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Financial frictions across the production network and the transmission of monetary policy

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Challenges for Monetary Policy Transmission in a Changing World Network (ChaMP)

This paper contains research conducted within the network “Challenges for Monetary Policy Transmission in a Changing World Network” (ChaMP). It consists of economists from the European Central Bank (ECB) and the national central banks (NCBs) of the European System of Central Banks (ESCB).

ChaMP is coordinated by a team chaired by Philipp Hartmann (ECB), and consisting of Diana Bonfim (Banco de Portugal), Margherita Bottero (Banca d’Italia), Emmanuel Dhyne (Nationale Bank van België/Banque Nationale de Belgique) and Maria T. Valderrama (Oesterreichische Nationalbank), who are supported by Melina Papoutsi and Gonzalo Paz-Pardo (both ECB), 7 central bank advisers and 8 academic consultants.

ChaMP seeks to revisit our knowledge of monetary transmission channels in the euro area in the context of unprecedented shocks, multiple ongoing structural changes and the extension of the monetary policy toolkit over the last decade and a half as well as the recent steep inflation wave and its reversal. More information is provided on its website.

Abstract

We show that monetary policy transmission is shaped not only by a sector's own financial frictions but also by those prevailing in the broader production network. The latter, indirect frictions amplify the output and price effects of monetary policy and empirically dominate the direct ones. The amplification results from a downstream demand channel, as customers respond to tighter policy by purchasing fewer inputs. This is partly offset by an upstream cost channel, reflecting that suppliers raise prices to protect margins when financing costs rise. We inspect the mechanism in a multi-sector general equilibrium model with input-output linkages and working-capital constraints.

JEL: C32, C67, E31, E32, E52

Keywords: Monetary policy, production networks, input-output linkages, financial frictions

Non-technical summary

We study how financial frictions in different parts of the production network shape the transmission of monetary policy. To this end, we construct novel measures of financial frictions not only for individual sectors but also for their suppliers and customers within the production network. We do this by combining firm-level balance sheet data with sectoral input-output tables for a broad panel of euro area countries and sectors. These measures are then used to estimate how the frictions prevailing at different points along the supply chain affect the response of output and prices to policy rate shocks.

The results reveal that the network dimension of financial frictions is quantitatively more important than the direct effect of a sector's own financial position. At its peak, the amplification of monetary policy operating through financial frictions in the broader production network is roughly three times as large as the direct effect in absolute terms.

Moreover, we find that upstream and downstream financial frictions – that is, the financial conditions of suppliers and customers respectively – operate through distinct and opposing channels. When downstream customers face more restrictive financial conditions following a monetary policy tightening, they reduce their demand for intermediate inputs, reinforcing the contractionary impact on output and prices. When upstream suppliers face tight financial conditions, they respond by raising prices to protect margins, partially offsetting the disinflationary effect. While these two forces work in opposite directions, the downstream demand channel dominates in the aggregate. Overall, network-level financial frictions thus amplify rather than dampen monetary policy transmission.

To shed light on the underlying mechanisms, we develop a multi-sector general equilibrium model with input-output linkages and sector-specific financial frictions. In this model, we show analytically how the upstream cost channel and the downstream demand channel arise and interact within the production network.

Taken together, our findings imply that two economies with similar average leverage can respond very differently to the same interest rate change, depending on where financial frictions are concentrated along their supply chains. Monitoring financial conditions at key nodes of the production network, rather than only at the aggregate level, can therefore improve assessments of the strength of transmission and thus inform the conduct of monetary policy.

1 Introduction

The past two decades have offered important lessons about the transmission of financial and macroeconomic shocks. The Global Financial Crisis brought leverage back to the fore as a key amplifier of such shocks, while the Covid-19 pandemic showed that these propagate across sectors in ways that purely aggregate frameworks do not adequately capture. In the current paper, we connect these two insights and use them to study the transmission of monetary policy, a third key factor driving macroeconomic dynamics. In particular, we ask how financial frictions distributed across the production network shape the effects of monetary policy on output and prices and find that the answer depends critically on whether those frictions reside upstream or downstream of the sectors being studied.

To this end, the paper makes three key contributions. First, we construct novel measures of upstream and downstream financial frictions for a euro area country-sector panel, combining firm-level balance sheet data with sectoral input-output tables to capture the financial conditions prevailing among each sector's suppliers and customers. Second, we embed these measures in a panel local projections framework to provide systematic empirical evidence on how direct, upstream, and downstream financial frictions shape the transmission of monetary policy to prices and output. Third, we develop a multi-sector general equilibrium model that interprets the empirical findings, deriving analytically an upstream cost channel and a downstream demand channel by which monetary policy transmits through the production network.

The empirical results show that monetary policy tightening has significant contractionary effects on output and prices. While the price response is slightly attenuated in sectors with higher leverage, the network dimension amplifies transmission and turns out to be quantitatively more important: at its peak, the amplification stemming from indirect financial frictions – operating through the balance sheets of the sector's trading partners in the production network – is roughly three times as large as the direct effect of a sector's own leverage.

Moreover, decomposing the indirect channel we find an interesting asymmetry. Downstream financial frictions amplify transmission, as highly-leveraged customers reduce their demand for intermediate inputs. Upstream financial frictions, by contrast, work in the opposite direction: highly-leveraged suppliers raise prices to protect margins, partially offsetting the contractionary effect. The downstream demand channel dominates, so that the net indirect effect is amplifying. But the opposing forces within the indirect channel speak to the importance of distinguishing the direction of network linkages when studying the transmission of monetary policy.

To interpret these findings, we develop a multi-sector general equilibrium model with input–output linkages and sector-specific working-capital frictions, building on Bigio and La'O (2020). The key theoretical result is a closed-form decomposition of each sector's price response into three distinct channels: a direct channel operating through the sector's own financing costs, an indirect upstream cost channel arising when suppliers pass on higher financing costs through input prices, and a downstream demand channel operating when customers facing financial frictions reduce their purchases from the sector in question. This decomposition maps directly onto the empirical specification and provides a coherent mechanism through which heterogeneity in leverage, interpreted following Anderson and Cesa-Bianchi (2024) as a proxy for exposure to external finance premia, shapes the transmission of monetary policy across the production network.

Related literature. The paper sits at the intersection of three strands of literature: the empirical analysis of monetary policy transmission through production networks, the role of financial frictions in that transmission, and the broader theory of shock propagation in network economies.

A growing body of work documents that input-output linkages are important for how monetary policy propagates through the economy. Ozdagli and Weber (2025) use high-frequency stock return data around Federal Reserve press releases and find that a substantial part of the overall monetary policy effect is attributable to indirect network effects. Ghassibe (2021) reaches a similar conclusion using monthly US sectoral consumption data, estimating that at least 30 percent of the aggregate consumption response to monetary shocks reflects amplification through input-output linkages, with network effects increasing in price stickiness and intermediate input intensity. Despite this progress, evidence for economies outside the United States remains scarce; a notable early exception is Peersman and Smets (2005), who adopt a sectoral perspective for European industries, though without accounting for intersectoral linkages. We contribute to this strand by providing euro area evidence that explicitly incorporates the interaction between production network structure and financial frictions, a dimension absent from existing work.

Methodologically, we relate to recent studies using disaggregated price data to trace monetary policy transmission to consumer prices (Borağan Aruoba and Drechsel, 2024; Allayioti et al., 2024), but focus on the production side and on financial conditions as the key moderating factor. In doing so, we document a novel cost channel running through the propagation of sectoral financial frictions across supply chains, extending the aggregate cost-channel evidence of BarthIII and Ramey (2002). Our paper also complements Demir et al. (2024), who show that supply shocks are amplified by low firm liquidity in production networks. We establish an analogous amplification for monetary policy shocks and additionally disentangle a cost channel running from suppliers to customers on the one hand and a demand channel running from customers to suppliers on the other.

Turning to the second strand, financial frictions have been central to the analysis of monetary policy transmission since Bernanke and Gertler (1995) and Bernanke et al. (1999). More recently, firm-level studies have shifted attention to heterogeneity in that transmission. Ottonello and Winberry (2020) show that financially constrained firms invest significantly less following a monetary policy shock than unconstrained ones, rationalizing the finding in a heterogeneous-firm New Keynesian model. Jeenas (2023) highlights corporate liquidity management as a complementary margin, finding that firms with low liquid asset holdings cut investment more sharply after rate increases, independently of leverage or size. We build on this literature by moving beyond the individual firm and show that financial frictions propagate through supplier-customer relationships: a sector's output and price response to monetary policy depends not only on its own balance sheet but also on the balance sheet conditions among its trading partners. Our findings also connect with Adelino et al. (2023), who document that trade credit shapes the transmission of unconventional monetary policy. We broaden that perspective to conventional interest rate policy and use the production network structure to separately identify demand and cost channels.

Finally, our theoretical framework builds on the network macroeconomics literature initiated by Acemoglu et al. (2012), which established that different types of shocks – including to productivity, government spending, and financial variables – travel through supply chains and affect aggregate outcomes. Subsequent work has extended this framework to monetary policy specifically. La'O and Tahbaz-Salehi (2022) show that optimal monetary stabilization requires a price index that places greater weight on larger, stickier, and more upstream industries. Rubbo (2023) demonstrate that input-output linkages break the "divine coincidence", so that simultaneous stabilization of output and prices is generally infeasible in multi-sector economies. On the financial frictions side, Bigio and La'O (2020) show that the US input-output structure amplified financial distortions approximately twofold during the Global Financial Crisis, and Su (2024) demonstrates that input-output linkages amplify sectoral financial distortions to generate aggregate TFP losses. Our model extends Bigio and La'O (2020) by introducing heterogeneous monetary policy pass-through across sectors, governed by sector-specific leverage, which allows us to derive a closed-form decomposition into direct, upstream, and downstream financial friction channels that maps directly onto our empirical specification.

The remainder of the paper is organized as follows. Section 2 describes the euro area country-sector panel and, in particular, the construction of the upstream and downstream financial friction measures. Section 3 presents the empirical results, documenting the direct, upstream, and downstream channels and their relative magnitudes. Section 4 develops the theoretical model and derives the closed-form decomposition of sectoral price responses. Section 5 concludes.

2 Data

We construct a monthly country-sector panel spanning the period from January 1999 to December 2024, covering 20 euro area countries and 64 sectors. The main novelty is that we build a comprehensive set of network-based financial frictions measures combining detailed data on input-output linkages across euro area countries and sectors with granular information on firm balance sheets. In the following, we first present the production network data (section 2.1) and then describe how we feed financial frictions into this structure (section 2.2). Finally, we complement these network-based variables with a set of standard macro-financial outcome and control variables, as well as high-frequency identified monetary policy shocks, which we describe in section 2.3. The section is complemented by appendix A, which provides additional detail on the main components of our dataset.

2.1 Production network

We obtain information on the EU inter-country and industry-by-industry input-output relations from the Eurostat's "Full International and Global Accounts for Research in Input-Output Analysis (FIGARO)" database. The sectoral breakdown broadly corresponds to the two-digit level of the NACE Rev. 2 classification.[1] These

[1]The NACE Rev. 2 classification comprises 88 sectors, whereas the FIGARO input–output tables distinguish only 64 sectors, as several sectors are aggregated in FIGARO (see table A.2).

tables are available at an annual frequency from 2010 to 2022. To cover the entire period of interest, we set the values for years prior to 2010 equal to those observed in 2010, while values for years after 2022 are set equal to their 2022 levels. We take comfort in doing so from the fact that IO linkages are changing only very gradually over time. To illustrate the key metrics we derive from the IO matrix, table 1 provides a schematic example of an IO matrix with just two countries, A and B, and two industries, 1 and 2.

Table 1: Stylized multi-country input-output table

	A_1	A_2	B_1	B_2	*Final consumption*	*Total*
A_1	$x_{11}^{A,A}$	$x_{12}^{A,A}$	$x_{11}^{A,B}$	$x_{12}^{A,B}$	c_1^A	$y_1^A = \sum_{k,d} x_{1k}^{A,d} + c_1^A$
A_2	$x_{21}^{A,A}$	$x_{22}^{A,A}$	$x_{21}^{A,B}$	$x_{22}^{A,B}$	c_2^A	$y_2^A = \sum_{k,d} x_{2k}^{A,d} + c_2^A$
B_1	$x_{11}^{B,A}$	$x_{12}^{B,A}$	$x_{11}^{B,B}$	$x_{12}^{B,B}$	c_1^B	$y_1^B = \sum_{k,d} x_{1k}^{B,d} + c_1^B$
B_2	$x_{21}^{B,A}$	$x_{22}^{B,A}$	$x_{21}^{B,B}$	$x_{22}^{B,B}$	c_2^B	$y_2^B = \sum_{k,d} x_{2k}^{B,d} + c_2^B$
Labor	l_1^A	l_2^A	l_1^B	l_2^B		$l = \sum_{k,d} l_k^d$
Value added	v_1^A	v_2^A	v_1^B	v_2^B		$v = \sum_{k,d} v_k^d$
Taxes	t_1^A	t_2^A	t_1^B	t_2^B		$t = \sum_{k,d} t_s^d$
Total	y_1^A	y_2^A	y_1^B	y_2^B		$y = \sum_{k,d} y_s^d$

In this notation, $x_{jk}^{A,B}$ is the flow of goods produced by sector j in country A and sold to sector k in country B,[2] c_1^A indicates the amount of output produced by sector 1 in country A going into final consumption, l_1^A is the amount of labor used, v_1^A stands for value added and t_1^A for taxes net of subsidies. The sum of the elements in a given row j is equal to the total output produced (for intermediate and final consumption) by sector j. The sum of the elements in a given column k gives the total cost of inputs (including labor, taxes, and value added) used in the production of sector k. By construction, the total output produced is equal to the total cost of inputs.

Based on these definitions, we first compute the share of inputs not sourced from the production network, as the ratio of labor costs, taxes, and value added to total

To ensure consistency with the sectoral disaggregation of the FIGARO tables, all datasets used in the analysis are aggregated accordingly and mapped to the corresponding 64-sector classification.

[2]Note that part of the sector's output could be used by the same sector for production, so one can have $x_{jj}^{X,X} > 0$.

input expenditures. For industry 1 in country A, for instance, this share is given by:[3]

$$\alpha_{1A} = \frac{l_1^A + v_1^A + t_1^A}{x_{11}^{A,A} + x_{21}^{A,A} + x_{11}^{B,A} + x_{21}^{B,A} + l_1^A + v_1^A + t_1^A} \tag{1}$$

Second, we compute the share of output directed outside the production network as the ratio of final consumption over total output. For sector 1 in country A, this is equal to:

$$\tilde{\alpha}_{1A} = \frac{c_1^A}{x_{11}^{A,A} + x_{12}^{A,A} + x_{11}^{A,B} + x_{12}^{A,B} + c_1^A} \tag{2}$$

The matrix $\mathbf{X}$ denotes the square input–output matrix, which records the flows of goods and services across sectors and countries:

$$\mathbf{X} = \begin{bmatrix} x_{11}^{A,A} & x_{12}^{A,A} & x_{11}^{A,B} & x_{12}^{A,B} \\ x_{21}^{A,A} & x_{22}^{A,A} & x_{21}^{A,B} & x_{22}^{A,B} \\ x_{11}^{B,A} & x_{12}^{B,A} & x_{11}^{B,B} & x_{12}^{B,B} \\ x_{21}^{B,A} & x_{22}^{B,A} & x_{21}^{B,B} & x_{22}^{B,B} \end{bmatrix} \tag{3}$$

Starting from matrix $\mathbf{X}$, we construct the direct-coefficient matrix $\mathbf{\Omega}$ by normalizing each element $x_{jk}^{c,d}$ by the total output of sector j in country c, denoted by y_j^c. Formally, each coefficient is defined as $\omega_{jk}^{c,d} = x_{jk}^{c,d}/y_j^c$. Each element $\omega_{jk}^{A,B}$ therefore measures the share of output produced by sector j in country A that is absorbed by sector k in country B, capturing the relative importance of sector k as a customer of sector j.

$$\mathbf{\Omega} = \begin{bmatrix} \omega_{11}^{A,A} & \omega_{12}^{A,A} & \omega_{11}^{A,B} & \omega_{12}^{A,B} \\ \omega_{21}^{A,A} & \omega_{22}^{A,A} & \omega_{21}^{A,B} & \omega_{22}^{A,B} \\ \omega_{11}^{B,A} & \omega_{12}^{B,A} & \omega_{11}^{B,B} & \omega_{12}^{B,B} \\ \omega_{21}^{B,A} & \omega_{22}^{B,A} & \omega_{21}^{B,B} & \omega_{22}^{B,B} \end{bmatrix} \tag{4}$$

The matrix $\mathbf{\Omega}$ can be used to capture the direct (first-order) effects of shocks, such as a change in demand for the output of a given sector, on the production of its immediate suppliers.[4] However, it does not account for the indirect (higher-order) effects that arise through subsequent rounds of production adjustments along the supply chain. To take into consideration these higher-order effects, we follow the approach of Acemoglu et al. (2016) and define the Leontief inverse matrix as:

[3] For a mapping of these definitions into the variables contained in the FIGARO tables, see appendix A.

[4] Equivalently, in its transposed form, it characterizes the propagation of cost shocks.

$$\mathbf{Z} \equiv (\mathbf{I} - \mathbf{\Omega})^{-1} \tag{5}$$

where $\mathbf{I}$ denotes the identity matrix. The matrix $\mathbf{Z}$ summarizes the total (direct and indirect) effects of a demand shock propagating from customers to suppliers through production linkages. Analogously, we define the Leontief inverse based on the transposed coefficient matrix as:

$$\tilde{\mathbf{Z}} \equiv (\mathbf{I} - \mathbf{\Omega}^T)^{-1} \tag{6}$$

which captures the total (direct and indirect) effects of cost shocks propagating from suppliers to customers through the production network. Table A.4 provides an overview of the notation and definitions used throughout the paper.

2.2 Network-based financial frictions measures

2.2.1 Definitions and measurement

Our main hypothesis is that the response of output and prices to monetary policy shocks may be shaped not only by the financial frictions faced by a given sector, but also those faced by its suppliers and customers. To quantify the exposure of a given sector j in country c to the financial frictions of its counterparts in the production network, we combine sectoral measures of financial frictions with the input–output structure of the economy. Specifically, we define the exposure of sector j to financial frictions of its direct and indirect upstream suppliers ($\Phi_{jc,t_{12}}$) and downstream customers ($\tilde{\Phi}_{jc,t_{12}}$) as follows:

$$\begin{aligned}
\Phi_{jc,t_{12}} &= \sum_{k,d} (z_{kd,jc,t_{12}} - 1_{k=j,d=c}) \times \varphi_{kd,t_{12}} \\
\tilde{\Phi}_{jc,t_{12}} &= \sum_{k,d} (\tilde{z}_{jc,kd,t_{12}} - 1_{k=j,d=c}) \times \varphi_{kd,t_{12}}
\end{aligned} \tag{7}$$

where $\varphi_{kd,t_{12}}$ denotes the financial frictions of sector k in country d at time t, measured either by total leverage or by the working capital ratio (see below).[5] The

[5] To distinguish between frequencies, variables observed at annual frequency are indexed with the subscript t_{12}.

terms $z_{kd,jc,t_{12}}$ and $\tilde{z}_{jc,kd,t_{12}}$ denote, respectively, the elements of the Leontief inverse and its transpose. Under this formulation, upstream and downstream financial frictions are computed as the weighted sum of sector j's exposure to its suppliers and customers, respectively. The weights are given by the bilateral linkages between sector j and each sector $k \in K$ in country $d \in D$, as captured by the relevant elements of the Leontief matrices, while the intensity of exposure is determined by the level of financial frictions in sector k in country d, $\varphi_{kd,t_{12}}$. Following Acemoglu et al. (2016), we isolate the contribution of a sector's own financial frictions by removing the within-effect embedded in the network representation. In practice, this is achieved by subtracting one from the diagonal elements of the Leontief matrices (equation 7) for the case $k = j$ and $d = c$.

To measure these financial frictions, we use firm-level data from Orbis and aggregate these up to the country-sector level.[6] In the baseline specification of our empirical analysis, financial frictions are proxied by the total leverage ratio. As a robustness check, we additionally consider the working capital ratio, given its close conceptual alignment with the representation of financial frictions in the theoretical framework developed in section 4.[7] To ensure consistency with the level of aggregation of the remaining variables, we compute financial frictions at the level of the 64 sectors in our dataset by taking sector-specific weighted averages of the firm-level ratios, where weights are given by firm sales. As a robustness exercise, we alternatively construct these measures using the sectoral median of the corresponding firm-level financial frictions indicators.

2.2.2 Stylized facts

The left panel in figure 1 presents the heatmap illustrating the Leontief inverse given by equation 5. For ease of exposition, the coefficients are presented for the euro area as a whole and at NACE 1-digit sectors. The heatmap shows that the euro area production network can be broadly characterized as "diagonal", with internal (roundabout) exchange of inputs and outputs in a respective sector being relatively

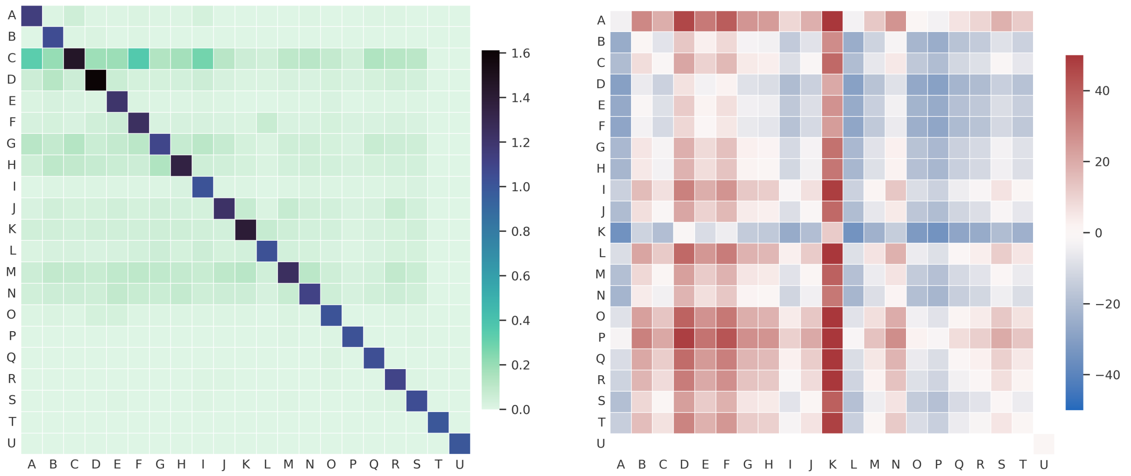
[6]We clean Orbis data following the routines and conventions proposed by Gopinath et al. (2017) and Kalemli-Özcan et al. (2024). Moreover, as our Orbis sample includes data with satisfactory coverage only up until 2023, we assume that 2024 figures are equal to those recorded in 2023.

[7]The total leverage ratio is defined as total liabilities, excluding shareholders' funds, divided by total assets. The working capital ratio is measured as working capital over total assets, where working capital is defined as inventories (stocks) plus trade receivables (trade debit) minus trade payables (trade credit).

important. However, some sectors such as manufacturing (C) or wholesale/retail trade (G) are important suppliers to and/or customers of other sectors.

The right panel in figure 1 illustrates how the relative significance of upstream (supplier) and downstream (customer) financial frictions across NACE-1 sectors in the euro area changes once IO entries are interacted with total leverage. A blue cell indicates that the corresponding row sector has a lower relative importance as a supplier compared to when only the flow of goods and services is considered. Similarly, a red cell signifies that the corresponding column sector's relative importance as a buyer of the row sector increases when leverage is taken into account. As the figure illustrates, the relative importance of certain sectors can vary considerably, with some sectors gaining up to about 40 percent in importance as leverage transmitters, while others experience a decline of more than 20 percent when compared to an analysis based solely on intersectoral flows of goods and services.

Figure 1: Input–output linkages and the leverage-induced reweighting of upstream and downstream exposures, NACE 1-digit, euro area



(a) Leontief inverse **(b)** Delta accounting for leverage (%)

Notes: Data refer to 2022 for the euro area (fixed composition), obtained by aggregating across countries and sectors. Left panel reports the Leontief Inverse **Z** given by equation 5. Right panel reports the leverage-induced change in the relative importance of sectors, computed as the difference between the leverage-weighted exposures and the standard input–output exposures shown in the left panel. Sector definitions follow the NACE 1-digit categorisation: *A: Agriculture, B: Mining, C: Manufacturing, D: Electricity/Gas, E: Water/Waste, F: Construction, G: Wholesale/Retail, H: Transport/Storage, I: Accommodation/Food, J: IT/Communication, K: Financial/Insurance, L: Real Estate, M: Professional/Scientific, N: Admin/Support, O: Public Admin, P: Education, Q: Health/Social Work, R: Arts/Entertainment, S: Other Services, T: Household Activities, U: Intl. Organizations.*

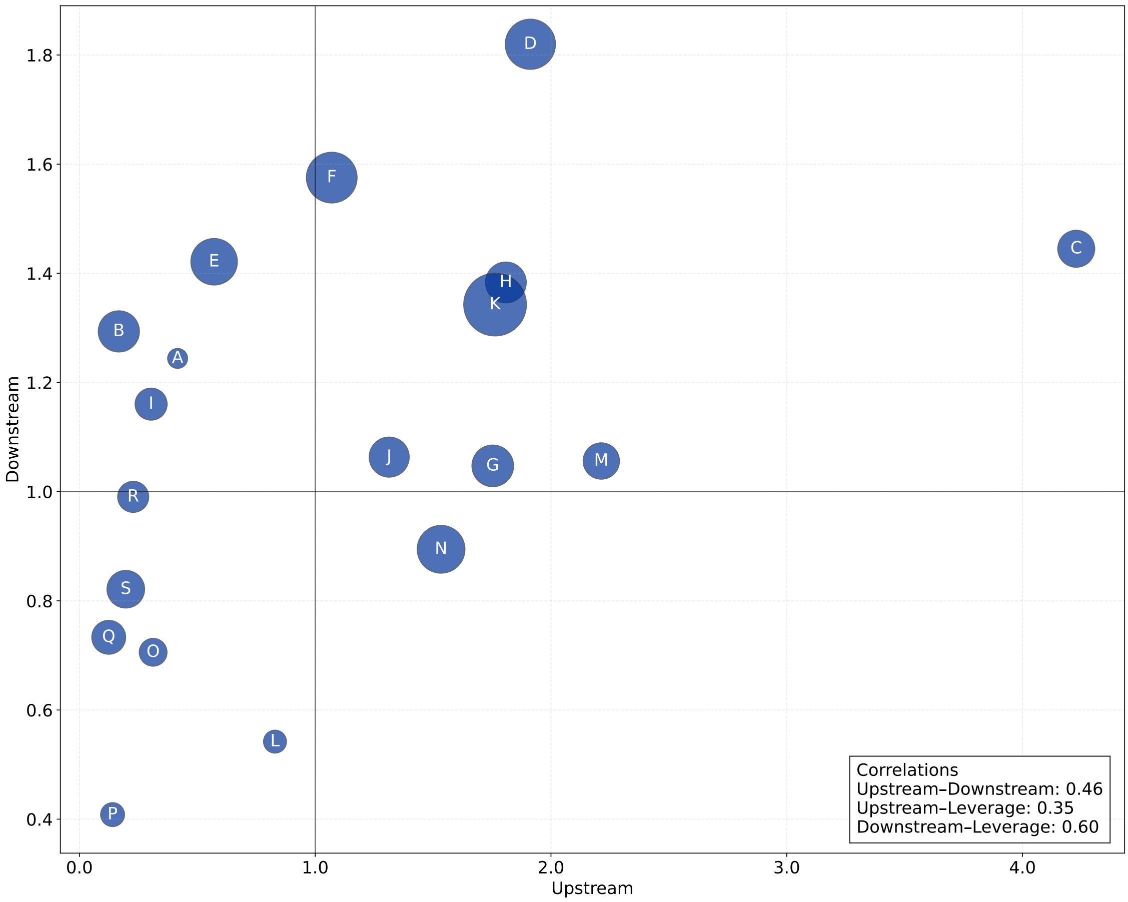
Figure 2 illustrates the relationship between a sector's own financial frictions, proxied by its leverage, and the financial frictions of customers and suppliers to which the sector is directly and indirectly exposed.[8] Overall, bubbles are scattered mainly across the two left and the upper-right quadrants delineated by the lines at upstream and downstream exposure equal to one. The correlation between sectors' upstream and downstream financial-friction exposures is moderate, standing at 0.46.

Several sectors are highly exposed on one side of the network but not on the other, suggesting that the two channels are complementary rather than overlapping. In the upper-right quadrant (high upstream and high downstream), Manufacturing (C) stands out as the most upstream-exposed sector by a wide margin, with above-average downstream exposure as well. Electricity/Gas (D) shows the highest downstream exposure combined with high upstream exposure, while Construction (F) ranks second in terms of downstream exposure and just above average on the upstream dimension. Transport/Storage (H), Financial/Insurance (K), Professional/Scientific (M), Wholesale/Retail (G) and IT/Communication (J) cluster at moderately high upstream values with downstream slightly above one. In the upper-left quadrant, Mining (B), Water/Waste (E), Agriculture (A) and Accommodation/Food (I) are mainly exposed via their customers rather than their suppliers. The lower-left quadrant gathers less central sectors with low exposure on both sides, including Real Estate (L), Education (P), Health (Q), Public Administration (O), Other Services (S) and Arts (R). Finally, Admin/Support (N) lies in the lower-right quadrant, exposed mainly through their suppliers.

Bubble size, which captures a sector's own leverage, is weakly correlated with the upstream position (0.35) and more correlated with the downstream one (0.6). Taken together, a sector's own leverage is a poor predictor of its network-based financial-friction exposure, and upstream and downstream exposures themselves are moderately related, with most sectors tilted towards one side of the network only.

[8] The figure shows the results for the euro area as a whole and at NACE-1 digit level, while we obtain comparable patterns also for individual euro area countries and the NACE-2 digit level of disaggregation.

Figure 2: Sectoral leverage and production-network financial exposure in the euro area



Notes: Values refer to the euro area aggregate at the NACE 1-digit sectoral level in 2023. Euro area aggregates are constructed by summing values across sectors and countries using Orbis firm-level leverage data and the FIGARO input-output matrix. Each bubble represents a sector, with bubble size proportional to the sector's own leverage. The x-axis measures upstream exposure, while the y-axis measures downstream exposure. Variables are normalized by the cross-sectoral mean, such that values above (below) 1 indicate levels above (below) average. Sector definitions follow the applied at NACE-1 level categorization: *A: Agriculture, B: Mining, C: Manufacturing, D: Electricity/Gas, E: Water/Waste, F: Construction, G: Wholesale/Retail, H: Transport/Storage, I: Accommodation/Food, J: IT/Communication, K: Financial/Insurance, L: Real Estate, M: Professional/Scientific, N: Admin/Support, O: Public Admin, P: Education, Q: Health/Social Work, R: Arts/Entertainment, S: Other Services, T: Household Activities, U: Intl. Organizations*. Sectors *T* (Household Activities) and *U* (International Organizations) are omitted from the figure, as their upstream and downstream exposures are zero by construction, reflecting their negligible role in the production network.

2.3 Monetary policy shocks and sectoral data

Monetary policy shocks are identified using high-frequency movements in the three-month OIS rate within a narrow window around ECB Governing Council meetings, drawn from the event-study database of Altavilla et al. (2025). To separate monetary policy shocks from information shocks, we apply the sign restriction approach of Jarociński and Karadi (2020), retaining only those OIS rate movements that are accompanied by stock price movements of the opposite sign. The outcome variables of interest – producer price indices and industrial production – are drawn from the Eurostat Short-Term Business Statistics (STS) dataset, supplemented for specific sectors where standard STS coverage is incomplete. Full details of the construction of all series, including sectoral adjustments, seasonal adjustment procedures, and data cleaning steps, are provided in appendix A. Summary statistics for all variables are reported in table A.3.

3 Empirical analysis

3.1 Econometric strategy

In what follows, we employ a panel local projection framework, similar to that of Jordà and Taylor (2016),[9] to study the transmission of monetary policy while explicitly accounting for the role of production networks and financial frictions. Concretely, we estimate the following model:

[9]This framework is particularly suited for studying the transmission of exogenously identified shocks and requires only few assumptions regarding the data generating process.

$$y_{jc,t+h} - y_{jc,t-1} = \underbrace{\beta_1^h \varphi_{jc,t_{12}-1} \times s_t}_{\substack{\text{Direct financial frictions}\\ \text{effect}}} + \underbrace{\overbrace{\beta_2^h \Phi_{jc,t_{12}-1} \times s_t}^{\text{Upstream effect}} + \overbrace{\beta_3^h \tilde{\Phi}_{jc,t_{12}-1} \times s_t}^{\text{Downstream effect}}}_{\substack{\text{Indirect financial frictions}\\ \text{effect}}} + \tag{8}$$

$$\underbrace{\beta_4^h \alpha_{jc,t_{12}-1} \times s_t + \beta_5^h \tilde{\alpha}_{jc,t_{12}-1} \times s_t + \beta_6^h s_t}_{\text{Non-network effect}}$$

$$+ \sum_{l=0}^{L} \gamma^{\mathbf{h}} \mathbf{H_{t-l}} + \sum_{l=1}^{L} \delta^{\mathbf{h}} \mathbf{K_{t-l}} + \sum_{l=0}^{L} \eta^{\mathbf{h}} \mathbf{\Delta X_{t-l}} + \theta_m + \kappa_{t+h} + \epsilon_{jc,t+h}$$

$$\text{with} \qquad h = 1, 2, ..., H \quad \text{and} \quad L = 3$$

$$\mathbf{H_t} = \begin{bmatrix} \alpha_{jc,t_{12}-1} \\ \tilde{\alpha}_{jc,t_{12}-1} \\ \varphi_{jc,t_{12}-1} \\ \Phi_{jc,t_{12}-1} \\ \tilde{\Phi}_{jc,t_{12}-1} \end{bmatrix}, \mathbf{K_t} = \begin{bmatrix} \Delta y_{jc,t} \\ \varphi_{jc,t_{12}-1} \times s_t \\ \alpha_{jc,t_{12}-1} \times s_t \\ \tilde{\alpha}_{jc,t_{12}-1} \times s_t \\ \Phi_{jc,t_{12}-1} \times s_t \\ \tilde{\Phi}_{jc,t_{12}-1} \times s_t \\ s_t \end{bmatrix}$$

We follow Jordà and Taylor (2024) and estimate the model in long-differences, with $\Delta x_t = x_t - x_{t-1}$.[10] The specification allows us to study interactions between production networks and financial frictions for the overall transmission of monetary policy shocks s_t. In our model, the full impact of the shock is determined by the sum of three separate transmission channels: a direct, an indirect and a non-network channel. We capture the "direct financial frictions" channel by the coefficient β_1^h on the interaction of the monetary policy shock s_t with the respective sector's level of financial frictions as measured by $\varphi_{jc,t}$.[11] This channel can be interpreted as a sector-level representation of the traditional balance sheet channel (Bernanke and Gertler, 1995).

We then identify an additional "indirect financial frictions" channel taking into account how a monetary policy tightening may be amplified via balance sheet channel conditions in other parts of the production network that sector j interacts with. To this end, we interact the up- and downstream financial frictions measures $\Phi_{jc,t_{12}}$

[10] As shown in appendix section D.2, our main results are robust to estimating the model in levels.

[11] Similarly to the annual notation introduced in section 2.2, the notation $t_{12} - 1$ refers to the one-year lag of a variable at annual frequency reported for the previous year. We lag the financial frictions measures by one year to avoid contemporaneous endogeneity concerns.

and $\tilde{\Phi}_{jc,t_{12}}$ given by equation 7 with the monetary policy shock s_t, again lagging the financial frictions measure by one year. This approach also allows us to disentangle the overall "indirect financial frictions channel" into downstream (β_2^h) and upstream (β_3^h) financial frictions effects. We also control for the degree to which the transmission of the monetary policy shock to aggregate output and prices depends on a sector j's activity taking place outside the production network. To account for the importance of non-network customers of sector j, we interact the policy shock s_t with $\tilde{\alpha}_{jc,t}$, the share of production sold to final customers outside the network as given by equation 2 (β_4^h). Likewise, we account for the importance of obtaining inputs from outside the network in the transmission of the monetary policy shock by interacting s_t with $\alpha_{jc,t}$, the share of production inputs purchased by sector j from outside the network, as given by equation 1 (β_5^h). Finally, coefficient β_6 account for all other possible channels through which monetary policy shocks may transmit to the real economy, i.e. independent of up- and downstream financial frictions and the broader production network.[12] In the main results presented in section 3.2, we refer to coefficients β_4^h to β_6^h jointly as "non-network effects".

Finally, matrix $\mathbf{H_t}$ contains the remaining single elements in our interaction terms unrelated to the monetary policy shock s_t which are not of first-order interest in our analysis. Matrix $\mathbf{K_t}$ collects lags of first-differences of the dependent variable $\Delta y_{jc,t}$ and the shock variables. Matrix $\mathbf{X_t}$ contains a set of macro-financial control variables including the euro area three-month OIS rate, a GDP-weighted 10y composite euro area sovereign bond yield, the euro-dollar exchange rate, and log-levels of the euro area Composite Indicator of Systemic Stress (CISS), the IMF Commodities Price Index, the euro area harmonized index of consumer prices (HICP) and the euro area unemployment rate, as well as sectoral employment which we add as an additional sector-specific macroeconomic control. θ_m denotes month fixed-effects,[13] and we control for the Covid-19 pandemic by adding a forward dummy κ_{t+h} entering the model at the same horizon as the dependent variable.[14]

[12] Such channels may include the interest rate channel, the exchange rate channel, asset price channels, risk-taking and expectation channels. See for instance Beyer et al. (2017) for an overview of traditional transmission channels of monetary policy.

[13] We only account for month fixed-effects as long-differencing eliminates entity (i.e. country-sector) fixed effects. As shown in appendix section D.2, we obtain almost identical results when estimating the model in levels including both month and country-sector fixed-effects.

[14] The forward dummy takes a value of one for the pandemic period being defined as lasting from March 2020 to April 2023, in line with the World Health Organization' declaring the end of Covid-19 as a global health emergency in May 2023.

3.2 Main results

In this section, we report our main findings on the role of production networks and financial frictions obtained with the model presented in section 3. We show results using total leverage as our financial friction measure of interest, $\varphi_{jc,t_{12}-1}$, and we ensure consistency in units between $\varphi_{jc,t_{12}-1}$, $\Phi_{jc,t_{12}-1}$, $\tilde{\Phi}_{jc,t_{12}-1}$, $\alpha_{jc,t_{12}-1}$ and $\tilde{\alpha}_{jc,t_{12}-1}$ by scaling the beta coefficients of the interaction terms such that a one-unit change refers to a z-score change in leverage, i.e. a one-standard deviation of leverage from the mean.[15] All impulse responses are scaled to the impact of a monetary policy shock that leads to a 25 basis point peak increase of the three-month OIS rate within the first year after the shock.[16] Standard errors in all specifications are clustered at the country-sector and time level.[17]

Finding 1. *Prices and production fall in response to a monetary policy tightening shock, with financial frictions amplifying the contractionary effect over the medium term.*

As shown in figure 3, both producer prices and industrial production fall significantly in response to a monetary policy tightening shock. For producer prices, the average effect of a monetary policy shock (β_6^h in equation 8), with all financial frictions measures at the mean (black line in the LHS panel of figure 3), amounts to a trough decline of 0.8 percent between 6 to 12 months after the shock. The blue line in figure 3 reports the *additional* overall effect of a one standard deviation increase in network-wide financial frictions above the mean, as measured by the linear combination of coefficients β_1^h, β_2^h, and β_3^h. Initially, an increase in network-wide financial frictions partly counteracts the decline in prices and output in the first year, before it starts adding to the downward economic effects induced by a monetary policy tightening. As we discuss below, the difference in short-to medium financial frictions effects can be attributed to additional indirect downstream demand effects materializing over the medium term – and overcompensating short-term cost-channel effects. At the trough, the additional disinflationary effect of a monetary policy

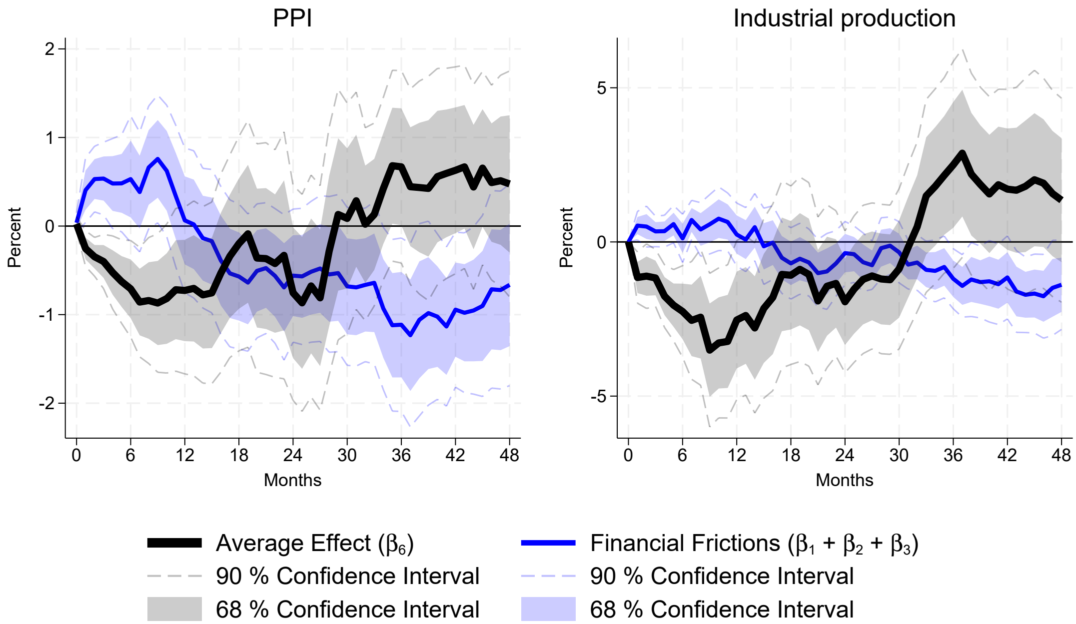
[15]Formally, the z-score is defined as $z_{jc,t} = \frac{x_{jc,t}-\mu}{\sigma}$, with $x_{jc,t}$ being the country-sector-time observation, and μ and σ indicating the population mean and standard deviation, respectively.

[16]See appendix section B for details on the scaling routine.

[17]Following Jordà and Taylor (2024), we use cluster-robust standard errors as the default, as using for instance Driscoll and Kraay (1998) standard errors to account for heteroscedasticity and autocorrelation would require a large time-series dimension T compared to the cross-sectional dimension N, which is not the case in our setup.

shock due to a one standard deviation increase of network-wide leverage amounts to approximately one percent. The average decline in production stands at approximately 3 percent 9 months after the shock (black line in the RHS panel of figure 3), with an additional contractionary effect of the monetary policy shock due to financial frictions materializing after approximately 3 years (blue line in the RHS panel of figure 3).

Figure 3: Impulse responses to monetary policy tightening shock

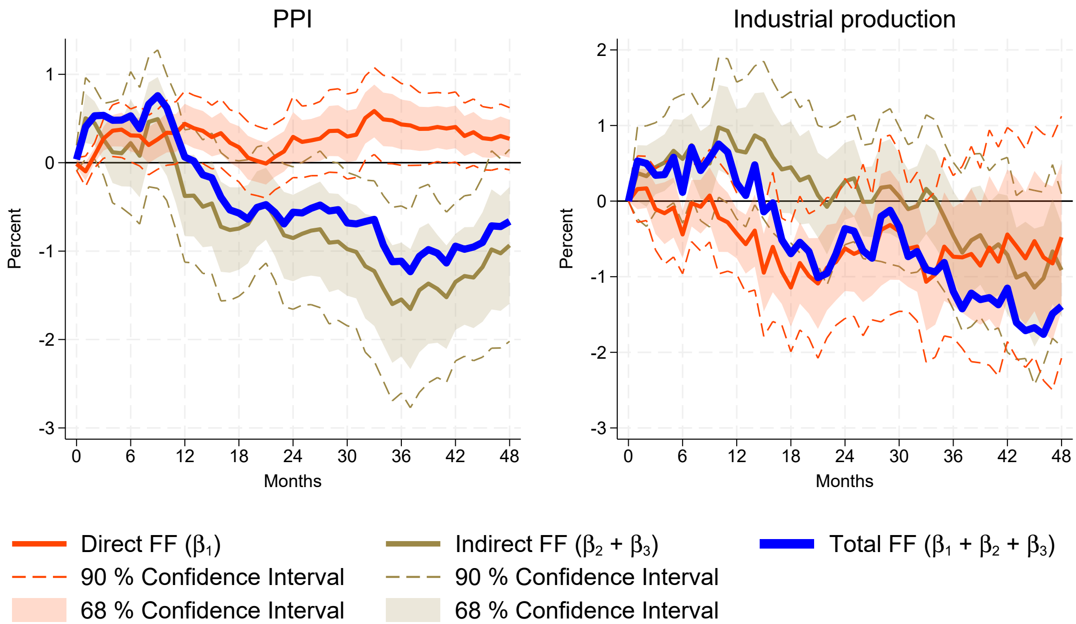


Notes: Impulse responses to a monetary policy tightening shock scaled to a peak increase in the 3m OIS rate of 25bp. Estimates for model 8 on the full country-sector euro area panel.

Finding 2. *Effects stemming from network-related financial frictions are statistically significant and economically sizable.*

Figure 4 decomposes the overall financial frictions effect reported in blue in figure 3 into the direct financial frictions effect related to the interaction of the monetary policy shock with the respective sector's own leverage (as measured by β_1^h in equation 8), and the indirect effect of financial frictions in the transmission of monetary policy shocks (as measured by the linear combination of β_2^h and β_3^h in equation 8). It shows that direct financial frictions induce significant upward pressure on prices and dampen activity, whereas indirect financial frictions decrease producer prices particularly over the medium term. While the upward effect from direct financial frictions on prices proves persistent throughout the horizon, the trough impacts for indirect financial frictions effects are reached approximately 3 years after the shock (red and brown lines in LHS panel).[18] The direct effect explains a large share of the inflationary impact of overall network-induced financial frictions in the short term, while price-dampening indirect effects dominate in the total financial frictions effect over the medium-term horizon. At its peak, the amplification effect of indirect frictions is around three times as high, in absolute terms, as the dampening effect of direct frictions. As for industrial production (RHS panel in figure 4), the trough impact of direct financial frictions is reached after around 1.5 years and only after 46 months in the case of indirect financial frictions. Regarding the respective shares in the total financial frictions effects (blue line), indirect effects emerge as the dominating factor, reinforcing the disinflationary effect of monetary tightening over the medium term.

Figure 4: Impulse responses to monetary policy tightening shock - direct vs. indirect financial frictions effects



Notes: Impulse responses to a monetary policy tightening shock scaled to a peak increase in the 3m OIS rate of 25bp. Estimates for model 8 on the full country-sector euro area panel.

[18] As for the cross-sectoral exposures discussed in more detail in the following paragraph and figure 5, potentially counterbalancing cost and demand channels stemming from within-sector firm exposures determine the overall shape and sign of the direct effect. In particular, the fact that some firms may act primarily as buyers from or sellers to firms *within the same sector* may induce the same channels as described in the following, with downstream demand effects appearing to dominate across specifications.

Finding 3. *Downstream (upstream) frictions reinforce (dampen) the drop in prices, consistent with upstream cost and downstream demand channels.*

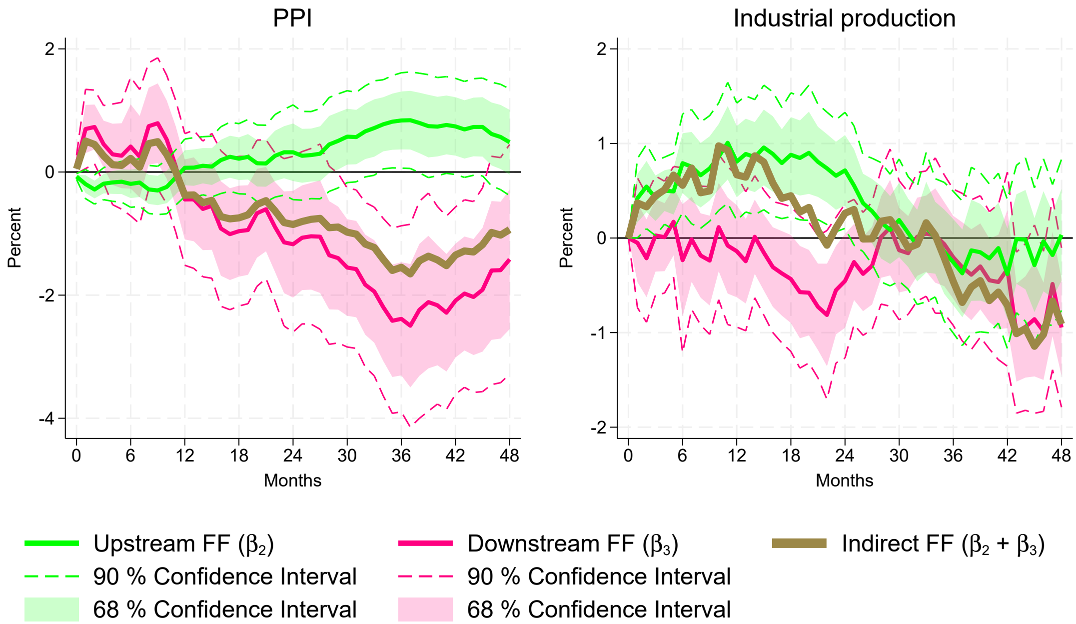
Figure 5 provides a breakdown of the indirect financial frictions effects reported by the brown lines in figure 4 into up- and downstream channels (as measured by β_2^h and β_3^h in equation 8, respectively). Interestingly, these work in opposite directions: downstream financial frictions reinforce the decline in prices and output following a monetary policy tightening shock (pink lines in figure 5); by contrast, upstream financial frictions tend to mitigate these effects, though only partly (green lines in figure 5). As a consequence, the overall drop in prices over the medium term stemming from the interaction of the monetary policy shock and the indirect financial frictions (brown line LHS panel of figure 5) can be largely attributed to downstream financial frictions. Similarly, upstream financial frictions mitigate the drop in industrial production over the first two years after the shock (green line RHS panel of figure 5), while downstream financial frictions aggravate the additional drop (pink line RHS panel of figure 5). Taken together, the results thus indicate that a tightening of financial conditions lowers downstream customers' demand for intermediate goods produced by sector i (in line with a sector-specific "demand channel"), while fostering incentives for upstream suppliers to raise prices to alleviate financial frictions (in line with a sector-specific "cost channel").

Finally, to quantify the importance of different monetary policy transmission channels, we decompose the variance of the model-implied monetary policy effect into contributions from its underlying components.[19] Specifically, we construct the monetary policy-related fitted value as the sum of all terms in regression 8 that involve the monetary policy shock captured by coefficient β_1 to β_6, including all interaction terms and their lags. We then attribute the variance of this monetary policy-induced component to individual channels based on their covariance with the overall fitted value. This covariance-based decomposition captures the extent to which each channel contributes to fluctuations in the monetary policy effect.

Figure 6 plots the resulting variance shares across horizons. For producer prices, the average effect, downstream financial frictions, and non-network effects account for the largest share of the monetary policy-induced variation over the projection horizon. The average monetary effect contributes positively throughout, indicating that it co-moves strongly with the aggregate response and is a key driver of its fluctuations. The downstream financial frictions channel exhibits a pronounced sign

[19] See appendix C for details.

Figure 5: Impulse responses to monetary policy tightening shock - upstream vs. downstream financial frictions effects



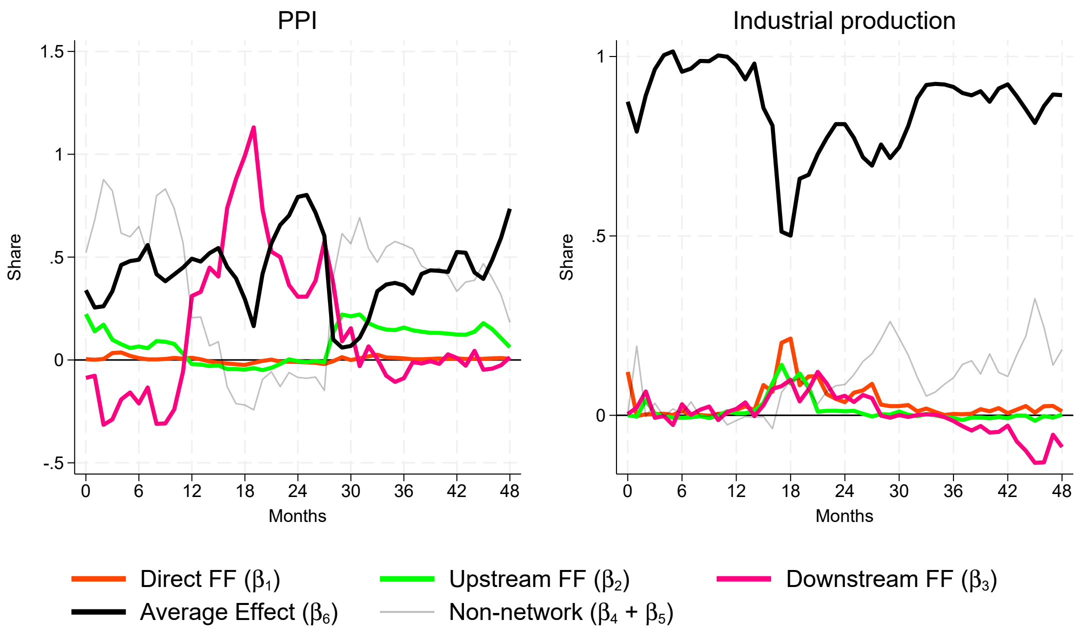
Notes: Impulse responses to a monetary policy tightening shock scaled to a peak increase in the 3m OIS rate of 25bp. Estimates for model 8 on the full country-sector euro area panel.

change over time: it contributes negatively to the variance at short horizons, implying that it initially moves against the aggregate response and offsets fluctuations in the first year after the shock. At longer horizons, however, its contribution turns strongly positive, indicating that it becomes an important driver of the variation in the monetary policy effect in the second and third year after the shock. As we discuss in section 4.5, this evidence is qualitatively consistent with the results of a comparable variance decomposition derived from simulated data using the theoretical model presented in section 4. Direct and upstream financial frictions channels contribute comparatively little to the overall variation in prices induced by monetary policy shocks. For industrial production, similar patterns emerge, with the average effect dominating the variation, while the remaining channels play a more limited role.

3.3 Robustness

We report a detailed set of robustness checks in appendix D and provide a brief summary here. First, we assess whether our main results change when using the

Figure 6: Variance decomposition of monetary policy channels



Notes: Variance decomposition of monetary policy-related components in model 8.

working capital share instead of leverage to define the financial frictions measure $\varphi_{t_{12}}$. Second, we assess how differences in aggregating firm-level data on financial frictions to the sectoral level affect the result. Third, we discuss robustness to an alternative specification accounting for distortions as in Baqaee and Farhi (2020). While the main results in figures 3 to 5 were derived using sales-based weighted averages of firm level data to generate sectoral measures, we re-estimate the model with sectoral leverage defined as the median firm's leverage.

Overall, the results presented in appendix D show that our main findings remain intact in these alternative specifications. In all cases, prices and output fall in response to a monetary policy tightening shock, with financial frictions adding to the dampening impact. Moreover, the robustness checks confirm the opposing directions in which upstream and downstream financial frictions affect the price impact of a monetary policy tightening.

4 Theoretical model

To shed light on potential mechanisms underlying the empirical findings in section 3.2 and to provide theoretical interpretation of the estimated direct, upstream and downstream financial friction effects, we develop a multi-sector general equilibrium model with production networks and sector-specific financial frictions. The model is designed to capture, in a tractable reduced-form way, how heterogeneous exposure to monetary-policy-induced movements in credit costs affects firms' pricing and production decisions, and how these effects propagate along the supply chain.

Our framework builds on Bigio and La'O (2020). We depart from their framework in two ways. We introduce labor market segmentation; and we introduce a policy rate i that has heterogeneous effects across sectors. The strength of this sector-specific transmission is governed by an exogenous parameter indexing the tightness of financing conditions. This parameter captures the sensitivity of firms' effective borrowing costs to monetary policy and is the key object linking the theoretical model to the empirical analysis.

The interpretation of this parameter is guided by recent empirical and theoretical work showing that firms with higher leverage experience a more pronounced increase in credit spreads following a monetary tightening. Anderson and Cesa-Bianchi (2024) show that this pattern can be rationalized in a heterogeneous-firm model with default risk, financially constrained intermediaries, and segmented financial markets, highlighting the central role of intermediary balance-sheet constraints in shaping firm-level transmission. In that environment, leverage acts as a sufficient statistic for firms' exposure to intermediary-driven fluctuations in external finance premia. We adopt the same interpretation: Sectoral leverage in the data proxies for exposure to changes in the cost of financing, and we represent this exposure in the model through sector-specific working-capital frictions. A monetary tightening raises the cost of financing variable production inputs, with the magnitude of this increase being proportional to the sector's friction parameter. This reduced-form representation allows us to abstract from the detailed modeling of default and financial intermediation while preserving the key mechanism through which leverage amplifies monetary transmission.

The model delivers three transmission channels that map directly into the empirical decomposition. A *direct cost channel* operates when rising financing costs raise a sector's own marginal cost and therefore its price. An *indirect cost channel* arises when upstream suppliers face higher financing costs and pass these on through higher input prices. A *downstream demand channel* operates when indebted downstream customers reduce their intermediate purchases, compressing upstream revenues and prices. The model quantifies each of these channels in closed form and qualitatively confirms the regression results in section 3.2.[20]

The benchmark model is static and uses the consumption price index as the numéraire, so the price decomposition is stated in terms of sectoral semi-elasticities relative to an aggregate normalization. Appendix G shows that this normalization does not drive the main result. In a two-period flexible-price version, the household Euler equation pins down nominal expenditure: a monetary tightening lowers the current aggregate price level through a common nominal scale effect. The sectoral response can still be decomposed into the same direct, upstream, and downstream financial friction channels, together with a common aggregate term that is absorbed by country-time fixed effects in the empirical specification.

Nominal rigidities are an important part of monetary transmission. We abstract from them to focus on the financial friction channels operating through production networks. Sticky prices would affect the timing and persistence of aggregate and sectoral responses, but they should not change the main decomposition emphasized here. The two-period flexible-price extension shows that, even when the aggregate price level is pinned down by the Euler equation, the cross-sector response still separates into direct financial frictions, upstream cost pressures, and downstream demand effects.

4.1 The model economy

In the following, we write $\hat{x} \equiv d\log x/di$ for the semi-elasticity of variable x with respect to changes in the interest rate. The economy consists of n production sectors indexed by $j \in \{1, \ldots, n\}$. Each sector produces a differentiated good using sector-specific labor and a composite of intermediate inputs purchased from all other sectors. A representative household supplies labor inelastically and consumes a CES composite final good aggregated from sectoral goods.[21]

Production. Each sector j operates a representative firm under perfect competition with a constant returns-to-scale Cobb–Douglas technology

[20] We report the full set of model derivations in appendix E.

[21] In the following, we align the notation of input-output tables to conventions in the FIGARO database, see section 2.

$$y_j = \tilde{a}_j \, l_j^{\alpha_j} \prod_{k=1}^{n} x_{kj}^{\omega_{kj}}, \qquad \alpha_j + \sum_{k=1}^{n} \omega_{kj} = 1, \tag{9}$$

where l_j and x_{kj} denote, respectively, the quantity of labor and of sector k's output used as input in sector j's production, $\tilde{a}_j$ is total factor productivity, α_j is the labor share, and ω_{kj} is the cost share of good k in the production of good j.

Firms must borrow in advance to cover variable production costs before generating sales revenues. The effective cost of borrowing rises with the nominal interest rate i, and does so heterogeneously across sectors: Parameter $\varphi_j \geq 0$ captures the sensitivity of sector j's effective borrowing costs to a change in the policy rate, reflecting the pass-through of policy rates onto lending rates accruing to firms in that sector. This wedge scales total production costs by the factor $1 + i\varphi_j$, such that firm profits are given by

$$\pi_j = p_j y_j - (1 + i\varphi_j)\Big(w_j l_j + \sum_{k=1}^{n} p_k x_{kj}\Big). \tag{10}$$

Cost minimization with Cobb–Douglas technology yields the non-financial marginal cost $\tilde{mc}_j = a_j^{-1} w_j^{\alpha_j} \prod_k p_k^{\omega_{kj}}$, where $a_j \equiv \tilde{a}_j \alpha_j^{\alpha_j} \prod_k \omega_{kj}^{\omega_{kj}}$ collects constant terms from Shephard-lemma factor demands. The total marginal cost including the financing markup is $mc_j = (1 + i\varphi_j)\tilde{mc}_j$, and perfect competition implies

$$p_j = (1 + i\varphi_j)\, \tilde{mc}_j \qquad \forall\, j. \tag{11}$$

Cost minimization implies that the labor bill is proportional to total non-financial production costs, i.e. $w_j l_j = \alpha_j \tilde{mc}_j y_j$. We define nominal revenue as $r_j \equiv p_j y_j = (1 + i\varphi_j)\tilde{mc}_j y_j$, from which, as shown in appendix E, we can derive the following wage identity:

$$w_j l_j = \alpha_j \frac{r_j}{1 + i\varphi_j} \tag{12}$$

Households. The representative household has CES preferences over sectoral goods:

$$c = \left(\sum_{j=1}^{n} \nu_j^{1/\sigma} \, c_j^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}, \tag{13}$$

where $\sigma > 0$ is the elasticity of substitution and ν_j are expenditure weights normalized such that $\sum_j \nu_j = 1$. The household supplies a fixed labor endowment $\bar{l}_j$

to each sector. Expenditure minimization yields the standard CES demand system $p_j c_j = e\,\nu_j (p_j/p^c)^{1-\sigma}$, where e is total income and $p^c = (\sum_j \nu_j p_j^{1-\sigma})^{1/(1-\sigma)}$ is the CES price index. We take the composite consumption good as the numéraire, so $p^c = 1$.

Market clearing and equilibrium. Output market clearing requires each sector's output to be absorbed by final consumption and intermediate demand

$$y_j \;=\; c_j + \sum_{k=1}^{n} x_{jk}, \qquad \forall\, j, \tag{14}$$

and labor market clearing requires $l_j = \bar{l}_j$ for all j. An equilibrium is a set of prices $\{p_j, w_j\}$ and quantities $\{y_j, x_{kj}, c_j\}$ satisfying the firm optimality conditions 11, the household demand system, and the market clearing conditions, together with the numéraire normalization $p^c = 1$.

4.2 Input-output structure and vector notation

We express prices and revenues in vector form to facilitate subsequent derivations of the price decomposition.

Cost shares and revenue shares. Let $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_n)^\top$ denote the vector of labor shares and let $\mathbf{D}_\alpha \equiv \mathrm{diag}(\alpha_1, \ldots, \alpha_n)$ denote the corresponding diagonal matrix. Elements ω_{kj} in the *cost-share matrix* $\boldsymbol{\Omega}$ equal the share of sector j's non-financial production costs spent on inputs from supplier k. Due to the Cobb–Douglas structure, they directly map into the technology parameters in equation 9. The elements in the *revenue-share matrix* $\tilde{\boldsymbol{\Omega}}$ are given by $\tilde{\omega}_{kj} = p_k x_{kj}/(p_j y_j)$, the share of sector j's nominal output paid to sector k for intermediates. Since $p_j y_j = (1 + i\varphi_j)\tilde{mc}_j y_j$, the two matrices are linked via

$$\tilde{\omega}_{kj} \;=\; \frac{\omega_{kj}}{1 + i\varphi_j}. \tag{15}$$

The financing wedge thus compresses revenue shares below cost shares whenever $i > 0$.[22]

[22] See appendix E.

Leontief inverses. The analysis requires two distinct Leontief inverses. The *cost-based inverse*

$$\mathbf{H} \;\equiv\; (\mathbf{I}-\boldsymbol{\Omega}^{\top})^{-1} \;=\; \mathbf{I}+\boldsymbol{\Omega}^{\top}+(\boldsymbol{\Omega}^{\top})^{2}+\cdots \tag{16}$$

has element h_{kj} equal to the total cost importance of sector j in the production of sector k, summing all supply-chain paths of arbitrary length. It uses the transpose $\boldsymbol{\Omega}^{\top}$ as costs flow from upstream suppliers to downstream customers in $\boldsymbol{\Omega}$, while $\mathbf{H}$ traces costs from downstream customers back to upstream suppliers. The Cobb–Douglas accounting identity $\alpha_j + \sum_k \omega_{kj} = 1$ implies

$$\boldsymbol{\alpha} = (\mathbf{I}-\boldsymbol{\Omega}^{\top})\mathbf{1}, \qquad \mathbf{H}\boldsymbol{\alpha} = \mathbf{1}. \tag{17}$$

The *revenue-based inverse*

$$\mathbf{Z} \;\equiv\; (\mathbf{I}-\tilde{\boldsymbol{\Omega}})^{-1} \tag{18}$$

has element z_{kj} capturing the propagation of demand changes through the production network via revenue-share linkages. At $i=0$, $\tilde{\boldsymbol{\Omega}} = \boldsymbol{\Omega}$ and $\mathbf{Z}_0 = (\mathbf{I}-\boldsymbol{\Omega})^{-1} = \mathbf{H}^{\top}$, up to the row-column convention. The two inverses diverge for $i>0$ as the financing wedge compresses $\tilde{\boldsymbol{\Omega}}$.

Nominal revenues. Multiplying the goods market clearing conditions by p_j and using the Cobb–Douglas structure, one can show that the vector of sectoral revenues $\mathbf{r}$ satisfies

$$\mathbf{r} \;=\; \mathbf{Z}\, e\big(\boldsymbol{\nu}\circ\mathbf{p}^{1-\sigma}\big), \tag{19}$$

where $\mathbf{p}^{1-\sigma}$ denotes the vector of $p_j^{1-\sigma}$. For general values of σ, revenues depend on prices through the CES demand term $\mathbf{p}^{1-\sigma}$, and prices depend on revenues through the wage identity 12. Consequently, the equilibrium is a fixed point in $(\mathbf{p},\mathbf{r})$ that in general has no closed-form solution. However, with $\sigma=1$, the Cobb-Douglas case, prices drop out of equation 19, and the system becomes block-triangular, delivering a closed-form solution as derived in section 4.3.

4.3 Decomposing the price-interest rate elasticity

We now derive the impact of a change in the interest rate i on sectoral log-prices and decompose it into five economically distinct terms. The derivation proceeds in three steps, with details provided in appendix E.[23]

Step 1: The log-price equation. Taking logarithms of equation 11 and substituting the wage identity 12 to eliminate w_j, one obtains a scalar equation linking $\log p_j$ to the financing markup $\log(1+i\varphi_j)$, sectoral revenue $\log r_j$, and intermediate input prices $\{\log p_k\}_{k=1}^n$. Stacking across all sectors and inverting the input-output block yields

$$\log p_k \;=\; \sum_{j=1}^{n} h_{kj}\big[(1-\alpha_j)\log(1+i\varphi_j)+\alpha_j\log r_j+\mathrm{const}_j\big], \tag{20}$$

where the constant collects terms invariant to i. The Leontief weight h_{kj} measures the total cost importance of sector j in sector k's production, indicating that sector k's price reflects the financing and revenue conditions of all its upstream suppliers, propagated through the entire network.

Step 2: Revenue response with $\sigma = 1$. With $\sigma = 1$, the revenue equation 19 simplifies to $r_j = e\rho_j$ where $\boldsymbol{\rho} \equiv \mathbf{Z}\boldsymbol{\nu}$. As the factor $p_j^{1-\sigma}$ equals one in the revenue equation, revenues depend on i only through aggregate income e and the revenue-share matrix $\tilde{\boldsymbol{\Omega}}$ entering $\mathbf{Z}$. Differentiating $\boldsymbol{\rho}$ with respect to i applying the matrix inverse derivative formula gives

$$\frac{d\boldsymbol{\rho}}{di} \;=\; -\mathbf{Z}\,\tilde{\boldsymbol{\Omega}}\,\mathrm{diag}(\mathbf{f})\,\boldsymbol{\rho}. \tag{21}$$

We define the *downstream customer-friction exposure* of sector j as the proportional rate of revenue contraction:

$$\beta_j \;\equiv\; -\frac{1}{\rho_j}\frac{d\rho_j}{di} \;=\; \frac{\big[\mathbf{Z}\,\tilde{\boldsymbol{\Omega}}\,\mathrm{diag}(\mathbf{f})\,\boldsymbol{\rho}\big]_j}{\rho_j} \;\geq\; 0. \tag{22}$$

[23] Appendix E.10 derives the corresponding response to productivity shocks. Unlike an interest-rate shock, which changes prices through financing wedges, upstream cost pass-through, downstream revenue compression, and the income term, a productivity shock enters directly as a sectoral unit-cost shift; at $\sigma = 1$, it propagates through the cost Leontief inverse, up to the common numéraire adjustment.

Economically, β_j measures how strongly sector j's revenues are compressed by the financial frictions of its downstream customers: A large value indicates that sector j sells to severely constrained customers, propagating through all demand chains via revenue Leontief inverse $\mathbf{Z}$. Backward substitution results in the revenue semi-elasticity $\hat{r}_j = \hat{e} - \beta_j$, where $\hat{e}$ is the aggregate income semi-elasticity.

Step 3: Solving the numéraire condition for $\hat{e}$. Since $p^c = 1$ at all times, differentiating the CES price index at $\sigma = 1$ yields $\boldsymbol{\nu}^\top \hat{\mathbf{p}} = 0$. Imposing this constraint on the differentiated price system yields

$$\hat{e} \;=\; \boldsymbol{\nu}^\top \mathbf{H}\big[\mathbf{D}_\alpha \boldsymbol{\beta} - (\mathbf{I} - \mathbf{D}_\alpha)\mathbf{f}\big], \tag{23}$$

where we used $\mathbf{H}\boldsymbol{\alpha} = \mathbf{1}$ and $\boldsymbol{\nu}^\top \mathbf{1} = 1$. Writing $\boldsymbol{\lambda} \equiv \mathbf{H}^\top \boldsymbol{\nu}$ for the vector of Domar weights, this is $\hat{e} = \sum_j \lambda_j[\alpha_j\beta_j - (1-\alpha_j)f_j]$, with $\sum_j \lambda_j \alpha_j = 1$. The term $(1-\alpha_j)f_j$ is the positive cost-push component in sector j's price equation, whereas $\alpha_j\beta_j$ is the demand-contraction component. The income semi-elasticity adjusts so that these aggregate forces are consistent with the numéraire condition $p^c = 1$. Crucially, $\hat{e}$ depends only on economy-wide averages of frictions and bears no sector-specific information about k's suppliers or customers.

Price-interest rate elasticity decomposition. Combining the three steps and applying the identity $\mathbf{H} = \mathbf{I} + (\mathbf{H} - \mathbf{I})$ to separate own-sector $(j = k)$ from network $(j \neq k)$ effects, we obtain the main result of the model:

$$\begin{aligned} \frac{d \log p_k}{di} \;=\; & \underbrace{(1-\alpha_k)\, f_k}_{\text{(i) Direct cost}} + \underbrace{\sum_j (h_{kj} - \mathbf{1}_{j=k})(1-\alpha_j)\, f_j}_{\text{(ii) Supplier cost}} \\ & - \underbrace{\alpha_k\, \beta_k}_{\substack{\text{(iii) Own} \\ \text{customers}}} - \underbrace{\sum_j (h_{kj} - \mathbf{1}_{j=k})\alpha_j\, \beta_j}_{\substack{\text{(iv) Suppliers'} \\ \text{customers}}} + \underbrace{\hat{e}}_{\text{(v) Income}} \end{aligned} \tag{24}$$

This decomposition is *exact* at $\sigma = 1$ and exhaustive, i.e. with no residual terms. All five terms are explicit functions of the structural parameters $\omega_{kj}, \alpha_j, \varphi_j, \nu_j$ and the two Leontief inverses, computed with no fixed-point iteration.

Direct financial frictions The first term (i) in equation 24 reflects the direct effect of a higher policy rate on sector k's production costs, with $f_k = \frac{d \log(1+i\varphi_k)}{di}$

defined as in appendix equation E.1. Since the financing wedge $1 + i\varphi_k$ scales all variable input costs, a rise in i raises marginal cost and ultimately p_k. The effect is attenuated by the labor share α_k as, through the wage identity 12, the wage cost component adjusts with sector revenues rather than with the policy rate directly. Intuitively, a rise in the policy rate i raises the sector-specific financing cost by φ_k, feeding directly into sector k's price via a "direct cost channel". Equation 24 shows that, while keeping the marginal costs in other sectors fixed, an increase in the interest rate will have a direct effect on the price in sector k through sector k's own financial frictions (φ_k).

Upstream financial frictions Term (ii) accumulates the analogous cost increases of every upstream supplier j, weighted by h_{kj} for all supply-chain paths connecting j to k. Higher borrowing costs in sectors j supplying inputs to sector k increase their prices, which are passed through to sector k's own marginal costs in proportion to the Leontief inverse weights h_{kj}. Intuitively, in addition to direct increases in interest payments for sector k, an "indirect cost channel" stemming from suppliers' financial frictions affects p_k, with the strength of this channel depending on sector k's input purchases from other sectors.

Downstream financial frictions Term (iii) captures a customer effect: As defined in equation 22, β_k captures the revenue compression in sector k stemming from the financial frictions of its customers in a Leontief sense. Consequently, higher financing costs for sector k's customers reduce their intermediate demand from k, compressing k's revenues and hence its wages, which lowers prices p_k. This effect is proportional to the labor share α_k, as the wage is the only component of marginal costs that directly moves with nominal output. Term (iv) captures that the same mechanism also applies to all upstream suppliers j via the Leontief weights: If sector j's customers are constrained, its revenues fall as well, inducing a decline in wages, with the resulting costs relief propagating back to sector k. Both demand-channel terms vanish if $\varphi_m = 0$ applies to all downstream customers.

Income effects Term (v) is the *income channel*, entering the model with an ambiguous sign. Because $\mathbf{H}\boldsymbol{\alpha} = \mathbf{1}$, a change in aggregate income e enters every sector's price equation one-for-one as the common term $\hat{e}$. Since this term is common to all sectors in a given period, it is absorbed by country-time fixed effects in the empirical specification.

4.4 Mapping from theory to empirical proxies

While equation 24 yields a model-exact decomposition of sectoral price semi-elasticities into five channels, the objects f_j, β_j, and $\hat{e}$ are nonlinear functions of the interest rate i and the full equilibrium network structure, and are therefore not directly observable. We now show how the three proxies in the empirical specification 8 arise as leading-order approximations of the model-exact channels as $i \to 0$, and what each estimated coefficient identifies.

The starting point is the exact decomposition 24, which we can write as:

$$\frac{d \log p_k}{di} = \underbrace{[\mathbf{H}(\mathbf{I}-\mathbf{D}_\alpha)\mathbf{f}]_k}_{\text{cost channels (i)+(ii)}} - \underbrace{[\mathbf{H}\mathbf{D}_\alpha\boldsymbol{\beta}]_k}_{\text{customer channels (iii)+(iv)}} + \underbrace{\hat{e}}_{\text{income (v)}} . \tag{25}$$

As $i \to 0$, two simplifications occur simultaneously: $f_j \to \varphi_j$ for all j, and $\tilde{\boldsymbol{\Omega}} \to \boldsymbol{\Omega}$ so that $\mathbf{Z} \to \mathbf{Z}_0 \equiv (\mathbf{I}-\boldsymbol{\Omega})^{-1}$.

Cost channels and the upstream proxy. Substituting $f_j \to \varphi_j$ into the cost block of equation 25 gives the exact leading-order cost contribution

$$[\mathbf{H}(\mathbf{I}-\mathbf{D}_\alpha)\mathbf{f}]_k \;\to\; (1-\alpha_k)\varphi_k \;+\; \sum_j (h_{kj} - \mathbf{1}_{j=k})(1-\alpha_j)\varphi_j \;\geq\; 0, \tag{26}$$

where non-negativity follows from $h_{kj} - \mathbf{1}_{j=k} \geq 0$, $(1-\alpha_j) \geq 0$, and $\varphi_j \geq 0$. The first term is the direct cost effect of sector k's own friction, and the second term aggregates upstream suppliers' cost increases through all supply-chain paths, including indirect feedback loops captured by the diagonal of $\mathbf{H}-\mathbf{I}$. The empirical proxies for these two terms are φ_k and $\Phi_k = [(\mathbf{H}-\mathbf{I})\boldsymbol{\varphi}]_k = \sum_j (h_{kj} - \mathbf{1}_{j=k})\varphi_j$, respectively, obtained by absorbing the approximately constant factor $(1-\alpha_j)$ into the estimated coefficients β_1^h and β_2^h. This approximation is valid when α_j has limited cross-sectional variation. The theoretical model thus predicts $\beta_1^h > 0$ and $\beta_2^h > 0$ for the empirical specification 8.

Customer channels and the downstream proxy. At $i = 0$, substituting $f_j \to \varphi_j$ and $\tilde{\boldsymbol{\Omega}} \to \boldsymbol{\Omega}$ into equation 22 gives $\beta_j|_{i=0} = [(\mathbf{Z}_0 - \mathbf{I})\,\mathrm{diag}(\boldsymbol{\varphi})\boldsymbol{\rho}_0]_j / \rho_{0,j}$, where we have used the Leontief identity $\mathbf{Z}_0\boldsymbol{\Omega} = \mathbf{Z}_0 - \mathbf{I}$. Under the approximation that Domar weights $\rho_{0,j}$ are approximately equal across sectors, this simplifies to $\beta_j|_{i=0} \approx [(\mathbf{Z}_0 - \mathbf{I})\boldsymbol{\varphi}]_j = \tilde{\Phi}_j$. Substituting into the customer block of equation 25 yields

$$-[\mathbf{H}\mathbf{D}_\alpha\boldsymbol{\beta}]_k \;\approx\; -[\mathbf{H}\mathbf{D}_\alpha(\mathbf{Z}_0-\mathbf{I})\boldsymbol{\varphi}]_k \;\leq\; 0, \tag{27}$$

where non-negativity of the bracketed term follows from non-negativity of all entries of $\mathbf{H}$, $\mathbf{D}_\alpha$, $(\mathbf{Z}_0-\mathbf{I})$, and $\boldsymbol{\varphi}$. The empirical proxy for this combined customer channel is $\tilde{\Phi}_k = [(\mathbf{Z}_0-\mathbf{I})\boldsymbol{\varphi}]_k$, obtained by absorbing the weights α_j and the outer matrix $\mathbf{H}$ into the estimated coefficient β_3^h. The theoretical model therefore predicts $\beta_3^h < 0$ in the empirical specification 8.[24]

Income channel and time fixed effects. The income channel (v) equals $\hat{e} \times s_t$ because $\mathbf{H}\boldsymbol{\alpha} = \mathbf{1}$. The scalar $\hat{e}$ is common to all sectors in a given country-period and is therefore absorbed by the country-time fixed effects θ_{ct} in specification 8. No additional sector-level proxy is needed to account for this channel in the Cobb–Douglas benchmark.

4.5 Simulation results

In the following, we provide simulation evidence on whether the first-order proxy mapping in section 4.4 remains informative once the CES elasticity σ deviates from unity and the linear approximation is no longer exact.[25] We address this by first running a Monte Carlo simulation over $R = 500$ random calibrations of model-implied economies with $n = 70$ sectors each, solving the full CES equilibrium numerically. For each simulated economy, we draw sector-level parameters $(\alpha_j, \omega_{kj}, \varphi_j, \nu_j)$ and solve the CES equilibrium at a baseline interest rate $i_0 = 5\%$. We then draw $n_s = 10$ monetary policy shocks $s_t \sim |N(25\,\text{bp}, 15\,\text{bp})|$ per economy and estimate the proxy regression

$$\Delta \log p_{k,t} \;=\; \gamma_1\varphi_k \times s_t + \gamma_2\Phi_k \times s_t + \gamma_3\tilde{\Phi}_k \times s_t + \varepsilon_{k,t}, \tag{28}$$

pooled across all sectors, economies, and shocks, mirroring the panel structure of the empirical model 8. We also estimate a decomposition regression of CES prices on the exact model channels as in equation 24, which should return coefficients exactly equal to $[1, 1, 1, 1, 1]$ at $\sigma = 1$.

[24]Note that channels (iii) and (iv) are not separately identified in the empirical model, such that $\tilde{\Phi}_k$ captures their combined effect. Moreover, channel (iv) shares the upstream Leontief structure h_{kj} with channel (ii), so it is partially correlated with Φ_k and partially absorbed into β_2^h.

[25]See appendix F for details and the complete set of simulation results.

The simulation yields four main validation checks regarding the mapping between the empirical model in equation 8 and the theory-based proxies in section 4.4. First, the exact decomposition as in equation 24 explains 99.97% of the price variation at $\sigma = 0.9$ ($R^2 = 0.9997$), with coefficients close to unity across all channels, confirming that this decomposition is a credible approximation even for alternative configurations of CES preferences (see table F.1 in appendix F). Second, the proxy regression recovers the predicted signs $\hat{\gamma}_1 > 0$, $\hat{\gamma}_2 > 0$, $\hat{\gamma}_3 < 0$ with substantial explanatory power, validating that the linear proxies capture the main sources of heterogeneity in price responses (table F.1 and figure F.1 in appendix F). Third, sign analysis confirms a perfectly correct sign recovery for the four unambiguous channels – direct cost, supplier cost, within-sector customers, and suppliers' customers – across all simulated economies. The common income channel shows mixed signs, consistent with its theoretical ambiguity and its absorption by time fixed effects (table F.2 in appendix F). Fourth, the variance decomposition derived from the fully model-consistent decomposition of the price-interest rate elasticity in equation 24 indicates that downstream customer channels account for the largest share of price variations, as shown in the right column of appendix table F.3. This is consistent with the large producer price variance share for the downstream financial frictions effect reported in figure 6. The within-sector downstream customer effects (iii) account for the largest share in variation in table F.3, but are partly counteracted by a negative variance share for the supplier's customers effect (iv). While, as described in section 4.4, we cannot directly disentangle these channels in the empirical measures, the changing sign in the downstream financial frictions effect over time in figure 6 is generally consistent with these counterbalancing effects obtained in the theoretical setting.

5 Conclusion

This paper provides new evidence showing that financial frictions distributed across the production network play a quantitatively important role in shaping the transmission of monetary policy. By combining firm-level balance-sheet data with information on sectoral input-output linkages, we show that both a sector's own financial position and that of its suppliers and customers materially affect the response of prices and real activity to monetary policy shocks.

A contractionary monetary policy shock leads to a persistent and heterogeneous decline in producer prices and output across sectors. These responses are amplified by financial frictions and, crucially, not only by those prevailing in a given sector. Indirect financial frictions arising from the leverage of other sectors in the production network exert an independent and sizeable effect, accounting for a substantial share of the overall amplification and implying a markedly larger disinflationary impact of monetary policy than specifications that abstract from network linkages.

Upstream and downstream frictions operate through distinct and opposing channels. Downstream sectors reduce demand for intermediate inputs, reinforcing the decline in prices and activity. Upstream sectors raise prices to stabilize cash flows, partially offsetting disinflationary pressures. The downstream demand channel dominates, so that higher network-level leverage leads to a stronger and more persistent fall in prices and output following a tightening.

Our theoretical model provides a coherent framework for these findings, showing analytically how the upstream cost channel and the downstream demand channel arise and interact within the production network. From a policy perspective, the results underscore the value of incorporating network-aware financial indicators into monetary policy analysis: monitoring leverage at key nodes of the production network can improve assessments of the strength of transmission and thereby inform the conduct of monetary policy.

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A Dataset construction

Euro area and country level data

We collect a set of standard macroeconomic variables at both the euro area and country levels from Eurostat and the IMF, which serve as controls in the empirical analysis. To identify common euro area monetary policy shocks, we exploit high-frequency movements in the three-month overnight index swap (OIS) rate within a narrow window (approximately 135 minutes) surrounding the publication of the press release and the subsequent press conference following ECB Governing Council meetings. These data are drawn from the euro area extended monetary policy event-study database constructed by Altavilla et al. (2025). To disentangle monetary policy shocks from information shocks, we implement the "poor man's sign restriction" approach proposed by Jarociński and Karadi (2020). Specifically, we identify monetary policy shocks as high-frequency changes in the three-month OIS rate within the event window that are associated with contemporaneous movements in stock prices of the opposite sign.[26]

Country-sector level data

We collect data on prices, industrial production, turnover, employment, hours worked, and wages from the "Short-Term Business Statistics" (STS) dataset of Eurostat.[27] As no information on price indices for the trade sector (NACE codes G00, G45, G46, G47) are reported in the STS, we compute them by dividing nominal turnover by real turnover, consistently with the methodology employed by Eurostat.[28] In addition, we proxy the producer price index (PPI) for the construction sector (NACE code F00) with the PPI for residential buildings construction.[29] Moreover, as STS do not cover the agriculture sector (A01), we complement PPI data for this sector with quarterly information from the "Price Indices of Agricultural Products" (APRI) dataset of Eurostat, which we linearly interpolate to obtain monthly observations.[30] We do not have data on agricultural industrial production as Euro-

[26]The results reported in section 3.2 are robust to an alternative identification strategy based on the first principal component of high-frequency changes in 1-, 3-, 6-, and 12-month OIS rates, rather than relying solely on the three-month OIS rate (Jarociński and Karadi, 2020).

[27]More details on STS data can be found here.

[28]More precisely, we divide turnover values by the volume of sales (deflated turnover).

[29]This refers to the statistical classification of products by activity (CPA) code F411X, the main component of sector F41 which covers "new buildings" only.

[30]The "Price indices of agricultural products (apri_pi)" data which can be found here.

stat data are only reported at an annual frequency,[31] so that a monthly interpolation would not be sufficiently accurate.

In STS, data on employment, hours worked, and wages are mainly available at quarterly frequency, and only partly provided at monthly frequency on a voluntary basis by national statistical offices.[32] Data on prices, industrial production, and turnover are available both at monthly and quarterly frequency, depending on the respective country, sector and time period.[33] To maximize the number of observations while preserving the consistency of the dataset, we linearly interpolate quarterly series to monthly and either take the original monthly series or the quarterly interpolation, depending on which of the two has more observations.

For all indices, we use seasonally adjusted series and perform a three-step cleaning procedure.[34] First, we exclude all observations of a specific index variable reported with a value of zero. Second, we drop all observations of a series with exactly identical data entries for more than six subsequent months (if the series is monthly) or more than four quarters (if the series is quarterly). Third, we drop entirely data series exhibiting an implausibly high level of volatility or poor data quality, i.e. due to implausibly large discrete jumps at random intervals.

Aggregation from NACE at 88 sectors to 64 sectors

The input-output matrices available from Eurostat are disaggregated at NACE 2 digits, but with few difference compared to the standard NACE 2 digits classification. Indeed, the latter has 88 sectors, while in the input-output tables some sectors are aggregated, resulting in a 64 sectors breakdown. Table A.2 shows the sectors which are aggregated in the input-output tables.

Since STS data are at 88 sectors, we aggregate the variables contained in STS (e.g. PPI, industrial production, etc.) to match the 64 sectors breakdown of the input-output matrices. For sectors B00, F00 and I00 we don't perform any aggregation, since variables are already available also at 1 digit. For the other sectors listed in table A.2 we perform the aggregation calculating a weighted average of the sub

[31]See the "Economic accounts for agriculture (aact_eaa)" data which can be found here.

[32]See STS regulations here.

[33]This implies that for a given country-sector pair we may be able to retrieve only monthly or only quarterly observations, or both. Moreover, monthly and quarterly observations, if both available, are not perfectly consistent at all times in STS.

[34]We derive seasonally adjusted series in cases for which the seasonal adjustment is not directly carried out by Eurostat by performing a LOESS transformation.

NACE code (88 sectors)	NACE code (64 sectors)
B05	B00
B06	B00
B07	B00
B08	B00
B09	B00
C10	C10-12
C11	C10-12
C12	C10-12
C13	C13-15
C14	C13-15
C15	C13-15
C31	C31_32
C32	C31_32
F41	F00
F42	F00
F43	F00
I55	I00
I56	I00
J59	J59_60
J60	J59_60
J62	J62_63
J63	J62_63
M69	M69_70
M70	M69_70
M74	M74_75
M75	M74_75
N80	N80-82
N81	N80-82
N82	N80-82

Table A.2: Mapping of NACE code 88 to NACE code 64.

sectors, using as weights the "Production value" (variable code V12120) of each sector obtained from the Eurostat Structural Business Statistics (SBS) dataset. SBS describes the detailed structure, economic activity, and performance of businesses over time and covers the countries and sectors of our interest. However, SBS only goes from 2005 to 2020, with some gaps and some countries not reporting in all the years. To have a complete series of weights we perform the following steps. For each country and sector:

1. we linearly interpolate missing values to fill gaps in the time series;
2. we extrapolate missing values filling forward (after the latest observation) and backward (before the earliest observation), effectively assuming a constant value for the latest available observation;
3. in case a given country-sector is completely missing, we assume its weight is zero;
4. if in a given country all "subsectors" (e.g. C13, C14, C15) that are part of a given aggregation at 64 sector (e.g. C13-C15) are missing, then we assume that they have the same weight (i.e. we make a simple average). In practice, this happens only in the case of Luxembourg for sectors C13, C14, C15.

Finally, for each sector in the FIGARO tables, we compute the share of input sourced outside the production network as the sum of labor, value added and taxes (i.e. rows *"W2_D21X31 - Taxes less subsidies on products"*, *"W2_OP_NRES - Purchases of non-residents in the domestic territory"*, *"W2_OP_RES - Direct purchase abroad by residents"*, *"W2_D1 - Compensation of employees"*, *"W2_D29X39 - Other net taxes on production"*, *"W2_B2A3G - Gross operating surplus"*) divided by the total input cost of production. Analogously, for each sector, we compute the share of output going to final consumption as the sum of government and household consumption, capital formation and inventories (i.e. columns *"P3_S13 - Final consumption expenditure of general government", "P3_S14 - Final consumption expenditure of households", "P3_S15 - Final consumption expenditure of non-profit institutions serving households", "P51G - Gross fixed capital formation", "P5M - Changes in inventories and acquisition less disposals of valuables"*) divided by total output.

Table A.3: Summary statistics

	Unit	Freq.	Obs.	Mean	St. dev.	Median
Euro area level						
EUR/USD exchange rate	rate	M	312	1.18	0.16	1.17
Composite 10y sov. bond yield	%	M	312	3.03	1.60	3.41
IMF Commodity Index	idx	M	256	134.08	38.25	126.39
CISS	p.n.	M	312	0.18	0.15	0.12
3-month OIS	%	M	305	1.47	1.75	0.78
Monetary policy shock	bps	M	312	0.16	2.67	0.00
Country level						
Unemployment	%	M	7,115	8.79	4.14	7.98
HICP	idx	M	6,409	95.51	15.81	97.96
Country-sector level						
Producer price	idx	M	145,986	52.81	48.01	74.04
Industrial production	idx	M	197,951	84.83	88.09	92.59
Turnover	idx	M	220,521	101.23	2533.46	83.60
Employment	idx	M	98,710	42.77	53.98	0.00
Total leverage	ratio	A	249,144	0.59	0.16	0.60
Working capital	ratio	A	249,144	0.15	0.15	0.14

Sources: Altavilla et al. (2025), ECB, Eurostat, IMF, Orbis.

Notes: "idx", "bps" and "p.n." stand for index, basis points and pure number, while M and A denote monthly and annual frequency.

Table A.4: Summary of notation and definitions

Object	**Formula**	**Definition**
Upstream effect	From supplier to customer $x_{j \to k}$	Transmission from upstream supplier j to downstream customer k, capturing the propagation of cost shocks along the production network.
Downstream effect	From customer to supplier $x_{j \leftarrow k}$	Transmission from downstream customer k to upstream supplier j, capturing the propagation of demand shocks along the production network.
Input-output matrix	$\mathbf{X} = [x_{jk}^{c,d}]$	Bilateral flows of intermediate goods and services, where each element denotes sales from sector j in country c to sector k in country d.
Total output	$y_j^c = \sum_{k,d} x_{jk}^{c,d} + c_j$	Total output of sector j in country c, defined as the sum of intermediate sales to all countries (d) and sectors (k), as well as final demand.
Direct-coefficients matrix	$\mathbf{\Omega} = \left[x_{jk}^{c,d} / y_j^c\right]$	Matrix of input coefficients, where each element measures the share of sector j's output sold to sector k.
Leontief inverse	$\mathbf{Z} = (\mathbf{I} - \mathbf{\Omega})^{-1}$	Matrix summarizing the total (direct and indirect) effects of a demand shock propagating from customers to suppliers via production linkages.
Leontief inverse (transposed-based)	$\tilde{\mathbf{Z}} = (\mathbf{I} - \mathbf{\Omega}^T)^{-1}$	Matrix capturing the total (direct and indirect) effects of a cost shock propagating from suppliers to customers via production linkages.

B Impulse response scaling

We scale the size of a monetary policy tightening shock s_t in model 8 to imply a peak increase in the 3m OIS rate – the market-based monetary policy rate proxy from which monetary policy shocks are identified – by 25 basis points in the first year after the shock. We then scale the impulse response functions for the macroeconomic variables of interest to be consistent with such a 25bps peak impact monetary policy tightening shock. To do so, we proceed in two steps. First, we estimate a euro area local projection model including broadly the same control variables as in the country-sector panel model 8, but at the aggregate level, to account for the fact that the dependent variable y_t is observed at the euro area level only in this setting. We then derive a scaling parameter $\tau \equiv \frac{0.25}{\psi}$, with ψ referring to the peak of the impulse response of the OIS 3m rate to a monetary policy tightening shock within the first year after the shock, expressed in percentage points. We finally use τ as a scaling parameter in the impulse response functions of the dependent variable of interest shown in section 3.2.

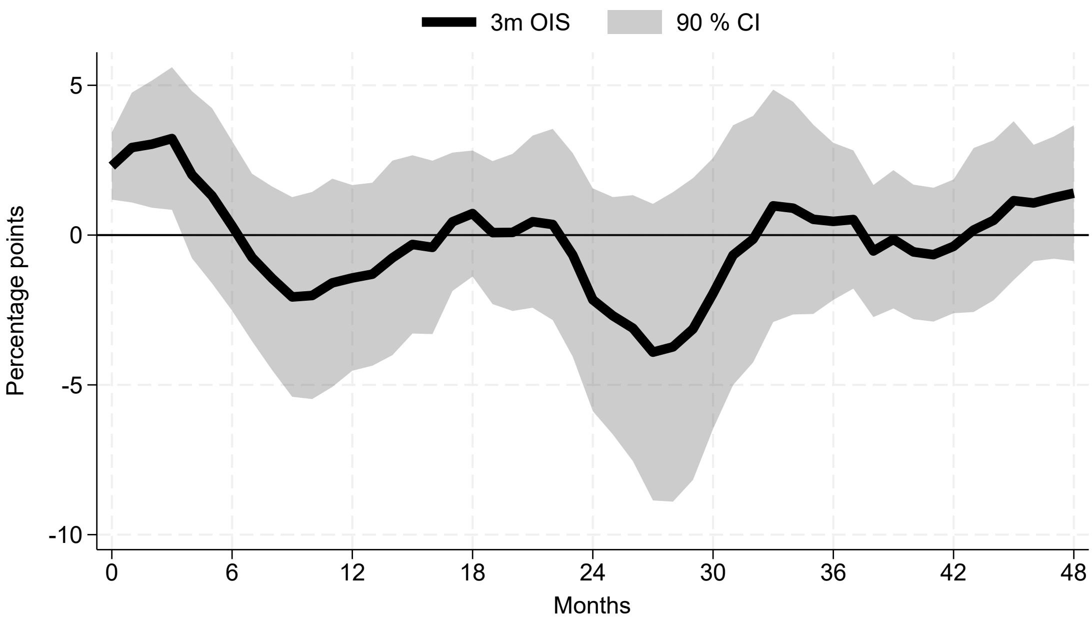
The euro area aggregate model is given by:

$$\begin{aligned} y_{t+h} =& \beta_1^h s_t + \sum_{l=1}^{L} \delta^{\mathbf{h}} \mathbf{K_{t-l}} + \sum_{l=0}^{L} \eta^{\mathbf{h}} \mathbf{X_{t-l}} + \epsilon_{t+h} \\ \text{with} \qquad & h = 1, 2, ..., H, \mathbf{K_t} = \begin{bmatrix} y_t \\ s_t \end{bmatrix} \end{aligned} \tag{B.1}$$

with vector $\mathbf{K_t}$ indeed collecting lags of the dependent variable y_t and of the shock s_t. Matrix $\mathbf{X_t}$ contains the contemporaneous values and lags of the same set of macro-financial control variables as included in model 8, i.e. a GDP-weighted 10y composite euro area sovereign bond yield, the euro-dollar exchange rate, and log-levels of the euro area Composite Indicator of Systemic Stress (CISS), the IMF Commodities Price Index, the euro area harmonized index of consumer prices (HICP) and the euro area unemployment rate. It also includes our main variables of interest, industrial production and producer prices, now measured at the euro area aggregate level.

Figure B.1 shows the impulse response function to a monetary policy tightening shock as obtained from model B.1. Without scaling, the shock refers a one percentage point increase in the shock series from the mean. Given that the mean monetary policy shock in our sample only amounts to 0.2 basis points, the shock impact as measured by β_1^h in equation B.1 and shown in figure B.1 turns out large.[35] Within the first year, the peak increase in the three-month OIS rate is reached three months after the shock and amounts to 3.2 percentage points. This implies a the scaling parameter $\tau \approx 0.078$.

Figure B.1: Impulse responses to monetary policy tightening shock - aggregate model



Notes: Impulse responses to a monetary policy tightening shock. Estimates for model B.1 estimated on aggregate euro area data with Newey and West (1987) standard errors.

[35] As discussed in section 2, we identify monetary policy shocks by applying the Jarociński and Karadi (2020) "poor man's" sign restrictions to the innovations in the 3m OIS rate around policy events as identified by Altavilla et al. (2025).

C Variance decomposition of monetary policy effects

This appendix describes the covariance-based decomposition used to quantify the contribution of different transmission channels to the variance of the monetary policy effect. It also discusses robustness of the obtained variance shares to a sequential Shapley (1953)-style decomposition.

C.1 Covariance-based decomposition

C.1.1 Model-implied components

We start by deriving the monetary policy-related fitted value of the outcome variable obtained from the local projection model

$$\hat{y}_t^{MP} = \sum_{k \in \mathcal{K}} C_{k,t}, \tag{C.1}$$

where $C_{k,t}$ denotes the contribution of component k associated with the monetary policy shock s_t. In our application, the set of components is given by

$$\mathcal{K} = \{\text{Direct}, \text{Upstream}, \text{Downstream}, \text{Average}, \text{Non-network}\},$$

captured by the coefficients β_1 to β_6 in model 8. Each component aggregates the effects of contemporaneous and lagged interactions between the monetary policy shock and the corresponding transmission channel, as described in the empirical model. The monetary policy-only fitted value $\hat{y}_t^{MP}$ therefore captures the total contribution of the monetary policy shock to the dependent variable, abstracting from other controls and fixed effects.

C.1.2 Variance decomposition

The total variance of the monetary policy-induced component is given by

$$\text{Var}(\hat{y}_t^{MP}) = \text{Var}\left(\sum_{k \in \mathcal{K}} C_{k,t}\right). \tag{C.2}$$

Using the identity

$$\mathrm{Var}\left(\sum_k C_{k,t}\right) = \sum_k \mathrm{Cov}(C_{k,t}, \hat{y}_t^{MP}), \tag{C.3}$$

we can decompose total variance into the sum of covariances between each component and the overall monetary policy-related fitted value $\hat{y}_t^{MP}$.

We define the variance share of component k as

$$S_k = \frac{\mathrm{Cov}(C_{k,t}, \hat{y}_t^{MP})}{\mathrm{Var}(\hat{y}_t^{MP})}. \tag{C.4}$$

In the empirical implementation, these objects are estimated using sample moments:

$$\widehat{\mathrm{Var}}(\hat{y}_t^{MP}) = \frac{1}{T-1}\sum_{t=1}^{T}(\hat{y}_t^{MP} - \bar{y}^{MP})^2, \tag{C.5}$$

$$\widehat{\mathrm{Cov}}(C_{k,t}, \hat{y}_t^{MP}) = \frac{1}{T-1}\sum_{t=1}^{T}(C_{k,t} - \bar{C}_k)(\hat{y}_t^{MP} - \bar{y}^{MP}). \tag{C.6}$$

By construction,

$$\sum_{k\in\mathcal{K}} S_k = 1. \tag{C.7}$$

The variance share S_k measures the contribution of channel k to fluctuations in the monetary policy effect. A larger value of S_k indicates that variation in component $C_{k,t}$ is strongly aligned with variation in the overall monetary policy-induced response. Conversely, a small value indicates that the component contributes little to overall fluctuations.

Because the components are not orthogonal, the decomposition accounts for their joint comovement with the overall monetary policy-related fitted value $\hat{y}_t^{MP}$ rather than isolating purely independent contributions. As a result, the shares reflect both the magnitude of each component and its correlation with other channels. In particular, a component that is highly volatile but weakly correlated with the overall response will receive a smaller share than a component whose movements closely track the total effect.

Importantly, all components are constructed using the estimated coefficients and all included lags of the monetary policy shock and its interactions. The decomposition therefore captures the full dynamic propagation of monetary policy through the different transmission channels implied by the model.

C.2 Robustness: Sequential variance decomposition

As an alternative approach to the above covariance-based decomposition, we implement a sequential variance decomposition based on the model-implied components broadly following Shapley (1953). Specifically, we again construct the fitted value associated with the monetary policy shock, $\hat{y}_t^{MP}$, as the sum of its constituent components as in equation C.1. We then evaluate the *incremental contribution* of each component to the variance of the fitted values. This approach captures how much each transmission channel contributes to overall fluctuations, *conditional on previously included components.*

C.2.1 Variance decomposition

To assess the contribution of each component to the variance of $\hat{y}_t^{MP}$, we construct a sequence of partial fitted values. Let $\Xi = (k_1, k_2, \ldots, k_K)$ denote an ordering of the components. Defining

$$\hat{y}_t^{MP,(m)} = \sum_{j=1}^{m} C_{k_j,t}, \quad m = 1, \ldots, K, \tag{C.8}$$

with $\hat{y}_t^{MP,(0)} = 0$, the marginal contribution of component k_m is then given by

$$\Delta V_{k_m} = \text{Var}(\hat{y}_t^{MP,(m)}) - \text{Var}(\hat{y}_t^{MP,(m-1)}). \tag{C.9}$$

The total variance is defined as

$$V = \text{Var}(\hat{y}_t^{MP}) \tag{C.10}$$

and the variance share of component k_m is

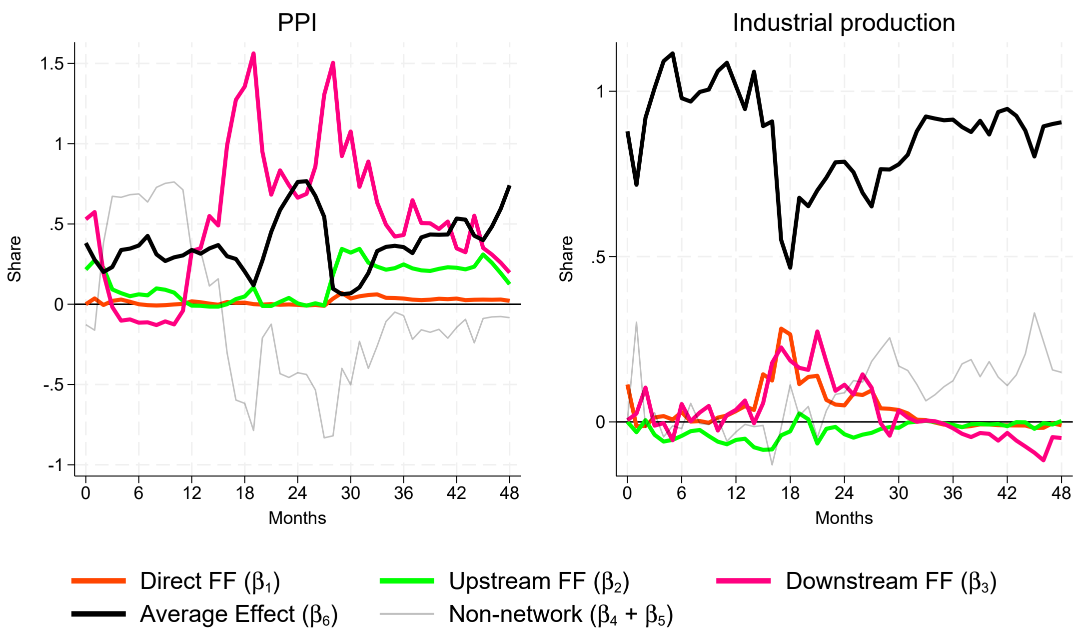
$$S_{k_m} = \frac{\Delta V_{k_m}}{V}. \tag{C.11}$$

By construction, the shares again satisfy

$$\sum_{k \in \mathcal{K}} S_k = 1. \tag{C.12}$$

As above, the variance share S_k measures how much component k contributes to overall fluctuations in the monetary policy-related fitted value $\hat{y}_t^{MP}$ when added to the set of previously included components. Figure C.1 shows that the respective patterns for variance shares are qualitatively consistent with the ones obtained by the covariance-based approach reported in figure 6 in the main text.

Figure C.1: Sequential variance decomposition of monetary policy channels



Notes: Sequential Shapley (1953)-type variance decomposition of monetary policy-related components in model 8.

C.2.2 Ordering of components

As components may be correlated, the ordering Ξ affects the allocation of shared variation across components. The reported decomposition therefore reflects one economically motivated ordering of transmission channels and can be interpreted as an approximation to a full Shapley (1953) decomposition, which averages over all possible orderings.

In the sequential variance decomposition in figure C.1, we adopt the following ordering:

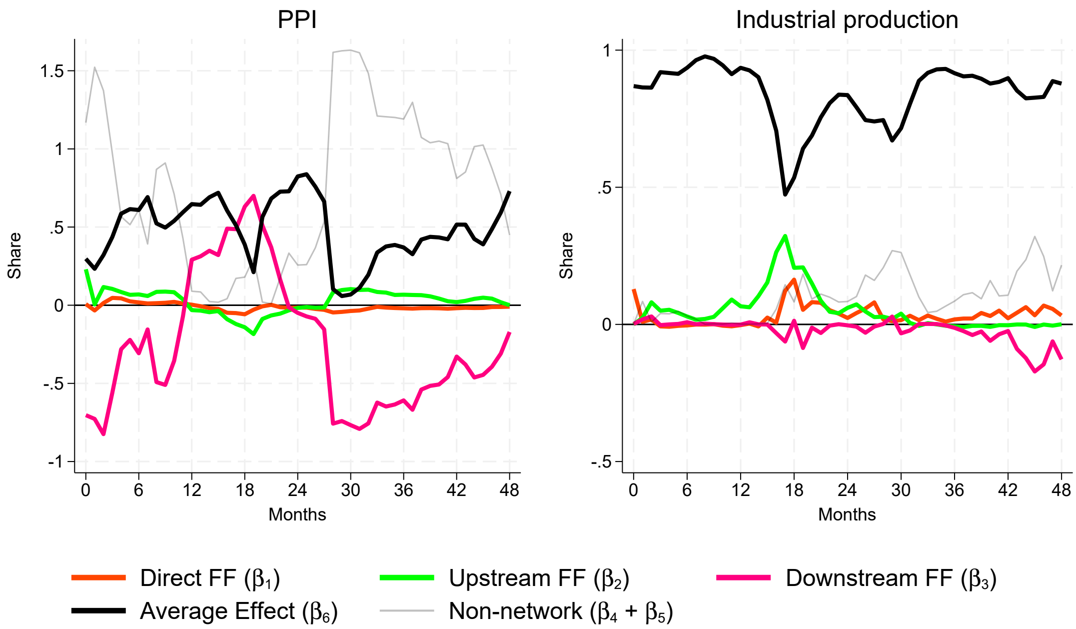
$$\Xi = (\text{Average}, \text{Direct}, \text{Upstream}, \text{Downstream}, \text{Non-network}). \tag{C.13}$$

This ordering reflects an economically motivated hierarchy of transmission channels. We first include the average effect of a monetary policy shock, i.e. barring network-induced nonlinearities, given by β_6 in model 8. We then straightforwardly add the direct effect governed by β_1 and upstream and downstream network effects given by β_2 and β_3, and finally the non-network related interactions described by β_4 and β_5. Under this ordering, each component's contribution is interpreted as the incremental increase in the variance of the fitted values when the component is added to the set of previously included components. Figure C.2 shows respective results for the reversed ordering

$$\tilde{\Xi} = (\text{Non-network}, \text{Downstream}, \text{Upstream}, \text{Direct}, \text{Average}). \tag{C.14}$$

Results remain broadly robust under complete reversal to those obtained under both the covariance-based and main sequential decomposition as shown in figures 6 and C.1, with the average, downstream financial frictions and non-network effects still explaining the largest shares of the variation in producer prices and industrial production.

Figure C.2: Sequential variance decomposition of monetary policy channels - reverse ordering



Notes: Sequential Shapley (1953)-type variance decomposition of monetary policy-related components in model 8, based on reverse ordering C.14.

D Robustness checks

In the following, we provide robustness checks to the results obtained with empirical model presented in section 3.2. We particularly assess the robustness of our main empirical results in figures 3 to 5 to using an alternative financial frictions measure, namely firms' working capital, to using different means of aggregating firm-level leverage data to the sectoral level, and to running different model specifications.

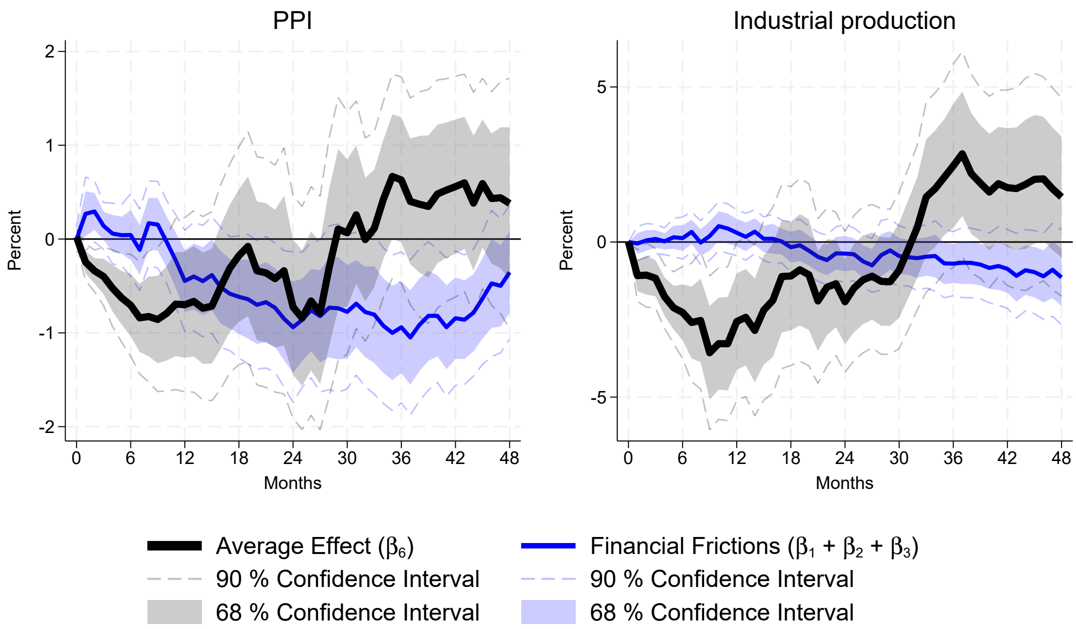
D.1 Financial frictions measures

In this section, we show that our results remain broadly robust when using an alternative empirical financial frictions measure for φ in equations 7 and 8. Figures D.1 to D.3 show the same set of impulse response as depicted in figures 3 to 5 when sector-aggregates of firm level data on the working capital share, defined as working capital expenses/total assets instead of total leverage is used. We assess robustness to working capital also in light of its importance in the theoretical analysis we carry out in section 4. Overall, results are robust to using the working capital share, with results still indicating the same diverging effects of up- and downstream financial frictions in the transmission of monetary policy over the medium term as displayed in section 3.2. However, using the working capital share results in a slight tilting of results in the short term, with downstream demand effects potentially dominating direct financial constraints effects in the first year after the shock. As shown in figure D.2, the direct financial frictions effect is negative in the first year after the shock, before upstream cost effects become the dominant driver of direct financial constraints effects as in the baseline specification (figure 4). This in turn results in a muting of the inflationary short-term financial constraints effects reported in the baseline results when reyling on working capital shares instead of total leverage (figure 3 vs figure D.1).

We also test for differences in aggregating firm-level data on financial frictions measures to the sectoral level. While main results in figures 3 to 5 where derived using sales-based weighted averages of firm level data to generate sectoral measures, figures D.4 to D.6 show that results are robust to changing aggregation such that sectoral levels of total leverage reflect the median firm's leverage holdings.

Finally, we adjust our baseline financial frictions measure in equation 7 to proxy for additional possible distortions requiring a differentiation between cost- and revenue-based input–output matrices as in Baqaee and Farhi (2020). Our main

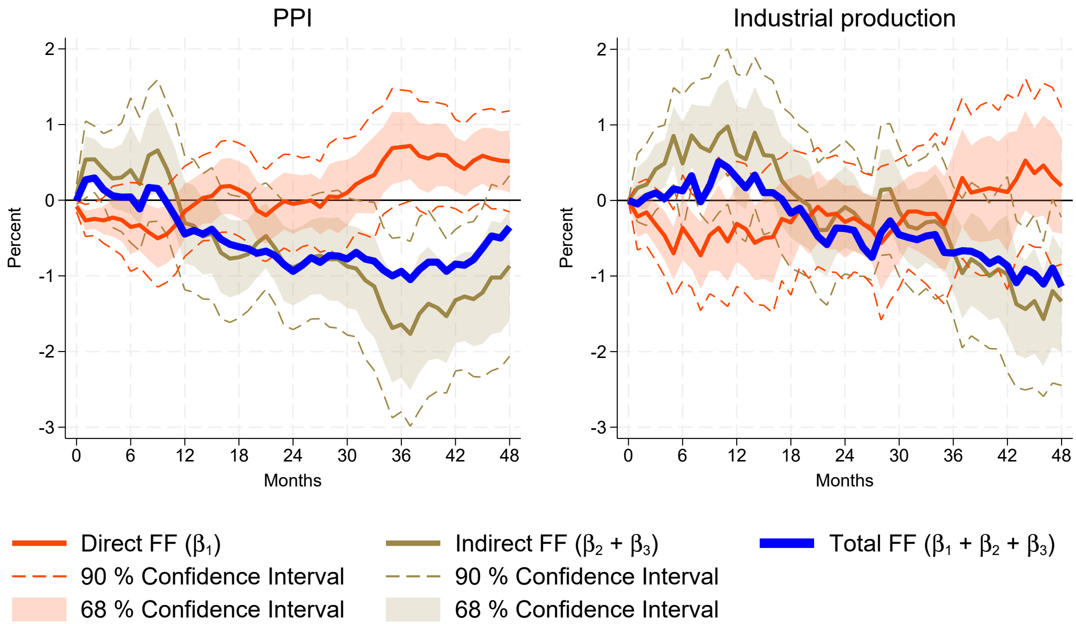
Figure D.1: Impulse responses to monetary policy tightening shock - working capital



Notes: Impulse responses to a monetary policy tightening shock scaled to a peak increase in the 3m OIS rate of 25bp. Estimates for model 8 estimated on the full country-sector euro area panel.

empirical measures in equations 7 follow the theoretical model in assuming perfect competition and no further distortions beyond the financial friction introduced in equation 10. Our empirical and theoretical measures are thus consistent with the approach in Acemoglu et al. (2012), in which, as a result of the absence of other frictions, cost and revenue shares would ultimately coincide, such that both the cost- and revenue based Leontief inverses $\mathbf{Z}$ and $\tilde{\mathbf{Z}}$ would be based on the same matrix $\mathbf{\Omega}$, as stated in table A.4. We thus also run our baseline regression 8 using versions of the financial frictions measures 7 adjusted for additional frictions as in Baqaee and Farhi (2020). However, as we are not able to empirically observe the actual cost-based input–output matrix defined by the elasticity of sector j's marginal cost with respect to the price of sector k as prescribed by the theoretical setup in Baqaee and Farhi (2020), we approximate the distortion component by using observed input cost shares as reported in input-output tables. Concretely, in comparison to the definitions in table A.4, we now differentiate between a cost-share matrix and a revenue-share matrix, with the two matrices being given by

Figure D.2: Impulse responses to monetary policy tightening shock - direct vs. indirect financial frictions effects - working capital



Notes: Impulse responses to a monetary policy tightening shock scaled to a peak increase in the 3m OIS rate of 25bp. Estimates for model 8 estimated on the full country-sector euro area panel.

$$\mathbf{\Omega} = \left[\frac{x_{ij}^{c,d}}{\sum_{j,d} x_{kj}^{d,c} + l_j} \right] \tag{D.1}$$

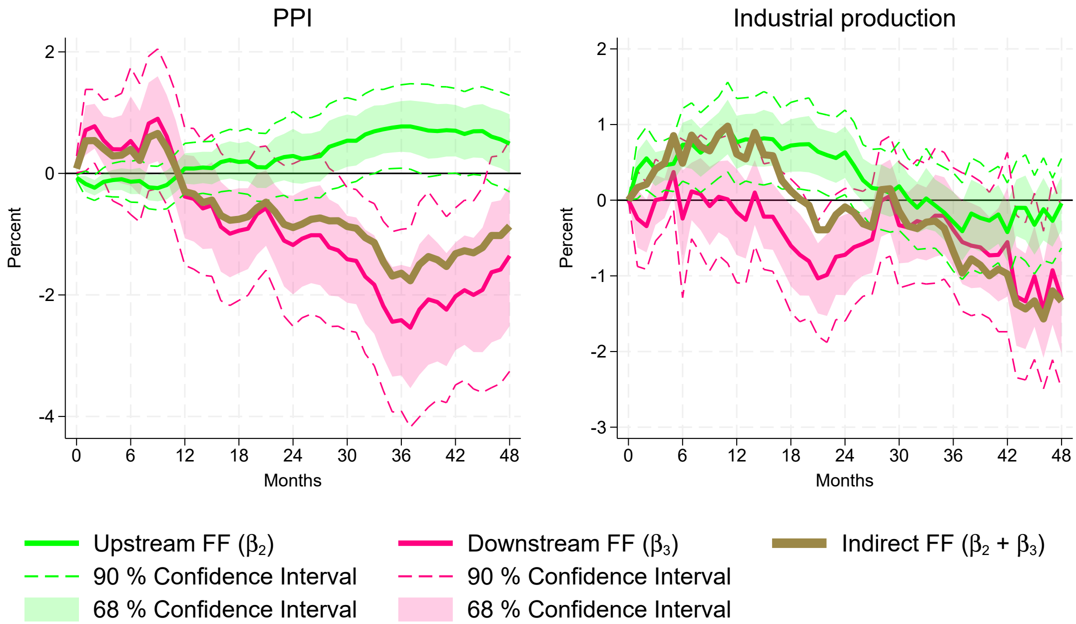
$$\tilde{\mathbf{\Omega}} = \left[\frac{x_{jk}^{c,d}}{y_j^c} \right]. \tag{D.2}$$

The two matrices differ in terms of the denominators, where input costs are reported in the denominator of $\mathbf{\Omega}$, while total revenue $y_j^c = \sum_{k,d} x_{jk}^{c,d} + c_j = \sum_{j,d} x_{kj}^{d,c} + l_j + v_j + t_j$ is reported in the denominator of matrix $\tilde{\mathbf{\Omega}}$. Thus, the revenue-share matrix $\tilde{\mathbf{\Omega}}$ accounts also for value added and tax terms v_j and t_j, which determine our input-output based proxies for additional distortions such as market power and distortive tax policies. Consistently, the adjusted cost- and revenue-based Leontief inverses are now given by:

$$\mathbf{Z} = (\mathbf{I} - \mathbf{\Omega}^T)^{-1} \tag{D.3}$$

$$\tilde{\mathbf{Z}} = (\mathbf{I} - \tilde{\mathbf{\Omega}})^{-1} \tag{D.4}$$

Figure D.3: Impulse responses to monetary policy tightening shock - upstream vs. downstream financial frictions effects - working capital



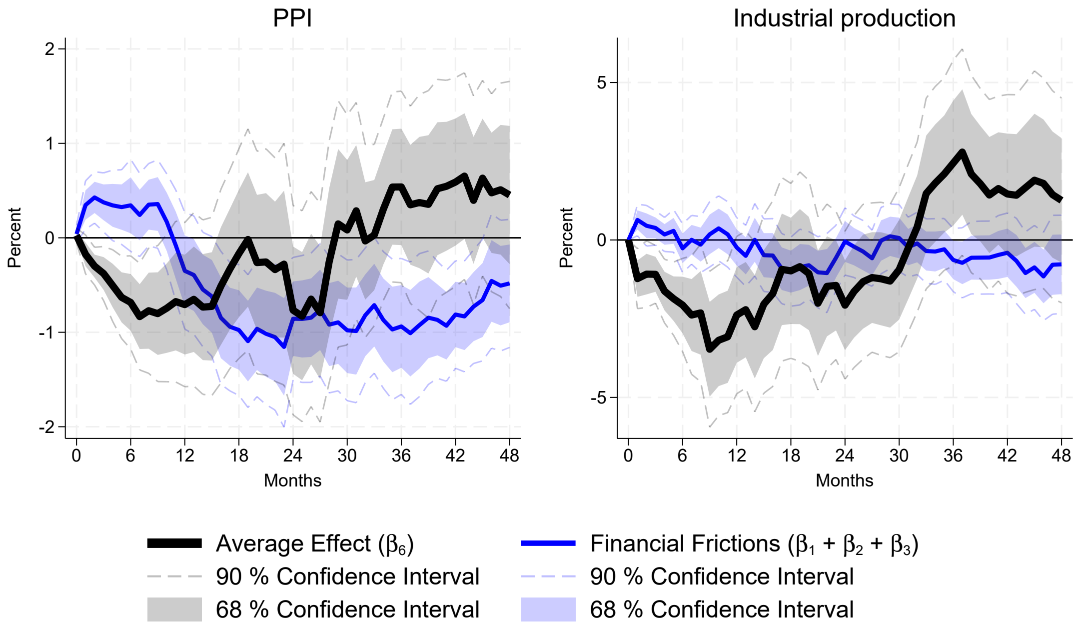
Notes: Impulse responses to a monetary policy tightening shock scaled to a peak increase in the 3m OIS rate of 25bp. Estimates for model 8 estimated on the full country-sector euro area panel.

As shown in figures D.7 to D.9, our main results in figures 3 to 5 are robust to those obtained when estimating model 8 with the adjusted cost- and revenue-based Leontief inverses.

D.2 Model specification

Finally, we also assess the robustness of our results across different specifications of the model, beyond the choice of financial frictions measures and the representation of input-output linkages. In particular, we assess whether estimating the model in level terms instead of long-differences. The level variant of the long-difference model 8 is given by:

Figure D.4: Impulse responses to monetary policy tightening shock - median level of sectoral total leverage



Notes: Impulse responses to a monetary policy tightening shock scaled to a peak increase in the 3m OIS rate of 25bp. Estimates for model 8 estimated on the full country-sector euro area panel.

$$y_{jc,t+h} = \underbrace{\beta_1^h \varphi_{jc,t_{12}-1} \times s_t}_{\text{Direct financial frictions effect}} + \underbrace{\overbrace{\beta_2^h \Phi'_{jc,t_{12}-1} \times s_t}^{\text{Upstream effect}} + \overbrace{\beta_3^h \tilde{\Phi}'_{jc,t_{12}-1} \times s_t}^{\text{Downstream effect}}}_{\text{Indirect financial frictions effect}} + \tag{D.5}$$

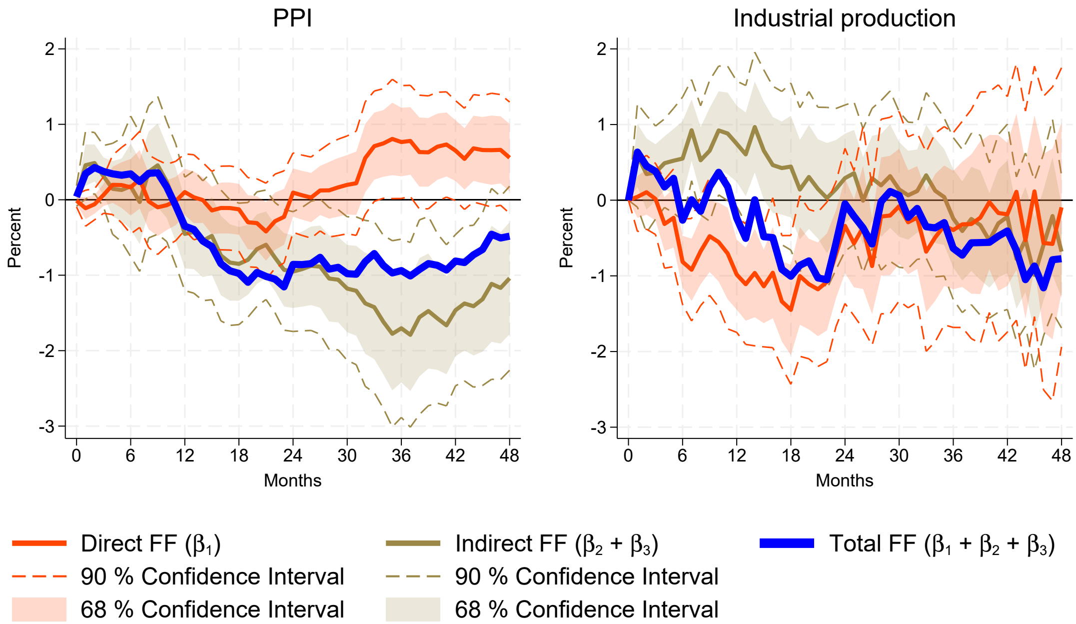
$$\underbrace{\beta_4^h a_{jc,t_{12}-1} \times s_t + \beta_5^h \tilde{a}_{jc,t_{12}-1} \times s_t + \beta_6^h s_t}_{\text{Non-network effect}}$$

$$+ \sum_{l=0}^{L} \gamma^{\mathbf{h}} \mathbf{H_{t-l}} + \sum_{l=1}^{L} \delta^{\mathbf{h}} \mathbf{K_{t-l}} + \sum_{l=0}^{L} \eta^{\mathbf{h}} \mathbf{X_{t-l}} + \phi_{jc} + \theta_m + \kappa_{t+h} + \epsilon_{jc,t+h}$$

$$\text{with} \quad h = 1, 2, ..., H \quad \text{and} \quad L = 3$$

$$\mathbf{H_t} = \begin{bmatrix} a_{jc,t_{12}} \\ \tilde{a}_{jc,t_{12}} \\ \varphi_{jc,t_{12}} \\ \Phi'_{jc,t_{12}} \\ \tilde{\Phi}'_{jc,t_{12}} \end{bmatrix}, \mathbf{K_t} = \begin{bmatrix} y_{jc,t} \\ \varphi_{jc,t_{12}} \times s_t \\ a_{jc,t_{12}} \times s_t \\ \tilde{a}_{jc,t_{12}} \times s_t \\ \Phi'_{jc,t_{12}} \times s_t \\ \tilde{\Phi}'_{jc,t_{12}} \times s_t \\ s_t \end{bmatrix}$$

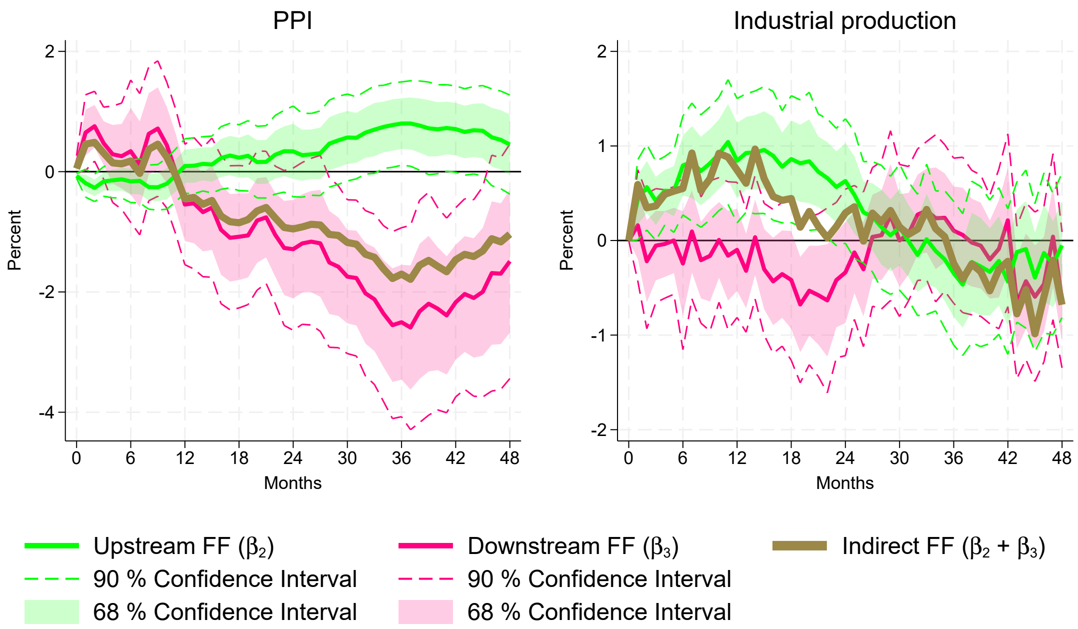
Figure D.5: Impulse responses to monetary policy tightening shock - direct vs. indirect financial frictions effects - median level of sectoral total leverage



Notes: Impulse responses to a monetary policy tightening shock scaled to a peak increase in the 3m OIS rate of 25bp. Estimates for model 8 estimated on the full country-sector euro area panel.

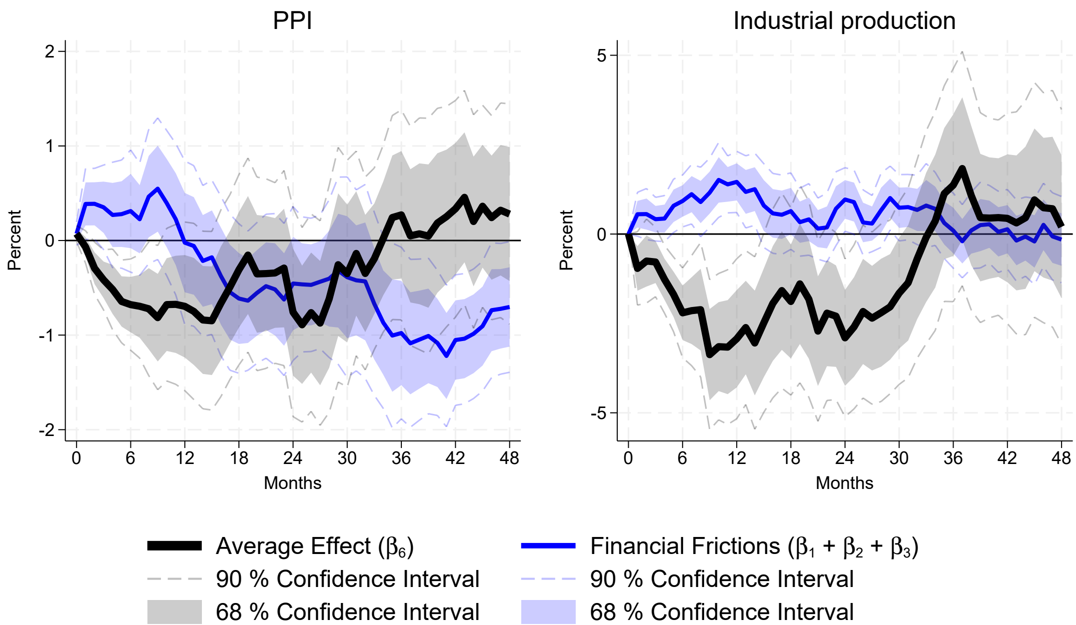
Compared to the long-difference variant, we include country-sector fixed effects ϕ_{ic}, which are not present in the differenced version of the model. Jordà and Taylor (2024) suggest using long-differences to mitigate concerns regarding small sample biases, and comparing results in figures D.10 to D.12 with our main results in figures 3 to 5 confirm that our findings would be broadly robust to such concerns when estimating the model in levels.

Figure D.6: Impulse responses to monetary policy tightening shock - upstream vs. downstream financial frictions effects - median level of sectoral total leverage



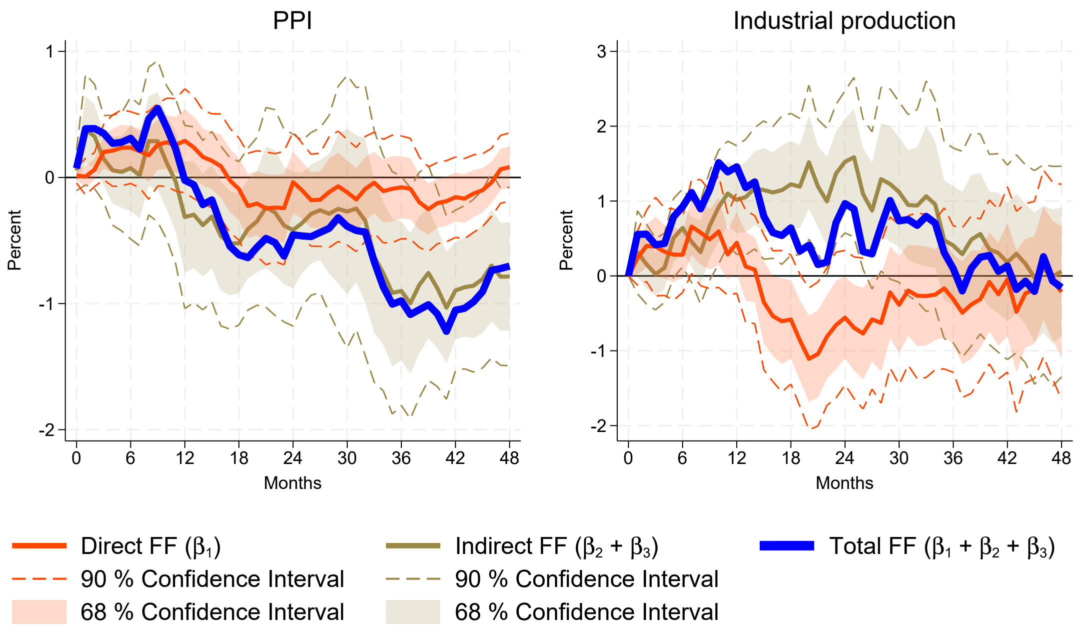
Notes: Impulse responses to a monetary policy tightening shock scaled to a peak increase in the 3m OIS rate of 25bp. Estimates for model 8 estimated on the full country-sector euro area panel.

Figure D.7: Impulse responses to monetary policy tightening shock - distortion-adjusted Leontief inverses



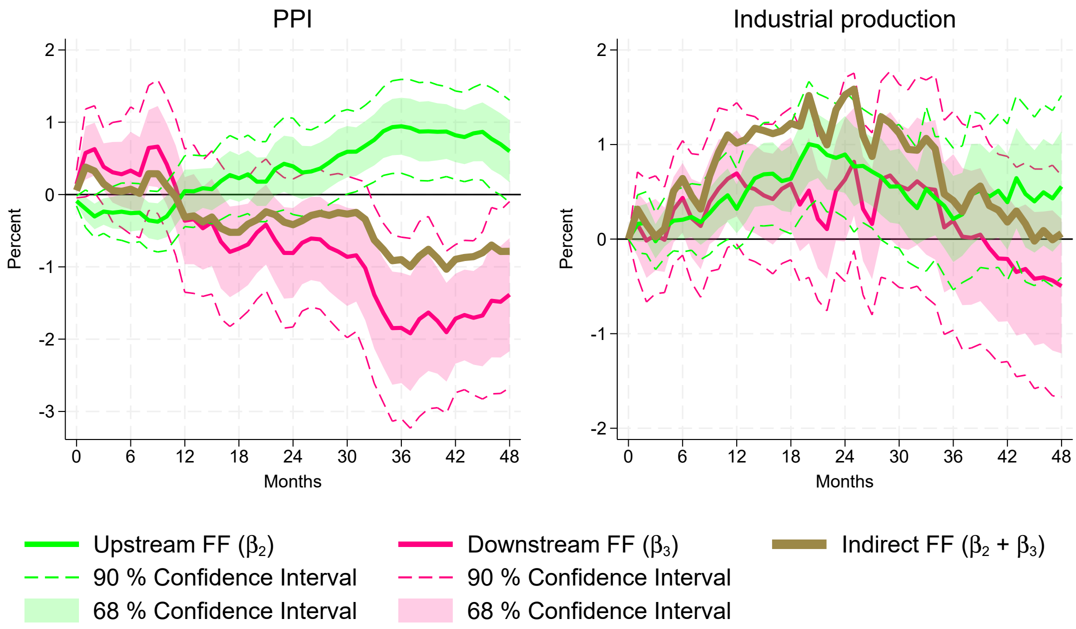
Notes: Impulse responses to a monetary policy tightening shock scaled to a peak increase in the 3m OIS rate of 25bp. Estimates for model 8 estimated on the full country-sector euro area panel.

Figure D.8: Impulse responses to monetary policy tightening shock - direct vs. indirect financial frictions effects - distortion-adjusted Leontief inverses



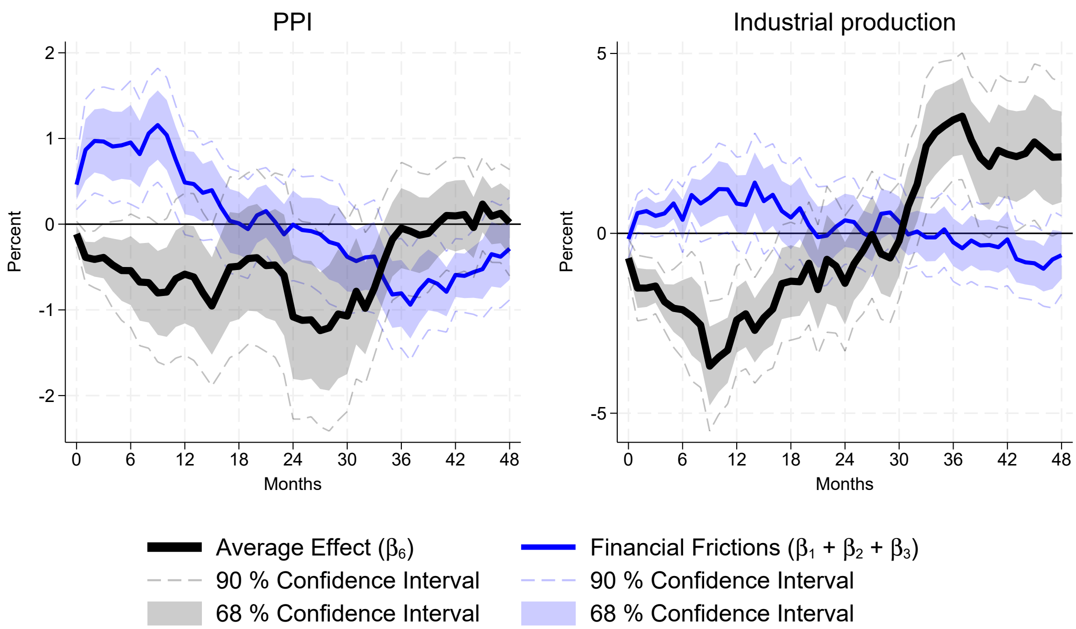
Notes: Impulse responses to a monetary policy tightening shock scaled to a peak increase in the 3m OIS rate of 25bp. Estimates for model 8 estimated on the full country-sector euro area panel.

Figure D.9: Impulse responses to monetary policy tightening shock - upstream vs. downstream financial frictions effects - distortion-adjusted Leontief inverses



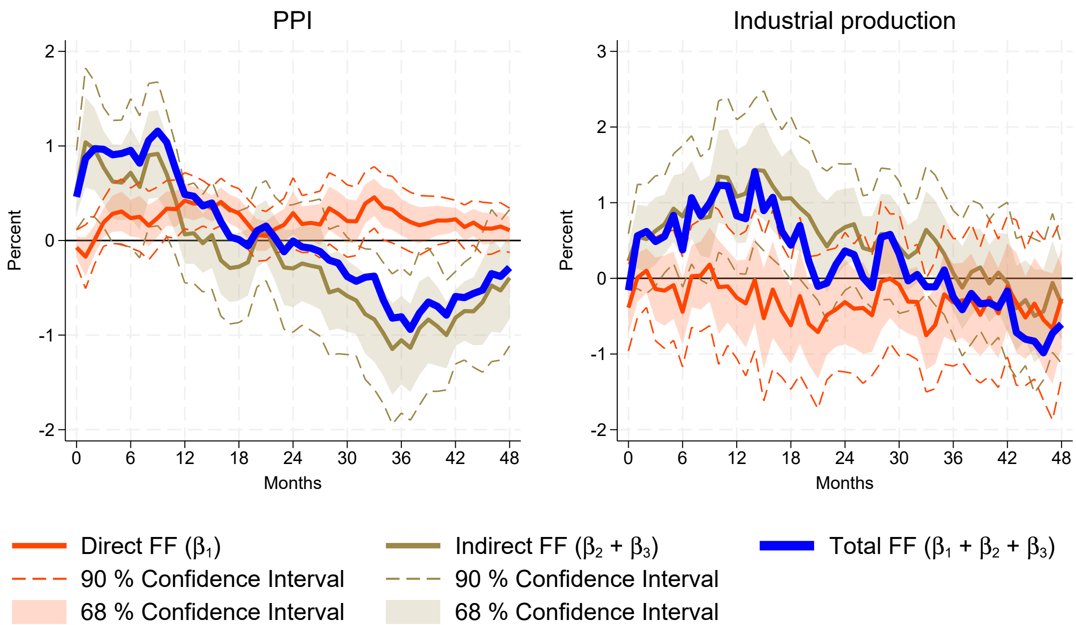
Notes: Impulse responses to a monetary policy tightening shock scaled to a peak increase in the 3m OIS rate of 25bp. Estimates for model 8 estimated on the full country-sector euro area panel.

Figure D.10: Impulse responses to monetary policy tightening shock - level specification



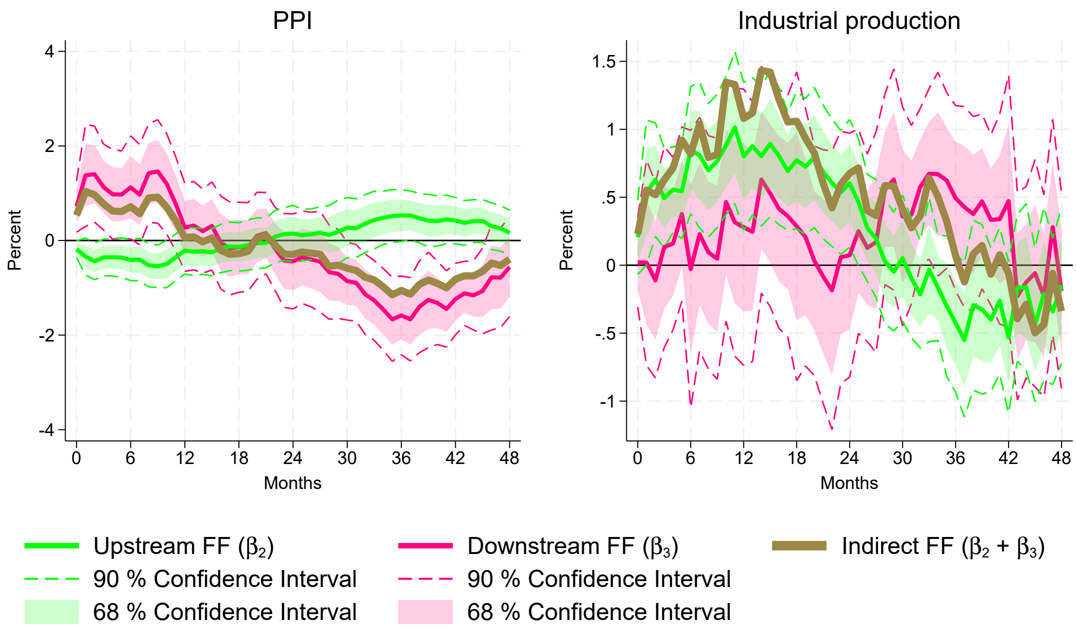
Notes: Impulse responses to a monetary policy tightening shock scaled to a peak increase in the 3m OIS rate of 25bp. Estimates for model 8 estimated on the full country-sector euro area panel.

Figure D.11: Impulse responses to monetary policy tightening shock - direct vs. indirect financial frictions effects - level specification



Notes: Impulse responses to a monetary policy tightening shock scaled to a peak increase in the 3m OIS rate of 25bp. Estimates for model 8 estimated on the full country-sector euro area panel.

Figure D.12: Impulse responses to monetary policy tightening shock - upstream vs. downstream financial frictions effects - level specification



Notes: Impulse responses to a monetary policy tightening shock scaled to a peak increase in the 3m OIS rate of 25bp. Estimates for model 8 estimated on the full country-sector euro area panel.

E Model derivations

This appendix provides the complete set of derivations for results stated in section 4. We first report the firm's optimization problem and the market clearing conditions for completeness, before describing in more detail the closed-form price decomposition 24, the corresponding response to productivity shocks, and the leading-order proxy mapping of section 4.4. All results are derived at $\sigma = 1$, i.e. with Cobb–Douglas demand, unless otherwise stated.[36]

We use the notation $\hat{x} \equiv d \log x / di$ for semi-elasticities, and define two friction derivatives recalled throughout the derivations:

$$f_j \equiv \frac{\varphi_j}{1 + i\varphi_j} = \frac{d \log(1 + i\varphi_j)}{di}, \qquad g_j \equiv \frac{\varphi_j}{(1 + i\varphi_j)^2} = \frac{f_j}{1 + i\varphi_j}. \tag{E.1}$$

Both f_j and g_j collapse to φ_j at $i = 0$. The quantity f_j governs the rate at which revenue shares shrink with i (see section E.4), while g_j appears in intermediate steps, without entering the final price decomposition. Throughout, $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_n)^\top$ denotes the vector of labor shares and $\mathbf{D}_\alpha \equiv \mathrm{diag}(\alpha_1, \dots, \alpha_n)$ denotes the corresponding diagonal matrix.

E.1 Firm optimization and optimality conditions

Cost minimization problem

Sector j takes output prices $\{p_k\}$, the wage w_j, the interest rate i, and its friction parameter φ_j as given. It chooses labor l_j and intermediate inputs $\{x_{kj}\}_{k=1}^n$ to minimize the total cost of producing a given output y_j, subject to the production technology 9.

The financing wedge $1 + i\varphi_j$ is a multiplicative markup on all variable inputs, thus not affecting relative input prices. The cost minimization problem is therefore given by:

$$\min_{\{l_j, x_{kj}\}} (1 + i\varphi_j)\Big(w_j l_j + \sum_{k=1}^n p_k x_{kj}\Big) \quad \text{s.t.} \quad \tilde{a}_j\, l_j^{\alpha_j} \prod_{k=1}^n x_{kj}^{\omega_{kj}} \ \geq\ y_j. \tag{E.2}$$

[36] We discuss the revenue equation for general levels of σ in section E.6 below.

As the financing wedge is a common scalar multiplier, the first-order conditions for cost minimization are identical to those of a standard Cobb–Douglas problem without financing frictions. The Lagrangian is given by

$$\mathcal{L} = (1 + i\varphi_j)\Big(w_j l_j + \sum_k p_k x_{kj}\Big) - \mu_j\Big(\tilde{a}_j l_j^{\alpha_j} \prod_k x_{kj}^{\omega_{kj}} - y_j\Big), \tag{E.3}$$

where μ_j is the Lagrange multiplier on the output constraint. The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial l_j} = 0 \implies \quad (1 + i\varphi_j)\, w_j \;=\; \mu_j\, \alpha_j\, \frac{y_j}{l_j}, \tag{E.4}$$

$$\frac{\partial \mathcal{L}}{\partial x_{kj}} = 0 \quad \forall\, k \implies \quad (1 + i\varphi_j)\, p_k \;=\; \mu_j\, \omega_{kj}\, \frac{y_j}{x_{kj}}. \tag{E.5}$$

Dividing equation E.4 by equation E.5 eliminates μ_j:

$$\frac{w_j l_j}{p_k x_{kj}} \;=\; \frac{\alpha_j}{\omega_{kj}} \qquad \forall\, k. \tag{E.6}$$

This is the standard Cobb–Douglas optimal input ratio showing that the cost shares are exactly equal to the technology exponents, confirming that ω_{kj} is both the exponent in equation 9 and the cost share defined in equation E.6.

Non-financial marginal cost

From the first-order condition E.5, the Lagrange multiplier satisfies

$$\mu_j = (1 + i\varphi_j)\, p_k x_{kj} / (\omega_{kj} y_j) \tag{E.7}$$

for any k. Using the cost shares and summing over all inputs plus labor, the non-financial total cost is

$$w_j l_j + \sum_k p_k x_{kj} \;=\; \tilde{mc}_j\, y_j, \tag{E.8}$$

where the *non-financial marginal cost* is

$$\tilde{mc}_j \;=\; \frac{\mu_j}{1 + i\varphi_j} \;=\; \frac{1}{a_j}\, w_j^{\alpha_j} \prod_{k=1}^{n} p_k^{\omega_{kj}}, \qquad a_j \;\equiv\; \tilde{a}_j\, \alpha_j^{\alpha_j} \prod_{k=1}^{n} \omega_{kj}^{\omega_{kj}}. \tag{E.9}$$

Here a_j collects all constant terms from the Shephard-lemma factor demands: the coefficient $\alpha_j^{\alpha_j}$ arises from the labor demand $l_j^* = (\alpha_j/w_j)\tilde{mc}_j y_j$, and each $\omega_{kj}^{\omega_{kj}}$ arises from the intermediate demand $x_{kj}^* = (\omega_{kj}/p_k)\tilde{mc}_j y_j$.

Optimal pricing under perfect competition

Under perfect competition, firms price at marginal cost. Total marginal cost including the financing wedge is $mc_j = (1 + i\varphi_j)\tilde{mc}_j$, so the zero-profit condition gives

$$p_j \;=\; (1 + i\varphi_j)\,\tilde{mc}_j \;=\; \frac{1 + i\varphi_j}{a_j}\, w_j^{\alpha_j} \prod_{k=1}^{n} p_k^{\omega_{kj}}. \tag{E.10}$$

This is equation 11 in the main text. Note that the financing wedge $1+i\varphi_j$ raises the price proportionally for all levels of output, consistent with perfect competition.

Household optimality

The representative household minimizes expenditure $\sum_j p_j c_j$ subject to achieving utility c from the CES aggregator 13. The first-order conditions give the standard CES demand system:

$$p_j c_j \;=\; e\,\nu_j \left(\frac{p_j}{p^c}\right)^{1-\sigma}, \tag{E.11}$$

where $e = \sum_j p_j c_j$ is total nominal expenditure and $p^c = (\sum_j \nu_j p_j^{1-\sigma})^{1/(1-\sigma)}$ is the CES price index. Demand for good j falls with its relative price at elasticity σ, and expenditure shares are $p_j c_j/e = \nu_j (p_j/p^c)^{1-\sigma}$.

E.2 The Cobb–Douglas wage identity

A key identity used throughout the derivations follows directly from the above cost minimization. From the first-order condition E.4, the labor bill satisfies

$$w_j l_j = \alpha_j \frac{\mu_j}{1 + i\varphi_j} y_j = \alpha_j \tilde{mc}_j\, y_j$$

since

$$\tilde{mc}_j = \frac{\mu_j}{1 + i\varphi_j}.$$

Since $r_j \equiv p_j y_j = (1+i\varphi_j)\tilde{mc}_j y_j$, we obtain $\tilde{mc}_j y_j = r_j/(1+i\varphi_j)$, and therefore:

$$w_j l_j = \frac{\alpha_j r_j}{1+i\varphi_j}. \tag{E.12}$$

Identity E.12 represents equation 12 in the main text and indicates that the wage bill is a constant fraction α_j of revenue, adjusted downward by the financing wedge. It directly follows from combining the Cobb–Douglas technology fixing cost shares with the working-capital friction driving a wedge between revenue and non-financial cost.

Equation E.12 allows for *eliminating the sector wage* from the log-price equation. From labor market clearing $l_j = \bar{l}_j$, we get

$$w_j = \frac{\alpha_j r_j}{(1+i\varphi_j)\bar{l}_j}, \tag{E.13}$$

such that $\log w_j = \log \alpha_j + \log r_j - \log(1+i\varphi_j) - \log \bar{l}_j$. This substitution converts the log-price equation from a system in (p_j, w_j) to a system in (p_j, r_j), where revenues are determined directly by the demand side.

E.3 The log-price equation

In the following, we describe the derivation steps for obtaining equation 20 in the main text.

Step 1: Taking logarithms of the pricing condition

Taking logarithms of equation E.10

$$p_j = \frac{1+i\varphi_j}{a_j} w_j^{\alpha_j} \prod_{k=1}^{n} p_k^{\omega_{kj}}. \tag{E.14}$$

yields

$$\log p_j = \log(1+i\varphi_j) - \log a_j + \alpha_j \log w_j + \sum_{k=1}^{n} \omega_{kj} \log p_k. \tag{E.15}$$

Step 2: Eliminating the sectoral wage

Substituting $\log w_j = \log \alpha_j + \log r_j - \log(1 + i\varphi_j) - \log \bar{l}_j$ into equation E.15 yields

$$\log p_j = \log(1 + i\varphi_j) - \log a_j \\ + \alpha_j \left[\log \alpha_j + \log r_j - \log(1 + i\varphi_j) - \log \bar{l}_j\right] + \sum_{k=1}^{n} \omega_{kj} \log p_k. \tag{E.16}$$

Collecting the $\log(1 + i\varphi_j)$ terms yields

$$\log p_j \;=\; \underbrace{(1 - \alpha_j)}_{\text{net markup weight}} \log(1+i\varphi_j)+\alpha_j \log r_j+\underbrace{\left(-\log a_j + \alpha_j \log \alpha_j - \alpha_j \log \bar{l}_j\right)}_{\equiv c_j,\ \text{constant}}+\sum_{k=1}^{n} \omega_{kj} \log p_k. \tag{E.17}$$

Economically, the weight $1 - \alpha_j$ indicates that an increase in i raises the cost of all variable inputs through the wedge $1 + i\varphi_j$. However, the wage identity 12 shows that the wage already adjusts proportionally as revenues and hence the wage bill are determined on the demand side, such that the *net* cost-push on prices only stems from the non-labor share of inputs, determined by the weight $1 - \alpha_j$.

Step 3: Stacking into a vector system

We define the following vectors: $\mathbf{p} = (\log p_j)$, $\mathbf{v} = (\log(1 + i\varphi_j))$, $\mathbf{r} = (\log r_j)$, and $\mathbf{c} = (c_j)$. Equation E.17 stacked across all j gives

$$\mathbf{p} \;=\; (\mathbf{I} - \mathbf{D}_\alpha)\mathbf{v} + \mathbf{D}_\alpha \mathbf{r} + \mathbf{c} + \boldsymbol{\Omega}^\top \mathbf{p}. \tag{E.18}$$

The input-output term $\sum_k \omega_{kj} \log p_k$ becomes $[\boldsymbol{\Omega}^\top \mathbf{p}]_j$ as ω_{kj} has row = seller k and column = customer j, such that $\boldsymbol{\Omega}^\top$ has row = customer j and $[\boldsymbol{\Omega}^\top \mathbf{p}]_j = \sum_k \omega_{kj} \log p_k$.

Step 4: Solving for the price vector

Rearranging equation E.18 and premultiplying by $\mathbf{H} = (\mathbf{I} - \boldsymbol{\Omega}^\top)^{-1}$ yields

$$\mathbf{p} \;=\; \mathbf{H}\left[(\mathbf{I} - \mathbf{D}_\alpha)\mathbf{v} + \mathbf{D}_\alpha \mathbf{r} + \mathbf{c}\right] \tag{E.19}$$

which for a single sector k implies

$$\log p_k \;=\; \sum_{j=1}^{n} h_{kj}\big[(1-\alpha_j)\log(1+i\varphi_j) + \alpha_j \log r_j + c_j\big]. \qquad \text{(E.20)}$$

This is equation 20 in the main text. The element h_{kj} of the cost Leontief inverse weighs sector j's contribution to sector k's price, aggregating all supply-chain paths of arbitrary length connecting j to k upstream.

E.4 The revenue-share matrix and its derivative

Cost and revenue shares

The cost-share matrix $\mathbf{\Omega}$ has element $\omega_{kj} = p_k x_{kj}/(\tilde{mc}_j y_j)$, the share of sector j's non-financial production costs paid to upstream supplier k. The Cobb–Douglas technology and cost minimization imply that this equals the exponent ω_{kj} in equation 9.

The revenue-share matrix $\tilde{\mathbf{\Omega}}$ has element $\tilde{\omega}_{kj} = p_k x_{kj}/(p_j y_j)$, the share of sector j's nominal output sold to sector k as intermediate inputs. Since $p_j y_j = (1 + i\varphi_j)\tilde{mc}_j y_j$, we obtain

$$\tilde{\omega}_{kj} \;=\; \frac{\omega_{kj}}{1+i\varphi_j}. \qquad \text{(E.21)}$$

The financing wedge compresses the revenue share below the cost share whenever $i > 0$. The column sum of $\tilde{\mathbf{\Omega}}$ is $\sum_k \tilde{\omega}_{kj} = (1-\alpha_j)/(1+i\varphi_j) < 1-\alpha_j$, so $\tilde{\mathbf{\Omega}}$ is strictly more compressed than $\mathbf{\Omega}$.

Derivative of the revenue-share matrix

Differentiating equation E.21 with respect to i yields

$$\frac{d\tilde{\omega}_{kj}}{di} \;=\; -\frac{\omega_{kj}\varphi_j}{(1+i\varphi_j)^2} \;=\; -\tilde{\omega}_{kj} \times \frac{\varphi_j}{1+i\varphi_j} \;=\; -\tilde{\omega}_{kj}\, f_j, \qquad \text{(E.22)}$$

where $f_j = \varphi_j/(1 + i\varphi_j)$ is the friction derivative defined in equation E.1. The second equality uses $\omega_{kj}/(1 + i\varphi_j)^2 = \tilde{\omega}_{kj} \times \varphi_j/(1 + i\varphi_j)$. In matrix form:

$$\frac{d\tilde{\mathbf{\Omega}}}{di} \;=\; -\tilde{\mathbf{\Omega}}\,\mathrm{diag}(\mathbf{f}). \qquad \text{(E.23)}$$

Importantly, this derivative involves f_j, not $g_j = \varphi_j/(1+i\varphi_j)^2$, such that the ratio $-d\tilde{\omega}_{kj}/di = \tilde{\omega}_{kj} f_j$, and not $\tilde{\omega}_{kj} g_j \times (1+i\varphi_j)$. The factor $(1+i\varphi_j)$ already appears in the denominator of $\tilde{\omega}_{kj}$, leaving f_j as the correct multiplier.

E.5 Leontief inverses and derivatives

Definitions

The cost-based Leontief inverse is defined as

$$\mathbf{H} \;\equiv\; (\mathbf{I}-\mathbf{\Omega}^\top)^{-1} \;=\; \sum_{s=0}^{\infty} (\mathbf{\Omega}^\top)^s \;=\; \mathbf{I} + \mathbf{\Omega}^\top + (\mathbf{\Omega}^\top)^2 + \cdots \tag{E.24}$$

The element h_{kj} counts the total intermediate cost importance of sector j in the production of sector k: At order s, the term $(\mathbf{\Omega}^\top)^s_{kj}$ accounts for paths of exactly s input-output links from k to j upstream. Since $\mathbf{\Omega}$ has row = supplier and column = customer, $\mathbf{\Omega}^\top$ has row = customer and the matrix product chains cost flows from customers back to their suppliers.

The revenue-based Leontief inverse is

$$\mathbf{Z} \;\equiv\; (\mathbf{I}-\tilde{\mathbf{\Omega}})^{-1} \;=\; \mathbf{I} + \tilde{\mathbf{\Omega}} + \tilde{\mathbf{\Omega}}^2 + \cdots \tag{E.25}$$

The element z_{kj} captures the propagation of demand shocks from customer j forward to upstream supplier k through revenue-share linkages. Here $\tilde{\mathbf{\Omega}}$ is used without transpose because demand flows from customer upstream to suppliers, and in $\tilde{\mathbf{\Omega}}$ the column j already describes customer j's expenditure shares on its suppliers.

At $i = 0$, $\tilde{\mathbf{\Omega}} = \mathbf{\Omega}$ and the two inverses satisfy $\mathbf{Z}_0 = \mathbf{H}_0^\top$ up to the row/column convention, while they diverge for $i > 0$. Moreover, the Cobb–Douglas accounting identity $\alpha_j + \sum_k \omega_{kj} = 1$ implies

$$\boldsymbol{\alpha} = (\mathbf{I}-\mathbf{\Omega}^\top)\mathbf{1}, \qquad \mathbf{H}\boldsymbol{\alpha} = \mathbf{1}. \tag{E.26}$$

Derivative of Z with respect to i

We apply the matrix inverse derivative identity, stating that if $\mathbf{A}(i)$ is invertible and differentiable in i, then $d(\mathbf{A}^{-1})/di = -\mathbf{A}^{-1}(d\mathbf{A}/di)\mathbf{A}^{-1}$, with $\mathbf{A} = \mathbf{I}-\tilde{\mathbf{\Omega}}$ such that

$$\frac{d\mathbf{Z}}{di} \;=\; -\mathbf{Z}\,\frac{d(\mathbf{I}-\tilde{\mathbf{\Omega}})}{di}\,\mathbf{Z} \;=\; \mathbf{Z}\,\frac{d\tilde{\mathbf{\Omega}}}{di}\,\mathbf{Z}. \tag{E.27}$$

Substituting equation E.23 yields

$$\frac{d\mathbf{Z}}{di} = -\mathbf{Z}\,\tilde{\boldsymbol{\Omega}}\,\mathrm{diag}(\mathbf{f})\,\mathbf{Z}. \tag{E.28}$$

Derivative of H with respect to i

The cost-share matrix $\boldsymbol{\Omega}$ is a technology parameter and does not depend on i. Thus, matrix $\mathbf{H}$ also does not depend on i directly. However, for completeness and given that it enters through the revenue side with $\sigma \neq 1$), the formula analogous to equation E.28 would be

$$\frac{d\mathbf{H}}{di} = \mathbf{0}, \tag{E.29}$$

since $\boldsymbol{\Omega}$ is fixed. This depicts an important asymmetry between matrices $\mathbf{H}$ and $\mathbf{Z}$: The cost Leontief inverse is invariant to i, whereas the revenue Leontief inverse changes with i as $\tilde{\boldsymbol{\Omega}}$ depends on i through the financing wedge.

E.6 Nominal revenues and the revenue equation

Deriving the revenue equation

Multiplying the goods market clearing condition 14 by p_j gives $p_j y_j = p_j c_j + \sum_k p_j x_{jk}$. Sector j's revenue satisfies

$$r_j = p_j c_j + \sum_{k=1}^{n} \tilde{\omega}_{jk}\, r_k, \tag{E.30}$$

where $\tilde{\omega}_{jk} r_k = p_j x_{jk}$ determines sector j's revenue from selling to customer k and $p_j c_j$ is household expenditure on good j. Stacking across all j in vector form implies $\mathbf{r} = \mathbf{p} \circ \mathbf{c}_h + \tilde{\boldsymbol{\Omega}}\mathbf{r}$, where $\mathbf{p} \circ \mathbf{c}_h$ denotes household expenditure by sector. We further obtain

$$\mathbf{r} = (\mathbf{I} - \tilde{\boldsymbol{\Omega}})^{-1}(\mathbf{p} \circ \mathbf{c}_h) = \mathbf{Z}(\mathbf{p} \circ \mathbf{c}_h). \tag{E.31}$$

Substituting the household demand system E.11 with $p^c = 1$ gives

$$\mathbf{r} = \mathbf{Z}\, e\big(\boldsymbol{\nu} \circ \mathbf{p}^{1-\sigma}\big). \tag{E.32}$$

Simplification at $\sigma = 1$

With $\sigma = 1$, $p_j^{1-\sigma} = p_j^0 = 1$ for all j, such that

$$\mathbf{r} \;=\; e\,\mathbf{Z}\boldsymbol{\nu} \;=\; e\,\boldsymbol{\rho}, \qquad \boldsymbol{\rho} \;\equiv\; \mathbf{Z}\boldsymbol{\nu}. \tag{E.33}$$

Consequently, prices cancel from the revenue equation entirely, yielding the key property for making the Cobb–Douglas case with $\sigma = 1$ analytically tractable: Revenues are proportional to $\boldsymbol{\rho}$, which depends on the interest rate i only through $\mathbf{Z}$ and not through prices directly, while prices depend on revenues through equation E.20. Consequently, the system becomes block-triangular, and we solve for $\boldsymbol{\rho}$ and hence revenues before substituting into the price equation.

The case of $\sigma \neq 1$

For $\sigma \neq 1$, differentiating equation E.32 gives

$$\hat{r}_j \;=\; \hat{e} + (1-\sigma)\sum_k \zeta_{jk}\hat{p}_k - \beta_j^{(\sigma)}, \tag{E.34}$$

where ζ_{jk} are revenue-weighted price elasticities and $\beta_j^{(\sigma)}$ captures a modified customer-friction exposure. The term $(1-\sigma)\sum_k \zeta_{jk}\hat{p}_k$ reintroduces $\hat{\mathbf{p}}$ on the right-hand side of the revenue equation, creating a genuine simultaneous system without a closed-form solution for general values of σ. Alternatively, the CES equilibrium must be solved numerically, as carried out in the simulations shown in appendix F.

E.7 Revenue response to interest rate changes with $\sigma = 1$

Differentiating $\boldsymbol{\rho}$

Taking logs of equation E.33 gives $\log r_j = \log e + \log \rho_j$, and differentiating with respect to i yields

$$\hat{r}_j \;=\; \hat{e} + \frac{1}{\rho_j}\frac{d\rho_j}{di}. \tag{E.35}$$

We compute $d\boldsymbol{\rho}/di = d(\mathbf{Z}\boldsymbol{\nu})/di$ using equation E.28 to obtain

$$\frac{d\boldsymbol{\rho}}{di} \;=\; \frac{d\mathbf{Z}}{di}\boldsymbol{\nu} \;=\; -\mathbf{Z}\,\tilde{\boldsymbol{\Omega}}\,\mathrm{diag}(\mathbf{f})\,\mathbf{Z}\boldsymbol{\nu} \;=\; -\mathbf{Z}\,\tilde{\boldsymbol{\Omega}}\,\mathrm{diag}(\mathbf{f})\,\boldsymbol{\rho}. \tag{E.36}$$

Customer-friction exposure

We define the *customer-friction exposure* of sector j as

$$\beta_j \;\equiv\; -\frac{1}{\rho_j}\frac{d\rho_j}{di} \;=\; \frac{\big[\mathbf{Z}\,\tilde{\boldsymbol{\Omega}}\,\mathrm{diag}(\mathbf{f})\,\boldsymbol{\rho}\big]_j}{\rho_j} \;\geq\; 0. \tag{E.37}$$

Writing out the numerator of the last term in equation E.37 explicitly gives

$$[\mathbf{Z}\,\tilde{\boldsymbol{\Omega}}\,\mathrm{diag}(\mathbf{f})\,\boldsymbol{\rho}]_j \;=\; \sum_l z_{jl} \sum_m \tilde{\omega}_{lm}\, f_m\, \rho_m. \tag{E.38}$$

The inner sum $\sum_m \tilde{\omega}_{lm} f_m \rho_m$ measures how quickly sector l's revenue share is contracting due to its customers' financing costs: The term $f_m\rho_m$ defines the rate at which ρ_m falls, weighted by how much of sector l's revenues come from customer m. The outer Leontief inverse $\mathbf{Z}$ propagates this contraction upstream to sector j through all demand chains of arbitrary length. Dividing by ρ_j, β_j is the proportional rate at which sector j's revenue contracts when i rises by one unit. It equals zero if all of j's customers (and their customers, etc.) have $\varphi_m = 0$.

Substituting back into equation E.35 gives

$$\hat{r}_j \;=\; \hat{e} - \beta_j. \tag{E.39}$$

Each sector's revenue semi-elasticity has two components: A common income effect $\hat{e}$ shared by all sectors, and a sector-specific demand contraction $-\beta_j$ driven by downstream financial frictions.

E.8 The income semi-elasticity $\hat{e}$

Differentiating the price system

Differentiating equation E.20 with respect to i, and noting that c_j is invariant to i:

$$\hat{p}_k \;=\; \sum_{j=1}^{n} h_{kj}\big[(1-\alpha_j)\, f_j + \alpha_j\, \hat{r}_j\big]. \tag{E.40}$$

Substituting $\hat{r}_j = \hat{e} - \beta_j$ from equation E.39 gives

$$\hat{\mathbf{p}} \;=\; \mathbf{H}\big[(\mathbf{I}-\mathbf{D}_\alpha)\mathbf{f} - \mathbf{D}_\alpha\boldsymbol{\beta}\big] + \hat{e}\,\mathbf{1}, \tag{E.41}$$

The numéraire condition

Since $p^c = 1$ at all times, differentiating the CES price index with $\sigma = 1$ gives $\log p^c = \boldsymbol{\nu}^\top \mathbf{p} = \text{const}$, hence $\boldsymbol{\nu}^\top \hat{\mathbf{p}} = 0$.

Solving for $\hat{e}$

Premultiplying equation E.41 by $\boldsymbol{\nu}^\top$ and imposing $\boldsymbol{\nu}^\top \hat{\mathbf{p}} = 0$ gives

$$0 \;=\; \boldsymbol{\nu}^\top \mathbf{H}\big[(\mathbf{I} - \mathbf{D}_\alpha)\mathbf{f} - \mathbf{D}_\alpha \boldsymbol{\beta}\big] + \hat{e}\,\boldsymbol{\nu}^\top \mathbf{1}. \tag{E.42}$$

Using $\boldsymbol{\nu}^\top \mathbf{1} = 1$, solving for $\hat{e}$ yields equation 23 of the main text:

$$\hat{e} \;=\; \boldsymbol{\nu}^\top \mathbf{H}\big[\mathbf{D}_\alpha \boldsymbol{\beta} - (\mathbf{I} - \mathbf{D}_\alpha)\mathbf{f}\big]. \tag{E.43}$$

Defining $\boldsymbol{\lambda} \equiv \mathbf{H}^\top \boldsymbol{\nu}$ as the vector of Domar weights, which measures each sector's total importance in final demand through all supply chains, implies

$$\hat{e} \;=\; \sum_j \lambda_j \big[\alpha_j \beta_j - (1 - \alpha_j) f_j\big], \qquad \sum_j \lambda_j \alpha_j = 1. \tag{E.44}$$

This expression is the Domar-weighted net effect of monetary tightening on the aggregate price level. The term $(1 - \alpha_j) f_j$ is the positive cost-push component in sector j's price equation, while $\alpha_j \beta_j$ captures the offsetting demand-contraction component from constrained customers' lower revenues and hence prices. Since the numéraire pins $p^c = 1$, $\hat{e}$ must adjust to balance these two forces in the aggregate. If cost pressures dominate, $\hat{e} < 0$ and aggregate real income falls, dampening demand and stabilizing prices. Crucially, $\hat{e}$ depends on the economy-wide distribution of frictions $\{\varphi_j, \beta_j, \alpha_j\}$ through the Domar weights and carries no sector-specific information about sector k's own suppliers or customers.

E.9 The price-interest rate elasticity decomposition

In the following, we provide the derivations leading to the decomposition 24 in the main text.

For a single sector k, we get from equation E.41

$$\hat{p}_k = \sum_{j=1}^{n} h_{kj}(1-\alpha_j)f_j - \sum_{j=1}^{n} h_{kj}\alpha_j\beta_j + \hat{e}. \tag{E.45}$$

We apply the identity $\mathbf{H} = \mathbf{I} + (\mathbf{H} - \mathbf{I})$ to each of the two sums. For any vector $\mathbf{x}$:, this yields

$$\sum_j h_{kj}x_j = x_k + \sum_j (h_{kj} - \mathbf{1}_{j=k})\, x_j, \tag{E.46}$$

where the first term isolates the own-sector ($j = k$) contribution and the second collects all network ($j \neq k$) effects through paths of length ≥ 1. This is valid because $\mathbf{H} - \mathbf{I} = \sum_{s=1}^{\infty} (\mathbf{\Omega}^\top)^s \geq 0$ element-wise, with the diagonal entries $h_{kk} - 1 \geq 0$.

Applying to the cost sum implies

$$\sum_j h_{kj}(1-\alpha_j)f_j = (1-\alpha_k)f_k + \sum_j (h_{kj} - \mathbf{1}_{j=k})(1-\alpha_j)f_j, \tag{E.47}$$

and applying to the demand sum gives

$$\sum_j h_{kj}\alpha_j\beta_j = \alpha_k\beta_k + \sum_j (h_{kj} - \mathbf{1}_{j=k})\alpha_j\beta_j. \tag{E.48}$$

Substituting equations E.47 and E.48 into equation E.45, and writing $f_j = \varphi_j/(1 + i\varphi_j)$ explicitly yields

$$\begin{aligned} \frac{d\log p_k}{di} &= \underbrace{(1-\alpha_k)\,\frac{\varphi_k}{1+i\varphi_k}}_{\text{(i) Direct cost}} + \underbrace{\sum_j (h_{kj} - \mathbf{1}_{j=k})(1-\alpha_j)\,\frac{\varphi_j}{1+i\varphi_j}}_{\text{(ii) Supplier cost}} \\ &\quad - \underbrace{\alpha_k\,\beta_k}_{\substack{\text{(iii) Own}\\ \text{customers}}} - \underbrace{\sum_j (h_{kj} - \mathbf{1}_{j=k})\alpha_j\,\beta_j}_{\substack{\text{(iv) Suppliers'}\\ \text{customers}}} + \underbrace{\hat{e}}_{\text{(v) Income}} \end{aligned} \tag{E.49}$$

with β_j and $\hat{e}$ given by equations E.37 and E.43. This is equation 24 in the main text.

E.10 Price response to productivity shocks

The decomposition above characterizes the reaction of sectoral prices to an interest-rate shock. It is useful to contrast this with the reaction to a sectoral productivity shock. Let $\mathbf{a}^{\log} \equiv (\log a_1, \ldots, \log a_n)^\top$ denote the vector of log productivities entering the non-financial marginal cost in equation E.9. Keeping the dependence on productivity explicit, the log-price system can be written as

$$\mathbf{p} \;=\; \mathbf{H}\Big[(\mathbf{I} - \mathbf{D}_\alpha)\mathbf{v} + \mathbf{D}_\alpha \mathbf{r} - \mathbf{a}^{\log} + \mathbf{c}_0\Big], \tag{E.50}$$

where $\mathbf{c}_0$ collects terms that are independent of prices, revenues, interest rates, and productivity. Consider a small productivity change $d\mathbf{a}^{\log}$, holding i, $\boldsymbol{\varphi}$, $\boldsymbol{\Omega}$, and final expenditure weights fixed. Differentiating equation E.50 gives

$$d\mathbf{p} \;=\; -\mathbf{H}\, d\mathbf{a}^{\log} + \mathbf{H}\mathbf{D}_\alpha\, d\mathbf{r}. \tag{E.51}$$

At $\sigma = 1$, the revenue equation is $r_j = e\rho_j$, with $\boldsymbol{\rho} = \mathbf{Z}\boldsymbol{\nu}$. Since $\mathbf{Z}$ depends on the revenue-share matrix and the financing wedges, but not directly on productivity, $\boldsymbol{\rho}$ is unchanged by $d\mathbf{a}^{\log}$ when i and $\boldsymbol{\Omega}$ are held fixed. Therefore

$$d\mathbf{r} = d\log e\, \mathbf{1}. \tag{E.52}$$

Using $\mathbf{H}\boldsymbol{\alpha} = \mathbf{1}$, equation E.51 becomes

$$d\mathbf{p} \;=\; -\mathbf{H}\, d\mathbf{a}^{\log} + d\log e\, \mathbf{1}. \tag{E.53}$$

The numéraire condition $p^c = 1$ implies, at $\sigma = 1$, $\boldsymbol{\nu}^\top d\mathbf{p} = 0$. Applying this condition to equation E.53 yields

$$d\log e \;=\; \boldsymbol{\nu}^\top \mathbf{H}\, d\mathbf{a}^{\log} \;=\; \boldsymbol{\lambda}^\top d\mathbf{a}^{\log}, \qquad \boldsymbol{\lambda} \equiv \mathbf{H}^\top \boldsymbol{\nu}. \tag{E.54}$$

Thus the full equilibrium price response to an arbitrary vector of productivity shocks is

$$d\mathbf{p} \;=\; -\mathbf{H}\, d\mathbf{a}^{\log} + \mathbf{1}\, \boldsymbol{\lambda}^\top d\mathbf{a}^{\log}. \tag{E.55}$$

For a productivity shock only in sector k, this gives

$$\frac{\partial \log p_j}{\partial \log a_k} \;=\; -h_{jk} + \lambda_k. \tag{E.56}$$

The first term, $-h_{jk}$, is the cost-network effect: higher productivity in sector k lowers the unit cost of every sector j that uses k, directly or indirectly, as an input.

The strength of this pass-through is governed by the same cost Leontief inverse that appears in the upstream cost channel of the interest-rate decomposition. The second term, λ_k, is common across all destination sectors j for a given shocked sector k. It is not a technological pass-through term; it is the adjustment in nominal expenditure required to keep the consumption price index fixed at the numéraire $p^c = 1$. Without imposing this numéraire adjustment, or equivalently holding nominal expenditure fixed, the partial cost effect would simply be

$$\left.\frac{\partial \log p_j}{\partial \log a_k}\right|_{d \log e = 0} = -h_{jk}. \tag{E.57}$$

This highlights the difference between productivity and interest-rate shocks. An interest-rate shock changes sectoral prices through the financing wedge f_j, the induced change in revenue shares and downstream customer demand summarized by β_j, and the common income term $\hat{e}$. A productivity shock, by contrast, does not directly affect the financing wedge or the revenue-share matrix at $\sigma = 1$. Its relative-price effect is therefore a pure cost-network propagation, up to the common numéraire adjustment. A sector with a high Domar weight λ_k has a large aggregate effect on the consumption price index; after imposing $p^c = 1$, sectors with weak input dependence on k may therefore experience a positive nominal price response through the common λ_k term even though the underlying cost effect $-h_{jk}$ is non-positive.

E.11 Proof of first-order proxy mappings

In the following, we provide proofs for the mappings presented in section 4.4. We derive each approximation step by step as $i \to 0$.

Step 1: Cost channels at $i = 0$

At $i = 0$, the friction derivative collapses to $f_j|_{i=0} = \varphi_j/(1+0) = \varphi_j$. Substituting directly into terms (i) and (ii) of equation E.49 yields

$$\text{(i)}: \quad (1-\alpha_k)f_k \;\to\; (1-\alpha_k)\varphi_k, \tag{E.58}$$

$$\text{(ii)}: \quad \sum_j (h_{kj} - \mathbf{1}_{j=k})(1-\alpha_j)f_j \;\to\; \sum_j (h_{kj} - \mathbf{1}_{j=k})(1-\alpha_j)\varphi_j \tag{E.59}$$

without need for further approximation for the cost channels.

Step 2: Customer channels at $i = 0$

At $i = 0$, $\tilde{\boldsymbol{\Omega}} = \boldsymbol{\Omega}$, such that the revenue-based Leontief inverse reduces to $\mathbf{Z}|_{i=0} = \mathbf{Z}_0 = (\mathbf{I} - \boldsymbol{\Omega})^{-1}$ and $f_j = \varphi_j$. Substituting this into equation E.37 yields

$$\beta_j\big|_{i=0} = \frac{[\mathbf{Z}_0\,\boldsymbol{\Omega}\,\mathrm{diag}(\boldsymbol{\varphi})\,\boldsymbol{\rho}_0]_j}{\rho_{0,j}}, \qquad \boldsymbol{\rho}_0 = \mathbf{Z}_0\boldsymbol{\nu}. \tag{E.60}$$

We use the Leontief inverse identity $\mathbf{Z}_0\boldsymbol{\Omega} = \mathbf{Z}_0 - \mathbf{I}$, which follows from $(\mathbf{I}-\boldsymbol{\Omega})\mathbf{Z}_0 = \mathbf{I}$ rearranged as $\mathbf{Z}_0\boldsymbol{\Omega} = \mathbf{Z}_0(\mathbf{I} - (\mathbf{I} - \boldsymbol{\Omega})) = \mathbf{Z}_0 - \mathbf{I}$ and substitute such that

$$[\mathbf{Z}_0\boldsymbol{\Omega}\,\mathrm{diag}(\boldsymbol{\varphi})\boldsymbol{\rho}_0]_j = [(\mathbf{Z}_0 - \mathbf{I})\,\mathrm{diag}(\boldsymbol{\varphi})\boldsymbol{\rho}_0]_j = \sum_m (z_{0,jm} - \mathbf{1}_{j=m})\,\varphi_m\,\rho_{0,m}. \tag{E.61}$$

To obtain the proxy $[(\mathbf{Z}_0 - \mathbf{I})\boldsymbol{\varphi}]_j$, we factor out $\rho_{0,m}$ by using that Domar weights $\rho_{0,m}$ are approximately equal across sectors at $i = 0$, i.e. $\rho_{0,m} \approx \bar{\rho}_0$ for all m, where $\bar{\rho}_0 = \boldsymbol{\nu}^\top\mathbf{Z}_0^\top\mathbf{1}/n$:

$$\beta_j|_{i=0} \approx \frac{\bar{\rho}_0}{\rho_{0,j}}\,[(\mathbf{Z}_0 - \mathbf{I})\boldsymbol{\varphi}]_j \approx [(\mathbf{Z}_0 - \mathbf{I})\boldsymbol{\varphi}]_j = \tilde{\Phi}_j, \tag{E.62}$$

where the second approximation absorbs $\bar{\rho}_0/\rho_{0,j}$ into the estimated coefficient as at leading order, this ratio is approximately constant across sectors and therefore captured by the regression constant or absorbed by the regression coefficient β_3^h.

Substituting $\beta_j \approx \tilde{\Phi}_j$ into the combined customer channels gives

$$-[\mathbf{H}\mathbf{D}_\alpha\boldsymbol{\beta}]_k \approx -[\mathbf{H}\mathbf{D}_\alpha(\mathbf{Z}_0 - \mathbf{I})\boldsymbol{\varphi}]_k. \tag{E.63}$$

Step 3: Income term

The income term is $\hat{e} \times s_t$ because $\mathbf{H}\boldsymbol{\alpha} = \mathbf{1}$. The factor $\hat{e}$ depends on economy-wide averages $\{\varphi_j, \beta_j, \alpha_j\}$ through Domar weights (see equation E.44) and is therefore a scalar common to all sectors in a given period. In the empirical specification, country-time fixed effects θ_{ct} absorb any term of the form $f(t) \times s_t$ that does not vary across sectors. Hence the income channel is fully absorbed in the Cobb–Douglas benchmark.

Step 4: From exact channels to observable proxies

The exact upstream term (ii) is $\sum_j (h_{kj} - \mathbf{1}_{j=k})(1 - \alpha_j)\varphi_j$. The empirical upstream proxy is $\Phi_k = [(\mathbf{H} - \mathbf{I})\boldsymbol{\varphi}]_k = \sum_j (h_{kj} - \mathbf{1}_{j=k})\varphi_j$. The difference is the weight $(1 - \alpha_j)$. The proxy does not account for this weight, thereby introducing an approximation error proportional to the cross-sectional variation in α_j. This approximation is valid when α_j has limited cross-sectional variation, in which case $1 - \alpha_j$ is approximately constant across j and absorbed into the estimated coefficient β_2^h. The approach of including $\alpha_k \times s_t$ as a control in the empirical specification partially corrects for this, as mentioned in the main text.

For the downstream proxy, the exact term is $-[\mathbf{H}\mathbf{D}_\alpha(\mathbf{Z} - \mathbf{I})\boldsymbol{\varphi}]_k$ at $i = 0$. The empirical proxy $\tilde{\Phi}_k = [(\mathbf{Z} - \mathbf{I})\boldsymbol{\varphi}]_k$ uses the full revenue-based Leontief inverse $(\mathbf{Z} - \mathbf{I})$ rather than just one-step linkages. This choice is justified as the customer channel propagates through all demand chains of arbitrary length via matrices $\mathbf{Z}$ and $\mathbf{H}$, and not only stems from direct customers relations alone. The $\mathbf{D}_\alpha$ weight and the outer matrix $\mathbf{H}$ application are absorbed into the coefficient β_3^h under the approximation that α_j is approximately constant.

Sign predictions

The sign predictions follow directly from non-negativity: All entries of matrices $\mathbf{H} - \mathbf{I}$ and $\mathbf{Z} - \mathbf{I}$ are non-negative, as Leontief inverses have only non-negative entries and diagonal entries exceeding one, and all $\varphi_j \geq 0$ and $\alpha_j \in [0, 1]$. Therefore:

$$
\begin{aligned}
(1 - \alpha_k)\varphi_k \geq 0 \quad &\Rightarrow \quad \beta_1^h > 0 \text{ (direct)}, \\
\Phi_k = [(\mathbf{H} - \mathbf{I})\boldsymbol{\varphi}]_k \geq 0 \quad &\Rightarrow \quad \beta_2^h > 0 \text{ (upstream)}, \\
-[\mathbf{H}\mathbf{D}_\alpha \tilde{\Phi}]_k \leq 0 \quad &\Rightarrow \quad \beta_3^h < 0 \text{ (downstream)}.
\end{aligned}
$$

E.12 Regularity Conditions

The derivations above require the following conditions to hold:

1. **Cobb–Douglas technology.** Constant returns-to-scale Cobb–Douglas production with fixed cost-share parameters $\{\alpha_j, \omega_{kj}\}$ satisfying $\alpha_j + \sum_k \omega_{kj} = 1$ for all j. This ensures the wage identity E.12 holds exactly and that cost shares equal technology exponents.

2. **Multiplicative working-capital friction.** The friction $(1 + i\varphi_j)$ is multiplicative and applies to all variable inputs, with $\varphi_j \geq 0$ and $i \geq 0$. This ensures the financing wedge enters the pricing equation in a log-linear way as required for the decomposition.

3. **Invertibility of Leontief matrices.** Both $(\mathbf{I} - \boldsymbol{\Omega}^\top)$ and $(\mathbf{I} - \tilde{\boldsymbol{\Omega}})$ must be invertible. A sufficient condition is that the spectral radii of $\boldsymbol{\Omega}^\top$ and $\tilde{\boldsymbol{\Omega}}$ are both strictly less than one, which is the standard input-output productivity condition ensuring that finite amounts of primary inputs produce finite outputs. Since $\tilde{\boldsymbol{\Omega}} = \boldsymbol{\Omega}\,\mathrm{diag}((1 + i\varphi_j)^{-1}) \leq \boldsymbol{\Omega}$ element-wise, if $\boldsymbol{\Omega}$ satisfies the condition, then so does $\tilde{\boldsymbol{\Omega}}$ for all $i \geq 0$.

4. **Interior equilibrium.** All equilibrium prices, wages, and revenues are strictly positive. This ensures that log-linearization around equilibrium is valid and that the Leontief expansion converges.

5. **First-order approximation accuracy.** The limits $f_j \to \varphi_j$ and $\tilde{\boldsymbol{\Omega}} \to \boldsymbol{\Omega}$ as $i \to 0$ generate approximation errors of order $O(i)$. Additional proxy errors arise from replacing weighted network objects such as $[\mathbf{H}(\mathbf{I} - \mathbf{D}_\alpha)\boldsymbol{\varphi}]_k$ and $[\mathbf{H}\mathbf{D}_\alpha(\mathbf{Z} - \mathbf{I})\boldsymbol{\varphi}]_k$ with the observable proxies φ_k, Φ_k, and $\tilde{\Phi}_k$. These errors are small when labor shares and Domar weights do not vary too strongly across sectors, as assessed in the simulation evidence in appendix F.

F Simulation-based validation of empirical proxies

This appendix reports the full set of results from the simulation exercise sketched in section 4.5. We simulate $R = 500$ variants of the heterogeneous CES economies prescribed by the model in section 4, with $n = 70$ sectors each. We solve the model equilibrium, and subsequently estimate two regressions, one reflecting the full decomposition in equation 24, and a second one based on the proxy regression 28 mapping the theoretical proxies to the empirical regression model 8.

F.1 Simulation setup

The simulation draws $R = 500$ random economies, each with $n = 70$ sectors. For each economy, we proceed along the following steps.

1. Parameterization

The labor share α_j is drawn uniformly from $[0.3, 0.9]$ for each sector j. The input-output cost-share matrix ω_{kj} (share of seller k in buyer j's costs) is random and row-stochastic: columns sum to $(1 - \alpha_j)$ so that labor plus intermediate inputs equal unity. Friction parameters φ_j are drawn uniformly from $[0, 1]$ to represent the range of sectoral leverage observed in data. Exogenous final demand ν is drawn from a uniform distribution on the simplex (after normalization).

2. Equilibrium solution: CES at baseline policy rate

For a baseline policy rate $i_0 = 5\%$, we solve the nonlinear CES equilibrium system via `fsolve` (MATLAB) using a numerical tolerance of 10^{-12} on both the function residual and the change in iterates. The initial guess for `fsolve` comes from the Cobb-Douglas closed-form solution at i_0, which is highly accurate and reduces the iteration count substantially. We adopt the CPI numeraire $\nu^\top \log \mathbf{p} = 0$ to ensure comparability across all elasticities σ.

3. Price derivative via finite difference

To compute the price response to a small interest rate increase $di = 10^{-6}$, we solve the equilibrium again at $i_1 = i_0 + di$ and compute

$$\frac{d \log p_k}{di} \approx \frac{\log p_k(i_1) - \log p_k(i_0)}{di}. \tag{F.1}$$

This finite-difference derivative is used as the dependent variable in Regression 1.

4. Decomposition channels

Using the equilibrium at i_0, we compute the decomposition channels (direct, supplier, own-customers, suppliers' customers, income) as prescribed by equation 24. These channels are evaluated at the Cobb-Douglas closed-form solution (Step 1), which provides the Leontief inverses $\mathbf{H}$ and $\mathbf{Z}$ consistent with the CPI numeraire.

5. Shock distribution and multiple observations per economy

To generate cross-sectional variation comparable to the time-series variation of monetary shocks in the empirical panel, we draw $n_{\text{shocks}} = 10$ monetary policy shocks per economy from a normal distribution $s_i \sim \mathcal{N}(0.0025, 0.0015^2)$, representing mean 25bp and standard deviation 15bp. Shocks are floored at 1bp to avoid negligible perturbations. Each shock is added to the baseline rate: $i_s = i_0 + s_i$. We solve the equilibrium at i_s, compute the price change $\Delta \log p_k = \log p_k(i_s) - \log p_k(i_0)$, and derive the regressors $(s_i \varphi_k, s_i[\mathbf{H}\boldsymbol{\varphi}]_k, s_i[(\mathbf{Z} - \mathbf{I})\boldsymbol{\varphi}]_k)$ mirroring the empirical design.

This yields approximately $500 \times 10 = 5000$ observations for Regression 2. Observations with negligible price responses ($|\Delta \log p| > 1\%$) or non-finite values are dropped as numerical failures.

Summary

The procedure yields two regression datasets: Regression 1 has ≈ 500 observations, one per economy, the finite-difference derivative and five channels. Regression 2 has ≈ 5000 observations (ten shocks per surviving economy, the price change and three proxy regressors).

F.2 Regression 1: Channel decomposition at $\sigma = 1$

We first validate that the decomposition from equation 24 accurately tracks CES prices:

$$\Delta \log p_k = \beta_1 \,\mathrm{Direct}_k + \beta_2 \,\mathrm{Supplier}_k + \beta_3 \,\text{Own-cust}_k + \beta_4 \,\text{Supp.-cust}_k + \beta_5 \,\mathrm{Income}_k + \varepsilon_k. \tag{F.2}$$

The β-coefficients measure how well each model channel predicts the equilibrium price when channels are exact (at $\sigma = 1$). Given the model structure, all coefficients should equal one by construction, with deviations indicating either numerical errors or CES-specific effects prevalent even with $\sigma = 1$.

Results: As shown in table F.1, coefficients obtained from regression F.2 are indeed very close to one, with a high $R^2 = 0.9997$. These results confirm that the decomposition of equation 24 exactly captures equilibrium price responses in the simulation, with minor deviations representing numerical rounding.

F.3 Regression 2: Empirical proxy regression

We now estimate the reduced-form proxy regression that mirrors the empirical specification in equation 8:

$$\Delta \log p_k = \gamma_1 \left(s_t \varphi_k\right) + \gamma_2 \left(s_t [\mathbf{H}\boldsymbol{\varphi}]_k\right) + \gamma_3 \left(s_t [(\mathbf{Z} - \mathbf{I})\boldsymbol{\varphi}]_k\right) + \varepsilon_k. \tag{F.3}$$

This regression tests whether the empirical proxy system in section 4.4 recovers the model's structural parameters in finite samples with observationally relevant noise and collapsing dimensions.

The predicted signs from theory are: $\gamma_1 > 0$ (direct cost), $\gamma_2 > 0$ (upstream supplier cost), $\gamma_3 < 0$ (downstream customer demand). The magnitude of R^2 reflects how much heterogeneity in prices is explained by the three proxy channels.

Results: As shown in table F.1, the predicted signs are recovered in the results for regression model F.3, with $\hat{\gamma}_1 = 1.610 > 0$, $\hat{\gamma}_2 = 0.912 > 0$, and $\hat{\gamma}_3 = -2.163 < 0$. The $R^2 = 0.5128$ indicates that these three channels explain approximately half of cross-sectional price variation, a strong result for a three-parameter parsimonious design. The fitted scatter plot (right panel of figure F.1) shows tight correspondence between predicted and actual prices.

Table F.1: Exact model vs. simulation-based regression results

	Regression 1: Channel decomposition	Regression 2: Empirical proxies
Direct cost $(+)$	1.016	
Supplier cost $(+)$	1.014	
Own customers $(-)$	0.997	
Suppliers' customer $(-)$	1.015	
Income	1.300	
$i \cdot \varphi_k$ (direct, $+$)		0.339
$i \cdot [(\mathbf{H} - \mathbf{I})\boldsymbol{\varphi}]_k$ (upstream, $+$)		0.879
$i \cdot [(\mathbf{Z} - \mathbf{I})\boldsymbol{\varphi}]_k$ (downstream, $-$)		-2.086
R^2	0.999	0.482

Notes: Coefficients from decomposition regression on exact model channels (Regression 1), and simulation-based regression (Regression 2), following equation 28.

F.4 Channel sign analysis

To assess whether the model's sign predictions hold uniformly or only on average, we compute the share of observations where each channel has the predicted sign.

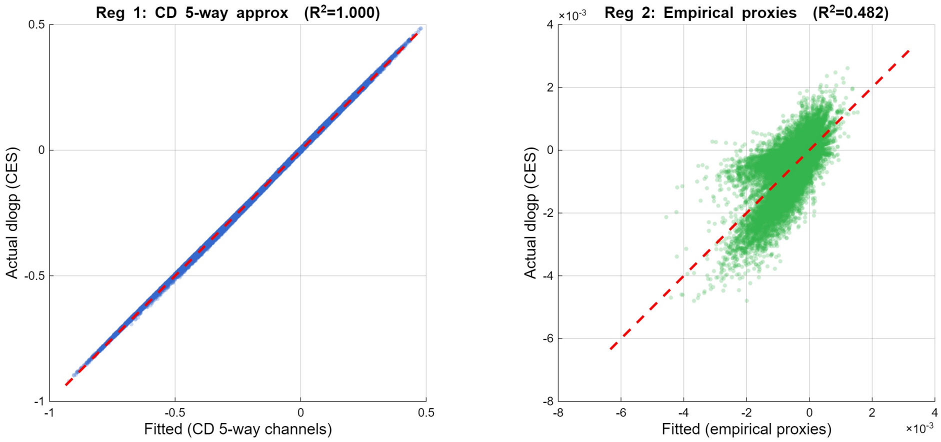
Interpretation: As reported in table F.2, the four non-ambiguous channels in equation 24 (direct, supplier, own-customers, suppliers' customers) display perfect sign recovery. The income channel shows mixed signs as it depends on the balance of cost-push and demand-pullback forces, which varies across sectors. In the empirical estimation, time fixed effects absorb this aggregate income term, leaving it unidentified.

F.5 Channel contribution diagnostics

This subsection discusses the relative quantitative contribution of each channel to explaining price variation. Figure F.2 and table F.3 summarize the channel-contribution decomposition used to benchmark the robustness exercises.

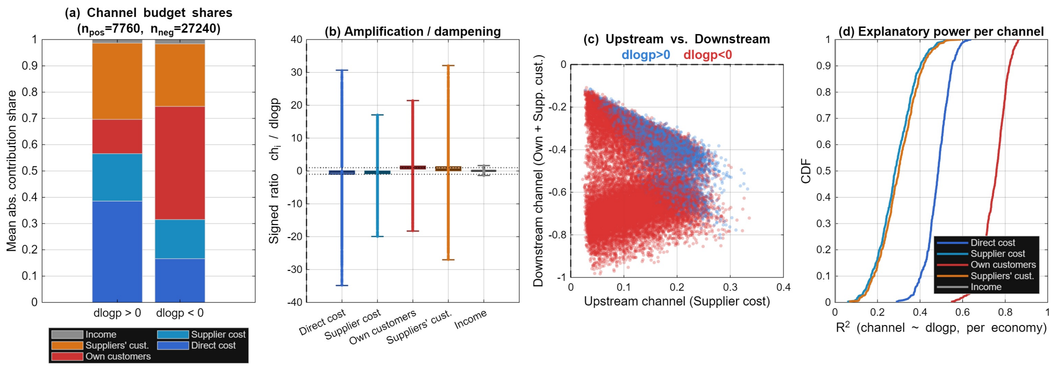
Key findings:

Figure F.1: Predicted versus simulated prices (Regressions 1 and 2)



Notes: Fitted model from decomposition regression on exact model channels (Regression 1, left panel), and simulation-based regression (Regression 2, right panel), following equation 28.

Figure F.2: Channel contributions: distribution and composition



Notes: Left panel: histogram of contributions by channel. Middle and right panels: stacked contributions showing which channels dominate price responses across the distribution.

1. **Symmetric cost-demand split:** Direct and supplier channels contribute approximately equally to price variation, as do own-customer and suppliers' customer channels. This reflects the structural balance between cost-push and demand-pullback forces.

2. **Income channel smaller but structural:** The income effect contributes less on average but plays a material role in reconciling the aggregate constraint across sectors. Its covariance share shows the expected negative correlation with other channels as when income rises, other channels contract to satisfy the numéraire condition.

Table F.2: Sign analysis by decomposition channel

Channel	Expected sign	% correct	% positive	Mean
(i) Direct cost	$+$	100.0	100.0	0.195
(ii) Supplier cost	$+$	100.0	100.0	0.130
(iii) Own customers	$-$	100.0	0.0	-0.312
(iv) Suppliers' cust.	$-$	100.0	0.0	-0.209
(v) Income	n/a	n/a	0.0	-0.012

Notes: Share of sector-shock observations with correctly model-predicted signs (third column), unconditional share of positive sign (fourth column), mean channel contribution (fifth column).

3. **Negative covariance shares:** Covariance shares can be negative when a channel offsets price variation in other channels. This reflects the structural interdependence of channels in general equilibrium.

4. **Consistency with empirical variance decomposition:** As shown in table F.3, downstream customer channels account for the largest share of price variations when considering a fully model-consistent decomposition of the price-interest rate elasticity.

F.6 Robustness: Sign patterns across elasticity values

To check that our results are not driven by the choice $\sigma = 1$, we verify that sign patterns remain stable as σ varies. We re-run the full simulation exercise and regressions 1 and 2 for elasticity values spanning $\sigma \in \{0.5, 0.75, 1.0, 1.25, 1.5, 2.0\}$ and report the coefficient estimates and model fit statistics.

Key findings

Table F.4 shows the decomposition channel coefficients (Regression 1) across elasticity values. The regression coefficients remain tightly clustered around their theoretical values (direct/supplier/supp.-cust. near $+1$, own-customers near -1) for all σ values, with $R^2 > 0.99$ throughout. This confirms that decomposition 24 is appropriate for actual CES price responses even when elasticities deviate substantially from unity.

Table F.3: Channel decomposition

Channel	Mean contribution	Mean |contribution| (%)	Covariance share (%)
(i) Direct cost	0.198	74.4	38.4
(ii) Supplier cost	0.132	49.7	11.3
(iii) Own customers	-0.310	116.8	68.2
(iv) Suppliers' cust.	-0.212	79.7	-18.0
(v) Income	-0.016	5.9	0.1
Total	-0.208	326.4	100.0

Notes: Mean contribution in units of log prices (second column), mean absolute contribution share (explanatory power of each channel in $|p|$, third column), covariance share (share of channel's covariance in total price variance, fourth column). Covariance shares can be negative due to potentially offsetting channels.

Table F.5 reports the four-proxy empirical regression (Regression 2). Importantly, the predicted signs – positive for direct and upstream channels, and negative for downstream proxies – hold uniformly across the entire range of elasticities. While magnitudes vary – e.g., the downstream coefficient ranges from -1.6 to -2.8 as σ increases– , the sign patterns persist across the range of values for σ. The R^2 remains in the range 0.45–0.55 for all values, indicating stable explanatory power.

Table F.4: Robustness: Decomposition coefficients across elasticity values

Channel	$\sigma = 0.50$	$\sigma = 0.75$	$\sigma = 1.00$	$\sigma = 1.25$	$\sigma = 1.50$	$\sigma = 2.00$
Direct cost	1.0984	1.0428	1.0000	0.9635	0.9315	0.8828
Supplier cost	1.0851	1.0428	1.0000	0.9406	0.9483	0.9219
Own customers	0.9801	0.9921	1.0000	1.0085	1.0134	1.0249
Suppliers' customers	1.0974	1.0448	1.0000	0.9529	0.9475	0.9251
Income	2.7724	1.7828	1.0000	0.2624	-0.3953	-1.3895
R^2	0.9884	0.9976	1.0000	0.9985	0.9946	0.9839
Obs.	10500	10500	10500	10500	10500	10500

Notes: Regression 1 coefficients (five CD channels) estimated for each elasticity value σ. "Obs." is the number of sector observations.

Table F.5: Robustness: Proxy regression coefficients across elasticity values

Regressor	$\sigma = 0.50$	$\sigma = 0.75$	$\sigma = 1.00$	$\sigma = 1.25$	$\sigma = 1.50$	$\sigma = 2.00$
$s \times \varphi_j$	0.3804	0.3528	0.3454	0.3246	0.2954	0.2879
$s \times (\mathbf{H} - \mathbf{I}) * \varphi_j$	0.9007	0.8752	0.8657	0.8744	0.8453	0.8284
$s \times (\mathbf{Z} - \mathbf{I}) * \varphi_j$	-2.1951	-2.1189	-2.0843	-2.0376	-1.9748	-1.9331
R^2	0.4425	0.4278	0.4077	0.4075	0.3728	0.3635
Obs.	10500	10500	10500	10500	10500	10500

Notes: Regression 2 coefficients (three empirical proxies) estimated for each elasticity value σ. All three predicted signs (direct: $+$, upstream: $+$, downstream: $-$) hold across every elasticity.

G A two-period flexible-price extension

The benchmark model in section 4 is static and uses the consumption price index as the numéraire. This appendix shows that the sectoral price decomposition derived there is not an artifact of this normalization. We extend the environment to two periods while keeping prices fully flexible. The only dynamic force we add is the household Euler equation, which pins down current nominal expenditure relative to future nominal expenditure. As a result, a monetary tightening lowers the current aggregate price level through a common nominal scale effect. At the sector level, however, prices continue to move according to the same direct, upstream, and downstream financial-friction channels, plus this common aggregate component.

G.1 The two-period economy

The economy lasts for two periods, $t = 0, 1$. Monetary policy affects sector j's effective variable input costs through the same working-capital wedge used in the benchmark model:

$$q_{jt} = 1 + i_t \varphi_j, \qquad \varphi_j \geq 0. \tag{G.1}$$

Production is otherwise identical to the static environment. Sector j operates a constant-returns Cobb–Douglas technology,

$$y_{jt} = a_j l_{jt}^{\alpha_j} \prod_k x_{kj,t}^{\omega_{kj}}, \qquad l_{jt} = \bar{l}_j, \tag{G.2}$$

with constant returns to scale,

$$\alpha_j + \sum_k \omega_{kj} = 1. \tag{G.3}$$

As in the main text, ω_{kj} denotes the share of sector j's costs spent on inputs from sector k, so columns of $\mathbf{\Omega}$ correspond to customer sectors. The vector of labor shares therefore satisfies

$$\boldsymbol{\alpha} = (\mathbf{I} - \mathbf{\Omega}')\mathbf{1}. \tag{G.4}$$

The corresponding cost-based Leontief inverse is

$$\mathbf{H} = (\mathbf{I} - \boldsymbol{\Omega}')^{-1}. \tag{G.5}$$

The Cobb–Douglas adding-up condition implies the same identity used throughout the benchmark derivation:

$$\mathbf{H}\boldsymbol{\alpha} = \mathbf{1}. \tag{G.6}$$

Finally, define

$$\mathbf{D}_\alpha = \mathrm{diag}(\boldsymbol{\alpha}). \tag{G.7}$$

G.2 Firm optimality and prices

Let

$$R_{jt} = p_{jt} y_{jt} \tag{G.8}$$

denote nominal revenue in sector j and period t. Cost minimization implies that the financing wedge scales both labor and intermediate-input costs:

$$q_{jt} w_{jt} \bar{l}_j = \alpha_j R_{jt}, \tag{G.9}$$

and

$$q_{jt} p_{kt} x_{kj,t} = \omega_{kj} R_{jt}. \tag{G.10}$$

Equivalently,

$$w_{jt} = \frac{\alpha_j R_{jt}}{q_{jt} \bar{l}_j}, \tag{G.11}$$

and

$$p_{kt} x_{kj,t} = \frac{\omega_{kj} R_{jt}}{q_{jt}}. \tag{G.12}$$

Substituting these factor demands into the unit-cost condition yields a log-price equation that is the dynamic counterpart of equation 20 in the main text:

$$\log p_{jt} = \chi_j + (1 - \alpha_j) \log q_{jt} + \alpha_j \log R_{jt} + \sum_k \omega_{kj} \log p_{kt}, \tag{G.13}$$

where $\chi_j = -\log a_j - \alpha_j \log \bar{l}_j - \alpha_j \log \alpha_j - \sum_k \omega_{kj} \log \omega_{kj}$ collects sector-specific constants. Stacking sectors and inverting the input-output block gives

$$\log \mathbf{p}_t = \mathbf{H}\left[\boldsymbol{\chi} + (\mathbf{I} - \mathbf{D}_\alpha)\log \mathbf{q}_t + \mathbf{D}_\alpha \log \mathbf{R}_t\right]. \tag{G.14}$$

G.3 Revenue system

Households allocate final expenditure across sectoral goods with Cobb–Douglas shares $\boldsymbol{\nu}$:

$$D_{jt} = \nu_j E_t, \qquad E_t = P_t^c C_t, \qquad \sum_j \nu_j = 1. \tag{G.15}$$

Sectoral revenue equals final demand plus sales to all downstream producers,

$$R_{jt} = \nu_j E_t + \sum_m p_{jt} x_{jm,t}. \tag{G.16}$$

Using the input demand of customer sector m,

$$p_{jt} x_{jm,t} = \frac{\omega_{jm} R_{mt}}{q_{mt}}. \tag{G.17}$$

Thus the wedge affects the revenue system by reducing the revenue share paid to intermediate-input suppliers:

$$R_{jt} = \nu_j E_t + \sum_m \omega_{jm} \frac{R_{mt}}{q_{mt}}. \tag{G.18}$$

In vector form,

$$\mathbf{R}_t = E_t \boldsymbol{\nu} + \boldsymbol{\Omega}\mathbf{Q}_t^{-1}\mathbf{R}_t, \tag{G.19}$$

where

$$\mathbf{Q}_t = \mathrm{diag}(q_{1t}, \ldots, q_{Nt}). \tag{G.20}$$

Solving for revenues gives

$$\mathbf{R}_t = E_t \left(\mathbf{I} - \boldsymbol{\Omega}\mathbf{Q}_t^{-1}\right)^{-1} \boldsymbol{\nu}. \tag{G.21}$$

It is useful to collect the dependence of revenue shares on the wedge vector in the vector

$$\mathbf{z}(\mathbf{q}_t) \equiv \left(\mathbf{I} - \boldsymbol{\Omega}\mathbf{Q}_t^{-1}\right)^{-1} \boldsymbol{\nu}. \tag{G.22}$$

Then sectoral revenues can be written as

$$\mathbf{R}_t = E_t \mathbf{z}(\mathbf{q}_t), \tag{G.23}$$

and therefore

$$\log \mathbf{R}_t = \log E_t \, \mathbf{1} + \log \mathbf{z}(\mathbf{q}_t). \tag{G.24}$$

G.4 Nominal scale and relative prices

The central object in this appendix is the separation between aggregate nominal scale and sectoral relative prices. Substituting equation G.24 into the price system G.14 gives

$$\log \mathbf{p}_t = \mathbf{H} \left[\boldsymbol{\chi} + (\mathbf{I} - \mathbf{D}_\alpha) \log \mathbf{q}_t + \mathbf{D}_\alpha \log \mathbf{z}(\mathbf{q}_t) + \boldsymbol{\alpha} \log E_t \right]. \tag{G.25}$$

Using the identity $\mathbf{H}\boldsymbol{\alpha} = \mathbf{1}$, this becomes

$$\log \mathbf{p}_t = \log E_t \, \mathbf{1} + \widetilde{\mathbf{p}}(\mathbf{q}_t), \tag{G.26}$$

where

$$\widetilde{\mathbf{p}}(\mathbf{q}_t) \equiv \mathbf{H} \left[\boldsymbol{\chi} + (\mathbf{I} - \mathbf{D}_\alpha) \log \mathbf{q}_t + \mathbf{D}_\alpha \log \mathbf{z}(\mathbf{q}_t) \right]. \tag{G.27}$$

Equation G.26 is the key link to the benchmark decomposition. Nominal expenditure E_t shifts all sectoral prices one-for-one, while the vector $\widetilde{\mathbf{p}}(\mathbf{q}_t)$ captures the relative-price effects generated by financial wedges and production-network linkages.

G.5 Aggregate consumption

The Cobb-Douglas consumption price index satisfies, up to constants,

$$\log P_t^c = \boldsymbol{\nu}' \log \mathbf{p}_t. \tag{G.28}$$

Using the decomposition in equation G.26,

$$\log P_t^c = \log E_t + \boldsymbol{\nu}' \widetilde{\mathbf{p}}(\mathbf{q}_t). \tag{G.29}$$

Since $E_t = P_t^c C_t$,

$$\log C_t = \log E_t - \log P_t^c. \tag{G.30}$$

Therefore,

$$\log C_t = -\boldsymbol{\nu}' \widetilde{\mathbf{p}}(\mathbf{q}_t). \tag{G.31}$$

Real consumption is therefore pinned down by the relative price distortion induced by $\mathbf{q}_t$. The aggregate nominal scale E_t affects nominal prices and nominal expenditure, but not feasible real consumption directly.

G.6 Household Euler equation

The intertemporal margin is introduced through the household Euler equation,

$$\frac{1}{C_0} = \beta(1+i_0)\frac{P_0^c}{P_1^c}\frac{1}{C_1}. \tag{G.32}$$

Using $P_t^c = E_t/C_t$, this condition can be rewritten as

$$\frac{1}{C_0} = \beta(1+i_0)\frac{E_0/C_0}{E_1/C_1}\frac{1}{C_1}. \tag{G.33}$$

The real-consumption terms cancel. Hence the Euler equation determines the relative nominal expenditure levels across the two periods:

$$1 = \beta(1+i_0)\frac{E_0}{E_1}. \tag{G.34}$$

Hence

$$\frac{E_0}{E_1} = \frac{1}{\beta(1+i_0)}. \tag{G.35}$$

Equivalently,

$$\log E_0 = \log E_1 - \log\beta - \log(1+i_0). \tag{G.36}$$

This is the source of monetary non-neutrality in the two-period flexible-price extension. A higher current interest rate lowers current nominal expenditure relative to future nominal expenditure. Through equation G.26, this generates a common decline in current sectoral prices, while real consumption continues to be governed by the relative-price distortion in equation G.31.

G.7 Exact response to an interest-rate change

Consider a change in the period-0 interest rate from i to i', holding future nominal expenditure E_1 and the future policy rate i_1 fixed. Since

$$q_j(i) = 1 + i\varphi_j, \tag{G.37}$$

the Euler equation implies

$$\Delta \log E_0 = -\log \frac{1+i'}{1+i}. \tag{G.38}$$

Real consumption changes by

$$\Delta \log C_0 = -\boldsymbol{\nu}' \left[\widetilde{\mathbf{p}}(\mathbf{q}(i')) - \widetilde{\mathbf{p}}(\mathbf{q}(i))\right]. \tag{G.39}$$

The consumption price index changes according to the accounting identity $P_0^c = E_0/C_0$:

$$\Delta \log P_0^c = \Delta \log E_0 - \Delta \log C_0. \tag{G.40}$$

Therefore,

$$\Delta \log P_0^c = -\log \frac{1+i'}{1+i} + \boldsymbol{\nu}' \left[\widetilde{\mathbf{p}}(\mathbf{q}(i')) - \widetilde{\mathbf{p}}(\mathbf{q}(i))\right]. \tag{G.41}$$

Similarly, sectoral prices satisfy

$$\Delta \log \mathbf{p}_0 = -\log \frac{1+i'}{1+i}\mathbf{1} + \left[\widetilde{\mathbf{p}}(\mathbf{q}(i')) - \widetilde{\mathbf{p}}(\mathbf{q}(i))\right]. \tag{G.42}$$

Equations G.41 and G.42 show that a monetary tightening has two effects. The first is a common nominal scale effect, which moves all sectoral prices in the same direction. The second is a relative price effect, which is generated by the same network-induced financial wedges that appear in the static decomposition.

G.8 Sectoral quantity responses

The decomposition also clarifies how sectoral quantities adjust. Sectoral revenues satisfy

$$\mathbf{R}_0 = E_0 \mathbf{z}(\mathbf{q}_0). \tag{G.43}$$

Hence

$$\Delta \log \mathbf{R}_0 = \Delta \log E_0 \, \mathbf{1} + \Delta \log \mathbf{z}(\mathbf{q}_0). \tag{G.44}$$

Sectoral household consumption is

$$c_{j0} = \frac{\nu_j E_0}{p_{j0}}. \tag{G.45}$$

The common nominal scale effect cancels between E_0 and p_{j0}, so

$$\Delta \log \mathbf{c}_0 = -\Delta \widetilde{\mathbf{p}}(\mathbf{q}_0). \tag{G.46}$$

Sectoral output is

$$y_{j0} = \frac{R_{j0}}{p_{j0}}. \tag{G.47}$$

Thus its response combines the revenue-share adjustment and the relative price adjustment:

$$\Delta \log \mathbf{y}_0 = \Delta \log \mathbf{z}(\mathbf{q}_0) - \Delta \widetilde{\mathbf{p}}(\mathbf{q}_0). \tag{G.48}$$

Intermediate inputs satisfy

$$x_{kj,0} = \frac{\omega_{kj} R_{j0}}{q_{j0} p_{k0}}, \tag{G.49}$$

so their response reflects the customer's revenue response, the customer's financing wedge, and the supplier's price:

$$\Delta \log x_{kj,0} = \Delta \log R_{j0} - \Delta \log q_{j0} - \Delta \log p_{k0}. \tag{G.50}$$

Finally, wages satisfy

$$w_{j0} = \frac{\alpha_j R_{j0}}{q_{j0} \bar{l}_j}. \tag{G.51}$$

Hence

$$\Delta \log w_{j0} = \Delta \log R_{j0} - \Delta \log q_{j0}. \tag{G.52}$$

G.9 Local response around an arbitrary interest rate

We now differentiate the exact expressions around an arbitrary initial interest rate i. The relevant semi-elasticity of the financing wedge with respect to $\log(1+i)$ is

$$s_j(i) \equiv \frac{\partial \log q_j(i)}{\partial \log(1+i)} = \frac{(1+i)\varphi_j}{1+i\varphi_j}, \tag{G.53}$$

and we collect these terms in

$$\mathbf{s}(i) = (s_1(i), \ldots, s_N(i))'. \tag{G.54}$$

The revenue share objects evaluated at the initial interest rate are

$$\mathbf{Z}(i) = \left(\mathbf{I} - \boldsymbol{\Omega}\mathbf{Q}(i)^{-1}\right)^{-1}, \tag{G.55}$$

$$\mathbf{z}(i) = \mathbf{Z}(i)\boldsymbol{\nu}, \tag{G.56}$$

and

$$\mathbf{B}(i) = \operatorname{diag}(\mathbf{z}(i))^{-1}\mathbf{Z}(i)\boldsymbol{\Omega}\mathbf{Q}(i)^{-1}\operatorname{diag}(\mathbf{z}(i)). \tag{G.57}$$

Differentiating the revenue-allocation vector gives

$$d\log \mathbf{z}(i) = -\mathbf{B}(i)d\log \mathbf{q}(i). \tag{G.58}$$

Combining this derivative with the relative price object in equation G.27 yields

$$d\widetilde{\mathbf{p}}(i) = \mathbf{H}\left[(\mathbf{I} - \mathbf{D}_\alpha) - \mathbf{D}_\alpha\mathbf{B}(i)\right] d\log \mathbf{q}(i). \tag{G.59}$$

Since

$$d\log \mathbf{q}(i) = \mathbf{s}(i)d\log(1+i), \tag{G.60}$$

the relative price response is

$$d\widetilde{\mathbf{p}}(i) = \mathbf{H}\left[(\mathbf{I} - \mathbf{D}_\alpha) - \mathbf{D}_\alpha\mathbf{B}(i)\right]\mathbf{s}(i)d\log(1+i). \tag{G.61}$$

The consumption-weighted relative price response is summarized by

$$\kappa(i) \equiv \boldsymbol{\nu}'\mathbf{H}\left[(\mathbf{I}-\mathbf{D}_{\alpha})-\mathbf{D}_{\alpha}\mathbf{B}(i)\right]\mathbf{s}(i). \tag{G.62}$$

Then aggregate real consumption and the aggregate price index satisfy

$$d\log C_0 = -\kappa(i)d\log(1+i_0), \tag{G.63}$$

and

$$d\log P_0^c = \left[\kappa(i)-1\right]d\log(1+i_0). \tag{G.64}$$

Sectoral prices respond according to

$$d\log \mathbf{p}_0 = -d\log(1+i_0)\mathbf{1} + \mathbf{H}\left[(\mathbf{I}-\mathbf{D}_{\alpha})-\mathbf{D}_{\alpha}\mathbf{B}(i)\right]\mathbf{s}(i)d\log(1+i_0). \tag{G.65}$$

The first term is the common nominal-scale effect implied by the Euler equation. The second term is the sector-specific relative-price component. It has the same economic content as the benchmark decomposition: direct financing costs, upstream cost propagation, and downstream revenue compression.

G.10 The zero-interest case

The comparison with the benchmark model is especially transparent around a zero-interest baseline. Set

$$i = 0. \tag{G.66}$$

Then

$$q_j(0) = 1, \qquad \mathbf{Q}(0) = \mathbf{I}, \tag{G.67}$$

and

$$s_j(0) = \varphi_j. \tag{G.68}$$

Moreover,

$$\mathbf{Z}(0) = (\mathbf{I} - \boldsymbol{\Omega})^{-1} \equiv \mathbf{Z}, \tag{G.69}$$

and

$$\mathbf{z}(0) = \mathbf{Z}\boldsymbol{\nu}. \tag{G.70}$$

The revenue-derivative matrix becomes

$$\mathbf{B}_0 = \operatorname{diag}(\mathbf{z})^{-1}\mathbf{Z}\boldsymbol{\Omega}\,\operatorname{diag}(\mathbf{z}). \tag{G.71}$$

The relative-price derivative is

$$d\widetilde{\mathbf{p}}(0) = \mathbf{H}\,[(\mathbf{I} - \mathbf{D}_\alpha) - \mathbf{D}_\alpha \mathbf{B}_0]\,\boldsymbol{\varphi}\,di_0. \tag{G.72}$$

At $i = 0$, the undistorted equilibrium satisfies the envelope condition

$$\boldsymbol{\nu}'\mathbf{H}\,[(\mathbf{I} - \mathbf{D}_\alpha) - \mathbf{D}_\alpha \mathbf{B}_0] = \mathbf{0}'. \tag{G.73}$$

Therefore,

$$\kappa(0) = 0. \tag{G.74}$$

It follows that

$$d\log C_0 = 0. \tag{G.75}$$

A small monetary tightening around a zero-interest baseline has no first-order effect on feasible aggregate consumption.

Since $d\log(1 + i_0) = di_0$ at $i_0 = 0$, the aggregate price index satisfies

$$d\log P_0^c = -di_0. \tag{G.76}$$

Sectoral prices satisfy

$$d\log \mathbf{p}_0 = -di_0\,\mathbf{1} + \mathbf{H}\,[(\mathbf{I} - \mathbf{D}_\alpha) - \mathbf{D}_\alpha \mathbf{B}_0]\,\boldsymbol{\varphi}\,di_0. \tag{G.77}$$

The second term changes relative prices across sectors, but its consumption-weighted aggregate is zero:

$$\boldsymbol{\nu}' d\widetilde{\mathbf{p}}(0) = 0. \tag{G.78}$$

Thus the aggregate price-index response is entirely given by the common nominal scale effect:

$$d \log P_0^c = -di_0. \tag{G.79}$$

The sectoral deviations around this aggregate decline are governed by the same network-induced financial forces emphasized in the main model. The dynamic extension therefore changes the aggregate normalization–a tightening lowers the current price level–but leaves the sectoral decomposition intact.

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