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FINANCIAL MARKET FRICTIONS IN A MODEL OF THE EURO AREA

by Giovanni Lombardo and Peter McAdam

In 2012 all ECB publications feature a motif taken from the €50 banknote.

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Abstract

We build a model of the euro area incorporating financial market frictions at the level of firms and households. Entrepreneurs borrow from financial intermediaries in order to purchase business capital, in the spirit of the "financial accelerator" literature. We also introduce two types of households that differ in their degree of time preference. All households have preferences for housing services. The impatient households are faced with a collateral constraint that is a function of the value of their housing stock. Our aim is to provide a unified framework for policy analysis that emphasizes financial market frictions alongside the more traditional model channels. The model is estimated by Bayesian methods using euro area aggregate data and model properties are illustrated with simulation and conditional variance and historical shock decomposition.

JEL Classification: C11, C32, E32, E37.

Keywords: Financial Frictions, euro area, DSGE modeling, Bayesian estimation, simulation, decompositions.

Non-Technical Summary

This paper builds on recent attempts to model the euro area using the current generation of micro-founded Dynamics Stochastic General Equilibrium (DSGE) models, following Smets and Wouters (2003) and the "New Area Wide Model" (NAWM) of Christoffel et al. (2008), but supplemented with a number of standard financial frictions. For instance, in its current state, the NAWM emphasizes international trade channels by modeling the euro area as a small open economy vis-a-vis the rest of the world. Financial markets, though, are modeled in relatively standard way, except that trade in domestic bonds and international bond takes place at an exogenous premium. Recent episodes of financial market turbulence have increased the demand for general equilibrium models that can account for the interaction between these markets, inflation and the real economy. Central banks' staff, for example, are increasingly confronted with questions concerning the interaction between asset prices, inflation monetary policy. Our work is a first step in the direction of putting existing and well-known euro area models in the position of being able to address this type of questions.

We extend the baseline model by introducing financial market frictions that can give rise to an interaction between financial market variables and the rest of the economy. In particular we consider two types of financial market frictions. First, we assume that a fraction of households has access to the financial market only to the extent that it can post collateral. We assume, realistically, that collateral consists of housing. This implies that drops in the value of housing will affect the amount of funds that this type of households can borrow. Second, we assume that entrepreneurs must borrow from financial intermediaries part of the resources necessary to run their business. We assume that the cost of borrowing is an increasing function of the leverage of the entrepreneurs: more leveraged entrepreneurs will face higher external finance premia. We also assume that this financial contract between entrepreneurs and financial intermediaries is subject to stochastic shocks, aimed at capturing the type of turbulence that has characterized the recent global recession.

We estimate the model using both real and financial variables using Bayesian techniques. Our results show that, although - relative to a model without the same financial frictions - the simulation properties are mostly not qualitatively affected, the model's ability to track and enhance our understanding of the evolution of financial variables and the strength of financial channels, makes the model a valuable addition to modeling work in the euro area.

1 Introduction

The global financial crisis which began around the turn of 2007/2008 has – amongst other things – prompted a re-evaluation of modeling strategies as regards financial linkages. It has long been known that financial markets are and were highly imperfect. This reflects information asymmetries between lenders and borrowers, costly verification of financial contracts, and the possibilities of bankruptcies and contagions etc. Consequently, a feature of financial markets is that lenders tend to demand a premium (or spread) over risk-less interest rates as compensation against such uncertainties. In the data, that premium, tends to be counter-cyclical (i.e., it tightens in economic downturns) thus amplifying the effect of economic downturns. Premia aside, borrowers may also be restricted in the absolute amount of funds available to them, for example as in mortgage loans.

The strength of such "financial frictions" and the soundness of the financial system have implications for how central banks conduct monetary policy and assess inflationary pressures and risks. The widening of spreads and deterioration in private lending from late 2007 onwards in many countries prompted a number of central banks to loosen monetary policy and engage in various forms of enhanced credit support, reflecting concerns that tensions in financial markets would spill-over to the wider economy.

Nevertheless, many policy models largely assume *frictionless* financial markets (with a few notable exceptions, Christiano et al. (2003)). This reflects, to some degree, likely academic and empirical controversy as to the importance of financial channels. Some analysis stress them as a key amplifier and source of business-cycle fluctuations (see e.g. Bernanke et al. (1999), hereafter BGG) whilst others suggest their impact may be relatively minor (see Meier and Mueller (2006)) or strongest during extreme and particular financial distress such as the Great Depression, the Asian Crisis (see, Gertler et al. (2007)) as well as presumably the most recent global financial turbulence.

Notwithstanding, our work builds on recent attempts to model the euro area using the current generation of micro-founded Dynamics Stochastic General Equilibrium (DSGE) models, following Smets and Wouters (2003) and the "New Area Wide Model" (NAWM) of Christoffel et al. (2008), but supplemented with a number of standard financial frictions. For instance, in its current state, the NAWM emphasizes international trade channels by modeling the euro area (EA) as a small open economy vis-a-vis the rest of the world. Financial markets, though, are modeled in relatively standard way, except that trade in domestic bonds and international bond takes place at an exogenous premium. Recent episodes of financial market turbulence have increased the demand for general equilibrium models that can account for the interaction between these markets, inflation and the real economy. Central banks' staff, for example, are increasingly confronted with questions concerning the interaction between asset prices, inflation monetary policy. Our work is a first step in the direction of putting existing and well-known euro area models in the position of being able to address this type of questions.

We extend the baseline model by introducing financial market frictions that can give rise to an interaction between financial market variables and the rest of the economy. There are two common ways of modeling financial constraints: *i*) via limited enforceability and collateralized debt (Iacoviello (2005)) and *ii*) via costly state verification and default risk (e.g. BGG). Here we use both. The first is used to model the financial constraints faced by households. The second is used to model the constraint faced by firms.

Both financing schemes generate a link between the net worth of agents and their creditworthiness, and so would be equally suitable to describe both household finance and firms finance. Nevertheless the collateral constraint model generates quantitative rationing leaving the cost of funds at the risk-less rate level. The costly-state-verification model, instead does not limit the level of debt, but generates instead a cost of funds that is larger than the risk-less rate. Both features are of interest and should ideally be combined: the quantitative constraint could be more powerful in generating spillovers to the real economy; the premium effect has the benefit of reflecting real world interest rate spreads. Aoki et al. (2004) use the BGG for the housing market in a simplified general equilibrium model. Christensen et al. (2007) estimate a DSGE model for Canada with borrowing constraints for both firms and households.

We show that the introduction of financial market frictions in the typical DSGE framework can provide important insights on the response of the economy to financial market shocks, although it need not alter dramatically the predictions on the response of the economy to non-financial shocks. Furthermore, our extensions allow us to explain the effects of non-financial shocks on important financial variables as external-finance premia, house prices and residential investment.

The paper proceeds as follows. Section 2 describes the model. Section 3 takes a general look at the data and underlying calibration and shock structure of the model. We then examine, in section 4, the Bayesian estimation of variants of the model. This is followed by an examination of the model properties: simulation exercises; stylized facts matching; conditional variance and historical decompositions. Finally, we conclude.

2 The model

The following sections describes the formal representation of the model. The Appendix provides a concise list of symbols and definitions of variables as well as the list of (non-linear) equations and functional forms that make up the model.

2.1 Households

The population consists of an infinite number of households, which mass is normalized to one. A fraction **p** are patient households, the rest being impatient. The former type is denoted with subscript 1, the second with subscript 2, if not otherwise stated.

2.1.1 Patient Households

Patient households solve the following program:

$$\max_{C_{1,t},\mathcal{H}_{1,t},I_t,B^*_{t+1},B_t,W_{1,t},D^H_t,D^B_t} \mathbb{E}_t \sum_{k=0}^{\infty} \left(\tilde{\beta}_1 G_C\right)^k U\left(C_{1,t+k} - \kappa \bar{C}_{1,t-1+k}, \mathcal{H}_{1,t+k}, N_{1,t+k}\right) \tag{1a}$$

subject to

$$
(1+\tau^C)C_{1,t}+\frac{P_{I,t}}{P_{C,t}}I_t+(\epsilon_t^{RP}R_t)^{-1}\frac{B_{t+1}}{P_{C,t}}+\left(R_t^*\left(1-\Gamma_{B^*}\left(S_{t+1}B_{t+1}^*\right)\right)\right)^{-1}\frac{S_t}{P_{C,t}}B_{t+1}^*+\frac{\Xi_t+\Phi_t}{P_{C,t}}=
$$

$$
\frac{B_t+S_tB_t^*}{P_{C,t}}-q_{h,t}\mathcal{H}_{1,t}+q_{h,t}\left(1-\delta_h\right)\mathcal{H}_{1,t-1}+\frac{\left(1+R_t^{DH}\right)D_{t-1}^H-D_t^H+\left(1+R_t^{DB}\right)D_{t-1}^B-D_t^B}{P_{C,t}}+\Omega_t
$$

$$
+\left(1-\tau^N-\tau^W\right)\frac{W_{1,t}}{P_{C,t}}N_{1,t}-\frac{Q_t}{P_{C,t}}\left(1-\delta\right)K_t+\frac{Q_t}{P_{C,t}}K_{t+1}+\left(1-\gamma_t\right)\left(T_t^{\gamma}-T_t^e\right) \tag{1b}
$$

and to

$$
K_{t+1}=(1-\delta)K_t+\epsilon_t^I\left(1-\Gamma_I\left(\frac{I_t}{I_{t-1}}\right)\right)I_t \tag{1c}
$$

where the instantaneous utility function is defined as

$$
U\left(C_{1,t}-\kappa C_{t-1},\mathcal{H}_t,N_{1,t}\right)\equiv\left(1-\frac{\kappa}{G_C}\right)\ln\left(C_{1,t}-\kappa\bar{C}_{1,t-1}\right)+j_{1,t}\ln\mathcal{H}_{1,t}-\frac{1}{1+\zeta}N_{1,t}^{1+\zeta}, \tag{2}
$$

and where δ is the capital depreciation rate, $\tilde{\beta}_1$ is the households time preference parameter, G_C is the long-run growth rate of consumption (identical to the long-run growth rate of technology), $C_{1,t}$ is a CES index of domestic and foreign goods, described later, $\bar{C}_{1,t}$ is the aggregate consumption of patient households (with habit-persistence parameter κ),[1] $\mathcal{H}_{1,t}$ is the stock of housing, $N_{1,t}$, is labor supply (with elasticity $\frac{1}{\zeta}$, I_t is investment, K_t is capital, B_t is domestic bonds, B_t^* is foreign bonds, S_t is the nominal exchange rate, D_t^H is deposits at the Building Societies, D_t^B is deposits at the Banks, Ξ_t and Φ_t are profits rebated by firms and banks to households including rent of land to residential investment, $W_{1,t}$ is nominal wages, $q_{h,t}$ is the price of houses in terms of households' consumption units, Γ_{B^*} is a premium paid on foreign bonds transactions (proportional to the size of foreign borrowing), Γ_I is the investment adjustment cost,[2] $P_{C,t}$ is the consumer price index, $P_{I,t}$ is the price of investment goods, R_t is the policy rate (return on domestic bonds), R_t^* is the return on foreign bonds, R_t^{DH} is the return on deposits at Building Societies, R_t^{DB} is the return on deposits at Banks, Q_t is the price of capital, τ^C is a consumption tax, τ^N is a labor income tax, τ^W is an additional payroll tax, T_t is a lump-sum tax,[3] ϵ_t^{RP} is a risk premium shock, ϵ_t^I is an investment specific shock, $j_{1,t}$ is a housing preference shock, γ_t is the entrepreneurs' survival shock discussed further below, T_t^{γ} is a transfer from exiting entrepreneurs to households, T_t^e is a transfer from households to new entrepreneurs (start-ups), finally Ω_t is a transfer to household consisting of the monitoring costs of the net worth of defaulting firms.[4]

[1] We assume that habit formation is "external" to the household but "internal" to the household type. Notice also that we are assuming log-preferences in consumption (cum habits) and in housing.

[2] Functional forms are presented in the Appendix.

[3] See also Christoffel et al. (2008) on assumptions concerning taxes.

[4] We can think of the default costs as a tax transferred to households.

First order conditions

For simplicity re-define $\beta_1 \equiv \tilde{\beta}_1 G_C$. The FOCs to the household problem are:

$$B_{t+1} : \lambda_t^{\mathbf{P}} = \beta_1 \mathbb{E}_t \left[\lambda_{t+1}^{\mathbf{P}} \frac{R_t}{\pi_{t+1}} \right] \tag{3a}$$

$$D_t^H : \lambda_t^{\mathbf{P}} = \beta_1 \mathbb{E}_t \left[\lambda_{t+1}^{\mathbf{P}} \frac{(1 + R_t^{DH})}{\pi_{t+1}} \right] \tag{3b}$$

$$D_t^B : \lambda_t^{\mathbf{P}} = \beta_1 \mathbb{E}_t \left[\lambda_{t+1}^{\mathbf{P}} \frac{(1 + R_t^{DB})}{\pi_{t+1}} \right] \tag{3c}$$

$$B_{t+1}^* : \lambda_t^{\mathbf{P}} \left(\left(1 - \Gamma_{B^*} \left(S_{t+1} B_{t+1}^* \right) \right) \right)^{-1} S_t = \beta_1 \mathbb{E}_t \left[\lambda_{t+1} \frac{S_{t+1} R_t^*}{\pi_{t+1}} \right] \tag{3d}$$

$$C_{1,t} : \lambda_t^{\mathbf{P}} = \left(1 - \frac{\kappa}{G_C} \right) \left(C_{1,t} - \kappa C_{t-1} \right)^{-1} \tag{3e}$$

$$\mathcal{H}_t : j_{1,t}^H \mathcal{H}_{1,t}^{-1} + \beta_1 \mathbb{E}_t \left[\lambda_{t+1}^{\mathbf{P}} (1 - \delta_h) q_{h,t+1} \right] = \lambda_t^{\mathbf{P}} q_{h,t} \tag{3f}$$

$$\begin{aligned} I_t : \lambda_t^{\mathbf{P}} \frac{P_{I,t}}{P_{C,t}} = \frac{Q_t}{P_{C,t}} \lambda_t^{\mathbf{P}} \epsilon_t^I \left(\left(1 - \Gamma_I \left(\frac{I_t}{I_{t-1}} \right) \right) - \Gamma_I'(\cdot) \frac{I_t}{I_{t-1}} \right) + \\ \mathbb{E}_t \left[\frac{Q_{t+1}}{P_{C,t+1}} \beta_1 \lambda_{t+1}^{\mathbf{P}} \epsilon_{t+1}^I \left(\Gamma_I' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right) \right], \end{aligned} \tag{3g}$$

where $\lambda^{\mathbf{P}}$ is the Lagrange multiplier associated to the budget constraint of the patient household.

Wage setting

The wage setting problem is identical to the one in the NAWM (Christoffel et al., 2008). In particular we (implicitly) assume that households of the same type trade in state contingent assets that insure them from idiosyncratic income fluctuations due to wage stickiness. Furthermore, the marginal disutility of labor is independent of the investment decisions. These two facts allow us to aggregate across wage setters in the standard way.

The details are discussed further below.

2.1.2 Impatient Households

The Impatient Households solve the following program:

$$\max_{C_{2,t}, \mathcal{H}_{2,t}, W_{2,t}, N_{H,t}, B_t^H} \mathbb{E}_t \sum_{k=0}^{\infty} \left(\tilde{\beta}_2 G_C \right)^k U \left(C_{2,t+k} - \kappa \bar{C}_{2,t-1+k}, \mathcal{H}_{2,t+k}, N_{2,t+k}, N_{H,t+k} \right) \tag{4a}$$

subject to

$$\begin{aligned} 0 = -(1 + \tau^C) P_{C,t} C_{2,t} + B_t^H - q_{h,t} \mathcal{H}_{2,t} + q_{h,t} \left(1 - \delta_h \right) \mathcal{H}_{2,t-1} + \\ - \left(1 + R_t^{DH} \right) B_{t-1}^H + \left(1 - \tau^N - \tau^W \right) W_{2,t} N_{2,t} + \left(1 - \tau^N - \tau^W \right) W_{H,t} N_{H,t} \end{aligned} \tag{4b}$$

and to

$$0 \leq m_t^B \mathbb{E}_t \left(\frac{q_{h,t+1}\pi_{t+1}\mathcal{H}_{2,t}}{(1+R_t^{DH})} \right) - \frac{B_t^H}{P_{C,t}} \tag{4c}$$

where the intraperiod utility function is

$$U\left(C_{2,t} - \kappa \bar{C}_{2,t-1}, \mathcal{H}_{2,t}, N_{2,t}, N_{H,t}\right) = \tag{5}$$

$$\left(1 - \frac{\kappa}{G_C}\right) \ln\left(C_{2,t} - \kappa \bar{C}_{2,t-1}\right) + j_{2,t} \ln \mathcal{H}_{2,t} - \frac{1}{1+\zeta_2} N_{2,t}^{1+\zeta_2} - \frac{1}{1+\zeta_H} N_{H,t}^{1+\zeta_H}$$

where $N_{H,t}$ is labor supply to the construction sector, π_t is the consumer price inflation rate and m^B is the loan-to-value ratio parameter.[5] The meaning of the remaining symbols is analogous to that given to the patient household's variables. Notice that while we assume sticky prices in the non-construction sector, the housing sector is assumed to have a competitive (flexible-wage) labor market.

First order conditions

Define $\beta_2 = G_C \tilde{\beta}_2$. Then the FOCs are:

$$B_t^H : \lambda_t - \beta_2 \mathbb{E}_t \left[\lambda_{t+1} \frac{\left(1 + R_t^{DH}\right)}{\pi_{t+1}} \right] - \lambda_t^B = 0 \tag{6a}$$

$$C_{2,t} : -\lambda_t (1 + \tau^C) + \left(1 - \frac{\kappa}{G_C}\right) \left(C_{2,t} - \kappa \bar{C}_{2,t-1}\right)^{-1} = 0 \tag{6b}$$

$$\mathcal{H}_t : -\lambda_t q_{h,t} + j_{2,t}^H \mathcal{H}_{2,t}^{-1} + \beta_2 \mathbb{E}_t \left[\lambda_{t+1}(1 - \delta_h) q_{h,t+1}\right] + m_t^B \lambda_t^B \mathbb{E}_t \left(\frac{q_{h,t+1}\pi_{t+1}}{(1+R_t^{DH})} \right) = 0 \tag{6c}$$

Notice that since the collateral constraint is assumed to be always binding the Lagrange multiplier associated with the borrowing constraint λ_t^B will be positive.

2.1.3 Wage setting

We assume that wages are set à la Calvo (1983) with indexation. The probability of not resetting is denoted by $\xi_{W,p}$.

Patient households

Patient households solve the following problem:

$$\max_{W_{1,i,t}} \mathbb{E}_t \sum_{s=t}^{\infty} \left(\xi_{W,p} \bar{\beta}_1\right)^{s-t} \left[\lambda_s^{\mathbf{p}} \left(1 - \tau^N - \tau^W\right) \frac{W_{1,i,t} g_{z,t-1|s-1} \Pi_{t-1|s-1}^{\dagger}}{P_{C,s}} N_{1,t} - \frac{1}{1+\zeta} N_{1,i,t}^{1+\zeta} \right]$$

where

$$\Pi_{t-1|s-1}^{\dagger} \equiv \begin{cases} \prod_{j=t}^{s} \pi_{j-1}^{\chi_{W,p}} \bar{\pi}_j^{1-\chi_{W,p}} & t < s \\ 1 & t = s \end{cases}$$

[5]Notice that the bank loans (B^H) are positive, hence the sign in the constraint.

$\bar{\pi}$ being the steady-state inflation rate and $g_{z,t|s} \equiv \frac{z_s}{z_t}$ being the cumulative growth rate of labor-augmenting technology (discussed further below).[6]

Notice that demand for the i^{th} labor type is

$$N_{1,i,s} = \mathbf{p}^{-1} \left(g_{z,t-1|s-1} \Pi^{\dagger}_{t-1|s-1} \frac{W_{1,i,t}}{W_{1,t}} \frac{W_{1,t}}{W_{1,s}} \right)^{-\theta_{W,1}} \mathcal{N}_{1,s} \tag{7}$$

where $\mathcal{N}_{1,t}$ is the total demand for (patient) labor. The solution to this problem yields a relative price

$$\left(\bar{W}_{1,i,t}\right)^{\theta_{W,1}\zeta_1+1} \bar{W}_{1,t} \equiv \left(\frac{W_{1,i,t}}{W_{1,t}}\right)^{\theta_{W,1}\zeta_1+1} \bar{W}_{1,t} = \varphi_t^{W,1} \frac{\tilde{\mathbb{Q}}^{W,1}_{1,t}}{\tilde{\mathbb{Q}}^{W,1}_{2,t}} \tag{8}$$

where (a bar above a variable except inflation means "real", for inflation means "central bank target")

$$\begin{aligned} \tilde{\mathbb{Q}}^{W,1}_{1,t} &= \left(\mathbf{p}^{-1}\mathcal{N}_{1,t}\right)^{\zeta_1+1} \\ &+ \xi_{W,p}\mathbb{E}_t\beta_1 \left(g_{z,t|t-1} \pi_t^{\chi_{W,p}} \bar{\pi}_{t+1}^{1-\chi_{W,p}} \right)^{-(\zeta_1+1)\theta_{W,1}} \pi_{t+1}^{(\zeta_1+1)\theta_{W,1}} \\ &\times \left(\frac{\bar{W}_{1,t+1}}{\bar{W}_{1,t}}\right)^{(\zeta_1+1)\theta_t^{W,1}} \tilde{\mathbb{Q}}^{W,1}_{1,t+1}, \end{aligned} \tag{9}$$

and

$$\begin{aligned} \tilde{\mathbb{Q}}^{W,1}_{2,t} &= \left(\mathbf{p}^{-1}\mathcal{N}_{1,t}\right) \lambda_t^{\mathbf{p}} \left(1 - \tau_s^N - \tau_s^W\right) \\ &+ \xi_{W,p}\mathbb{E}_t\beta_1 \left(g_{z,t|t-1} \pi_t^{\chi_{W,p}} \bar{\pi}_{t+1}^{1-\chi_{W,p}} \right)^{1-\theta_{W,1}} \pi_{t+1}^{\theta_{W,1}-1} \\ &\times \left(\frac{\bar{W}_{1,t+1}}{\bar{W}_{1,t}}\right)^{\theta_t^{W,1}} \tilde{\mathbb{Q}}^{W,1}_{2,t+1}, \end{aligned} \tag{10}$$

$\varphi_t^{W,1} = \frac{\theta_t^{W,1}}{\theta_t^{W,1}-1}$ is the firm's mark-up.

The real wage index of labor-type 1 is

$$\bar{W}_{1,t} \equiv \frac{W_{1,t}}{P_{C,t}} = \left[(1-\xi_{W,p}) \left(\bar{W}_{W,1,t}\bar{W}_{1,t}\right)^{1-\theta_{W,1}} + \xi_{W,p} \left(\frac{\left(g_{z,t|t-1} \pi_{t-1}^{\chi_{W,p}} \bar{\pi}_t^{1-\chi_{W,p}} \right)}{\pi_t} \bar{W}_{1,t-1} \right)^{1-\theta_{W,1}} \right]^{\frac{1}{1-\theta_{W,1}}}.$$

2.1.4 Impatient households

The equations are identical to those of the patient household. We omit them here, though they are reported in the list of non-linear equations further below.

[6] Notice that in order to have a balanced-growth path, all variables measured in consumption units will grow at a common trend.

2.2 Home Intermediate-goods Firms

Firms produce using the following production function

$$Y_{f,t} = \max \left[\varepsilon_t z_t^{\alpha_{N1}+\alpha_{N2}} \left(K_{f,t}^s\right)^{\alpha_K} \left(\mathcal{N}_{1,t}\right)^{\alpha_{N1}} \left(\mathcal{N}_{2,t}\right)^{\alpha_{N2}} - z_t\psi,\, 0 \right] \tag{11}$$

where $\alpha_K + \alpha_{N1} + \alpha_{N2} = 1$ and where

$$\mathcal{N}_{1,t} = \left(\int_0^{\mathbf{p}} \left(N_{1,t}^i\right)^{\frac{1}{\varphi_t^W}} \mathrm{di} \right)^{\varphi_t^W} \tag{12}$$

and

$$\mathcal{N}_{2,t} = \left(\int_{\mathbf{p}}^1 \left(N_{2,t}^i\right)^{\frac{1}{\varphi_t^W}} \mathrm{di} \right)^{\varphi_t^W} \tag{13}$$

where $\mathcal{N}_{i,t}$ are labor-input aggregators, ε_t is a transitory productivity shock, z_t is a permanent labor-augmenting technology shock with (possibly stochastic) trend and where ψ is a fixed cost parameter.

2.2.1 Conditional factor demands

Cost minimization yield the usual factor demands

$$K_{f,t}^s : \alpha_K \frac{Y_{f,t} + z_t\psi}{K_{f,t}^s} MC_{f,t} = r_t^{K\alpha_K} \tag{14a}$$

$$\mathcal{N}_{1,t} : \alpha_{N1} \frac{Y_{f,t} + z_t\psi}{\mathcal{N}_{1,t}} MC_{f,t} = \left(1 + \tau^{Wf}\right) W_{1,t} \tag{14b}$$

$$\mathcal{N}_{2,t} : \alpha_{N2} \frac{Y_{f,t} + z_t\psi}{\mathcal{N}_{2,t}} MC_{f,t} = \left(1 + \tau^{Wf}\right) W_{2,t} \tag{14c}$$

where τ^{Wf} is the tax rate on labor input and where marginal costs are

$$MC_t = \frac{r_t^{K\alpha_K} (1 + \tau^{Wf})^{\alpha_{N1}+\alpha_{N2}} W_{1,t}^{\alpha_{N1}} W_{2,t}^{\alpha_{N2}}}{\left(\varepsilon_t z_t^{\alpha_{N1}+\alpha_{N2}} \alpha_K^{\alpha_K} \alpha_{N1}^{\alpha_{N1}} \alpha_{N2}^{\alpha_{N2}}\right)} \tag{15}$$

2.2.2 Price setting

Firms producing final goods set prices only at random intervals of time. These firms charge different prices at home and abroad. In each quarter a fraction ξ_H of firms sells goods at home at the price posted in the previous quarter, after updating it in part to the sectoral inflation observed in the past quarter and in part to trend inflation (with indexation parameter χ_H). The remaining firms are able to post the optimal price.

With probability ξ_X the same story holds for prices charged to foreign importers (with indexation parameter χ_X).

Firms are owned by the patient domestic households. Therefore, each firm chooses the optimal price in order to maximize the expected discounted dividends accruing to households.

In what follows we define the cumulative inflation rate between period t and s of price j by $\pi_{j,t|s}$, where for convenience $\pi_{j,t|t+1} \equiv \pi_{j,t+1}$.

Domestic prices

Firm f chooses its price ($P_{H,f,t}$) by solving the following profit-maximization problem:

$$\max_{P_{H,f,t}} \mathbb{E}_t \sum_{s=t}^{\infty} (\xi_H)^{s-t} \bar{R}_{s,t} \left[\frac{P_{H,f,t} \left(\pi_{t-1|s-1}\right)^{1-\chi_H} \left(\pi_{H,t-1|s-1}\right)^{\chi_H}}{P_{C,s}} Y_{f,s} - \frac{TC_s}{P_{C,s}} \right]$$

where TC denotes total costs of production and $\bar{R}_{s,t} \equiv \beta_1^{s-t} \frac{\lambda_s^{\mathrm{P}}}{\lambda_t^{\mathrm{P}}}$ is the patient household nominal discount factor between period s and $t \leq s$.

Notice that the demand for this type of final goods is given by

$$Y_{f,s} = \left(\left(\bar{\pi}_{t|s}\right)^{1-\chi_H} \left(\pi_{H,t-1|s-1}\right)^{\chi_H} \frac{P_{H,f,t}}{P_{H,t}} \right)^{-\theta_H} Q_t^C, \tag{16}$$

where Q_t^C denotes domestic aggregate demand for domestic intermediate goods.

The solution to this problem yields a relative price

$$\bar{P}_{H,f,t} = \varphi_t^H \frac{\tilde{\mathbb{Q}}_{1,t}}{\tilde{\mathbb{Q}}_{2,t}}$$

where

$$\tilde{\mathbb{Q}}_{1,t} = (1 - \xi_H \beta_1) \overline{mc}_t \left(\bar{P}_{H,t}\right)^{\theta_t^H} Q_t^C + \xi_H \mathbb{E}_t \overline{R}_{t+1,t} \left(\pi_{H,t}{}^{\chi_H} \bar{\pi}_{t+1}^{1-\chi_H}\right)^{-\theta_H} \pi_{t+1}^{\theta_H} \tilde{\mathbb{Q}}_{1,t+1},$$

$$\tilde{\mathbb{Q}}_{2,t} = (1 - \xi_H \beta_1) \left(\bar{P}_{H,t}\right)^{\theta_t^H} Q_t^C + \xi_H \mathbb{E}_t \bar{R}_{t+1,t} \left(\pi_{H,t}{}^{\chi_H} \bar{\pi}_{t+1}^{1-\chi_H}\right)^{1-\theta_H} \pi_{t+1}^{\theta_H - 1} \tilde{\mathbb{Q}}_{2,t+1},$$

$\varphi_t^H = \frac{\theta_t^H}{\theta_t^H - 1}$ is the firm's mark-up.

The (real) producer price index of intermediate goods is

$$\bar{P}_{H,t} \equiv \frac{P_{H,t}}{P_{C,t}} = \left[(1 - \xi_H) \left(\bar{P}_{H,f,t}\right)^{1-\theta_t^H} + \xi_H \left(\frac{\left(\bar{\pi}_t\right)^{1-\chi_H} \left(\pi_{t-1}^H\right)^{\chi_H}}{\pi_t} \bar{P}_{H,t-1} \right)^{1-\theta_t^H} \right]^{\frac{1}{1-\theta_t^H}}.$$

Export prices

Exporting firms choose prices in order to solve the following profit-maximization problem:

$$\max_{P_{X,f,t}} \mathbb{E}_t \sum_{s=t}^{\infty} (\xi_X)^{s-t} \bar{R}_{s,t} \left[\frac{P_{X,f,t} \left(\pi_{t-1|s-1}\right)^{1-\chi_X} \left(\pi_{X,t-1|s-1}\right)^{\chi_X}}{P_{C,s}} Y_{f,s} - \frac{TC_s^X}{P_{C,s}} \right]$$

where TC denotes total costs of production and $\bar{R}_{s,t} \equiv \beta_1^{s-t} \frac{\lambda_s^{\mathrm{P}}}{\lambda_t^{\mathrm{P}}}$ is the patient household nominal discount factor between period s and $t \leq s$.

Notice that export demand faced by each individual firm is

$$Y_{X,f,s} = \left(\left(\bar{\pi}_{t|s}\right)^{1-\chi_X} \left(\pi_{X,t-1|s-1}\right)^{\chi_X} \frac{P_{X,f,t}}{P_{X,t}} \right)^{-\theta_X} X_t \tag{17}$$

The solution to this problem yields a relative price

$$\bar{P}_{X,f,t} = \varphi_t^X \frac{\tilde{\mathbb{Q}}_{1,t}^X}{\tilde{\mathbb{Q}}_{2,t}^X}$$

where X_t denotes aggregate demand for exports and where

$$\tilde{\mathbb{Q}}_{1,t}^X = (1 - \xi_X \beta_1)\overline{mc}_t \left(\bar{P}_{X,t}\right)^{\theta_t^X} X_t + \xi_X \mathbb{E}_t \overline{R}_{t+1,t} \left(\pi_{X,t}{}^{\chi_X} \bar{\pi}_{t+1}^{1-\chi_X}\right)^{-\theta_X} \pi_{t+1}^{\theta_X} \tilde{\mathbb{Q}}_{1,t+1}^X,$$

$$\tilde{\mathbb{Q}}_{2,t}^X = (1 - \xi_X \beta_1) \left(\bar{P}_{X,t}\right)^{\theta_t^X} X_t + \xi_X \mathbb{E}_t \bar{R}_{t+1,t} \left(\pi_{X,t}{}^{\chi_X} \bar{\pi}_{t+1}^{1-\chi_X}\right)^{1-\theta_X} \pi_{t+1}^{\theta_X - 1} \tilde{\mathbb{Q}}_{2,t+1}^X,$$

where $\varphi_t^X = \frac{\theta_t^X}{\theta_t^X - 1}$ is the firm's mark-up and $\overline{mc}_t$ is the marginal cost in consumption units.

The (real) producer price index of intermediate goods is

$$\bar{P}_{X,t} \equiv \frac{P_{X,t}}{P_{C,t}} = \left[(1 - \xi_X)\left(\bar{P}_{X,f,t}\right)^{1-\theta_t^X} + \xi_X \left(\frac{\left(\bar{\pi}_t\right)^{1-\chi_X}\left(\pi_{t-1}^X\right)^{\chi_X}}{\pi_t} \bar{P}_{X,t-1}\right)^{1-\theta_t^X}\right]^{\frac{1}{1-\theta_t^X}}.$$

Import prices

Foreign exporters set export prices in home-country currency, $P_{IM,f,t}$, to solve the following problem

$$\max_{P_{IM,f,t}} \mathbb{E}_t \sum_{s=t}^{\infty} (\xi_{IM})^{s-t} \bar{R}_{s,t}^* \left[\frac{P_{IM,f,t}\left(\bar{\pi}_{t-1|s-1}^*\right)^{1-\chi_{IM}}\left(\pi_{IM,t-1|s-1}\right)^{\chi_{IM}}}{S_s P_{C,s}^*} Y_{f,s} - \frac{TC_s^{IM}}{P_{C,s}^*}\right]$$

where TC^{IM} denotes total costs of production in home-country currency and $\bar{R}_{s,t}^* \equiv \beta_1^{s-t}\frac{\lambda_s^*}{\lambda_t^*}$ is the foreign household nominal discount factor between period s and $t \leq s$.

Notice that export demand faced by each individual firm is

$$Y_{IM,f,s} = \left(\left(\bar{\pi}_{t|s}^*\right)^{1-\chi_{IM}}\left(\pi_{IM,t-1|s-1}\right)^{\chi_{IM}} \frac{P_{IM,f,t}}{P_{IM,t}}\right)^{-\theta_{IM}} IM_t \tag{18}$$

The solution to this problem yields a relative price (expressed here in terms of home consumption basket)

$$\bar{P}_{IM,f,t} \equiv \frac{P_{IM,f,t}}{P_{C,t}} = \varphi_t^{IM} \frac{\tilde{\mathbb{Q}}_{1,t}^{IM}}{\tilde{\mathbb{Q}}_{2,t}^{IM}}$$

where

$$\begin{aligned}\tilde{\mathbb{Q}}_{1,t}^{IM} &= (1 - \xi_{IM}\beta_1) mc_t^F \left(\bar{P}_{IM,t}\right)^{\theta_t^{IM}} IM_t \\ &\quad + \xi_{IM}\mathbb{E}_t \bar{R}_{t+1,t}^* \left(\pi_{IM,t}{}^{\chi_{IM}}\left(\bar{\pi}_{t+1}^*\right)^{1-\chi_{IM}}\right)^{-\theta_{IM}} \left(\pi_{t+1}\right)^{\theta_{IM}} \tilde{\mathbb{Q}}_{1,t+1}^{IM},\end{aligned} \tag{19}$$

$$\tilde{\mathbb{Q}}^{IM}_{2,t} = (1 - \xi_{IM}\beta_1)\frac{P_{C,t}}{S_t P^*_{C,t}}\left(\bar{P}_{IM,t}\right)^{\theta^{IM}_t} IM_t$$
$$+ \xi_{IM}\mathbb{E}_t \bar{R}^*_{t+1,t}\left(\pi_{IM,t}{}^{\chi_{IM}}\left(\bar{\pi}^*_{t+1}\right)^{1-\chi_{IM}}\right)^{1-\theta_{IM}}\left(\pi_{t+1}\right)^{\theta_{IM}-1}\tilde{\mathbb{Q}}^{IM}_{2,t+1}, \tag{20}$$

$\varphi^{IM}_t = \frac{\theta^{IM}_t}{\theta^{IM}_t - 1}$ is the firm's mark-up. IM_t denotes aggregate demand for exports and $mc^F$is the real foreign marginal cost. Following Christoffel et al. (2008) we assume that the foreign marginal cost is a log-linear function of the foreign relative price of oil (with elasticity denoted by ω^*).

The (real) producer price index of intermediate goods is

$$\bar{P}_{IM,t} \equiv \frac{P_{IM,t}}{P_{C,t}} = \left[(1-\xi_{IM})\left(\bar{P}_{IM,f,t}\right)^{1-\theta^{IM}_t} + \xi_{IM}\left(\frac{\left(\bar{\pi}^*_t\right)^{1-\chi_{IM}}\left(\pi_{IM,t-1}\right)^{\chi_{IM}}}{\pi^*_t}\bar{P}_{IM,t-1}\right)^{1-\theta^{IM}_t}\right]^{\frac{1}{1-\theta^{IM}_t}}.$$

2.3 Final-goods firms

Finally we have a further set of firms that assemble the different types of goods and sell them to different types of agents.

2.3.1 Non-tradable consumer-final-goods producers

These firms put together domestically produced goods with imported goods and sell them at competitive prices to final consumers.

$$Q^C_t = \left(\nu_C^{\frac{1}{\mu_C}}\left(H^C_t\right)^{1-\frac{1}{\mu_C}} + (1-\nu_C)^{\frac{1}{\mu_C}}\left(\overline{IM}^C_t\right)^{1-\frac{1}{\mu_C}}\right) \tag{21}$$

where H^C_t is demand for goods produced at home, IM^C_t is demand for imported consumption goods and where

$$\overline{IM}^C_t \equiv \left(1 - \Gamma_{IM^C}\left(\frac{IM^C_t}{Q^C_t}\epsilon^{IM}_t\right)IM^C_t\right)$$

which includes an import cost function denoted by Γ_{IM^C}, and where

$$H^C_t \equiv \left(\int_0^1 \left(H^C_{i,t}\right)^{\frac{1}{\varphi^H_t}} \mathrm{di}\right)^{\varphi^H_t}$$

$$IM^C_t \equiv \left(\int_0^1 \left(IM^C_{i,t}\right)^{\frac{1}{\varphi^*_t}} \mathrm{di}\right)^{\varphi^*_t}$$

2.3.2 Non-tradable productive-capital investment-goods producers

Same as before after replacing superscript C with I (including same elasticity of substitution). In particular the demand for intermediate goods will be H^I_t (domestic) and IM^I_t (imported).

2.3.3 Non-tradable public-consumption-goods producers

Same as before except that only domestic goods are used (with same elasticity). In particular the demand for inputs will be H_t^G.

2.3.4 Non-tradable housing-investment-goods producers

The housing investment sector produces new housing-units that augment the existing stock of housing and replaces the depreciated hosing stock. We assume that the capital used in the housing-investment sector is the same as that used in the intermediate-goods sector.

$$IH_t = \varepsilon_{H,t} z_{h,t} \left(K_t^H\right)^{\omega_K} \left(z_t(1-\mathbf{p})N_{H,t}\right)^{\omega_N} \left(H_t^H\right)^{\omega_H} \left(z_t land_t\right)^{\omega_L} \tag{22}$$

where $\varepsilon_{H,t}$ is a temporary shock in the housing sector. Notice that the permanent productivity shock ($z_{h,t}$) can differ from the one in the intermediate goods sector (as in Iacoviello and Neri (2010)).

Aggregate housing evolves according to[7]

$$\mathcal{H}_t = (1-\delta_h)\mathcal{H}_{t-1} + IH_t \tag{23}$$

where δ_h is the depreciation rate of housing.

Given that there is no differentiation between capital used in construction and capital used in production, the possible trend in the construction sector does not affect capital, labor or intermediate inputs in that sector.

Detrended production is then

$$ih_t \equiv \frac{IH_t}{z_{h,t} z_t^{\omega_N+\omega_L}} = \varepsilon_{H,t} \left(g_z k_t^H\right)^{\omega_K} \left((1-\mathbf{p})N_{H,t}\right)^{\omega_N} \left(h_t^H\right)^{\omega_H} \left(land_t\right)^{\omega_L}$$

Conditional factor demands

Cost minimization implies the following demands for factors of production:

$$r_t^K = MC_{IH,t}\omega_K \frac{IH_t}{K_t^H} \tag{24a}$$

$$\left(1+\tau^{Wf}\right) W_{H,t} = MC_{IH,t}\omega_N \frac{IH_t}{N_{H,t}} \tag{24b}$$

$$P_{H,t} = MC_{IH,t}\omega_H \frac{IH_t}{H_t^H} \tag{24c}$$

$$R_t^L = MC_{IH,t}\omega_L \frac{IH_t}{land_t} \tag{24d}$$

[7] We experiment also with the case of investment adjustment costs in the housing sector. The adjustment costs are of the same type used for business capital and are not reported here for conciseness.

where R_t^L is the return to land. By replacing these conditions into the production function (22) we obtain an expression for the marginal cost in the residential sector

$$MC_{IH,t} = \frac{\left(r_t^K\right)^{\omega_K} W_{N,t}^{\omega_N} P_{H,t}^{\omega_H} \left(R_t^L\right)^{\omega_L}}{\varepsilon_t^H z_t^{\omega_N+\omega_L} z_{h,t} \omega_K^{\omega_K} \omega_N^{\omega_N} \omega_H^{\omega_H} \omega_L^{\omega_L}} \tag{25}$$

or, by the normalization $land_t = 1$ have

$$MC_{IH,t} = \left(\frac{\left(r_t^K\right)^{\omega_K} \left(1+\tau^{Wf}\right)^{\omega_N} W_{N,t}^{\omega_N} P_{H,t}^{\omega_H}}{\varepsilon_t^H z_t^{\omega_N+\omega_L} z_{h,t} \omega_K^{\omega_K} \omega_N^{\omega_N} \omega_H^{\omega_H}} IH^{\omega_L} \right)^{\frac{1}{1-\omega_L}}$$

The housing-investment market is competitive, which implies

$$q_{h,t} \equiv \frac{Q_{h,t}}{P_{C,t}} = MC_{IH,t}$$

2.4 Equilibrium in the goods markets

Domestic demand for home intermediate-goods

$$H_{f,t} = H_{f,t}^C + H_{f,t}^I + H_{f,t}^G + H_{f,t}^H = \left(\frac{P_{H,f,t}}{P_{H,t}}\right)^{\frac{\varphi_t^H}{1-\varphi_t^H}} H_t$$

Domestic demand for foreign intermediate-goods

$$IM_{f^*,t} = IM_{f^*,t}^C + IM_{f^*,t}^I = \left(\frac{P_{IM,f^*,t}}{P_{IM,t}}\right)^{\frac{\varphi_t^*}{1-\varphi_t^*}} IM_t$$

2.5 Entrepreneurs

Entrepreneurs are risk-neutral agents. In equilibrium they all take the same decision regarding the purchase and supply of capital.[8]

At the end of period t entrepreneurs purchase the new capital stock that is produced by households for production in period t+1. This purchase is financed with the entrepreneurs net worth and by bank loans:

$$q_{K,t} K_{t+1} = N_{t+1} + \frac{D_t^B}{P_t} \tag{26}$$

The amount of capital that the entrepreneurs can bring to the market is subject to idiosyncratic shocks ϖ_t observable only by each individual entrepreneur.

Furthermore, each entrepreneur decides about the amount of capital that can be used in each period. Varying the utilization of capital is costly. The amount of effective capital brought to the market by each entrepreneur is

$$K_t^e \equiv u_t \varpi_t K_t, \tag{27}$$

[8]See Bernanke et al. (1999). Our specification of the entrepreneurs and of their debt contract is similar to Christiano et al. (2008), Gertler et al. (2007), Aoki et al. (2004) and Christensen and Dib (2008).

where u_t is the degree of capital utilization. We define the function $a(u_t)$ as the costly-capital-utilization function such that $a' > 0$, $a'' > 0$ and $a(1) = 1$. The capital utilization cost could be expressed in terms of final consumption/investment goods or in terms of energy (oil) as in Finn (1995). The total cost of using capital is then

$$P_t^a a(u_t)\varpi_t K_t \tag{28}$$

where P_t^a is the price of the goods used up in changing utilization (relative to the numeraire).

The efficient choice of utilization rate sets the (real) marginal return to utilization r_t^K equal to the marginal cost of utilization, i.e.

$$r_t^K = a'(u_t) P_t^a \tag{29}$$

At the end of period t entrepreneurs sell the un-depreciated part of capital to households and extinguish their debt with the bank. Those entrepreneurs whose net worth is sufficient to pay the interest and principal to banks will do so. The others will have all their remaining net worth seized by the bank.

The bank faces monitoring costs, so that a fraction μ_B of the net worth of the insolvent entrepreneurs is consumed (e.g. legal costs etc). We will need this expression once aggregating the resource constraint in subsection A of the appendix.

Then a fraction $1 - \gamma_t$ of entrepreneurs exits the market. Their net worth is transferred lump-sum to the patient households. At the same time a fraction $1 - \gamma_t$ of new entrepreneurs enters the market with a small endowment T_t^e paid to them by the patient households.

Define as R_t^{NK} the entrepreneurs' cost of borrowing. Then the efficient capital choice for the entrepreneurs requires that (in real terms)

$$\mathbb{E}_t\left[R_{t+1}^K\right] \equiv \mathbb{E}_t\left[\frac{\left((1-\tau_{t+1}^k)\left(r_{t+1}^k u_{t+1} - a(u_{t+1})\right) + (1-\delta)q_{k,t+1}\right) + \tau_{t+1}^k \delta q_{k,t}}{q_{k,t}}\right] \tag{30}$$

which amounts to the entrepreneurs expected real gross return on capital (assuming the cost of utilization is in terms of final consumption goods).

We don't specify the details of the debt contract of the entrepreneurs and the related agency problems, instead, following e.g. Gertler et al. (2007) we assume that the external finance premium over the risk free rate (opportunity cost of funds), $\chi_t(\cdot)$, is an increasing function of the aggregate entrepreneurs leverage ratio, i.e.

$$\chi_t \equiv \chi\left(\frac{\frac{D_t^B}{P_t}}{N_{t+1}}; \epsilon_t^\chi\right) \tag{31}$$

where $\chi' > 0$, $\chi(0) = 1$ and $\chi(\infty) = \infty$

The variable ϵ_t^χ is a stochastic shock that summarizes exogenous variations of the premium (e.g. changes in the distribution of the idiosyncratic shocks).

At the end of period t, entrepreneurs have to decide how much capital they want to purchase from households, given their net worth the expected return on capital and the cost of external finance. Optimality requires that the expected return to the entrepreneur (equation (30)) equals the expected cost of borrowing, i.e.

$$\mathbb{E}_t\left[R_{t+1}^K\right] = (\chi_t(\cdot))\mathbb{E}_t\left[\frac{\left(1+R_t^{DB}\right)}{\pi_{t+1}}\right] \tag{32}$$

At the end of period t, the net worth of the surviving entrepreneurs plus the wealth of the $1-\gamma_t$ newborn entrepreneurs amounts to

$$N_{t+1} = \gamma_t\left\{R_t^K q_{K,t-1}K_t - \left[\chi_{t-1}(\cdot)\frac{\left(1+R_{t-1}^{DB}\right)}{\pi_t}\right]\frac{D_{t-1}}{P_{t-1}}\right\} + T_t^e \tag{33}$$

The default cost is given by

$$\Omega_t = \left(R_{t-1}^K - \frac{R_{t-1}}{\pi_t}\right)(q_{K,t-1}K_t - N_t)$$

2.6 Banks

2.6.1 Commercial banks

There is a large number of identical banks operating in perfectly competitive markets. At the beginning of period t, after the shocks have been realized, these banks receive deposits D_t^B from $\mathbf{p}$ households, pay back the deposits lodged in period t-1, D_{t-1}^B, together with the interest R_{t-1}^{DB}, lend D_t^B to each entrepreneur and receive the principal and interest from the entrepreneurs who borrowed in period t-1.

2.6.2 Building societies

These banks intermediate between patient and impatient households. Equilibrium requires that $\mathbf{p}D_t^H = -(1-\mathbf{p})B_t^H$.

2.7 The Government Budget Constraint

The government budget constraint implies:

$$\begin{aligned} P_{G,t}G_t + B_t = {} & \tau^C P_{C,t}C_t + \left(\tau^N + \tau^W + \tau^{Wf}\right)\left(\mathbf{p}W_{1,t}\mathcal{N}_{1,t} + (1-\mathbf{p})\left(W_{2,t}\mathcal{N}_{2,t} + W_{H,t}N_{H,t}\right)\right) \\ & + \tau^K P_{C,t}\left(\left(r_t^K u_t - \delta\right)q_{K,t} - (a(u_t))\right)K_t + \tau^D R_t^{DB}\left(D_t^B + D_t^{BH}\right) + T_t + R_t^{-1}B_{t+1} \end{aligned} \tag{34}$$

We experiment with the case of the default losses and the transfer from the exiting entrepreneurs to be transferred to the government. The benchmark case is that the default losses are resources demanded to the domestic intermediate sector (same type of goods demanded by the government).

2.8 Monetary Policy

Monetary policy follows the same specification of the rule used in Christoffel et al. (2008), i.e. (in log-terms)

$$R_t = \lambda_R R_{t-1} + (1 - \lambda_R)\lambda_\pi (\pi_{t-1} - \pi) + \lambda_{\Delta\Pi} (\pi_t - \pi_{t-1}) + \lambda_{\Delta Y} (Y_t - Y_{t-1}) + \epsilon_t^R \quad (35)$$

where ϵ_t^R is assumed to be an *iid* shock.

3 Aspects of the Data

In this section we present the data used in the estimation of the model. We also discuss how some of the parameters of the model are calibrated.

3.1 Data

In estimating the model, we used some basic euro area times series taken from the current vintage of the Area Wide database (updated from Fagan et al. (2001)) as well as some separate financial series. The former group comprises: real GDP (Y); total employment (E); private consumption (C); compensation per head (W); total investment (I); euro area policy rate (nominal interest rate R); government consumption (G); nominal effective exchange rate (S); extra-euro area exports (X); foreign demand (Y^*)†; extra-euro area imports (IM); foreign prices (P*Y)†; GDP deflator (PY); foreign interest rate (R*)†; consumption deflator (PC); competitors' export prices (P^{CX})†; extra-euro area import deflator (P_{IM}); oil prices (P_O)†.[9]

The extra financial variables used as observables in the model are:

- Residential investment (Log Differences)
- The external finance premium: The difference between the rate on MFIs loans to NFCs of maturity up to one year and the policy rate.
- House prices (Log Differences)

The series on residential investment is constructed courtesy of the New MCM model, see Dieppe et al. (2011), and weights the corresponding country data of Spain, Italy, Netherlands, France and Germany, with fixed GDP weights.

The short rate on MFI loans to firms is derived from in-house country aggregated sources. The policy rate is the euribor taken from the AWM database.

House prices are taken from historical OECD sources and again are aggregated to a euro-area aggregate using fixed country weights.

We experimented with several other observables – such as real loan volumes from MFIs to entrepreneurs and households (for the purpose of residential investment) but the estimation performance and information content appeared fragile. This has of course implications for estimation since the relatively limited number of financial observables

[9] Series with a dagger (†) are modeled, as in Christoffel et al. (2008), using a structural VAR, details of which can be also found there.

(and the lack of quantity flows) can constrain identification and limit the number of structural shocks that might be included and econometrically identified.

The sample used was 1980q2 to 2010q2 (with 19 quarters used for training the Kalman filter). Figures 1 to 3 show the full set of observables, financial and non-financial. Further, Figures 4 and 5 show, respectively, the clear counter-cyclicality of the premium (i.e., rising in downturns) and the substantially greater volatility (although largely similar turning points) of the growth of the residential investment series relative to growth in total investment (and their combined greater volatility relative to real output growth).

3.2 Structural Shocks

To summarize, the model has 21 observables. The number of structural shocks in the model depends – as we see below – on the model variant, although the preferred case has 21 structural shocks supplemented by measurement errors in extra-euro (volume and price) trade data.

Of the structural shocks all except the policy rate shock are assumed to follow a stationary AR(1) process. The structural shocks are the following:

Shock Symbol and Type			
ϵ^{RP}	*domestic risk premium*	ϵ^{RP*}	*external risk premium*
ϵ^{G_c}	*permanent technology*	ϵ^{prod}	*transitory neutral technology*
ϵ^{I}	*investment-specific technology*	ϵ^{φ_W}	*wage markup*
ϵ^{φ_H}	*domestic price markup*	ϵ^{φ_X}	*export price markup*
$\epsilon^{\varphi_{IM}}$	*import price markup*	ϵ^{IM}	*import demand*
ϵ^{ν_*}	*export preference*	ϵ^{G}	*government consumption*
ϵ^{R}	*domestic policy interest rate*	ϵ^{PiY*}	*foreign price*
ϵ^{Y*}	*foreign demand*	ϵ^{R*}	*foreign interest rate*
ϵ^{p_O}	*oil price*	ϵ^{pCX}	*competitors' export price*
ϵ^{j}	*housing Demand*	ϵ^{χ}	*premium*
$\epsilon^{G_{qh}}$	*permanent technology in res. sector*		

However, we also have the possibility of the additional structural innovations:

- ϵ^{m_B} (loan-to-value); ϵ^{γ} (entrepreneurs' survival rate)
- ϵ^{T_e} (start-up transfers to entrepreneurs); ϵ^{prod_h} (transitory neutral technology in residential sector)

Finally, we also have measurement errors on trade variables (volumes and prices) with a cross-correlation of -1 imposed on volumes.

3.3 Calibration

In the cases of ratios or growth rates or tax rates these were, as in Christoffel et al. (2008), set to broadly match features of the euro area data and to ensure balanced growth (see Table 1). Thus, the trend growth rate of the model economy is 0.5% per quarter (2% per annum), which is the growth rate of all (or most) real variables. This rate is decomposed into a labor productivity growth rate and a labor-force/population growth rate.

As already mentioned, a separate growth rate was imposed for residential investment. As in Iacoviello and Neri (2010), the most sensible value for this implies a degree of regression over time of technical progress in the residential sector, which is necessary to ensure that growth in house prices are matched.

The two discount factors were taken from Iacoviello (2005), whilst the impatient household's collateral constraint (loan to value ratio) was set to 0.75 on the basis of averaging Calza et al. (2009)'s country data: Finland and France (75%), Germany, Spain and Ireland (70%), Italy (50%), Netherlands (90%) (see also Sorensen and Lichtenberger (2007)). The share of patient consumers is set to 0.80, following standard ranges. The value of the leverage ratio and the share of surviving entrepreneurs was taken from Bernanke et al. (1999). Over our sample, the spread of firms' financing rate over the riskless rate was around 130 basis points. The annual real equilibrium interest rate is 2.5% and the inflation objective is assumed to be just under 2%.

The 'Calvo' employment parameter linking unobserved hours worked with the labor input is set at around 0.85, consistent with both Smets and Wouters (2003) and Christoffel et al. (2008), and quite consistent with its freely estimated value.

Finally, a word about parameters relating to the household types. Given the lack of (e.g., consumption, wage) data on different households, we found it difficult to credibly identify Calvo, habit and Frisch elasticity parameters across constrained and unconstrained household types. We therefore estimated these as a single parameter, whilst in the latter case, we imposed a value of 2.

3.4 Trends

In Iacoviello and Neri (2010) there are three different deterministic trends: consumption (production of goods), housing (production of housing investment goods) and nonresidential investment. Here we follow Iacoviello and Neri (2010) noting that the NAWM single trend is a nested case.

4 Estimation

Tables 2 and 3 show, respectively, a full set of posterior estimations for the core model and for variants of it. We take the main framework (the two households, the external finance premium) as essentially given, and so our variants refer to the testing for various shock processes – such as whether the data can detect transfer and survival shocks to entrepreneurial activity.

4.1 Prior Distributions

We largely follow Christoffel et al. (2008) and indeed standard practise in setting our priors (see Table 2 for the core case). The habit parameter is centered a 0.5 Beta process with 0.05 standard error. All Calvo parameters are set as a Beta process with first and second moments of 0.7 and 0.05, whilst indexation parameters are set at 0.5 and 0.10, respectively. All parameters relating to final goods production are set to Gamma(1.5, 0.25) distributions. The same distribution – with slightly more specific prior moments – is also used for the adjustment costs. All auto-regressive shock processes are set as a Beta with mean set at around 0.7 and a standard deviation typically set at 0.1. The standard deviations of the shocks follow diffuse Inverse Gamma processes.

4.2 Posterior Distributions

Table 2 reports the results obtained with 250,000 draws and two chains of the Monte-Carlo Markov-Chain (MCMC) algorithm. The average "acceptance rate" of the two chains is around 0.3%. [10] Although our core case is not necessarily the scenario chosen on the basis of model odds, it is favored since the additional parameters associated with perturbations of the core case seem very weakly identified.

Normalizing on the core case, we find some interesting results: a premium elasticity of just under 0.02; Calvo parameters around 0.75 (suggesting average price stickiness lasting a quite plausible 4 quarters); and a relatively persistent housing demand shock (around 0.95). The premium shock is small in relative value but it is relatively persistent.

Figure 6 displays "Monte Carlo chain multivariate diagnostics". The red and blue lines on the charts represent specific measures of the parameter vectors both within and between the (two MCMC) chains. These should be relatively constant and should converge (as they do, in our case).[11]

Figures 7 to 12 show the distributions of the priors and posteriors. It can be seen that in most cases the estimation data is quite informative in the sense that the posterior parameter distribution is pulled away from that of the prior.

5 Model Properties

Figures 13 to 22 depict the dynamic responses of selected variables over a 40-quarter horizon to an increase by one estimated standard error in the innovation relating to the policy shock, the neutral productivity shock, the housing demand shock, the premium elasticity shock, and the investment specific shock. The black line is the responses of the model when parameters are set to their mean values and the grey shaded areas represent the 95% confidence intervals.

In the panels, we show the model responses of GDP; both consumption types; total and residential investment; employment; volume trade; the real effective exchange rate; the policy rate; the premium; measures of inflation (GDP, CPI, and housing based); loans to firms and households; and finally entrepreneurs' net worth.

[10]Estimation was done with Dynare (4.2.0). The mode was first obtained using a simulated annealing algorithm (Goffe (1996)). That mode is then used as starting point for the MCMC draws.

[11]We report three measures: "interval", being constructed from an 80% confidence interval around the parameter mean, "Variance", being a measure of the variance and "Third Moment" based on third moments. For space, we suppress these figures relating to individual parameters and show the "multivariate diagnostic" which presents results of the same nature, except that they reflect an aggregate measure based on the eigenvalues of the variance-covariance matrix of each parameter.

5.1 Interest Rate shock

This simulation (see Figures 13 and 14) follows the standard mechanisms of an unanticipated tightening in the policy rate. Real demand components (output, consumption) and employment all display a protracted decline followed by a gradual return to base following the removal of the shock. By definition, impatient consumers witness a far greater drop in their expenditures given their inability to borrow, and their tighter real home borrowing constraint. The two investment series are more volatile than output and consumption, although, in this shock scenario, non-residential investment drops by a slightly greater amount. As a result of the output contraction, firms' premium rises, further exacerbating the downturn. This tightening of the premia clearly reflects the drop in entrepreneurs' net worth.

5.2 Temporary Productivity Shock

By contrast, a temporary productivity shock (see Figure 15 and 16) raises output and consumption. Here – in line with New-Keynesian mechanisms – total employment falls due to the presence of nominal and real rigidities which imply than output growth grows less then that of technology. The various indices of Inflation fall given that core real marginal costs have fallen.

The premium of the entrepreneurs rises mainly due to the Fisher's effect of an unexpected drop in inflation which increases the real value of debt.

There is a qualitative difference between patient and impatient consumers. The latter benefit in the same way as the economy benefits since the technical possibilities of the economy have increased as have the returns to real productive assets. Patient consumers now find themselves able to extract a high premium from entrepreneurs for their savings. Impatient consumers, however, see a substantial fall in their consumption growth given that – as borrowers on nominal contracts – the decrease in inflation has increased the real value of their liabilities.

5.3 Housing Demand Shock

Figure 17-18 shows the response to a housing preference shock, i.e., a shift in preference for housing with respect to consumption and leisure. Since it generates an increase in both house prices and the returns to housing investment this shock is commonly interpreted as a housing *demand* shock. As a result of the rise in housing prices, impatient consumers face looser credit constraints and increase their consumption expenditures. They do not do so however as a result of curtailing other consumption purchases. The positive response of consumption to the housing demand shock – which is witnessed in VAR studies and event studies of data – cannot be reproduced without collateral effects. GDP rises given the very large increase in liquidity constrained households' consumption and the resulting increase in residential investment and employment (total investment is however barely changed). The shock has a small negative impact on the behavior of Patient consumers. In the absence of collateralized debt, patient households substitute current consumption for housing services.

5.4 Premium Shock

Figure 19-20 shows the response to a premium shock. This generates a reduction in non-residential investment akin to that of the policy shock. The negative effect of the investment offsets to some degree by the slightly higher consumption profiles (the impatient consumers faces a relaxation in his real borrowing constraint given the increase in house prices). The slightly higher returns to housing generate a small but positive expansion of residential investment.

By definition the rise in the premia – coupled with the decline in overall economic activity – decreases the borrowing activities of firms and entrepreneurs and reduces net worth.

5.5 Investment-Specific Shock

The investment specific shock – in our context – can be considered as working in a similar manner to the temporary productivity shock. Whilst output, consumption, investment, and employment increase, the expansion of borrowing activities intertemporally implies a rising premium and declining net worth.

6 Model with and without financial friction

In this section, we illustrate how the model would behave to a standard monetary policy shock with (individually and jointly) its two main financial channels shut down: namely where the share of financially constrained consumers is set to zero, and where the BGG mechanism is shut down. For comparability across these three cases, we break with normal data-based convention and replace the value of σ_{policy} with a value that ensure that a 25 basis points increase in the first-period annualized nominal interest rate is achieved. Figures 23, 24, 25 shows the results.

Overall, the simulation, as before, follows the standard mechanisms of an unanticipated tightening in the policy rate. Real demand components (output, consumption, investment) and employment all display a protracted decline followed by a gradual return to base following the removal of the shock.

The two investment series are (as to be expected) more volatile than output and consumption, although, in this shock scenario, non-residential investment drops by a slightly greater amount. As a result of the output contraction, firms' premium rises, further exacerbating the downturn. This tightening of the premium clearly reflects the drop in entrepreneurs' net worth (not shown).

With the prolonged contraction of demand, current and expected inflation fall. This induces a negative income effect on indebted households since the real service cost of nominal debt rises. The monetary shock also affects the credit constraint: For any given level of the housing stock and expected house prices the drop in inflation tightens the borrowing constraint and at the same time reduces the marginal utility of further borrowing due to the higher future service cost of debt.

All these effects reduce borrowers' consumption and housing demand and lead to a decrease in house prices. The latter, in turn, reinforces the negative effects on the credit constraint just described and hence, magnifies the drop in borrowers' consumption demand.

If financial markets were frictionless, the economy would exhibit a weaker drop in inflation after a negative monetary shock since the negative effects on consumption demand operating via nominal debt and the credit constraint would be absent. It can be shown that the higher the fraction of borrowers, the more pronounced the negative effect of a monetary tightening on current inflation. Note also that since this model assumes that the liquidity constrained consumers all work in the residential sector, any demand contraction is especially damaging for the consumption profiles. This is why the aggregate consumption profile reduces sharply on impact following the shock (reflecting a large collapse in liquidity constrained consumers' consumption). When there are no constrained agents, consumption follows the more familiar hump-shaped profile.

Figure 24: Here – in comparison to the full model benchmark – entrepreneurs borrow at the riskless rate (hence there is no dashed line in the premium panel). The effects are not greatly different. Having no premium allows total investment to suffer a smaller decline, this leads directly to a less contractionary GDP profile. Since non-residential investment is determined more by borrowing conditions than residential investment (which is directly related to borrowing constraints and house prices), it makes sense that non-residential investment should be more affected by the absence of a premium channel. The better performance for non-residential investment, draws resources away from the residential sector which in turn produces slightly more negative outturns compared to the full model, with a consequently more negative profile for house prices.

7 Model and Data Sample Moments

Table 4 shows some comparisons of the first and second moments of the data, compared to that generated by the model for a selection of observables. The final column is the second moment of each variable relative to GDP.

The mean values of the model can be seen simply to embody balanced growth closures, as discussed earlier in section 3.3. The second moments show a variety of hits and misses. The values of the policy rate, the premium, residential investment growth, and house prices seem not unreasonable. Clearly, however there is a considerable upward bias in the estimation of the second moment of aggregate consumption. This is not an entirely surprising result – the NAWM reports a data (model) standard deviation of 0.48(0.74). This reflects in part the traditional weakness of the Euler-equation approach to modeling consumption, even when supplemented with a quantitatively significant habit and consumption smoothing parameter values. In our case, the failure is more drastic with the model (data) standard deviations for consumption at 0.5(1.6). This gap widens the larger share is attributed to liquidity constrained consumers – which in itself goes some (but relatively little) way to improve relative housing price and volume volatilities.

Clearly the inclusion of the non-Ricardian household is a useful device in some dimensions – for example in incorporating collateral constraints in a tractable manner and in reconciling housing demand shocks with the data. But there is a price to be paid by assuming that some fraction of consumers are permanently liquidity constrained. More recent financial frictions literatures have tried to make such shares state dependent and at least hold out the hope of reducing this excessive consumption feature. A similar argument holds with real investment where – again similarly so with the NAWM - the data produces more volatility than the data – ostensibly related to the models' excessive sensitivity to movements in the real user cost.

8 Variance Decomposition

Table 5 shows the conditional variance decomposition over an immediate (1 quarter) and medium run (20-quarter) horizon.

In the short run we see that output is driven in almost equal measure by all the shock groups. Financial shocks have a relatively large effect on output, consumption and specifically financial variables like house prices, residential investment and the Premium. In the case of the premium it is almost completely accounted for by its stochastic shock in the short run (i.e., the model is essentially uninformative about that shock over the short horizon). But as the horizon widens, variation in the premium is mostly (2/3rds) by variation in other model elements. Real residential investment and house prices tend however to be dominated by their own shocks.

As the horizon increases monetary policy shocks have a smaller effect on all observables, except inflation. The same is true for demand shocks

9 Historical Decompositions

One of the most interesting products of the DSGE frameworks is the production of historical decompositions. This involves taking observables and decomposing them into the contribution associated with the structural shocks. The figures below show contribution charts for key variables in growth rates (measured in deviation from a mean growth rate that needs to be added to obtain the realized values). We omit the effect of initial conditions and measurement errors for convenience.

Since the number of structural shocks is relatively high, we group them into the following categories:

- Financial: Net Worth; Premium; Housing Demand;
- Foreign: external risk premium; export preference; import price; foreign variables (foreign demand, foreign interest rate)
- Mark-Ups: All mark-up shocks.
- Demand: Domestic Risk Premium Shocks; Government Expenditure Shocks; Preference Shocks; Import Demand Shocks.
- Technology: Permanent neutral technology shock; Transitory neutral technology shock; Investment-specific technology shock.
- Monetary Policy: Innovation on Taylor feedback rule

Note, that this distinction of shocks is by no means unique (investment-specific shocks could also be considered as financial shocks), but it does illustrate well the workings of the model. The financial block, note, is deliberately intended to isolate shocks which would not be found in more standard models without financial frictions.

Figures 26 to 29 show the (annual cumulated) historical decomposition of real GDP growth, Inflation, the premium and house prices from 2005q1 until the end of the estimation sample (2010q2).

GDP Growth We see that financial shocks played some small role in the cumulated downturn in real GDP but they were by no means dominant, with movements in technology and foreign shocks seemingly more important. Interestingly, monetary policy shocks were not supportive around the downturn: this reflected the fact that short-term nominal monetary policy were on a tightening cycle in the run up to the crisis and although policy rates corrected themselves rapidly (see the positive contribution to GDP growth after the nadir), the contribution was rather muted reflecting perhaps the lower-bound constraint as well as the past effect of monetary tightening, and perhaps (to the extent that we can meaningfully capture non-credit effects), the enhanced credit support of the central bank. Given that the premium paid by firms went up over the crisis (as theory would predict), there was a persistently negative contribution of financial shocks to real output growth (except for the last 2 quarters).

Inflation The GDP-deflator inflation behaved not dissimilarly to output growth: being mainly driven down towards the end of the sample by large negative productivity and foreign shocks. Monetary policy was also not supportive around the crisis although its particular contribution is quite muted. Generally the effects of financial shocks on inflation are small and – around the crisis – contributing negatively to inflation. The main positive contribution to prices was variations in mark-up shocks.

Premium Financial shocks (as well as foreign shocks) were the dominant contributor to the rising premium in the latter part of our sample, with technology and (to a far lesser extent monetary) shocks playing something of an offsetting role.

House Prices Financial shocks played a dominant (and highly pro-cyclical) role in the growth of house prices. Again monetary policy shocks though mildly unsupportive around the largest drop in house, become (although again mildly) supportive thereafter.

10 Conclusions

We estimated a model of the euro area following the recent contributions of Smets and Wouters (2003) and Christoffel et al. (2008), with the addition and distinction of allowing for a number of financial frictions. These related to the allowance of an external finance premia on entrepreneurs' purchases of capital from unconstrained households, and from the existence of collateral constraints on household purchases by unconstrained households.

We presented estimation results obtained from a linear state-space representation of the model using Bayesian methods. We then explored various aspects of the model's properties – such as standard impulse response analysis, its implied sample moments relative to the observed data and forecast error variance decompositions and historical shock decompositions. Although – relative to a model without the same financial frictions – the simulation properties are mostly not qualitatively affected, the model's ability to track and enhance our understanding of the evolution of financial variables and the strength of financial channels, makes the model a valuable addition to modeling work in the euro area.

Recently, the literature on financial frictions and financial crises has expanded dramatically. Many of these extensions include substantial non-linearities (for example, in "occasionally binding" constraints); in modeling a monopolistically competitive banking sector and banking entry and exit costs; in modeling the channels involved in "unconventional" monetary policy (i.e., central banks inter-temporally directly taking private assets onto their balance sheet); accounting for imperfect pass-through between riskless and lending rates; debt-deflation spirals; and sudden stops etc. The challenge for future modeling in this area will be to assess to which extent large production policy and projection models can go beyond the incorporation of simpler financial frictions (as analyzed here) to these more extensive features whilst still retaining tractability.

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Table 1: Calibrated Parameters

Parameter	Description	Value
β_1	Discount Factor (Patient)	$G_c/\left(R^{ss}\right)^{\frac{1}{4}} = 0.9968$
β_2	Discount Factor (Impatient)	0.9750
m_B	Loan to Value Ratio	0.7500
γ_D_N	Leverage Ratio	0.5000
ρ	Share of Patient HHs	0.8000
$-$	Labor Share (Patient Household)	0.6000
δ	Depreciation Rate (Capital)	0.0250
δ_h	Depreciation Rate (Housing)	0.0100
τ^C	Consumption Tax	0.1830
τ^N	Labor Income Tax	0.1220
τ^{W_h}	Social Security Contribution (Worker)	0.1180
τ^{Wf}	Social Security Contribution (Firm)	0.2190
τ^K	Capital Tax	0.5080
$-$	Trend Growth rate (%)	$Gc + Ge$
Gc	Trend Labor Prod. Growth Rate (%)	$(1.012)^{\frac{1}{4}} = 1.0030$
Ge	Trend Labor Force Growth Rate (%)	$(1.008)^{\frac{1}{4}} = 1.0020$
Gqh	Trend Res. Inv. Prod. Growth rate (%)	$(0.9904)^{\frac{1}{4}} = 0.9976$
C/Y	Consumption/Output	0.5750
I/Y	Investment/Output	0.2300
H/Y	Housing Stock/Output	0.7500
G/Y	Government Expenditure/Output	0.2150
IM_C/Y	Imports in Consumption/Output	0.1000
IM_I/Y	Imports in Investment/Output	0.0600
$IM/Y,\ X/Y$	Imports/Output, Exports/Output	0.1600
$\bar{\pi},\ \bar{\pi}^*$	Inflation Target	1.00475
R^{SS},$R^{*,SS}$	Steady State Nominal Interest Rate	$\pi_i^{ss} + \frac{Gc}{\beta_1} - 1 = 1.0110$
RR^{SS},$RR^{*,SS}$	Annual Real Steady State Interest Rate (%)	$1 + 4 \cdot \left(R^{SS} - \bar{\pi}\right) = 1.0250$
γ	Share of Surviving Entrepreneurs	0.9700
γ_T_e	Start Up Transfer as share of consumption	0.0100
$\chi\_$	Steady State Premia	$128bp$
$\zeta_1, \zeta_2, \zeta_H$	Inverse labor Supply elasticity	2
γ_B^*	External Intermediation Premium Elasticity	0.0100
ρ_G, σ_G	Auto-Regressive (Standard error) Process for G	0.9700 (0.4305)
ξ_E	Calvo Employment Parameter	0.850

Note: Numbers refer to quarterly-frequency.

Table 2: Posterior Distributions of the Structural Parameters: Core Case

	Prior Distribution	Posterior Distribution			
		mode	mean	5%	95%
		Preferences			
κ Habit Formation	$B(0.50, 0.05)$	0.565	0.573	0.495	0.646
		Wage and Price Setting			
ξ_W Calvo Wages	$B(0.75, 0.05)$	0.702	0.706	0.645	0.767
χ_W Indexation Wages	$B(0.50, 0.05)$	0.460	0.461	0.383	0.539
ξ_H Calvo Dom. Prices	$B(0.75, 0.05)$	0.801	0.801	0.765	0.839
χ_H Indexation Dom. Prices	$B(0.50, 0.05)$	0.392	0.395	0.317	0.472
ξ_X Calov Export Prices	$B(0.75, 0.05)$	0.818	0.800	0.759	0.850
χ_X Indexation Export Prices	$B(0.75, 0.10)$	0.376	0.407	0.258	0.561
ξ_{IM} Calvo Import Prices	$B(0.75, 0.05)$	0.502	0.508	0.446	0.567
χ_{IM} Indexation Import Prices	$B(0.50, 0.05)$	0.428	0.432	0.355	0.507
ω_* Oil Import Shares	$B(0.15, 0.05)$	0.175	0.175	0.145	0.203
		Final Good Production			
μ_C Subst. Elast. Cons.	$\Gamma(1.50, 0.25)$	3.033	3.027	2.567	3.473
μ_I Subst. Elast. Inv.	$\Gamma(1.50, 0.25)$	2.014	2.057	1.520	2.593
μ_{ast} Export Market Share	$\Gamma(1.50, 0.25)$	1.193	1.181	0.968	1.378
		Financial Accelerator			
χ Premium Elasticity	$B(0.05, 0.01)$	0.017	0.017	0.013	0.022
Adjustment Costs					
γ_I Investment	$\Gamma(4.00, 0.50)$	3.337	3.454	2.719	4.155
γ_{IMC} Import Content of Cons.	$\Gamma(5.00, 0.25)$	4.784	4.815	4.424	5.200
γ_{IMI} Import Content of Inv.	$\Gamma(1.50, 0.50)$	4.146	4.293	3.269	5.308
γ_{ast} Export Market Share	$\Gamma(2.50, 1.00)$	2.453	2.516	1.968	3.031
γ_{IH} Residential Investment	$\Gamma(2.50, 1.00)$	1.470	1.310	0.992	1.618
		Monetary Policy			
λ_R: Smoothing	$B(0.6, 0.05)$	0.860	0.858	0.832	0.885
λ_π Reaction to Infl.	$N(1.7, 0.1)$	1.654	1.647	1.516	1.778
$\lambda_{\Delta\pi}$ Reaction to Change in Infl.	$N(0.3, 0.1)$	0.193	0.194	0.123	0.264
$\lambda_{\Delta gdp}$ Reaction to Output Growth	$N(0.063, 0.05)$	0.214	0.223	0.188	0.256
		Auto-Regressive Coefficients			
ρ_{RP} Risk Premia Foreign	$B(0.80, 0.10)$	0.834	0.839	0.787	0.892
ρ_{Gc} Permanent Technology Shock	$B(0.80, 0.10)$	0.758	0.769	0.649	0.890
ρ_{prod} Transitory technology Shock	$B(0.75, 0.05)$	0.856	0.842	0.811	0.875
ρ_I Inv-spec. Tech.	$B(0.75, 0.05)$	0.666	0.659	0.587	0.730
ρ_{φ_H} Price Mark-up	$B(0.50, 0.10)$	0.931	0.926	0.891	0.961
ρ_{φ_X} Export Price Mark-up	$B(0.50, 0.10)$	0.291	0.315	0.191	0.437
$\rho_{\varphi_{IM}}$ Import Price Mark-up	$B(0.50, 0.10)$	0.577	0.571	0.402	0.730
ρ_{IM} Import Demand	$B(0.80, 0.10)$	0.486	0.482	0.395	0.569
ρ_{RP_dom} Risk Premium	$B(0.50, 0.10)$	0.812	0.792	0.733	0.856
ρ_{Gqh} Transitory Residential Tech	$B(0.80, 0.10)$	0.543	0.531	0.446	0.614
ρ_j Housing Demand	$B(0.80, 0.10)$	0.950	0.946	0.941	0.950
ρ_χ Premium	$B(0.80, 0.10)$	0.876	0.868	0.800	0.940
ρ_w Wage Mark-up	$B(0.80, 0.10)$	0.835	0.816	0.748	0.885
		Standard Deviations			
σ_{RP} Risk Premia Foreign	$\Gamma^{-1}(0.15, Inf)$	0.659	0.680	0.491	0.854
σ_{Gc} Permanent Technology Shock	$\Gamma^{-1}(0.15, Inf)$	0.277	0.273	0.179	0.368
σ_{prod} Temporary Productivity Shock	$\Gamma^{-1}(0.15, Inf)$	1.174	1.220	1.040	1.397
σ_I Investment Spec. Technology shock	$\Gamma^{-1}(0.15, Inf)$	2.905	3.030	2.336	3.737
σ_{φ_H} Price Mark-up	$\Gamma^{-1}(0.15, Inf)$	0.402	0.411	0.350	0.469
σ_{φ_X} Export Price Mark-up	$\Gamma^{-1}(0.15, Inf)$	1.942	2.025	1.754	2.290
$\sigma_{\varphi_I M}$ Import Price Mark-up	$\Gamma^{-1}(0.15, Inf)$	0.822	0.851	0.606	1.101
σ_{IM} Import Demand	$\Gamma^{-1}(0.15, Inf)$	12.876	13.133	11.374	14.845
σ_{policy} Policy Rate	$\Gamma^{-1}(0.15, Inf)$	0.127	0.134	0.112	0.155
σ_{RP_dom} Risk Premia Foreign	$\Gamma^{-1}(0.15, Inf)$	2.553	2.596	2.125	3.069
σ_χ Premium	$\Gamma^{-1}(0.15, Inf)$	0.071	0.072	0.064	0.081
σ_{Gqh} Transitory Residential Tech.	$\Gamma^{-1}(0.15, Inf)$	0.694	0.706	0.619	0.796
σ_j Housing Demand	$\Gamma^{-1}(0.15, Inf)$	10.000	9.517	8.945	10.000
σ_w Wage Mark-up	$\Gamma^{-1}(0.15, Inf)$	0.260	0.274	0.211	0.334

Table 3: Posterior Distributions of the Structural Parameters: Variants

	core	v1	v2	v3	v4	v5	v6
	Preferences						
κ Habit Formation	0.573	0.579	0.577	0.571	0.580	0.595	0.581
	Wage and Price Setting						
ξ_W Calvo Wages	0.706	0.746	0.718	0.707	0.709	0.729	0.711
χ_W Indexation Wages	0.461	0.492	0.414	0.460	0.460	0.454	0.457
ξ_H Calvo Dom. Prices	0.801	0.782	0.781	0.802	0.801	0.780	0.800
χ_H Indexation Dom. Prices	0.395	0.408	0.401	0.392	0.395	0.393	0.395
ξ_X Calvo Export Prices	0.800	0.804	0.791	0.800	0.799	0.797	0.798
χ_X Indexation Export Prices	0.407	0.410	0.395	0.402	0.413	0.413	0.406
ξ_{IM} Calvo Import Prices	0.508	0.516	0.497	0.507	0.507	0.504	0.507
χ_{IM} Indexation Import Prices	0.432	0.430	0.410	0.432	0.434	0.429	0.429
ω_* Oil Import Shares	0.175	0.180	0.170	0.175	0.175	0.173	0.175
	Final Good Production						
μ_C Subst. Elast. Cons.	3.027	2.825	3.187	3.034	3.030	3.036	3.051
μ_I Subst. Elast. Inv.	2.057	2.350	1.914	2.074	2.083	2.049	2.066
μ_* Export Market Share	1.181	1.129	1.198	1.189	1.185	1.178	1.180
	Financial Accelerator						
χ Premium Elasticity	0.017	0.009	0.018	0.017	0.017	0.018	0.017
	Adjustment Costs						
γ_I Investment	3.454	2.724	3.387	3.470	3.463	3.438	3.470
γ_{IMC} Import Content of Cons.	4.815	4.819	4.793	4.806	4.812	4.807	4.809
γ_{IMI} Import Content of Inv.	4.293	4.873	4.376	4.312	4.273	4.251	4.297
γ_* Export Market Share	2.516	2.675	2.385	2.514	2.517	2.531	2.515
γ_{IH} Residential Investment	1.310	1.047	1.255	1.317	1.301	1.317	1.304
	Monetary Policy						
λ_R Smoothing	0.858	0.914	0.849	0.858	0.859	0.848	0.857
λ_π Reaction to Infl.	1.647	1.560	1.697	1.649	1.652	1.689	1.656
$\lambda_{\Delta\pi}$ Reaction to Change in Infl.	0.194	0.085	0.228	0.195	0.195	0.219	0.200
$\lambda_{\Delta Y}$ Reaction to Output Growth	0.223	0.244	0.196	0.224	0.224	0.216	0.225
	Auto-Regressive Coefficients						
ρ_{RP} Risk Premia Foreign	0.839	0.864	0.838	0.837	0.837	0.836	0.839
ρ_{Gc} Permanent Technology Shock	0.769	0.741	0.716	0.771	0.769	0.712	0.764
ρ_{prod} Transitory technology Shock	0.842	0.802	0.855	0.841	0.842	0.850	0.841
ρ_I Inv-spec. Tech.	0.659	0.669	0.720	0.659	0.657	0.659	0.648
$\rho_{\varphi H}$ Price Mark-up	0.926	0.934	0.942	0.926	0.925	0.943	0.929
$\rho_{\varphi X}$ Export Price Mark-up	0.315	0.329	0.397	0.314	0.313	0.318	0.316
$\rho_{\varphi IM}$ Import Price Mark-up	0.571	0.575	0.570	0.572	0.569	0.571	0.573
ρ_{IM} Import Demand	0.482	0.505	0.494	0.483	0.479	0.491	0.482
$\rho_{RP_{dom}}$ Risk Premium	0.792	0.893	0.833	0.795	0.798	0.825	0.800
ρ_{Gqh} Transitory Residential Tech	0.531	0.628	0.764	0.535	0.531	0.752	0.533
ρ_j Housing Demand	0.946	0.790	0.944	0.946	0.946	0.945	0.946
ρ_χ Premium	0.868	0.941	0.860	0.867	0.866	0.839	0.840
ρ_w Wage Mark-up	0.816	0.689	0.819	0.814	0.813	0.802	0.812
ρ_{m_B} Loan-to-value	-	-	-	-	0.791	0.804	0.792
ρ_{T_e} Transfer to Entrepreneurs	-	-	-	0.796	-	0.798	0.800
ρ_γ Firms' Survival Rate	-	0.998	-	-	-	0.769	0.763
ρ_{prod_H} Residential Productivity	-	-	0.860	-	-	0.804	-
	Standard Deviations						
σ_{RP} Risk Premia Foreign	0.680	0.643	0.675	0.682	0.681	0.691	0.678
σ_{Gc} Permanent Technology Shock	0.273	0.279	0.289	0.275	0.273	0.276	0.279
σ_{prod} Temporary Productivity Shock	1.220	1.395	1.094	1.225	1.210	1.133	1.200
σ_I Investment Spec. Technology shock	3.031	1.962	2.869	3.027	3.035	3.029	3.039
$\sigma_{\varphi H}$ Price Mark-up	0.411	0.419	0.399	0.412	0.410	0.404	0.411
$\sigma_{\varphi X}$ Export Price Mark-up	2.025	2.064	2.078	2.024	2.038	2.035	2.026
$\sigma_{\varphi IM}$ Import Price Mark-up	0.851	0.868	0.832	0.853	0.859	0.857	0.857
σ_{IM} Import Demand	13.133	13.717	12.840	13.146	13.172	13.033	13.144
σ_{policy} Policy Rate	0.134	0.124	0.127	0.134	0.134	0.135	0.134
$\sigma_{RP_{dom}}$ Risk Premia Foreign	2.596	2.740	2.544	2.583	2.556	2.585	2.562
σ_χ Premium	0.072	0.078	0.070	0.072	0.073	0.071	0.071
σ_{Gqh} Transitory Residential Tech.	0.706	0.697	0.414	0.706	0.707	0.443	0.709
σ_j Housing Demand	9.517	0.156	10.000	9.554	9.459	9.522	9.423
σ_w Wage Mark-up	0.274	0.346	0.272	0.276	0.278	0.282	0.277
σ_{m_B} Loan-to-value	-	-	-	-	0.185	0.149	0.197
σ_{T_e} Transfer to Entrepreneurs	-	-	-	0.144	-	0.150	0.151
σ_γ Firms' Survival Rate	-	0.250	-	-	-	0.123	0.124
σ_{prod_H} Residential Productivity	-	-	0.429	-	-	0.424	-
Log Density	-3029.753	-3066.690	-3111.885	-3029.639	-3029.296	-3017.823	-3029.681
Model odds	0.135	0.003	0.000	0.137	0.142	0.446	0.136

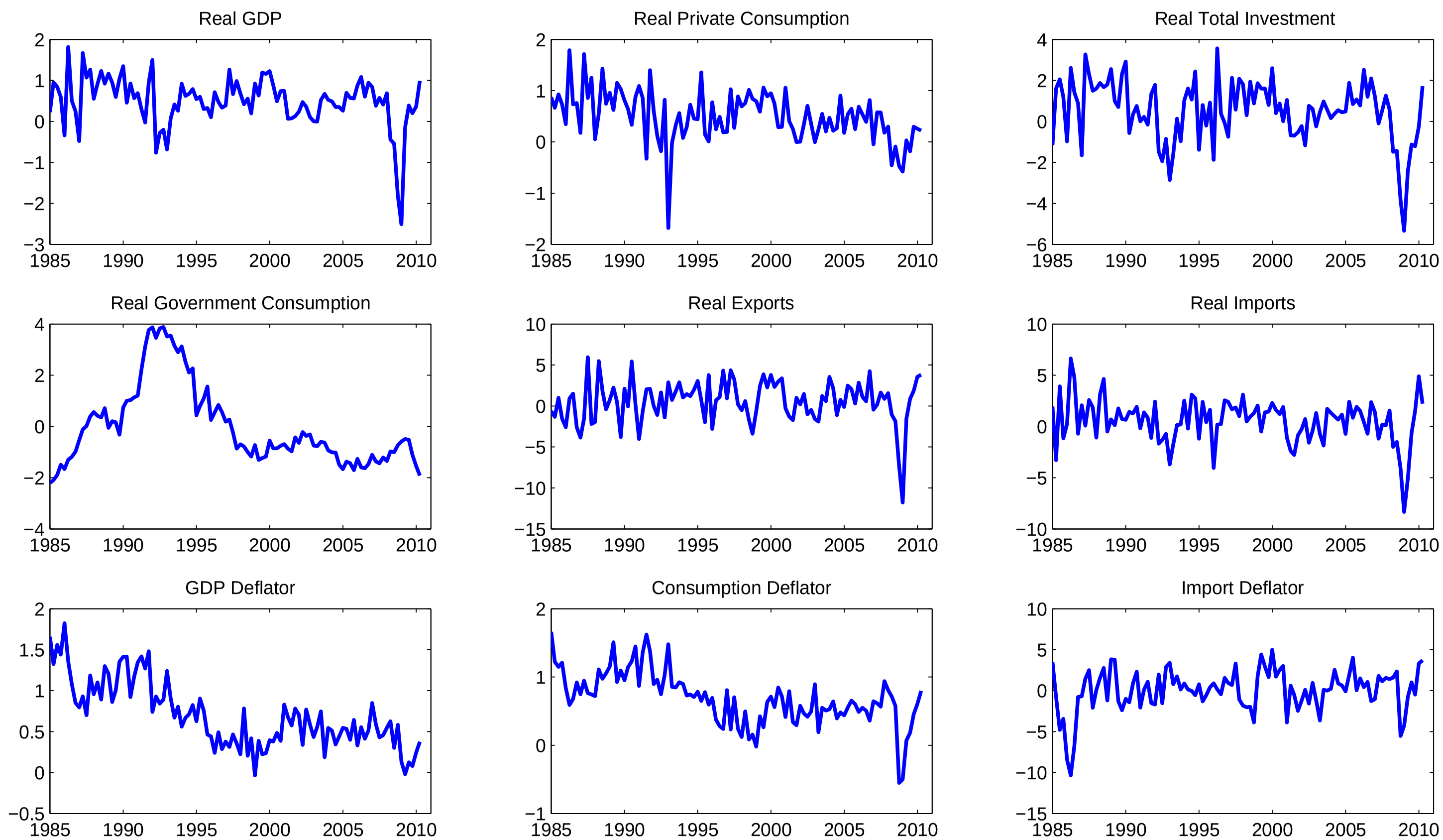
Table 4: Selected Moments of Data and Model

	Data			Model		
	mean	std. dev		mean	std. dev	
Real GDP	0.46	0.6	—	0.5	1.09	—
Consumption	0.44	0.5	0.85	0.5	1.61	1.48
Total Investment	0.36	1.47	2.47	0.5	2.56	2.35
GDP Def.	0.93	0.68	1.14	0.48	0.92	0.84
Policy Rate	6.89	3.94	6.57	4.41	2.42	2.22
Premium	1.28	0.68	1.15	1.28	1.16	1.07
Residential Inv.	-0.04	2.34	3.92	0.26	3.32	3.05
House Prices	1.44	1.51	2.53	0.72	1.24	1.14

Table 5: Conditional Variance Decomposition

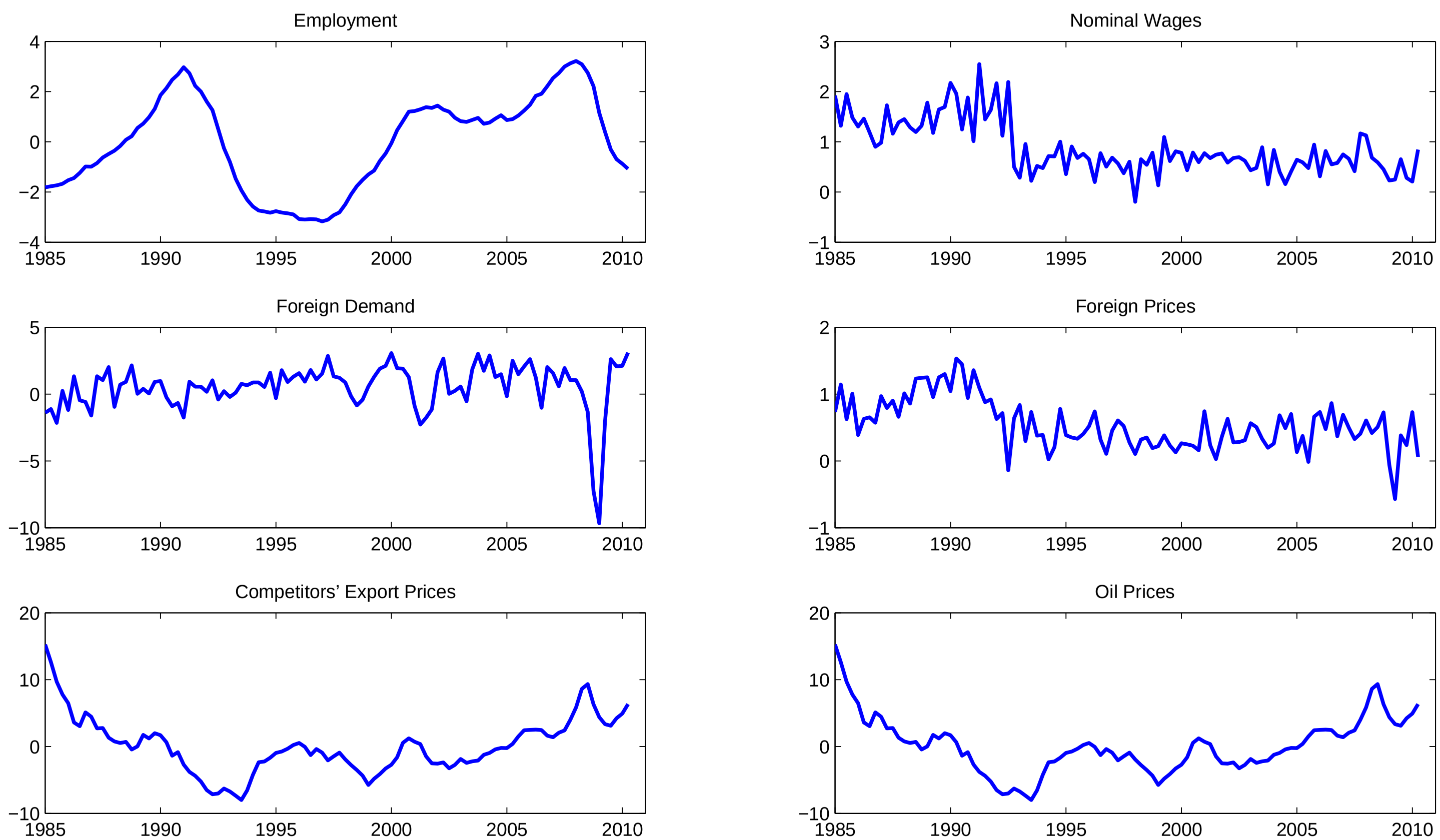
	Financial	Foreign	Markups	Demand	Technology	Mon. Policy
	t = 1					
Real Output Growth	17.53	13.96	7.09	21.66	25.38	14.38
GDP Deflator	0.33	4.37	64.93	1.65	26.94	1.76
Real Consumption Growth	29.47	2.94	4.75	38.12	11.49	13.22
Real Investment Growth	3.04	7.65	12.77	11.47	59.68	5.4
Premium	85.06	0.42	2.26	0.27	9.55	2.46
Real Residential Investment Growth	69.97	5.03	9.72	6.22	6.08	2.96
House Prices	86.3	0.42	2.88	0.88	5.92	3.59
	t = 20					
Real Output Growth	13.92	12.33	22.54	15.74	27.33	8.15
GDP Deflator	0.45	15.06	48.96	4.07	27.71	3.77
Real Consumption Growth	27.48	3.42	13.67	31.88	14.82	8.72
Real Investment Growth	2.24	21.03	24.37	11.94	37.27	3.17
Premium	33.42	14.07	16.58	4.51	30.06	1.34
Real Residential Investment Growth	64.08	5.62	15.97	4.95	7.31	2.05
House Prices	74.84	8.1	3.68	3.03	6.43	3.92

Figure 1: The Data



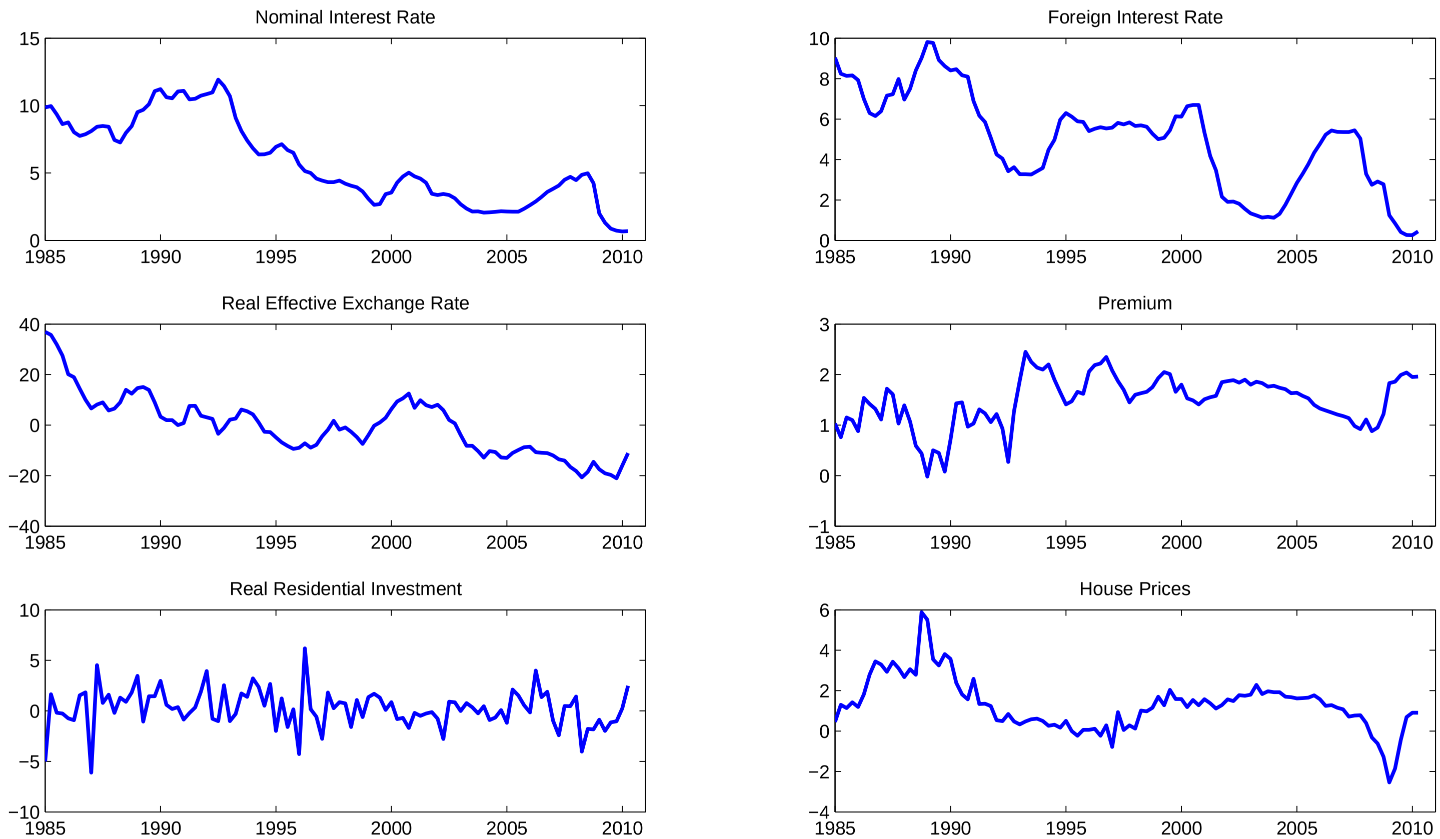
The figure shows the observable variables used in estimation of the model. See the main text for more details on the derivation of these variables.

Figure 2: The Data (Cont.)



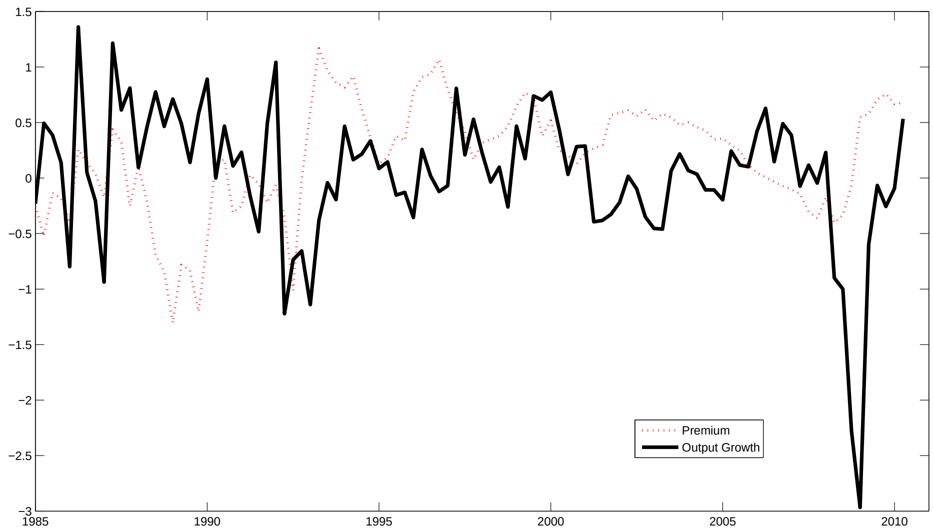
The figure shows the observable variables used in estimation of the model. See the main text for more details on the derivation of these variables.

Figure 3: The Data: Financial series



The figure shows the observable variables used in estimation of the model. See the main text for more details on the derivation of these variables.

Figure 4: Premia and Real Output Growth (Demeaned).



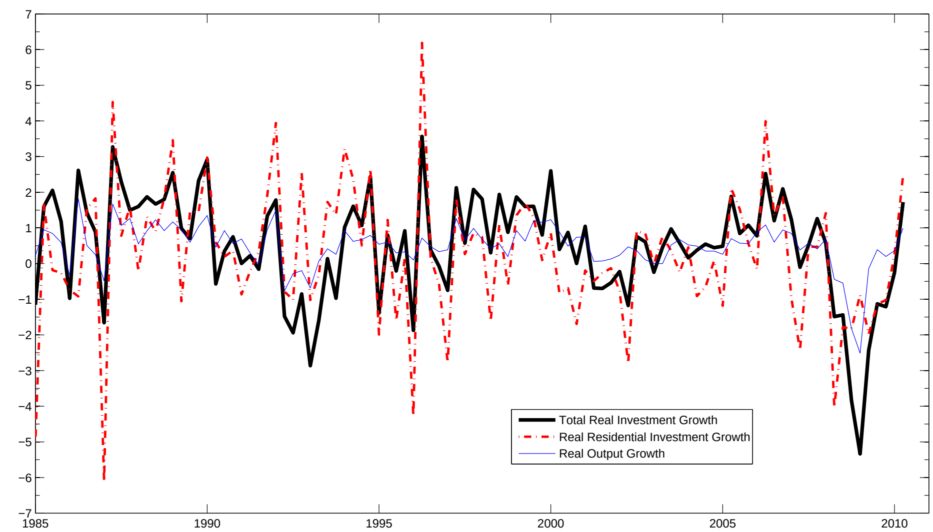
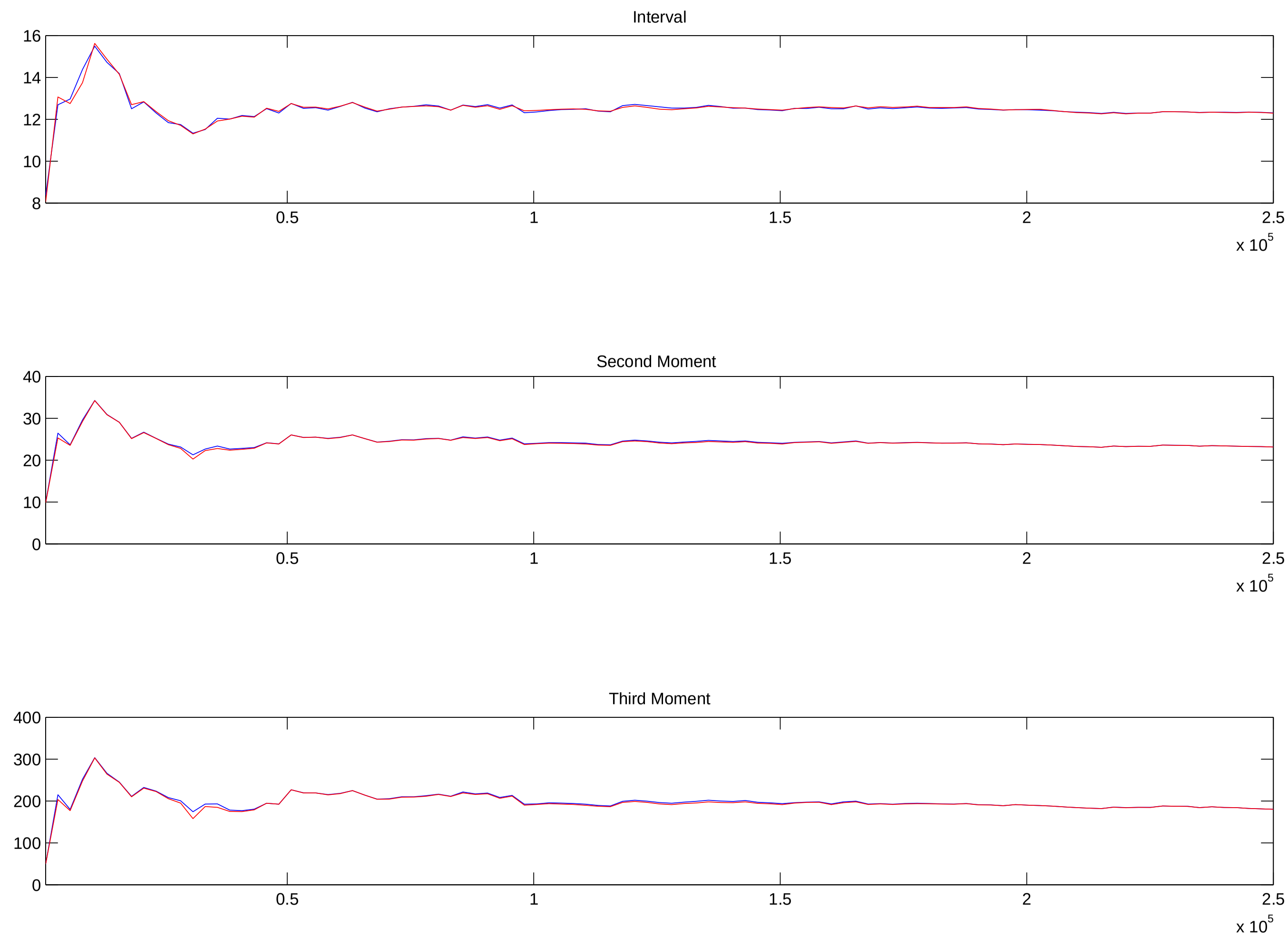


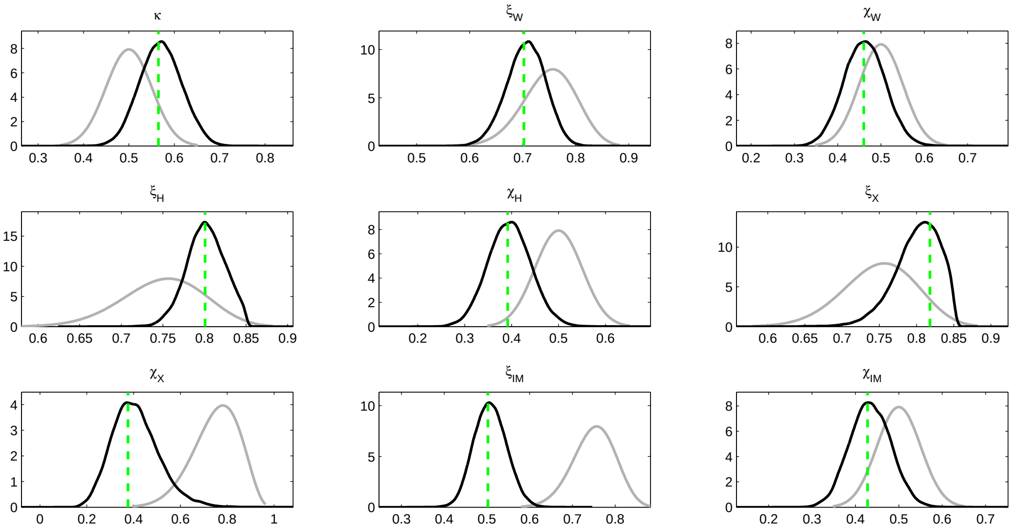
Figure 5: Total, Residential Real Investment Growth, and Real Output Growth

Figure 6: Multivariate Diagnostics



Note: the horizontal axis represents the number of Metropolis-Hastings iterations that have been undertaken, and the vertical axis the measure of the parameter moments, with the first, corresponding to the measure at the initial value of the Metropolis-Hastings iterations.

Figure 7: Priors and Posteriors



Note: The figures shows the marginal distribution of the model's structural parameters based on a Markov Chain with 250,000 draws (black line) against their marginal prior distribution (grey line).

Figure 8: Priors and Posteriors (Cont.)

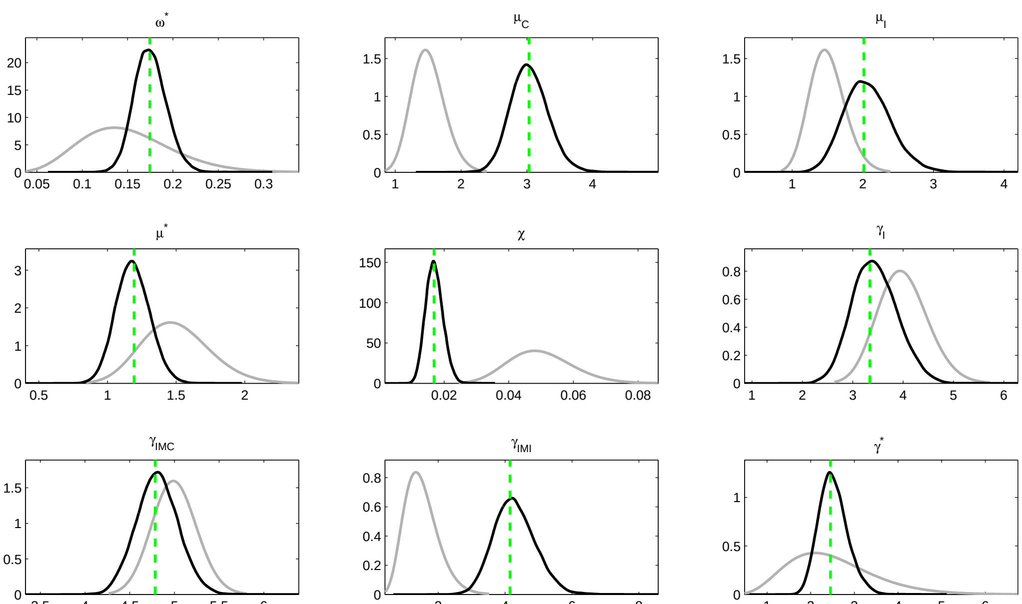


Figure 9: Priors and Posteriors (Cont.)

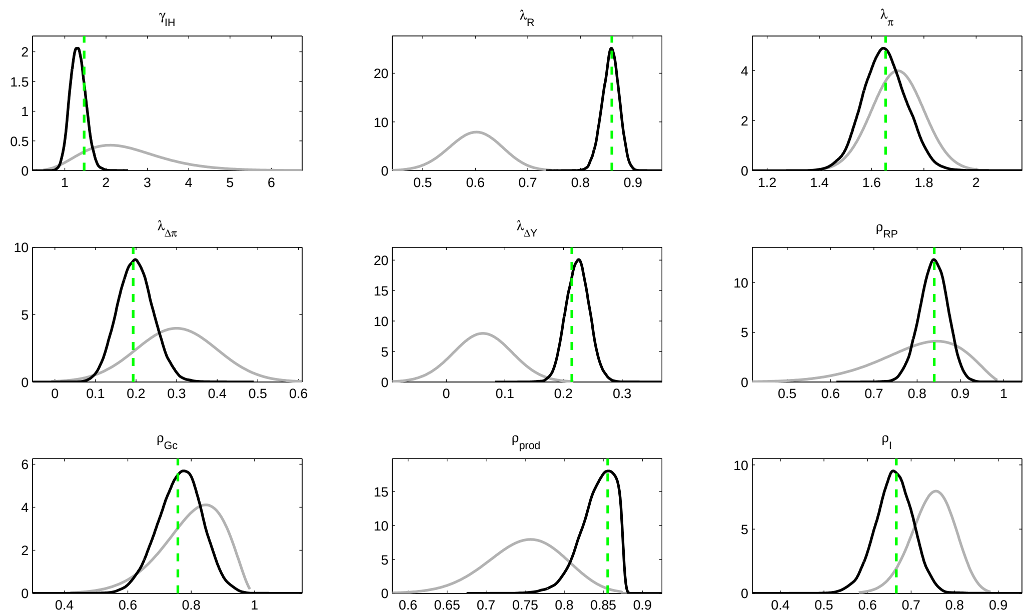


Figure 10: Priors and Posteriors (Cont.)

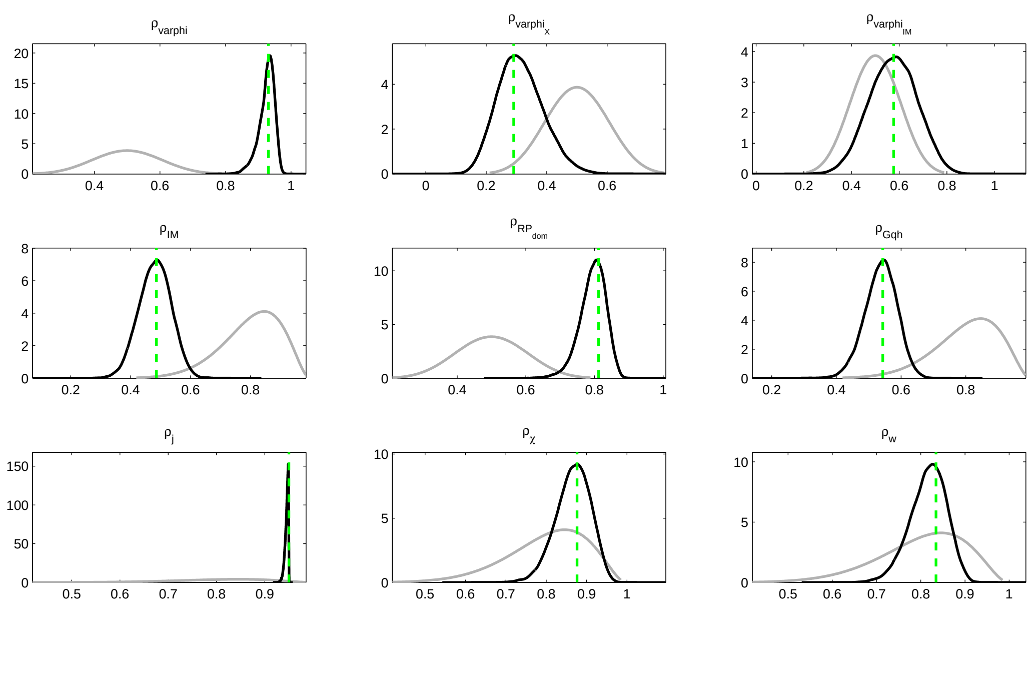


Figure 11: Priors and Posteriors (Cont.)

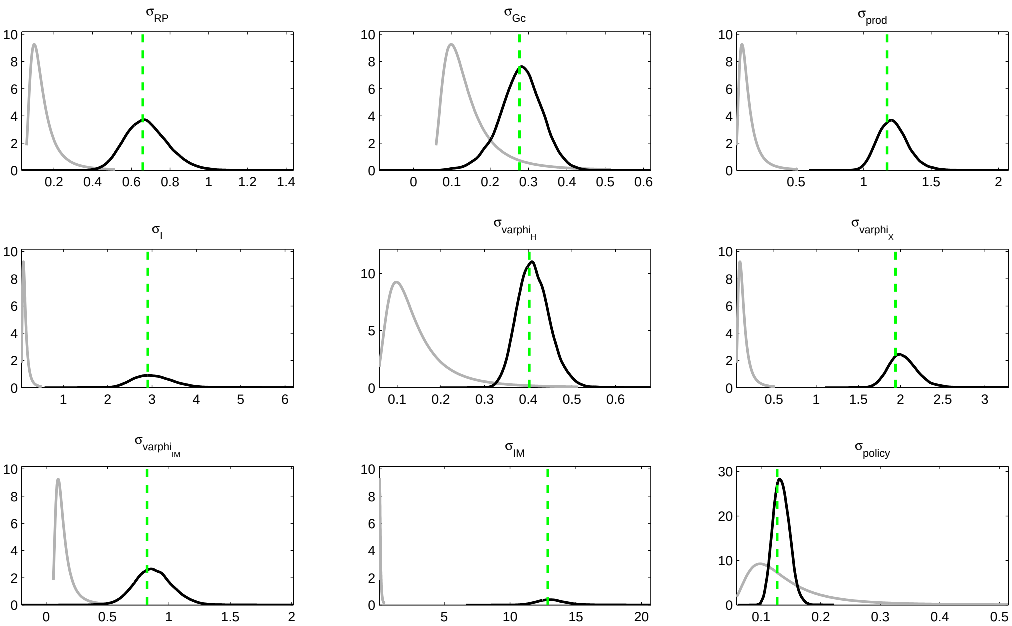


Figure 12: Priors and Posteriors (Cont.)

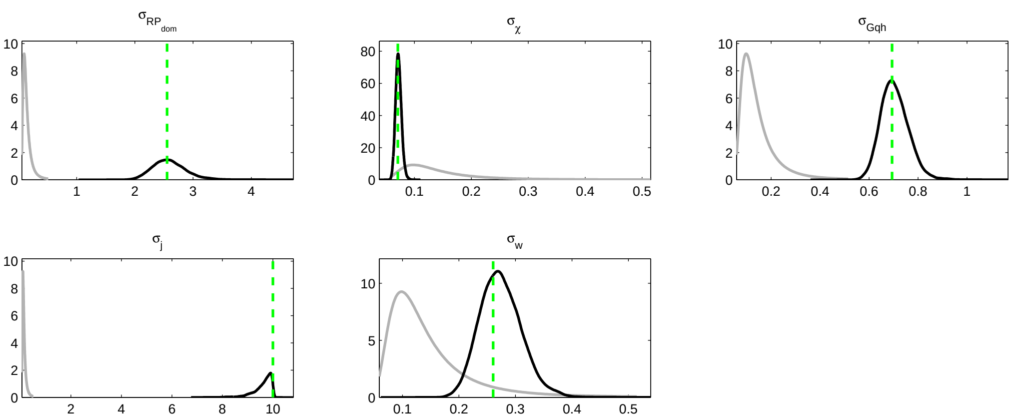
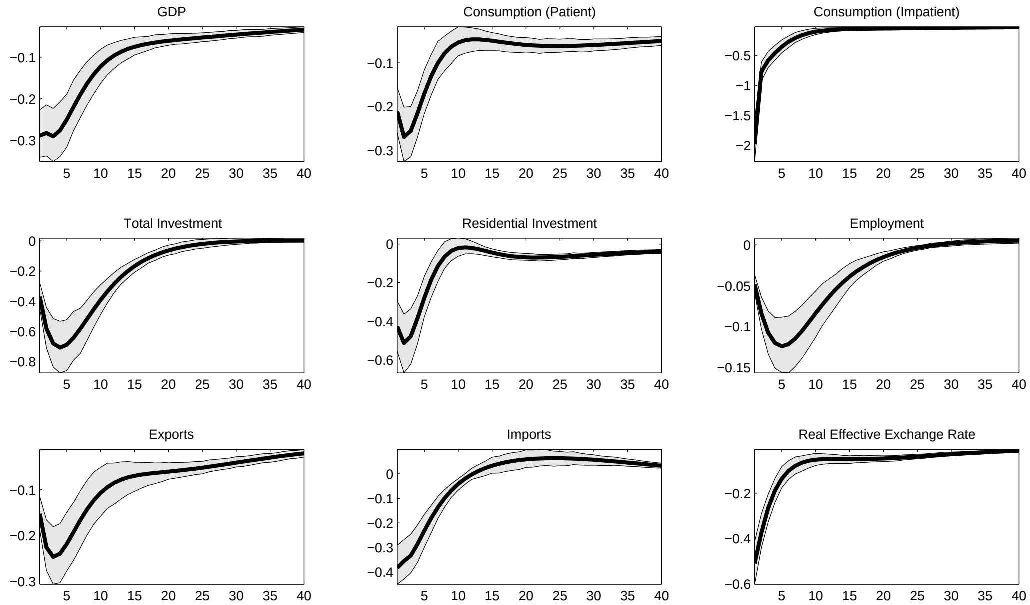
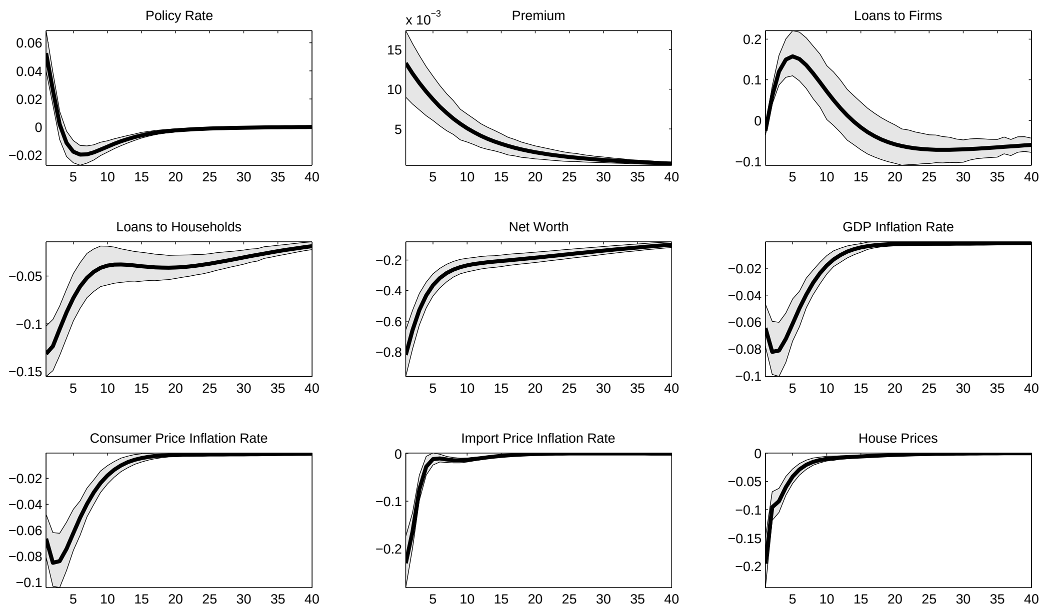


Figure 13: Temporary Monetary Policy Shock





Note: All responses are reported as percentage deviations from the non-stochastic steady, except inflation and interest rates. Shaded areas represent 95% probability bands.

Figure 14: Temporary Monetary Policy Shock (Cont.)

Figure 15: Temporary Neutral Productivity Shock

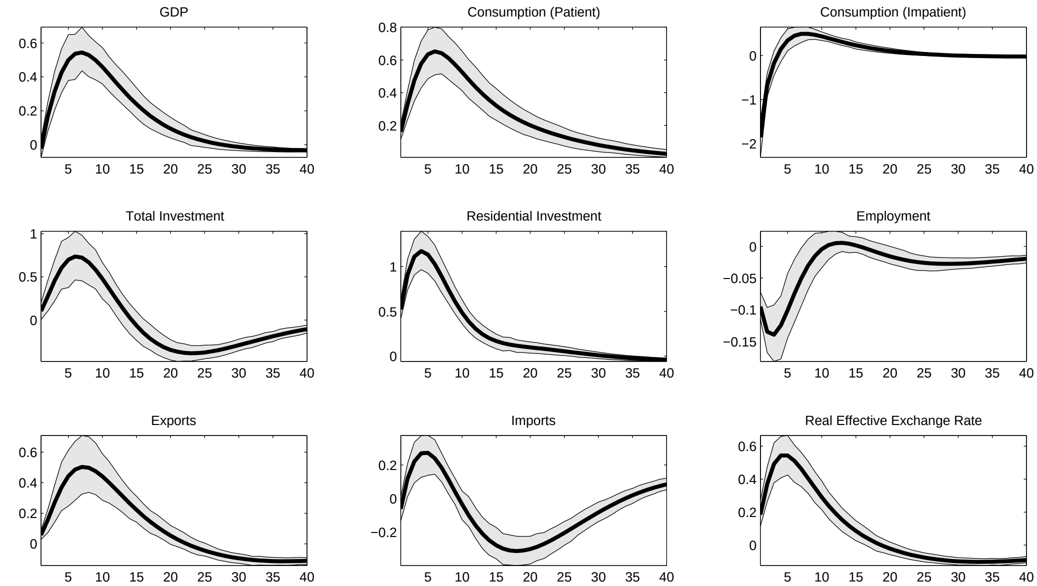
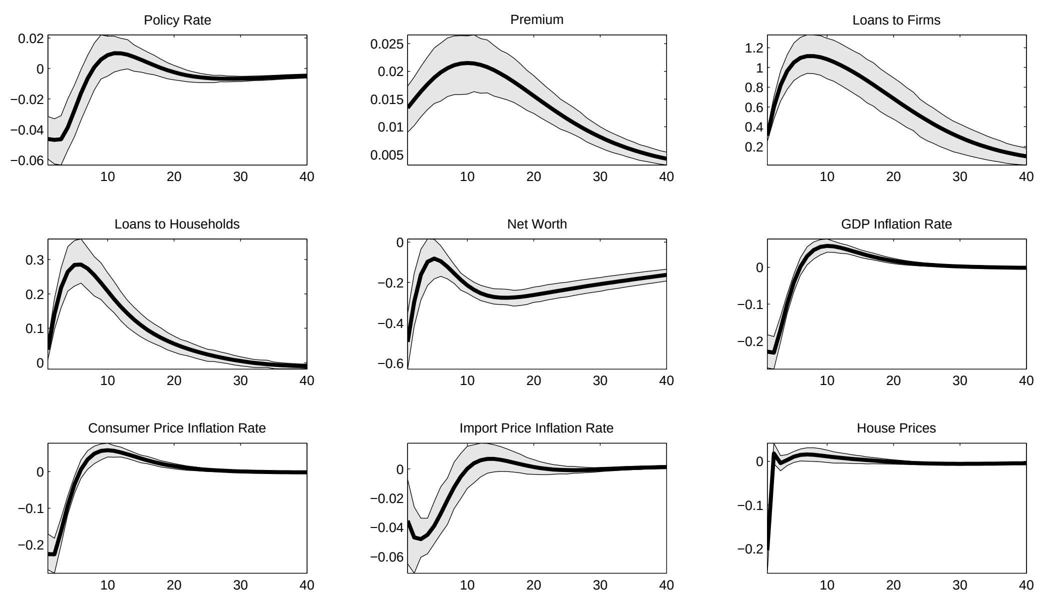
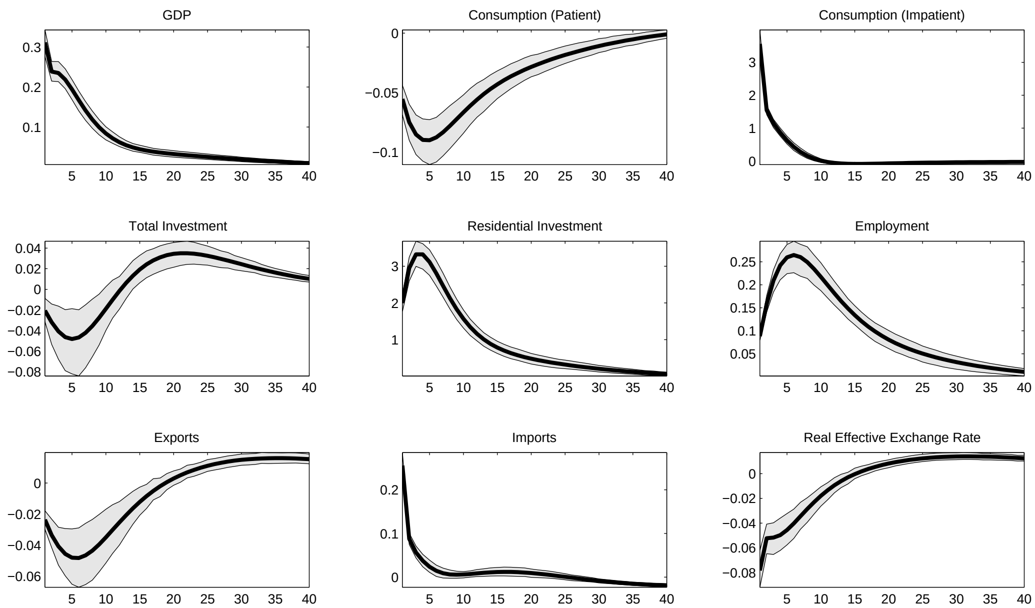


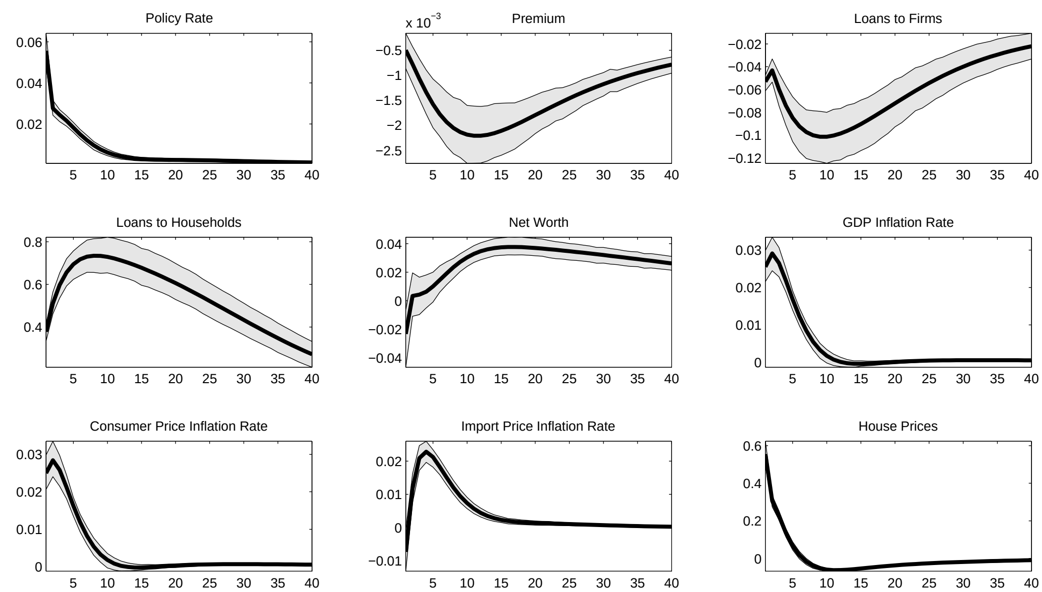
Figure 16: Temporary Neutral Productivity Shock (Cont.)



Note: See above.

Figure 17: Temporary Housing Demand Shock

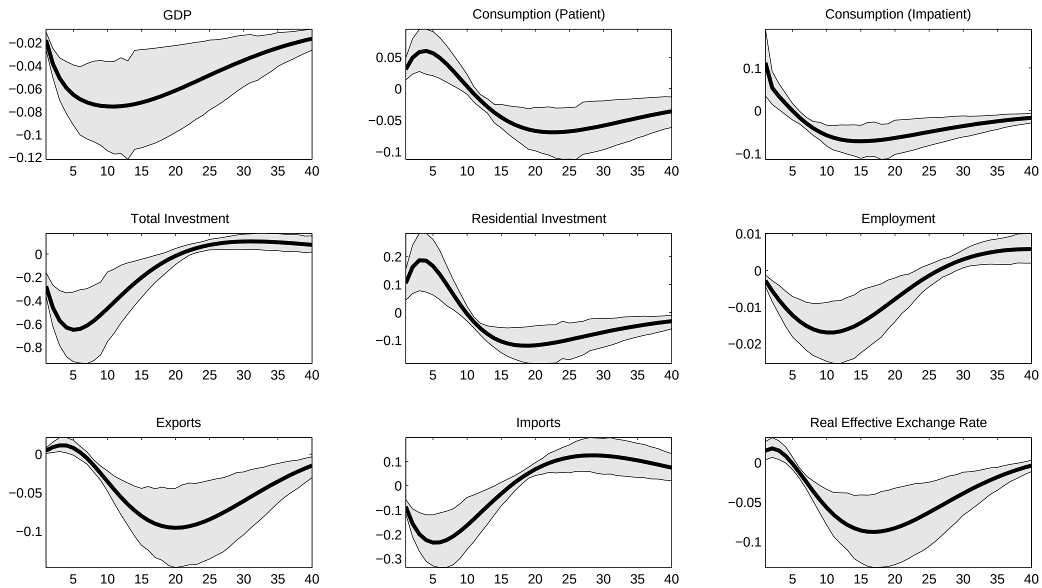


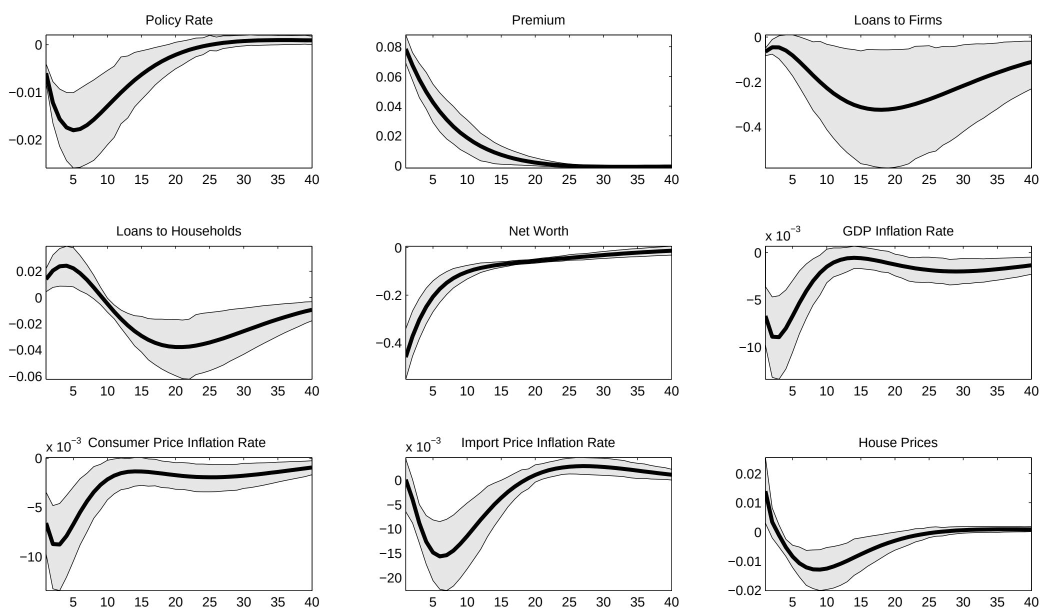


Note: See above.

Figure 18: Temporary Housing Demand Shock (Cont.)

Figure 19: Temporary Premium Shock

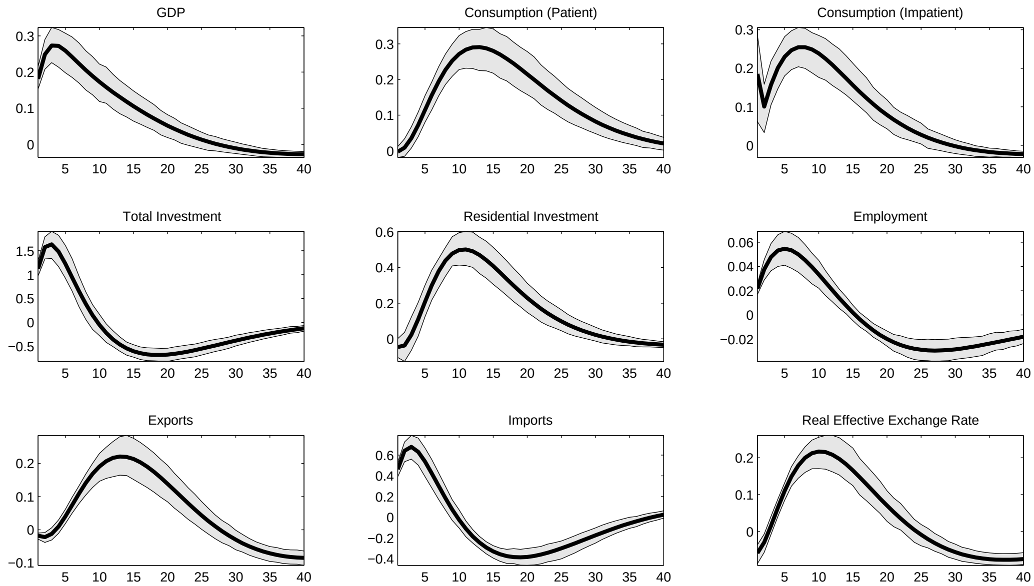


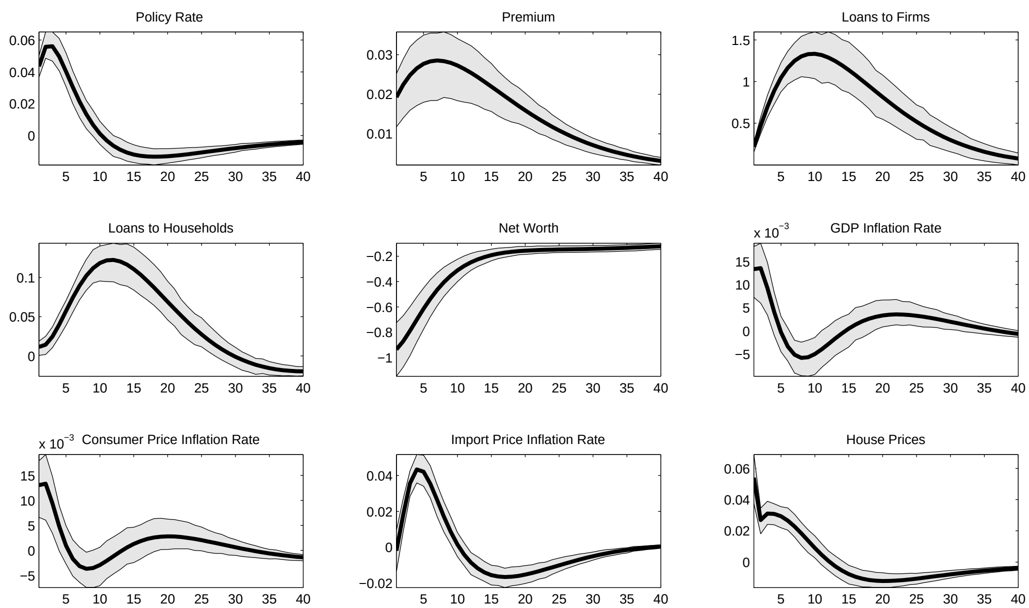


Note: See above.

Figure 20: Temporary Premium Shock (Cont.)

Figure 21: Investment-Specific Shock





Note: See above.

Figure 22: Investment-Specific Shock (Cont.)

Figure 23: Temporary Monetary Policy Shock with and without liquidity constrained agents

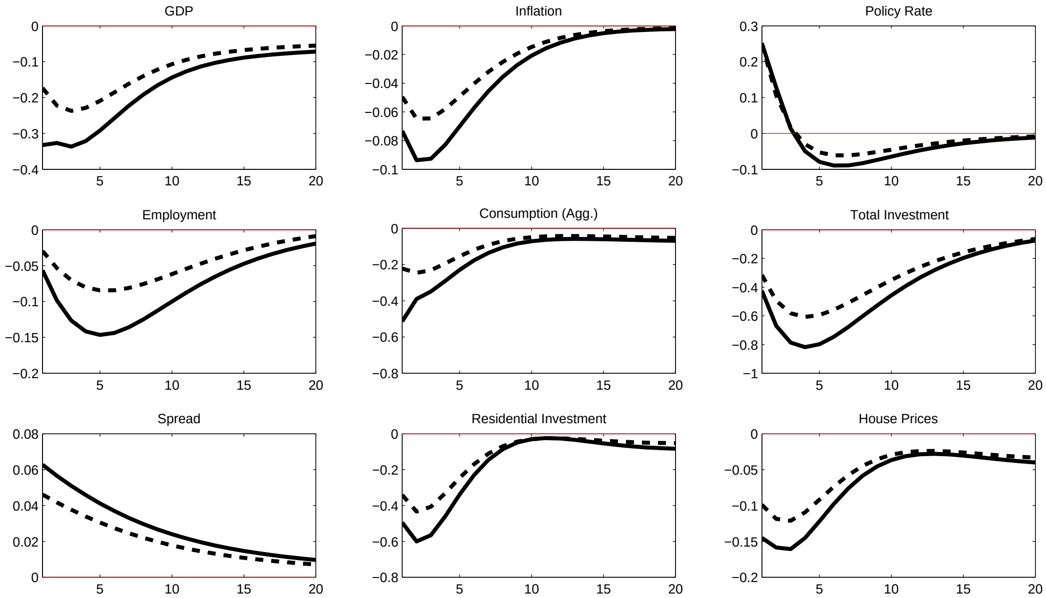
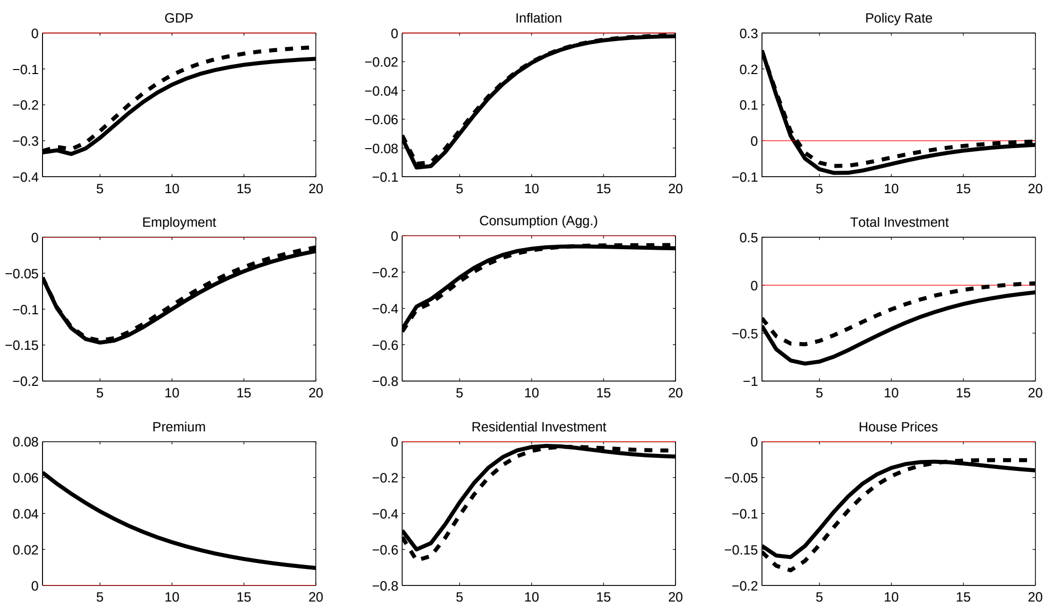
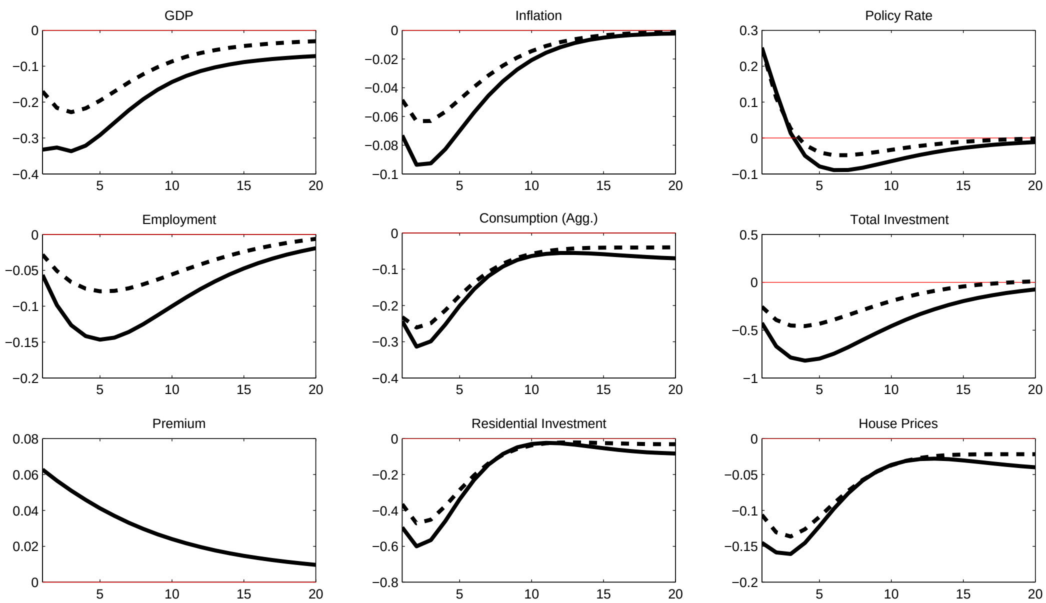


Figure 24: Temporary Monetary Policy Shock with and without External Finance Premium



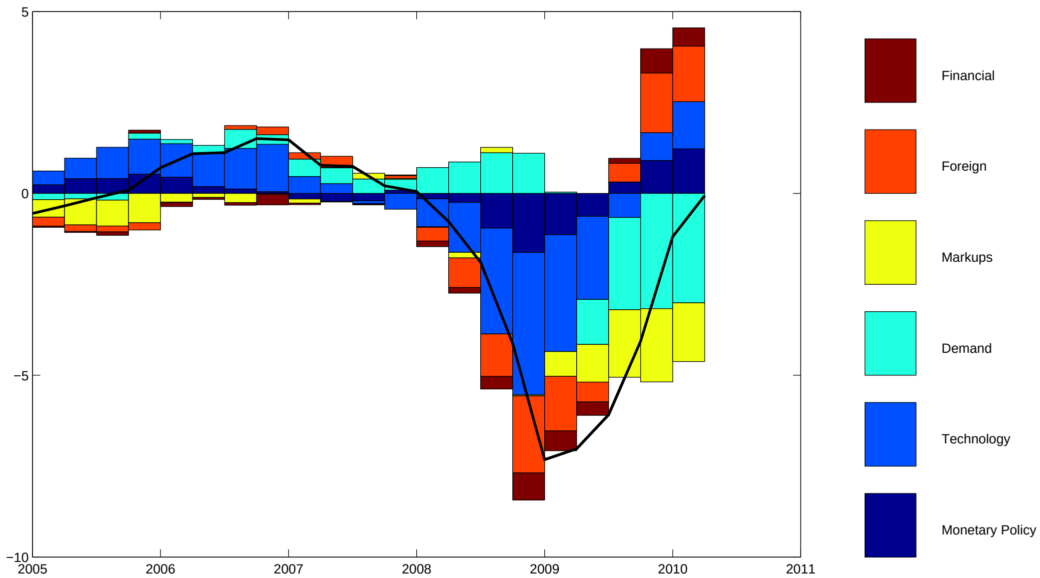
Note: See above.

Figure 25: Temporary Monetary Policy Shock with and without Financial Frictions



Note: See above.

Figure 26: Historical Decomposition of Annualized Real GDP Growth



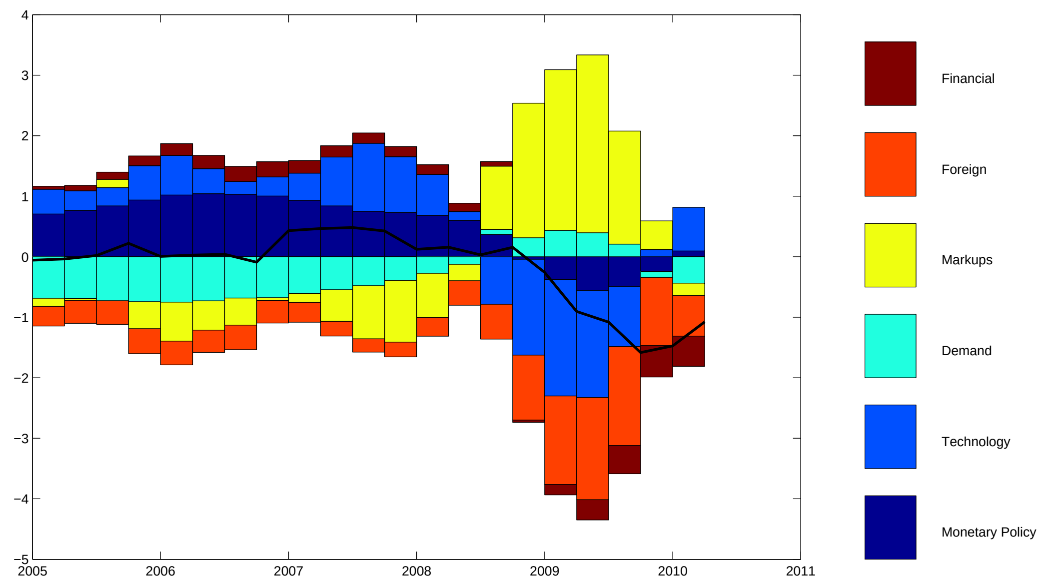
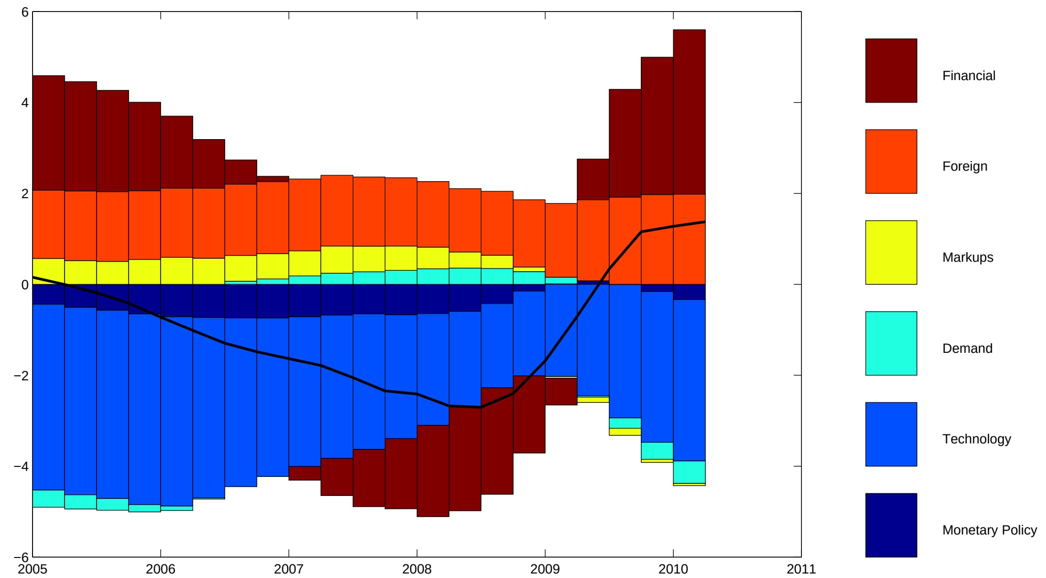


Figure 27: Historical Decomposition of Annualized GDP Deflator Inflation

Figure 28: Historical Decomposition of Annualized premium



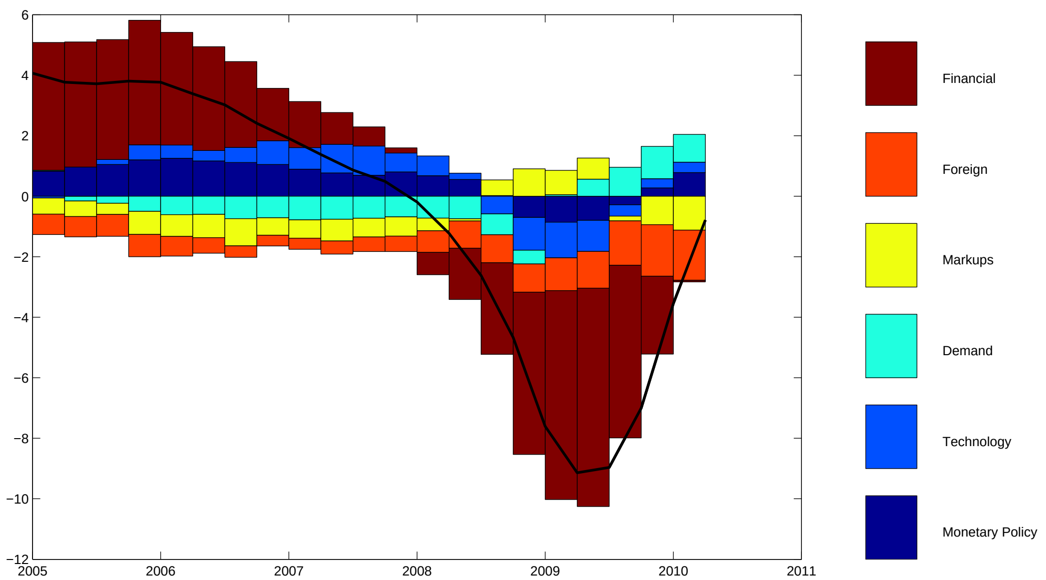


Figure 29: Historical Decomposition of Annualized House Prices

A Aggregation and market clearing

Aggregate consumption and final goods market equilibrium

$$C_t = \mathbf{p}C_{1,t} + (1-\mathbf{p})C_{2,t}; \tag{36}$$

Assuming that the cost of capital utilization is paid in final goods,

$$P_{C,t}Q_t^C = P_{C,t}C_t + P_{C,t}a(u_t)\varpi_t K_t \tag{37}$$

Aggregate intermediate goods demand

$$H_t = H_t^C + H_t^I + H_t^G + H_t^H \tag{38}$$

where

$$H_t^C = \nu_C \left(\frac{P_{H,t}}{P_{C,t}}\right)^{-\mu_c} Q_t^C \tag{39}$$

$$Q_t^C = \left(\nu_C^{\frac{1}{\mu_C}} H_t^{C\,1-\frac{1}{\mu_C}} + (1-\nu_C)^{\frac{1}{\mu_C}} \left(\left(1-\Gamma_{IM^C}\left(IM_t^C/Q_t^C;\epsilon_t^{IM}\right)\right) IM_t^C\right)^{1-\frac{1}{\mu_C}}\right)^{\frac{\mu_C}{\mu_C-1}} \tag{40}$$

where Γ_{IM^C} is a cost of import adjustment

$$\Gamma_{IM^C}\left(IM_t^C/Q_t^C;\epsilon_t^{IM}\right) \equiv \frac{\gamma_{IM}^C}{2}\left(\epsilon_t^{IM}\frac{IM_t^C/Q_t^C}{IM_{t-1}^C/Q_{t-1}^C} - 1\right)^2 \tag{41}$$

$$P_{C,t} = \left(\nu_C P_{H,t}^{1-\mu_C} + (1-\nu_C)\left(\frac{P_{IM,t}}{\Gamma_{IM^C}^{\dagger}\left(IM_t^C/Q_t^C;\epsilon_t^{IM}\right)}\right)^{1-\mu_C}\right)^{\frac{1}{1-\mu_C}} \tag{42}$$

and where

$$\Gamma_{IM^C}^{\dagger}\left(\cdot\right) \equiv 1 - \Gamma_{IM^C}\left(\cdot\right) - \Gamma'_{IM^C}\left(\cdot\right) IM_t^C \tag{43}$$

Aggregate demand for imports

$$IM_t = IM_t^C + IM_t^I \tag{44}$$

where

$$IM_t^C = (1-\nu_C)\left(\frac{P_{IM,t}}{P_{C,t}\Gamma^{\dagger}_{IM^C}\left(IM_t^C/Q_t^C;\epsilon_t^{IM}\right)}\right)^{-\mu_C}\frac{Q_t^C}{1-\Gamma_{IM^C}\left(IM_t^C/Q_t^C;\epsilon_t^{IM}\right)} \tag{45}$$

$$IM_t^I = (1-\nu_I)\left(\frac{P_{IM,t}}{P_{I,t}\Gamma^{\dagger}_{IM^I}\left(IM_t^I/Q_t^I;\epsilon_t^{IM}\right)}\right)^{-\mu_I}\frac{Q_t^I}{1-\Gamma_{IM^I}\left(IM_t^I/Q_t^I;\epsilon_t^{IM}\right)} \tag{46}$$

Capital market clearing condition

$$u_t K_t = K_t^s + K_t^H \tag{47}$$

Intermediate-goods market clearing

$$P_{Y,t}Y_t = P_{H,t}H_t + P_{X,t}X_t \tag{48}$$

Exports

$$X_t = \nu_t^*\left(\frac{S_t P_{X,t}}{P_{X,t}^{c,*}\Gamma_X^{\dagger}\left(\frac{X_t}{Y_t^{d,*}};\epsilon_t^X\right)}\right)^{-\mu^*}\frac{Y_t^{d,*}}{1-\Gamma_X\left(X_t/Y_t^{d,*};\epsilon^{d,*}\right)} \tag{49}$$

where

$$\Gamma_X(\cdot) \equiv \frac{\gamma^*}{2}\left(\epsilon_t^X\frac{X_t/Y_t^{d,*}}{X_{t-1}/Y_{t-1}^{d,*}}-1\right)^2 \tag{50}$$

and

$$\Gamma_X^{\dagger}(\cdot) \equiv 1-\Gamma_X(\cdot)-\Gamma_X'(\cdot)X_t \tag{51}$$

Private-Sector Aggregate Budget Constraint: NFA position

$$\mathbf{p}\left(R_t^*\left(1-\Gamma_{B^*}\left(S_tB_{t+1}^*\right)\right)\right)^{-1}\frac{S_t}{P_{C,t}}B_{t+1}^* = \mathbf{p}\frac{S_tB_t^*}{P_{C,t}} + P_{H,t}X_t - P_{IM,t}IM_t$$

B The non-linear model

Here we list all the variables and equations that constitute the non-linear model.

B.1 List of variables

I)	Consumption Aggregated	C_t
II)	Consumption Patient H/h	$C_{1,t}$
III)	Consumption Impatient H/h	$C_{2,t}$
IV)	Housing Patient H/h	$\mathcal{H}_{1,t}$
V)	Housing Impatient H/h	$\mathcal{H}_{2,t}$
VI)	Housing Aggregated	$\mathcal{H}_t$
VII)	Lagrange multiplier borrowing constraint	λ_t^B
VIII)	Lagrange multiplier H/h patient	$\lambda_t^{\mathbf{P}}$
IX)	Lagrange multiplier H/h impatient	λ_t
X)	Capital Intermediate goods	K_t^s
XI)	Capital Housing investment sector	K_t^H
XII)	Capital Aggregated	K_t
XIII)	Housing investment	IH_t
XIV)	Capital goods investment	I_t
XV)	Capital utilization	u_t
XVI)	Net worth	N_t
XVII)	Transfer to entrepreneurs	T_t^γ
XVIII)	Labor Aggregated type 1	$\mathcal{N}_{1,t}$
XIX)	Labor Aggregated type 2	$\mathcal{N}_{2,t}$
XX)	Labor Aggregated housing	$N_{h,t}$
XXI)	Wage Aggregated housing	$W_{h,t}$
XXII)	Import consumption goods sector	IM_t^C
XXIII)	Import investment goods sector	IM_t^I
XXIV)	Import Aggregated	IM_t
XXV)	Demand final consumption goods	Q_t^C
XXVI)	Demand final investment goods	Q_t^I
XXVII)	Demand/supply Government goods	H_t^G
XXVIII)	Demand/supply intermediate housing goods	H_t^H
XXIX)	Demand consumption goods inputs	H_t^C
XXX)	Demand investment goods inputs	H_t^I
XXXI)	Aggregate demand for intermediate goods	H_t
XXXII)	Marginal return on capital	r_t^K
XXXIII)	Price of housing	$q_{h,t}$
XXXIV)	Marginal cost intermediate firms	MC_t
XXXV)	Marginal cost housing investment	$MC_{IH,t}$
XXXVI)	Total return on capital	R_t^K
XXXVII)	Tobin's q	$q_{k,t}$
XXXVIII)	Risk free interest rate	R_t
XXXIX)	Entrepreneurs' loans	D_t^B
XL)	Housing credit	B_t^H

XLI)	NFA position	B_t^*
XLII)	Government bonds	B_t
XLIII)	Government spending	G_t
XLIV)	Price final intermediate goods	$P_{H,t}$
XLV)	Price Investment goods	$P_{I,t}$
XLVI)	Price of export	$P_{X,t}$
XLVII)	Price of import	$P_{IM,t}$
XLVIII)	REAL exchange rate	REX_t
XLIX)	Calvo's numerator intermediate	$\mathbb{Q}_{1,t}$
L)	Calvo's denominator intermediate	$\mathbb{Q}_{2,t}$
LI)	Calvo's numerator export	$\mathbb{Q}_{1,t}^X$
LII)	Calvo's denominator intermediate	$\mathbb{Q}_{2,t}^X$
LIII)	Calvo's numerator import	$\mathbb{Q}_{1,t}^{IM}$
LIV)	Calvo's denominator import	$\mathbb{Q}_{2,t}^{IM}$
LV)	Calvo's numerator Wage type 1	$\mathbb{Q}_{1,t}^{W,1}$
LVI)	Calvo's denominator Wage type 1	$\mathbb{Q}_{2,t}^{W,1}$
LVII)	Calvo's numerator Wage type 2	$\mathbb{Q}_{1,t}^{W,2}$
LVIII)	Calvo's denominator Wage type 2	$\mathbb{Q}_{2,t}^{W,2}$
LIX)	Calvo's numerator Wage type H	$\mathbb{Q}_{1,t}^{W,H}$
LX)	Calvo's denominator Wage type H	$\mathbb{Q}_{2,t}^{W,H}$
LXI)	Aggregate Wage type 1	$\bar{W}_{1,t}$
LXII)	Aggregate Wage type 2	$\bar{W}_{2,t}$
LXIII)	Export	X_t
LXIV)	Intermediate goods price index	$P_{Y,t}$
LXV)	Intermediate goods output	Y_t
LXVI)	Inflation home	π_t
LXVII)	Inflation export goods	π_t^X
LXVIII)	Real ex-ante rate foreign	RR_t^*
LXIX)	Wage inflation type 1	$\pi_t^{W,1}$
LXX)	Wage inflation type 2	$\pi_t^{W,2}$
LXXI)	Intermediate goods inflation	$\pi_{H,t}$
LXXII)	Import goods inflation	$\pi_{IM,t}$

B.2 List of non-linear equations in detrended real form

Here we allow for a possibly stochastic trend so that e.g. $G_{C,t} = \ln z_t - \ln z_{t-1}$. With abuse of notation the Lagrange multipliers have the same symbol for detrended and trending variable. The same is for housing and all the variables that were denoted with small letters. Detrended variables are denoted with small letters, e.g. $x = \frac{X}{z_x}$ where $z_x$is the trend in X. Everything is expressed in real terms, i.e. relative to consumer goods.

$$k_{t+1} = (1-\delta)G_{C,t}^{-1}k_t + \epsilon_t^I \left(1 - \Gamma_I\left(G_{C,t}\frac{i_t}{i_{t-1}}\right)\right) i_t \tag{I}$$

$$\lambda_t^{\mathbf{P}} = \beta_1 \mathbb{E}_t \left[\lambda_{t+1}^{\mathbf{P}} G_{C,t+1}^{-1} \frac{R_t}{\pi_{t+1}}\right] \lambda_t^{\mathbf{P}} \tag{II}$$

$$REX_t = \beta_1 \left(\left(1 - \Gamma_{B^*} \left(s_{B^*,t+1} \right) \right) \right) \mathbb{E}_t \left[\lambda_{t+1} G_{C,t+1}^{-1} REX_{t+1} R_t^* \right] \tag{III}$$

$$\lambda_t^{\mathbf{p}} (1 + \tau^C) = \left(1 - \frac{\kappa}{G_C} \right) \left(c_{1,t} - \frac{\kappa}{G_{C,t}} c_{t-1} \right)^{-1} \tag{IV}$$

$$j_{1,t}^H \mathcal{H}_{1,t}^{-1} + \beta_1 \mathbb{E}_t \left[G_{C,t+1}^{-1} G_{QH,t+1}^{-1} \lambda_{t+1}^{\mathbf{p}} (1 - \delta_h) q_{h,t+1} \right] = \lambda_t^{\mathbf{p}} q_{h,t} \tag{V}$$

$$\begin{aligned} \lambda_t^{\mathbf{p}} p_{I,t} \quad = \quad & q_{k,t} \lambda_t^{\mathbf{p}} \epsilon_t^I \left(\left(1 - \Gamma_I \left(G_{C,t} \frac{i_t}{i_{t-1}} \right) \right) - \Gamma_I'(\cdot) G_{C,t} \frac{i_t}{i_{t-1}} \right) \\ & + \mathbb{E}_t \left[q_{K,t+1} \beta_1 \frac{\lambda_{t+1}^{\mathbf{p}}}{G_{C,t+1}} \epsilon_{t+1}^I \left(\Gamma_I' \left(G_{C,t+1} \frac{i_{t+1}}{i_t} \right) \left(G_{C,t+1} \frac{i_{t+1}}{i_t} \right)^2 \right) \right] \end{aligned} \tag{VI}$$

$$b_t^H = m_t^B \mathbb{E}_t \left(\frac{q_{h,t+1} \mathcal{H}_{2,t}}{G_{QH,t+1} \left(1 + R_t^{DH} \right)} \right) \tag{VII}$$

$$\lambda_t (1 + \tau^C) = \left(1 - \frac{\kappa}{G_C} \right) \left(C_{2,t} - \frac{\kappa}{G_{C,t}} C_{t-1} \right)^{-1} \tag{VIII}$$

$$\lambda_t = \beta_2 \mathbb{E}_t \left[G_{C,t+1}^{-1} \lambda_{t+1} \frac{\left(1 + R_t^{DH} \right)}{\pi_{t+1}} \right] + \lambda_t^B \tag{IX}$$

$$\begin{aligned} \lambda_t q_{h,t} = & \, j_{2,t}^H \mathcal{H}_{2,t}^{-1} + \beta_2 \mathbb{E}_t \left[G_{C,t+1}^{-1} G_{QH,t+1}^{-1} \lambda_{t+1} (1 - \delta_h) q_{h,t+1} \right] \\ & + m_t^B \lambda_t^B \mathbb{E}_t \left(G_{QH,t+1}^{-1} \frac{q_{h,t+1} \pi_{t+1}}{\left(1 + R_t^{DH} \right)} \right) \end{aligned} \tag{X}$$

$$N_{H,t}^{\zeta} = \lambda_t w_{H,t} \tag{XI}$$

$$r_t^K = \alpha_K \frac{y_t + \psi}{k_t^s} mc_t \tag{XII}$$

$$\left(1 + \tau^{Wf} \right) w_{1,t} = \alpha_{N1} \frac{y_t + \psi}{\mathcal{N}_{1,t}} mc_t \tag{XIII}$$

$$\left(1 + \tau^{Wf} \right) w_{2,t} = \alpha_{N2} \frac{y_t + \psi}{\mathcal{N}_{2,t}} mc_t \tag{XIV}$$

$$mc_t = \frac{r_t^{K \alpha_K} \left(1 + \tau^{Wf} \right)^{\alpha_{N1} + \alpha_{N2}} w_{1,t}^{\alpha_{N1}} w_{2,t}^{\alpha_{N2}}}{\left(\varepsilon_t \alpha_K^{\alpha_K} \alpha_{N1}^{\alpha_{N1}} \alpha_{N2}^{\alpha_{N2}} \right)} \tag{XV}$$

$$1 = \left(\nu_C p_{H,t}^{1-\mu_C} + (1 - \nu_C) \left(\frac{p_{IM,t}}{\Gamma_{IM^C}^{\dagger} \left(IM_t^C / Q_t^C ; \epsilon_t^{IM} \right)} \right)^{1-\mu_C} \right)^{\frac{1}{1-\mu_C}} \tag{XVI}$$

$$p_{I,t} = \left(\nu_I p_{H,t}^{1-\mu_I} + (1 - \nu_I) \left(\frac{p_{IM,t}}{\Gamma_{IM^I}^{\dagger} \left(IM_t^I / Q_t^I ; \epsilon_t^{IM} \right)} \right)^{1-\mu_I} \right)^{\frac{1}{1-\mu_I}} \tag{XVII}$$

$$\mathcal{H}_t = (1-\delta_h)\left(G_{C,t}G_{QH,t}\right)^{-1}\mathcal{H}_{t-1} + ih_t \tag{XVIII}$$

$$r_t^K = mc_{IH,t}\omega_K G_{C,t}\frac{ih_t}{k_t^H} \tag{XIX}$$

$$(1+\tau^{Wf})w_{H,t} = mc_{IH,t}\omega_N\frac{ih_t}{N_{H,t}} \tag{XX}$$

$$p_{H,t} = mc_{IH,t}\omega_H\frac{ih_t}{h_t^H} \tag{XXI}$$

$$MC_{IH,t} = \left(\frac{\left(r_t^K\right)^{\omega_K} w_{N,t}^{\omega_N} p_{H,t}^{\omega_H}}{\varepsilon_t^H \omega_K^{\omega_K}\omega_N^{\omega_N}\omega_H^{\omega_H}} ih_t^{\omega_L}\right)^{\frac{1}{1-\omega_L}} \tag{XXII}$$

$$q_{h,t} = mc_{IH,t} \tag{XXIII}$$

$$\mathbb{E}_t\left[R_{t+1}^K\right] = \mathbb{E}_t\left[\frac{\left((1-\tau_{t+1}^k)\left(r_{t+1}^k u_{t+1} - a(u_{t+1})\right) + (1-\delta)q_{k,t+1}\right) + \tau_{t+1}^k\delta q_{k,t+1}}{q_{k,t}}\right] \tag{XXIV}$$

$$\mathbb{E}_t\left[R_{t+1}^K\right] = (\chi_t(\cdot))\mathbb{E}_t\left[\frac{1+R_t^{DB}}{\pi_{t+1}}\right] \tag{XXV}$$

$$n_{t+1}G_{C,t} = \gamma_t\left\{R_t^K q_{K,t-1}k_t - \left[\chi_{t-1}(\cdot)\frac{\left(1+R_{t-1}^{DB}\right)}{\pi_t}\right]d_{t-1}^B\right\} + (1-\gamma_t)t_t^e \tag{XXVI}$$

$$d_t^B = q_{k,t}k_{t+1} - n_{t+1} \tag{XXVII}$$

$$t_t^\gamma G_{C,t} = (1-\gamma_t)\left\{R_t^K q_{K,t-1}k_t - \left[\chi_{t-1}(\cdot)\frac{\left(1+R_{t-1}^{DB}\right)}{\pi_t}\right]d_{t-1}^B\right\} \tag{XXVIII}$$

$$u_t k_t = k_t^s + k_t^H \tag{XXIX}$$

$$r_t^K = a'(u_t) \tag{XXX}$$

$$h_t^C = \nu_C\left(p_{H,t}\right)^{-\mu_c} q_t^C \tag{XXXI}$$

$$h_t^I = \nu_I\left(p_{H,t}\right)^{-\mu_I} q_t^I \tag{XXXII}$$

$$im_t^C = (1-\nu_C)\left(\frac{p_{IM,t}}{\Gamma_{IM^C}^\dagger\left(im_t^C/q_t^C;\epsilon_t^{IM}\right)}\right)^{-\mu_C}\frac{q_t^C}{1-\Gamma_{IM^C}\left(im_t^C/q_t^C;\epsilon_t^{IM}\right)} \tag{XXXIII}$$

$$im_t^I = (1-\nu_I)\left(\frac{p_{IM,t}}{p_{I,t}\Gamma_{IM^I}^\dagger\left(im_t^I/q_t^I;\epsilon_t^{IM}\right)}\right)^{-\mu_I}\frac{q_t^I}{1-\Gamma_{IM^I}\left(im_t^I/q_t^I;\epsilon_t^{IM}\right)} \tag{XXXIV}$$

$$x_t = \nu_t^*\left(\frac{REX_t p_{X,t}}{p_{X,t}^{c,*}\Gamma_X^\dagger\left(\frac{X_t}{Y_t^{d,*}};\epsilon_t^X\right)}\right)^{-\mu^*}\frac{y_t^{d,*}}{1-\Gamma_X\left(x_t/y_t^{d,*};\epsilon^{d,*}\right)} \tag{XXXV}$$

$$\mathbf{p}\left(R_t^*\left(1-\Gamma_{B^*}\left(s_{B^*,t+1}\right)\right)\right)^{-1}\frac{S_t}{P_{C,t}}B_{t+1}^* = \mathbf{p}\frac{S_t B_t^*}{P_{C,t}} + P_{H,t}X_t - P_{IM,t}IM_t \tag{XXXVI}$$

$$\bar{p}_{H,f,t} = \varphi_t^H \frac{\tilde{\mathbb{Q}}_{1,t}}{\tilde{\mathbb{Q}}_{2,t}}$$

$$\tilde{\mathbb{Q}}_{1,t} = (1-\xi_H\beta_1)mc_t\left(p_{H,t}\right)^{\theta_t^H} q_t^C + \xi_H \mathbb{E}_t \overline{R}_{t+1,t}\left(\pi_{H,t}{}^{\chi_H}\bar{\pi}_{t+1}^{1-\chi_H}\right)^{-\theta_H}\pi_{t+1}^{\theta_H} G_{C,t+1}\tilde{\mathbb{Q}}_{1,t+1} \tag{XXXVII}$$

$$\tilde{\mathbb{Q}}_{2,t} = (1-\xi_H\beta_1)\left(p_{H,t}\right)^{\theta_t^H} q_t^C + \xi_H \mathbb{E}_t \bar{R}_{t+1,t}\left(\pi_{H,t}{}^{\chi_H}\bar{\pi}_{t+1}^{1-\chi_H}\right)^{1-\theta_H}\pi_{t+1}^{\theta_H-1} G_{C,t+1}\tilde{\mathbb{Q}}_{2,t+1} \tag{XXXVIII}$$

$$p_{H,t} = \left[(1-\xi_H)\left(\bar{p}_{H,f,t}\right)^{1-\theta_t^H} + \xi_H\left(\frac{\left(\bar{\pi}_t\right)^{1-\chi_H}\left(\pi_{t-1}^H\right)^{\chi_H}}{\pi_t}p_{H,t-1}\right)^{1-\theta_t^H}\right]^{\frac{1}{1-\theta_t^H}} \tag{XXXIX}$$

$$p_{X,f,t} = \varphi_t^X \frac{\tilde{\mathbb{Q}}_{1,t}^X}{\tilde{\mathbb{Q}}_{2,t}^X}$$

$$\tilde{\mathbb{Q}}_{1,t}^X = (1-\xi_X\beta_1)mc_t\left(p_{X,t}\right)^{\theta_t^X} x_t + \xi_X \mathbb{E}_t \overline{R}_{t+1,t}\left(\pi_{X,t}{}^{\chi_X}\bar{\pi}_{t+1}^{1-\chi_X}\right)^{-\theta_X}\pi_{t+1}^{\theta_X} G_{C,t+1}\tilde{\mathbb{Q}}_{1,t+1}^X \tag{XL}$$

$$\tilde{\mathbb{Q}}_{2,t}^X = (1-\xi_X\beta_1)\left(p_{X,t}\right)^{\theta_t^X} x_t + \xi_X \mathbb{E}_t \bar{R}_{t+1,t}\left(\pi_{X,t}{}^{\chi_X}\bar{\pi}_{t+1}^{1-\chi_X}\right)^{1-\theta_X}\pi_{t+1}^{\theta_X-1} G_{C,t+1}\tilde{\mathbb{Q}}_{2,t+1}^X \tag{XLI}$$

$$p_{X,t} = \left[(1-\xi_X)\left(p_{X,f,t}\right)^{1-\theta_t^X} + \xi_X\left(\frac{\left(\bar{\pi}_t\right)^{1-\chi_X}\left(\pi_{t-1}^X\right)^{\chi_X}}{\pi_t}p_{X,t-1}\right)^{1-\theta_t^X}\right]^{\frac{1}{1-\theta_t^X}} \tag{XLII}$$

$$p_{IM,f,t} \equiv \frac{P_{IM,f,t}}{S_t P_{C,t}^*} = \varphi_t^{IM}\frac{\tilde{\mathbb{Q}}_{1,t}^{IM}}{\tilde{\mathbb{Q}}_{2,t}^{IM}}$$

$$\begin{aligned}\tilde{\mathbb{Q}}_{1,t}^{IM} &= (1-\xi_{IM}\beta_1)mc_t\left(p_{IM,t}\right)^{\theta_t^{IM}} im_t + \\ &\xi_{IM}G_{C,t+1}\mathbb{E}_t\bar{R}_{t+1,t}^*\left(\pi_{IM,t}{}^{\chi_{IM}}\left(\bar{\pi}_{t+1}^*\right)^{1-\chi_{IM}}\right)^{-\theta_{IM}}\left(\pi_{t+1}^*\right)^{\theta_{IM}}\tilde{\mathbb{Q}}_{1,t+1}^{IM}\end{aligned} \tag{XLIII}$$

$$\begin{aligned}\tilde{\mathbb{Q}}_{2,t}^{IM} &= (1-\xi_{IM}\beta_1)\left(p_{IM,t}\right)^{\theta_t^{IM}} im_t + \\ &\xi_{IM}G_{C,t+1}\mathbb{E}_t\bar{R}_{t+1,t}^*\left(\pi_{IM,t}{}^{\chi_{IM}}\left(\bar{\pi}_{t+1}^*\right)^{1-\chi_{IM}}\right)^{1-\theta_{IM}}\left(\pi_{t+1}^*\right)^{\theta_{IM}-1}\tilde{\mathbb{Q}}_{2,t+1}^{IM}\end{aligned} \tag{XLIV}$$

$$p_{IM,t} \equiv \frac{P_{IM,t}}{S_t P_{C,t}^*} = \left[(1-\xi_{IM})\left(p_{IM,f,t}\right)^{1-\theta_t^{IM}} + \xi_{IM}\left(\frac{\left(\bar{\pi}_t^*\right)^{1-\chi_{IM}}\left(\pi_{t-1}^{IM}\right)^{\chi_{IM}}}{\pi_t^*}p_{IM,t-1}\right)^{1-\theta_t^{IM}}\right]^{\frac{1}{1-\theta_t^{IM}}} \tag{XLV}$$

$$\left(w_{1,i,t}\right)^{\theta_{W,1}\zeta_1+1}\bar{w}_{1,t} = \varphi_t^{W,1}\frac{\tilde{\mathbb{Q}}_{1,t}^{W,1}}{\tilde{\mathbb{Q}}_{2,t}^{W,1}}$$

$$\tilde{\mathbb{Q}}^{W,1}_{1,t} = \left(\mathbf{p}^{-1}\mathcal{N}_{1,t}\right)^{(\zeta_1+1)} + \\ + \xi_{W,p}\mathbb{E}_t\beta_1\left(\pi^{\chi_{W,p}}_{W1,t}\left(G_{C,t}\bar{\pi}_{t+1}\right)^{1-\chi_{W,p}}\right)^{-(\zeta_1+1)\theta_{W,1}}\pi^{(\zeta_1+1)\theta_{W,1}}_{t+1} \\ \times\left(G_{C,t+1}\frac{\bar{w}_{1,t+1}}{\bar{w}_{1,t}}\right)^{(\zeta_1+1)\theta^{W,1}_t}\tilde{\mathbb{Q}}^{W,1}_{1,t+1} \tag{XLVI}$$

$$\tilde{\mathbb{Q}}^{W,1}_{2,t} = \left(\mathbf{p}^{-1}\mathcal{N}_{1,t}\right)\lambda^{\mathbf{p}}_t\left(1-\tau^N_s-\tau^W_s\right) \\ + \xi_{W,p}\mathbb{E}_t\beta_1\left(\pi^{\chi_{W,p}}_{W1,t}\left(G_{C,t}\bar{\pi}_{t+1}\right)^{1-\chi_{W,p}}\right)^{1-\theta_{W,1}}\pi^{\theta_{W,1}-1}_{t+1} \\ \times\left(G_{C,t+1}\frac{\bar{w}_{1,t+1}}{\bar{w}_{1,t}}\right)^{\theta^{W,1}_t}G^{-1}_{C,t+1}\tilde{\mathbb{Q}}^{W,1}_{2,t+1}, \tag{XLVII}$$

$$w_{1,t} = \Bigg[\left(1-\xi_{W,p}\right)\left(w_{1,i,t}w_{1,t}\right)^{1-\theta^W_t} \\ + \xi_{W,p}\left(\frac{\left(\pi^{\chi_{W,p}}_{W1,t-1}\left(G_{C,t-1}\bar{\pi}_t\right)^{1-\chi_{W,p}}\right)}{\pi_t}G^{-1}_{C,t}w_{1,t-1}\right)^{1-\theta^W_t}\Bigg]^{\frac{1}{1-\theta^W_t}} \tag{XLVIII}$$

$$\left(w_{2,i,t}\right)^{\theta^{W,2}\zeta_2+1}\bar{w}_{2,t} = \varphi^{W,2}_t\frac{\tilde{\mathbb{Q}}^{W,2}_{1,t}}{\tilde{\mathbb{Q}}^{W,2}_{2,t}}$$

$$\tilde{\mathbb{Q}}^{W,2}_{1,t} = \left(\mathbf{1}-\mathbf{p}^{-1}\mathcal{N}_{2,t}\right)^{(\zeta_2+1)} + \\ + \xi_{W,2}\mathbb{E}_t\beta_2\left(\pi^{\chi_{W,2}}_{W2,t}\left(G_{C,t}\bar{\pi}_{t+1}\right)^{1-\chi_{W,2}}\right)^{-(\zeta_2+1)\theta_{W,2}}\pi^{(\zeta_2+1)\theta_{W,2}}_{t+1} \\ \times\left(G_{C,t+1}\frac{\bar{w}_{2,t+1}}{\bar{w}_{2,t}}\right)^{(\zeta_2+1)\theta^{W,2}_t}\tilde{\mathbb{Q}}^{W,2}_{1,t+1} \tag{XLIX}$$

$$\tilde{\mathbb{Q}}^{W,2}_{2,t} = \left((\mathbf{1}-\mathbf{p})^{-1}\mathcal{N}_{2,t}\right)\lambda_t\left(1-\tau^N_s-\tau^W_s\right) \\ + \xi_{W,2}\mathbb{E}_t\beta_2\left(\pi^{\chi_{W,2}}_{W2,t}\left(G_{C,t}\bar{\pi}_{t+1}\right)^{1-\chi_{W,2}}\right)^{1-\theta_{W,2}}\pi^{\theta_{W,2}-1}_{t+1} \\ \times\left(G_{C,t+1}\frac{\bar{w}_{2,t+1}}{\bar{w}_{2,t}}\right)^{\theta^{W,2}_t}G^{-1}_{C,t+1}\tilde{\mathbb{Q}}^{W,2}_{2,t+1}, \tag{L}$$

$$w_{2,t} = \Bigg[\left(1-\xi_{W,2}\right)\left(w_{2,i,t}w_{2,t}\right)^{1-\theta^{W,2}_t} \\ + \xi_{W,2}\left(\frac{\left(\pi^{\chi_{W,2}}_{W2,t-1}\left(G_{C,t-1}\bar{\pi}_t\right)^{1-\chi_{W,2}}\right)}{\pi_t}G^{-1}_{C,t}w_{2,t-1}\right)^{1-\theta^{W,2}_t}\Bigg]^{\frac{1}{1-\theta^{W,2}_t}}, \tag{LI}$$

$$\left(w_{H,i,t}\right)^{\theta^W\zeta_H+1}\bar{w}_{H,t} = \varphi^{W,H}_t\frac{\tilde{\mathbb{Q}}^{W,H}_{1,t}}{\tilde{\mathbb{Q}}^{W,H}_{2,t}}$$

$$\tilde{\mathbb{Q}}^{W,H}_{1,t} = \left(\mathbf{1} - \mathbf{p}^{-1}\mathcal{N}_{H,t}\right)^{(\zeta_H+1)} + \tag{LII}$$
$$+ \xi_{W,H}\mathbb{E}_t\beta_2 \left(\pi^{\chi_{W,H}}_{WH,t}\left(G_{C,t}\bar{\pi}_{t+1}\right)^{1-\chi_{W,H}}\right)^{-(\zeta_H+1)\theta_{W,H}} \pi^{(\zeta_H+1)\theta_{W,H}}_{t+1}$$
$$\times \left(G_{C,t+1}\frac{\bar{w}_{H,t+1}}{\bar{w}_{H,t}}\right)^{(\zeta_H+1)\theta^{W,H}_t} \tilde{\mathbb{Q}}^{W,H}_{1,t+1}$$

$$\tilde{\mathbb{Q}}^{W,2}_{2,t} = \left((\mathbf{1}-\mathbf{p})^{-1}\mathcal{N}_{H,t}\right)\lambda_t\left(1-\tau^N_s-\tau^W_s\right) + \tag{LIII}$$
$$\xi_{W,H}\mathbb{E}_t\beta_2 \left(\pi^{\chi_{W,H}}_{WH,t}\left(G_{C,t}\bar{\pi}_{t+1}\right)^{1-\chi_{W,H}}\right)^{1-\theta_{W,H}} \pi^{\theta_{W,H}-1}_{t+1}$$
$$\times \left(G_{C,t+1}\frac{\bar{w}_{H,t+1}}{\bar{w}_{H,t}}\right)^{\theta^{W,H}_t} G^{-1}_{C,t+1}\tilde{\mathbb{Q}}^{W,H}_{H,t+1}$$

$$w_{H,t} = \left[\left(1-\xi_{W,H}\right)\left(w_{H,i,t}w_{H,t}\right)^{1-\theta^{W,H}_t} \right. \tag{LIV}$$
$$\left. + \xi_{W,H}\left(\frac{\left(\pi^{\chi_{W,H}}_{WH,t-1}\left(G_{C,t-1}\bar{\pi}_t\right)^{1-\chi_{W,H}}\right)}{\pi_t}G^{-1}_{C,t}w_{H,t-1}\right)^{1-\theta^{W,H}_t}\right]^{\frac{1}{1-\theta^{W,H}_t}}$$

$$p_{Y,t} = \frac{p_{H,t}h_t + p_{X,t}x_t}{y_t} \tag{LV}$$
$$y_t = h_t + x_t \tag{LVI}$$
$$im_t = im^C_t + im^I_t \tag{LVII}$$
$$h_t = h^C_t + h^I_t + h^G_t + h^H_t \tag{LVIII}$$
$$q^C_t = c_t + a(u_t)k_tG^{-1}_{C,t} \tag{LIX}$$
$$q^I_t = i_t \tag{LX}$$
$$h^G_t = g_t\mathcal{H}_t = \mathbf{p}\mathcal{H}_{1,t} + (1-\mathbf{p})\mathcal{H}_{2,t} \tag{LXI}$$
$$c_t = \mathbf{p}c_{1,t} + (1-\mathbf{p})c_{2,t} \tag{LXII}$$

$$p_{G,t}g_t + G^{-1}_{C,t}b_t = \tau^C c_t + \left(\tau^N + \tau^W + \tau^{Wf}\right)\left(\mathbf{p}w_{1,t}\mathcal{N}_{1,t} + (1-\mathbf{p})\left(w_{2,t}\mathcal{N}_{2,t} + w_{H,t}N_{H,t}\right)\right)$$
$$+ \tau^K\left(\left(r^K_t u_t - \delta\right)q_{K,t} - \left(a(u_t)\right)\right)G^{-1}_{C,t}k_t +$$
$$\tau^D R^{DB}_t G^{-1}_{C,t}\left(d^B_{t-1} + d^{BH}_{t-1}\right) + tr_t + R^{-1}_t b_{t+1} \tag{LXIII}$$

$$\left(1+\tau^C\right)\left(1-\mathbf{p}\right)c_{2,t} + b^H_t + \left(1-\mathbf{p}\right)q_{h,t}\mathcal{H}_{2,t} +$$
$$-\left(1-\mathbf{p}\right)q_{h,t}\left(1-\delta_h\right)G^{-1}_{C,t}G^{-1}_{QH,t}\mathcal{H}_{2,t-1} = \tag{LXIV}$$
$$R_t b^H_{t-1}G^{-1}_{C,t} + \left(1-\tau^N-\tau^W\right)w_{2,t}\left(1-\mathbf{p}\right)\mathcal{N}_{2,t} +$$
$$+ \left(1-\tau^N-\tau^W\right)w_{H,t}\left(1-\mathbf{p}\right)N_{H,t}$$

$$\pi_t^H = \frac{p_{H,t}}{p_{H,t-1}}\pi_t \tag{LXV}$$

$$\pi_t^X = \frac{p_{X,t}}{p_{X,t-1}}\pi_t \tag{LXVI}$$

$$\pi_t^{W,1} = \frac{w_{1,t}}{w_{1,t-1}}\pi_t \tag{LXVII}$$

$$\pi_t^{W,2} = \frac{w_{2,t}}{w_{2,t-1}}\pi_t \tag{LXVIII}$$

$$\pi_t^{IM} = \frac{p_{IM,t}}{p_{IM,t-1}}\pi_t \tag{LXIX}$$

$$\text{monetary policy rule home} \tag{LXX}$$

$$\text{monetary policy rule foreign} \tag{LXXI}$$

$$\text{Fiscal policy rule home} \tag{LXXII}$$

B.3 Functions used in non-linear equations

$$\Gamma_X(\cdot) \equiv \frac{\gamma^*}{2}\left(\epsilon_t^X \frac{X_t/Y_t^{d,*}}{X_{t-1}/Y_{t-1}^{d,*}} - 1\right)^2 \tag{52}$$

$$\Gamma_X^\dagger(\cdot) \equiv 1 - \Gamma_X(\cdot) - \Gamma'_X(\cdot)\, X_t \tag{53}$$

$$\Gamma_{IM^C}\left(IM_t^C/Q_t^C; \epsilon_t^{IM}\right) \equiv \frac{\gamma_{IM}^C}{2}\left(\epsilon_t^{IM} \frac{IM_t^C/Q_t^C}{IM_{t-1}^C/Q_{t-1}^C} - 1\right)^2 \tag{54}$$

$$\Gamma_{IM^C}^\dagger(\cdot) \equiv 1 - \Gamma_{IM^C}(\cdot) - \Gamma'_{IM^C}(\cdot)\, IM_t^C \tag{55}$$

$$\Gamma_{IM^I}\left(IM_t^I/Q_t^I; \epsilon_t^{IM}\right) \equiv \frac{\gamma_{IM}^I}{2}\left(\epsilon_t^{IM} \frac{IM_t^I/Q_t^I}{IM_{t-1}^I/Q_{t-1}^I} - 1\right)^2 \tag{56}$$

$$\Gamma_{IM^I}^\dagger(\cdot) \equiv 1 - \Gamma_{IM^I}(\cdot) - \Gamma'_{IM^I}(\cdot)\, IM_t^I \tag{57}$$

$$\chi_t(\cdot) \equiv \chi\left(\frac{\frac{D_t^B}{P_t}}{N_{t+1}}; \epsilon_t^\chi\right) = \chi + \epsilon_t^\chi\, e^{\chi^2\left(\frac{D_t^B}{N_{t+1}} - \frac{D^B}{N}\right)} \frac{N}{D^B} - \frac{N}{D^B} \tag{58}$$

$$a(u_t) \equiv r^K\left(\frac{\gamma_1^u u_t^2}{2} + (1-\gamma_1^u)u_t + \frac{\gamma_1^u}{2} - 1\right) \tag{59}$$

$$\Gamma_{B^*}(\cdot) \equiv \gamma_{B^*}\left(\epsilon^{RP^*} exp\left(\frac{S_t B_{t+1}^*}{P_{Y,t} Y_t}\right) - 1\right) \tag{60}$$

$$\Gamma_I(\cdot) \equiv \frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}} - G_C\right)^2 \tag{61}$$

Notice that the external finance premium function (equation 58) need not be specified. All we need to now is a value at the steady state (χ) and a value of its first derivative at the steady state (χ'). The function used above has the property that the elasticity of the premium with respect to the leverage is χ^2 while the steady state premium is χ. Notice also that in the NAWM the capital-utilization cost function is $\gamma_{u,1}(u_t - 1) + \frac{\gamma_{u,2}}{2}(u_t - 1)^2$. We use the function used by Iacoviello and Neri (2010) as it makes the return on capital independent of the function's parameters.