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NATURAL RATE DOUBTS

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2 The views in this paper do not necessarily represent those of the European Central Bank.

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Abstract

We study the low frequency comovements in unemployment, inflation and the federal funds rate in the U.S. From 1970 through 1979 all three series trended up together; after 1979 they all trended down. The conventional explanation for the buildup of inflation in the 1970’s is that the Fed reacted to an increase in the natural rate of unemployment by conducting an overly passive monetary policy. We show that this explanation is difficult to reconcile with the observed comovement of the fed funds rate and inflation. We argue instead that the source of the inflation buildup in the 1970’s was a downward drift in the real interest rate that was translated into a simultaneous increase in unemployment and inflation by passive Fed policy. Our explanation relies on the existence in the data of an upward sloping long run Phillips curve.

JEL Classification: C32, E3, E43, E58
Key words: Natural rate, Phillips curve, real interest rate, cointegration
Non-technical Summary

According to the Natural Rate Hypothesis (NRH), the long-run unemployment rate is independent of the inflation rate. Since this proposition was advanced by Milton Friedman and Edmund Phelps, it has become a central tenet of monetary economics and is built into the structure of most theoretical and empirical analyses of macroeconomic policy. Advocates of the natural rate hypothesis recognize that there may be a short-run relationship between inflation and unemployment, a so called “statistical Phillips curve”, but the reason for its existence is attributed to errors in the expectations of agents.

In the monetary confusion model of Lucas expectational errors may cause households to supply more labor than justified by economic fundamentals as they confuse real and nominal price changes. In the contract models of Fischer and Taylor, or the price setting model of Calvo, expectational errors have persistence since they are built into wage contracts or pricing decisions. In all three of these models the existence of a relationship between unemployment and inflation is a temporary phenomenon and one would not expect such a relationship to characterize the data at low frequencies.

We do not, however, observe a vertical Phillips curve in low frequency data because - according to most existing work in the area - the natural rate of unemployment has been changing over time. In the period before 1980 the natural rate of unemployment slowly increased and after 1980 it began to fall. We are aware of two explanations, by Orphanides and by Ireland, for the observed low frequency comovements of inflation and the nominal interest rate with the unemployment rate. Although these theories can explain why inflation and unemployment move together, they are unable to account for the low frequency movements in the interest rate and inflation that we find in the data. In contrast to the view of a unit root in the natural rate of unemployment we offer an alternative explanation for the experience of the 1970’s and 1980’s using the idea that observed nonstationarity arises from drift in the underlying real rate of interest.
1 Introduction

According to the Natural Rate Hypothesis (NRH), the long-run unemployment rate is independent of the inflation rate. Since this proposition was advanced by Milton Friedman [11] and Edmund Phelps [20], it has become a central tenet of monetary economics and is built into the structure of most theoretical and empirical analyses of macroeconomic policy. Advocates of the natural rate hypothesis recognize that there may be a short-run relationship between inflation and unemployment, a so called “statistical Phillips curve”, but the reason for its existence is attributed to errors in the expectations of agents.

In the monetary confusion model of Lucas [16] expectational errors may cause households to supply more labor than justified by economic fundamentals as they confuse real and nominal price changes. In the contract models of Fischer [8] and Taylor [22], or the price setting model of Calvo [5], expectational errors have persistence since they are built into wage contracts or pricing decisions. In all three of these models the existence of a relationship between unemployment and inflation is a temporary phenomenon and one would not expect such a relationship to characterize the data at low

![Figure 1: Inflation and Unemployment (Decade Averages)](image-url)
frequencies.

![Graph showing the relationship between average T-Bill rates and unemployment rates for different decades.](image)

**Figure 2: The Federal Funds Rate and Unemployment (Decade Averages)**

In Figure 1 we have drawn a scatter plot of the inflation rate against the unemployment rate. Each point on this graph represents a decade average of quarterly data. Figure 2 presents a similar plot of decade averages, this time for the nominal interest rate against the unemployment rate. These figures present a puzzle for advocates of the natural rate hypothesis since one would expect that over a decade there would be as many quarters in which inflation was above average as there were quarters in which it was below average. A scatter plot of average inflation against average unemployment should reveal a vertical line at the position of the long run natural rate of unemployment. Since economic theory predicts that the average real interest rate should be approximately constant one would also expect a plot of the average nominal interest rate against the average unemployment rate to reveal a vertical line at the natural rate of unemployment.

How might one reconcile Figures 1 and 2 with the natural rate hypothesis? According to most existing work in the area we do not observe a vertical Phillips curve in low frequency data because the natural rate of unemployment has been changing over time. In the period before 1980 the natural rate of unemployment slowly increased and after 1980 it began to fall. We
are aware of two explanations for the observed low frequency comovements of inflation and the nominal interest rate with the unemployment rate. According to Orphanides [18], [19], the Fed mistook an increase in the natural rate of unemployment for a recession. This mistake caused policy makers at the Fed to overstimulate the economy leading to a buildup of inflation and a concurrent increase in the nominal interest rate. Ireland [12] uses the Barro-Gordon [2] model of monetary policy as a dynamic game to argue that a unit root in the natural rate was transmitted to a unit root in inflation as a consequence of time inconsistency in the monetary authority’s optimal policy. In a related argument, Sargent [21] models the buildup of inflation in the 1970’s with a model that replaces the rational expectations assumption with the notion of a self-confirming equilibrium.

In this paper we take a different approach. Section 2 discusses the characteristics of the data and develops a statistical model that can account for these characteristics. Section 3 lays out the results of our data analysis. Our main finding is that when the sample is split in 1980, each subsample is well described by a vector equilibrium correction model with a single common trend and two cointegrating equations. We estimate the parameters of the cointegrating equations for each regime and find that one of them is stable across regimes but the other is different before and after 1980. In Sections 4 – 7 we interpret our statistical results using a class of three equation structural models that embody the natural rate hypothesis. We argue that this class cannot explain the data and we are led to reject existing theories such as those of Orphanides, Ireland and Sargent. Although these theories can explain why inflation and unemployment move together, they are unable to account for the low frequency movements in the interest rate and inflation that we find in the data.

If the Orphanides, Ireland or Sargent explanation were correct, we would expect to see the Fisher equation holding as a cointegrating equation. Our statistical evidence rejects the existence of a single stable Fisher equation for the entire period and we are thus led to reject the standard interpretation of American inflationary experience in the 1970’s. According to this interpretation, nonstationarity in unemployment, inflation and the federal funds rate is due to the presence of a unit root in the natural rate of unemployment. In Section 8 we offer an alternative explanation for the experience of the 1970’s and 1980’s using the idea that observed nonstationarity arises from drift in the underlying real rate of interest. Section 9 presents a short conclusion.
2 Characteristics of the Data and a Statistical Model

In Figure 3 we plot data for the federal funds rate and the unemployment rate and in Figure 4 we plot the federal funds rate and the rate of change in the GDP deflator. All three series represent U.S. data from 1970Q1 to 1999Q3.¹

![Graph showing U.S. Unemployment Rate and U.S. Federal Funds Rate](image)

Figure 3: Unemployment and the Federal Funds Rate

We want to draw attention to two features of this data that will be important for our arguments. The first is that although unemployment and

¹In preliminary work we looked at data beginning in the 1950’s when a time series on the federal funds rate first becomes available. We chose to exclude this initial period and to begin our analysis instead in 1970Q1 because our methodology requires that we are able to fit a stable parameter model over an extended time period and data before 1970Q1 behaves quite differently from the data in our study. Since 1970Q1 is the date at which Arthur Burns took over as chairman of the board of governors of the Fed, it is perhaps unsurprising that our preliminary results indicated a break in parameter stability at this time. In our most recent work (still in progress) we have included a third regime in our analysis covering the period from 1958Q1 through 1969Q4. We expect to report the results of this work in a forthcoming working paper.
the interest rate move in opposite directions at high frequency, they move together at low frequencies: In the period before 1980 both series are trending up; in the period after 1980 they are trending down. The same low frequency comovement can be seen clearly in the inflation and interest rate data in Figure 4.

![Diagram: Inflation and the Federal Funds Rate](image)

Figure 4: Inflation and the Federal Funds Rate

A second feature of the data that will be significant for our later argument is that the difference between the inflation rate and the nominal interest rate is larger after 1980 than before. Since the difference in these series measures the real interest rate, this fact implies that the average real interest rate was higher in the second part of the sample than the first.

Our approach is to use cointegration analysis to uncover low frequency comovements in unemployment, inflation and the federal funds rate. Let \( u_t, i_t, \) and \( \pi_t \) be the unemployment rate, the interest rate and the inflation rate, define \( X_t = \{u_t, i_t, \pi_t\} \) and consider the vector autoregressive (VAR) representation

\[
X_t = \Pi_1 X_{t-1} + \Pi_2 X_{t-2} + \ldots + \Pi_k X_{t-k} + \Phi D_t + \epsilon_t, \quad t = 1, \ldots, T. \tag{1}
\]
The terms $\Pi_i$ and $\Phi$ represent conformable matrices of coefficients and $D_t$ is a vector of deterministic variables. We assume that $\{\epsilon_t\}$ is a sequence of independent Gaussian variables with zero mean and covariance matrix $\Omega$ and we write the system as an observationally equivalent Vector Equilibrium Correction Model (VEqCM)

$$\Delta X_t = \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Pi X_{t-1} + \Phi D_t + \epsilon_t$$

(2)

where the matrices $\Pi = \sum_{i=1}^{k} \Pi_i - I$ and $\Gamma_i = -\sum_{j=i+1}^{k} \Pi_j$ are each $p \times p$.

The key idea behind cointegration is that the matrix $\Pi$ that premultiplies the levels variables in equation (2) may not be of full rank. To see why this is interesting, suppose instead that $\Pi$ did have full rank and consider the simple case where $D_t$ is a vector of constants. In this case the variables in the system would all be stationary and one could represent their long run means as follows,

$$\bar X = -\Pi^{-1} \Phi D.$$  

One can think of $\bar X$ as the non-stochastic steady state of the system and one could recover estimates of the elements of $\bar X$ by taking averages of the data.

The situation where $\Pi$ is of full rank and $X_t$ is stationary is in contrast to the assumption made in the cointegration literature. Here, one assumes that the $\Pi$ matrix has rank $r < p$. In this case $\Pi$ can be decomposed as the product of two matrices $\alpha \beta^T$ where $\alpha, \beta$ are each $(p \times r)$ and have rank $r$.

The $r$ columns of $\beta$ are called cointegrating vectors and $\alpha$ is referred to as the loading matrix (see e.g. Johansen, [13]).

For the 3-dimensional process $\{u_t, i_t, \pi_t\}$; in the case that $\Pi$ has full rank the data clusters around a point (the long run steady state) defined by the intersection of three planes. In the reduced rank model the data clusters around a subspace referred to as $\beta_\perp$ (beta orthogonal) defined by the intersection of two planes. Our focus in this paper is to draw inferences about the class of structural economic models that are consistent with the space $\beta_\perp$ uncovered by our analysis of the data.

\footnote{We use boldface to denote matrices and superscript $T$ to denote transpose.}
3 A Description of the Results

Section 3 describes the results of our statistical analysis.

3.1 Breaking the Sample

In a preliminary study of the data we attempted to fit a stable VAR with 3 lags over the period 1970Q1 - 1999Q3. In this stage of our analysis we found considerable evidence of misspecification. For example, recursive CHOW tests such as the one based on one step ahead forecasts in Figure 5 showed strong evidence of a break around 1980. Other misspecification tests on a model estimated over the whole sample also led us to reject a constant parameter model.

![Figure 5: A Structural Break Test](image)

Table 1 presents test statistics for a range of misspecification tests for the full data set from 1970Q1 to 1999Q3. Square brackets represent p-values. These statistics include tests of the residuals of the VAR for the presence of autocorrelation, normality of the errors, and the presence of ARCH effects.

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3 The empirical analysis in this paper was conducted using PcGive10, [9], and Cats in Rats [10].
\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
$H_0$ & No AR (1 – 5) & Normality & No ARCH $X^2_i$ & No ARCH 4 \\
\hline
$\Delta u$ & 1.58[0.17] & 10.48[0.00] & 4.04[0.00] & 1.78[0.13] \\
$\Delta i$ & 4.54[0.00] & 47.50[0.00] & 2.89[0.00] & 4.70[0.00] \\
$\Delta \pi$ & 1.91[0.09] & 2.47[0.28] & 1.97[0.01] & 3.02[0.02] \\
Distribution & $F(5, 104)$ & $\chi^2(2)$ & $F(18, 90)$ & $F(4, 101)$ \\
\hline
$VAR$ & 1.23[0.15] & 44.05[0.00] & 2.40[0.00] & n.a. \\
Distribution & $F(45, 265)$ & $\chi^2(6)$ & $F(108, 494)$ & \\
\hline
\end{tabular}
\caption{Misspecification Tests for the reduced Form 1970-1999}
\end{table}

We performed these tests for each individual equation as well as for the entire system.

Most of the test statistics in Table 1 clearly reject the null hypothesis reported at the head of the column. These rejections caused us to conclude that a VAR with well behaved residuals could not be fitted to the full sample and we chose instead to try fitting two separate regimes using the third quarter of 1979 to break the sample.\footnote{We also tried to represent the data with a single stable VEqCM by including a variety of dummy variables particularly around the apparent break in 1980. This strategy also was unsuccessful.} This date, suggested by our structural break tests, corresponds to the period when Paul Volcker took over from Arthur Burns’ successor William Miller as Chairman of the Fed. Using this break point our two samples run from 1970Q1 to 1979Q3 and 1979Q4 to 1999Q3.

\section{3.2 Developing a Statistical Model}

Our next step was to establish data congruent models for each subperiod. We used three criteria, (Hannan-Quinn, Schwarz and Akaike) to test for the optimal lag length in each of our subsamples. Beginning with 4 lags, all three criteria suggested a lag length of 3 for each sub-period. Further restricting the system to 2 lags was rejected by F tests in each case. In order to ensure well behaved residuals, we included three impulse dummies in the system of the second period (fourth quarter of 1980 and 1981 and second quarter of 1981). The period from 1979Q3 to 1982Q4 corresponds to one in which the Fed temporarily abandoned interest rate control and tried instead to control
<table>
<thead>
<tr>
<th>$H_0$</th>
<th>$No$</th>
<th>$AR (1-5)$</th>
<th>$Normality$</th>
<th>$No$</th>
<th>$ARCH X_i^2$</th>
<th>$No$</th>
<th>$ARCH 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta u$</td>
<td>1.20[0.33]</td>
<td>2.66[0.26]</td>
<td>1.10[0.45]</td>
<td>0.69[0.60]</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$\Delta i$</td>
<td>0.16[0.97]</td>
<td>3.99[0.13]</td>
<td>0.32[0.98]</td>
<td>0.60[0.66]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \pi$</td>
<td>0.96[0.45]</td>
<td>0.90[0.63]</td>
<td>0.69[0.76]</td>
<td>0.45[0.76]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distribution</td>
<td>$F(5, 24)$</td>
<td>$\chi^2(2)$</td>
<td>$F(18, 10)$</td>
<td>$F(4, 21)$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Misspecification Tests for the Reduced Form: 1970-1979

<table>
<thead>
<tr>
<th>$H_0$</th>
<th>$No$</th>
<th>$AR (1-5)$</th>
<th>$Normality$</th>
<th>$No$</th>
<th>$ARCH X_i^2$</th>
<th>$No$</th>
<th>$ARCH 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta u$</td>
<td>0.47[0.79]</td>
<td>1.63[0.44]</td>
<td>1.34[0.20]</td>
<td>0.51[0.72]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta i$</td>
<td>2.87[0.02]</td>
<td>2.50[0.28]</td>
<td>3.45[0.00]</td>
<td>1.00[0.41]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \pi$</td>
<td>1.26[0.28]</td>
<td>6.51[0.04]</td>
<td>0.60[0.87]</td>
<td>0.97[0.42]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distribution</td>
<td>$F(5, 62)$</td>
<td>$\chi^2(2)$</td>
<td>$F(18, 48)$</td>
<td>$F(4, 59)$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Misspecification Tests for the Reduced Form: 1979-1999

the rate of growth of the money supply and it is perhaps unsurprising that this period displays considerable instability.

Tables 2 and 3 show results of misspecification tests on the two subperiods. In the first period none of these tests is significant at the 5% level. In the second period there is some evidence of ARCH effects in the interest rate equation but otherwise the residuals are consistent with the existence of two separate well behaved models, one for each period. We decided to proceed on the assumption there were two separate monetary policy regimes by investigating the cointegration properties of the data over each subperiod.

### 3.3 Testing for Nonstationarity

To test for nonstationarity, we ran Augmented Dickey Fuller (ADF) tests for each of the variables \{\(u, i, \pi\)\} over the two regimes. In each period we allowed for a constant and a linear trend and, apart from inflation in the second period where the ADF test statistic is borderline at the 5% level, we were
unable to reject the null hypothesis that the variables contain a unit root. Given that ADF tests have comparatively low power we also ran multivariate tests by checking whether each of the variables was $I(0)$ and hence could be represented as a trivial cointegrating vector within a three variable system. In each case this possibility was rejected by the corresponding $\chi^2(1)$ statistic and we report the results of our tests in Table 4. These results confirm the results of our ADF tests that all of the variables are $I(1)$.

3.4 Establishing the Cointegrating Rank

Having established this fact our next step was to test for cointegration by investigating the rank of the $\Pi$ matrix. In Figure 6, we present recursive estimates of the eigenvalues of the $\Pi$ matrix together with 2-standard error bounds. Notice that two of these eigenvalues are different from zero by at least two standard errors for both sub-periods and the third is insignificantly different from zero.\(^5\)

In Table 5 we present the trace statistics,

$$Q_r = -T \sum_{i=r+1}^{p} \ln(1 - \hat{\lambda}_i),$$

To interpret Table 5, the rows should be read sequentially. The row labeled $r = i$ ($i = 0, 1, 2$) reports the statistic $Q_r$ and its 95% confidence

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\(^5\)In order to allow for the possibility that the data contains a trend in the level we did not restrict the constant to lie in the cointegrating space, although our conclusions regarding the rank of $\Pi$ are not sensitive to this assumption.
Figure 6: Recursive Estimates of the Roots of $\Pi$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0 : \text{rank } \Pi = r )</td>
<td>( \lambda_i )</td>
<td>( Q_r )</td>
<td>( \lambda_i )</td>
</tr>
<tr>
<td>( r = 0 )</td>
<td>0.66</td>
<td>56.94</td>
<td>0.46</td>
</tr>
<tr>
<td>( r \leq 1 )</td>
<td>0.30</td>
<td>14.6</td>
<td>0.14</td>
</tr>
<tr>
<td>( r \leq 2 )</td>
<td>0.02</td>
<td>0.67</td>
<td>0.06</td>
</tr>
<tr>
<td>\VAR: { (X_t = u_t, \ i_t, \ \pi_t) } , 3 lags, unrestricted constant.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Trace Test Statistics \(Q_r\) for Cointegrating Rank

value under the null hypothesis that \( \Pi \) has rank \( i \) against the alternative of full rank. Reading across the row \( r = 0 \) it is clear that we can reject the hypothesis of rank 0 for each sub-period and thus at least one eigenvalue is non zero. Reading across row \( r \leq 1 \), for the first period the \( Q_r \) statistic is close to its 95% confidence value for both sub-periods. Finally, reading the row \( r \leq 2 \) allows us to accept the hypothesis of one zero eigenvalue (rank \( \leq 2 \)) for the first sub-period but for the second sub-period the hypothesis of rank \( \leq 2 \) is borderline at the 5% significance level.

Our interpretation of Table 5, in conjunction with Figure 6, is that the data for each subperiod is consistent with the assumption that \( \Pi \) has two non-zero eigenvalues and one eigenvalue equal to zero.

### 3.5 Estimating the Cointegrating Space

Our next step was to estimate the cointegrating matrix \( \beta^i \) for the two subperiods, \( i = 1, 2 \). To identify the space we imposed two zero restrictions,

\[
(\beta^i)^T X = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & \beta_{13}^i \\ & i & \beta_{23}^i \end{pmatrix} \begin{pmatrix} u \\ i \\ \pi \end{pmatrix}, \quad i = 1, 2
\]

and we estimated the parameters \( \beta_{13}^i \) and \( \beta_{23}^i \), using reduced rank regression (see e.g. Johansen and Juselius [15]). We refer to \( \beta^i \) (normalized in this way) as the reduced form cointegrating matrix to distinguish it from the structural cointegrating matrix from an economic model. In Section 4 we will introduce the notation \( \tilde{\beta}^i \) to refer to this second concept.

Table 6 shows our estimates of \( \beta^i \) with standard errors in parentheses and Figures 7 and 8 show the recursive estimates of the freely estimated
coefficients $\hat{\beta}_j^i$ together with their $\pm 2$ standard error bands.\(^6\) Our estimates of the first cointegrating vector show that in the first subperiod the interest rate cointegrates with inflation with a coefficient of 0.76. In the second subperiod it cointegrates with inflation with a coefficient of 1.5. In both periods the low frequency comovements of unemployment with inflation are similar; we find a cointegrating coefficient of 0.58 in the first period and 0.75 in the second. We discuss these estimates further below.

4 A Class of Structural Models

This section describes a class of structural models that we will use to capture the cointegrating properties of the data for each of the two subperiods.

\(^6\)The notation $\hat{\beta}_j$ refers to cointegrating vector $j$ in subperiod $i$ and for $i, j \in \{1, 2\}$, $\hat{\beta}_j^i$ is our estimate of $\beta_j$.
Figure 8: Recursive Estimates of the Cointegrating vectors (Second Subsample)

4.1 Stationary Models

We begin with a class of stationary structural models that is broad enough to include most of the theoretical approaches that have been applied in the literature to explain the time series properties of inflation, unemployment and the interest rate:

\[
A_2 E_t[X_{t+1}] + A_0 X_t + A_1(L) X_{t-1} + v_t + \bar{v} = 0, \\
E_t[v_{t+1}] = 0, \\
E_t[v_{t+1}v'_{t+1}] = \Sigma.
\]

More compactly we write this system as

\[
A(L) E_t[X_{t+1}] + v_t + \bar{v} = 0. \quad (3)
\]

The terms \(A_2, A_0\), are \(3 \times 3\) matrices of coefficients, \(A_1(L)\) and \(A(L)\) are matrix polynomials in the lag operator and \(\bar{v}\) is a vector of constants.

Although Equation (3) defines a large class of models, it cannot account for the non-stationary behavior of the data since it implies the existence of
<table>
<thead>
<tr>
<th>$u$</th>
<th>$i$</th>
<th>$\pi$</th>
<th>$\text{const}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1^1$</td>
<td>0</td>
<td>1</td>
<td>$-0.76$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$(0.17)$</td>
</tr>
<tr>
<td>$\beta_2^1$</td>
<td>1</td>
<td>0</td>
<td>$-0.58$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$(0.10)$</td>
</tr>
<tr>
<td>$\beta_1^2$</td>
<td>0</td>
<td>1</td>
<td>$-1.50$</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>$(0.29)$</td>
</tr>
<tr>
<td>$\beta_2^2$</td>
<td>1</td>
<td>0</td>
<td>$-0.75$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$(0.15)$</td>
</tr>
</tbody>
</table>

Table 6: Identified Cointegrating Relationships

A vector of long-run means, $\tilde{X}$, given by the expression

$$\tilde{X} = -A (0)^{-1} \tilde{v}.$$ 

Since the data on unemployment, the interest rate and the inflation rate is well described as a non-stationary cointegrated system with a single common trend Equation (3) is not a suitable model.

### 4.2 Nonstationary Models

To arrive at a class of structural models that can be described by a VEqCM we amend Equation (3) by assuming that the error term associated with one of the equations in the system is a random walk. By appropriate ordering of the equations we can always choose the nonstationary error to be in the third equation. Then, by differencing the third equation and rewriting the other two equations in differences and levels, one arrives at the “equilibrium correction” representation described in Equation (4).\(^7\)

$$B(L)E_t[X_{t+1}] + \tilde{\alpha} \tilde{\beta}^T X_t + w_t + \bar{w} = 0. \tag{4}$$

In Equation (4) the vector of errors $w_t$ is stationary with variance-covariance

\(^7\)We place the terms “equilibrium correction” in parentheses because Equation (4) describes a structural model that contains expectations of future variables. It does not yet describe the data generation process.
matrix $\tilde{\Sigma}$ and the error vectors $w_t$ and $v_t$ are related by the expression

$$
\begin{bmatrix}
w^1_t \\
w^2_t \\
w^3_t
\end{bmatrix} =
\begin{bmatrix}
v^1_t \\
v^2_t \\
v^3_t - v^3_{t-1}
\end{bmatrix}.
$$

The parameters of the matrix $B(L)$, are functions of the corresponding parameters of $A(L)$. The matrices $\tilde{\alpha}$ and $\tilde{\beta}$ represent the structural loading matrix and the matrix of structural cointegrating vectors as opposed to the reduced form matrices $\alpha$ and $\beta$ that we introduced in Section 2.

To arrive at a description of the data generation process one must supplement Equation (4) with a description of how expectations are formed. In this paper we take the stand that expectations are rational in a very weak sense. We do not require that the subjective probability distribution of future variables should coincide with the actual probability distribution at all points in time. We require only that there should be no systematic long run biases in the mechanism generating expectations. Our assumption is consistent with a wide variety of learning mechanisms, as well as with pure rational expectations models, and it implies that we would expect to see the structural cointegrating vectors ($\tilde{\beta}$ from Equation (4)) holding in the data.

5 Naming the Equations

In this section we consider a subset of models in the class defined by the system of equations (4). This subset consists of an aggregate demand equation, an aggregate supply equation and a policy rule. The main idea of our paper is to address the question: Can this class of models explain the cointegrating relationships uncovered in data?

In the following discussion we define the following elements of the matrix $B(L)$

$$
B(L) \equiv \begin{bmatrix}
b^D_u(L) & b^D_i(L) & b^D_\pi(L) \\
b^S_u(L) & b^S_i(L) & b^S_\pi(L) \\
b^P_u(L) & b^P_i(L) & b^P_\pi(L)
\end{bmatrix}.
$$

where the superscripts $D$, $S$ and $P$ refer to the equations; Demand, Supply and Policy and the subscripts $u$, $i$, and $\pi$ refer to the variables; unemployment, the interest rate and inflation.
5.1 Aggregate Demand

We model the aggregate demand equation as follows

\[ E_t \left[ b_u^D (L) \Delta u_{t+1} \right] + \tilde{\alpha}^D (i_t - \pi_t - \rho) - \tilde{v}^D = \nu_t^D, \tag{5} \]

where \( b_u^D (L) \) is a scalar polynomial in the lag operator that defines the response coefficients of the aggregate demand equation to all lags of differences in the unemployment rate. A similar equation that replaces unemployment with output has been widely used in the literature. Such an equation can be derived from a representative agent model with a single commodity and no capital. This equation, known as an optimization based IS curve,\(^8\) is obtained from the Euler equation in a representative agent model by making the assumption that all output is consumed. The fact that differences of output rather than levels enters the standard demand equation follows from the fact that, in a linearized Euler equation, the coefficients on future and current consumption are the same.

Since we are not using output in our study we assume instead that output is inversely related to unemployment through a production function and we replace output by the negative of unemployment. Our specification allows for fairly general distributed lags of unemployment to enter the equation and thus it can capture the effects of assumptions such as habit persistence that have been proposed as possible business cycle propagation mechanisms. A key feature of Equation (5) is that it implies, in the long run, that the real interest rate should be constant, that is, the Fisher equation should characterize the data. This feature is a property of the representative agent assumption and it does not hold in more complicated general equilibrium models.

The Fisher equation is a strong assumption to impose on data and there are a variety of alternative models that impose a weaker long run restriction. For example, simple Keynesian macro models imply that there should be an upward sloping relationship between unemployment and the real interest rate in the long run (a downward sloping IS curve in output-interest rate space). A similar implication follows from models, like the overlapping generations model, in which Ricardian equivalence fails to hold. To allow for models in this class we also consider a weaker specification of the aggregate demand equation.

\(^8\)This is the notation used by McCallum and Nelson [17].
equation

\[ E_t \left[ b_u^D (L) \Delta u_{t+1} \right] + \tilde{\alpha}^D \left( -\tilde{\beta}_u^D u_t + i_t - \pi_t - \rho \right) - \tilde{\nu}^D = v_t^D, \] (6)

where the level of unemployment as well as differences of unemployment appear in the equation. In Equation (6) the coefficient \( \tilde{\beta}_u^D \) refers to the coefficient on unemployment (subscript \( u \)) in the structural cointegrating equation associated with the demand equation (superscript \( D \)).

5.2 Aggregate Supply

We model the supply equation with the following fairly general version of an expectations augmented Phillips curve.

\[ E_t \left[ b_L^S (L) \Delta \pi_{t+1} \right] + \tilde{\alpha}^S \left( u_t - u^{NR} \right) - \tilde{\nu}^S = v_t^S, \] (7)

where \( u^{NR} \) is a constant that represents the natural rate of unemployment.\(^9\)

According to the natural rate hypothesis there is no long-run trade-off between inflation and unemployment. To capture this hypothesis we assume that only differences of the inflation rate enter the supply equation. If the equation were written in levels, our assumption would imply that the coefficients on all lags of inflation sum to zero. As a consequence of this assumption our model will have the property that the same constant unemployment rate \( u^{NR} \) will be associated with any constant inflation rate.

5.3 Policy Rule

The final equation of our system is a policy rule of the form,

\[ E_t \left[ b_P^P (L) \Delta i_{t+1} \right] + \tilde{\alpha}^P \left( \tilde{\beta}_i^P \left( u_t - u^{NR} \right) + i_t - \tilde{\beta}_\pi^P \pi_t + \gamma \right) - \tilde{\nu}^P = v_t^P. \] (8)

The terms \( \tilde{\beta}_i^P \) and \( \tilde{\beta}_\pi^P \) represent the coefficients of the Fed’s reaction function to inflation and to deviations of unemployment from its natural rate and \( \gamma \) is a constant in the policy rule.

\(^9\)There is a large literature on the microfoundations of aggregate supply that discusses whether lagged or expected future inflation should appear in this equation. This literature derives the influence of excess capacity (or unemployment) on inflation in a class of models in which there are nominal rigidities in price or wage setting. See the survey by Clarida, Gali and Gertler [6].
A number of authors have worked with versions of this equation that allow the interest rate to depend on more complicated distributed lags of past or expected future endogenous variables. Since our analysis relies only on the assumption that the policy rule induces a low frequency relationship amongst the three endogenous variables, it is consistent with these more complicated specifications of the dynamics of the policy response.

6 The Source of Nonstationarity

We have argued that unemployment, the interest rate and inflation are non-stationary but cointegrated with two cointegrating relationships. If a structural model in the class of (3) is to be consistent with these facts, one of the error terms, \( v^1_t, v^2_t \) or \( v^3_t \) must be nonstationary: But which one? If all three error terms were stationary then the following linear combinations of \( u, i \) and \( \pi \) would also be stationary. We refer to these stationary linear combinations as \( \tilde{\beta}^D, \tilde{\beta}^S \) and \( \tilde{\beta}^P \).

\[
AD : \quad \left( \tilde{\beta}^D \right)^T X = i - \pi - \rho, \quad \rho > 0, \quad (9)
\]
\[
AS : \quad \left( \tilde{\beta}^S \right)^T X = u - u^{NR}, \quad u^{NR} > 0, \quad (10)
\]
\[
PR : \quad \left( \tilde{\beta}^P \right)^T X = \tilde{\beta}_u^P (u - u^{NR}) + i - \tilde{\beta}_\pi^P \pi + \gamma; \quad \tilde{\beta}_u^P > 0, \quad \tilde{\beta}_\pi^P > 0, \quad (11)
\]

If we replace Equation (5) with the weak form of the aggregate demand equation given by Equation (6), the term \( \tilde{\beta}^D X \) would be given by the expression,

\[
AD : \quad \left( \tilde{\beta}^D \right)^T X = -\tilde{\beta}_u^D u + i - \pi - \rho; \quad \tilde{\beta}_u^D > 0. \quad (12)
\]

If any one of the error terms, \( v^D, v^S \) or \( v^P \) is nonstationary then the cointegrating vectors associated with the other two equations are the ones that one would expect to see in the data. For example, if \( v^D \) is nonstationary then we would expect to see that \( \tilde{\beta}^S \) and \( \tilde{\beta}^P \) should appear as cointegrating vectors. From this argument we see that if either \( v^D \) or \( v^P \) were nonstationary, \( \left( \tilde{\beta}^S \right)^T X \) should appear as one of the two cointegrating equations and hence
unemployment itself should be stationary. Since unemployment is \( I(1) \) in the
data we can rule out the possibility that either \( u_t^D \) or \( u_t^P \) is nonstationary.
This leaves only the possibility that nonstationarity arises from the supply
equation. We turn to this possibility next by examining the implications of
assuming that the natural rate of unemployment is a random walk.

6.1 Modeling Nonstationarity in the Natural Rate

To model drift in the natural rate of unemployment, let the natural rate \( u^{NR}_t \)
be indexed by \( t \) and suppose that it follows the process,

\[
\hat{\alpha}^S (u^{NR}_t - u^{NR}_{t-1}) = w_t^S + \bar{w}^S,
\]

where \( w_t^S \) is an \( I(0) \) variable, \( \bar{w}^S \) is a drift parameter and \( \hat{\alpha}^S \) is the structural
loading factor in the supply equation. Suppose further that there is no other
shock hitting the aggregate supply equation so that \( v_t^S \) is identically zero.\(^{10}\)

Under this specification we can rewrite Equation (7) as follows

\[
E_t \left[ b_n^S (L)^\Delta \pi_{t+1} \right] + \hat{\alpha}^S u_t = \hat{\alpha}^S u^{NR}_t. \tag{13}
\]

Since \( \hat{\alpha}^S u^{NR}_t \) is nonstationary we must take differences of Equation (13) to
arrive at an equation within a stationary error term

\[
E_t \left[ \Delta b_n^S (L) \Delta \pi_{t+1} + \hat{\alpha}^S \Delta u_t = w_t^S + \bar{w}^S. \tag{14}
\]

If we accept that the natural rate of unemployment is nonstationary we
must explain how a drift in the natural rate was transmitted to a drift in
inflation and the fed funds rate. In the following two subsections we will
discuss two alternative explanations of this process that have been put for-
ward in the literature and we will explain why we find these explanations
lacking. The first is due to Peter Ireland [12] and the second to Athanasios
Orphanides [18].\(^{11}\)

\(^{10}\)Relaxing this assumption would add an additional stationary moving average error
term to Equation (14).

\(^{11}\)Sargent [21] presents a third argument also based on time inconsistency of the optimal
policy. Sargent’s argument is more sophisticated than Ireland’s since it can explain the
fall in inflation endogenously in a model in which agents form expectations using a least
squares learning rule. Sargent’s explanation, like that of Ireland and Orphanides, implies
that the Fisher equation should hold in the data.
6.2 Ireland’s Explanation

In a recent paper Peter Ireland [12] constructs a bivariate model of inflation and unemployment using the Barro-Gordon [2] model of time inconsistent monetary policy. In his work, the Fed plays a game against the public. In this policy game it directly picks the mean of the inflation rate in an attempt to minimize a quadratic loss function. Ireland shows that if the natural rate of unemployment is non-stationary then time inconsistency in the policy game will cause the equilibrium unemployment rate and the equilibrium inflation rate both to inherit non-stationarity from the natural rate of unemployment. However, a linear combination of unemployment and the inflation rate will be stationary. Hence the Barro-Gordon model can account for why the policy maker might transfer a unit root in unemployment into a unit root in inflation and it can also explain why these variables are cointegrated in the data. Notably, Ireland does not model the interest rate.

6.3 Orphanides’ Explanation

Athanasios Orphanides [18] has proposed a different mechanism to explain why inflation and unemployment both went up (and came down) together. His explanation relies on the fact that, during the 1970’s, most economists did not know that the natural rate of unemployment had increased and he substantiates this claim by looking at real time estimates of potential output. These estimates were much more optimistic about the trend growth path of the economy than were subsequent revisions of the same series. According to the Orphanides explanation, the Fed overstimulated the economy in the 1970’s by reducing the Fed Funds Rate because it mistook an increase in the natural rate of unemployment for a recession.

6.4 Are these Explanations Correct?

In our discussion of results in Section 7 we will make use of the Orphanides assumption that the Fed erroneously responded to the unemployment rate instead of to deviations of unemployment from its natural rate. In the absence of this assumption our model has no hope of explaining why inflation and unemployment appear cointegrated. With this assumption we will be able to replicate a version of Orphanides’ argument and also explain why we find his argument unconvincing. We will show that if nonstationarity in the
data arose because of a unit root in the natural rate of unemployment then one of the cointegrating equations in the data should be the Fisher equation. Our data analysis strongly rejects the hypothesis that the Fisher equation holds across the two subperiods and so we are led to look for an alternative explanation of the facts.

Our dissatisfaction with Ireland’s explanation is based on the same idea. We differ from Ireland’s study since we have included the federal funds rate in our analysis whereas Ireland looked at a bivariate model of inflation and unemployment. Since Ireland did not directly model the interest rate, it is always possible that a richer version of his analysis might be able to account for all of the facts; but we find this unlikely since a richer version of Ireland’s model is likely to include the Fisher equation just like the models we study in this paper.

7 Some Implications of Nonstationarity in the Natural Rate

What are the implications of the assumption that the natural rate of unemployment is a random walk? In the following two subsections we study this question using two alternative assumptions about the aggregate demand curve.

7.1 Implications of the Strong Form of Aggregate Demand

In our empirical analysis we identify the cointegrating space by excluding unemployment from one cointegrating vector and excluding the interest rate from the other. If we impose this identification scheme on the structural cointegrating vectors $\beta^D$ and $\beta^P$, we arrive at the following mapping that represents the reduced form cointegrating vectors $\beta^1$ and $\beta^2$ in terms of the
parameters $\tilde{\beta}_j^i$ of the structural cointegrating vectors:

\[
(\beta^1)^T X = \begin{pmatrix}
0 & 1 & -1 \\
1 & 0 & \frac{1 - \tilde{\beta}_1^P}{\beta_u}
\end{pmatrix}
\begin{pmatrix}
u \\ i \\
\pi
\end{pmatrix},
\]

\[
(\beta^2)^T X = \begin{pmatrix}
0 & 1 & -1 \\
1 & 0 & \frac{1 - \tilde{\beta}_2^P}{\beta_u}
\end{pmatrix}
\begin{pmatrix}
u \\ i \\
\pi
\end{pmatrix}.
\]

Consider $\beta^i_2$: $i = 1, 2$, which represents the cointegrating relationship between unemployment and the inflation rate in each subperiod. Our model implies that this vector should be given by

\[
(\beta^i_2)^T X = \begin{pmatrix}
1 & 0 & \frac{1 - \tilde{\beta}_i^P}{\beta_u}
\end{pmatrix}
\begin{pmatrix}
u \\ i \\
\pi
\end{pmatrix},
\]

and our estimates of $\beta^i_2$ are given by

\[
\begin{pmatrix}
\tilde{\beta}^1_2 \\
\tilde{\beta}^2_2
\end{pmatrix}^T X = \begin{pmatrix}
1 & 0 & -0.58 \\
(0.10)
\end{pmatrix}
\begin{pmatrix}
u \\ i \\
\pi
\end{pmatrix},
\]

\[
\begin{pmatrix}
\tilde{\beta}^2_2 \\
\tilde{\beta}^3_2
\end{pmatrix}^T X = \begin{pmatrix}
1 & 0 & -0.75 \\
(0.15)
\end{pmatrix}
\begin{pmatrix}
u \\ i \\
\pi
\end{pmatrix}.
\]

The numbers in parentheses are standard errors. These estimates are consistent with Orphanides’ argument if the Fed responded to inflation by raising the interest rate with a reaction coefficient $\tilde{\beta}_\pi$ that was greater than one in each period and if the response of the Fed to increased unemployment was positive. Evidence from our estimates of the first cointegrating equation (the relationship between unemployment and the inflation rate) is thus supportive of Orphanides’ explanation. However, things do not look so good when we consider the first cointegrating vector (the relationship between the federal funds rate and inflation).
Our model implies that $\beta_i^i: i = 1, 2$ should be equal in both sub-periods and should be given by

$$(\beta_1)^T = (\beta_2)^T = (0, 1, -1).$$

In words, the Fisher equation should hold in both sub-periods. But our point estimates of the elements of these vectors are given by

$$
(\beta_1)^T X = \begin{pmatrix}
0 & 1 & -0.76 \\
\frac{1}{17} & \frac{1}{17} & \frac{1}{17}
\end{pmatrix}
\begin{pmatrix}
u \\
i \\
\pi
\end{pmatrix},
$$

$$
(\beta_2)^T X = \begin{pmatrix}
0 & 1 & -1.50 \\
\frac{1}{29} & \frac{1}{29} & \frac{1}{29}
\end{pmatrix}
\begin{pmatrix}
u \\
i \\
\pi
\end{pmatrix}.
$$

![Cointegrating Equation Between the Fed. Funds Rate and Inflation](image1)

![Cointegrating Equation Between Unemployment and Inflation](image2)

Figure 9: Estimates of the Cointegrating Vectors with 2-Standard Error Bands

Figure 9 illustrates the full sample estimates of the two cointegrating vectors for both sub-periods. The Fisher hypothesis requires that the coefficient on inflation in the first cointegrating vector should equal $-1$ in each subsample and it further requires that the two subsample estimates should be equal.
Notice from the left panel of the figure that one cannot reject the Fisher hypothesis in either period since the dashed line indicating a coefficient of \(-1\) is marginally within the 2-standard error confidence bound in each case. However, the recursive estimates in Figures 7 and 8 show that for the second period this restriction is only accepted at the end of the sample period and clearly rejected over the 80s and the beginning of the 90s.

Further evidence against the Fisher hypothesis comes from the fact the point estimates in each sub sample in Figure 9 lie well outside of the 2-standard error bounds for the other one. This implies that although one cannot reject the hypothesis that the Fisher equation holds over either separate subsample, one can reject the joint hypothesis that it holds in both sub-samples together. We are led to reject both the Orphanides and the Ireland explanations for the comovements of inflation and unemployment because their explanations are inconsistent with the observed comovements of the interest rate with inflation.

### 7.2 Implications of the Weak Form of Aggregate Demand

Consider an alternative weak form of the aggregate demand equation given in Equation (6). If this weak form of the aggregate demand equation holds then the cointegrating equations we would expect to see in the data would be given by (12) and (11). Once again we can impose our identification scheme on the structural cointegrating vectors to arrive at the following mapping from structural to reduced form parameters:

\[
\begin{align*}
(\beta^1)^T X &= \begin{pmatrix} 0 & 1 & -\left(\frac{\hat{\beta}_D^1 \hat{\beta}_u^D + \hat{\beta}_u^1}{\beta_u^D + \beta_u^1}\right) \\
0 & 1 & -\left(\frac{\hat{\beta}_D^2 \beta_u^D + \hat{\beta}_u^2}{\beta_u^D + \beta_u^1}\right)
\end{pmatrix} \begin{pmatrix} u \\
i \\
\pi \end{pmatrix},
\end{align*}
\]

\[
\begin{align*}
(\beta^2)^T X &= \begin{pmatrix} 0 & 1 & -\left(\frac{\hat{\beta}_D^2 \beta_u^D + \hat{\beta}_u^2}{\beta_u^D + \beta_u^1}\right) \\
0 & 1 & -\left(\frac{\hat{\beta}_D^1 \hat{\beta}_u^D + \hat{\beta}_u^1}{\beta_u^D + \beta_u^1}\right)
\end{pmatrix} \begin{pmatrix} u \\
i \\
\pi \end{pmatrix},
\end{align*}
\]
where the parameters $\tilde{\beta}_\pi^P, \tilde{\beta}_u^P$ and $\tilde{\beta}_u^D$ are all non-negative. Recall that the estimates of $\beta^1$ and $\beta^2$ are given by:

\[
(\hat{\beta}^1)^T X = \begin{pmatrix}
0 & 1 & -0.76 \\
1 & 0 & -0.58 \\
1 & 0 & -0.75
\end{pmatrix}
\begin{pmatrix}
u \\
i \\
\pi
\end{pmatrix},
\]

\[
(\hat{\beta}^2)^T X = \begin{pmatrix}
0 & 1 & -1.50 \\
1 & 0 & -0.75
\end{pmatrix}
\begin{pmatrix}
u \\
i \\
\pi
\end{pmatrix}.
\]

In the first subperiod, the point estimate of

\[
-\frac{(\tilde{\beta}_\pi^P \tilde{\beta}_u^D + \tilde{\beta}_u^P)}{\tilde{\beta}_u^D + \tilde{\beta}_u^P}
\]

(the coefficient on inflation in the first cointegrating vector) is equal to $-0.76$. But the numerator and denominator of Equation (15) are both weighted sums of the same positive numbers $\tilde{\beta}_u^D$ and $\tilde{\beta}_u^P$ that differ only in the weight $\tilde{\beta}_\pi^P$ attached to $\tilde{\beta}_u^D$ in the numerator. It follows that $\tilde{\beta}_\pi^P$ must be a positive number between 0 and 1. But if $\tilde{\beta}_\pi^P$ is between zero and 1 then the coefficient on inflation in the second cointegrating vector, given by the expression

\[
\frac{1 - \tilde{\beta}_\pi^P}{\tilde{\beta}_u^D + \tilde{\beta}_u^P},
\]

must be positive. Our point estimate of this parameter is equal to $-0.58$ with a standard error of 0.1 and hence, under our maintained assumptions, the weak form of the aggregate demand curve is inconsistent with data from the first subperiod.

8 An Alternative Explanation of the Data

We previously rejected the hypothesis that nonstationarity arises from either the demand equation or the policy rule since we could not explain nonstationarity of the unemployment rate under either of these assumptions. But if
the natural rate hypothesis were to be replaced by an equation linking unem-
ployment and the inflation rate, we would need to reassess these possibilities.

In this section we look at the possibility that there is a long run Phillips
curve in the data that leads to a cointegrating equation of the form

$$AS: \quad \left( \beta^S \right)^T X = u - \beta^S \pi - u^{NR}; \quad \beta^S, u^{NR} > 0. \quad (16)$$

Notice that our hypothesis is that higher inflation is associated with higher
unemployment at low frequencies, the opposite slope to the traditional short
run Phillips curve.

If (16) holds in the data then if either the demand curve or the policy
equation were non-stationary then we could explain the comovement in in-
flation and unemployment since Equation (16) implies that these variables
should be cointegrated. Our preferred specification is one in which $v^D$, (the
error term hitting the aggregate demand curve) is nonstationary since if the
policy equation were drifting we would expect to see the Fisher equation in
the data. We think it more likely that the break in 1980 was caused by a
break in policy than by a break in the behavioral equations of the private
sector and if this is the case then the policy equation must be one of the
cointegrating equations of the model.

### 8.1 Is a Long Run Phillips Curve Consistent with Theory?

The idea of a long run relationship between unemployment and inflation
is consistent with a model in which money has non-superneutral effects. Non
superneutralities are relatively easy to identify in countries that expe-
rience hyperinflations since hyperinflation is typically accompanied by high
unemployment and severe recessions. Our claim in this paper is that non-
superneutralities can also account for the low frequency movements in infla-
tion and unemployment that occurred in the last thirty years in the United
States, even though inflation barely reached double digits over this period.

There are many possible mechanisms that might cause high interest rates
to be transmitted to the labor market. One explanation is given by the
monetary model of Benhabib and Farmer [3] in which the effects of money on
equilibrium output can be substantial. But there are many other possibilities.
Non neutralities in the tax code would cause changes in the equilibrium
supply of labor in an equilibrium model like the monetary real business cycle
model studied by Cooley and Hansen [7]. In search models with liquidity effects such as those studied by Ramey, Den Haan and Watson [4], or in models with hysteresis effects like the one studied by Ball [1], one would expect there to be permanent effects on the equilibrium unemployment rate resulting from changes in monetary policy.

8.2 Why Did Inflation and Unemployment Move Together?

In this subsection we will discuss an interpretation that attributes the common trend in unemployment, the inflation rate and the federal funds rate to non stationarity in the error term $v^D$. This assumption implies that Equations (17) and (18) should hold as cointegrating equations in the data;

$$\text{AS: } \begin{align*}
    (\tilde{\beta}^S)^T X &= u - \tilde{\beta}_\pi^S \pi - u^{ NR}; \\
    \tilde{\beta}_\pi^S, u^{ NR} &> 0,
\end{align*}$$

$$\text{PR: } \begin{align*}
    (\tilde{\beta}^P)^T X &= \tilde{\beta}_u^P (u - u^{ NR}) + i - \tilde{\beta}_\pi^P \pi + \gamma; \\
    \tilde{\beta}_u^P &> 0, \quad \tilde{\beta}_\pi^P > 0.
\end{align*}$$

Our explanation of the buildup of inflation does not require us to assume that the Fed mistakenly targeted the unemployment rate as in the Orphanides explanation and so we revert in this section to the assumption that the Fed correctly targeted deviations of unemployment from its natural rate.

Mapping the structural cointegrating Equations (17) and (18) into the identified reduced form gives the following expressions, one for each subperiod;

$$\begin{align*}
    (\beta^1)^T X &= \begin{pmatrix} 0 & 1 - (\tilde{\beta}_\pi^{P1} + \tilde{\beta}_\pi^S \tilde{\beta}_u^{P1}) \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u \\ \pi \end{pmatrix} = \begin{pmatrix} u \\ i \end{pmatrix},
\end{align*}$$

$$\begin{align*}
    (\beta^2)^T X &= \begin{pmatrix} 0 & 1 - (\tilde{\beta}_\pi^{P2} + \tilde{\beta}_\pi^S \tilde{\beta}_u^{P2}) \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u \\ \pi \end{pmatrix} = \begin{pmatrix} u \\ i \end{pmatrix}.
\end{align*}$$

\[12\]In a representative agent model our assumption would imply that the agent’s rate of time preference is a random walk. In more complex general equilibrium models the equilibrium real interest rate might be non-stationary as a result of cohort effects in an overlapping generations model or as a result of non stationary fiscal policies.
According to these equations the slope of the long run Phillips curve, $\tilde{\beta}_S^\pi$, is exactly identified from $\tilde{\beta}_2$. The parameters $\tilde{\beta}^\piu$ and $\tilde{\beta}^\piPi$ are not, however, separately identified and it is not possible to disentangle the effects of a policy response to inflation from the response to unemployment.

Recall that the estimates of the reduced form relationships in each regime are given by

$$
(\hat{\beta}_1)^T X = \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix} \begin{pmatrix}
-0.76 \\
-0.58
\end{pmatrix}
\begin{pmatrix}
u \\
i
\end{pmatrix},
$$
$$
(\hat{\beta}_2)^T X = \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix} \begin{pmatrix}
-1.50 \\
-0.75
\end{pmatrix}
\begin{pmatrix}
u \\
i
\end{pmatrix}.
$$

If our model were correct we would expect to see the same cointegrating relationship between unemployment and inflation in both subperiods. According to our hypothesis, this equation is a structural equation representing the long-run Phillips curve. Since we cannot reject the hypothesis that the estimated value of $\tilde{\beta}_S^\pi$ in the first period (equal to $0.58 \pm 0.1$) is equal to the estimated coefficient in the second period ($0.75 \pm 0.15$) we conclude that our explanation is not contradicted by the data. Our interpretation of the data is also consistent with a change in the cointegrating equation between inflation and the federal funds rate from $-0.76$ to $-1.5$. We interpret this break in the cointegrating vector as the consequence of a change in Fed policy.

### 8.3 How Did the Fed Tame Inflation?

What led to the buildup of inflation under Arthur Burns in the 1970’s and the subsequent taming of inflation in the Volcker-Greenspan regimes? According to our explanation, the real interest rate is non-stationary. If we take first differences of the aggregate demand curve we arrive at the following equation

$$
\Delta i - \Delta \pi = \Delta \rho,
$$

(19)
where $\Delta \rho$ is a stationary variable with mean $\overline{\Delta \rho}$ where $\overline{\Delta \rho}$ represents drift in changes to the real rate.

Suppose that the Fed follows a simple Taylor rule that induces the stationary cointegrating vector $i - \tilde{\beta}_\pi^P \pi$ in the data so that the expression\(^{13}\)

$$\Delta i - \tilde{\beta}_\pi^P \Delta \pi$$

is stationary with a zero mean. Putting together equations (19) and (20) leads to the expression

$$\Delta \pi = - \left( \frac{1}{1 - \tilde{\beta}_\pi^P} \right) \Delta \rho. \quad (21)$$

What caused inflation in the 1970’s and how did the Fed tame inflation after 1979? Our explanation is that $\overline{\Delta \rho}$ is negative and so there has been a downward drift in $\rho$ at least since the beginning of the 1970’s. In the 1970’s under Arthur Burns, Fed policy was over accommodative in the sense that $\tilde{\beta}_\pi^P$ was less than unity. Since $\tilde{\beta}_\pi^P$ was between zero and one, the coefficient $\left( \frac{1}{1 - \tilde{\beta}_\pi^P} \right)$ was positive and downward drift in the real rate was translated into an upward drift in inflation.

After 1979 the Fed became more aggressive in its response to inflation and the parameter $\tilde{\beta}_\pi^{P2}$ in the second regime was greater than one. This switch in policy stance caused $\left( \frac{1}{1 - \tilde{\beta}_\pi^{P2}} \right)$ to be negative. Although the real interest rate continued to drift down, under an active monetary policy this downward drift in the real rate was translated into a downward drift in inflation.

### 8.4 Why Did the Real Rate Increase After 1980?

The alert reader will have noticed an apparent inconsistency in our explanation of the history of U.S. inflation. If the parameter $\rho$ was drifting down over the whole period, why does the real rate appear to increase in 1980? The explanation lies with the constant in the cointegrating equation induced by Fed policy. Let the policy rule be given by

$$i = \tilde{\beta}_\pi^{P1} \pi + \gamma_1 \quad (22)$$

\(^{13}\)To keep our notation to a minimum we assume that the Fed does not respond to unemployment and so $\tilde{\beta}_\pi^P = 0$. The argument for non zero $\tilde{\beta}_\pi^P$ is identical with more complicated expressions for the coefficients.
before 1980 and

\[ i = \tilde{\beta}_\pi^P \pi + \gamma_2 \]  \hspace{1cm} (23)

afterwards. Given our assumption that the parameter \( \rho \) is nonstationary, the real rate will be equal to

\[ i - \pi = \left( \tilde{\beta}_\pi^{Pj} - 1 \right) \pi + \gamma_j, \hspace{1cm} j = 1, 2. \]  \hspace{1cm} (24)

In the period before 1980 inflation is drifting up and the real rate is drifting down. The direction of drift is opposite because \( \left( \tilde{\beta}_\pi^{P1} - 1 \right) \) is less than zero. In the period after 1980 inflation and the real rate are both drifting down. The direction of drift is the same because \( \left( \tilde{\beta}_\pi^{P2} - 1 \right) \) is greater than zero. At the date of the policy shift the real rate jumps up because the parameter \( \gamma_2 \) is greater than \( \gamma_1 \).

9 Conclusion

We have argued that the inflation rate, the unemployment rate and the nominal interest rate can be well described as non-stationary but cointegrated variables in U.S. data from 1959 through 1999. The first cointegrating equation linking inflation with the fed funds rate displays a much larger response of the interest rate to inflation after 1980 than before 1980. The cointegrating equation linking unemployment with inflation has been stable over the entire period.

If one accepts our statistical representation of the data, how might one respond to the evidence? In the paper we have made two separate claims, both based on the assumption that one should seek a common cause for the break in data that appears in 1980. The first is that the source of non-stationarity is a unit root in the shock to the aggregate demand equation. The second is that the natural rate hypothesis is false. If one is willing to accept the coincidental and simultaneous change in two different structural equations, then our arguments break down. For example, the trend in the inflation rate may have been caused by a Fed policy that reversed itself in 1980 at the same time that fundamental factors caused a reversal in the upward trend in unemployment. We find dual cause explanations of this kind implausible.
Our primary reason for rejecting alternative models of American inflation in the 1970’s is that the alternatives that we have considered imply that the Fisher equation should hold in the data and our statistical analysis rejects this hypothesis. Although one can find alternative explanations for the failure of the Fisher equation we believe that our explanation is the most obvious candidate since it leads to a unified explanation of the American inflation experience. We attribute the buildup of inflation and its subsequent demise to the effects of two different Fed policies in a world in which shocks to aggregate demand are nonstationary. In the period before 1970 a passive monetary policy led these shocks to be transmitted into an upward drift in the inflation rate. After 1980 policy was reversed and a more active policy caused them to be translated in to a downward drift in the inflation rate. A non stationary inflation rate was transmitted to the unemployment rate since the data is characterized by the existence of a long run upward sloping Phillips curve. The evidence leads us to be skeptical of theories that incorporate superneutrality as a maintained assumption of an economic model, hence the title of our paper, natural rate doubts.
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