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INTERBANK MARKET INTEGRATION UNDER ASYMMETRIC INFORMATION

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Abstract

While domestic interbank markets are often considered to work in an efficient way, cross-country bank lending appears to be subject to market imperfections leading to persistent interest rate differentials. In a model where banks need to cope with liquidity shocks by borrowing or by liquidating assets, we study the scope for international interbank market integration with unsecured lending when cross-country information is noisy. We find that an equilibrium with integrated markets need not always exist, and that it coexists with one characterized by segmentation. A repo market reduces interest rate spreads and improves upon the segmentation equilibrium. However, it may destroy the unsecured integrated equilibrium.

JEL Classifications: G15, G20, F36
Non-Technical Summary

A crucial ingredient of the Economic and Monetary Union (EMU) is to have a unified money market across the euro area. Indeed, the interest rates on the inter-bank market have adjusted within a few days after the euro was introduced in 1999. Since then, the average differences between money market rates in EMU member states remain stable at a low level of approximately 3 basis points. Consequently, the introduction of the euro successfully implemented uniform liquidity conditions across the euro area.

A closer look at inter-bank relationships within the euro area reveals that banks are not homogeneous players in the inter-bank market for liquidity. Instead, only a few, larger banks operate in the international market. The bulk of banks, on the other hand, are relying exclusively on domestic channels to manage their liquidity needs.

This paper attempts to explain these empirical findings with a theory of asymmetric information. It starts from the presumption that information on foreign banks is coarser than on domestic peers, with whom inter-bank relations might have developed over a far longer time period. On this basis, the paper develops a model of banking in which inter-bank lending is an important ingredient in banks’ liquidity management. The model is one of a perfectly competitive banking sector in which banks offer demand-deposit contracts to their consumers. Consumers are uncertain with respect to the timing of consumption needs, and banks are exposed to liquidity shocks because the fraction of consumers withdrawing liquidity at a certain point in time is stochastic. These shocks occur both on the individual as well as on the country level. As a result, a country might find itself in a situation with excess liquidity or with a liquidity shortage. An inter-bank market for liquidity is used to provide liquidity smoothing across banks and in this way, avoid costly liquidation of profitable investment projects.

The model analyses equilibrium in the market for unsecured loans, in which lenders assume credit risk in their credit operations. This risk is reflected, among other factors, in the market interest rates. For cross border lending, this risk-premium is higher as it reflects the assumption that the quality of information on the borrowing banks is lower than in domestic lending transactions. It is shown that an integrated market in which all banks of high credit standing are able to participate is not always possible. Only if the cross-country liquidity shocks are large enough, can the adverse selection problem in the international market be overcome. Furthermore, even if an equilibrium with integrated markets exists, it is always possible that the market remains at another equilibrium in which markets are segmented. Therefore, in the presence of asymmetric information across borders, having a single currency can not guarantee the integration of money markets.
When a repo market is introduced into this model, the paper reaches two different conclusions: first, starting from the equilibrium with segmented markets, a collateralised market for liquidity is able achieve some degree of integration in spite of the asymmetry in information. Second, however, the integrated equilibrium might break down as a consequence of having a repo market. This is because the repo market reduces the liquidity imbalances across countries and reduces the gain from establishing an integrated market.

The paper then considers different scenarios with heterogeneous banks. It analyses a situation in which banks with a good international standing act as correspondent banks for their domestic peers, by obtaining liquidity in the international market and relending domestically. These banks could be either multinational banks which have access to information in several countries, or else very large banks which are either well-known across borders, or considered to be Too-Big-To-Fail by the market. It is shown that banks of this type are able to achieve cross-border liquidity smoothing, thus overcoming any welfare-reducing effects that may arise because of asymmetric information. This scenario seems to describe well the situation in the euro area.
1 Introduction

The objective of this paper is to study the effects of cross-country asymmetric information on the structure of financial markets. Our main concern is the design of money markets and the role of repo and (unsecured) interbank markets in an international framework, but our results carry over to a more general framework of the analysis of cross-country direct investment, covering the cross-country market both for bonds and equity. The creation of an integrated interbank market is particularly relevant in order for banks to cope efficiently with liquidity shocks. Interbank markets are instrumental in allowing for a smooth working of the payment systems (so that a bank that is lacking liquidity in the payment system is able to borrow from another bank), and in channeling liquidity to the banks and countries that need it most. Both repo and unsecured interbank lending allow to cope with liquidity shocks. Still, because unsecured markets are based on peer monitoring, they introduce market discipline, thus playing the role unsecured deposits may play when depositors receive information (Calomiris and Kahn, 1991).

In this paper, we are particularly interested in investigating the effects that the European Monetary Union has had on the development of European interbank markets. Still, the paper might also prove useful in analysing the crisis in Eastern Asia, where interbank lending by foreign countries had played a key role in these countries’ funding structure, and the collapse of a well functioning unsecured interbank market proved to be crucial. Finally, the issue has direct policy implications since, if the interbank market’s provision of liquidity is inefficient, this calls for regulatory intervention.

The role of the interbank market to cope with bank specific liquidity shocks and avoid unnecessary liquidation of long term investments was first acknowledged in Bhattacharya and Gale (1987). Later contributions built upon this role while introducing either moral hazard (Rochet and Tirole), aggregated liquidity risk (Allen and Gale 2000) or else by introducing credit risk (Freixas, Parigi and Rochet 2000). These studies yield similar results pointing at the potential contagion provided by an interbank market as well as the effect the network of interbank lending may have on financial fragility. Furthermore, Bhattacharya and Fulghieri (1994) analyze the efficiency of an interbank market in a framework where banks face uncertain timing of liquidity returns, and Holmström and
Tirole (1998) discuss the role of liquidity provision by the public sector.

Although our paper does not focus on financial crises, we also consider two different interbank lending networks, segmentation vs. integration, where the collapse of integration may be interpreted as a financial crisis.

As in Rochet and Tirole, we consider "peer monitoring" as a key factor in improving the efficiency of the interbank market; still we are concerned with asymmetric information and therefore about the quality of signals rather than about moral hazard and the way to discipline borrowers. From that perspective, our work is related to Broecker (1990) and to Flannery (1996) who consider models of asymmetric information and credit risk. An important difference to their work is that the value of a signal is given exogenously in their model, while in our model it is endogenous, as it results from the equilibrium behavior of borrowers.

Our model uses a Diamond and Dybvig type of framework, where consumers are uncertain about the timing of their consumption needs. This generates liquidity shocks, which we assume are present both at the individual and at the aggregate level. To be able to cope with these shocks, banks can invest in a storage technology. Because this technology has a lower return than alternative investment opportunities, it is efficient for banks to use the interbank market. We consider both an unsecured interbank market and a repo-market where government bills are traded. In order to introduce credit risk, the model assumes that banks have some risk of failure. As in Rochet and Tirole (1996), banks monitor each other in the interbank market, thus obtaining a signal on the solvency probability of each of their peers. The key assumption of our model is that cross border information about banks is less precise than home country information. Hence, when a bank tries to borrow from a foreign bank, it does so either because it belongs to a liquidity short country or else because it has generated a "bad" signal at a domestic level and is therefore unable to borrow in his home country. As it is intuitive, depending on the equilibrium probability distribution of the two types of motivations for borrowing abroad, an integrated interbank market may exist or not.

Our contribution is to show, using this framework, that having a single currency is no guarantee for having a single uninsured interbank market. Namely, a segmented interbank market is always an equilibrium, while the emergence
Figure 1: *Spanish and Italian rates in the overnight market for unsecured loans in comparison to the German rates (in percentage points)*

of an integrated international market is only possible when the quality of cross-border information is sufficiently good. A further result is that the integration of markets does not always yield a more efficient outcome.

On the other hand, the repo-market provides a perfect medium to channel liquidity between banks and across countries. However, the secured nature of the repo reduces banks’ incentives for peer monitoring. As a result, banks with a low probability of solvency are able to obtain liquidity via the repo market and to avoid liquidation, although their liquidation value might exceed their expected continuation value. Surprisingly, we also establish that the combination of both types of markets need not yield a more efficient allocation, as it may lead to the collapse of the unsecured integrated market.

After the introduction of the Single Currency, money market rates in the EMU have very quickly converged. In figure 1, the Spanish and Italian overnight rates in the unsecured interbank market are displayed as differences to the German rate. From the beginning of Stage 3 in January 1999, differences between national rates have become very small and average around only 3 basis points. At first glance, it seems that money markets are almost perfectly integrated across Europe. A closer investigation however shows that only very few banks are actively participating in the international market. For instance, in a recent study Ciampolini and Rhode (2000) report the results of a survey conducted among European Banks according
to which most institutions still have to rely on domestic markets to manage their liquidity needs. This paper tries to analyze this situation from a theoretical point of view.

The paper is organized as follows. In section 2 we set up the basic model of interbank credit and the structure of signals. Section 3 analyzes the unsecured interbank market in a two country setting, while section 4 is devoted to the general and more complex case when the two markets -unsecured and repo- are coexistent. Section 5 extends the analysis by allowing for the introduction of correspondent banking, transnational banking and too-big-to-fail banks. Section 6 offers some concluding remarks on the policy implications of our results. All proofs are in the appendix.

2 The Model

We consider an economy with two countries.

Consumers. In each country, there is a continuum of consumers of a total measure of one, who possess one unit of endowment each at time 0. Consumers are risk neutral and face liquidity shocks as in a Diamond and Dybvig (1988) type of model: they need to consume either at time 1 or at time 2. At time 0, consumers deposit their endowments in a bank, and can withdraw funds at the time they need to consume. Deposits are fully insured by a deposit insurance, so no bank runs occur.\(^1\) We assume that the demand-deposit contract promises them a consumption of 1 in either period, \(C_1 = 1\) or \(C_2 = 1\).

Banks’ Investment. There is an infinite number of risk neutral banks. Each receives the endowments of a continuum of consumers at time 0, and invest them either in a risky technology or in reserves (storage technology). Furthermore, a bank can buy government T-Bills, which are issued at price \(B_0\), and yield 1 in the second period with certainty. Denote the investment in the risky technology \(I\), the one in T-bills \(T_0B_0\) and the one in reserves \(s_0 = 1 - I - T_0B_0\). We simplify the analysis by assuming that the amount of banks’ equity is negligible but that banks are, nevertheless, residual claimants.

\(^1\)The deposit insurance company is assumed to raise funds at unit cost. The deposit insurance is fair in that banks have to bear the expected cost of their failures.
Each unit invested in the risky technology yields an uncertain payoff $\tilde{R}$ at time 2, where $\tilde{R} = R$ (solvent) with probability $p \geq \frac{1}{2}$, and $\tilde{R} = 0$ (insolvency) with probability $1 - p$. Investment in the technology is assumed to be ex-ante efficient in that $pR > 1$. This risky asset can also be (partially) liquidated at time 1, with the following technology: liquidation of $\Delta I$ units gives a liquidation value of $l(\Delta I)$, where $l(\Delta I)$ is increasing and concave. For simplicity we use a logarithmic liquidation function $l(\Delta I) = \ln(\Delta I + 1)$.\(^2\)

Because banks have limited liability they have incentives to forbear no matter how bad the prospects of the risky technology are. As a consequence, in our model bank closure will only occur if it is triggered by a liquidity shock.

**Liquidity Shocks.** Our model combines bank specific liquidity shocks with country-wide ones. Banks are uncertain about the liquidity demand they face at time 1. For a fraction $q$ of all banks, a high fraction of consumers $\pi_H$ is impatient and wishes to withdraw at time 1. A fraction $1 - q$, on the other hand, faces a low liquidity demand $\pi_L$, $\pi_L < \pi_H$. The remaining consumers are impatient and withdraw at time 2.

The variable $q$ reflects the country wide aggregate demand for liquidity and is uncertain as well. We assume liquidity shocks to occur with probability $1/2$ and restrict our comparative statics analysis to changes in the probability of solvency. Thus with probability $1/2$, $q = q_B$, in which case a country is in a state of high aggregate liquidity demand, because many banks face high time-1 withdrawals ($\pi_H$). On the other hand, with probability $1/2$ the country faces a low liquidity demand with $q = q_A < q_B$, so that fewer banks face a high amount of withdrawals. For the sake of simplicity we assume $q_B + q_A = \overline{q}$. The probability of solvency and liquidity are uncorrelated.

Banks can manage their liquidity needs at time 1 by borrowing or lending in the interbank market, by buying or selling T-Bills on the repo market, by liquidating assets, and by storing reserves until the next period. Throughout the base model, we assume that a bank cannot split its loan demand among several banks. If illiquid banks fail to meet their creditors’ demand, they are forced into

\(^2\)We assume that liquidation of an asset is equivalent to selling the asset to someone outside the banking sector, e.g. to institutional investors who are not participating in the interbank market.
bankruptcy, liquidating all assets. In this case, the proceeds are absorbed by the
deposit insurance company.

The timing of the model is the following:

<table>
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<th></th>
<th>$t = 0$</th>
<th>$t = 1$</th>
<th>$t = 2$</th>
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<tbody>
<tr>
<td></td>
<td>consumers deposit</td>
<td>$q, \pi$ and signals observed</td>
<td>returns $\bar{R}$ realized</td>
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<tr>
<td></td>
<td>banks invest</td>
<td>liquidation takes place</td>
<td>patient consumers withdraw</td>
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<tr>
<td></td>
<td></td>
<td>repo and interbank market operations realized</td>
<td>interbank loans repaid</td>
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<td></td>
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<td>impatient consumers withdraw</td>
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**Information.** At time 0, the ex-ante probability of being solvent, $p$, is common knowledge. At time 1, all banks in a given country receive a non-verifiable signal $s_D$ about the solvency of their domestic counterparts. We assume that the same signal is observed about each bank from all its peers in a given country. The signal can either be good ($\bar{s}$) or bad ($s$), it is defined as

$$prob(s_D = \bar{s}|\bar{R} = R) = prob(s_D = s|\bar{R} = 0) = \alpha.$$  

Our setting thus reflects the existence of ”soft” domestic information regarding the different banks’ strategies, their risk-taking behavior, and their accounting strategy (loan-loss provisions, window dressing and so on). This information is not directly observable, even at a cost by other banks that are not members of the place.

Denote $\theta \equiv p\alpha + (1 - p)(1 - \alpha)$ the ex-ante probability that the good signal is received about a bank.\(^3\) We assume that the signal is informative, i.e., $\alpha \in (\frac{1}{2}, 1]$.

The above expressions allow us to compute the probability $\bar{p}$ of success condition on the bank having produced the signal $s_D = \bar{s}$. It is given by $\bar{p} \equiv prob(\bar{R} = R|s_D = \bar{s}) = \frac{\alpha}{\theta}$ and we denote, symmetrically, $p \equiv prob(\bar{R} = R|s) = \frac{\alpha(1 - \alpha)}{1 - \theta}$.

The signals received in the foreign country can only be observed with some noise. We make the following assumption:

**Assumption 1** (Noisy cross-country information)

$$prob(s_F = \bar{s}|s_D = \bar{s}) = prob(s_F = s|s_D = s) = 1 - \beta$$

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\(^3\)Because we are dealing with a continuum of banks, the ex-ante probability is equal to the ex-post fraction of banks of this type.
This holds regardless of the bank’s solvency, where $\beta \in (0, \frac{1}{2})$ denotes the probability that the domestic signal is received wrongly in the foreign country.

Thus, the lower $\beta$, the better is the information flow between countries. Each bank is then characterized by a pair $(s_D, s_F)$, denoting the signals that have been received by domestic and foreign banks about this particular bank. Assumption 1 implies that $s_D$ is a sufficient statistics for $(s_D, s_F)$.

Note that a bank cannot observe its own solvency, but only the signal. Therefore, it has no informational advantage over the other market participants regarding its own solvency. This assumption is required in order to leave aside the additional issues of moral hazard.

The second assumption we make regarding the signal structure is that it is not profitable to lend to a low-signal bank (in either country). Because $\bar{s}$-banks have a higher probability of failure than $\bar{\bar{s}}$-banks, lenders need to demand a higher interest rate from those banks in order to break even. We assume here that this rate would need to be so high that it is impossible for a bank to meet its interest rate payments at time 2.

**Assumption 2** (No lending to $\bar{s}$-banks) The expected net present value of a loan to a bank is positive if $s_F = \bar{\bar{s}}$ and negative if $s_F = \bar{s}$.

Assumption 2 is stated for the foreign signal, however, because the domestic signal is more precise than the foreign one, this implies that the same result holds for $s = s_D$.

Finally, we assume that when a bad signal is observed, it is efficient to close down the bank. That is, we assume

**Assumption 3** (Efficient closure of bad-signal-banks)

$$\text{prob}(\hat{R} = R|s_F = \bar{s})R \leq l'(I)$$

Because of concavity of $l(\cdot)$, this assumption implies that it is efficient to liquidate the entire risky technology of a bank when the bad signal has been received in the foreign country. Assumption 3 obviously implies that it is also

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4This can easily be checked that for any signal combination, for example for $(\bar{s}, \bar{\bar{s}})$, we have

$$p(\hat{R} = R|s_D = \bar{s}) = \frac{p\bar{s}(1-\beta)}{p\bar{s}(1-\beta) + (1-p)(1-\alpha)(1-\beta)} = p(\hat{R} = R|s_D = \bar{s})$$
efficient to close down this bank with better quality of information, e.g. after observing the domestic bad signal \( (\tilde{p} \leq l'(I)) \).

On the other hand, the assumption \( pR > 1 \) implies that \( \tilde{p}R > 1 \), so that (partial) liquidation of a high-signal bank is never efficient.

3 Cross Country unsecured interbank market integration

In this and all subsequent sections, we focus on the banks’ time 1 problem of managing their liquidity needs.\(^5\) We regard two countries with different aggregate liquidity demands at time 1. W.l.o.g., we assume that for \( q = q_A \), there is excess liquidity, while for \( q = q_B \), the liquidity shortage is so high that both lenders and borrowers liquidate. Notice that while there are country-wide aggregate shocks, the assumption \( q_B + q_A = \bar{q} \) implies that there is no aggregate liquidity shock when both countries are taken together.

![Diagram](image)

**Figure 2: Liquidity shocks in countries A and B**

Let us denote \( A \) the country with excess liquidity, and \( B \) the one with a liquidity deficit. We assume that there are no legal or infrastructural barriers to the emergence of an international money market.

3.1 Structure of equilibrium

At time 1, when the liquidity shock occurs, banks with liquidity needs can find themselves with one of the four pairs of signals: \( (s_D, s_F) = (\bar{s}, \bar{s}), (\bar{s}, \bar{s}), \bar{s}, \bar{s}) \), or \( (s, s) \). Because of Assumption 2, only banks with the signal pair \( (\bar{s}, \bar{s}) \) are

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\(^5\)The time 0 investment problem is not considered.
Figure 3: Borrowing choices for country-B banks: banks can only obtain a loan in the country where the good signal $\overline{\pi}$ has been obtained about them. In equilibrium, a fraction $1 - \psi$ of $(\overline{\pi}, \overline{\pi})$-banks borrows domestically while $\psi$ borrow abroad.

able to choose in which country to borrow. The other banks are constrained to using either the domestic market if they are $(\overline{\pi}, \overline{s})$, or the foreign market if they are $(\overline{s}, \overline{\pi})$. Banks of type $(\overline{s}, \overline{s})$ are not able to borrow at all. This is illustrated in figure 3. Moreover, in equilibrium, lenders will be able to update their information on foreign borrowers’ solvency on the basis of their strategic behavior and on equilibrium interest rates. Because of the limited access of $(\overline{\pi}, \overline{s})$ banks to the foreign interbank market and of $(\overline{s}, \overline{\pi})$ to the local interbank market, the equilibrium is not a fully integrated market, and insurance against liquidity shocks is only partial.

In order to characterize the equilibrium, notice, first that the interbank interest rates, $r_i, i = A, B$ are crucial in determining how much of the long run technology the banks will liquidate. In equilibrium, in each country $i$ the demand for liquidity $\Omega_i$ equals the supply of liquidity $\Lambda_i$. These are given by the following expressions:

$$\Omega_i \equiv \sigma_i^L \pi_L + \sum_{k=D,F} \sigma_i^k \pi_H$$  \hspace{1cm} (1)

$$\Lambda_i \equiv \sum_{k=L,D,F} \sigma_i^k \left[ s_0 + l(\Delta I_k^i) \right]$$  \hspace{1cm} (2)

where the superindex $i = A, B$ denotes the country, and subindex $k$ refers to the different agents active in the local interbank market, $k = L, D, F$. Here, $L$ denotes lenders, $D$ refers to domestic borrowers, and $F$ to foreign borrowers, and $\sigma_i^k$ is the (equilibrium) measure of each type of bank in country $i$. Using the bank’s first order decisions, we can establish the following Lemma:
**Lemma 1** The supply of liquidity in country \( i \), \( \Lambda_i \), is non-decreasing in the domestic interest rate \( r_i \), where \( i = A, B \).

**Proof.** See appendix. ■

For low levels of interbank interest rates, only reserves are used and the credit market does not develop. For larger levels of interest rate, a market begins to develop where lending banks do not liquidate, but lend out their reserves and have a zero expected yield. When we consider higher interest rates, both borrowing and lending banks liquidate their long term assets and the expected yield for the lending banks is strictly positive.

Unsurprisingly, the interbank interest rate is an increasing function of the aggregate demand for liquidity. However, markets do not provide for a perfect smoothing of liquidity shocks: banks with a high amount of early withdrawals \( \pi_H \) liquidate less than banks facing \( \pi_L \). The reason is that the expected cost of borrowing is higher than the expected return from lending. This can be seen by computing the expected return on lending and the expected cost of borrowing conditional on the banks’ survival: lenders obtain an expected return of \( \bar{p}(1 + r) \) on loans, while borrowers pay \( (1 + r) \) on each unit borrowed.

Equilibrium in both countries requires

\[
F_A = \Omega_A - \Lambda_A = 0 \\
F_B = \Omega_B - \Lambda_B = 0
\]

It is quite intuitive that the equilibrium will be characterized by a unilateral flow of borrowers from the country with high liquidity needs and high interest rates to the country with excess liquidity and low interest rates. The following lemma establishes this point. Denoting by \( \psi_i \) the fraction of \((\overline{\pi}, \overline{\pi})\)-banks in country \( i \) that choose to borrow abroad, we prove that \( \psi_A = 0 \).

**Lemma 2** Borrowers from the excess liquidity country \( A \) borrow only in country \( A \) \( (\psi_A = 0) \). In addition, in the case of segmented markets \( (\psi_A = \psi_B = 0) \), interest rates in country \( B \) are higher than in country \( A \) \( (r_A < r_B) \).

**Proof.** See appendix. ■

This will simplify our notations, since it implies having only one foreign market, the one for borrowers of country \( B \) that wish to borrow in country \( A \). Thus,
denote $\psi \equiv \psi_B$, and $r_A$, $r_B$, and $r_F$ the domestic rates of countries $A$ and $B$ and the rate for foreign borrowers, respectively. A perfect Bayesian equilibrium in the inter-bank market is a quintuple $(\psi^*, r_A^*, r_B^*, r_F, p_F^*)$, specifying the fraction $\psi^*$ of $(\bar{\pi}, \bar{\pi})$-banks from country $B$ borrowing abroad as well as the interest rates demanded and the corresponding rationally updated probability $p_F^*$. Integration will be defined as the case where in equilibrium $\psi^* > 0$, while segmentation is defined as the opposite case when $\psi^* = 0$.

In an integrated equilibrium, with $0 < \psi^* < 1$, two conditions need to be satisfied: on the one hand, lenders in country $A$ should be indifferent between lending to either country:

$$\overline{\pi}(1 + r_A) = p_F(1 + r_F).$$  

On the other hand, borrowers able to borrow at home or abroad should be indifferent where to borrow, thus

$$r_B = r_F.$$

In the segmented equilibrium, $\psi^* = 0$, $p_F = \text{prob}(\bar{R} = R|\bar{\pi}) = \frac{1}{2}$ and $r_F$ is indeterminate. The equilibrium results simply from the two market clearing conditions (3) and (4), one for each country.

**Proposition 3** Under integration we have $r_B(\psi^*) \geq r_F(\psi^*) > r_A(\psi^*)$, $p_F(\psi)(1 + r_F^*) = \overline{\pi}(1 + r_A^*)$. Under segmentation we have $r_F(0) > r_B(0)$, $p_F = \text{prob}(\bar{R} = R|\bar{\pi}) = \frac{1}{2}$.

**Proof.** See appendix.

Notice that in our terminology, integration includes the case where $\psi = 1$, and $r_B(1) > r_F(1)$: the interest rate charged abroad is still lower than the domestic one, and all banks who are able to, will borrow abroad. Only ”captive” $(\bar{\pi}, \bar{\pi})$ banks will be forced to borrow domestically at the higher rate $r_B(1)$.

When integration occurs, we have the following implications:

- Lenders in the $B$ country draw an informational rent from the liquidity shortage.
- Lending banks in the $A$ country finance a heterogenous population of foreign banks, consisting of all the ”bad risks” $(\bar{\pi}, \bar{\pi})$, and a fraction $\psi$ of the ”good risks”, $(\bar{\pi}, \bar{\pi})$.  

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3.2 Integration versus Segmentation

We will now proceed to establish under what conditions segmentation and integration occur, that is to determine the equilibrium values of $\psi$. A crucial element in the analysis is the information updating of foreign lenders. Observing the measure of banks trying to borrow abroad, lenders can infer $\psi$ and therefore the fraction of $(\mathcal{S}, \mathcal{S})$ and $(\mathcal{S}, \mathcal{R})$ banks among the foreign borrowers. As it is intuitive, a higher fraction of $(\mathcal{S}, \mathcal{S})$ banks implies a higher updated probability of solvency $p_F$. Moreover, from (5) it follows that the premium on foreign loans, $r_F - r_A$ is decreasing in $\psi$.

Lemma 4 A foreign bank’s updated probability of solvency $p_F(\psi)$ is increasing in $\psi$. For $\psi > 0$, the premium charged to foreign borrowers $r_F - r_A$ is decreasing in $\psi$.

Proof. See appendix. ■

In order to obtain $\psi^*$, we first derive the demand and supply of liquidity in the domestic and foreign markets for given values of $\psi$. Market clearing then allows us to characterize interest rates in both markets as functions of $\psi$. The results of this analysis are presented in the following Lemma:

Lemma 5 The domestic interest rate in country B, $r_B$, is decreasing in $\psi$.

Proof. See appendix. ■

Unsurprisingly, a higher level of cross-country borrowing eases the liquidity shortage in country B and leads to a lower interest rate $r_B$.

For $\psi = 0$, interest rates in country B are lower than the foreign interest rates, $r_B(0) < r_F(0)$. This, together with the results from Lemma 5, has two implications: first, it might be the case that $r_F(\psi) > r_B(\psi)$ for all values of $\psi$. In this case, borrowing abroad is never an equilibrium strategy. Second, if $r_F(\psi) \leq r_B(\psi)$ for some $\psi$, an equilibrium with integration exists. However, nothing precludes the segmentation equilibrium to be obtained as well.

Figure 4 illustrates the interest rates $1 + r_B$ and $1 + r_F$ as functions of $\psi$ for different parameter constellations. In all of them, a segmented equilibrium with separated inter-bank markets (point A) is possible. In case (i), it is the only
Figure 4: Interest rates for country-B borrowers as functions of $\psi$: Liquidity short banks in country $B$ can either borrow domestically at rate $r_B$, or in the foreign market at rate $r_F$. Both rates depend on the fraction of $(\overline{\pi}, \underline{\pi})$-banks borrowing abroad, $\psi$. The existence of an integrated equilibrium depends on the relative location of both rates.

equilibrium because for all values of $\psi$, the rate charged abroad is strictly higher than the one in country $B$.

In cases (ii) and (iii), the curves $1+r_B(\psi)$ and $1+r_F(\psi)$ cross for some $\psi > 0$. In case (ii), three equilibria coexist, two of them with an active international market (at the crossing points $B$ and $C$): borrowers face the same interest rates in both countries, and they are indifferent as to where to demand a loan. Still, $C$ implies a higher level of integration than $B$. Finally, in case (iii), there is an equilibrium with integration at point $D$, where all banks prefer to borrow abroad ($\psi = 1$) and the only banks in country $B$ to borrow domestically are the $(\overline{\pi}, \underline{\pi})$ ones.

Under which circumstances can an equilibrium with an integrated market be obtained? Since the foreign interest rate is increasing in the informational premium, integration becomes impossible for very high $\beta$. Another important parameter is the difference in liquidity needs across countries, $\Delta q \equiv q_B - q_A$: Only when $\Delta q$ is sufficiently large, integration is possible.

The following proposition outlines the results of this market imperfection.

**Proposition 6 (Multiple equilibria)** If the difference in aggregate liquidity demand in the two countries is large enough, both the integrated and segmented market equilibrium exist. Otherwise, only the segmented market equilibrium exists.
**Proof.** See appendix. ■

Proposition 6 implies first, that there is a threshold $\bar{\Delta q}$ (which depends on the exogenous parameters $\alpha$, $\beta$, and $\overline{\rho}$) such that only for $\Delta q \geq \bar{\Delta q}$ an integrated equilibrium exists. Second, and more important for its policy implications, it implies that whenever an integrated market exists the segmented market also exists. So even if integration is possible it is not necessarily reached, and even if integration is reached, the possibility to revert to the segmentation equilibrium is always present. This possible collapse of the integrated equilibrium is somewhat reminiscent of Flannery’s (1996) argument for the collapse of a domestic interbank market. In both cases it is the existence of excessively noisy information that drives the collapse of the market.

In this model, segmentation is quite a ”robust” equilibrium because there exist self-fulfilling beliefs that support it, independently of the existence of an integrated interbank market equilibrium. This is of concern because usually, but not necessarily, integration will Pareto dominate segmentation, a point the following proposition establishes.

**Proposition 7** The integrated equilibrium does not always dominate the segmented equilibrium, but it does so for $\beta$ sufficiently small.

**Proof.** See appendix. ■

As is intuitive, for most parameter constellations integration leads to a higher expected welfare than segmentation because of higher cross-country interest rate smoothing. However, if the signals in the foreign country are sufficiently imprecise, foreign lenders make more ‘mistakes’ in granting loans to insolvent banks, which decreases welfare. Still, parameter values for which this effect dominates are hard to find since the integrated equilibrium exists only for small $\beta$.

The analysis shows that a high level of cross-border information (i.e. low $\beta$) is essential for an integrated inter-bank market to exist. However, even when the difference in information across borders is sufficiently low, there is no guarantee that private market forces reach the most efficient equilibrium.

Furthermore, in all possible equilibria, an inefficiency remains that is due to the informational asymmetry between countries. Because of the concavity of the liquidation technology, the most efficient outcome would involve all banks with the domestic good signal to liquidate the same amount. However, since borrowers
from country $B$ pay a premium that reflects the asymmetry in information, they liquidate more than banks in country $A$ in either one of the equilibria.

4 Coexistence of unsecured and repo markets in an international setting

Finally, we discuss the general case where both types of markets, repo and unsecured, coexist. We focus on the more interesting case where banks hold few T-Bills so that borrowers have to rely on both markets, and borrowing banks without access to unsecured markets have to close down (The opposite case where banks are able to cope with liquidity shocks by selling T-Bills is trivial). In this way we are able to examine the combined effect of integration through the repo market and market discipline by peer monitoring.

In this context, as it is intuitive, it is possible to show that the excess liquidity banks of the liquidity short country will never hold T-Bills until time 2. This is in line with the fact that these lenders have an informational rent from lending to the interbank market because of more accurate information, so that they can lend the liquidity they obtain through the sale of T-Bills. Therefore, equilibrium is now obtained with a transfer of liquidity equal to $\Delta s_0$ through the sale of T-Bills (notice that T-Bills are not redeemable and therefore do not produce liquidity before time $t = 2$). That is,

$$\Omega_B = \Lambda_B + \Delta s_0$$

$$\Omega_A = \Lambda_A - \Delta s_0$$

The following Proposition extends Proposition 3 and characterizes the integrated equilibrium

**Proposition 8** The equilibrium in the interbank market is characterized by $r_B^* > r_A^*$ and either

1. Integration if $0 < \psi^* \leq 1$, $r_B(\psi^*) \geq r_F(\psi^*)$, $p_F(1 + r_A^*) = \tilde{p}(1 + r_A^*) = \frac{1}{\tilde{p}} < \tilde{p}(1 + r_B)$, and country-B banks will sell all their T-Bills to the A country ones.

2. Integration with unsecured markets segmentation if $r_B^* = r_A^*$ and $\psi^* = 0$ in which case $p_F = \text{prob}(\tilde{R} = R|\mathcal{S})$. This will occur when the aggregate liquidity shocks are small with regard to the T-Bill market.
3. Segmentation if $\psi^* = 0$ and $r_B^* > r_A^*$, in which case $p_F = \text{prob}(\tilde{R} = R|s)$, and the B country banks will sell all their T-Bills.

**Proof.** See appendix. ■

Proposition 8 considers a situation where banks with excess liquidity are willing to both buy repos and lend in the unsecured market. Therefore, we assume that the price for liquidity on both markets is the same one for lenders in country $A$, i.e. $p(1 + r_A) = \frac{1}{H}$. This implies that for borrowers in both countries, the cost of obtaining liquidity through the repo market, $\frac{1}{H}$, is smaller than the cost it faces in the unsecured market, which is $1 + r_s$, $s = A, B$. The borrowing banks will resort to the unsecured market only after having sold all their T-Bills, i.e. $T_H^{s*} = 0$. Thus, in an international framework, the repo-facility will be used to transfer liquidity from the excess liquidity country to the liquidity short one, while the unsecured market will be used to provide liquidity to the $s$ borrowers.

However, because the repo market results in a lower interest rate spread of domestic rates, the potential gain from integration of unsecured markets is now lower. At the same time, the cost that is associated with integration remains unchanged, namely the fact that credit is extended to insolvent foreign banks. Indeed, recalling Proposition 6, we saw that only if the difference in liquidity needs was sufficiently high could an equilibrium with $\psi > 0$ be obtained. But, the existence of an international repo market reduces $\Delta q$, and hence makes the integration of unsecured markets less likely. This point is established in the following Proposition.

**Proposition 9** The introduction of a cross-border repo market implies

- for a segmented equilibrium, $(\psi^* = 0)$, $r_B - r_A$ is reduced.
- the integrated equilibrium $(\psi^* > 0)$ collapses for $\Delta s_0$ sufficiently high.

**Proof.** See appendix. ■

The possible equilibria in the unsecured market are illustrated in figure 5. The left graph illustrates the case where $\Delta s_0$ is relatively low. Here, equilibria with both separated and integrated unsecured markets exist. In the right graph, on the other hand, $\Delta s_0$ is so high that integration in the unsecured market is not possible any longer.
Figure 5: Equilibria with different levels of T-Bill holdings.

It would be tempting to jump to the conclusion that the collapse of the integrated equilibrium decreases welfare. Still, in the light of Proposition 7, we know that the analysis is more en volved. Indeed, the following Proposition establishes that the collapse of an integrated equilibrium need not be welfare decreasing.

Proposition 10 The break-down of the international unsecured interbank market due to a higher $\Delta s_0$ can lead to an increase in welfare only for high $\beta$.

Proof. See appendix. ■

5 Extensions

Up to now we have restricted our focus to a simplified world where banks were either domestic or foreign, correspondent banking services were excluded and the population of banks was homogeneous in regard of their credit risk. In the following three sections we consider these extensions, starting with correspondent banking, then turning to transnational banks and concluding with the introduction of safe banks.

5.1 Correspondent Banking

Correspondent banking will develop when some banks are able to borrow from the liquid country and lend to the illiquid one. These are liquidity long banks from the $B$-country with a $\pi$ foreign signal that will borrow at the rate $r_F$, and lend at the domestic rate $r_B$. As correspondent banking develops, some aggregated
amount of liquidity $Z$ is channeled into the illiquid country and this will result in new equilibrium interest rates $r_B(Z)$ and $r_F(Z)$.

For correspondent banking to be profitable, the cost of borrowing in country $A$, $1 + r_F(Z)$, cannot exceed the average return from lending to borrowers in country $B$, $\bar{\pi}(1 + r_{CB}(Z))$, where $r_{CB}(Z)$ represents the interest rate charged by correspondent banks. Therefore, it is required that

$$1 + r_F(Z) \leq \bar{\pi}(1 + r_B(Z)).$$

(9)

Condition (9) implies that $r_F(Z) < r_B(Z)$, requiring that the equilibrium is of the $\psi = 1$ type, which corresponds to the case where all $(\bar{s}, \bar{s})$—banks of country $B$ borrow abroad. In addition, corresponding banking is unable to reduce the spread $r_B - r_F$ to zero as the following proposition establishes

**Proposition 11** Correspondent banking will develop only in the $\psi = 1$ equilibrium, provided that the interest rate wedge $r_B(0) - r_F(0)$ is sufficiently large, and in equilibrium the wedge $r_B(Z^*) - r_F(Z^*)$ is strictly positive.

**Proof.** See appendix. ■

This interest rate differential will trigger the arbitrage operated by correspondent banking: those banks able to borrow abroad would do so, even if they are liquidity long, and then use the liquidity obtained to lend it to those banks in their home country.

Since operating in the interbank market does not deteriorate the credit rating of the bank, there is no limit in the amount correspondent banks face in order to borrow abroad. Therefore, the interbank market equilibrium will be such that competition among correspondent banks will lead to the limit point $Z^*$ where there are no more gains from correspondent banking:

$$1 + r_F(Z^*) = \bar{\pi}(1 + r_{CB}(Z^*)) = \bar{\pi}(1 + r_B(Z^*)).$$

(10)

From this discussion it is clear that correspondent banking has a positive effect on welfare, since it helps channeling liquidity to where it is most needed: it is efficient that banks with the signals $(\bar{s}, \bar{s})$ liquidate as little as possible. This fact together with a concave liquidation technology implies that having correspondent banking is welfare-improving. The difference with respect to the introduction of a repo market is that there is no possible switch to another equilibrium, as
correspondent banking requires $\psi = 1$. The inefficiency due to the asymmetry of information across countries is not completely removed since an interest rate differential continues to remain.

5.2 Transnational Banks

We will define a transnational bank as one which is part of the financial systems in the two countries and thus issues the same domestic signal in both countries. Therefore, there is no cross-country information asymmetry for transnational banks, so that they are able to operate in both markets when the signal they receive is good, and in none when it is bad. This implies that transnational banks borrow from the country with lower interest rates and lend in the country with the higher ones. Contrasting with the corresponding banking case that emerged only for $\psi = 1$, a transnational bank will operate whenever there is an interest rate differential, including the case of $\psi = 0$.

Because in our model there is no limit to the amount a bank can borrow other than its credit appraisal, transnational banks will borrow from country $A$ and lend to country $B$, as correspondent banks do.

The difference with respect to correspondent banking is that in the correspondent banking case, when arbitraging interest rate differentials is not profitable because condition (9) is not met, transnational banks will still choose to borrow from the cheapest source, in country $A$, if they are liquidity-short ($\pi_H$), and to lend at the best rates, in country $B$, if they are liquidity long ($\pi_L$). Consequently, we will have a "variable size market", as transnational banks will choose the market they enter. If we assume there is a measure $\mu$ of transnational banks in each country, the effect we will have is an increase in the supply of loans in the country with a liquidity shortage, and an increase in the demand for loans in the excess liquidity country. Equilibrium will occur with

$$\hat{\Omega}_B = \Lambda_B$$
$$\hat{\Omega}_A = \Lambda_A$$

where the expression for $\hat{\Omega}_B$ and $\hat{\Omega}_A$ are given by:

$$\hat{\Omega}_B = (1 - \mu)\Omega_B(\psi) + \mu(\sigma_L^A + \sigma_L^B)\pi_L$$
$$\hat{\Omega}_A = (1 - \mu)\Omega_A(\psi) + \mu(\sigma_D^A + \sigma_D^B)\pi_H$$
As a consequence, the effect of transnational banks is, even in the absence of cross-country arbitrage, to diminish the liquidity shocks by allowing a fraction of banks to choose the best rates without facing any information asymmetries.

It is also worth noticing that whenever transnational banks are present, correspondent banking is not profitable. This means that obviously, all transnational banks are able to act as correspondent banks. Still, if the amount of transnational banks is sufficiently large, banks with a good foreign signal will be indifferent between borrowing at home or borrowing abroad, as an integrated market with \( r_B = r_F \) can be achieved. This is in contrast to the result we obtain with correspondent banking (Proposition 11). If the amount of transnational banks is even larger, then the spread between \( r_B = r_F \) and \( r_A \) diminishes and the informational rents in the illiquid country begin to be eroded. This is so as a transnational bank is able to obtain liquidity in the country with excess liquidity at interest rate \( r_A \) (while a correspondent bank would borrow at \( r_F \)).

Notice that the presence of transnational banks is similar to an economy with a cross-border repo market: in both cases, the transfer of liquidity is facilitated and this is efficient provided there is no switch to an inferior equilibrium, as established in proposition 9.

5.3 Heterogeneous default risk levels

Assume now that banks are not homogeneous anymore, and that some banks in each country have a lower probability of failure which is common knowledge. For the sake of simplicity we take the extreme case where some banks are perfectly safe while others are risky. We consider this case to be particularly meaningful, as the too-big-to-fail argument implies that some banks have the unlimited support of the regulatory authority, while others face default with a non-zero probability.

The assumption \( \mathcal{P} = 1 \) immediately implies that such a bank is able to borrow freely from any market, as a transnational bank. But in addition, equation (10) implies a complete integration of the markets. The existence of noisy information for foreign banks will imply that lending unsecured to banks abroad is not profitable, as there is no spread to compensate for the risk. In equilibrium, liquidity is transferred from the excess liquidity country to the one experiencing the liquidity shortage by the safe banks at no cost. This equilibrium is efficient, (independently of the existence of an implicit transfer from the government to
the too-big-to-fail bank which is out of the focus of our analysis).

The implications of this finding are far reaching. It implies that a country that lacks the resources (or the credibility) to back up its major banks in case of distress will be at a disadvantage in obtaining liquidity in the international interbank market. For developed countries that are able to bail-out their banks, it may also imply that there is an interest on behalf of the country to build strong “too-big-to-fail” banks in order to compete in the international arena. If this was so, governments might have an interest in promoting national mergers by creating national banks rather than allowing for the creation of transnational banks that they would not be able to support.

6 Summary and Policy Conclusions

In this paper, we analyzed a model of interbank markets in an international context. We focused on the respective roles played by an unsecured money market and a repo-market on the domestic and international levels, and developed their welfare properties.

In an economy with unsecured markets, lending takes place on the basis of peer monitoring. This is shown to be efficient, as funds are channeled to the most efficient projects. In repo-markets, on the other hand, monitoring plays no role because all loans are collateralized. Therefore, markets are unable to achieve efficient liquidation of unprofitable projects, and insolvent banks forebear. Still, contrary to unsecured markets, a repo market is able to achieve liquidity smoothing across solvent market participants.

In an international context, interbank markets seem to work less efficiently, leading to market imperfections such as liquidity shortages or interest rate differentials. Although these differences could be attributed to exchange rate risk, we argue that an important barrier to an integrated international market is the existence of asymmetric information between different countries.

We have shown that as long as peer monitoring across borders is less efficient than on a domestic level, the integration of unsecured markets can never be perfect. In particular, cross-border lending involves the payment of interest rate premia which reflect the adverse selection of borrowers in the international market. This implies that a perfect liquidity smoothing across borders cannot take
place. As a consequence, we show, first, that an equilibrium with an integrated interbank market does not always exist. Second, even if it does exist, at the same time market segmentation is always an equilibrium. Therefore, even with monetary integration or currency pegging, market integration is not necessarily achieved. Interestingly, we also found that in an integrated equilibrium, welfare is not necessarily higher than in the segmented one.

A repo-market, on the other hand, is always able to function on an international basis, since it overcomes the problem of asymmetric information. Thus, it is able to achieve liquidity smoothing at least to some degree. Still, the welfare effects from a repo facility remain ambiguous: our analysis shows that a repo market reduces the benefits from peer monitoring and might even impede the integration of markets. Furthermore, even the combination of both types of markets is not necessarily beneficial since integration comes at the cost of a higher degree of inefficient forbearance.

Furthermore, the effects of correspondent banks, transnational banks, and banks with varying degrees of riskiness are analyzed. We show that these institutions can play, in varying degrees, a crucial role in the cross-border liquidity transfer.

The paper shed light on the situation in the European Monetary Union, where the differences in national overnight rates in the unsecured interbank market have decreased rapidly after the introduction of the Single Currency. Still, the interbank market seems to be characterized by a two-tier structure in which only large banks are able to use the international market for liquidity. The paper suggests that this could be the result of having banks that are well-known across the Euro area and therefore able to borrow at domestic rates in several countries. An alternative explanation is that these banks are very large so that they are perceived to be "Too-Big To Fail" in the sense that market participants expect these banks to be bailed out should it experience some trouble. This is in line with the empirical evidence found.

Concerning collateralized markets for liquidity, one should note that at the present stage repo markets are not yet well integrated within the Euro area. This is mainly due to infrastructural barriers on the security settlement level which lead to delays in the completion of repo transactions and imply substantial transaction costs. The paper predicts that once these barriers are removed, a
cross-border repo market is likely to develop in which also smaller or less known banks can participate.

Finally, we would like take a more dynamic viewpoint and consider possible changes in the informational structure. In particular, it is conceivable that over time, information becomes more evenly spread across the euro area. The paper has shown that in this case an integrated equilibrium in which also small banks participate in the international market is more likely to be obtained.

References


Appendix A: Proof of Lemmas and Propositions

Proof of Lemma 1. W.l.o.g., assume that $T_0 = 0$, as the repo market is inactive and T-Bills play no role. For a bank of type $k$, $k = L, D, F$, denote $\Delta I_k$ the amount liquidated, $L_k^S (L_k^D)$ the loan given to (demanded by) a bank of type $k$, and $s_k^1$ the storage of reserves. Denote $r_D$ and $r_F$ the domestic and foreign interbank market rates, and $p_F$ the expected probability of solvency of foreign banks. The expected profit at time-1 for a bank facing withdrawals $\pi_L$ is

$$p \left\{ R(I - \Delta I_L) + \overline{p}(1 + r_D)L_D^S + p_F(1 + r_F)L_F^S + s_L^1 - (1 - \pi_L) \right\} + (1 - p) \max \left\{ \overline{p}(1 + r_D)L_D^S + p_F(1 + r_F)L_F^S + s_L^1 - (1 - \pi_L), 0 \right\}$$

where the bank’s liquidity allocation has to satisfy $s_0 + l(\Delta I_L) \geq \pi_L + s_L^1 + L_D^S + L_F^S$. Here, we have assumed that profits are positive when the risky technology succeeds so that banking is profitable. Furthermore, the maximization problem depends on whether the bank has positive profits in the state of nature where the risky project is unsuccessful. For expositional reasons, let us assume that the bank has zero profits in this case, i.e.

$$\overline{p}(1 + r_D)L_D^S + p_F(1 + r_F)L_F^S + s_L^1 - (1 - \pi_L) \leq 0 \quad (13)$$

(the Lemma can be proved for the other case analogously). The bank then chooses $\{\Delta I_k, s_k^1, L_D^S, L_F^S\}$ to maximize the Lagrangian

$$L = p \left\{ R(I - \Delta I_L) + \overline{p}(1 + r_D)L_D^S + p_F(1 + r_F)L_F^S + s_L^1 - (1 - \pi_L) \right\} - p\lambda \left[ \pi_L + s_L^1 + L_D^S + L_F^S - s_0 - l(\Delta, I_L) \right],$$

taking into account the appropriate non-negativity constraints, where we have taken $p\lambda$ to be the Lagrangian multiplier. Similarly, a bank of type $k = D, F$ with withdrawals $\pi_H$ faces the problem

$$\max_{\{\Delta I_k, s_k^1, L_k^D\}} p \left\{ R(I - \Delta I_k) - (1 + r_k)L_k^D + s_k^1 - (1 - \pi_H) \right\}$$

s.t. $s_0 + L_k^D + l(\Delta I_k) \geq \pi_H + s_k^1$. 

The first-order conditions for the Lagrangian for an $L$-bank are the following:

\[
\begin{align*}
\lambda (1 + r_D) - \lambda & \leq 0 \\
p_F (1 + r_F) - \lambda & \leq 0 \\
-R + \lambda l'(\Delta I_L) & \leq 0 \\
1 - \lambda & \leq 0
\end{align*}
\]  

while in a state of nature $k = D, F$ (and multiplier $p\mu$) they are characterized by:

\[
\begin{align*}
-(1 + r_k) + \mu & \leq 0 \\
-R + \mu l'(\Delta I_k) & \leq 0 \\
1 - \mu & \leq 0
\end{align*}
\]

In order for $L$-banks to lend to liquidity short banks from both countries, we need $\lambda (1 + r_D) = p_F (1 + r_F)$. If $\lambda (1 + r_D) \leq 1$, only reserves are used, $\Delta I_k = 0$ for every $k$, and the interbank markets are inactive. Banks with a liquidity shortage liquidate positive amounts only for $1 + r_k > 1/R$, while excess liquidity banks offer loans for $\lambda (1 + r_D) > 1$, and liquidate for $\lambda (1 + r_D) > 1/R$. The optimal liquidation decisions are

\[
\begin{align*}
l(\Delta I_L) &= \max \left\{ 0, \ln \left( \frac{\lambda (1 + r_D)}{R} \right) \right\} \\
l(\Delta I_D) &= \max \left\{ 0, \ln \left( \frac{1 + r_D}{R} \right) \right\} \\
l(\Delta I_F) &= \max \left\{ 0, \ln \left( \frac{1 + r_F}{R} \right) \right\} = \max \left\{ 0, \ln \left( \frac{\lambda (1 + r_D)}{p_F R} \right) \right\}.
\end{align*}
\]

From (2), it follows that $\Lambda_i$ is non-decreasing in the local interest rate. Notice that the result also holds in the case that (13) is not satisfied. In this case, liquidation would be $l(\Delta I_k) = \max \{ l(\Delta I_k) + \ln p \}$ for $k = L, D, F$, which is non-decreasing in the interest rate. 

**Proof of Lemma 2.** We will prove the lemma in two steps. First we prove that only one $\psi_i$ can be non-zero, the one corresponding to the country with high interest rates. Second, we prove that if $\psi_A > 0$, the interest rates differentials between the two countries would imply a contradiction.

1) Assume by way of contradiction that $r_i \geq r_j$, and $\psi_j > 0$. Denote by $r_{iF}$ the rate offered to foreign banks that want to borrow in country $i$. Because
country \( i \) banks have access to coarser information on foreign borrowers, we have \( r_i < r_{iF} \) for \( i = A, B \).

On the other hand, a necessary condition for \( \psi_j > 0 \) is that \( r_{iF} \leq r_j \) (borrowing in country \( i \) is attractive for \( (\bar{s}, \bar{\pi}) \) borrowers in country \( j \)) but this would imply \( r_j > r_i \), a contradiction, so that \( r_i \geq r_j \), implies \( \psi_j = 0 \).

2) To prove this second point, consider first the interest rates when \( \psi_A = \psi_B = 0 \). Equilibrium demands \( \Omega_i = \Lambda_i(r_j) \) for \( i = A, B \). Since the liquidation technology is the same in the two countries, so is the supply function, and as we have \( \Omega_A < \Omega_B \) and the supply is increasing in \( r_j \), we obtain \( r_A < r_B \).

Consider now the case \( \psi_A > 0 \) and \( \psi_B = 0 \). In this case, the demand \( \Omega'_B \) in country \( B \) is larger than in the \( \psi_A = \psi_B = 0 \) case, while the demand for \( \Omega'_A \) is lower than before. This would imply therefore that the market clearing interest rates satisfy again \( r_A < r_B \). But then , using the argument in 1), yields a contradiction, since we have proved in 1) that the in order to have \( \psi_A > 0 \) we need \( r_A > r_B \). □

**Proof of Proposition 3.** A necessary condition for \( \psi > 0 \) is that borrowing abroad is no more expensive than borrowing domestically, so that \( r_B \geq r_F \), with \( \psi = 1 \) in case of strict inequality. On the other hand, (5) implies \( r_F \geq r_A \).

If \( \psi = 0 \), there is segmentation, since any potential borrower is identified as a \((\underline{s}, \bar{\pi})\)-type and therefore, \( p_F = \bar{p} \). But, if \( r_F < r_B \), we would have \( \psi > 0 \), a contradiction. □

**Proof of Lemma 4.** The updated probability of solvency of a foreign borrower is

\[
p_F(\psi) = \frac{\psi \text{prob}(\hat{R} = R \text{ and } (\bar{s}, \bar{\pi})) + \text{prob}(\hat{R} = R \text{ and } (\underline{s}, \bar{\pi}))}{\psi \text{prob}(\bar{s}, \bar{\pi}) + \text{prob}(\underline{s}, \bar{\pi})}
\]

(17)

Taking into account that \( \theta = \alpha \bar{p} + (1 - \alpha)(1 - p) \), it is easy to see that \( p_F'(\psi) > 0 \).

From (5), the foreign premium is given by \( r_F - r_A = (1 + r_A) \left( \frac{\bar{p}}{p_F(\psi)} - 1 \right) \). Because \( p_F'(\psi) > 0 \), it is decreasing in \( \psi \). □

**Proof of Lemma 5.** In country \( B \), there are \( \sigma^L_B = 1 - q_B \) lenders, \( \sigma^D_B(\psi) = q_B \theta [1 - \psi(1 - \beta)] \equiv q_B \xi^B \) domestic borrowers, and no foreign borrowers, \( \sigma^F_B = 0 \).
Using the first order conditions (16), we can express liquidity demand and supply, (1) and (2), as functions of \( \psi \),

\[
\begin{align*}
\Omega_B(\psi) &= \sigma^B_L \pi_L + \sigma^B_D(\psi) \pi_H \\
\Lambda_B(\psi) &= \sigma^B_L \left[ s_0 + \ln \left( \frac{m(1 + r_B)}{R} \right) \right] + \sigma^B_D(\psi) \left[ s_0 + \ln \left( \frac{1 + r_B}{R} \right) \right].
\end{align*}
\]

Since \( \frac{m}{R} > 1 \), it is never efficient to choose \( s_0 \) so that \( \Omega_B(\psi) > s_0 \left( \sigma_L + \sigma^B_D(\psi) \right) \). This implies that some banks (i.e. domestic borrowers) will always liquidate positive amounts. The interest rate \( r_B \) adjusts so that

\[
F_B = \Lambda_B(\psi) - \Omega_B(\psi) = 0. \tag{18}
\]

Then, \( \frac{\partial F_B}{\partial r_B} = \frac{\sigma^B_L}{1 + r_B} > 0 \) (resp. \( \frac{\partial F_B}{\partial r_A} = \frac{\sigma^B_D}{1 + r_B} > 0 \) if lenders do not liquidate) and \( \frac{\partial F_B}{\partial \pi_B(\psi)} = l(\Delta t^B_H) - (\pi_H - s_0) \equiv X^B_H \) which is negative because \( \pi_H \)-banks borrow. Then, the Implicit Function Theorem implies \( \frac{dF_B}{d\pi_B(\psi)} > 0 \), and together with \( \frac{dF_B(\psi)}{d\psi} < 0 \), we obtain \( \frac{dr_B}{d\psi} = \frac{dF_B}{d\pi_B(\psi)} \frac{d\pi_B(\psi)}{d\psi} < 0. \]

**Proof of Proposition 6.** The segmented equilibrium exists since for \( \psi = 0 \), any \( (s, s) \) bank would prefer to borrow at rate \( r_B \) rather than not obtaining any credit. Therefore, \( \psi = 0 \) is consistent with their behavior, \( F_A = 0 \) and \( F_B = 0 \) determine \( r_A \) and \( r_B \), respectively, while \( r_F \) is undetermined since there is no cross-border interbank lending.

An integrated equilibrium does not exist if and only if

\[
\min_\psi r_F(\psi) - r_B(\psi) > 0 \tag{19}
\]

Let \( \psi^*(\Delta q) \) be the solution to (19) with equality. We will show that if \( \Delta q \) satisfies \( \min_\psi r_F(\psi^*(\Delta q)) - r_B(\psi^*(\Delta q)) = 0 \), then for any \( \Delta q' \) such that \( \Delta q' \leq \Delta q \), condition (19) holds, i.e. the integrated equilibrium does not exist. We only have to prove that \( \frac{d(r_F - r_B)}{d\Delta q} < 0 \).

Since \( q_B + q_A = Q \), we have \( dq_A = dq_B \), so that \( d\Delta q = 2dq_B \). Consider first \( \frac{\partial r_B}{\partial \Delta q} \). In country \( B \), \( r_B \) solves \( \Lambda_B = \Omega_B \) (as in the proof of Lemma 5). From (18), denote \( F_B \equiv \Lambda_B - \Omega_B = (1 - q_B) X^B_L + q_B \xi^B X^B_B = 0 \) where \( X^B_L \geq 0 \) and \( X^B_B \leq 0 \). From the proof of Lemma 5, we have \( \frac{\partial F_B}{\partial r_B} = \frac{1}{1 + r_B} \left( \sigma^B_L + \sigma^B_D \right) \). On the other hand,

\[
\frac{\partial F_B}{\partial \Delta q} = 2 \frac{\partial F_B}{\partial q_B} = 2 \left\{ -X^B_L + \xi^B X^B_B \right\} = -\frac{2}{q_B} X^B_B \leq 0
\]
Proof of Proposition 7. Suppose the banks hold a small amount of T-Bills at time 0, and that the integrated equilibrium exists. Denoting by index $i = A, B$ the country, and $k$ the bank type, expected welfare can be shown to be

$$W = I(\lambda_R-1)+T_0^b+\sum_{k=A,B} \sum_{i=1}^{m_k} \int [\Delta \lambda_i - (\Delta \lambda_i)]$$

(21)

where $p_k$ denotes the expected probability of solvency for a given type $k$, i.e.

$$P_k \in [p, \overline{p}].$$

The first term is positive. Furthermore, the second term exceeds the third term, first, $X^b > X^A$ because $r^b > r^A$ implies $(\Delta F_i) > (\Delta F_i)$. Second, it is easy to show that $\frac{\partial F_i}{\partial X_i} < \frac{\partial F_i}{\partial X_i}$ as it is equivalent to $(\Delta X_i) = 0$. Hence, $\frac{\partial F_i}{\partial X_i} < 0$. [$\blacksquare$]

Using the Implicit Function Theorem and that $r^B = r^A$, for $\Delta g$ belonging to

where the last equality follows from (20),

$$\frac{\partial F_i}{\partial X_i} = \frac{\partial F_i}{\partial X_i} = 2 \{X^A - (\theta X^B) [(a_b + a_A) - a_X]\}$$

(20)

with $X^A = 0$, $X^B = 0$ and $X^i = 0$. Then, $\frac{\partial F_i}{\partial X_i} = \frac{\partial F_i}{\partial X_i}$ and

$$\frac{\partial F_i}{\partial X_i} = \frac{\partial F_i}{\partial X_i} = 2 \{X^A - (\theta X^B) [(a_b + a_A) - a_X]\}$$

(20)

Next, consider $\frac{\partial F_i}{\partial X_i}$. Liquidity demand and supply in country $A$ are
Denote the integrated equilibrium \( I \) and the separated one \( S \). In country \( A \), suppose that even in the integrated equilibrium, \( \bar{\pi}(1 + r_A) = 1 \). This implies that country \( A \) banks liquidate the same amount in both equilibria, so we can neglect them in the welfare comparison. For country \( B \), denote interest rates in the segmented equilibrium \( r_B(S) \) and those in the integrated one \( r_B(I) = r_F(I) \), where \( r_B(S) > r_B(I) \). A change from a separated to an integrated equilibrium then affects the following banks in country \( B \): \( \sigma^B_k \equiv 1 - q_B \) lenders and \( \sigma^B_k \equiv q_B \theta \) borrowers with the good domestic signal will face higher interest rates \( 1 + r_B \). Thus, \( \Delta I^B_k(S) > \Delta I^B_k(I), k = L, D \). Furthermore, \( \sigma^B_k \equiv q_B (1 - \theta) \beta \) borrowers with the signal pair \((\underline{\sigma}, \overline{\sigma})\) will go bankrupt in the equilibrium with separation, having to liquidate \( \Delta I^B_k(S) \geq \Delta I^B_k(S) \), but obtain a foreign loan with integration and liquidate \( \Delta I^B_D(I) \).

For \( p_L = p, p_D = \overline{\pi} \), and \( p_C = \underline{\pi} \), we can manipulate the change in welfare from (21) so that

\[
\Delta W \equiv W(S) - W(I) = \sum_{k=L,D,C} \sigma^B_k \left\{ [p_k R \Delta I^B_k(S) - l(\Delta I^B_k(S))] - [p_k R \Delta I^B_k(I) - l(\Delta I^B_k(I))] \right\}
\]

\[
= \sum_{k=L,D,C} \sigma^B_k \left\{ \int_{\Delta I^B_k(I)} \Delta I^B_k(S) l'(\Delta I) d\Delta I - \int_{\Delta I^B_k(I)} \Delta I^B_k(S) p_k R d\Delta I \right\}
\]

\[
= \sum_{k=L,D,C} \sigma^B_k \left\{ \int_{\Delta I^B_k(I)} \Delta I^B_k(S) [l'(\Delta I) - p_k R] d\Delta I \right\}.
\]

Our model assumptions imply \( \overline{\pi} R \geq p R \geq l'(\Delta I) \) and hence that the terms corresponding to \( k = L, D \) are negative, while Assumption 3 implies \( \sum R \leq l'(\Delta I) \) so that the term for \( k = C \) is positive.

The sign of \( \Delta W \) depends therefore on parameters. Suppose first that \( \beta \to 0 \), so that \( \sigma^B_C \to 0 \). Then, \( \Delta W < 0 \), and the the break-down of the integrated equilibrium decreases welfare.

On the other hand, consider the following set of parameters: \( R = 2, p = 0.8, \alpha = 0.9, \beta = 0.45, I = 0.5, T_0 = 0, q_B = 0.55, q_A = 0.425, \pi_H = 0.8 \), and \( \pi_L = 0.3 \). Calculations show that expected welfare (21) is \( W^{seg} = 0.2968 \) in the segmented equilibrium. Furthermore, an integrated equilibrium exists for \( \psi \approx 0.77 \), leading to expected welfare of \( W^{int} = 0.2964 < W^{seg} \).
Proof of Proposition 8. We know that \( p_F(1 + r^*_F) = \bar{p}(1 + r^*_A) < \underline{p}(1 + r^*_B) < 1 + r^*_B \). Now, depending on the T-Bills yield \( \frac{1}{B_t} \), banks will be willing to buy and sell T-Bills rather than making loans. Notice first that if \( \frac{1}{B_t} < \bar{p}(1 + r^*_A) \) there is an excess demand in the T-Bill market as no agent wants to sell T-Bills, and if \( \frac{1}{B_t} > \bar{p}(1 + r^*_A) \) there is an excess supply as no one wants to buy them. As a consequence, for possible equilibria we have to focus on the following cases:

1. \( p_F(1 + r^*_F) = \underline{p}(1 + r^*_A) = \frac{1}{B_t} < \underline{p}(1 + r^*_B) \)
2. \( \underline{p}(1 + r^*_A) = \frac{1}{B_t} < \underline{p}(1 + r^*_B) \)
3. \( \underline{p}(1 + r^*_A) < \frac{1}{B_t} = \underline{p}(1 + r^*_B) \)
4. \( \underline{p}(1 + r^*_A) = \frac{1}{B_t} = \underline{p}(1 + r^*_B) \)

Case 1 corresponds to integration. country B banks will all prefer to sell their T-Bills and use the liquidity thus obtained either to borrow less or to lend more in the interbank market. Case 2 corresponds to segmentation. For a higher T-Bill yield, we would reach case 3. This, however, cannot be sustained as an equilibrium as we have assumed that the T-Bill holdings are insufficient to guarantee survival by borrowing on the T-Bills market only. Consequently, liquidity short L-banks will bid higher interest rates rather than being liquidated. If this occurs, the interest rate hike either restores the case 1-2 type of equilibrium or else it involves only the domestic interbank market. In the latter case, we reach case 4 which corresponds to the case where unsecured interbank market segmentation occurs so that there is no strict inequality between \( r^*_A \) and \( r^*_B \). In case 4, the T-Bill market leads to interest rate equalization without any cross-country unsecured borrowing. If the yield on interest rates is higher, then only liquidity short H-banks will sell T-Bills. Competition among buyers will then restore the equality of either 1,2 or 4. ■

Proof of Proposition 9. Suppose the cross country trading of T-Bills leads to a transfer of liquidity of \( \Delta s_0 \) as in (7) and (8). It is sufficient to show that \( \frac{\partial (r_B - r_A)}{\partial \Delta s_0} < 0 \). Consider country A. From (8), equilibrium with the market now requires a lower supply of liquidity \( \Lambda_A \). Lemma 1 implies that \( \frac{\partial r_A}{\partial \Delta s_0} \geq 0 \). Consider now country B. Notice first that without the T-Bill market, there is a liquidity shortage in country B so that in equilibrium, a positive measure of banks need to liquidate positive amounts. W.l.o.g. assume that \( \Delta I^B_L = 0 \). From (1), (7),
and (16) it follows $1 + r_B = R \exp \left\{ \frac{\Omega_B - \Delta s_0}{\sigma_B^2} \right\}$ so that $\frac{\partial r_B}{\partial \Delta s_0} < 0$. This proves the first part of the proposition.

To establish the second part, suppose that $T_0$ is so high that an equilibrium of the second type in Proposition 8 is obtained, where $r_A(\psi = 0) = r_B(\psi = 0)$. Since $r_F \geq r_A$, $r_A$ is non-decreasing and $r_B$ non-increasing in $\psi$ (see Lemma 5), if follows that $r_F(\psi) > r_B(\psi)$ for all $\psi$. ■

**Proof of Proposition 10.** An integrated equilibrium does not exist if and only if (19) holds. Suppose that $\Delta s_0$ is such that

$$\min_{\psi(\Delta s_0)} r_F(\psi(\Delta s_0)) - r_B(\psi(\Delta s_0)) = 0. \quad (23)$$

We first show that if $\Delta s_0$ satisfies (23), then for any $\Delta s'_0$ such that $\Delta s'_0 > \Delta s_0$, condition (19) holds, i.e. the integrated equilibrium does not exist. For this, we have to prove that $\frac{d(r_F - r_B)}{d \Delta s_0} > 0$. From (7) and (8), define $\hat{F}_A \equiv F_A - \Delta s_0$ and $\hat{F}_B \equiv F_B - \Delta s_0$ with $F_B$ and $F_A$ as defined in (18) and (20). From the proof of Lemma 5 and Proposition 6, we have $\frac{\partial F_A}{\partial r_B} > 0$ and $\frac{\partial F_A}{\partial r_A} < 0$. The Implicit Function Theorem then indeed yields $\frac{dr_B}{d \Delta s_0} = -\frac{\partial F_B}{\partial r_B} > 0$ and $\frac{dr_F}{d \Delta s_0} = -\frac{\partial F_A}{\partial r_A} \frac{\partial F_A}{\partial r_F} < 0$.

As a second step, we consider $\Delta s_0$ so that (23) holds, and consider an infinitesimal increase in $\Delta s_0$, $\Delta s'_0 = \Delta s_0 + \varepsilon$. We will show under which circumstances welfare is decreased.

Because the difference in welfare between the integrated and the segmented equilibrium is independent of $\Delta s_0$, we are entitled to take a sufficiently small change in $\Delta s_0$ whose effect on welfare will be negligible in regard to the effect of the equilibrium switch.

Finally, suppose $\Delta s_0$ satisfies (23). The change in welfare is given by (22). From Proposition (7) follows that for $\beta$ sufficiently small, the switch to a segmented equilibrium reduces welfare. ■

**Proof of Proposition 11.** Using the fact that $r'_F(Z) \geq 0$ and $r'_B(Z) \leq 0$, condition (9) for $Z = 0$ implies

$$\frac{1 - \bar{p}}{\bar{p}} (1 + r_F(0)) \leq r_B(0) - r_F(0).$$

On the other hand, the same inequality will hold in equilibrium replacing 0 by $Z$, so that the wedge $r_B(Z) - r_F(Z)$ has a lower bound. ■
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