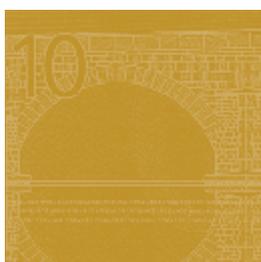




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LARGE SHOCKS IN MENU COST MODELS

by Peter Karadi
and Adam Reiff

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Abstract

How do prices react to large aggregate shocks? Our new micro-data evidence on value-added tax changes shows that prices react *(i)* flexibly and *(ii)* asymmetrically to large positive and negative shocks. We use it to quantitatively evaluate the performance of prominent pricing models. We show that standard time-dependent models are unable to reproduce either of these facts. A realistically calibrated state-dependent menu cost model, in contrast, is successful in matching the observed price responses. Its success lies in its ability to capture the exploding fraction of price changes for large shocks. The evidence facilitates comparison of different menu cost models and raises doubts on alternative pricing models with information or search frictions as sole reasons for price rigidity.

JEL codes: E31, E52

Keywords: Inflation Asymmetry, State-Dependent Pricing, Time-Dependent Pricing, Value-Added Tax Shock

Non-Technical Summary

Price rigidity is a major reason why monetary policy has real effects. Realistic price setting models are therefore essential for understanding the monetary transmission mechanism. However, different models are almost observationally equivalent when faced with small shocks. To discriminate among them, we study their predictions in reaction to large shocks, and compare such predictions to the evidence from a natural experiment.

The standard New-Keynesian literature overwhelmingly uses 'time-dependent' pricing models (Calvo (1983)) to model price rigidities. In these models, it is exogenous when a firm is allowed to change its price. This simplifying assumption is justifiable, if the models have similar conclusions to more micro-founded 'state-dependent' models, where the timing of price-changes is endogenous. Recent state-dependent 'menu cost' literature (including Midrigan (2011) and Gertler and Leahy (2008)) support the similarity of the frameworks. For small business cycle shocks, both imply large and persistent real effects of monetary policy shocks. The similarity of the models, however, necessarily breaks down for *large* enough shocks. The reason is intuitive. In time-dependent models, the fraction of price changes stays constant by assumption even for large shocks. As a result, large shocks imply large real effects. In state-dependent models, however, a large fraction of firms will find it optimal to adjust their prices. As a result, the price level becomes endogenously more flexible, implying small real effects for large aggregate shocks.

In this paper, we provide empirical evidence supporting the validity of state-dependent models for large shocks. We study the effects of large Value-Added Tax (VAT) changes in Hungary. The Hungarian government sequentially closed the gap between two VAT-rates in 2006. Before the election, it had decreased the 25% rate to 20%, and it increased the 15% rate to 20% following the election. As gross prices are quoted in Hungary, these shocks were large and easily identifiable cost push shocks. We find that the price level responds *i) flexibly* and *ii) asymmetrically* to these shocks. In particular, short-run inflation pass-through of the positive and negative VAT shocks were approximately 100% and 30%, respectively.

Both of these observations are in line with state-dependent menu cost models. Price level becomes flexible, because a large fraction of firms adjusts their prices. The asymmetry is also a natural prediction of menu cost models with trend inflation. For negative aggregate shocks, firms can save on their menu costs by keeping their prices constant and allowing the trend inflation to reduce their relative prices for free. They do not have the same option for positive shocks. We find, furthermore, that our evidence supports the menu cost model of Midrigan (2011) over the standard menu cost model of Golosov and Lucas (2007). We show that the key in the relative success of Midrigan's model is its ability to capture the highly non-linear relationship between the shock size and the fraction of price changes. For small shocks, the model implies substantial price rigidity, but the price level rapidly becomes flexible as the shock size reaches a certain threshold.

Our model can be used to assess where this threshold might be. We find that in our baseline

model, shock sizes need to be exceptionally large for the fraction of price changes to start adjusting. These shock sizes are only realistic during rare events of large exchange rate depreciations, large credit crises, or substantial changes in the tax code. To be more specific, our calibration suggests that with well anchored trend-inflation expectations, even the recent US downturn was not large enough to make the time-dependent Calvo (1983) model an invalid approximation to menu-cost models. The US Great Depression, however, with an exceptionally large nominal contraction and unanchored trend-inflation expectation would imply very flexible price developments, making the application of the Calvo model to that episode questionable. Thus we find that for standard business cycle fluctuations and monetary policy shocks in a low inflation environment, the Calvo (1983) model is still a good approximation of the menu cost model, in line with the results of Midrigan (2011) and Gertler and Leahy (2008).

1 Introduction

Prices change infrequently, as was forcefully documented by Bils and Klenow (2004), Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008). An important strand of literature in New Keynesian macroeconomics debates the extent to which infrequent price changes provide a mechanism through which nominal shocks affect real economic activity. This literature¹ studies the responses of an economy to small changes in monetary policy. Our goal is to study the role of price rigidities in shaping the response of the economy to *large shocks*. Such shocks are abundant in the data: financial crises, monetary policy shocks, nominal exchange rate movements, like the Great Recession and the Great Depression, and are arguably more important.

The standard New-Keynesian literature overwhelmingly uses 'time-dependent' pricing models (Calvo (1983)) to model price rigidities. In these models, it is exogenous when a firm is allowed to change their prices. This simplifying assumption is justifiable, if the models have similar conclusions to more micro-founded² 'state-dependent' models, where the timing of price-changes is endogenous. Recent state-dependent 'menu cost' literature (including Midrigan (2011) and Gertler and Leahy (2008)) support the similarity of the frameworks for small business cycle shocks, when both models imply large and persistent real effects of monetary policy shocks.

The similarity of time-dependent and state-dependent models, however, necessarily breaks down for *large* aggregate shocks, for two reasons. First, in time-dependent models, the fraction of price changes stays constant by assumption even for large shocks. As a result, large shocks imply large real effects. In state-dependent models, however, a large fraction of firms will find it optimal to adjust their prices if hit by a large shock. As a result, the price level becomes endogenously more *flexible* implying small real effects for large aggregate shocks. Second, perhaps less intuitively, state-dependent menu cost models imply *asymmetry* between the inflation effects of positive and negative shocks for positive trend inflation rates, as was argued by Ball and Mankiw (1994). Time-dependent models, on the contrary, imply symmetry. The reason is that in menu cost models, after negative aggregate shocks, firms can save on their menu costs by keeping their prices constant and allowing the trend inflation to reduce their relative prices for free; while inflation does not help them with positive shocks. Firms in time-dependent models do not have this waiting option by assumption.

But it is not only between time-dependent and state-dependent pricing models, where large aggregate shocks predict distinct responses. In our theoretical contribution, we show that they imply substantial quantitative differences between prominent menu cost models as well. The comparison of these menu cost models is interesting because they reach conflicting conclusions

¹Fischer (1977), Taylor (1980) and Calvo (1983) are prominent examples of 'time-dependent' pricing models; Golosov and Lucas (2007), Dotsey, King, and Wolman (1999), Gertler and Leahy (2008), Midrigan (2011) are recent quantitative 'menu cost' models; and Mankiw and Reis (2002), Woodford (2003), and Mackowiak and Wiederholt (2009) are relevant models with information frictions.

²Menu costs were directly observed and quantified in the data by Zbaracki, Ritson, Levy, Dutta, and Bergen (2004).

on the real effects of small monetary policy shocks. Midrigan (2011) finds large real effects of monetary shocks supporting the predictions of time-dependent pricing models. Golosov and Lucas (2007), in contrast, find essentially money neutrality. The difference in their results is the consequence of their different assumptions on the idiosyncratic technology shock distribution: Golosov and Lucas (2007) assume Gaussian shocks, while Midrigan (2011) introduces fat-tailed shocks.³

In this paper, we present predictions of our baseline fat-tailed idiosyncratic shock menu cost model based on Midrigan (2011) on the effects of permanent aggregate shocks of various sizes. We compare them to alternative sticky price models: the time-dependent Calvo-model and the Gaussian shock menu cost model Golosov and Lucas (2007). We solve the models numerically using global methods and calibrate their parameters to match standard data moments. We show that for large shocks (*i*) the baseline menu cost model with fat-tailed shocks implies even *higher price flexibility* than the standard menu cost model with Gaussian shocks. Note that under small shocks, it is the Gaussian model that predicts more flexibility. (*ii*) Our baseline model implies significantly *higher asymmetry* between large positive and negative shocks under positive trend inflation than the standard model. The asymmetry is essentially missing in either models under small shocks.⁴

We compare the models' predictions to pricing responses to large value-added tax (VAT) changes, which provide clear examples of measurable aggregate cost shocks. The shocks happened in Hungary, where, importantly, posted prices include VAT,⁵ so stores need to reset their prices when they want to respond to these changes. Within a short time span in 2006, the Hungarian government sequentially closed the gap between two VAT-rates. It had decreased the 25% rate by 5 percentage points before the general election, and increased the 15% rate by 5 percentage points following it. The changes were preannounced and widely publicized to both stores and consumers. Because of these features of the tax-changes, we believe that alternative

³The fat-tailed distribution assumption of Midrigan (2011) is confirmed by micro-data evidence on the dispersion of price changes. In both instances, idiosyncratic shocks are introduced to account for the observed large average absolute size of price changes, that small aggregate fluctuations do not explain.

⁴Burstein (2006) also documents flexibility and asymmetry in a related framework. He presents a non-linear solution to the Dotsey, King, and Wolman (1999) menu cost model with random menu costs. He documents high price flexibility and asymmetry for large shocks. There are some notable differences between his paper and ours. His focus is mainly on the qualitative differences of his sticky plan model from a standard sticky price model; while our focus is on the quantitative effects of the idiosyncratic shock distribution. Differently from our paper, he assumes zero trend-inflation rate, so asymmetry in his case is coming exclusively from the asymmetry of the profit function. This channel of asymmetry, as we argue, is quantitatively negligible for reasonable parameterizations. His sticky price plan model, differently from our model, would underestimate the asymmetric response to large shocks, as price plans would allow firms to disregard the expected trend inflation when responding to aggregate shocks.

⁵Differently from the U.S. practice, where prices are posted net of sales tax. It is, however, similar to most other European countries.

price rigidity explanations based on information frictions⁶ or uncertainty on the cost shocks⁷ can only play a minor role in explaining the outcomes.

To study the price effects of these large tax shocks, we use micro-level price data underlying the Hungarian consumer price index. As an immediate response to the tax changes, we observed a major jump in the fraction of price changes in affected prices (44% from 14%).⁸ This observation provides clear support to state-dependent menu cost models over time-dependent or flexible price models. Time-dependent models have no chance to account for this large increase, as they assume an unchanged fraction of price changes. But the evidence also challenges flexible price models, which could not explain why not *all* affected firms adjusted their prices after such a large shock. Our baseline model with fat-tailed shocks predicts the increase almost precisely.

We also observed a substantial asymmetry between the short-run inflation pass-throughs of the positive (close to 100%) and the negative shocks (around 30%). Both of these observations are predicted remarkably well by our baseline menu cost model with fat-tailed shocks and trend inflation. Quantitatively, it dominates the standard Golosov and Lucas (2007) model, which underestimates both the frequency increase and the asymmetry of the pass-through.

What explains the success of our model? With fat-tailed shocks most idiosyncratic shocks are small, so menu costs do not need to be particularly large to match the observed low steady-state frequency of price changes. For small shocks, when the extensive margin is not yet effective, this contributes to the large real effects of monetary shocks by reducing the Caplin and Spulber (1987)-type selection effect, as argued by Midrigan (2011). But when a large shock hits, smaller menu costs will induce more firms to change their prices. This stronger extensive margin effect explains the excess price flexibility of our model. The excess asymmetry is also the result of the fat-tailed idiosyncratic shock assumption. Responses to aggregate shocks are asymmetric under trend inflation, because expected inflation increases the firms' desired price changes. As a result, more firms are ready to pay menu costs to increase their prices, than those who would pay to decrease them. Fat tails further increase their relative proportion, leading to high asymmetry for large aggregate shocks as observed in the data.

Related literature Our paper is related to the empirical literature comparing pricing models. Unlike papers that use micro-data collected during periods with low aggregate volatility and low inflation (see Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008), Klenow and Malin (2010), Costain and Nakov (2011)), our paper is closer to approaches that use special economic environments to learn about the models. A set of these papers use observations during

⁶For example Mackowiak and Wiederholt (2009), Mankiw and Reis (2002) and Woodford (2003).

⁷Search models like Cabral and Fishman (2012) and Yang and Ye (2008).

⁸As there can be differences between products facing the different tax rates, we concentrate our analysis on the (largest) processed food sector, where the moments of the two product groups are close to each other. An example of products in the two subgroups are 'cookies' facing lower tax rates and 'chocolate-chip-cookies' facing higher tax rates. The moments in this sector are also close to the moments in the sample of Midrigan (2011), who uses barcode data from a large US supermarket chain.

high trend inflation periods to provide supporting evidence to menu cost models (Goloso and Lucas (2007), Gagnon (2009), Alvarez, Gonzalez-Rozada, Neumeyer, and Beraja (2011)). They show that the fraction of price-changes increases with higher trend inflation rate, which supports state-dependent over time-dependent models. As we argue later, however, these episodes are of limited use to compare different menu cost models like the model of Midrigan (2011) and Goloso and Lucas (2007). The reason is that higher trend inflation implies a quantitatively similar frequency response in models with Gaussian and fat-tailed idiosyncratic shocks. Instead, our paper considers large and permanent aggregate shocks in a low trend-inflation environment, which allow stark comparison of the models.

Price responses to value-added tax shocks have been used in the literature to give support to state-dependent pricing assumptions (see Gagnon (2009) and the references therein). These papers document significant increases in the fraction of price changes in the months of the tax changes in line with state-dependent pricing assumptions. The evidence presented in this paper is favorable over earlier evidence because it contains a large positive and negative VAT change for a large set of products⁹, within a short time span.¹⁰ Furthermore, different from these descriptive papers, we use structural models to contrast the quantitative predictions of different state-dependent menu cost models.

The Hungarian episode provides new evidence that alternative pricing models need to be able to explain. Standard pricing models with *information frictions* (see e.g. Mackowiak and Wiederholt (2009), Mankiw and Reis (2002), Woodford (2003)), for example, assume that it is the costs of collecting information on shocks that limit firms' optimal response to aggregate shocks. Value-added tax shocks, however, are large, measurable and widely-publicized shocks. These costs should be minimal, and can be expected to be outweighed by the potential losses caused by suboptimal price setting. Still some 60% of the affected firms have chosen not to respond to the shocks immediately, suggesting that information frictions alone are not enough to explain the observed price rigidities. Similarly, a series of recent papers use *search frictions* to explain price stickiness and asymmetric price responses (Cabral and Fishman (2012), and Yang and Ye (2008)). Their common key assumption is that the marginal consumers are uninformed about the cost shocks faced by price setting firms. To avoid them starting a search, firms optimally do not respond to each cost changes. This is, however, not applicable to VAT-shocks: these were also known to most of the consumers. Still some affected firms did not change their prices, suggesting that the above explanation can not be the sole reason of infrequent price adjustments.¹¹

⁹The negative change in the sample of Jonker, Folkertsma, and Blijenberg (2004) in the Netherlands influences only 2 products

¹⁰In the sample of Baudry, Le Bihan, Sevestre, and Tarrieu (2007) for France, there were 5 years between the VAT increase (2%-points) and the VAT decrease (1% points).

¹¹Our evidence on inflation asymmetry is not in line with some alternative frameworks either. Consider, for example, an alternative state-dependent model assuming convex adjustment cost of price adjustment, like that

Our paper is also related to empirical research documenting asymmetric inflation responses to money shocks (see e.g. Cover (1992), Ravn and Sola (2004)) in reduced form estimations. Our paper is the first we know of, however, that uses a calibrated structural model to analyze the effects of the asymmetry. Furthermore, it is using value-added tax shocks which are arguably more easily measurable, exogenous and identifiable shocks than money growth shocks used by the previous papers. Our paper is considering the asymmetric effects of *aggregate* shocks, and not *firm-level* pricing asymmetries caused by idiosyncratic shocks (like Peltzman (2000), among many others). In a sticky price model with trend inflation we *do* observe firm-level asymmetry. In these frameworks, firms front-load their price changes by setting them above their static optimum to keep the gradually decreasing real price close to the optimum throughout the price spell. Therefore they will respond to a positive idiosyncratic shock by a larger price increase, and respond to a negative shock by a lower one (in absolute terms). This firm level asymmetry, however, does not necessarily translate into aggregate inflation asymmetry. In time-dependent pricing models with exogenous probability of price change, for example, the firm level asymmetry causes only trend inflation, while the *additional* inflation effects of aggregate shocks are symmetric. Therefore, our observed aggregate inflation asymmetry provides further evidence supporting menu cost pricing models over time-dependent models.

Structure: In section 2, we detail the model and its calibration. In section 3, we present the Hungarian VAT experiment, our data and show the predictive ability of our baseline model relative to the Golosov and Lucas (2007) model. In section 4, we run counterfactual experiments with our calibrated models. In particular, we show how the predictions of our model with fat-tailed shock compares to the Golosov and Lucas (2007) and the Calvo (1983) models for different monetary policy shock sizes and inflation rates. Section 5 shows the robustness of our results and section 6 concludes.

2 The Model

To investigate the effects of large shocks in menu cost models, we use a heterogenous firm menu cost model with fat-tailed idiosyncratic productivity shocks, as in Midrigan (2011)¹² and Gertler

of Rotemberg (1982). This model would also imply asymmetric inflation response, but with the opposite sign: in an inflationary environment, the additional response of firms to positive shocks would be *smaller* than to a negative one, because the marginal cost of price increase (additional to the trend-inflation induced optimal positive change) would be higher. Finally, the asymmetric inflation effect due to the asymmetric shape of the profit function (Devereux and Siu (2007), Ellingsen, Friberg, and Hassler (2006)) is in fact incorporated in our model, but quantitative results suggests that this type of asymmetry is negligible relative to the asymmetry caused by positive trend inflation.

¹²Besides fat-tailed shocks, Midrigan (2011) has also introduced (i) endogenous sales and (ii) multi-product firms into his model. Importantly, we disregard these in our baseline framework. We treat sales by sales-filtering the data. We concentrate on single-product firms, because according to Midrigan (2011), fat-tailed idiosyncratic shocks are more important than multi-product firms in generating large real effects of monetary policy shocks.

and Leahy (2008).¹³ We introduce nominal shocks into the model as exogenous permanent innovations to the money supply. We also introduce exogenously determined value-added tax rates.¹⁴ As we show in the Appendix, nominal shocks and value-added tax shocks lead to equivalent inflation-effects in our model. For simplicity, we assume away aggregate uncertainty, which means that consumers and firms always expect money supply to grow by its exogenously given rate, and expect tax rates to stay constant.¹⁵

2.1 Consumers

The representative consumer consumes a Dixit-Stiglitz aggregate (C_t) of a basket of individual goods i , holds real balances M_t/P_t and supplies labor L_t to maximize the expected present value of her utility:

$$\max_{\{C_t(i), L_t, M_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{\mu}{1+\psi} L_t^{1+\psi} + \nu \log \frac{M_t}{P_t} \right), \quad (1)$$

where β is the discount factor, μ is the disutility of labor, ψ is the inverse Frisch-elasticity of labor supply, ν is a utility parameter of real balances. The aggregate consumption $C_t = (\int C_t(i)^{(\theta-1)/\theta} di)^{\theta/(\theta-1)}$ is a constant elasticity of substitution aggregate (with elasticity parameter θ) of individual good consumptions $C_t(i)$. The measure of individual goods i is normalized to 1.

The consumer's budget constraint for each time period t is given by

$$\int P_t(i) C_t(i) di + B_t + M_t = R_{t-1} B_{t-1} + M_{t-1} + W_t L_t + \tilde{\Pi}_t + T_t, \quad (2)$$

where $P_t(i)$ is the nominal gross price, B_t is a nominal bond with gross return R_t , M_t is the nominal money balance, W_t is nominal wage, $\tilde{\Pi}_t$ is nominal profits, and T_t is a lump-sum transfer.

The aggregate price level in this economy is $P_t = (\int P_t(i)^{1-\theta} di)^{\frac{1}{1-\theta}}$, which implies that aggregate expenditure is given by $P_t C_t$. Then the representative consumer's demand for each individual good i can be expressed as

$$C_t(i) = C_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta}. \quad (3)$$

The Euler-equation implies that $1/R_t = \beta P_t C_t / (P_{t+1} C_{t+1})$. The labor supply and money demand equations are given by $\mu L_t^\psi C_t = W_t / P_t$ and $M_t / P_t = \nu C_t R_t / (R_t - 1)$.

This way, the model is also more directly comparable to the Golosov and Lucas (2007) model that also disregards both sales and multi-product firms. The assumption does not influence our results for large shocks, as we show in the robustness part of the paper.

¹³Differently from Gertler and Leahy (2008), we disregard real rigidities. Gertler and Leahy (2008) argued that without real rigidities, fat-tailed idiosyncratic shocks alone do not lead to realistic real effects of monetary shocks. However, the authors do not match the kurtosis of the price change size distribution, which can be one reason of their different conclusion from Midrigan (2011).

¹⁴In our framework with no intermediate production, value-added tax is equivalent to sales tax.

¹⁵This assumption simplifies the analysis, but has no qualitative implications on the results.

2.2 The government and the central bank

The central bank is assumed to follow a constant money supply growth rule g_M . We postulate that the nominal income $P_t Y_t$ is going to grow at a constant rate ($g_{PY} = g_M$) as well. Then from the Euler-equation, the gross nominal interest rate $R_t = \frac{e^{g_{PY}}}{\beta}$ will be constant over time, so nominal money demand will indeed be proportional to nominal output $P_t Y_t$ (by the money demand equation). The extra money supply M_t in the economy is redistributed in a lump-sum way.

All goods face the same value-added tax rate τ_t that is exogenously determined by the government. Tax revenues $\tau_t/(1 + \tau_t)P_t C_t$ are also redistributed in a lump-sum way. Without loss of generality, we assume balanced budgets: $M_t - M_{t-1} + \tau_t/(1 + \tau_t)P_t C_t = T_t$.

2.3 The firms

Each firm i is assumed to produce its product i in a monopolistically competitive market; post gross nominal prices $P_t(i)$ and satisfy all demand given this price. However, if firms choose to change their gross nominal prices, they must employ ϕ hours of extra labor input.

The firms' problem is to maximize the expected discounted present value of their profits $\max E_0 \sum_{t=0}^{\infty} \frac{1}{\prod_{q=0}^t R_q} \tilde{\Pi}_t(i)$, where the periodic profit level is the difference between nominal revenues and production costs: $\tilde{\Pi}_t(i) = 1/(1 + \tau_t)P_t(i)Y_t(i) - W_t L_t(i)$.

For the production process, we assume that firms use a constant returns to scale technology with a single production factor of labor and face idiosyncratic technology shocks $A_t(i)$. We introduce this to reproduce the observed large average absolute size price changes. Thus the production functions of the firms are given by $Y_t(i) = A_t(i)L_t(i)$. As the idiosyncratic technology process is assumed to be stationary, the growth rate of money supply and nominal expenditures (g_{PY}) will determine the trend inflation ($\pi = g_{PY} = g_M$).

We specify the log of the idiosyncratic productivity shock process as an AR(1) process:

$$\ln A_t(i) = \rho_A \ln A_{t-1}(i) + \varepsilon_t(i), \quad (4)$$

where ρ_A is the idiosyncratic shock persistence parameter, and innovations $\varepsilon_t(i)$ are mean-zero i.i.d. random variables with variance σ_A^2 . We assume a fat-tailed, leptokurtic distribution for the idiosyncratic technology shock innovations, in order to reproduce the excess kurtosis of the observed price change distribution, similarly to Gertler and Leahy (2008) and Midrigan (2011). Specifically, to generate this extra kurtosis we assume that $\varepsilon_t(i)$ is zero with probability p , but with probability $1 - p$ it is drawn from a normal distribution.¹⁶

$$\varepsilon_t(i) = \begin{cases} 0 & \text{with probability } p \\ N(0, \frac{\sigma_A^2}{1-p}) & \text{with probability } 1 - p \end{cases}$$

¹⁶The conditional variance ensures that the variance of $\varepsilon_t(i)$ is indeed σ_A^2 .

A higher parameter p will increase the kurtosis of the idiosyncratic productivity shock innovations. Note that by setting $p = 0$, we have the model with Gaussian idiosyncratic productivity innovations.

The production function implies an individual and aggregate labor demand $L_t(i) = \frac{Y_t(i)}{A_t(i)}$ and $L_t = \int L_t(i) di$. Substituting this individual labor demand and the households demand (equation (3)) into the periodic profit function $\tilde{\Pi}_t(i)$, using the equilibrium condition $Y_t(i) = C_t(i)$, and normalizing the resulting nominal profit function with the smoothly growing nominal GDP ($P_t Y_t$), we obtain a stationary period profit function

$$\Pi(p_t(i), A_t(i), w_t, \tau_t) = \frac{1}{1 + \tau_t} p_t(i)^{1-\theta} - p_t(i)^{-\theta} w_t A_t(i)^{-1}. \quad (5)$$

The variable $p_t(i) = \frac{P_t(i)}{P_t}$ is the relative price, w_t is the real wage. The normalized menu cost equals $\phi \tilde{w}_t = W_t \phi / P_t Y_t$.¹⁷

Denote the aggregate state variables $\Omega_t = (\tau_t, \Gamma_t)$, where Γ_t is the distribution of firms over their idiosyncratic state variables $(p_{t-1}(i), A_t(i))$. Given the normalized profit function $\Pi(\cdot)$, the value of the firm if it chooses not to change its price is

$$V^{NC}(p_{t-1}(i), A_t(i), \Omega_t) = \Pi\left(\frac{p_{t-1}(i)}{1 + \pi_t}, A_t(i), w_t, \tau_t\right) + \beta E_t V\left(\frac{p_{t-1}(i)}{1 + \pi_t}, A_{t+1}(i), \Omega_{t+1}\right), \quad (6)$$

where $A_{t+1}(i)$ is the next period's idiosyncratic productivity draw, and $\Omega_{t+1} = (\tau_{t+1}, \Gamma_{t+1})$ are next period's aggregate state variables. We also used the fact that if the firm decides to keep its nominal price constant, its beginning-of-period relative price $p_{t-1}(i)$ is going to depreciate by the inflation rate π_t .¹⁸

If the firm chooses to change its price, it chooses the new relative price $p_t(i)$ optimally, and it will have to pay a menu cost $\tilde{w}_t \phi$, so its value will be

$$V^C(A_t(i), \Omega_t) = \max_{p_t(i)} \{\Pi(p_t(i), A_t(i)) - \tilde{w}_t \phi + \beta E_t V(p_t(i), A_{t+1}(i), \Omega_{t+1})\}. \quad (7)$$

Finally, the value of the firm will be determined by its decision, i.e. whichever is higher from V^{NC} and V^C :

$$V(p_{t-1}(i), A_t(i), \Omega_t) = \max_{\{C, NC\}} [V^{NC}(p_{t-1}(i), A_t(i), \Omega_t), V^C(A_t(i), \Omega_t)]. \quad (8)$$

¹⁷In our calibration, we assume linear disutility of labor ($\psi = 0$), and hence the normalized wage \tilde{w}_t , by the labor supply equation, will be time-invariant and equal to the disutility of labor parameter μ . Then the normalized menu cost parameter will also be time-invariant $\mu \phi$.

¹⁸The expectation is taken over the future idiosyncratic and aggregate state variables, conditional on their current values.

2.4 The equilibrium

We consider a rational expectations equilibrium of the model. As there is no aggregate uncertainty in the model, firms know the equilibrium paths of aggregate variables state variables τ_t, Γ_t and prices w_t, π_t . The equilibrium conditions are the following:

1. The representative consumer chooses $C_t(i), L_t, M_t$ to maximize her utility function (1) given her budget constraint (2), taking goods prices $\{P_t(i)\}$, interest rates R_t and the nominal wage W_t as given.
2. The firms are assumed to set nominal prices $P_t(i)$ to maximize their value function (8), (6), (7), given their current relative prices $p_{t-1}(i)$ and exogenous state variable $(A_t(i))$, and the values and future paths of aggregate state variables $\{\tau_t, \Gamma_t\}$. The firms also form correct beliefs about the random process of the idiosyncratic productivity shock $\{A_t(i)\}$.
3. The central bank increases money supply M_t with a constant growth rate g_{PY} , with which it also keeps the nominal output growth constant. The seignorage revenue is redistributed in a lump-sum way.
4. The government sets value-added tax rates exogenously. It redistributes tax revenues in a lump-sum way.
5. Market clearing in all goods markets $C_t(i) = Y_t(i)$.
6. The net supply of nominal bonds is zero: $B_t = 0$.
7. Equilibrium in the labor market, implying that the nominal wage W_t equates aggregate labor demand and labor supply.

We solve for this equilibrium numerically using global solution techniques. Details of our numerical solution algorithm are in the Appendix.

2.5 Model variants

We compare our baseline model above with the model of Golosov and Lucas (2007) and Calvo (1983). The Golosov-Lucas model is in fact nested in the baseline model, when the probability of no idiosyncratic shock (p) is zero. We solve the Calvo-model globally. To do this, we set the menu cost (ϕ) to zero and introduce an exogenous probability of price change (λ). The value function in this case is

$$V(p_{t-1}(i), A_t(i), \Omega_t) = (1 - \lambda)V^{NC}(p_{t-1}(i), A_t(i), \Omega_t) + \lambda V^C(A_t(i), \Omega_t),$$

which reflects that price change is not the firms' decision any more, but it is an exogenous opportunity arriving with probability λ .

3 Empirical evidence

In this section, we present empirical evidence for a large inflation pass-through and a quantitatively significant asymmetry between positive and negative aggregate shocks. The experiment we use is two consecutive value-added tax changes in Hungary: a 5%-points decrease followed by a 5%-points increase, both affecting a wide range of products.

3.1 Tax shocks in Hungary

To simplify the tax system, the Hungarian authorities sequentially closed the gap between two different value-added tax rates in 2006. In January, before the general elections, the top rate was decreased from 25% to 20%, and in September, after the election, the lower rate was increased from 15% to 20%. Note, that both changes were permanent influencing different sets of products.

Changes in value added taxes are easily measurable and transparent cost push shocks, that directly influence the gross prices of affected firms. In Hungary gross prices are quoted, so changing them require paying menu costs – differently from the US sales tax practice, but similarly to most other European countries.¹⁹ The tax shocks dominated the inflation variation at the months of their introduction. Monetary policy did not respond to the shocks: the inflation targeting central bank had expressed in advance that it was "seeing through" the direct effects of the tax shocks, because they only affect the price level with temporary effects on the measured inflation rate.

The tax changes had substantial immediate effects on the consumer prices. Among the affected processed food products, the aggregate fraction of price-changing stores increased from 13.5% to 44.4% on average²⁰, while the average absolute size of price changes *dropped* from 9.9% to 8.9%. The tax shocks also had large and asymmetric immediate inflation effects: at the month of the tax changes, the inflation pass-through was 99% and 33% for the positive and negative tax shocks, respectively.

3.2 The data

To measure the price effects of VAT-shocks, we use a data set of store-level price quotes in Hungary between December 2001 and December 2006 underpinning the consumer price index.²¹ Among the many product categories, we focus on processed food items (128 different products, 98 affected by the tax increase and 30 by the tax decrease). This is the largest sector in our sample, with a CPI-weight of 16.1%. In this sector, the composition of the groups influenced

¹⁹This practice is also reinforced by a consumer protection law requiring that "consumers can not be forced to calculate prices in their head". 1997. CLV. Law on Consumer Protection and 7/2001. (III. 29.) Ministry of Economy decree and its explanation.

²⁰Gagnon (2009) documents similarly large frequency effect after large shocks in Mexico.

²¹For a detailed description of the data set, see Gabriel and Reiff (2010).

by the two tax changes were very similar.²² An example of similar products facing different tax rates is 'cookies' facing the lower rate and 'chocolate-chip cookies' facing the higher.

Each product in our data set is observed in 123 stores on average each month, and the number of item replacements and substitutions is small.²³ In our analysis, we focus on regular prices, so we sales-filter our data. To do this, we exclude price changes that are flagged as sales in the data. After this we also filter out any remaining price changes that are (1) at least 10 %, (2) and are completely reversed within 1 month.

3.3 Data moments

We calculate data moments first at the product level, and then aggregate them using the expenditure-based CPI-weights. The moments are sample averages disregarding the tax-change months. The set of moments we use to calibrate our models consists of (standard errors of the averages across products are in parentheses):

1. Trend inflation: 0.35% (0.018%) per month (4.2% per year).
2. Frequency of price changes: 13.5% (0.47%) per month.
3. Average absolute size of non-zero price changes: 9.9% (0.22%).
4. Kurtosis of the price change size distribution: 3.9768 (0.0015).

To test the predictions of our models after large aggregate shocks, we use moments at the months of the tax changes:

1. Frequency of price changes during tax increase and decrease: 62.0% (1.6%) and 26.9% (3.18%) (average 44.4%).
2. Average absolute size of non-zero price changes during tax increase and decrease: 9.0% (0.295%) and 8.6% (0.92%) (average 8.9%).
3. Inflation pass-through of the positive and negative tax shock: 98.9% (3.45%) and 32.9% (5.65%).

Finally, we have also calculated percentiles of the size distribution of (absolute) price changes in non tax-changing months to evaluate the fit of the models' distribution. In the data, the 10th, 25th, 50th, 75th and 90th percentiles of this distribution was found to be 2.75%, 4.56%, 7.66%, 12.69% and 19.43%, respectively.

²²The inflation rates were 4.27% vs. 4.06%; the frequency of price changes were 13.77% vs. 12.28%; the average absolute size of price changes were 9.67% vs. 10.79%, and the kurtosis were 3.982 and 3.957 in the products facing VAT-increase and VAT-decrease, respectively. In the services sector – the second largest with 8.8% of the CPI weight – we observed higher aggregate asymmetry with more price stickiness and higher trend inflation, in line with the theory. The moments of the subgroups facing the VAT-increase and the VAT-decrease, however, were substantially different, so we could not use that sector here to contribute to testing the predictions of our theory.

²³Even in the rare case of item or store substitution, there is a variable in the data set advising us about this.

3.4 Model calibration

For model calibration, we use the Hungarian data moments of the processed food sector presented in the previous section. We stress, however, that the moments (frequency, average absolute size of price changes and kurtosis of the price change size distribution) are close to the moments of Midrigan (2011) in a large US supermarket chain²⁴, and comparable to those used by Golosov and Lucas (2007).

We fix some parameters exogenously. In our monthly model we set $\beta = 0.96^{1/12}$ (implying 4% yearly real rate), and the value of θ (which determines the level of competition between goods) to 5, which is a usual number used in the industrial organization literature.²⁵ Further, we set the persistence of the idiosyncratic technology shock ρ_A equal to 0.95, the value chosen by Costain and Nakov (2011). This choice is also close to Midrigan (2011) and Gertler and Leahy (2008), who assume permanent idiosyncratic shocks with $\rho_A = 1$.²⁶ Finally, the inverse of the Frisch-elasticity (ψ) of the labor supply is set to zero, implying a perfect partial wage-elasticity of labor supply. In this case, nominal wages will move with the money supply similar to the menu cost models of Midrigan (2011) and Golosov and Lucas (2007). Furthermore, it is one of the necessary conditions to obtain full long term inflation pass-through with a value added shock, that we also see in the data.²⁷ The aggregate nominal growth rate (g_{PY}) is set equal to the inflation rate.

The remaining parameters of the model are calibrated to match basic micro facts from store-level price data in Hungary.

1. In the baseline model (“leptokurtic” model), we calibrate the normalized menu cost parameter $\mu\phi$, the standard deviation of the idiosyncratic productivity shock innovations σ_A and the kurtosis of idiosyncratic productivity shocks p to match the frequency and average (absolute) size of log price changes and the kurtosis of the observed price change size distribution.
2. In the Golosov and Lucas (2007) model (“normal”) we calibrate the normalized menu cost parameter $\mu\phi$ and the standard deviation of the idiosyncratic productivity shock innovations σ_A to match the frequency and average size of price changes, and set $p = 0$.

²⁴In Midrigan (2011)’s data, the frequency is 22%, the average absolute size is 11% and the kurtosis is 4.02.

²⁵There is no agreement about the value of θ in the menu cost literature, the values range from 3 (Midrigan (2011)) to 11 Gertler and Leahy (2008). Although the choice of θ influences our estimates of menu costs and the standard deviation of idiosyncratic shocks, it practically does not influence our estimates on inflation effects of nominal shocks.

²⁶Again, there is no agreement in the literature on the calibrated value of this parameter. In another set of papers, calibrated values are 0.45, 0.66 and 0.678 (Golosov and Lucas (2007), Nakamura and Steinsson (2008) and Klenow and Willis (2006)). We check the effect of this parameter in the robustness section by investigating the case of $\rho_A = 0.7$.

²⁷The assumption also simplifies our numerical procedure: by the labor supply equation it means that the wage rate $\frac{W_t}{P_t} = \mu Y_t$, and the normalized menu cost, $\tilde{w}_t\phi = \mu\phi$ will also be constant over time.

Obviously, in this case (with normally distributed idiosyncratic productivity shocks) we are unable to match the kurtosis of the price change size distribution.

3. In the Calvo-model, we set the price change frequency parameter λ equal to the frequency of price changes, and calibrate the standard deviation of the idiosyncratic productivity shock innovations σ_A to match the average absolute size of price changes. Again, we cannot match in this simple model the kurtosis of the price change size distribution.

Table 1: Calibration

Parameters	Leptokurtic	Normal	Calvo
$\mu\phi$	1.23%	2.015%	0
σ_A	4.04%	3.88%	7.27%
p	0.915	0	0
λ	–	–	13.46%

In all cases, we have exact identification. Table 1 contains the calibrated parameters in each model variants. The parameter estimates are fairly standard. The menu cost in the baseline model is estimated to be 1.23% of the revenues, when paid. But note that it is only paid in case of a price change which happens with 13.5% probability. It means that the yearly menu cost proportional to the firms' revenue is estimated to be 0.165%, which is in the order of magnitude found in empirical estimates (Levy, Bergen, Dutta, and Venable (1997) estimates menu costs to be 0.70% of yearly revenues, Klenow and Willis (2006) estimates a yearly cost of 1.4%, while Nakamura and Steinsson (2008) finds this measure to be 0.2%).

Note that the menu costs under the fat-tailed, leptokurtic model is approximately 60% of the menu costs under the Gaussian model. The reason is that with the fat-tailed model there will be more small shocks, so smaller menu costs will ensure the same steady-state fraction of price changes.

Similarly to previous quantitative menu cost models with idiosyncratic shocks, the model needs volatile idiosyncratic shocks to hit the large average absolute size of the price changes. The standard deviations are around 4% with a .95 persistence parameter. Note that while calibrated standard deviations are roughly equal in the menu cost models (normal and leptokurtic), the conditional standard deviation of a non-zero shock for the leptokurtic case is an order of magnitude higher $\sigma_A/\sqrt{1-p} = 47.4\%$. The lower menu costs, however, make sure that the average absolute size of price changes are also matched in this case. The Calvo-model requires much higher standard deviation for the idiosyncratic productivity shocks to be able to match the large average absolute size of price changes.

To match the kurtosis of the price change distribution, we need a fairly high probability of zero idiosyncratic shocks ($p = 0.915$), which is the same order of magnitude that Midrigan

(2011) has found. ²⁸

3.5 Results

Table 2 presents our results on our matched and unmatched moments in the three model variants. The targeted moments are perfectly matched.²⁹

Table 2: Matched and unmatched moments

Matched moments	Data	Leptokurtic	Normal	Calvo
Frequency (no tax, NT)	13.46%	13.46%	13.46%	13.46%
Avg abs size (NT)	9.91%	9.91%	9.91%	9.91%
Kurtosis (NT)	3.97	3.97	<i>1.50</i>	<i>4.55</i>
Unmatched moments	Data	Leptokurtic	Normal	Calvo
Frequency tax incr	61.96%	56.88%	29.21%	13.46%
Frequency tax decr	26.92%	25.87%	19.79%	13.46%
Avg abs size tax incr	8.95%	7.85%	10.87%	10.58%
Avg abs size tax decr	8.55%	7.92%	10.69%	10.58%
Infl path through tax incr	98.86%	87.38%	65.62%	13.91%
Infl path through tax decr	32.9%	43.5%	54.54%	13.91%
1st decile of size distr (NT)	2.75%	5.09%	7.56%	1.20%
1st quartile of size distr (NT)	4.56%	5.61%	8.36%	3.40%
Median of size distr (NT)	7.66%	7.11%	9.44%	7.40%
3rd quartile of size distr (NT)	12.69%	11.60%	10.95%	13.80%
9th decile of size distr (NT)	19.43%	19.05%	12.63%	22.00%

The moments at the months of tax changes are ‘unmatched’ in the sense that we do not have free parameters to hit them (the VAT shock sizes are given). The model with leptokurtic shocks (“leptokurtic” in the table) does remarkably well in hitting these moments. First, we almost perfectly hit the large frequency increase observed during the tax-changing months. The other models do not do nearly as well: the extensive margin is effective, but quantitatively small in the Golosov and Lucas (2007) model (frequency jumps to 29% instead of 62% for the positive shock), and there is no extensive margin effect in the Calvo model.

Second, our baseline model with leptokurtic shocks also generates a *decline* in the average absolute size of price changes during the tax-changing months, in line with our evidence. This

²⁸The kurtosis of the idiosyncratic distribution equals $3/(1-p) = 35.3$, where 3 is the kurtosis of the normal distribution.

²⁹Note that in the Golosov and Lucas (2007) and the Calvo models we have normally distributed productivity shocks, so we cannot hit the kurtosis. Numbers in *italics* in the upper (matched moment) panel of the table are in fact non-matched moments.

effect might be surprising, as the shock increases the desired price change of each firm. In line with this intuition, the Golosov and Lucas (2007) and the Calvo models both predict an increase in the average absolute size of price changes. The decline in the data is caused by a lot of new price changes of a size similar to the tax shock. The tax shocks, however, are smaller than the average price change size during normal times. The leptokurtic model is able to capture this effect, by correctly predicting the appearance of a large number of smaller than average price changes. We discuss this effect in more detail in the next section.

Third, the baseline model also does quite well in generating substantial asymmetry in the inflation pass-through predicting twice as high pass-through for the positive shock than for the negative one. The asymmetry is also present in the Golosov and Lucas (2007) model, but it quantitatively small, and there is no asymmetry in the Calvo model.

Looking at the distribution of the absolute sizes of price changes, we see that matching the kurtosis goes a long way to obtain a realistic price change distribution. With the exception of very small price changes, the baseline leptokurtic model hits the distribution remarkably well. Small price changes are missing because in our baseline model menu costs will hinder firms to make small changes. The Golosov and Lucas (2007) model with normal shock innovations does not do nearly as well as our baseline model, implying unrealistically concentrated distribution. The Calvo model, in turn, generates too many small and too many large price changes, so the whole size distribution is much more dispersed than in the data.

We view this empirical evidence of large and asymmetric pass-through of symmetric tax shocks in Hungary as a strong evidence favoring the menu cost pricing models, as it is inconsistent with standard time-dependent pricing models. Furthermore, the Hungarian experiment provides strong quantitative evidence supporting our model with leptokurtic idiosyncratic shocks similar to Midrigan (2011) and Gertler and Leahy (2008) over the standard Golosov and Lucas (2007) model.

3.6 A 5% tax increase in 2009

Although the product groups affected by the 2006 VAT-decrease and -increase were similar to each other (see footnote 22 in subsection 3.2), these groups were not identical. This may lead to composition bias in estimating data moments. In this subsection we use evidence from a general 5%-point VAT-increase in 2009 to evaluate the possible size of this bias in estimating the asymmetry in inflation effects in 2006. This extra evidence helps us, because now we have products that were hit by both a VAT-decrease and a VAT-increase, so we can directly compare their price responses. The difficulty, however is that during the 2009 increase, the economy was experiencing a serious recession that can be expected to have serious influence on the inflation pass-through. Controlling for the business cycle effects, as we show, implies comparable level and asymmetry of the pass-through as in our baseline experiment.

In July 2009, in an attempt to increase government revenues during the financial crisis,

Hungarian authorities decided to increase the unified VAT-rate of 20% to 25%. As the second column of Table 3 indicates, 102 of the 128 products in our original processed food sample were hit by this new 5%-point tax increase.³⁰ Our estimate for the inflation pass-through of this tax change is 56.6%, which is relatively small. One possible explanation for this moderate pass-through is the ongoing large recession (6.8% fall in Hungarian real GDP in 2009; as opposed to the 3.9% real GDP growth in 2006).

Table 3: Inflation pass-through in 2009 and 2006

	Tax inc in 2009	Inc09-Dec06	Inc09-Inc06
Number of items	102	29	73
CPI-weight	10.75	3.37	7.38
Infl PT in 2009 (+5%)	56.6%	68.8%	51.1%
Infl PT in 2006 Jan (-5%)	–	32.6%	–
Infl PT in 2006 Sep (+5%)	–	–	88.0%

The third column of Table 3 contains information about the inflation pass-through for those 29 processed food items that were hit by both the 5%-point tax decrease in January 2006 and the 5%-point tax decrease in July 2009. Even in this group (not subject to composition bias) we see substantial asymmetry, 32.6% vs 68.8%. This is despite the fact that the pass-through in 2009 was in general much smaller than in 2006: according to column 4, for the 73 items that were hit by both tax increases of September 2006 and July 2009, the respective pass-throughs were 51.1% and 88.0%. So the substantial asymmetry in column 3 is likely to be underestimated due to business cycle effects, thus our original estimate (32.9% vs 98.9%) seems reasonable.³¹

3.7 A 3% tax increase

In January 2004, the VAT rate of of the group that faced a VAT increase also in 2006 was increased from 12% to 15%. Its effect can be used to check the models' predictions. Table 4 presents the results.

The results show that for this 3% points shock, the normal and leptokurtic shock models predict similar inflation pass-throughs around 60% that are fairly close to the observed value of 73.5%. This pass-through, in reality, is coming from a high frequency increase and a sizeable drop in the average absolute size of price changes. The model with leptokurtic shock underestimates

³⁰The remaining 26 items (all of them basic food items) got into a newly created 18% VAT-category. In essence, this created again a multiple-rate VAT-system with rates 18% and 25%, but now a much smaller proportion of the consumption basket had the lower VAT-rate than before 2006.

³¹If we assumed that the 2009 pass-through was proportionally lower for each single product (i.e. only 51.1/88.0=58.1% of the 2006 pass-through), then the asymmetry in the third column would be 32.6% vs 118.5%, not far from our original estimate.

Table 4: The 3%-point VAT-increase in 2004

Unmatched moments	Data	Leptokurtic	Normal	Calvo
Frequency 3% tax incr	51.77%	31.09%	20.55%	13.46%
Avg abs size 3% tax incr	6.46%	8.29%	10.4%	10.19%
Infl path through 3% tax incr	73.5%	62.64%	59.5%	13.9

the frequency effect and overestimates the size effect, but still gets substantially closer to both of these moments than the model with normal shocks. This evidence suggests that even the leptokurtic model might overestimate the size of the menu costs: a 3%-points shock is enough to push a larger than predicted fraction of firms over their inaction thresholds.

4 Discussion: inflation effects of large monetary shocks

In the previous section, we calibrated our models to standard data moments and showed that our baseline model predicts remarkably well the responses to large value-added shocks. The aim of this section is to explain these results and the differences between the models, and show how prices would respond to large monetary policy shocks. Money shocks and tax shocks have equivalent inflation effects in our models, as we show in the Appendix.³² In our counterfactual experiments, we first assume away trend-inflation, and compare the models' (symmetric) predictions for various shock sizes. Later, we reintroduce a standard 2% trend inflation to examine its quantitative effects on the asymmetry of the inflation pass-through.

4.1 Inflation pass-through

In this section we show numerically, that in our baseline leptokurtic model the inflation pass-through is highly non-linear in the shock size. For small shocks, the pass-through is as low as in the time-dependent model of Calvo (1983); but for large shocks, the pass-through gets even higher (so implies even lower real-effects) than a similar shock in the standard menu cost model of Golosov and Lucas (2007) with Gaussian shocks. We quantitatively assess how large these shocks need to be for our menu cost model stopping to be a good micro-foundation for the time-dependent model of Calvo (1983).

In our experiments, we hit the model economies with an unexpected permanent money supply shock Δm_0 , after which the money supply growth is assumed to return to a constant deterministic growth path.³³

³²There are two critical assumptions for this result: (1) utility is proportional to log consumption and separable from labor, and (2) technology is constant returns-to-scale.

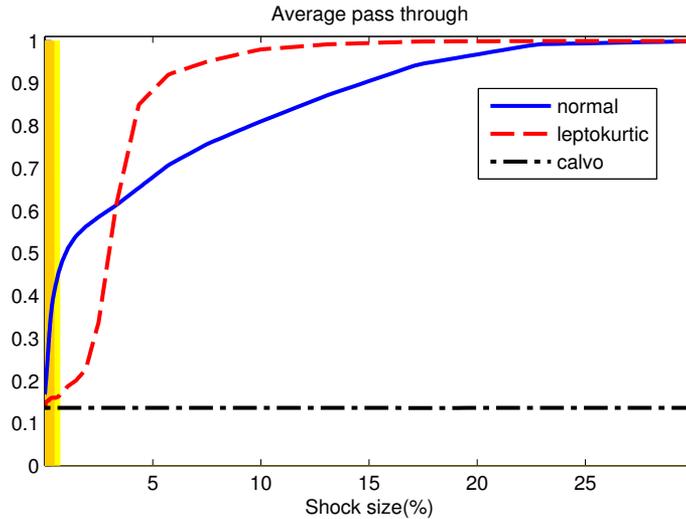
³³We experimented with a model with aggregate uncertainty matching the observed inflation variance. We have solved it by a numerical method advocated by Krusell and Smith (1998). The impulse responses of that models

The pass-through measure we report is the immediate inflation pass-through γ_0

$$\gamma_0 = \frac{\Delta\pi_0}{\Delta m_0} \quad (9)$$

where $\Delta\pi_t = \pi_t - \bar{\pi}$ is the deviation of the inflation rate from the deterministic trend inflation (zero in this subsection). In our case, this measure is a sufficient statistic to describe the dynamic reaction to a monetary policy shock: it determines both the initial real effect and its persistence. The reason is that *constant* marginal pass-through is a good approximation in our models, as we argue in the Appendix. Under constant marginal pass-through, it is easy to see that the initial real effects are proportional to the money shock with a proportionality factor of $1 - \gamma$: $\Delta y_0 = (1 - \gamma)\Delta m_0$, and the (log) output follows an AR(1) process with a parameter of $1 - \gamma$, implying a half-life of real effects of $-\log(0.5)/\gamma$. A high initial inflation pass-through, thus, means small, and quickly fading real effects, while a low inflation pass-through means both higher and more persistent real effects.

Figure 1: Pass-through for different shock sizes



The figure plots initial pass-throughs (γ_0) as a function of shock sizes for the Calvo model, and menu cost models with normal and leptokurtic idiosyncratic shocks. The bars on the left of the figures plot the 2 standard deviation bands for standard (small) monetary shocks in the Golosov and Lucas (2007) and Midrigan (2011) calibrations. The figure shows that for small shocks, the leptokurtic model implies small inflation pass-through, thus large real effects close to the Calvo model. For larger shock sizes, however, the leptokurtic model implies a non-linearly increasing pass-through that quickly overcomes that of the normal model; implying minimal real effects for large shocks.

Figure 1 shows the simulated immediate inflation pass-throughs (γ_0) of the models for different shock sizes (note that it is *not* an impulse response function): the time-dependent model are quantitatively very similar to the one presented in this paper.

(Calvo (1983)), the Golosov and Lucas (2007) model (normal) and our single-product variant of Midrigan (2011) with leptokurtic idiosyncratic shocks (leptokurtic). For the time-dependent model, the pass-through is constant. For the normal idiosyncratic shock calibration, the pass-through is quickly increasing from around 20% to close to 50% still within the 2 standard deviation bands of standard monetary policy shock calibrations. It suggests that even with zero inflation, the standard model would imply pass-throughs that are significantly higher than the Calvo model for small shocks, in line with the results of Golosov and Lucas (2007). After reaching 50%, the pass-through steadily increases with larger shock sizes. It reaches 95% average pass-through at around a 15% aggregate shock size.

The model with leptokurtic shocks, however, implies a low inflation pass-through for shock sizes to around 2.5%, close to that of the time-dependent Calvo-model. This is in line with the main result of Midrigan (2011), who argued that correctly accounting for the high kurtosis of the price change size distribution would imply a low pass-through and high real effects of monetary policy shocks. The 2.5% shock is a bit more than 5 times higher than the 0.45% standard deviation of money shocks used by Golosov and Lucas (2007). The figure shows, however, that for shocks higher than 2.5%, the pass-through implied by the model starts increasing quickly; it surpasses the pass-through of the Golosov and Lucas (2007) model at a shock-size of 3.5% and reaches a 95% immediate pass-through for an 8% shock. This highly non-linear development is very different from the behavior of the Golosov and Lucas (2007) model, but, as we showed in the last section, is closer to what we see in the data.

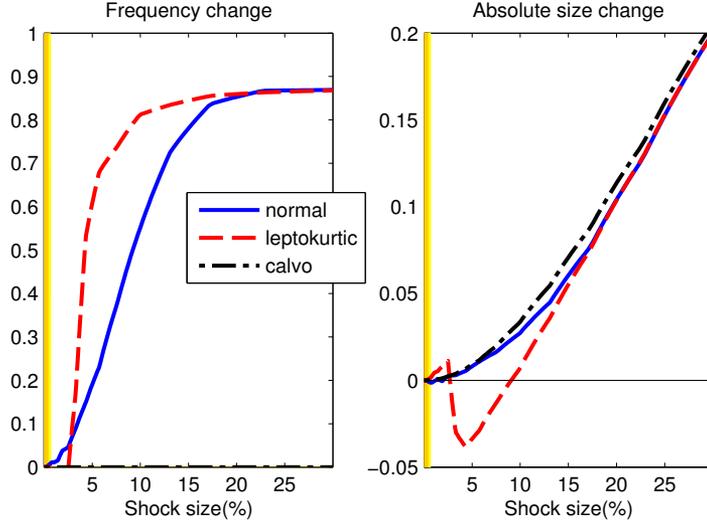
In order to gain some deeper insights on the mechanisms at play in the alternative models, it is instructive to look at the effects of the shock on the frequency and the average absolute size of price changes.

Figure 2 presents immediate deviations (from the steady state) of the frequency- and average absolute size of price changes for different shock sizes. For small shocks, the frequency deviations are minimal in all models. Note, however, that the aggregate frequency change in the leptokurtic model is smaller – effectively zero – for a shock size of around 2.5%. For larger shocks, however, we see a highly non-linear jump in the frequency of price changing firms for the leptokurtic model; much faster than the one observed in the standard model with normal idiosyncratic shocks. The increase in the aggregate frequency is mainly coming from a non-linear increase in the price increases (not-shown), and not so much from the decreases in the price decreases. As the second figure indicates, the average absolute size of price changes *falls* in the leptokurtic model, in parallel to this large increase in the frequency.

4.1.1 Decomposing the pass-through

We numerically decompose the pass-through into intensive margin, extensive margin and selection effects, following Costain and Nakov (2011). Our main aim here is to show that while for small shocks, the extensive margin effect is negligible, it dominates for large shocks. To do this,

Figure 2: Frequency and absolute size effects for different shock sizes (zero inflation)



The figure plots frequency and average absolute price change size as a function of shock sizes for the Calvo model, and menu cost models with normal and leptokurtic idiosyncratic shocks. The bars on the left of the figures plot the 2 standard deviation bands for standard (small) monetary shocks in the Golosov and Lucas (2007) and Midrigan (2011) calibrations. The figure shows that for small shocks the aggregate frequency does not change and the average size shows marginal differences between the models. For large shocks, however, the frequency increase in the leptokurtic model overcomes that of the standard model, and curiously, parallel with the frequency increase, the average absolute size of price changes decreases.

note that the inflation rate can be expressed as

$$\pi = \int \int x^*(p_{-1}, A) \lambda(p_{-1}, A) \psi(p_{-1}, A) dp_{-1} dA, \quad (10)$$

where x^* is the desired nominal size of price change, λ is the hazard function of price change (equal to 1 or 0), and ψ is the steady state distribution of individual firms. All functions depend on the individual state variables: the last period relative price (p_{-1}) and the idiosyncratic shock (A). Dependence on the aggregate states are suppressed for notational convenience.

The immediate pass-through of an aggregate shock Δm_0 is given by

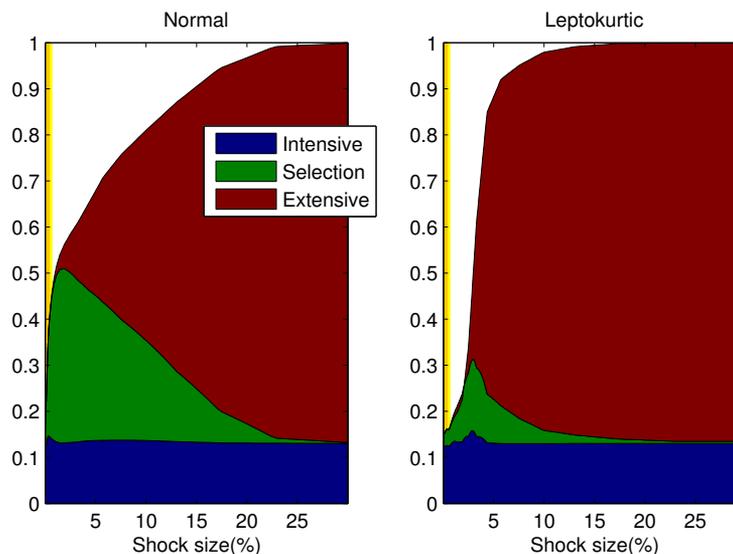
$$\frac{\pi' - \pi}{\Delta m_0} = \underbrace{\frac{\Delta \bar{x}^* \bar{\lambda}}{\Delta m_0}}_{\text{intensive}} + \underbrace{\frac{\Delta \bar{\lambda} \bar{x}^* + \Delta \bar{\lambda} \Delta \bar{x}^*}{\Delta m_0}}_{\text{extensive}} + \underbrace{\frac{\Delta \int_{p_{-1}, A} (x^* - \bar{x}^*) \lambda \psi}{\Delta m_0}}_{\text{selection}}, \quad (11)$$

where an average of variable y denoted by $\bar{y} = \int y(p_{-1}, A) \psi(p_{-1}, A) dp_{-1} dA$, and Δy denotes difference from the steady state $y' - y$. Dependence on individual states of functions in the selection effect here is suppressed for notational convenience.

The intensive margin is the product of the change in the average price change and the average frequency: in a Calvo model with fixed frequency and random selection, this would be

the only component of the pass-through. The extensive margin is defined here as the aggregate effects caused by the changes in the average price change probability (which is equal to the price change frequency). It is the sum of two products: the product of the frequency increase and the average desired price change and the product of the frequency change and the average desired price change. This second (cross) term has second order effects for small shocks, but plays a dominant role for large shocks. The third factor we are interested in is the selection effect, coming from the fact that 'new' price changers are going to have higher than average desired price changes. The third term expresses this by measuring the increased correlation between the desired price change and the adjustment hazard after a shock.

Figure 3: Components of the pass-through



The figure plots contributions of extensive, intensive and selection effects to the immediate pass-through for the normal and leptokurtic models as a function of the shock size. It shows that the small pass-through in the leptokurtic model for small shocks is the result of small selection effects; while the large pass-through for large shocks is the result of the extensive margin effect.

Figure 3 presents the decomposition of the initial pass-throughs to the three components. It shows that while the intensive margin effect is a constant fraction of the pass-through and equal in the two menu cost models, the selection effect is much lower in the model with leptokurtic shocks, in line with the results of Midrigan (2011).³⁴ For small shocks, furthermore, the extensive

³⁴The leptokurtic model implies lower selection effect, thus lower real effects of monetary shock for two reasons. (i) The model with leptokurtic shocks needs narrower inaction band to match the steady state price change frequency. This means that the firms that change their prices as a response to an aggregate shock (those close to the inaction bands) will face smaller idiosyncratic shocks, so will not respond by that large a price increase as in the normal model with wider inaction band. And (ii), the density of firms close to the inaction band (on the tail of the distribution) is lower than in the Golosov and Lucas (2007) model, so less firms are responding.

margin stays ineffective. For large shocks, however, the extensive margin gets effective and causes a highly nonlinear increase in the inflation pass-through in the leptokurtic model with much higher immediate pass-through than in the normal model.

4.1.2 A stylized example

In this section, our aim is to provide some intuition on the ineffectiveness of the extensive margin for small shocks, and explain why it becomes suddenly strong for large shocks in our model with leptokurtic idiosyncratic shocks.

To understand what is happening in the full quantitative model, it is instructive to develop a stylized graphical representation of the analyzed menu cost models. In a menu cost model, individual firms' behavior can be described by two objects: (*i*) the size of the price change they would desire if it was currently costless, and (*ii*) their (s,S) inaction thresholds expressing the desired price change that makes the firms willing to pay the menu cost. The firm's optimal policy is not to change its price, if its desired price change is within its inaction band, and change it by the desired price change if it is outside it. The aggregate behavior of the menu cost model can be similarly described by the (*i*) distribution of the desired price changes and (*ii*) the adjustment hazard, that describes the fraction of firms changing their prices for each desired price change size. The actual price change size distribution the product of these two: the realized desired price changes outside the inaction bands.

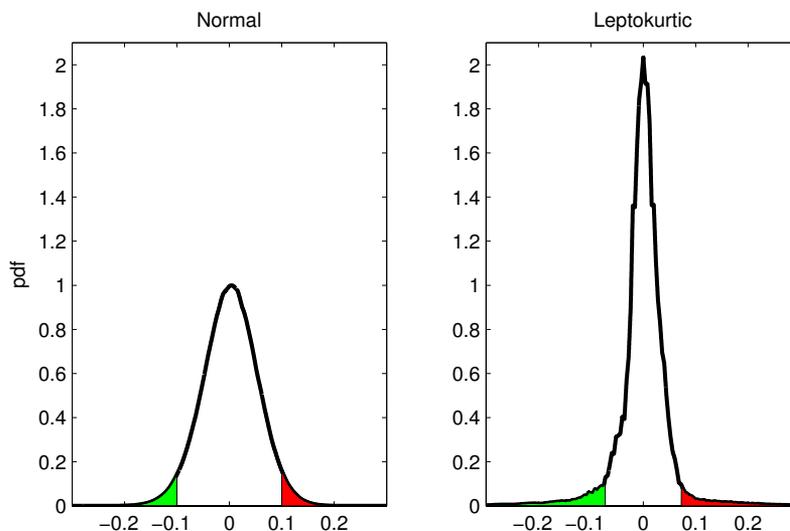
The leptokurtic idiosyncratic shock assumption in our baseline model means that the idiosyncratic shock distribution has (*i*) a sharp peak with a lot of firms facing small shocks and (*ii*) fat tails with some firms facing exceptionally large shocks. As Midrigan (2011) argues, excess kurtosis of the idiosyncratic technology shocks will imply excess kurtosis of his desired price change distribution. Figure 4 plots the desired price change distributions for the normal and leptokurtic idiosyncratic shock models; considers simplified, but quantitatively representative inaction (s,S) bands; and shows more heterogeneous actual price change distributions (shaded areas) in the leptokurtic model.

We argue that the same distributional assumptions that leads to lower inflation effect (because of selection) in the leptokurtic model, actually leads to higher inflation effects after large aggregate shocks (because of the extensive margin). As the extensive margin is missing from the time-dependent pricing model of Calvo (1983), significant extensive margin effects will provide quantitative arguments against its similarity to the menu cost model of Midrigan (2011) for these large shocks.

To understand the intuition of how the extensive margin becomes effective in the two menu cost models, Figure 5 considers the effect of a large positive aggregate shock. The shock increases each firm desired price change, so moves the distribution to the right relative to the inaction band.³⁵

³⁵We are keeping the inaction band unchanged in this stylized example, though they will optimally respond to

Figure 4: Desired price change distributions and inaction bands with normal and leptokurtic idiosyncratic shocks



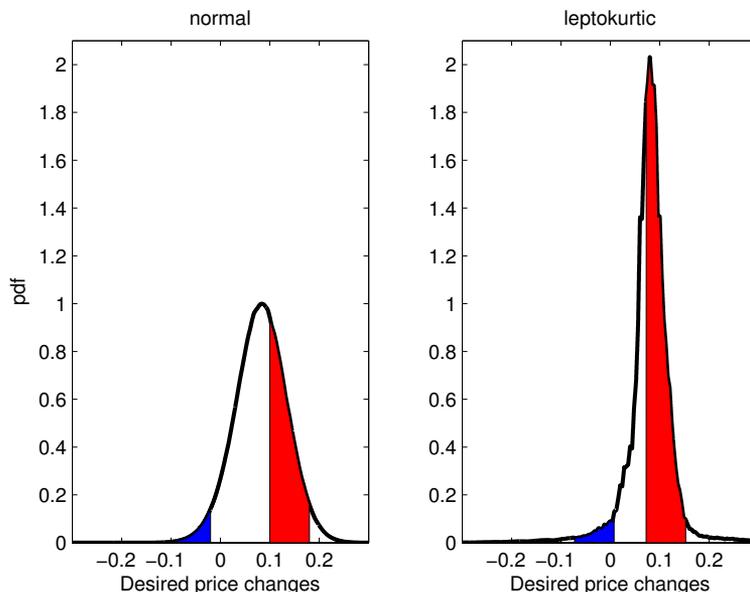
The figure plots steady state desired price change distributions obtained from calibrated models. It adds stylized inaction bands that represent important facts of the model. The shaded areas show the actual price change distribution, with price decreases on the left and price increases on the right.

The overall increase in the price change frequency is determined by the relative measure of the new price increases (shaded area to the right) to the new non-price-decreases (the shaded area to the left). The new price increasers would not have increased their prices without the aggregate shock, while the new non-decreasers would have decreased theirs. Note that the new increasers *increase* the aggregate frequency, while the new non-decreasers *decrease* it. For a marginal shock, these effects offset each other, and for small shocks their difference is still going to be insignificant. However, for large shocks, they can become quantitatively different, and as the figure shows, the shape of the distribution plays a key role in this.

For the normal model, where the slope of the desired price change distribution is changing slowly in the relevant region, increasing the shock size will generate a slowly increasing measure of new increasers and slowly decreasing measure of non-decreasers implying a steadily increasing extensive margin effect. For the leptokurtic model, however, the effects are much less linear. In the steady state, the inaction thresholds are going to be on the fat tails of the distribution, with moderate local slopes. For moderate aggregate shocks, the low slopes imply that the measure of new price increases are going to be similar to those of the new non-price decreases, so the frequency effect is going to be minimal, and lower than the effect in the normal model. However, for large shocks, the nonlinearly increasing slope of the desired price change distribution close

the shock in our full model. The change in the inaction bands, however, is much smaller than the shift in the desired price change distribution.

Figure 5: Stylized effects of large aggregate shocks with normal and leptokurtic idiosyncratic shocks



The figure plots large shocks to the desired price change distributions obtained from the calibrated models. The stylized inaction bands are assumed to stay constant. The shaded areas show two groups of firms influenced by the shock: the new increasers (on the right) and the new non-decreasers (on the left). Both groups contribute to the aggregate inflation pass-through, but the frequency effect is determined by the *difference* between the measure of the two groups.

to its sharp peak means that the measure of price-increasing firms will quickly dominate the mass of non-price-decreasing firms in the lower tail. This implies large and highly non-linear extensive margin effects for large shocks.

4.2 Asymmetry

Trend inflation in a menu cost model can explain asymmetry of the inflation pass-through between positive and negative aggregate shocks, as was argued by Ball and Mankiw (1994). In this section, we are asking two questions. One is whether this asymmetry is significant in our baseline model with leptokurtic idiosyncratic shocks for a standard inflation rate of 2% and standard shock sizes. If it were, it would make the time-dependent model implying symmetry a weak approximation to it. Our second question is to quantify and explain the differences between the asymmetry in our leptokurtic baseline model and the standard model of Golosov and Lucas (2007) that assumes normally distributed shocks.

The theoretical argument of Ball and Mankiw (1994) rests on the menu cost assumption, and aggregate asymmetry would not be present either in a flexible price model, or in a time-

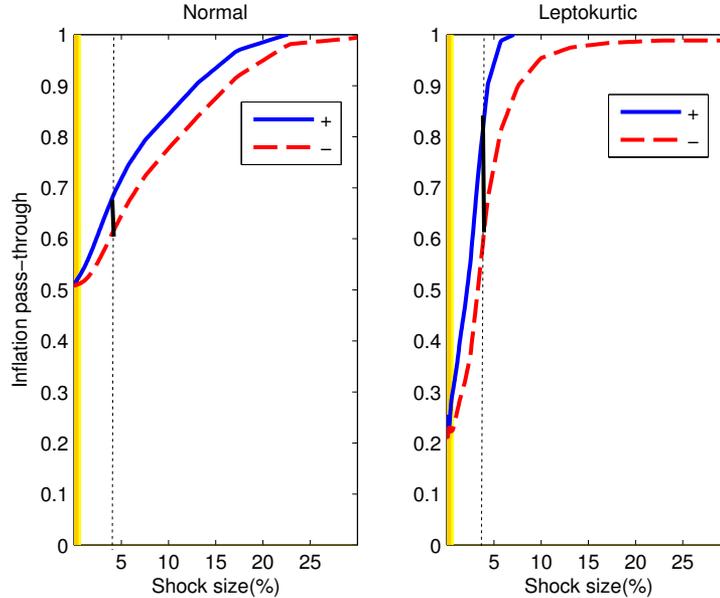
dependent pricing model like Calvo (1983). It is straightforward to see why the flexible price assumption would not interact with trend-inflation: firms would adjust their nominal prices every instance to follow the aggregate inflation rate and their *additional* response to aggregate shocks would imply a full and symmetric pass-through. In standard time-dependent pricing models, like Calvo (1983), the situation is a bit more complex. At the firm level, we will see asymmetry as a result of the trend inflation, but it will not translate to aggregate asymmetry. Firms are forward looking and know that they are setting their prices for several periods in advance. As a result, they will 'front-load' and implement a higher price change than would be required by their static optimum to keep their constantly decreasing relative price close to its static optimum throughout their price spell. This asymmetry in the desired price changes is present every period even without the aggregate shock: it maintains the trend inflation. The asymmetry of the aggregate pass-through, that we are interested in, is determined by the *additional* inflation responses over the trend inflation. These additional inflation effects, however, are no longer asymmetric in the Calvo model: the difference between the pass-through between a positive and a negative shock even in the fully non-linear model is negligible.³⁶ In these models, there is no endogenous frequency response which could introduce quantitatively significant asymmetry into the inflation pass-through for large shocks in menu cost models.

Figure 6 shows inflation pass-throughs for different aggregate shock sizes (both positive and negative) for the two alternative menu cost models under a 2% (yearly) trend inflation rate. It shows that for a marginal aggregate shock, both models imply symmetry; and the effects of very large shocks are also symmetric, as both the negative and the positive shocks reach full pass-through. For the immediate range, however, we can see that – at least up until a 10% shock – the asymmetry in the leptokurtic model gets much more pronounced than in the normal model. To get some idea about the size of the asymmetry, consider now a shock size of 3.6% (that implies a 60% immediate inflation pass-through for both the normal and the leptokurtic model). In the leptokurtic model a positive shock implies a 70% immediate inflation pass-through and the negative one a 50% pass-through. It would imply an immediate real effect of 30% of the monetary shock for the positive shock, but a 50% for the negative one, a 66% difference. It is true, however, that for such a large shock, none of these real effects are going to be long lived: their half lives are going to be 1 month and 1.4 months for the positive and negative shocks respectively. For the same shock, the normal model, that predicts a similar average inflation pass-through, implies a 63% positive and a 57% negative pass-through, a significantly smaller asymmetry (less than 20% difference in the real effects of 37% vs. 43%).

It is also interesting to note, that the increase in the asymmetry for the leptokurtic model is

³⁶The asymmetric shape of the profit function introduces some asymmetry between responses to positive and negative shocks. The reason is that *smaller* relative prices imply higher relative demands, and with it higher losses than higher relative prices. It means that price changing firms are willing to respond more to a positive shock and have a higher relative price than to a negative shock and have a relative price smaller than 1. The numerical importance of this channel, however is negligible.

Figure 6: Asymmetric inflation pass-through with 2% trend-inflation



The figures plot the positive and negative inflation pass-throughs for the normal and the leptokurtic models for different shock sizes. The bars on the left of the figures plot the 2 standard deviation bands for standard (small) monetary shocks in the Golosov and Lucas (2007) and Midrigan (2011) calibrations. The figures show that the effects are symmetric for a marginal aggregate shock; and also for very large shocks, where both positive and negative shocks imply full pass-through. In the intermediate range, the asymmetry gets substantially higher in the leptokurtic model.

very quick, so the asymmetry is present for even small shocks. However, as the average inflation pass-through is small in this range, so the real effects are large, the asymmetry in terms of the real effects are going to be small (for example, for a shock of 0.5%, the average real effects are 75% of the shock, the model predicts a real effect of 72.4% for a positive shock and 77.6% for a negative one).

So we find that even for a small 2% trend inflation, the asymmetry between positive and negative shocks can be substantial for large monetary shocks. Higher trend inflation, or steady state price distributions with higher kurtosis could lead to even higher asymmetries, as we saw in the Hungarian experiment. For standard shocks, however, the Calvo model that implies basically no asymmetry remains a good approximation.

4.2.1 Decomposing asymmetry

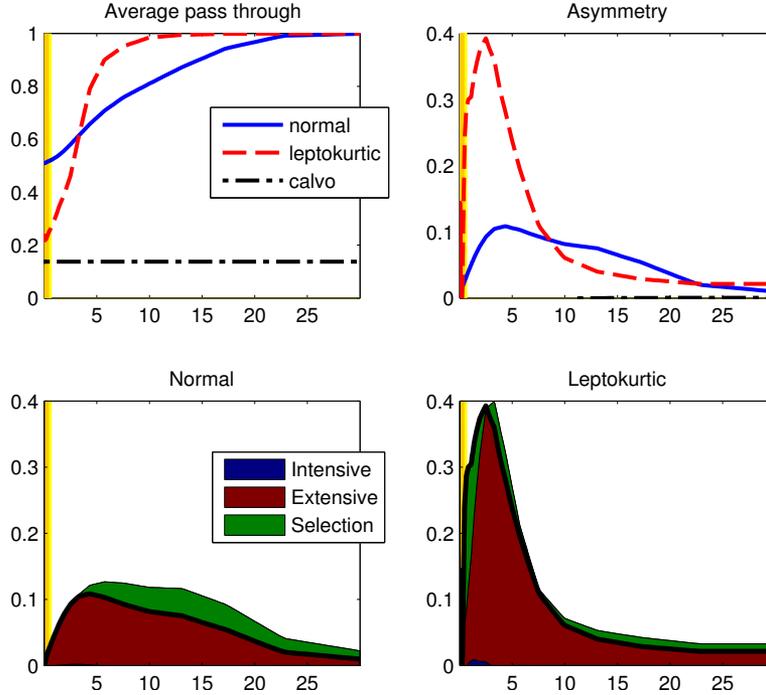
In this subsection, our question is what drives the asymmetry for large shocks. To calculate the contributions of the extensive, intensive and selection margins to the asymmetry, we measure the asymmetry here as the difference between the positive and the negative pass-throughs relative

to their average:

$$\frac{\gamma_0^+ - \gamma_0^-}{(\gamma_0^+ + \gamma_0^-)/2}, \quad (12)$$

where γ_0^\pm is the immediate inflation pass-through after a positive and negative aggregate shock.

Figure 7: Asymmetric inflation pass-through with 2% trend-inflation



The figures plot the average pass-through, measures of inflation asymmetry and contributions of the extensive, the intensive and the selection effects in the normal and leptokurtic models. The bars on the left of the figures plot the 2 standard deviation bands for standard (small) monetary shocks in the Golosov and Lucas (2007) and Midrigan (2011) calibrations. The figures show that the effects are symmetric for a marginal aggregate shock; and also for very large shocks, where both positive and negative shocks imply full pass-through. In the intermediate range, already for small shocks, the asymmetry gets substantially higher in the leptokurtic model. In the second row, the figures show the contributions of different margins. The solid line shows the aggregate effects. The selection effect decreases the asymmetry in many cases: when it happens its plotted area is negative, and the extensive margin effect should be increased by the area of the selection effect. The plots show that the asymmetric extensive margin effect is the main factor behind the observed inflation asymmetry.

The upper left panel of Figure 7 shows the average pass-throughs for a 2% inflation.³⁷

³⁷The positive inflation rate actually increases the average pass-through a bit for the normal model for small shocks, and also increases it away from the Calvo pass-through for the leptokurtic model. The reason is that trend inflation changes the relative position of the desired price change distribution to the inaction region, thus increases the selection effects. However, the leptokurtic model still implies a smaller pass-through, and thus larger real effects than the normal model for small shocks, and, also similarly to the zero inflation case, this relationship turns around for large shocks.

The upper right panel shows the asymmetry measures as defined above. The bottom panels decompose the asymmetry, by calculating the difference between the intensive margins, extensive margins and selection effects for the positive and negative shocks. The figures show that the main factor in explaining the inflation asymmetry is the asymmetry in the extensive margin effect. Asymmetries in the selection effect contribute to overall asymmetries until about it reaches its peak and reduce them afterwards, while the asymmetry of the intensive margin is numerically very small.

4.2.2 Explaining asymmetry

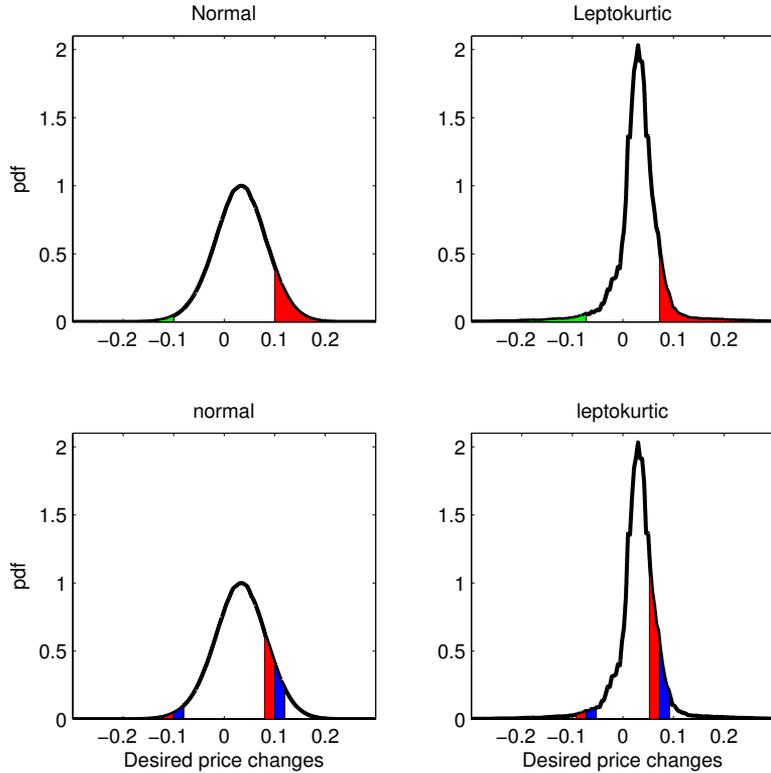
In this section, we use our stylized example to gain some insights into what drives the quantitative differences between the asymmetries found in our baseline model with leptokurtic shocks relative to the standard model with normally distributed shocks.

To understand the reasons behind these results, it is useful to get back again to the stylized example introduced earlier. With trend inflation, as the first row of Figure 8 shows, the median of the steady state desired price change distribution is to the right of the center of the inaction band. The reason is, similarly to the Calvo model, that firms front-load price changes to counteract the effect of the trend inflation reducing their relative price. Even under a zero idiosyncratic shock, their desired price change is going to be positive. Its magnitude is going to be influenced mainly by the trend inflation and the the expected duration of their price spell determined by the probability of their price changes.

We argue here that the asymmetric extensive margin effect under large shocks is influenced by the difference between the slopes of the desired price change distributions around the positive and the negative inaction thresholds. To see this, note that the extensive margin effect is influenced by two groups of firms: the *new* price changers, that would not have changed their prices absent the aggregate shock and the *new* non-price changers, that would have changed it, but facing the aggregate shock now choose not to. The new changers increase the aggregate frequency, while the new non-changers decrease it. With trend inflation, there will be a higher density of firms around the positive inaction threshold than around the negative one, but this fact, in itself is only responsible for the trend inflation, and does not cause asymmetry in the pass-throughs.

For a measure of the asymmetry in the extensive margin effects, we need to compare the frequency increase under a positive aggregate shock to the frequency decrease under a negative one. The bottom panels in Figure 8 show the relevant groups of firms. For easier presentation, instead of pushing the desired price change distributions, we are moving the inaction bands in opposite directions providing equivalent frequency effects. The frequency increase after a positive shock is going to be given by the new price increasers (shaded red bars on the right) minus the new price non-decreasers (shaded red bars on the left). The frequency decrease after a negative shock, is given by the difference between the price decreasers (dark blue bars on the left) minus the non-price increasers (dark blue bars on the right).

Figure 8: Stylized effects of a positive and negative shocks with a positive trend inflation



The figures plot the desired price change distributions and stylized inaction bands for the normal and leptokurtic idiosyncratic shocks. The shaded areas in the first row show the steady state actual price change distribution. In the second row, shaded areas show the new changers and new non-changers for a positive (red) and a negative (blue) aggregate shocks.

To determine the asymmetry, we need to look at the difference between the price increasers for the positive shock relative to the non-price decreaseers to the negative one (shaded red and dark blue bars on the right), and compare it to the difference between the price decreaseers as a response to a negative shock relative to the non-price increaseers to the positive one (shaded red and dark blue bars on the left). What these differences predominantly depend on is the absolute value of the slope of the desired price change distribution around the positive and the negative inaction thresholds. A higher absolute slope around the positive inaction threshold relative to the slope around the negative inaction threshold means that the frequency increase for a positive shock is going to be higher than the absolute value of the frequency decrease for a negative one. As the difference between the slopes is much higher in the leptokurtic model, we will find much higher asymmetry.

From this graphical argument we can also see that for a marginal shock, the differences of these areas will approach zero quadratically, implying zero extensive margin effects and thus

no extensive margin asymmetry. Also, under no trend inflation, the desired price change distribution is symmetric around the positive and the negative inaction thresholds implying equal absolute slopes. The number of new increases for a positive shock are equal to the number of new price decreases for the negative shock, and similarly, the disappearing price decreases for a positive shock are equal to the number of disappearing price increases for a negative one. It means that there is no asymmetry between the extensive margin effects for zero inflation rates.

5 Sensitivity analysis

5.1 Persistence of the idiosyncratic shocks

In our baseline calibration, we have set the idiosyncratic persistence parameter to 0.95. In this subsection we perform sensitivity analysis of our results by changing this parameter to $\rho_A = 0.7$. We find that even though it has some quantitative effects, our qualitative conclusions do not change.

Table 5: Calibration with $\rho_A = 0.7$

Parameters		leptokurtic	normal	Calvo
$\mu\phi$		1.116%	2.025%	0
σ_A		4.06%	4.98%	22.4%
p		0.912	0	0
λ		–	–	13.46%
Matched moments	data	leptokurtic	normal	Calvo
Frequency (no tax, NT)	13.46%	13.52%	13.46%	13.46%
Avg abs size (NT)	9.91%	9.89%	9.91%	9.91%
Kurtosis	3.9768	3.9809	<i>1.5865</i>	<i>3.4217</i>
Unmatched moments	data	leptokurtic	normal	Calvo
Frequency tax incr	61.96%	67.66%	28.42%	13.46%
Frequency tax decr	26.92%	33.02%	20.30%	13.46%
Avg abs size tax incr	8.95%	7.86%	10.69%	10.54%
Avg abs size tax decr	8.55%	6.53%	10.76%	10.54%
Infl path through tax incr	98.86%	101.54%	61.20%	13.88%
Infl path through tax decr	32.87%	42.37%	54.11%	13.98%
1st decile of size distr (NT)	2.75%	4.94%	7.58%	1.40%
1st quartile of size distr (NT)	4.56%	5.40%	8.28%	3.80%
Median of size distr (NT)	7.66%	7.11%	9.32%	8.00%
3rd quartile of size distr (NT)	12.69%	11.55%	10.89%	14.20%
9th decile of size distr (NT)	19.43%	17.05%	12.75%	21.00%

Table 5 shows the estimated parameters, the matched and the unmatched moments. The parameters of the leptokurtic shocks change only marginally implying a distribution with smaller kurtosis, and the normal model requires a slightly higher standard deviation and menu costs to hit the moments. With this calibration, the leptokurtic model again predicts a much higher frequency of price changing firms at the period of the tax shock than the Golosov and Lucas (2007) calibration, but now it slightly overestimates the true frequency jump. The model is surprisingly good at hitting the pass-through for the positive shock, but it somewhat overestimates the pass-through for the negative shock. More importantly, the estimated asymmetry for the model with leptokurtic shocks is much closer to those observed in data than in the Golosov and Lucas (2007) model.

5.2 Midrigan’s multi-product model

The full model of Midrigan (2011) considers two-product firms, where the cost of changing the price of the second good, when the price of the first good also changes, is zero. This increasing returns in the repricing technology provides him an additional mechanism – besides the leptokurtic distribution – that reduces the selection effect, thus increases the real effects of monetary shocks. This assumption also influences the extensive margin.

In effect, this additional mechanism introduces flexible prices to the economy: the firm that chooses to pay the menu costs, will have an opportunity to change its second price at no cost, irrespective of how large its desired price change is for this second product. Midrigan (2011) shows that this factor indeed reduces the selection effect: there will be a large number of small price changes. Its quantitative influence on reducing the selection effect, however, is much smaller than that of the leptokurtic shock assumption.

These results of Midrigan are valid for *small* money shocks. In order to investigate how this additional mechanism affects the selection effect, extensive margin effect and real effects of *large* monetary shocks, we calibrated the multi-product version of Midrigan’s model to match the same basic data moments as in our baseline single-product model, and exposed the model for the same large positive and negative nominal shocks as before. We calibrate the same model parameters – the size of the menu cost $\mu\phi$ (the payment of which now allows the firm to change both prices), the standard deviation of the idiosyncratic shock σ_A , and the parameter governing the kurtosis of the distribution of idiosyncratic shock innovations (p) – to match the frequency and average absolute size of price changes, and the kurtosis of the size distribution.³⁸ In this calibration, roughly half of the price changes will be “secondary” price changes, i.e. ones corresponding to a large price change of the first product. Table 6 compares this calibration with that of the baseline model featuring single-product firms; in both cases we assume leptokurtic technology

³⁸Additional model parameters were directly taken from Midrigan (2011). The within-firm correlation of idiosyncratic shocks is 0.6, and the within-firm elasticity a CES consumption aggregator is 1.1 (the between-firm elasticity remains 5, as in the baseline single-product model).

shock innovations.

Table 6: Calibration with multi-product vs single-product firms (both assuming leptokurtic shocks)

Parameters		multi-product	single-product
$\mu\phi$		2.75%	1.23%
σ_A		4.415%	4.04%
p		0.9196	0.915
Matched moments	data	multi-product	single-product
Frequency (no tax, NT)	13.46%	13.46%	13.46%
Avg abs size (NT)	9.91%	9.91%	9.91%
Kurtosis	3.9768	3.9780	3.9768
Unmatched moments	data	multi-product	single-product
Frequency tax incr	61.96%	51.96%	56.88%
Frequency tax decr	26.92%	24.44%	25.87%
Avg abs size tax incr	8.95%	8.36%	7.85%
Avg abs size tax decr	8.55%	8.42%	7.92%
Infl pass through tax incr	98.86%	82.54%	87.38%
Infl pass through tax decr	32.87%	41.22%	43.50%
1st decile of size distr (NT)	2.75%	1.39%	5.09%
1st quartile of size distr (NT)	4.56%	4.21%	5.61%
Median of size distr (NT)	7.66%	8.27%	7.11%
3rd quartile of size distr (NT)	12.69%	13.22%	11.60%
9th decile of size distr (NT)	19.43%	20.09%	19.05%

In terms of the calibrated parameters, only the menu cost parameter differs significantly from the single-product case. This is because now we have free secondary price changes along with the large primary price changes, so the frequency of these primary price changes can be substantially lower; hence the menu cost can be larger (2.75% vs 1.23%). The two parameters governing the shock distribution (standard deviation, kurtosis) are quite close to the baseline single-product case. Due to the larger menu cost, the inaction band will be larger; but this wider inaction band is apparently not wide enough to prevent the majority of the firms from being pushed outside of it for the large positive shock (+4.1%): the price change frequency at the tax increase gets only slightly smaller in the multi-product case (52% vs 57%). For the large negative shock (-4.1%), the frequency of price change jumps to around 24.5%, again only slightly smaller than in the single-product case (about 26%). Therefore, we have a similar asymmetry in the extensive margin effect as before, which is the main reason of the asymmetry in the inflation effects (82.54% and 41.22% for the positive and negative shocks in the multi-

product case). Hence the extent of asymmetry in the inflation pass-through remains the same, the only difference being that – consistently with Midrigan (2011) – the average inflation pass-through is somewhat smaller now than in the single-product case. This is because the presence of small price changes decreases the selection effect and the inflation pass-through, and increases real effect. However, this effect is quantitatively small, even smaller than for small shocks in Midrigan’s exercise.

Having seen that the full multi-product model has similar implications for the extent and asymmetry of inflation pass-through as the single-product model, we note that there is one aspect at which the multi-product variant performs much better: it almost exactly matches the observed price change size distribution.³⁹ While the single-product model is unable to generate small price changes (the first decile and quartile of the size distribution are 5.09% and 5.61%, respectively, which means that in the model-implied price change size distribution there is a big mass at around 5-6%, and nothing below), the multi-product model can do this: in that model every tenth price change is smaller than 1.39%, and every fourth is smaller than 4.21%. But this ability of the multi-product model seems to be independent of what it predicts for the size and asymmetry of inflation effects of large, 4% shocks.⁴⁰

5.3 Announcement

In practice, tax changes are usually announced in advance, so that agents can prepare for the changing conditions. In the Hungarian case, the tax increase of September was announced in early June, i.e. 3 months before it took effect. The tax decrease of January became public during the budget planning period for 2006, 4-5 months prior to the actual change. In this subsection we investigate whether our results are robust to this kind of pre-announcement, by studying the predictions of the baseline model under different pre-announcement leads.

Table 7 compares the baseline model’s predictions for the effect of tax changes with those under different pre-announcement dates. We consider three cases: the government announces the tax change 1, 3 or 6 months before it takes effect. Besides the non-matched moments reported earlier, in the last two rows of the table we add the initial inflation pass-through, i.e. the pass-through that takes place between the date of announcement and the actual tax change.

³⁹Another aspect is that the multi-product model gets closer to the average size of price changes at the tax-changing months.

⁴⁰We note two additional points. First, Midrigan (2011) also finds that from the two mechanisms that decrease the selection effect and hence increase the real effect – leptokurtic shocks and small price changes –, the leptokurtic shocks are quantitatively more important. We confirm this: the presence of small price changes only slightly decreases the selection effect. But in case of large shocks, the extensive margin effect becomes so large that this slight decrease in selection effect will become quantitatively unimportant when we look at the overall inflation effect. Second, and related, there is a range of shock sizes where the wider inaction band in the multi-product model actually leads to much smaller extensive margin effect (as the big mass of stores in the middle is still not pushed outside the inaction region) than the single-product model. For these shock sizes the single-product and multi-product models might have different predictions for the real effects of the shocks.

Table 7: Effect of pre-announcement

Unmatched moments	Announcement				
	data	baseline	1 mth	3 mth	6 mth
Frequency tax incr	61.96%	56.88%	57.87%	56.32%	57.58%
Frequency tax decr	26.92%	25.87%	21.09%	17.38%	16.83%
Avg abs size tax incr	8.95%	7.85%	8.10%	8.40%	8.42%
Avg abs size tax decr	8.55%	7.92%	8.71%	9.48%	9.55%
Infl pass through tax incr	98.86%	87.38%	92.00%	92.28%	94.93%
Infl pass through tax decr	32.87%	43.50%	35.38%	29.24%	27.56%
Initial infl PT tax incr	–	–	4.07%	8.95%	9.25%
Initial infl PT tax decr	–	–	14.60%	27.19%	32.74%

The main result emerging is that qualitatively, results do not change. The inflation pass-through remains to be highly asymmetric in the month of the tax change (rows 5-6), and this asymmetry is mainly driven by the asymmetry in the extensive margin (rows 1-2). In fact the inflation pass-through at the tax-changing months becomes a bit more asymmetric than in the baseline model, bringing the model prediction even closer to data.

Additionally, we also see asymmetry in the pass-through that takes place between the announcement and the actual tax change (rows 7-8): for the tax increase, this initial path-through is small (less than 10% even when the tax change is announced six months in advance). For the tax decrease, it is much larger (almost one third in case of a six-month announcement lead). The asymmetric price change frequency of the tax-changing months provides the explanation for this initial pass-through asymmetry. In case of the tax increase, firms know that in the month of the actual tax change they will change their price with relatively high probability (56-57%). Thus any initial price adjustment is going to last – in expected terms – for a relatively short period, which means that these initial price adjustments must provide the necessary profit gain within this relatively short period.⁴¹ Therefore firms will be reluctant to adjust, *unless* they are highly mispriced so that the immediate gains are large. Consequently, the model predicts falling price changing frequencies, for both price increases and decreases, and increasing price change sizes for the period between the announcement and the tax increase, leading to small initial pass-through. (In effect, firms are “waiting” for the tax change, when they are likely to adjust anyway.) We do not see a similar mechanism for the case of pre-announced tax decrease. In this case there will not be many adjusting firms in the month of tax decrease, and hence this waiting explanation does not work. The model therefore – perhaps intuitively – predicts increasing

⁴¹Remember that stores are undertaking price changes if and only if the gains – until the next price adjustment – in expected profits due to the price adjustment (or the marginal benefit from adjustment) exceed the menu cost (or the marginal cost of the price adjustment).

price decrease frequencies, and decreasing price increase frequencies for the period between the announcement and the tax change. The relatively large initial pass-through is mainly driven by the missing price increases in this period.

We have also studied the effect of pre-announcement in the Golosov-Lucas model with normally distributed idiosyncratic shocks. The results are similar: the effect of pre-announcement on the asymmetry of the pass-through at the month of the tax change is numerically small. For the 3-month pre-announcement, for example, the pass-through becomes 49% and 39.2%, which is similar to the baseline model with normally distributed shocks (65.6% vs 54.5%). In this case the initial pass-through is relatively large but only slightly asymmetric (26.7% vs 28.7% in the 3-month horizon), consistent with the fact that the extensive margin effect at the month of the tax change is relatively small and only slightly asymmetric (24.7% vs 16.5%).

5.4 Trend inflation and price change frequency

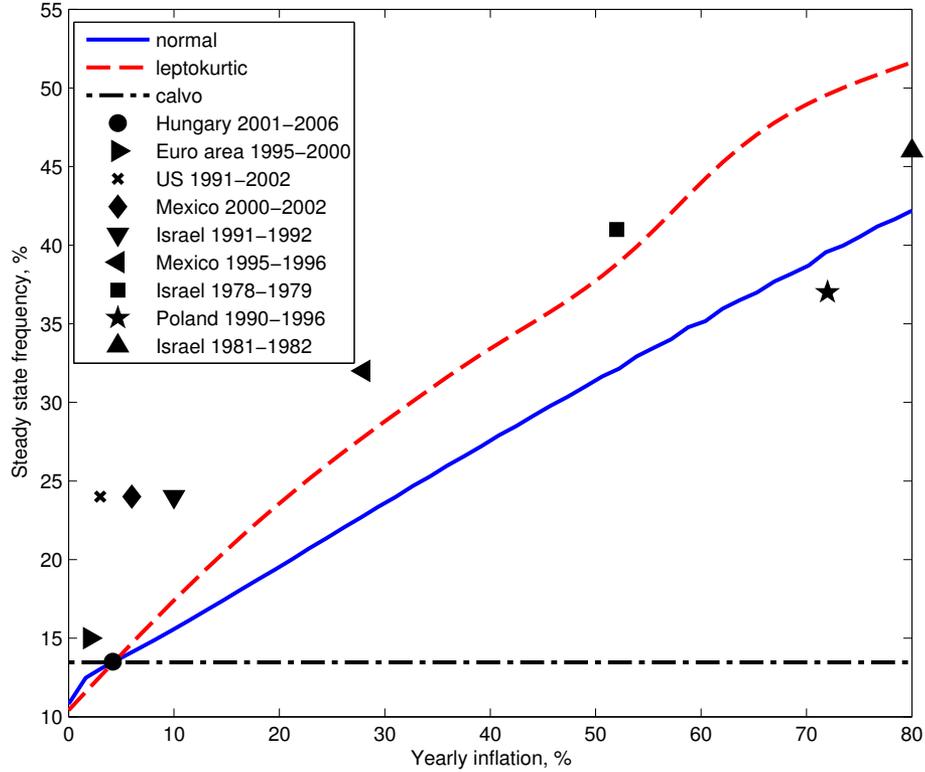
That inflation increases the number of price changes has long been an evidence supporting state-dependent pricing models.⁴² In this section, we look at the predicted fraction of price changes in our three calibrated models for various trend inflation rates. Our question is how useful this evidence is to compare the different state-dependent menu cost models.

Figure 9 plots the steady state frequency of price changes as a function of trend inflation rates. It shows that, contrary to the time-dependent Calvo model, price change frequency is increasing with inflation in the state-dependent menu cost models in line with evidence. It also shows, however, that the difference between the model with Gaussian and fat-tailed idiosyncratic shocks are much less pronounced than if we consider one-time permanent shocks as we did in our paper. Using scattered evidence available from different countries and time periods, it would be difficult to draw clear conclusions on the relative quantitative performance of the two menu cost models.

What is the main difference between a larger permanent shock and higher trend inflation? Trend inflation increases frequency not only in the short term – as a one-time permanent shock does – but increases future frequencies as well. As a result, the expected duration of a price spell gets shorter, influencing the firms' dynamic incentives. One-time shocks increase the firms' desired price change one-for-one and have only second order influence on their inaction bands. As a result, differences in the distribution of desired price changes imply stark differences in frequency responses to large shocks, as we have shown earlier. Higher trend inflation, in contrast,

⁴²For example, Golosov and Lucas (2007) use observations from moderate and high trend-inflation economies to confirm their predicted link between inflation and steady state price-change fraction. High inflation episodes in Mexico, provide Gagnon (2009) a special environment to show that the extensive margin becomes an effective channel after a certain trend inflation rate, that is well captured by the standard Golosov and Lucas (2007) framework. More recently, Alvarez, Gonzalez-Rozada, Neumeyer, and Beraja (2011) use hyper-inflationary episodes in Argentina to test their model's predictions on the sensitivity of frequency and size responses to changes in inflation for small and large inflation rates.

Figure 9: Frequency of price changes and trend-inflation



The figure plots the steady state frequency of price changes as a function of inflation rate for the time-dependent Calvo model, the menu cost model with normal and leptokurtic idiosyncratic shocks calibrated to the Hungarian evidence. It also plots some observations collected by Golosov and Lucas (2007).

increases not only the desired price changes, but has substantial influence on the width of the inaction bands as well. The reason is that firms set prices for shorter effective terms, so they are willing to respond only to larger idiosyncratic shocks, for which mispricing is costly already in the short term. The increase in the width of the inaction band, apparently, is quicker with fat-tailed shocks. As a result, differences in idiosyncratic distributions are not getting translated into substantial differences in frequency responses to higher steady state inflation rates.

6 Conclusion

The paper uses empirical evidence on large value-added tax shocks to confirm predictions of the state-dependent menu cost model of Midrigan (2011) over the standard model of Golosov and Lucas (2007). Its success lies in its ability to capture (i) the exploding fraction of price changes for large shocks and (ii) the large asymmetric reactions to positive and negative shocks under positive trend inflation. The well-publicized nature of the tax shocks also raises doubts

on information- and search-friction models as sole reasons of price rigidities.

Our baseline model implies highly non-linear effects of nominal shocks: prices react sluggishly for small shocks, but they turn flexible quickly as the shock size reaches a threshold. We find that this threshold is high and its position depends on the level of the trend-inflation. These quantitative conclusions can be used to form informed hypotheses on the price effects of some alternative large nominal shocks, like the 2007-2009 US Great Recession or the 1929-1933 Great Depression.⁴³ Sticking to our calibration, the model predicts qualitative differences between the price responses during these two episodes. During the 2007-2009 US crisis, the trend-inflation expectations stayed well-anchored at 2%. In this environment, a realistic *negative* nominal shock⁴⁴ was just not large enough to make the extensive margin of price adjustments effective. It suggests that using the standard time-dependent model of Calvo (1983) for the Great Recession period is still valid. Our predictions for the Great Depression would be very different, however. This is mainly because of the unanchored inflation expectations during that period: between 1929-33 the average yearly *deflation* rate was 5.5%. In this environment, prices respond very flexibly to negative shocks: the half-life of the real effects of even a small negative nominal shock would be less than a month. The shocks, that have led to a cumulative nominal contraction of over 50% between 1929-33, thereby, would imply highly flexible price-level responses. This makes using time-dependent price setting assumptions for this period highly questionable (e.g. Christiano, Motto, and Rostagno (2003)); and, in general, raises some doubts whether nominal rigidities could really be the main explanation for the contraction in real activity during the US Great Depression.

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⁴³Detailed analyzes of these periods are out of the scope of this paper.

⁴⁴Let's consider the hp-trend filtered nominal GDP level, and define the shock simply as the residual to an AR(1) process. Then in 2008Q4 the size of the *quarterly* shock was -2.5%. In our quantitative experiment, we considered permanent nominal shocks, and found that at 0 inflation rate, up to a 2.5% *monthly* nominal shock, the time-dependent Calvo (1983) model is still a good approximation to our baseline menu cost model. So for a 2% positive inflation rate, a quarterly 2.5% negative shock with an AR(1) parameter less than 1 would still imply sticky price developments.

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7 Appendix

7.1 Equivalent inflation effects of money and tax shocks

In this section, we show that for our baseline parametrization (similarly to those of Golosov and Lucas (2007) and Midrigan (2011)), money shocks and value added tax shocks have equivalent effects on the inflation path - even though their effects on the output is different. The proof is using a guess and verify method, showing that assuming the same price level paths, the money supply and the tax rate have equivalent effects on the optimal price choices; indeed justifying the equal price level assumption.

Nominal wage moves together with the money supply, under our assumptions on the labor supply equation with separable utility that is logarithmic in consumption and linear in labor ($\psi = 0$). In this case

$$W_t = \mu Y_t P_t = \xi M_t, \quad (13)$$

where $\xi = \mu(R - 1)/(\nu R)$. For the second equality, we used the money demand equation ($M = \nu P_t Y_t R / (R - 1)$) and the fact that, by the Euler equation, gross nominal interest rate ($R = g_M / \beta$) is constant under constant nominal output growth.

An individual firm's periodic nominal profit with value added taxes (τ_t) equals

$$\begin{aligned} \tilde{\Pi}_t(i) &= \frac{P_t(i)}{1 + \tau_t} Y_t(i) - W_t L_t(i) \\ &= \frac{P_t(i)}{1 + \tau_t} Y_t(i) - W_t \frac{Y_t(i)}{A_t(i)} \\ &= \left(\frac{P_t(i)}{1 + \tau_t} - \frac{W_t}{A_t(i)} \right) Y_t(i) \\ &= \frac{1}{1 + \tau_t} \left[\left(P_t(i) - \frac{\xi M_t (1 + \tau_t)}{A_t(i)} \right) Y_t(i) \right], \\ &= \frac{1}{1 + \tau_t} \left[\left(P_t(i) - \frac{\xi M_t (1 + \tau_t)}{A_t(i)} \right) \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t \right], \end{aligned} \quad (14)$$

where we used equation 13 to substitute out the nominal wage; and that demand for good i is given by $Y_t(i) = (P_t(i)/P_t)^{-\theta} Y_t$.

As before, it is instructive to normalize the nominal profits by the exogenous nominal output

$\Pi_t(i) = \tilde{\Pi}_t(i)/(P_t Y_t)$ to get

$$\Pi_t(i) = \frac{1}{1 + \tau_t} \left[\left(P_t(i) - \frac{\xi M_t (1 + \tau_t)}{A_t(i)} \right) P_t(i)^{-\theta} P_t^{\theta-1} \right]. \quad (15)$$

Let's guess that the present and future path of the price level $\{P_t\}$ is the same for a permanent tax shock and a permanent money level shock. The optimal price choices of firms depend on their normalized value functions that is a present discounted value of their future profits. As the derivation shows, the tax rate has independent influence the level of profits, but its influence on the optimal price choice is equivalent to that of the money supply.⁴⁵ As we also assume lump-sum redistribution of taxes, the variables will not influence the budget constraints either. It means that the assumption of equivalent price level development is indeed verified. So we are justified to use evidence gained from value added tax shocks to compare the predictions of the model for large monetary shocks.

7.2 The flexible price equilibrium

The algebraic solution for the flexible price equilibrium provides useful information about the long-term pass-through of the permanent tax- and money shocks. Money shocks, naturally, have no real effects under flexible prices, so we will have full pass-through to the price level. A permanent value-added tax shock, for our parametrization, will imply a unit drop in the real output, so under unchanged money supply, we will have full pass-through to the gross nominal prices in this case as well.⁴⁶

To gain some insight into why value-added tax shocks imply a unit drop in output, it is useful to look at the firms' static profit maximization problem. Under flexible prices, firms will choose prices to maximize this, implying the following optimal relative price:

$$p_t^*(i) = \frac{\theta w_t}{\theta - 1} \frac{1 + \tau_t}{A_t(i)}, \quad (16)$$

where $w_t = W_t/P_t$ is the real wage. The equation shows that each firms want to increase their *relative* prices as a response to a tax increase. As all firms can not do this in equilibrium, real wages have to endogenously drop. It requires lower labor demand and output; and as household wage income will drop in parallel, the aggregate demand will adjust sufficiently to satisfy general equilibrium.

More formally, let's calculate the Dixit-Stiglitz aggregate of relative prices, that needs to be equal to 1 by definition $[\int p_t(i)^{*(1-\theta)} di] = 1$. We find that

$$w_t = \frac{\theta - 1}{\theta} \frac{A_t}{1 + \tau_t}, \quad (17)$$

⁴⁵For that to be exactly true, we need to assume that menu costs are tax deductible, so their effective costs drop with higher value-added taxes together with the value functions.

⁴⁶We are also using the flexible price solution as starting values for our iterative procedure.

where the aggregate productivity $A_t = [\int A_t(i)^{\theta-1} di]^{\frac{1}{\theta-1}}$.

From the labor market equation, we know that $w_t = W_t/P_t = \mu Y_t$, and any demand is going to be satisfied at this wage. The equilibrium output is, thus, given by

$$Y_t = \frac{\theta - 1}{\theta \mu} \frac{A_t}{1 + \tau_t}. \quad (18)$$

The optimal relative prices are given by the firms' relative productivity: $p_t^*(i) = (A_t(i)/A_t)^{-1}$, and the relative outputs equal to $Y_t^*(i)/Y_t = (A_t(i)/A_t)^\theta$.

The nominal price level can be obtained from the money market equation:

$$P_t = \frac{M_t}{\nu Y_t} \frac{g_{PY}}{g_{PY} - \beta} \quad (19)$$

The expected growth rates are

$$E(g_{Y_t}) = -E(g_{1+\tau_t}), \quad E(\pi_t) = g_{PY} + E(g_{1+\tau_t}) \quad (20)$$

It shows that a permanent increase in the tax will imply a full and immediate inflation pass-through under flexible prices.

7.3 Numerical solution algorithm

This subsection describes our numerical solution algorithm. It consists of two parts.

First, we solve for the steady-state aggregate variables π^{SS} , w^{SS} and Γ^{SS} .⁴⁷ As we assumed no aggregate uncertainty, aggregate variables will converge to their steady-state values. The steady-state inflation rate is equal to the growth rate of money stock: $\pi^{SS} = g_{PY}$. Then we calculate the steady-state real wages (w^{SS}) and the distribution of firms over their idiosyncratic state variables (Γ^{SS}) with the following iterative procedure:

1. We start with a guess for w^{SS} , w_0 . Initially, this guess is equal to the flexible-price steady-state of w , that we can calculate analytically.
2. Given this guess and the steady-state inflation rate, we use a fine grid on relative prices and idiosyncratic productivity shocks to solve for the optimal pricing policies of individual firms. We use value function iteration.
3. With the resulting policy functions, we calculate the steady-state distribution of firms over their idiosyncratic state variables. For this, we use the same set of grids as for the value function iteration. We again do this numerically: starting from a uniform distribution, we calculate the resulting distribution after idiosyncratic productivity shocks hit, and also after firms re-price. Then again calculate the resulting distributions after a new set of idiosyncratic shocks and new re-pricing. We do this until the resulting distribution is the same as the distribution one period earlier.

⁴⁷There is one other relevant aggregate variable: R_t (return on nominal bonds). We showed earlier that our constant growth assumption on nominal income and money stock implies that $R_t = \frac{e^{g_{PY}}}{\beta}$.

4. We calculate the average relative price in the resulting steady-state distribution. If this is smaller (larger) than 1, then we increase (decrease) our initial guess (w_0) of the real wages.
5. We repeat these steps until the average relative price in the calculated steady-state distribution is exactly equal to 1.

In the second part of our numerical algorithm, we calculate equilibrium paths of aggregate variables after an unexpected shock at $t = 0$ to the money supply (and nominal income), assuming that initially all aggregate variables were in their steady states. We calculate the equilibrium paths of π (inflation), w (real wages) and Γ (distribution of firms over their idiosyncratic state variables) with the following shooting algorithm:⁴⁸

1. We assume that aggregate variables will reach their steady-state in a finite (large) number of periods, T .
2. We start with a guess for the equilibrium inflation path $\{\pi_1, \dots, \pi_T\}$.
3. Given this guess, we calculate the resulting equilibrium path of the real wages: $\{w_1, \dots, w_T\}$. As $w_t = \mu Y_t$, we do this by calculating the equilibrium real GDP path, which we know from the equilibrium inflation path (and the constant nominal growth assumption).
4. Given the inflation and real wage paths, we calculate the path of value and policy functions. We do this by backward iteration from T , where the economy and the value functions are assumed to converge to a steady state.
5. Starting from period 1, and using the steady-state distribution of firms over their idiosyncratic state variables as initial distribution, we use the sequence of policy functions (together with the idiosyncratic shock processes) to calculate the resulting path of Γ , the distribution of firms over their idiosyncratic state variables.
6. From the resulting sequence of distributions, we calculate the resulting inflation path, and compare it with our initial guess. If the two are different, we update our guess to the linear combination of our previous guess and the resulting inflation path.
7. We do these iterations until the resulting inflation path is the same as our initial guess.

7.4 Constant marginal inflation pass-through

We argue here that constant marginal pass-through of the inflation rate as a response to a permanent money supply shock is a good approximation in our models. It implies that using

⁴⁸The equilibrium path of R_t (gross return on nominal bonds) will be equal to their steady-state value. This follows from the Euler- equation and our assumption of no aggregate uncertainty: agents always expect constant growth rate in nominal expenditures.

the pass-through of the initial month provides a sufficient statistic for both the initial size and the persistence of the real effects of the monetary shocks.

To see this, let's define marginal pass-through as

$$\delta_t = \frac{\Delta\pi_t/\Delta m_0}{1 - \sum_{s=0}^{t-1} \Delta\pi_s/\Delta m_0}$$

The numerator is the time t pass-through, while the denominator is the distance of the time $t - 1$ cumulative pass-through from the full long term pass-through $\sum_{t=0}^{\infty} \Delta\pi_t/\Delta m_0 = 1$. Under the Calvo setup, the periodic marginal pass-through for our calibration is indeed constant.

In our case, the nominal marginal costs for each firms are $MC_t(i) = W_t/A_t(i)$, where W_t is the nominal wage. For our calibration, similarly to Golosov and Lucas (2007) and Midrigan (2011) the nominal wage is a constant proportion of the money supply. So a permanent shock to the money supply is going to imply a permanent shock to each firms' nominal marginal cost. The optimal new nominal price is given by $P_t^* = \sum_{k=0}^{\infty} (\lambda\beta)^k \theta / (\theta - 1) MC_{t+k}(i) P_{t+k}^\theta / \sum_{k=0}^{\infty} (\lambda\beta)^k P_{t+k}^\theta$. Up to a first order, it is going to change by the nominal shock, so each price changing firm is going to fully incorporate the shock. It implies a pass-through that is equal to the frequency of price changes λ . Next period, the distance from full pass-through is $(1 - \lambda)$, and $\lambda(1 - \lambda)$ firms are going to change their prices by the shock implying a constant marginal pass-through of λ , and similarly in later periods. So, up to a first order approximation, the periodic marginal pass-through is constant. For higher order approximations, we need to take into consideration that a positive aggregate shock causes a gradual increase in the price level, so a fully non-linear solutions will imply slightly higher price adjustments for positive shocks (and similarly to negative ones maintaining symmetry). Also, firms that adjusted earlier will revise their prices downwards after a positive shock when they have an opportunity. The joint influence of these two effects imply marginal pass-throughs that are somewhat higher than the frequency of price changes, and quantitatively close to constant for higher shock sizes.

In menu cost models, a constant marginal pass-through is also a very good approximation in the relevant range. We are using R^2 measure to calculate the goodness of fit of a constant marginal pass-through assumption over our impulse response functions. Up to around a shock size of 5% it is effectively 1, and it is still over 90% till around a 10% shock. When the size of the shock implies a very high immediate pass-through in the first period, our R^2 measures starts dropping. In this case, however, even though this high initial marginal pass-through is not repeated in later periods, we know that the overall real effects are very small and short lived.