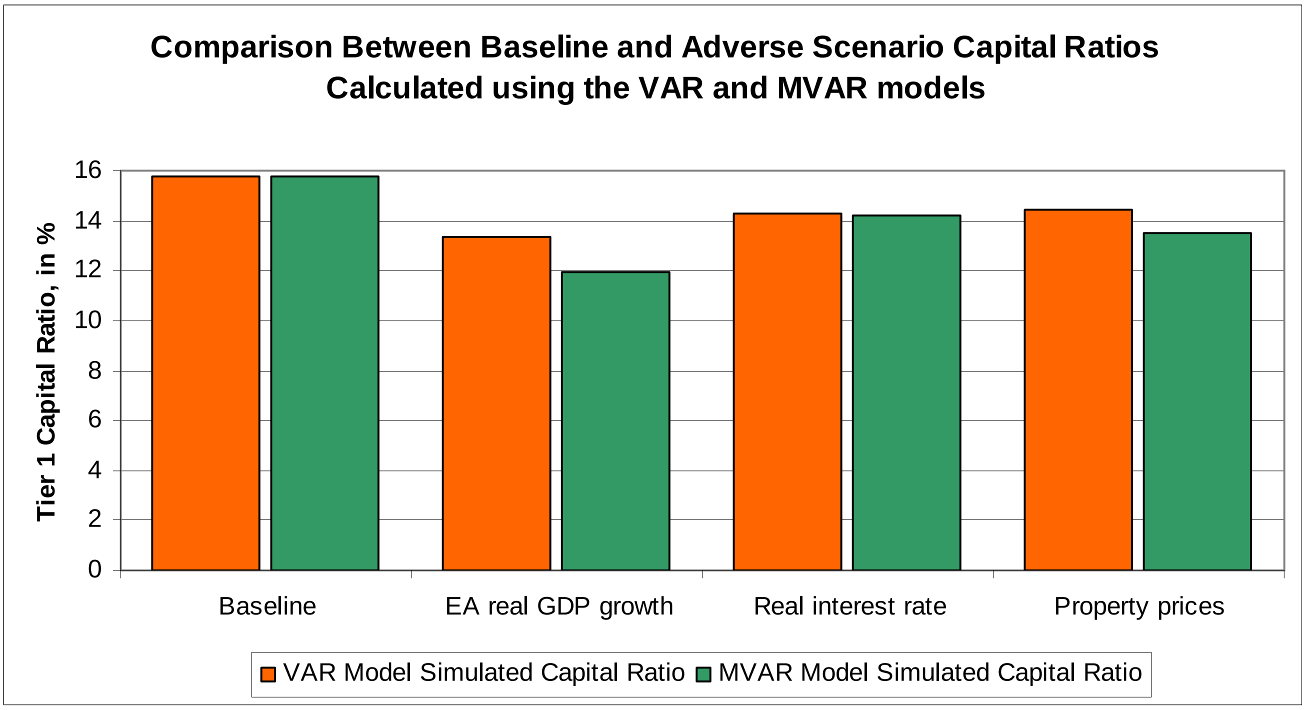


Table 1:

Coefficient and standard error estimates for the VAR model

	Dependent Variables			
	Δy_t	$\Delta \log(GDP^{EU})$	Δr_t	$\Delta prop_t$
Δy_{t-1}	-0.0236	**0.0187****	-0.0036	0.0366
	(0.1238)	((0.0071)	(0.0064)	(0.0376)
Δy_{t-2}	-0.0807	**0.0199****	0.0108	-0.0401
	(0.1335)	(0.0076)	(0.0068)	(0.0405)
$\Delta \log(GDP_{t-1}^{EU})$	3.6234	**0.4863****	-0.0964	1.2175
	(2.5123)	(0.1435)	(0.1288)	(0.7624)
$\Delta \log(GDP_{t-2}^{EU})$	**4.3652***	**-0.2679****	0.1536	-0.0230
	(2.3779)	(0.1358)	(0.1219)	(0.7216)
Δr_{t-1}	-0.0891	-0.02026	0.1128	-0.5150
	(2.5942)	(0.1481)	(0.1330)	(0.7873)
Δr_{t-2}	0.6769	0.0266	0.0101	-0.8704
	(2.6117)	(0.1492)	(0.1339)	(0.7926)
$\Delta prop_{t-1}$	0.1204	0.0112	**-0.0460****	0.1730
	(0.4461)	(0.0255)	(0.0229)	(0.1354)
$\Delta prop_{t-2}$	**1.1273****	-0.0031	**0.0373***	**0.3763****
	(0.4232)	(0.0242)	(0.0217)	(0.1284)
intercept	**-0.0441****	**0.0025****	-0.0006	0.0050
	(0.0176)	(0.0010)	(0.0009)	(0.0053)

Notes:

1. Standard error values are reported in brackets below their respective coefficient values. Entries in bold indicate statistically significant coefficients.

2. In the equations of the VAR(2) model, a dummy variable has been added in order to control for a structural break.

Table 2a:

Coefficient and standard error estimates for the first component distribution of the MVAR model (Distribution weight $\alpha_1 = 0.55672$)

VAR Component	Δy_t	$\Delta \ln\left(g_t^{EUR}\right)$	Δr_t	Δp_t
Δy_{t-1}	0.0723	**0.0203****	**-0.0166****	-0.0076
	(0.1324)	(0.0101)	(0.0079)	(0.0460)
Δy_{t-2}	0.1523	**0.0282****	0.0075	**-0.0858****
	(0.1207)	(0.0092)	(0.0072)	(0.0419)
$\Delta \ln\left(g_{t-1}^{EUR}\right)$	2.2067	0.1933	-0.0591	**1.8256***
	(2.7345)	(0.2079)	(0.1628)	(0.9493)
$\Delta \ln\left(g_{t-2}^{EUR}\right)$	0.6168	0.0966	0.1271	0.3841
	(3.7652)	(0.2854)	(0.2237)	(1.3058)
Δr_{t-1}	-4.2248	**-0.5274****	0.2215	**-2.2900****
	(2.8233)	(0.2151)	(0.1670)	(0.9765)
Δr_{t-2}	2.7157	0.2112	-0.0423	**-2.0653****
	(2.3822)	(0.1810)	(0.1418)	(0.8286)
Δp_{t-1}	0.7138	0.0236	-0.0447	**0.4955****
	(0.4899)	(0.0374)	(0.0292)	(0.1700)
Δp_{t-2}	**1.0410****	0.0028	**0.0570****	0.1065
	(0.3894)	(0.0296)	(0.0232)	(0.1351)
Intercept	**-0.0390****	0.0019	**-0.0025****	-0.0001
	(0.0177)	(0.0014)	(0.0011)	(0.0062)

Notes:

Standard error values are reported in italicized text below their respective coefficient estimates. The values of 1.645 (indicated by "*") and 1.96 (indicated by "**") can be used to estimate the significance of the ratio of the coefficient estimate to its standard error. Entries in bold indicate statistically significant coefficients. This component can be interpreted as the dynamics under "good" times.

Table 2b:

Coefficient and standard error estimates for the second component distribution of the MVAR model (Distribution weight $\alpha_2 = 0.44328$)

VAR Component	Δy_t	$\Delta \ln\left(g_t^{EUR}\right)$	Δr_t	Δp_t
Δy_{t-1}	-0.0363	**0.0249****	**0.0257****	**0.0724****
	(0.1533)	(0.0040)	(0.0033)	(0.0222)
Δy_{t-2}	**-1.1425****	0.0013	**0.0109****	**0.0739****
	(0.1903)	(0.0050)	(0.0041)	(0.0278)
$\Delta \ln\left(g_{t-1}^{EUR}\right)$	**18.4103****	**0.8470****	**0.3179****	-0.0953
	(3.4560)	(0.0894)	(0.0740)	(0.4754)
$\Delta \ln\left(g_{t-2}^{EUR}\right)$	**11.4872****	**-0.3583****	0.0841	-0.1098
	(2.3936)	(0.0619)	(0.0524)	(0.3322)
Δr_{t-1}	-0.3958	0.0350	-0.0029	**1.4734****
	(2.7286)	(0.0705)	(0.0570)	(0.3751)
Δr_{t-2}	-1.4371	-0.01394	-0.1036	-0.2324
	(3.8572)	(0.0999)	(0.0898)	(0.5534)
Δp_{t-1}	-0.6479	**0.0609****	**-0.0621****	**-0.2805****
	(0.5642)	(0.0146)	(0.0123)	(0.0778)
Δp_{t-2}	**2.9775****	-0.0232	0.0094	**0.8666****
	(0.5833)	(0.0151)	(0.0129)	(0.0806)
Intercept	**-0.1388****	0.0005	-0.0008	**0.0094****
	(0.0219)	(0.0006)	(0.0005)	(0.0031)

Notes:

Standard error values are reported in italicized text below their respective coefficient estimates. The values of 1.645 (indicated by "*") and 1.96 (indicated by "**") can be used to estimate the significance of the ratio of the coefficient estimate to its standard error. Entries in bold indicate statistically significant coefficients. This component can be interpreted as the dynamics under "bad" times.

Table 3:

Variance-Covariance Matrices Estimation (values are divided by 1×10^{-3})

Variance-Covariance Matrix for MVAR Component 1

	Δy_t	$\Delta\ln\left(g_t^{EUR}\right)$	Δr_t	Δp_t
Δy_t	**3,3630E+00****	**1.0995E-01****	8.7501E-03	-2.5822E-04
	(6.9256E-01)	(4.6668E-02)	(3.3851E-02)	(1.9758E-01)
$\Delta\ln\left(g_t^{EUR}\right)$		**1.9540E-02****	1.9530E-03	-2.7926E-02
		(4.6711E-03)	(2.5893E-03)	(1.5783E-02)
Δr_t			**1.1844E-02****	**3.0574E-02****
			(2.8088E-03)	(1.2885E-02)
Δp_t				**4.0461E-01****
				(9.3931E-02)

Variance-Covariance Matrix for MVAR Component 2

	Δy_t	$\Delta\ln\left(g_t^{EUR}\right)$	Δr_t	Δp_t
Δy_t	2,5673E+00	2,7722E-02	**-1,1048E-02****	**-1,0249E-02****
	(3,7104E+00)	(2,0105E+00)	(1,0415E+00)	(1,5139E-01)
$\Delta\ln\left(g_t^{EUR}\right)$		1,7068E-03	**1,2759E-04****	**2,3017E-03****
		(3,5145E+00)	(4,5757E-01)	(1,1953E+00)
Δr_t			1,1126E-03	**-1,9412E-03****
			(3,4867E+00)	(1,2632E+00)
Δp_t				4,7707E-02
				(3,5847E+00)

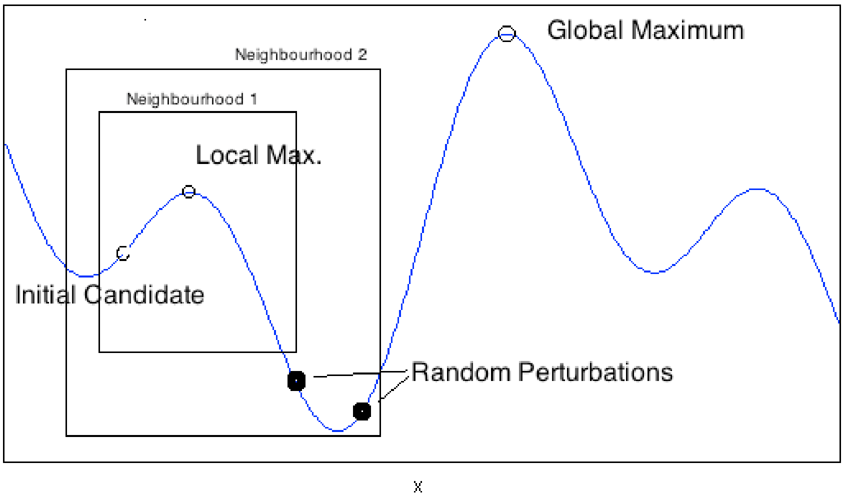
Notes:

Standard errors are provided in italicized brackets. The values of 1.645 (indicated by "*") and 1.96 (indicated by "**") can be used to estimate the significance of the ratio of the coefficient estimate to its standard error.

Appendix : The Variable Neighbourhood Search

Variable neighbourhood search methods have broad application in solving global optimization problems. The basic premise of the VNS approach as proposed by Mladenović and Hansen (1997) is to subject an initial solution candidate to a sequence of local changes such that this effects an improvement in the value of an objective function after each iteration. The search is continued in this fashion until a (local) optimum is located. Initially, a pre-defined neighbourhood, N_i, having i neighbourhood structures is defined where the set of solutions in the i^{th} neighbourhood of x is given by $\mathrm{N}_i(x)$. A graphical illustration of the principle behind the VNS method is provided in figure A1.

Figure A1: Illustration of the EMVNS search procedure



From the figure one can see how it is possible to combine both the EM and VNS algorithms. The resulting hybrid is termed the EMVNS algorithm and is described in detail by Bessadok et al. (2009). The novelty of the approach is to render the convergence of the EM routine independent of the initial starting – or candidate – solution. This may also help to overcome the problems posed by pathological likelihood functions with large attraction basins. In effect, the EMVNS uses the EM as a local search method that works in the larger context of a global optimization routine that seeks to maximize the log-likelihood function in order to obtain the model parameters. Under EMVNS, the maximization of the log-likelihood is performed under the condition that the estimated model parameters belong to the set of feasible neighbourhood solutions. As discussed in Bessadok et al. this means that the neighbourhood structures must be defined as subintervals derived from the observed distribution of the data.

In terms of its implementation, the EMVNS algorithm proceeds as follows. First, an initial candidate solution θ is used to initialize the EM algorithm. An initial solution can be found either by using a judicious choice of starting parameters or by an automatic initialization routine. For example, Biernacki et al. compare various methods for choosing the starting values of the EM algorithm for multivariate Gaussian mixture models. In terms of automatic initialization schemes, one popular method makes use of the *k*-means clustering algorithm proposed by Hartigan and Wong (1979). The aforementioned subinterval ranges of the data, indicated by I_p, can be extracted from the respective sample statistics of the means, covariances and mixing weight parameters of the input data. Next, the maximum number of embedded intervals in I_p is specified which gives I_{pk}, $k = 1,2,\ldots,k_{\max}$. Here k identifies a given search neighbourhood. The algorithm's complete pseudo code is provided in the accompanying box (A1). By formulating the search in this manner, the complete set of neighbourhood structures is searched for local optima and, based on the convergence criteria, the algorithm selects the best solution within the set of feasible solutions in the search space.

Repeat the following until the stopping condition is satisfied:

i. set $k \leftarrow 1$

ii. Repeat the following until $k = k_{\max}$

  a. Perturbation/shaking phase. Randomly draw a parameter vector θ' from the k^{th} neighbourhood of θ where $\theta' \in I_{pk}(\theta)$;

  b. Estimate the model using EM. Using θ' as the candidate solution, the EM algorithm is applied to obtain a local optimum denoted by θ'';

  c. Return to (i). If the local optimum is an improvement over the candidate, use this optimum so that $\theta \leftarrow \theta''$ and continue the search procedure setting $I_{p1}(k \leftarrow 1)$; otherwise

Box A1: EMVNS algorithm pseudo code.