EUROPEAN CENTRAL BANK

EUROSYSTEM

Working Paper Series

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Climate transition risk in the banking sector: what can prudential regulation do?

No 2910

Abstract

Climate-related risks are due to increase in coming years and can pose serious threats to financial stability. This paper, by means of a DSGE model including heterogeneous firms and banks, financial frictions and prudential regulation, first shows the need of climate-related capital requirements in the existing prudential framework. Indeed, we find that without specific climate prudential policies, transition risk can generate excessive risk-taking by banks, which in turn increases the volatility of lending and output. We further show that relying on microprudential regulation alone would not be enough to account for the systemic dimension of transition risk. Implementing macroprudential policies in addition to microprudential regulation, leads to a Pareto improvement.

Keywords: transition risk, financial frictions, prudential regulation

JEL classification: D58, E58, E61, Q54

Non-technical summary

Are the banking system and its regulatory framework ready for the transition to a low-carbon economy? The emerging evidence on the relevance of climate-related risks for financial stability has triggered a heated debate on whether the current regulatory framework can adequately capture transition risks (as well as physical risks), and how to address systemic aspects of climate change. With our analysis, we contribute to this debate by providing theoretical evidence that supports the view that banking capital requirements should explicitly incorporate risks arising from the transition to a low-carbon economy. We further show that employing macroprudential policies in addition to microprudential regulation is achieving a superior outcome.

To conduct our analysis, we employ a dynamic stochastic general equilibrium (DSGE) model which allows us to study the welfare implication of different policies and to capture systemic amplification effects. The economy in our model is composed of households, heterogeneous banks, heterogeneous firms, a tax authority, a monetary policy authority and a prudential authority. Firms produce the same type of goods but with different degrees of carbon intensity. They have to pay a carbon tax proportional to their emissions, which is the source of transition risk in our model. This additional cost increases the risk of default of more polluting firms with respect to less polluting ones. In line with empirical observations,[1] we assume that banks do not specifically identify the transition risk of their individual exposures when estimating the credit risk associated to their loans book.

The first part of our analysis shows the need of prudential policy intervention to account for climate transition risk. Our focus is on understanding whether such a policy intervention can be justified on the basis of potential financial risks arising from transition risk which are not addressed by other policy requirements. We show that without policy intervention, banks take excessive risk and this generates additional lending volatility relative to the case where banks are required to identify firm-specific transition risk. Climate prudential policies forcing banks to set capital requirements reflecting the specific transition risk of their exposures lead to a more efficient outcome (*i.e.* they are Pareto superior) compared to the outcome in a framework without these policies. In other words, the introduction of climate prudential policies allows the authorities to obtain more stable macroeconomic outcomes (in terms of credit volatility and

[1] See for example, Baranović et al. (2021) and ECB [2022].

output volatility) in the presence of transition shocks.

The second part of our analysis investigates whether climate prudential policies should rely only on microprudential instruments or whether macroprudential instruments are also necessary to address climate transition risk. Using a richer model setup with heterogeneous banks and firms and with transition shocks hitting part of the corporate sector and the banks exposed to these firms, we show that these shocks could spillover to the entire economy. Thereby, they also affect other banks not directly exposed to these shocks, and microprudential climate policies would not suffice to address systemic effects of transition risk. We show that the introduction of additional macroprudential policies would improve the responses of the banking sector and macroeconomic outcomes in the presence of transition risk shocks.

Our paper contributes to a recent growing literature showing the benefits of introducing climate policies (Carattini et al., 2023; Diluiso et al., 2020; Dunz et al., 2021). For the first time in the literature, we look at the need of introducing prudential policies focusing only on transition risk and its implications for financial stability risks. In addition, differently from the existing literature, we clearly distinguish between microprudential policy intervention and macroprudential policy intervention.

1 Introduction

Are the banking system and its regulatory framework ready for the transition to a low-carbon economy? The commitment by global leaders to reduce carbon emissions following the Paris agreement[2] and the measures to ensure that the agreed targets will be met may imply significant and abrupt changes in production activities, with potentially adverse consequences for the banking system. Transition risk shocks in the form of carbon taxes or other policy measures to reduce carbon emissions may be amplified by demand shocks as consumer habits and preferences may change,[3] or may lead to increased corporate default correlations and overlapping portfolios,[4] possibly representing a material risk to the banking system. The emerging evidence on the relevance of climate-related risks for financial stability has triggered a heated debate whether the current regulatory framework can adequately capture transition risks (as well as physical risks), and how to address systemic aspects of climate change.[5] Regulators and policymakers are currently considering policies to close potential regulatory gaps, including macroprudential measures.[6] With our analysis, we contribute to this debate by providing theoretical evidence that supports the view that banking capital requirements should explicitly incorporate risks arising from the transition to a low-carbon economy. We further show that employing macroprudential policies in addition to microprudential regulation is achieving a Pareto improvement.

We employ a dynamic stochastic general equilibrium (DSGE) model to investigate the efficiency of the portfolio allocation in terms of risk-taking by the banking sector in the presence of transition risk. The economy in our model is composed of households, heterogeneous banks, heterogeneous firms, a tax authority, a monetary policy authority and a prudential authority. Banks fund several types of firms that differ according to their emission intensity of the goods that they produce. The production of less carbon intensive products requires more expensive capital investments, while the production of carbon intensive goods is subject to the payment of taxes on emissions. Banks allocate their loan book to maximise their profits, subject to risk-sensitive capital requirements. We assume that transition risk is not the main source of credit risk and that the incentives for banks to internalize transition risks are low. This assumption is

[2] For example, the EU committed to reduce carbon emissions by 55% by 2030, see EC [2021a].

[3] See Batten (2018) and ECB [2021].

[4] See ECB/ESRB Project Team on climate risk monitoring (2022).

[5] See, for example, Baranović et al. (2021).

[6] For example, the Basel Committee on Banking Supervision (BCBS) is currently assessing potential gaps of the banking prudential framework in addressing climate risks and will consider potential measures to address them. See BCBS [2022]; The European Commission's proposal for amendments of the EU capital requirements directive suggests that the systemic risk buffer framework can be used to address climate risks. See EU Commission Banking Package: EC [2021b] and EC [2021c].

broadly realistic as transition risk originates from potential future extra taxes or regulation on emissions, whose path is uncertain but at the same time these taxes are not likely to be raised to a very high level owing to the high costs this would impose on consumers and firms. This makes the proper assessment of transition risk difficult for individual banks. In addition, the identification of credit risk driven by transition risk would imply additional costs for banks, such as investment in databases, the re-estimation or development of new credit risk models or the recruitment of specialised staff.

All firms are subject to standard idiosyncratic shocks determining credit risk (as in the literature starting from Bernanke et al. (1999)), but the default probability is higher at aggregate level for firms producing carbon intensive goods, as they are also subject to emission taxes which increase their expenses. As banks do not have sufficient incentives to distinguish the transition risk associated to different credit exposures, the credit contracts reflect the average level of risk across all firms. For this reason, the estimated credit risk of carbon intensive firms is underestimated (as they are actually characterized by higher than average transition risk) and overestimated for greener firms (who are offered a higher interest rate that covers partially for the transition risk emerging from carbon intensive firms). These differences between actual and estimated credit risk of individual exposures is reflected in the capital requirements. Due to the fact that capital requirements are a non-linear function of credit risk, banks do not hold enough capital to adequately cover transition risk and will be over-invested in carbon intensive firms. Therefore, transition risk generates excessive risk-taking by banks, which in turn increases the volatility of lending and output.

The first analysis presented in the paper aims to establish the case for explicitly incorporating climate risk into the regulatory framework. For the first time in the literature, we focus explicitly on the need of prudential policies to address climate transition risks by justifying their application solely from a financial stability perspective. With this objective and without loss of generality, we use a simplified version of the model where only two types of firms are present in the economy - firms which have either low or high carbon emissions. We furthermore assume that banks are homogeneous in the sense that they have the same allocation of lending and the same capital. We compare the allocation of the credit portfolio and the implied credit volatility in the presence of shocks with and without climate prudential policy intervention. In this part of our analysis, we abstract from the precise climate prudential policy instrument and we assume that this policy creates sufficient incentives for banks to fully internalize in their credit risk assessment the transition risk component. This could be achieved in practice by some transparency requirement, by differentiated risk weights across sectors, etc. In addition, the simplified version of the model used for this analysis is such that micro- and macroprudential instruments cannot be distinguished as banks are homogeneous, and therefore any change to prudential requirements affects the whole banking sector. In terms of policy objectives, we consider climate prudential policies that increase the resilience of the banking system against transition shocks rather than reducing the probability of these shocks to occur. This implies that the objective of prudential policies is to increase banks' loss absorption capacity and that the policy (or requirements) would need to be relaxed when a transition shock occurs. This assumption is quite common in the recent literature on the costs and benefits of prudential policies (see for example Budnik et al. (2021)) and is also aligned with policy practices, as we have seen both micro- and macroprudential policies being relaxed when shocks occurred.[7] Indeed, in the model, transition risk arises because of an exogenous carbon tax which would not be affected by prudential policies and some credit reallocation. Comparing the results of the model with and without climate prudential policy intervention, we find that the aggregate capital requirements do not cover all credit risk without policy intervention. This generates excessive risk-taking and additional lending volatility relative to the case where banks are required to internalise firm-specific transition risk. We show that climate prudential policies forcing banks to set capital requirements that distinguish between the two types of firms (for example in the form of differentiated risk weights) are Pareto superior to a framework without these policies as they would lower credit and output volatility. In addition, these policies would also be beneficial for the monetary policy authority, which would be able to achieve a better stabilization of the real economy.

The second analysis presented in this paper focuses on the type of prudential instruments that need to be used to address financial stability risks arising from transition shocks. In particular, we present a richer model version where both micro- and/or macroprudential instruments could be employed. A clear distinction between the two policies is not discussed in most of the academic literature (see Section 2), but this is crucial for the policy implementation. With this objective, we use the more general version of our model that features heterogeneous banks and firms as well as aggregate demand shocks. Microprudential climate policies are defined in our analysis as policies which are calibrated on the basis of the carbon intensity of the individual banks'

[7] For example, in the initial phase of the Covid-19 pandemic both macroprudential and microprudential buffers have been relaxed. See the press release entitled "ECB Banking Supervision provides temporary capital and operational relief in reaction to coronavirus", 12 March 2020, and "Macroprudential measures taken by national authorities since the outbreak of the coronavirus pandemic" on the ECB's website.

exposure. Instead, macroprudential climate policies account for the system-wide implications of transition risks. The underlying idea is that if banks have heterogeneous exposures to firms with different degrees of carbon intensity, microprudential policies are not sufficient when direct transition risk shocks coincide with aggregate demand shocks. To show this, we assume that there are three firms with different degrees of carbon intensity (high, medium and low) and two banking groups in the economy. In line with the often concentrated nature of banks' exposures to certain sectors, we assume that each banking group is exposed only to two of the three firms. As for the previous analysis, we compare the two policy frameworks by looking at the benefits achieved by the release of the policies in an adverse scenario. In this context, we assume that a transition shock hitting part of the corporate sector (for example the high carbon intensive firms) could trigger a demand shock, in the form of a shock to households consumption preferences. We show that such aggregate shocks are however not captured by prudential requirements calibrated on the basis of the carbon intensity of banks' individual exposures and that the transition risk hitting part of the corporate sector could then also indirectly affect banks that are not exposed to those (high carbon intensive) firms. For example, this can occur when the aggregate demand shock could reduce the demand of goods from medium carbon intensive firms. If capital requirements of banks exposed to medium carbon intensive firms rely on the carbon intensity of the individual firms, they would not account for this aggregate demand effect. Therefore, introducing a macroprudential climate policy on top of microprudential climate policies improves the equilibrium outcome by guarding banks against such potential systemic effect of transition risk.

The rest of the paper is organised as follows: section 2 provides an overview of the relevant literature. In section 3, the general structure of the model without climate prudential policies is presented. In section 4, we establish the case for explicitly incorporating climate risk into the prudential regulatory framework. In section 5, we make the case for explicitly incorporating climate macroprudential policies into the regulatory framework. Section 6 concludes.

2 Literature

Our paper contributes to a small but growing literature assessing the relevance of financial regulation to address climate-related risks to the financial system.[8] This literature includes

[8] For a broader literature review on the design and effect of climate policy – including climate prudential regulation – on business cycles, see Annicchiarico et al. (2022).

mainly theoretical models,[9] with a majority of macro-financial models developed recently.

In a micro-theoretical setting, Oehmke and Opp (2022) study the positive and normative implications of climate change for bank capital regulation under either a prudential mandate or a broader climate mandate. They find that climate-related risks that affect banks' stability can be optimally addressed by a combination of green supporting and dirty penalizing factors.

A few papers have used different types of macro-financial models to analyse the impact of green regulatory policies on climate-related risks and financial stability. For example, Dafermos and Nikolaidi (2021) use an agent-based stock-flow consistent model to study the impact of capital requirements that differentiate between loans to firms with different degrees of carbon emissions. In their setting, capital requirements are determined according to the 'degree of dirtiness' of the underlying investment and the 'dirty penalising factor' has a positive effect on financial stability only if applied jointly with climate-related fiscal policy. Also in a stock-flow consistent model, Dunz et al. (2021) study the impact of a carbon tax and of a green supporting factor on the banking sector's performance and show that although investments with low carbon emissions are boosted in the short-run, these policies might negatively impact financial stability. They further demonstrate that only if the government applies a redistributive welfare policy or if banks reallocated their portfolios towards low carbon emission investments, financial stability and a smooth transition to a low-carbon economy are achieved. Moreover, in another agent-based model, Lamperti et al. (2021) study the impact of using zero risk-weights for exposures to firms with low carbon emissions. The authors show that physical climate risks increase the probability of having a banking crisis and only a policy mix combining capital requirements more favourable towards low carbon emission exposures, public guarantees on credit to low carbon exposures and carbon-risk adjustment in credit ratings leads to a stable financial sector (and at the same time lower emissions and higher economic growth).

Punzi (2019) represents an early attempt to analyse different regulatory policies in an Environmental-DSGE (E-DSGE) context, with a focus on how to increase financing to low carbon emission firms. Compared to our paper, her analysis imposes non-risk based regulatory rules and considers each rule separately. In this setting, she finds that only differentiated capital requirements can encourage green financing. Also in an E-DSGE setting, Carattini et al. (2023) show that policies in the form of taxes or subsidies on banks' assets can help to guard against

[9] A notable exception is the paper of Thomä and Gibhardt (2019) in which several empirical models are used to quantify the impact of introducing green prudential regulation – in the form of a green supporting factor (GSF) or a dirty penalty (DP) – on bank capital, credit supply and the level of interest rates. They find that the GSF would have a limited effect on banks' capital requirements compared to the DP, as the latter applies to a larger set of assets.

transition risks. In a DSGE model with financial frictions and differentiated energy sectors, Diluiso et al. (2020) show that transition risk generated by an ambitious climate policy is limited and that green financial regulation via tax-subsidy schemes might increase financial instability. However, they find that unconventional monetary policy (green credit policy) in response to a financial crisis triggered by transition risk can stabilize the banking sector and at the same time encourage aggregate demand. Using a multi-sector DSGE model with default, Giovanardi and Kaldorf (2023) find only limited effects of using higher risk weights for brown assets as climate policy tool, whereas they find that differentiated risk weights can affect risk-taking and default rates of banks. In a E-DSGE model with bank default, Garcia Villegas and Martorell (2023) find that, in the presence of transition risk, increasing capital requirements for banks' fossil exposures are welfare superior to a general increase in capital requirements. Finally, Diluiso et al. (2023) show that countercyclical capital requirements – taking the form of reserve requirements – can reduce the uncertainty surrounding the functioning of carbon-pricing policies throughout the business cycle, using a DSGE model with financial frictions.

Compared to the aforementioned literature, we propose two main novelties. First, we explicitly show with our model that transition risk leading to higher riskiness of high polluting firms, can indeed have a negative impact on financial stability. This, in turn, would make the case for a policy intervention by the prudential authority. In the theoretical literature, this result is usually taken as given[10] and the objective of the papers is rather focused on studying the impact of different climate prudential regulations on (i) climate-related risks, (ii) the real economy, (iii) the financial sector or (iv) the climate itself (pollution and emissions). Three papers have however also looked at the financial stability implications of transition risk (Carattini et al., 2023; Diluiso et al., 2020; Dunz et al., 2021), but in a different manner than we do. Dunz et al. (2021) use a stock-flow consistent model, whereas we opt for a DSGE model with financial frictions and heterogeneous banks and firms. Carattini et al. (2023) show indeed that an ambitious climate (fiscal) policy can have implications for financial stability, but consider taxes and subsidies on banks' assets as regulatory policy options, whereas in our model we analyse prudential regulation based on risk-weighted assets, in line with the Basel III framework.[11] Finally, Diluiso et al.

[10] Note that several papers provide empirical evidence on the impact of transition risk on banks' portfolios and their losses (see for *e.g.* Alogoskoufis et al. (2021), Belloni et al. (2022), Jung et al. (2023), Nguyen et al. (2023)). In particular, Alogoskoufis et al. (2021) show that (physical and) transition risk can indeed generate *systemic* risk. This nascent empirical literature supports our idea that prudential regulation is needed to address the unexpected transition risk losses of banks.

[11] To show the existence of transition risk once a tax on carbon emissions is introduced, Carattini et al. (2023) compare a case where no prudential policy is present in the model with a scenario where (climate) prudential regulation is introduced to address the climate externality on the banking system. In our model, when comparing cases to show the benefits of introducing climate-related capital requirements, we use the more realistic configura-

(2020) use a DSGE model to show that transition risk arising from a credible medium-term emission reduction strategy has only a limited impact on banks' net worth, thus not necessarily showing the need of prudential policy intervention to deal with this type of climate risk.

Second, up to our knowledge, we are the first paper to distinguish between micro and macroprudential regulation in the context of a macro equilibrium model. From the policymakers' point of view, clearly distinguishing between micro and macroprudential policy instruments is particularly relevant as it is not always the same authority that is in charge of both regulations. Most of the papers consider that differentiated risk weights - the green supporting factor (GSF) and the dirty penalising factor (DPF) - are macroprudential tools, however this is not necessarily the case. In the models where these types of tools are introduced, the GSF and the DPF can be considered as being macroprudential tools only because they are assimilated to sectoral capital requirements applied to the production firms with different degree of carbon intensity. In addition, from a modeling perspective, in these papers, macro- and microprudential policies would coincide as they only consider a representative bank. In our framework, we introduce three types of firms - high, medium and low carbon intensive firms - allowing us to have differentiated banks and potentially heterogeneous requirements across banks (*i.e.* microprudential policies). In addition, we discuss a supplementary climate capital buffer applied to all banks that is designed to absorb aggregate transition risk (*i.e.* macroprudential policies). Introducing heterogeneous banks, exposed to three types of firms, allows us to clearly distinguish between micro and macroprudential regulation and explore the systemic dimension of transition risk, not accounted for in the previous literature.

Modelling wise, our paper is based on the DSGE literature featuring a financial sector and financial frictions in the spirit of Bernanke et al. (1999) and all the literature that developed afterwards (for *e.g.* Gerali et al. (2010), Gertler and Karadi (2011), Gertler and Kiyotaki (2010) and Darracq Paries et al. (2011), amongst many others). We build on the Darracq Paries et al. (2011) and Darracq Paries et al. (2019) models and add heterogeneous banks to it. Moreover, instead of two firms producing either residential and non-residential goods, we introduce heterogeneous firms which differ in their degree of carbon emissions. As in the rest of the E-DSGE literature, we assume emissions are a (non-linear) function of production.

tion where regular prudential policies are already imposed on bank vs. where both regular and climate prudential rules are enforced. Their scenario further implies that climate prudential regulation is implemented before the introduction of the (ambitious) carbon tax, whereas we introduce both policies at the same time. Moreover, unlike our paper, they do not study the joint effect of introducing micro and macroprudential policies to tackle transition risk in the banking system.

3 The model

Our paper develops a DSGE model with three main types of agents – households, heterogeneous firms and heterogeneous banks. Households offer their labor services to the different firms in the economy, consume an aggregate basket of goods produced by firms and save their money in the form of deposits. They play a major role in the analysis as their preferences are subject to a shock that can shift their consumption of goods from one sector to another. Firms that populate the economy produce differentiated goods and have different levels of carbon emissions. Less carbon intensive firms use more technologically advanced (and expensive) capital in their production process which allows them to be less polluting than more carbon intensive firms which use regular capital. Firms pay taxes depending on their carbon emissions which are the source of transition risk in the model. To finance their production, firms use in addition to their own net worth, funds coming from a bank. As firms are subject to idiosyncratic shocks, they might default on their loan commitment and cause losses to the bank (in the spirit of Bernanke et al. (1999), Darracq Paries et al. (2019), Gerali et al. (2010)). This asymmetric information between firms and banks implies that the entrepreneurs are financially constrained and that the interest rate (the external finance premium) established in the loan contract will depend on their leverage.

Without loss of generality, we assume that the differences between firms' credit risk arise only due to transition risk. Banks are operating in a perfectly competitive environment and collect deposits from households, provide funds to firms and are subject to risk-sensitive capital requirements. In the case of a firm's default, banks incur a monitoring cost which allows them to recover the collateral associated to the credit contract. Without any climate prudential regulation, these intermediaries have no incentive to distinguish between the different types of firms when setting financing conditions or capital requirements which can lead to higher exposures of banks to polluting firms, thus to higher transition risk in banks' portfolios.

The model includes also capital producers that provide differentiated capital to goods producing firms and retailers which operate in monopolistic competition to insure sticky prices (Calvo, 1983). Two authorities are also present in the model: the monetary policy authority sets the risk free rate, whereas the government collects taxes and transfers them to households. In our analysis we also assume that prudential regulation can be implemented, but we don't formulate an explicit prudential policy rule.

3.1 Households

Households supply labor to different types of firms which are characterized by different levels of carbon intensity (L_t^i, where $i = 1..I$ and indicates the classification of firms by carbon emission) for which they get paid an hourly wage (w_t^i), consume a mixed basket of goods from all firms (C_t^H) and save their income in the form of deposits D_t which are remunerated at the risk free rate R_t. Households are also the owners of banks and of retail firms from which they receive dividends (Π_t^H). In addition, they receive benefits G_t from the government. They maximize their utility so as to choose consumption, hours worked and deposits:

$$\max_{\{C_t^{H,i}, L_t^i, D_t^H\}} \mathbb{E}_t \left[\sum_{t=0}^{\infty} \beta^{H^t} \left(\frac{C_t^{H^{1-\gamma}}}{1-\gamma} - \sum_i \overline{L^i} \frac{L_t^{i^{1+\sigma^i}}}{1+\sigma^i} \right) \right] \tag{1}$$

subject to the budget constraint expressed in nominal terms:

$$\sum_i P_t^i C_t^{H,i} + P_t D_t^H \leq P_t \frac{1+R_{t-1}}{1+\pi_t} D_{t-1}^H + \sum_i w_t^i L_t^i + \Pi_t^H + G_t. \tag{2}$$

where π_t is the aggregate inflation rate and P_t is the aggregate price index. The parameters $\overline{L^i}$ in the utility function are calibrated such that at the steady state total labor is normalized to one. Household consume a basket C_t^H of goods produced by all firms and $C_t^{H,i}$ indicates the consumption of goods with a specific emission intensity. Overall, the households' preferences across different types of goods follow a CES form:

$$C_t^H = \left[\sum_i \epsilon_t^c \kappa^{i^{\frac{1}{\varepsilon}}} C_t^{H,i^{\frac{\varepsilon-1}{\varepsilon}}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

where $\sum_i \kappa^i = 1$ and ϵ_t^c is a shock to preferences log-normally distributed with mean 1.

3.2 Entrepreneurs

Firms produce differentiated goods of the same product category (*e.g.* electric and gas cars) and they are classified according to their level of carbon emission ($i = 1..I$). This classification could include both the production and the product, similarly to the Scope 3 ESG score. Less carbon intensive firms use more technologically advanced (and costly) capital than more carbon intensive firms allowing them to produce either with less emissions or more environmentally friendly products. Firms have to pay taxes for their carbon emissions, which are the source of transition risk in the model. Without loss of generality, we suppose that firms have otherwise the same solvency profile (*i.e.* the same 'traditional' credit risk) and differ only in their production technology and emission costs. Entrepreneurs own the firms and use their earnings for consumption, maximizing the utility from the consumption of a basket of differentiated goods:

$$\max_{\{C_t^{E,i,j},L_t^i,K_t^i,B_t^i,u_t^i,\overline{\omega}_t^i\}} \mathbb{E}_t\left[\sum_{t=0}^{\infty}\beta^{E^t}\frac{C_t^{E,i^{1-\gamma}}}{1-\gamma}\right] \tag{4}$$

where $C_t^{E,i}$ is the basket of differentiated goods consumed by entrepreneurs. As for the households, the consumption of goods with different degree of carbon intensity follows CES preferences:

$$C_t^{E,i} = \left[\sum_j \theta^{j^{\frac{1}{\varepsilon}}} C_t^{E,i,j^{\frac{\varepsilon-1}{\varepsilon}}}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{5}$$

The production function of each sector (Z_t^i) is Cobb-Douglas:

$$Z_t^i = \epsilon_t^i\left(u_t^i K_{t-1}^i\right)^a\left(L_t^i\right)^{1-a} \tag{6}$$

where u_t^i is the capacity utilization, K_t^i is the capital and ϵ_t^i is a technology shock following an AR(1) process. Each period, firms take loans $B_t^{E,i}$ from banks to finance their activities. The interest rate associate with the loan is $R_t^{E,i}$. However, firms are subject to idiosyncratic shocks affecting their revenues, ω_t^i, causing the default of some firms on the loan contract. If a firm does not repay its loan, the bank can seize the collateral $(1-\delta)Q_t^iK_{t-1}^i$, where Q_t^i is the nominal price of capital and δ a depreciation factor (see Gerali et al. (2010), Darracq Paries et al. (2011), Darracq Paries et al. (2019), amongst others). Calling $\overline{\omega}_t^i$ the idiosyncratic shock at which the entrepreneur is indifferent between defaulting or not, firms are willing to pay the interest rate implied by this threshold:

$$\frac{1+R_{t-1}^{E,i}}{1+\pi_t}P_tB_{t-1}^{E,i} = \overline{\omega}_t^i(1-\delta)Q_t^iK_{t-1}^i. \tag{7}$$

This also implies that the expected loan repayment can be written as (see Bernanke et al. (1999)):

$$H(\overline{\omega}_t^i)(1-\delta)Q_t^iK_{t-1}^i \equiv (1-\delta)Q_t^iK_{t-1}^i\left[\int_0^{\overline{\omega}_t^i}\omega dF(\omega)+\left(1-F\left(\overline{\omega}_t^i\right)\right)\overline{\omega}_t^i\right]. \tag{8}$$

Firms use different capital in their production functions which they acquire from the capital goods producers. The capital of less carbon intensive firms allows them to have lower emissions and, consequently, has higher capacity utilization adjustment costs $\Phi_i(u_t^i) = \phi^i r^{i,SS}(\exp[\frac{1}{\phi^i}(u_t^i - 1)] - 1)$. Firms also differ because they are subject to emission taxes $T(e_t)$ on the basis of their carbon intensity. In addition, each firm sells its goods to retailers, who will then distribute them to households at a price P_t^i charging a markup on the price at which they buy the intermediate goods from entrepreneurs. The inverse of the markup charged by retailers on the intermediate good price is indicated with μ_t^i. The budget constraint in nominal terms writes:

$$\begin{aligned}\sum_j P_t^j C_t^{E,i,j} + Q_t^i \left[K_t^i - (1-\delta)K_{t-1}^i\right] + w_t^i L_t^i + \\ \Phi_i(u_t^i)K_{t-1}^i + H(\overline{\omega}_t^i)(1-\delta)Q_t^i K_{t-1}^i + T(e_t) \\ \leq \mu_t^i P_t^i Z_t^i + P_t B_t^{E,i}\end{aligned} \tag{9}$$

where the tax on emissions is a non-linear function of production:

$$T(e_t^i) = \eta^i e_t^i \tag{10}$$

$$e_t^i = \left(Z_t^i\right)^{\zeta}. \tag{11}$$

where the parameter η^i can also be proportional to the level of carbon intensity. Firms in each sector are in perfect competition in producing their goods, this implies the following zero profit condition on production:

$$\mu_t^i P_t^i Z_t^i - T(e_t^i) = \Phi_i'(u_t^i)u_t^i K_{t-1}^i + w_t^i L_t^i \tag{12}$$

When maximizing their utility, firms also take into account the participation constrain of commercial banks to the lending contract. Commercial banks have financing costs $R_t^{F,i}$ for their loan contract and face some monitoring costs, ρ to collect the collateral in case of default. Their participation constraint can be written as:

$$\frac{1+R_{t-1}^{F,i}}{1+\pi_t} P_t B_{t-1}^{E,i} \leq (1-\delta)Q_t^i K_{t-1}^i G(\overline{\omega}_t^i) \tag{13}$$

where:

$$G(\overline{\omega}_t^i) \equiv \left[(1-\rho)\int_0^{\overline{\omega}_t^i} \omega dF(\omega) + (1-F(\overline{\omega}_t^i))\overline{\omega}_t^i\right]. \tag{14}$$

3.3 Capital good producers

There are capital good producers for each type of firms degree of carbon intensity. Capital good producers are in perfect competition and are owned by households. At the beginning of each period, capital producers buy at real prices the undepreciated capital $(1-\delta)K^i_{t-1}$ from entrepreneurs. Then they augment the capital using distributed goods and facing adjustment costs I^i_t. The augmented stocks are sold back at the end of the period to entrepreneurs at the same prices. The maximization problem of capital producers can be written as:

$$\max_{\{K^i_t, I^i_t\}} \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \beta^{H^t} \left[Q^i_t \left[K^i_t - (1-\delta)K^i_{t-1} \right] - I^i_t \right] \right\}, \tag{15}$$

subject to the law of motion of capital

$$K^i_t = (1-\delta)K^i_{t-1} + \left[1 - S\left(\frac{I^i_t \epsilon^I_t}{I^i_{t-1}} \right) \right] I^i_t \tag{16}$$

where $S(.)$ is the adjustment cost function defined as $S(x) = s0.5(x-1)^2$ and s is a parameter.

3.4 Retailers

Retailers are in monopolistic competition. They buy homogeneous intermediate goods $Z^i_t(j)$ at price $\mu^i_t P^i_t$ from the entrepreneurs, they differentiate the products and sell them to households and entrepreneurs. As aforementioned, μ^i_t represents the inverse of the gross markup of retail goods over intermediate goods, or otherwise stated the selling price of intermediate products. Retailers set their prices on a staggered basis à la Calvo (1983): in each period only a fraction $1-\chi$ of retailers is able to re-optimize its nominal prices $(P^i_t(j) = \tilde{P}^i_t(j))$, where $\tilde{P}^i_t(j)$ is the optimal price for retailer j, while the remaining fraction χ cannot re-optimize and will set the prices as in the previous period $(P^i_t(j) = P^i_{t-1}(j))$. The aggregate price dynamics is given by a weighted average of prices set by firms that re-optimize and prices from the previous period kept constant by non-reoptimizing firms:

$$P^i_t = \left[\chi {P^i_{t-1}}^{1-\epsilon} + (1-\chi)\tilde{P}^i_t(j)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \tag{17}$$

In aggregate, the production and price index functions of retailers follow a CES functional form:

$$Z_t^i = \left[\int_0^1 Z_t^i(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}} \tag{18}$$

$$P_t^i = \left[\int_0^1 P_t^i(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}} \tag{19}$$

which imply the following demand of intermediate goods of the individual retailer: $Z_t^i(j) = \left(\frac{\tilde{P}_t^i(j)}{P_t^i}\right)^{-\epsilon} Z_t^i$. Then, each retailer who can re-optimize at t maximizes the expected discounted stream of profits under the assumption that once the price $\tilde{P}_t^i(j)$ is set, he will not re-optimize with probability χ for a number of periods $k \in [0, \infty]$. The maximization of the retailer writes:

$$\max_{\{\tilde{P}_t^i(j)\}} \sum_{k=0}^{\infty} \chi^k \mathbb{E}_t \left[\lambda_{t+k} \left(P_{t+k}^i(j) - \mu_{t+k}^i P_{t+k}^i\right) Z_{t+k}^i(j)\right] \tag{20}$$

where $P_{t+k}^i(j) = \tilde{P}_t^i(j)$ for the assumptions above that the retailer will not re-optimize till $t+k$.

3.5 Market clearing conditions for goods

For all goods, the market clearing conditions are given by:

$$Z_t^i = C_t^{H,i} + \sum_j C_t^{E,j,i} + I_t^i + \Phi_B(u_t^i) K_{t-1}^i + \eta^i \left(Z_t^i\right)^\zeta \tag{21}$$

3.6 The banking sector

There are several banking groups ($k = 1...K$) in the economy each composed of three branches and operating in a perfectly competitive environment. The wholesale branch receives deposits from the retail deposit branch and allocates these funds to the commercial lending branch. The wholesale branch of the group also manages the capital position of the bank taking into account risk-sensitive capital requirements and incurs a cost whenever the capital to risk-weighted assets ratio deviates from a target value, CR. The deposit branch raises deposits from households to whom it pays the risk free rate and transfers them to the wholesale branch, which remunerates them at the risk free rate. The commercial lending branch obtains funds from the wholesale branch and uses them to give credits to firms.

As shown in Section 3.2, taxes on carbon emissions affect credit risk increasingly with the level of carbon emission as they represent an additional cost. As the idiosyncratic shock ω^i has the same distribution across types of firms, transition risk (*i.e.* the risk due to taxes on emissions) is the main source of differentiation of credit risk in the model. It is reasonable to assume that proportionally, credit risk due to transition risk is not more impactful than the more traditional source of credit risk as emission taxes cannot be raised abruptly to very high levels owing to the high costs this would impose on firms.

Internalizing the difference in credit risk of firms due to transition risk is costly for banks as this risk is quite challenging to identify and it would imply a number of additional costs for banks, such as investment in databases, the re-estimation or development of new credit risk models or the recruitment of specialized staff. Additionally, part of the information would potentially not even be available or sufficient considering the short history of emission taxes or emission measurements (reference). For these reasons, without specific climate prudential policies, banks have no incentive to internalize credit risk due to carbon taxes, *i.e.* transition risk, and therefore they do not distinguish between the degree of carbon intensity when lending. The estimation of these models is thus performed as if there was a single type of firm (indicated with E) in the economy. Banks estimate their models based on aggregate data (for example, on capital investments), which is the weighted average of data from all firms. Conversely, entrepreneurs assume that the bank can easily identify the carbon in tensity of firms and therefore believe that banks provide individualized contracts. In terms of modelling, this implies that the participation constrain of commercial banks to the lending contract in eq. 13 does not change (*i.e.* the participation constraint is specific to the type of firm).

In the next sections, we will describe each branch of the banking groups.

3.6.1 The deposit branch

The deposit branches operate in perfect competition. They collect deposits from households and give them to the wholesale branches. The deposit demand is entirely satisfied by the bank. No profits are made on this activity: the interest rate proposed to households on deposits is the same as the rate at which the deposit branch gets financed, *i.e.* the risk free rate R_t.

3.6.2 The wholesale branch

The wholesale branches receive deposits from the deposit branches and choose the allocations of loans. The wholesale branches maximize their profits, taking interest rates as given and taking into account a penalty if the risk-weighted capital ratio deviates from a target capital ratio. For the wholesale branch k of each banking group the profit maximization writes:

$$\max_{\{B_t^{E,k},D_t^{H,k}\}} R_t^{F,E} B_t^E - R_t D_t^{H,k} - PD_t^E LGD^E B_t^{E,k} - \frac{\Xi}{2}\left(\frac{Bankcap_t^k}{rw_t^E B_t^{E,k}} - CR\right)^2 Bankcap_t^k \quad (22)$$

which is subject to the balance sheet identity:

$$B_t^{E,k} = D_t^{H,k} + Bankcap_t^k \quad (23)$$

where $R_t^{F,E}$ represents the interest rate at which the wholesale branch gives funds to its commercial financing branch, CR is the target capital ratio and rw_t^E is the risk-weight associated to the total amount of credit $B_t^{E,k}$ distributed to the firms. In equilibrium, the credit given to firms coincides with the sum of credit given actually to entrepreneurs with different degree of carbon intensity, $\sum_k B_t^{E,k} = \sum_i B_t^i$. The term $PD_t^E LGD^E B_t^{E,k}$ corresponds to provisions for expected credit risk losses, with PD_t^E being the 1-year ahead probability of default of the undistinguished representative firm, LGD^E being a parameter capturing the loss given default. Given that there is a unique credit contract (the same financing conditions apply to all types of firms, irrespective of their transition risk), there is also a unique financing rate in equilibrium, $R_t^{F,i}$ identical for all firms. The probability of default, PD_t^E is a function of the cumulative distribution of the pooled idiosyncratic shocks $F\left(\omega_t^E\right)$ at the threshold $\overline{\omega}_t^E$ (see eq. 28). This threshold is estimated by banks as the threshold of the idiosyncratic shock at which they expect the representative firm of the pooled distribution of all firms to be indifferent between defaulting and repaying:

$$\frac{1 + R_{t-1}^E}{1 + \pi_t} P_t B_{t-1}^E = \overline{\omega}_t^E (1-\delta) Q_t^E K_{t-1}^E \quad (24)$$

where Q_t^E is the weighted average price of capital, $Q_t^E = \sum_i Q_t^i \frac{K_t^i}{K_t^E}$, and K_t^E the total amount of capital needed by entrepreneurs for their production $K_t^E = \sum_i K_t^i$. Therefore, according to banks, firms with different carbon intensity do not have different probabilities of default. Furthermore, the credit contract will reflect the average level of risk, which will be underestimated for more carbon intensive firms (as transition risk is not directly taken into account in this case) and overestimated for less carbon intensive firms (who are offered a higher interest rate that covers the transition risk emerging from higher carbon intensive firms). As capital requirements are a non-linear function of credit risk (PDs), then calculating the capital requirements based on the average credit risk will not lead to the same aggregate capital requirements as taking into account sector-specific credit risk.

Bank capital accumulates out of retained earnings from the bank group profits:

$$Bankcap_t^k = (1-\delta^{bk})Bankcap_{t-1}^k + \nu \Pi_t^{bk,k} \tag{25}$$

where δ^{bk} represents the resources used in managing bank capital, $\Pi_t^{bk,k}$ is the overall profit of the banking group which is partially distributed to households as dividends and ν is the share of retained earning.

The risk weights, rw_t^E are computed according the the Basel III IRB approach, as most of the euro area banking system non-financial corporations credit risk is calculated using this approach. According to the Basel III IRB approach, risk weights are defined as follows:

$$rw_t^E = 12.5LGD^E\left\{\Phi\left(\frac{\Phi^{-1}(PD_t^E) + (\tau^{rw^E})^{0.5}\Phi^{-1}(0.999)}{(1-\tau^{rw^E})^{0.5}}\right) - PD_t^E\right\} \tag{26}$$

$$\tau^{rw^E} = n_1^E\left[\frac{1-\exp(-n_3^E PD_t^E)}{1-\exp(-n_3^E)}\right] + n_2^E\left[1 - \frac{1-\exp(-n_3^E PD_t^E)}{1-\exp(-n_3^E)}\right] \tag{27}$$

$$PD_t^E = \alpha^{PD} PD_{t-1}^E + 4(1-\alpha^{PD})F\left(\overline{\omega}_t^E\right) \tag{28}$$

As aforementioned, PD_t^E is the 1-year ahead probability of default of the representative firm, with α^{PD} being the autoregressive coefficient of the PD process and τ^{rw^E} the asset-value correlation. The parameters $n_x, x = \{1,2,3\}$ are derived from the Basel III regulation for the non-financial corporate sector. From the maximization problem of the wholesale banks, we obtain the following relation on the interest rate spread between the refinancing rate and the risk free rate:

$$R_t^{F,E} - R_t = PD_t^E LGD^E - \Xi\left(\frac{Bankcap_t^k}{rw_t^E B_t^{E,k}} - CR\right)\left(\frac{Bankcap_t^k}{rw_t^E B_t^{E,k}}\right)^2 rw_t^E \tag{29}$$

3.6.3 The commercial lending branch

The commercial lending branches issue credit to entrepreneurs without distinguishing between different types of firms. These branches, which obtains funds from the wholesale parts of the groups, are also in perfect competition and has zero profits in equilibrium. Their profit maximization is given by:

$$\max_{R_t^E} \int_0^\infty \frac{(1+R_{t-1}^E)B_{t-1}^{E,k} - (1+R_{t-1}^{F,E})B_{t-1}^{E,k}}{1+\pi_t} P_t dF\left(\omega_t^E\right) - \rho\int_0^{\overline{\omega}_t^E}(1-\delta)Q_t^E K_{t-1}^E \omega_t^E dF\left(\omega_t^E\right) \tag{30}$$

where $\rho \int_0^{\overline{\omega}_t^E} (1-\delta) Q_t^E K_{t-1}^E \omega_t^E dF\left(\omega_t^E\right)$ are the (real) expected monitoring costs paid by the bank to seize collateral in case of the entrepreneur's default on its loan commitment. From the zero profit condition, the lending rate applied to the entrepreneur by the commercial lending bank is given by:

$$R_{t-1}^E = R_{t-1}^{F,E} + \frac{1+\pi_t}{B_{t-1}^E P_t} \left[H(\overline{\omega}_t^E) - G(\overline{\omega}_t^E)\right](1-\delta) Q_t^E K_{t-1}^E \tag{31}$$

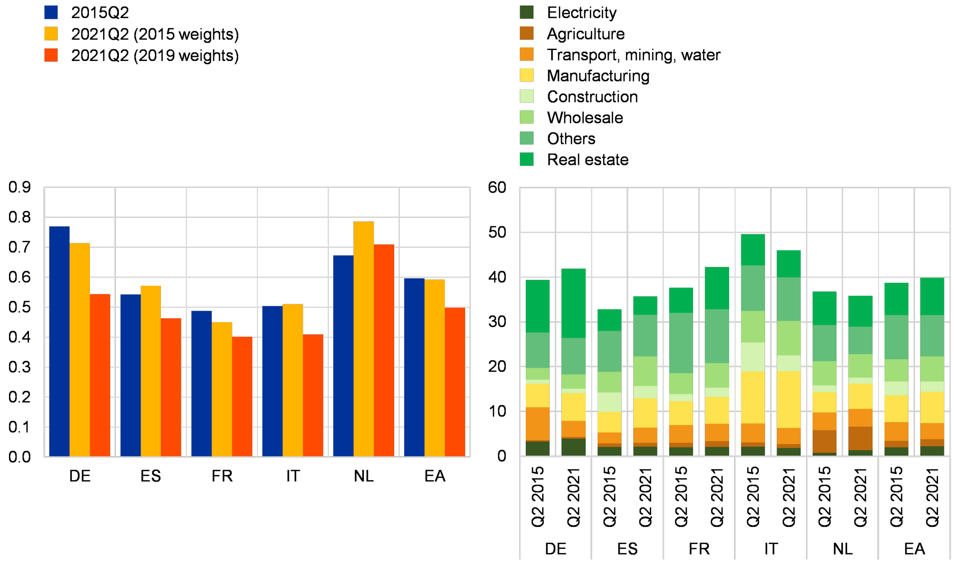
As banks do not distinguish between firms, in equilibrium the same financing conditions will be proposed to all types of firms, *i.e.* $R_t^E = R^{E,i}$, $\forall i$.

4 The need of climate prudential policies

For our first analysis, the objective is to use the above model to show the need of explicitly incorporating climate risks in the capital framework for banks. Indeed, despite the increasing attention on climate risks, banks' exposures to assets potentially affected by transition risk have not substantially changed in recent years (see Figure 1, left panel). Although the emission intensity of banks has decreased in the main euro area countries, in most of the countries this was only partially driven by a reallocation of exposures across sectors rather than a reduction in the emission intensity of their exposures. Exposures to the real estate sector increased, while exposures to the sectors most likely to be affected by transition risk have remained stable (Figure 1, right panel). This evidence raises the question as to whether banks are sufficiently taking these risks into account. Moreover, the presence of high exposures to high carbon intensive firms in banks' portfolios may expose banks to excessive credit risk and thereby pose serious threats to financial stability. The key question is then whether the regulator needs to oblige banks to internalize transition risks.

To show that climate prudential policies are indeed necessary, we compare the model setting where no climate prudential policies are implemented (as in Section 3.6) with a setting where these policies are implemented and banks have to distinguish between firms with different degrees of carbon intensity when calculating their capital requirements. As discussed above, if banks are not obliged to distinguish firms according to their carbon emission levels, they might be insufficiently capitalized when transition risk materializes. If climate prudential regulation is in place, banks have to distinguish (based on the ESG score, for example) between firms with different carbon intensity and apply risk weights which would mirror the transition risk specific

Figure 1: Average emission intensity of banks' exposures and exposures to different sectors of the economy



Sources: Eurostat (emission intensity per sector), ECB and ECB data calculations (sectoral exposures).
Notes: The bank sample includes the 167 largest euro area banks. The aggregation by country is done according to the country of the ultimate parent of each bank. The countries shown are (from left to right) Germany, Spain, France, Italy and the Netherlands; "EA" represents the overall sample average. The emission intensity is measured as the ratio between greenhouse gas emissions (CO2, N2O in CO2 equivalent, CH4 in CO2 equivalent, HFC in CO2 equivalent, PFC in CO2 equivalent, SF6 in CO2 equivalent, and NF3 in CO2 equivalent) and the value added. The left panel shows the average emission intensity using as weight the share of credit exposures to firms in each sector of the economy over the total loan book. The chart reports the average emission intensity of the exposures for 2015Q2 (blue bars), the average emission intensity of 2021Q2 exposures using the 2015Q2 emission intensity (yellow bars) and emission intensity of the exposures for 2021Q2 (red bars). The difference between the yellow and blue bars shows the impact of the exposure changes and the difference between the yellow and red bars shows the impact of the emission intensity change.

to the degree of carbon intensity of their exposures. As we do not focus on a precise policy instrument in this section,[12] in what follows we just assume that the climate prudential policy implies a perfect identification of the transition risk of different types of firms (first best). In practice, this could correspond to either some risk weight policy instrument (an add-on or a supporting factor) or the obligation to publish the transition risk implied by each exposures. The policy would also imply that the capital requirements would be relaxed on specific exposures when the transition risk would materialize (as for other capital requirements). This assumption is in-line with most of the recent institutional literature which focuses on the benefits of prudential policies in terms of banks' loss absorption capacity rather than taming the crisis (see for example Budnik et al. (2021)). Also as we have seen in practice, both micro- and macroprudential policies can be relaxed when shocks occur[13] with the calibration of some micro- and macroprudential buffers aimed at ensuring a sufficient loss absorption capacity in adverse scenarios. Our modeling framework reflects these observations. Moreover, in our model transition risk arises because

[12] As discussed in the next subsection, to show the general need of prudential policy intervention due to transition risk, it is sufficient to use a simple version of the model where banks are homogeneous. In this case, there is no difference between micro- and macroprudential policies.

[13] See footnote 7.

of an exogenous carbon tax which is not endogenously affected by prudential policies via its implications on credit re-allocation. In what follows, we first present the changes that are made to the model when we assume that banks have to identify firms on the basis of their carbon intensity and then we analyze the results obtained when comparing the two frameworks. The deposit branch presents no changes with respect to the case without climate prudential policies and for this reason we will not describe it again in the following sections.

4.1 The wholesale branch

As for the case without climate prudential policies, the wholesale branch receives deposits from households and chooses the allocation of credit between firms. However, giving credit to firms with different level of carbon intensity entails different risk weights reflecting individual transition risk. The wholesale branch maximizes the following profit function:

$$\begin{aligned}\max_{\{B_t^{E,i,k}, D_t^{H,k}\}} \quad & \sum_i R_t^{F,i} B_t^{E,i,k} - R_t D_t^{H,k} - \sum_i PD_t^i LGD^i B_t^{E,i,k} \\ & - \frac{\Xi}{2} \left(\frac{Bankcap_t^k}{\sum_i rw_t^i B_t^{E,i,k}} - CR \right)^2 Bankcap_t^k - \mathbb{T} \end{aligned} \tag{32}$$

subject to the following balance sheet identity:

$$\sum_i B_t^{E,i,k} = D_t^{H,k} + Bankcap_t^k \tag{33}$$

where CR is the target capital ratio and rw_t^i are the risk weights that apply to each type of firm and defined following the Basel III IRB approach, similarly to eq. 26 - 28. $\mathbb{T}$ is a sunk cost that banks are suffering when they have to identify the transition risk of each of their exposures (for example due to the hiring of new experts or the acquisition of data). The probabilities of default of entrepreneurs are different amongst firms with different degrees of carbon intensity and depend on different idiosyncratic shock thresholds $\overline{\omega}_t^i$. As described in Section 3.2, the threshold value $\overline{\omega}_t^i$ depends on the level of carbon emissions, implying a higher probability of default if the firm is more carbon intensive and, thus, higher credit (transition) risk. As in the case without regulation, bank capital accumulates out of retained earnings from the bank group profits (see eq. 25). Solving the maximization problem, we obtain the following spread between the financing rate for commercial lending branches and the risk free rate:

$$
\begin{aligned}
R_t^{F,i} - R_t &= PD_t^i LGD^i \\
&\quad -\Xi \left(\frac{Bankcap_t^k}{\sum_i rw_t^i B_t^{E,i,k}} - CR \right) \left(\frac{Bankcap_t^k}{\sum_i rw_t^i B_t^{E,i,k}} \right)^2 rw_t^i
\end{aligned} \tag{34}
$$

4.2 The commercial financing branches

Without loss of generality but keeping calculations simpler, each commercial lending branch specializes in giving credit to only one type of firm. As in the case without prudential policies, the commercial lending branches are in perfect competition and have zero profits in equilibrium. They get funds from the wholesale branch and distribute credit to entrepreneurs. Similarly to subsection 3.6.3, for each type of credit contract, the maximization problem of each commercial lending branch can be written as:

$$
\max_{R_t^{E,i}} \int_0^{\infty} \frac{(1 + R_{t-1}^{E,i}) B_{t-1}^{E,i,k} - (1 + R_{t-1}^{F,i}) B_{t-1}^{E,i,k}}{1 + \pi_t} P_t dF\left(\omega_t^i\right) - \rho \int_0^{\overline{\omega}_t^i} (1 - \delta) Q_t^i K_{t-1}^i \omega_t^i dF\left(\omega_t^i\right) \tag{35}
$$

From which we obtain:

$$
R_{t-1}^{E,i} = R_{t-1}^{F,i} + \frac{1 + \pi_t}{B_{t-1}^{E,i,k} P_t} \left[H(\overline{\omega}_t^i) - G(\overline{\omega}_t^i) \right] (1 - \delta) Q_t^i K_{t-1}^i \tag{36}
$$

4.3 Results

To make our case for introducing climate prudential policies, we compare the results of the two versions of the model described above – with and without prudential policy intervention. For this first analysis, we use a simplified version of the model, where only two types of firms ($i = 2$) are present in the economy – firms which have low carbon emissions intensity (LCI) and firms that present high levels of carbon emissions (HCI). Furthermore, assuming homogeneous banks is sufficient to make our point in this analysis. As mentioned earlier, the analysis presented in this section abstracts from the specific prudential policy instrument and therefore this simplified setting is sufficient. The calibrated values of the parameters follow Darracq Paries et al. (2019) and can be found in Annex A.8.

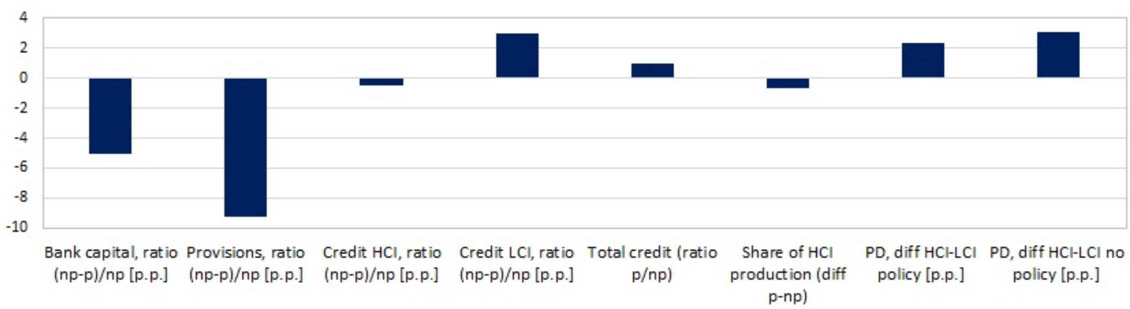
Figure 2 provides a comparison of steady state values for various financial variables and for the output of LCI and HCI firms. The bar charts highlight how the introduction of climate prudential regulation leads to better results for the financial and real sectors. However, there are some caveats to our analysis. First, the model does not have an endogenous level of target capital ratio and we assume in both cases that this ratio has the same level. In addition, the relative preference of consumers for the two types of products is assumed not to change between the two cases.

The level of bank capital is higher without climate prudential policies, as overall there is more risk (lending to the more carbon intensive firms is higher) implying that banks allocate capital more efficiently when they explicitly consider transition risk. Provisions to cover credit risk losses are about 9% lower relative to the case where banks are not submitted to climate prudential policies as more losses than expected materialize and banks do not correctly internalize the credit risk of their exposures. This result can be explained by the higher steady state default rates without climate prudential policies.[14]

When climate prudential policies apply, the steady state lending to HCI firms is lower as they are explicitly identified as being riskier, whereas LCI firms receive more credit. This is an important result as it implies that if banks were to fully internalize transition risk, they would have less exposures to HCI firms. Overall credit to the real economy increases when climate prudential policies are implemented.

Finally, the different allocation of lending between the two institutional settings is also reflected in the output of the two types of firms. The share of productions of HCI firms decreases with respect to the case where no climate prudential policy is enforced. The opposite is true for LCI firms who benefit from better credit conditions and produce more when climate policy is enforced.

Figure 2: Steady State Comparison policy [p] and no-policy [np] case



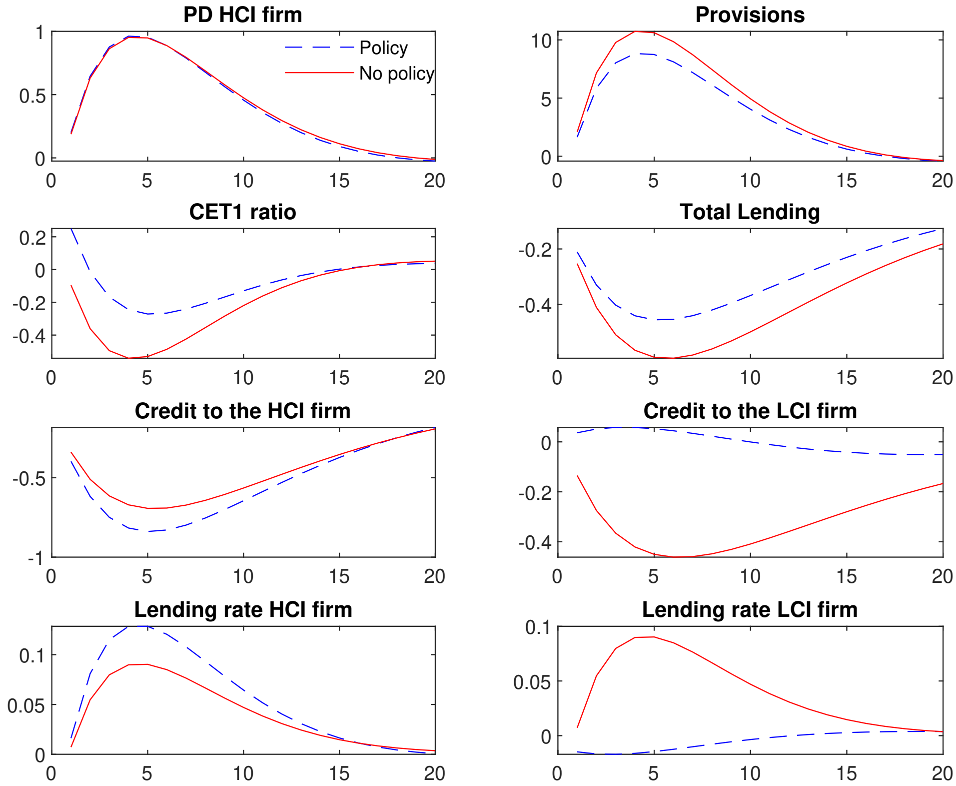
Notes: The bars represent the ratio (or difference) between the steady state value from the model where green prudential regulation in enforced with respect to the value from the model where prudential regulation does not take climate transition risk into account.

[14] Recall that provisions are computed as $PD * LGD * B$, where LGD is a parameter.

We turn next to impulse response functions. Figure 3 shows how financial variables respond to an unexpected increase in the riskiness of HCI firms. This shock is generated by an increase in transition risk for HCI borrowers. We start by looking at the overall dynamics of the model. The shock leads first to an increase in the probability of default of HCI firms. The higher default risk of these firms increases their financing cost which in turn reduces the amount of credit they can obtain from the bank. When no distinction is made by banks between HCI and LCI firms in terms of transition risk (no policy), lending to LCI firms decreases as well, implying that this type of firms are penalized even if their default risk does not change. The drop in lending can also be explained by the financial constraints that banks have once losses materialize. Indeed, risk-sensitive regulation calls for higher capital requirements when PDs increase, binding the bank to reduce its credit activity. Moreover, provisions considerably increase (up to 10% of their steady state value) in reaction to the rise in the PD of polluting firms, as more firms default on their credit engagements. When no climate prudential policy is implemented, the common equity tier 1 ratio (CET1) decreases. This can be explained by the fact that the higher default rate of HCI firms implies not only losses to bank capital, but also higher total risk weighted assets, with risk weights on both high and low carbon intensive firms increasing in the no policy scenario. Note that the impact on bank capital is limited as banks need to obey capital requirements, but the increase in the PD of polluting firms has a rather persistent impact on risk weighted assets.

When comparing the two cases – with and without climate prudential policies – we can see that the introduction of climate prudential policies weakens the negative effect of the transition shock. In response to an increase in the riskiness of the more polluting firms, banks deleverage more with respect to this type of firms when the policy is implemented. This clearly shows that regulation increases the sensitivity of banks to transition risk. At the same time, lending to LCI firms is rather stable with respect to its steady state value when climate prudential policies are implemented. The penalty seen before for this type of firm is thus alleviated with climate prudential policies in place. Notice that when banks are not obliged to distinguish between HCI and LCI firms, the lending rate is the same for both of them. With policy intervention, LCI firms obtain more favorable financing conditions than HCI firms for which the increase in the lending rate is more important than in the no policy case. Enforcing climate-related prudential policies leads thus to more equitable financing conditions, riskier firms having to pay higher interest rates on their credit. Total lending decreases less under the policy case (mainly driven by credit to more polluting companies). The difference is of approximately 15 basis points at its maximum, implying less consequences on the real economy when climate prudential policy is in place. Furthermore, in the case where these type of policies are enforced, we assume that regulation is relaxed with respect to polluting firms if a transition shock would materialize. This translates into a release of capital requirements allowing banks to diverge from the steady state ratio with lower bank capital for several periods. At the same time, risk weighted assets decrease for the first period as lending to HCI firms decreases, to increase immediately afterwards when the decrease in lending is more than compensated by the increase in the risk weights applied to polluting firms. In the simulations, it is assumed that the relaxation of prudential policies is in magnitude of about 30bps (which is close but below the release of other capital requirements in previous crises). Due to the relaxation of the capital requirements, in the policy case the CET1 ratio slightly increases in the beginning of the simulation period, to decrease immediately afterwards. Overall, implementing climate prudential policies to account for transition risk seems beneficial for the financial system as banks better internalize the actual credit risk of their exposures. Different values of the capital requirements release would lead to similar results. However, in this simulation we do not provide a precise estimate of the size of the excess in credit risk generated by transition risk. As mentioned earlier, the model is calibrated with values already used in the literature but due to lack of data we cannot estimate the distribution of transition risk shocks. In the current example, we assume that the size of the transition risk shock is of half the standard deviation of the calibrated riskiness of firms in Darracq Paries et al. (2019). This reflects the idea that transition risk is currently a less relevant source of credit risk, but at the same time it allows us to work with realistic magnitudes despite the lack of data for the calibration.

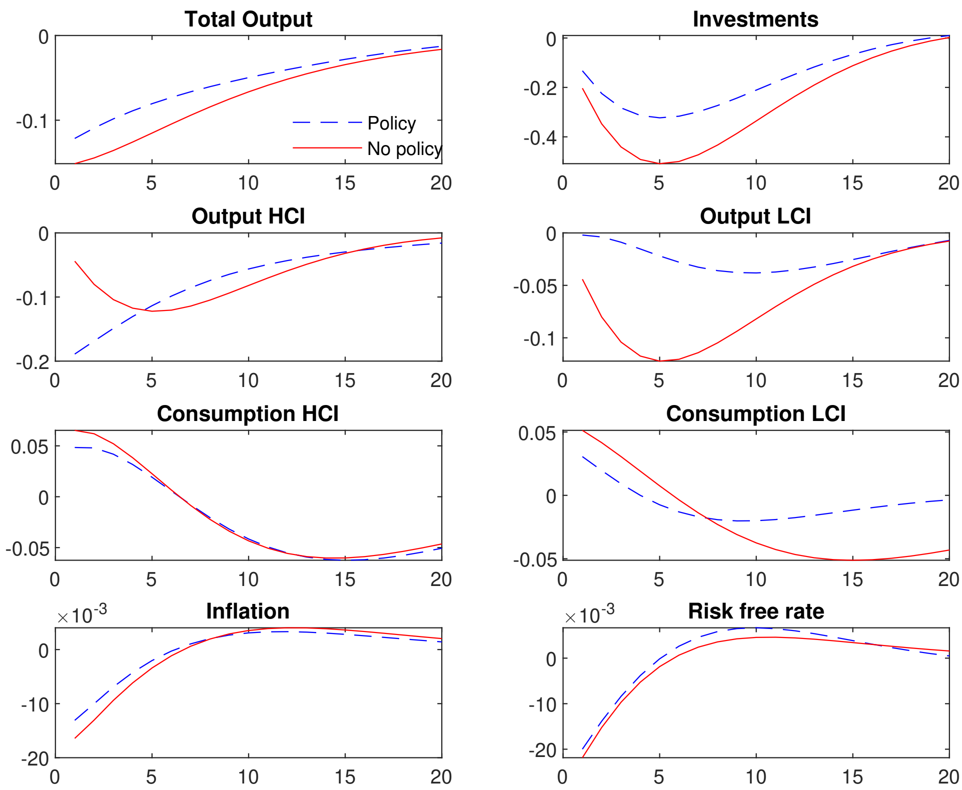
Figure 3: Impulse response functions for the financial sector [% deviation from the baseline; for the CET1 ratio pp difference from the baseline]



Note: The figure presents the IRFs of financial variables to a 1 standard deviation shock to $\sigma_{\omega^{HCI}}$, the riskiness of HCI firms.

Figure 4 shows the response of the real economy for the above mentioned exercise. As expected, the increase in transition risk of HCI borrowers translates in lower total output and investment, reflecting the deleveraging of banks and the higher cost of credit in both sectors. In the case where climate prudential policies are implemented, the impact on the real economy is better contained, as investment decreases less. The decline in output is mainly driven by the investment level, as consumption of both low and high carbon intensive goods tends to support output (recall that it is assumed here that consumers do not change their preferences). Looking at the production breakdown, it is relevant to notice that the output of polluting firms decreases a lot more when climate prudential policy is implemented, whereas the production of less polluting goods is only slightly impacted. However, without policies in place the production in both sectors would considerably diminish, as banks would deleverage in a similar way on all exposures. Finally, as inflation and output diminish in the economy, the monetary authority reduces its interest rate, but the impact is less pronounced when explicit transition risk policies are implemented.

Figure 4: Impulse response functions for the real sector [% deviation from the baseline]

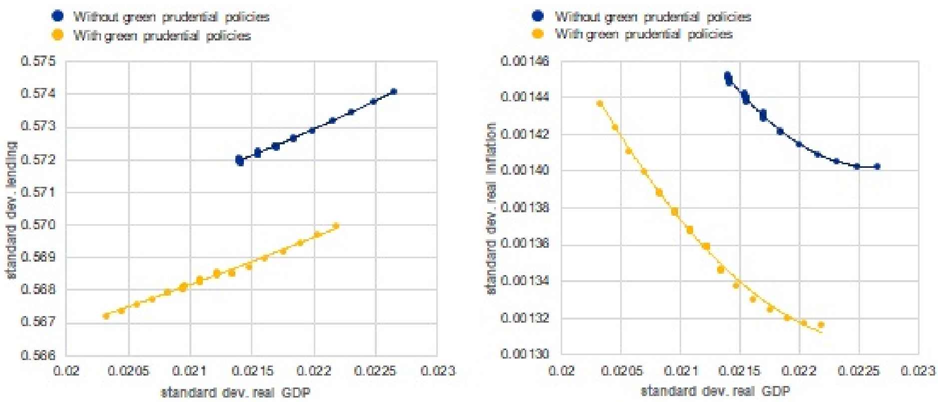


Note: The figure presents the IRFs of real variables to a 1 standard deviation shock to $\sigma_{\omega^{HCI}}$, the riskiness of HCI firms.

In order to highlight the stabilisation trade-offs, and show more general results, we compare the efficiency policy frontiers implied by the model with and without climate prudential policies. The efficiency policy frontier (EPF) portrays the surface where it is not possible to attain lower variance in one objective variable without increasing the one of the others. In the case under consideration, the efficiency policy frontiers reflect the three target variables of the monetary and macroprudential policy authorities: inflation, output gap and credit volatilities. The efficiency policy frontiers abstracts from the specific loss function and the strategic interaction between the monetary and macroprudential authorities and the 'best' allocation is given by the intersection with the efficiency policy frontiers and the authorities loss function/s. We conducted two simulations assuming either that climate prudential policies are implemented or that they are not. In each case, we optimized the monetary policy rule in terms of the coefficient of the inflation gap and of the output gap. The model does not specify any particular rule for the climate prudential policies, but we assume either that they are not implemented at all or that they allow a perfect identification of transition risk. In all cases, we activate the transition shocks. The resulting standard deviations for inflation, output gap and credit for each coefficient combination are illustrated in Figure 5. Depending on the assumptions about the loss function/s of the monetary authority and the activation of climate prudential policies, we can select a different point on each efficiency policy frontier to be the best.

In Figure 5, we can observe that the activation of climate prudential policies leads overall to lower output, inflation and lending volatility in equilibrium. We can derive some relevant policy conclusions from Figure 5 even without being specific about the loss functions of the policy authorities or about the strategic interactions among the authorities. Introducing climate prudential policies Pareto dominates the institutional configuration without them and this induces an inward shift in the efficiency frontier. In addition, this also enables monetary policy to achieve a superior performance in terms of macroeconomic stabilisation.

Figure 5: Efficiency frontier between output-lending policy outcomes (left) and inflation-output outcomes (right)



Note: The chart represents the efficiency policy frontier for given shocks to borrower riskiness. The efficiency frontier portrays, for all sets of policymaker's preferences, the lending, output and inflation volatilities implied by the corresponding optimised rules.

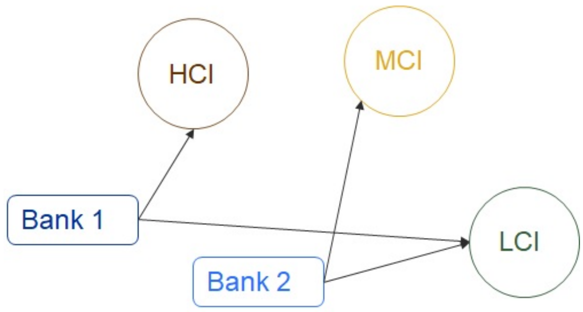
5 Macroprudential regulation to tackle transition risk

While in the previous section we showed the need of climate prudential policies, in this section we look more in detail at the specific type of policy instrument and more precisely at the benefits from macroprudential climate policies. In the previous section, we assumed that climate prudential policies would enforce the identification of firm-specific transition risks and we showed that even in the simplest case with homogeneous banks and two types of firms, this would generate a welfare improvement. In practice, this could be achieved by different types of policy instruments, but mostly microprudential instruments can lead to this objective. In this section, we investigate whether macroprudential instruments would also be needed to achieve a better outcome in terms of financial stability. We argue that transition risk can have potential systemic effects that cannot be captured by microprudential regulation alone and this would call for the implementation of climate macroprudential policies.

To illustrate the need of climate macroprudential policies in addition to microprudential ones, we use a version of the model with three types of firms and heterogeneous banking groups. The main idea is that if banks do not have the same loan book exposures, they might not fully internalize some possible systemic effect of transition risk when looking only at the transition risk of their individual exposures. For instance, transition shocks to part of the corporate sector can accelerate a shift in demand preferences which could negatively affect also other firms. In turn, this could affect banks not directly exposed to the firms hit by the transition shock. If capital requirements are based on the carbon intensity of each bank's exposures, then the banking system would not have sufficient capital to absorb the losses of the aggregate demand shock.

To show a possible case where this externality appears, it is sufficient to assume that there are three types of firms ($i = 3$) - firms with low, medium or high carbon intensity - and two types of banking groups. A key but realistic assumption is that banks have different loan book exposures such that each banking group is exposed only to two types of firms (Figure 6). For instance, both banks provide credit to the low carbon intensive firms (LCI), but each of them provides credit either to the high (HCI) or to the medium (MCI) carbon intensive firms. This assumption is realistic as European data shows that banks do not have homogeneous loan book exposures as they tend to specialize in some industries depending on their geographical location and their historical background (Figure 1, right panel). For example, German banks are more exposed to cement manufacturers than banks in other countries, Dutch banks are more exposed to oil producers than other euro area banks. This heterogeneity in banks' loan books implies that if only climate microprudential policies are implemented, banks exposed to MCI firms are required to hold less bank capital than banks giving credit to HCI firms. However, a shock to the HCI firm might have negative spillovers on MCI firms for example, via aggregate demand effects *i.e.* a drop in demand for goods produces by both HCI and MCI firms. Climate macroprudential policies would help hedging against these potential negative spillovers which might not be internalized by banks if they would only focus on the carbon intensity and transition risk of their own individual exposures.

Figure 6: Version of the model used to compare the institutional setting with only microprudential climate policies and with micro- and macroprudential climate policies



Note: HCI, MCI, LCI indicate firms with high, medium and low carbon intensity respectively.

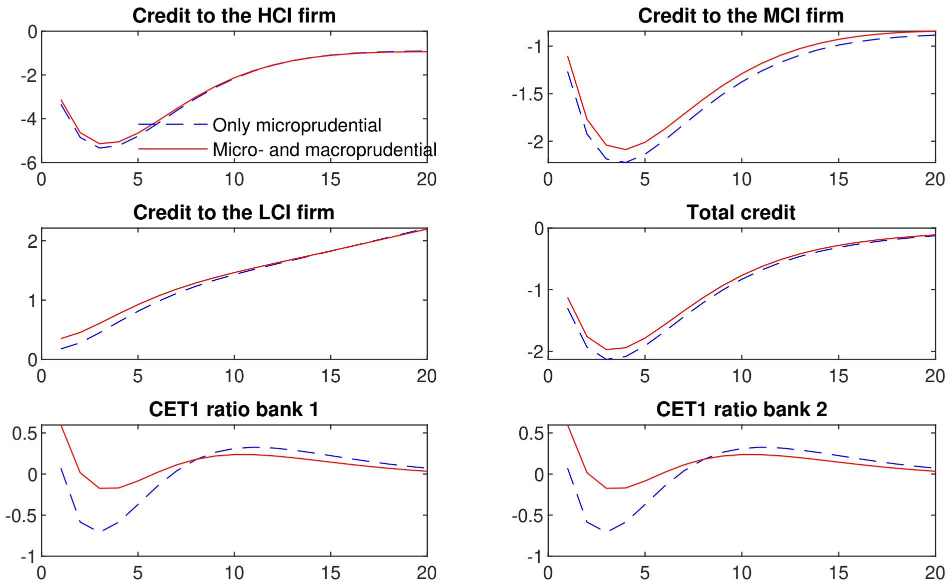
To conduct our analysis, we compare the results from the model where banks are subject only to microprudential climate policies and the case where banks are subject to supplementary macroprudential climate policies. In our framework, microprudential policies imply that risk weights fully reflect the transition risk on individual banks' exposures, whereas macroprudential policies imply an add-on across banks which reflects potential aggregate demand effects triggered by transition risk shocks. As in the previous section, we assume that when the transition shock materializes, prudential requirements are relaxed. When only microprudential policies are implemented, we assume that only capital requirements on the banks exposed to the firms hit directly by transition risk (HCI firms) are relaxed. When micro and macroprudential policies are applied, we assume that there is in addition an overall reduction in capital requirements for all banks. In the simulations, we assume the same magnitude of the release of the requirements as in the previous section for both micro- and macroprudential policies.[15] It is important to notice that in the model, households' preferences are subject to a shock ϵ^c (see eq. 3) which we assume here to be correlated with the transition risk of the HCI firms. However, this correlation is not perfectly observable by banks. In particular, we consider the case where a transition risk shock to HCI exposures would accelerate a shift in demand toward LCI exposures.[16] The results of these simulations are reported in Figure 7. The overall dynamic of the model's variables is similar in both cases to the one illustrated in the previous section. However, overall we can see that the introduction of additional macroprudential climate policies reduces the volatility of the responses

[15] As before, we do not present a calibration of the optimal requirements due to lack of data for the calibration.
[16] This implies thus that consumer turn away from both HCI and MCI goods.

of all the variables to the exogenous transition shock. In particular, with only microprudential climate policies, MCI firms would default more than what the bank would expect as they would not internalize that aggregate demand of MCI firms' products would decline. This leads to a stronger sensitivity of lending to MCI firms when only microprudential policies are implemented. This risk would be instead hedged by macroprudential climate policies. For this reason, when macroprudential climate policies are also implemented, the sensitivity of lending to MCI firms is lower when a transition shock to HCI firms materializes. Despite the increased demand of LCI goods and credit to LCI firms, the overall lending would initially decline driven by the increased riskiness of HCI and MCI firms. The relaxation of capital requirements in both cases leads to an initial increase in the CET1 ratio (driven by lower risk weights), but the increase is larger in the case where macroprudential policies are also implemented. The CET1 ratio then declines due to the increased riskiness of credit exposures and subsequently rebounds as banks deleverage and banks' profitability recovers.

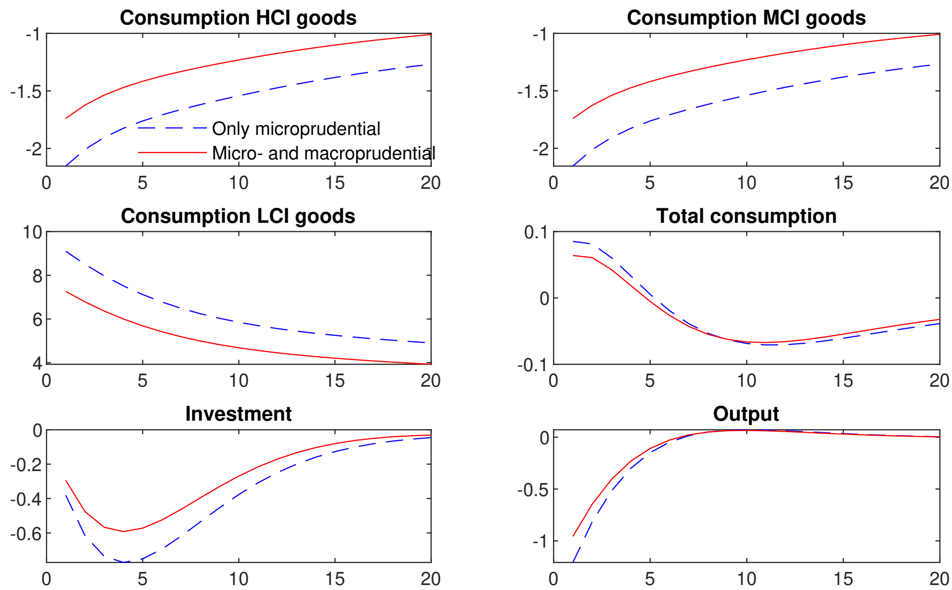
In Figure 8, we report the response of the real economy. As mentioned earlier, the transition shock triggers a shift in consumption towards LCI firms' products. However, overall consumption initially declines as well as investments. The decline is stronger for the case where only microprudential climate policies are implemented, reflecting banks' tighter credit conditions. As a consequence, we can also see a stronger decline in output when only microprudential climate policies are enforced.

Figure 7: Impulse response functions for the banking sector [% deviation from the baseline; for the CET1 ratio pp difference from the baseline]



Note: The figure presents the IRFs of banking sector variables to a 1 standard deviation shock to σ_{ω^B}, the riskiness of HCI firms assuming (i) that only climate prudential policies are implemented (dashed blue line) and (ii) that both micro and macroprudential policies are implemented (red solid line).

Figure 8: Impulse response functions for the real sector [% deviation from the baseline]



Note: The figure presents the IRFs of real variables to a 1 standard deviation shock to σ_{ω^B}, the riskiness of HCI firms assuming (i) that only climate prudential policies are implemented (dashed blue line) and (ii) that both micro and macroprudential policies are implemented (red solid line).

6 Conclusion

In this paper, we evaluate the need of introducing climate prudential policies and the design of such policies to tackle transition risk in the banking sector. Differently from most of the previous literature, our conclusions are based only on financial stability (credit risk) considerations, acknowledging that this is the unique objective of the prudential authority. To this end, we develop a DSGE model with heterogeneous firms and banks, financial frictions in the credit market and prudential regulation that accounts for climate transition risk. Firms in the model are classified based on their carbon emission intensity and they are subject to idiosyncratic shocks determining their credit risk. However, the default probability is higher for more carbon intensive firms, as they are also subject to emission taxes which increase their costs. This represents the main source of transition risk in the model.

By means of a stylized version of the model with two types of firms – high and low polluters – and a single banking group, we first compare the results of the model in the presence of a transition shock affecting the riskiness of the most carbon intensive firms with and without policy intervention. In the absence of climate prudential policies, we suppose that banks do not distinguish between firms with different degrees of carbon intensity when calculating their capital requirements. In our framework, we model this by assuming that banks estimate credit risk on the pool of high and low carbon intensive firms without identifying the impact of transition risk separately on each type of firm.

We show that without climate prudential regulation, banks are not covering fully for the credit risk driven by transition risk. At a macroeconomic level, output drops more when prudential regulation does not address climate transition risk. We also show that introducing climate prudential policies Pareto dominates the institutional configuration without it as this induces an inward shift in the efficiency frontier, lowering among other things the volatility of credit. In addition, this also enables monetary policy to achieve a superior performance in terms of macroeconomic stabilisation.

To argue that macroprudential climate policies are needed in addition to microprudential ones, we show that when banks have heterogeneous credit risk exposures to different types of firms, they might not internalize negative aggregate demand effects which could spillover from the materialization of transition shocks hitting only part of the production sector (high carbon intensive goods). For example, the materialization of transition risk can accelerate a shift in consumers' preferences reducing the demand of other carbon (medium) intensive goods. In this case, the credit risk of other banks would also be indirectly affected by transition shocks not hitting directly their own exposures. We show that climate macroprudential policies in addition to microprudential ones would lead to a better overall outcome than the simple utilization of microprudential policies.

Our work contributes to the overall policy debate on the need of introducing climate prudential policies in order to safeguard future financial stability. Moreover, concerning the design of policy, we show that regulators can achieve better outcomes if microprudential and macroprudential climate policies are applied jointly.

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A Annex

A.1 Households FOCs

The Lagrangian of the households writes:

$$\mathcal{L}_t^H = \mathbb{E}_t\Bigg\{\sum_{t=0}^{\infty}\beta^{H^t}\Bigg[\Bigg(\frac{C_t^{H^{1-\gamma}}}{1-\gamma}-\sum_i \overline{L^i}\frac{L_t^{i^{1+\sigma^i}}}{1+\sigma^i}\Bigg)-$$
$$\lambda_t^{H,BC}\Bigg(\sum_i P_t^i C_t^{H,i}+P_t D_t^H - P_t\frac{1+R_{t-1}}{1+\pi_t}D_{t-1}^H$$
$$-\sum_i w_t^i L_t^i - \Pi_t^H - G_t\Bigg)\Bigg]\Bigg\} \tag{37}$$

with:

$$C_t^H = \left[\sum_i \epsilon_t^{c,i}\kappa^{i^{\frac{1}{\varepsilon}}} C_t^{H,i^{\frac{\varepsilon-1}{\varepsilon}}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{38}$$

The maximization problem implies the following FOCs:

$$C_t^{H,i} \ : \ \epsilon_t^{c,i}\kappa^{i^{\frac{1}{\varepsilon}}} C_t^{H^{\frac{1}{\varepsilon}-\gamma}} C_t^{H,i^{-\frac{1}{\varepsilon}}} - \lambda_t^{H,BC}P_t^i = 0 \tag{39}$$

$$D_t^H \ : \ P_t\lambda_t^{H,BC} - \beta^H\mathbb{E}_t\left(\lambda_{t+1}^{H,BC}P_{t+1}\frac{1+R_t}{1+\pi_{t+1}}\right) = 0 \tag{40}$$

$$L_t^i \ : \ -\overline{L^i}L_t^{i^{\sigma^i}} + \lambda_t^{H,BC}w_t^i = 0 \tag{41}$$

$$\lambda_t^{H,BC} \ : \ \sum_i P_t^i C_t^{H,i} + P_t D_t^H - P_t\frac{1+R_{t-1}}{1+\pi_t}D_{t-1}^H - \sum_i w_t^i L_t^i - \Pi_t^H - G_t = 0 \tag{42}$$

At the steady state, the FOCs imply the following:

$$C^{H,i} \quad : \quad {\kappa^i}^{\frac{1}{\varepsilon}} {C^H}^{\frac{1}{\varepsilon}-\gamma} {C^{H,i}}^{-\frac{1}{\varepsilon}} - \lambda^{H,BC} P^i = 0 \tag{43}$$

$$D^H \quad : \quad 1 - \beta^H (1+R) = 0 \tag{44}$$

$$L^i \quad : \quad -\overline{L^i} {L^i}^{\sigma^i} + \lambda^{H,BC} w^i = 0 \tag{45}$$

$$\lambda^{H,BC} \quad : \quad \sum_i P^i C^{H,i} - P\ R\ D^H - \sum_i w^i L^i - \Pi^H - G = 0 \tag{46}$$

From eq. 43, we obtain the following condition for each pair of goods:

$$\frac{C^{H,i}}{C^{H,j}} = \frac{\kappa^i}{\kappa^j} \left(\frac{P^j}{P^i} \right)^{\varepsilon} \tag{47}$$

A.2 Entrepreneur FOCs

The Lagrangian of each type of entrepreneur i wirtes:

$$\begin{aligned}
\mathcal{L}_t^{E,i} &= \mathbb{E}_t \Bigg\{ \sum_{t=0}^{\infty} \beta^{E^t} \Bigg\{ \frac{{C_t^{E,i}}^{1-\gamma}}{1-\gamma} - \lambda_t^{E,i,BC} \Bigg[\sum_j P_t^j C_t^{E,i,j} + \\
&\quad + Q_t^i \left(K_t^i - (1-\delta) K_{t-1}^i \right) + w_t^i L_t^i + \Phi_i(u_t^i) K_{t-1}^i + H(\overline{\omega}_t^i)(1-\delta) Q_t^i K_{t-1}^i + \\
&\quad + \eta^i \left(\epsilon_t^i \left(u_t^i K_{t-1}^i \right)^a \left(L_t^i \right)^{1-a} \right)^{\zeta} - \mu_t^i P_t^i \epsilon_t^i \left(u_t^i K_{t-1}^i \right)^a \left(L_t^i \right)^{1-a} - P_t B_t^{E,i} \Bigg] + \\
&\quad - \lambda_t^{E,i,PC} \left[\frac{1 + R_{t-1}^{F,i}}{1+\pi_t} P_t B_{t-1}^{E,i} - (1-\delta) Q_t^i K_{t-1}^i G(\overline{\omega}_t^i) \right] \Bigg\} \Bigg\}
\end{aligned} \tag{48}$$

with:

$$C_t^{E,i} = \left[\sum_j {\theta^j}^{\frac{1}{\varepsilon}} {C_t^{E,i,j}}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{49}$$

and where:

$$H(\overline{\omega}_t^i) = \mathcal{N}_{\mathcal{CDF}} \left(\frac{log(\overline{\omega}_t^i) - 0.5\sigma_{\omega^i}^2}{\sigma_{\omega^i}} \right) + \overline{\omega}_t^i \left(1 - \mathcal{N}_{\mathcal{CDF}} \left(\frac{log(\overline{\omega}_t^i) + 0.5\sigma_{\omega^i}^2}{\sigma_{\omega^i}} \right) \right) \tag{50}$$

$$G(\overline{\omega}_t^i) = (1-\rho) \mathcal{N}_{\mathcal{CDF}} \left(\frac{log(\overline{\omega}_t^i) - 0.5\sigma_{\omega^i}^2}{\sigma_{\omega^i}} \right) + \overline{\omega}_t^i \left(1 - \mathcal{N}_{\mathcal{CDF}} \left(\frac{log(\overline{\omega}_t^i) + 0.5\sigma_{\omega^i}^2}{\sigma_{\omega^i}} \right) \right) \tag{51}$$

The maximization problem implies the following FOCs:

$$C_t^{E,i,j} \quad : \quad \theta^{j^{\frac{1}{\varepsilon}}} C_t^{E,i^{\frac{1}{\varepsilon}-\gamma}} C_t^{E,i,j^{-\frac{1}{\varepsilon}}} - \lambda_t^{E,i,BC} P_t^j = 0 \tag{52}$$

$$\begin{aligned} L_t^i \quad : \quad & w_t^i + \eta^i \left(\epsilon_t^i \left(u_t^i K_{t-1}^i \right)^a \right)^\zeta \zeta(1-a) \left(L_t^i \right)^{\zeta(1-a)-1} + \\ & -\mu_t^i P_t^i \epsilon_t^i \left(u_t^i K_{t-1}^i \right)^a (1-a) \left(L_t^i \right)^{-a} = 0 \end{aligned} \tag{53}$$

$$\begin{aligned} K_t^i \quad : \quad & -\lambda_t^{E,i,BC} Q_t^i - \beta^E \mathbb{E}_t \Bigg[\lambda_{t+1}^{E,i,BC} \Bigg[- Q_{t+1}^i (1-\delta) + \Phi_B(u_{t+1}^i) + \\ & + H(\overline{\omega}_{t+1}^i)(1-\delta) Q_{t+1}^i + \eta^i \left(\epsilon_{t+1}^i \left(u_{t+1}^i \right)^a \left(L_{t+1}^i \right)^{1-a} \right)^\zeta a\zeta \left(K_t^i \right)^{a\zeta-1} + \\ & - \mu_{t+1}^i P_{t+1}^i \epsilon_{t+1}^i \left(u_{t+1}^i \right)^a \left(L_{t+1}^i \right)^{1-a} a \left(K_t^i \right)^{a-1} \Bigg] \Bigg] + \\ & -\beta^E \mathbb{E}_t \Bigg[- \lambda_{t+1}^{E,i,PC} (1-\delta) Q_{t+1}^i G(\overline{\omega}_{t+1}^i) \Bigg] = 0 \end{aligned} \tag{54}$$

$$\begin{aligned} u_t^i \quad & : \Phi_i'(u_t^i) K_{t-1}^i + \eta \left(\epsilon_t^i \left(K_{t-1}^i \right)^a \left(L_t^i \right)^{1-a} \right)^\zeta a\zeta \left(u_t^i \right)^{a\zeta-1} + \\ & -\mu_t^i P_t^i \epsilon_t^i \left(K_{t-1}^i \right)^a \left(L_t^i \right)^{1-a} a \left(u_t^i \right)^{a-1} = 0 \end{aligned} \tag{55}$$

$$B_t^{E,i} \quad : \quad P_t \lambda_t^{E,i,BC} - \beta^E \mathbb{E}_t \left(\lambda_{t+1}^{E,i,PC} \frac{1+R_t^{F,i}}{1+\pi_{t+1}} P_{t+1} \right) = 0 \tag{56}$$

$$\overline{\omega}_t^i \quad : \quad -\lambda_t^{E,i,BC} H'(\overline{\omega}_t^i) + \lambda_t^{E,i,PC} G'(\overline{\omega}_t^i) = 0 \tag{57}$$

$$\begin{aligned} \lambda_t^{E,i,BC} \quad : \quad & \sum_j P_t^j C_t^{E,i,j} + Q_t^i \left(K_t^i - (1-\delta) K_{t-1}^i \right) + \\ & w_t^i L_t^i + \Phi_i(u_t^i) K_{t-1}^i + H(\overline{\omega}_t^i)(1-\delta) Q_t^i K_{t-1}^i + \\ & \eta^i \left(\epsilon_t^i \left(u_t^i K_{t-1}^i \right)^a \left(L_t^i \right)^{1-a} \right)^\zeta - \mu_t^i P_t^i \epsilon_t^i \left(u_t^i K_{t-1}^i \right)^a \left(L_t^i \right)^{1-a} \\ & -P_t B_t^{E,i} = 0 \end{aligned} \tag{58}$$

$$\tag{59}$$

$$\lambda_t^{E,i,PC} \quad : \quad \frac{1+R_{t-1}^{F,i}}{1+\pi_t} P_t B_{t-1}^{E,i} - (1-\delta) Q_t^i K_{t-1}^i G(\overline{\omega}_t^i) = 0 \tag{60}$$

At the steady state, the FOCs can be rewritten as:

$$C^{E,i,j} \quad : \quad \theta^{j\frac{1}{\varepsilon}} C^{E,i\frac{1}{\varepsilon}-\gamma} C^{E,i,j-\frac{1}{\varepsilon}} - \lambda^{E,i,BC} P^j = 0 \tag{61}$$

$$L^i \quad : \quad w^i L^i + (1-a)\left(\eta^i \zeta Z^{i^\zeta} - \mu^i P^i Z^i\right) = 0 \tag{62}$$

$$\begin{aligned} K^i \quad : \quad & -\lambda^{E,i,BC} Q^i K^i - \beta^E \Bigg[\lambda^{E,i,BC}\Bigg[-(1-\delta)Q^i K^i + \Phi_i(u^i)K^i + \\ & + H(\overline{\omega}^i)(1-\delta)Q^i K^i + a\left(\eta^i \zeta Z^{i^\zeta} - \mu^i P^i Z^i\right)\Bigg]\Bigg] + \\ & -\beta^E \Bigg[-\lambda^{E,i,PC}(1-\delta)Q^i K^i G(\overline{\omega}^i)\Bigg] = 0 \end{aligned} \tag{63}$$

$$u^i \quad : \Phi_i'(u^i) u^i K^i + a\left(\eta^i \zeta Z^{i^\zeta} - \mu^i P^i Z^i\right) = 0 \tag{64}$$

$$B^{E,i} \quad : \quad \lambda^{E,i,BC} - \beta^E \lambda^{E,i,PC}\left(1+R^{F,i}\right) = 0 \tag{65}$$

$$\overline{\omega}^i \quad : \quad -\lambda^{E,i,BC} H'(\overline{\omega}^i) + \lambda^{E,i,PC} G'(\overline{\omega}^i) = 0 \tag{66}$$

$$\begin{aligned} \lambda^{E,i,BC} \quad : \quad & \sum_j P^j C^{E,i,j} + \delta Q^i K^i + w^i L^i + \Phi_i(u^i)K^i + \\ & H(\overline{\omega}^i)(1-\delta)Q^i K^i + \left(\eta Z^{i^\zeta} - \mu^i P^i Z^i\right) - P\ B^{E,i} = 0 \end{aligned} \tag{67}$$

$$\lambda^{E,i,PC} \quad : \quad \left(1+R^{F,i}\right) P B^{E,i} - (1-\delta) Q^i K^i G(\overline{\omega}^i) = 0 \tag{68}$$

From eq. 61, we obtain for any couple of products k, j:

$$\frac{C^{E,i,k}}{C^{E,i,j}} = \frac{\theta^k}{\theta^j}\left(\frac{P^j}{P^k}\right)^\varepsilon \tag{69}$$

From eq. 62 and 64 and using the fact that $u^i = 1 \Rightarrow \Phi_i'(u^i) = r^{i,SS}$ at the steady state, we obtain:

$$\frac{u^i K^i}{L^i} = \frac{a w^i}{(1-a) r^{i,SS}} \tag{70}$$

which together with the the zero profit condition in eq. 12, implies:

$$\mu^i P^i - T(e_t^i) = \frac{{w^i}^{1-a} {r^{i,SS}}^a}{a^a (1-a)^{1-a}} \tag{71}$$

$$\Leftrightarrow w^i = \left((\mu^i P^i - T(e_t^i)) {r^{i,SS}}^{-a} a^a (1-a)^{1-a} \right)^{1/(1-a)} \tag{72}$$

From eq. 65 and 66, we obtain:

$$\frac{G'(\overline{\omega}^i)}{H'(\overline{\omega}^i)} \quad = \quad \beta^E \left(1 + R^{F,i}\right) \tag{73}$$

In addition using the fact that $\Phi_i(u^i) = 0$ at the steady state, we can rewrite eq. 63 as:

$$Q^i K^i [\beta^E (1-\delta) - 1] + \beta^E (1-\delta) Q^i K^i \left(\frac{\lambda^{E,i,PC}}{\lambda^{E,i,BC}} G(\overline{\omega}^i) - H(\overline{\omega}^i \right) -$$

$$\beta^E a \left(\eta^i \zeta {Z^i}^\zeta - \mu^i P^i Z^i \right) = 0 \tag{74}$$

Using the fact that $Q^i = 1$ and $\frac{\lambda^{E,i,PC}}{\lambda^{E,i,BC}} = \frac{H'(\overline{\omega}^i)}{G'(\overline{\omega}^i)}$ and that the marginal return to capital from the production, $r^{i,SS}$ is $(\mu^i P^i - \eta^i) a {K^i}^{a-1} {L^i}^{(1-a)} = (\mu^i P^i - \eta) a Z^i / K^i$ and for $\zeta = 1$, we obtain:

$$[\beta^E (1-\delta) - 1] + \beta^E (1-\delta) \left[\frac{H'(\overline{\omega}^i)}{G'(\overline{\omega}^i)} G(\overline{\omega}^i) - H(\overline{\omega}^i) \right] + \beta^E r^{i,SS} = 0 \tag{75}$$

$$\Leftrightarrow r^{i,ss} = \frac{1}{\beta^E} - (1-\delta) - (1-\delta) \left[\frac{H'(\overline{\omega}^i)}{G'(\overline{\omega}^i)} G(\overline{\omega}^i) - H(\overline{\omega}^i) \right] \tag{76}$$

A.3 Capital good producers FOCs

Substituting the law of motion of capital:

$$\max_{\{I_t^i\}} \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[Q_t^i \left[(1-\delta)K_{t-1}^i + \left[1 - S\left(\frac{I_t^i \epsilon_t^I}{I_{t-1}^i} \right) \right] I_t^i - (1-\delta)K_{t-1}^i \right] - I_t^i \right] \right\}. \tag{77}$$

FOC:

$$I_t^i \quad : \quad Q_t^i \left[1 - S\left(\frac{I_t^i \epsilon_t^I}{I_{t-1}^i} \right) \right] - \frac{I_t^i \epsilon_t^I}{I_{t-1}^i} S'\left(\frac{I_t^i \epsilon_t^I}{I_{t-1}^i} \right) + \beta \mathbb{E}_t \left[Q_{t+1}^i \left(\frac{I_{t+1}^i \epsilon_{t+1}^I}{I_t^i} \right)^2 S'\left(\frac{I_{t+1}^i \epsilon_{t+1}^I}{I_t^i} \right) \right] = 1 \tag{78}$$

At the steady state, the investment is given by:

$$I^i = \delta K^i \tag{79}$$

Furthermore, given that $S(1) = 0$ and $S'(1) = 0$, from eq. 78, we obtain the price of capital at steady state:

$$Q^i = 1 \tag{80}$$

A.4 Retailers FOCs

The maximization of the retailers, $i = \{B, G\}$ writes:

$$\max_{\{\tilde{P}_t^i(j)\}} \sum_{k=0}^{\infty} \chi^k \mathbb{E}_t \left[\lambda_{t+k}^{R,i} \left[\tilde{P}_t^i(j) - \mu_{t+1}^i P_{t+k}^i \right] \left(\frac{\tilde{P}_t^i(j)}{P_{t+k}^i} \right)^{-\epsilon} Z_{t+k}^i \right] \tag{81}$$

<u>Proof for the recursive form:</u>

The derivative with respect to $\tilde{P}_t^i(j)$ is equal to:

$$\tilde{P}_t^i(j) \quad : \quad \sum_{k=0}^{\infty} \chi^k \mathbb{E}_t \left\{ \lambda_{t+k}^{R,i} \left[\left(\frac{\tilde{P}_t^i(j)}{P_{t+k}^i} \right)^{-\epsilon} Z_{t+k}^i - \epsilon \left(\tilde{P}_t^i(j) - \mu_{t+k}^i P_{t+k}^i \right) \frac{(\tilde{P}_t^i(j))^{-\epsilon-1}}{(P_{t+k}^i)^{-\epsilon}} Z_{t+k}^i \right] \right\} = 0 \tag{82}$$

Equivalently:

$$\sum_{k=0}^{\infty}\chi^k\mathbb{E}_t\left\{\lambda_{t+k}^{R,i}\left[\left(\frac{\tilde{P}_t^i(j)}{P_{t+k}^i}\right)^{-\epsilon}Z_{t+k}^i-\epsilon\left(\frac{\tilde{P}_t^i(j)}{P_{t+k}^i}\right)^{-\epsilon}Z_{t+k}^i+\epsilon\mu_{t+k}^i\left(\frac{\tilde{P}_t^i(j)}{P_{t+k}^i}\right)^{-\epsilon-1}Z_{t+k}^i\right]\right\}=0 \quad (83)$$

Or:

$$\sum_{k=0}^{\infty}\chi^k\mathbb{E}_t\left\{\lambda_{t+k}^{R,i}\left[(1-\epsilon)\left(\frac{\tilde{P}_t^i(j)}{P_{t+k}^i}\right)^{-\epsilon}Z_{t+k}^i+\epsilon\mu_{t+k}^i\left(\frac{\tilde{P}_t^i(j)}{P_{t+k}^i}\right)^{-\epsilon-1}Z_{t+k}^i\right]\right\}=0 \tag{84}$$

From the individual demand equation, we can substitute $\left(\frac{\tilde{P}_t^i(j)}{P_{t+k}^i}\right)^{-\epsilon}Z_{t+k}^i$ with $Z_{t+k}^i(j)$ and $\left(\frac{\tilde{P}_t^i(j)}{P_+kt^i}\right)^{-\epsilon-1}Z_{t+k}^i$ with $Z_{t+k}^i(j)\left(\frac{\tilde{P}_t^i(j)}{P_{t+k}^i}\right)^{-1}$ to obtain:

$$\sum_{k=0}^{\infty}\chi^k\mathbb{E}_t\left\{\lambda_{t+k}^{R,i}\left[(1-\epsilon)Z_{t+k}^i(j)+\epsilon\mu_{t+k}^i\left(\frac{\tilde{P}_t^i(j)}{P_{t+k}^i}\right)^{-1}Z_{t+k}^i(j)\right]\right\}=0 \tag{85}$$

Multiplying this equation by $\frac{\tilde{P}_t^i(j)}{1-\epsilon}$, we obtain:

$$\sum_{k=0}^{\infty}\chi^k\mathbb{E}_t\left\{\lambda_{t+k}^{R,i}Z_{t+k}^i(j)\left[\tilde{P}_t^i(j)-\frac{\epsilon}{\epsilon-1}\mu_{t+k}^iP_{t+k}^i\right]\right\}=0, \tag{86}$$

where $\frac{\epsilon}{\epsilon-1}$ represents the markup rate. Last, we divide by P_{t-1}^i in order to obtain the FOC as a function of inflation, $\Pi_{t-1,t+k}^i=\frac{P_{t+k}^i}{P_{t-1}^i}$:

$$\sum_{k=0}^{\infty}\chi^k\mathbb{E}_t\left\{\lambda_{t+k}^{R,i}Z_{t+k}^i(j)\left[\frac{\tilde{P}_t^i(j)}{P_{t-1}^i}-\frac{\epsilon}{\epsilon-1}\mu_{t+k}^i\Pi_{t-1,t+k}^i\right]\right\}=0 \tag{87}$$

To write the last equation in a recursive way, it is useful to separate it in two terms:

$$\sum_{k=0}^{\infty}\chi^k\mathbb{E}_t\left(\lambda_{t+k}^{R,i}Z_{t+k}^i(j)\frac{\tilde{P}_t^i(j)}{P_{t-1}^i}\right)=\sum_{k=0}^{\infty}\chi^k\mathbb{E}_t\left(\lambda_{t+k}^{R,i}Z_{t+k}^i(j)\frac{\epsilon}{\epsilon-1}\mu_{t+k}^i\Pi_{t-1,t+k}^i\right) \tag{88}$$

Replacing individual demand by aggregate demand, we obtain:

$$\frac{\tilde{P}_t^i(j)}{P_{t-1}^i}\sum_{k=0}^{\infty}\chi^k\mathbb{E}_t\left(\lambda_{t+k}^{R,i}\left(\frac{\tilde{P}_t^i(j)}{P_{t+k}^i}\right)^{-\epsilon}Z_{t+k}^i\right)=\frac{\epsilon}{\epsilon-1}\sum_{k=0}^{\infty}\chi^k\mathbb{E}_t\left(\lambda_{t+k}^{R,i}\left(\frac{\tilde{P}_t^i(j)}{P_{t+k}^i}\right)^{-\epsilon}Z_{t+k}^i\mu_{t+k}^i\Pi_{t-1,t+k}^i\right) \tag{89}$$

Defining Γ^i_{1t} and Γ^i_{2t} as follows:

$$\Gamma^i_{1t} = \sum_{k=0}^{\infty} \chi^k \mathbb{E}_t \left(\lambda^{R,i}_{t+k} \left(\frac{\tilde{P}^i_t(j)}{P^i_{t+k}} \right)^{-\epsilon} Z^i_{t+k} \mu^i_{t+k} \Pi^i_{t-1,t+k} \right) \tag{90}$$

$$\Gamma^i_{2t} = \sum_{k=0}^{\infty} \chi^k \mathbb{E}_t \left(\lambda^{R,i}_{t+k} \left(\frac{\tilde{P}^i_t(j)}{P^i_{t+k}} \right)^{-\epsilon} Z^i_{t+k} \right) \tag{91}$$

we can rewrite the FOC of the retailers:

$$\frac{\tilde{P}^i_t(j)}{P^i_{t-1}} = \frac{\epsilon}{\epsilon - 1} \frac{\Gamma_{1t}}{\Gamma_{2t}} \tag{92}$$

For the next step, we need to write Γ^i_{1t} and Γ^i_{2t} in a recursive way. For Γ^i_{1t}, we have:

$$\begin{aligned}
\Gamma^i_{1t} &= \sum_{k=0}^{\infty} \chi^k \mathbb{E}_t \left(\lambda^{R,i}_{t+k} \left(\frac{\tilde{P}^i_t(j)}{P^i_{t+k}} \right)^{-\epsilon} Z^i_{t+k} \mu^i_{t+k} \Pi^i_{t-1,t+k} \right) = \\
&= \left(\frac{\tilde{P}^i_t(j)}{P^i_t} \right)^{-\epsilon} \lambda^{R,i}_t Z^i_t \mu^i_t \Pi^i_{t-1,t} + \chi \mathbb{E}_t \left(\left(\frac{\tilde{P}^i_t(j)}{P^i_{t+1}} \right)^{-\epsilon} \lambda^{R,i}_{t+1} Z^i_{t+1} \mu^i_{t+1} \Pi^i_{t-1,t+1} \right) + \\
&\quad + \chi^2 \mathbb{E}_t \left(\left(\frac{\tilde{P}^i_t(j)}{P^i_{t+2}} \right)^{-\epsilon} \lambda^{R,i}_{t+2} Z^i_{t+2} \mu^i_{t+2} \Pi^i_{t-1,t+2} \right) + \dots \\
&= \left(\frac{\tilde{P}^i_t(j)}{P^i_t} \right)^{-\epsilon} \lambda^{R,i}_t Z^i_t \mu^i_t \Pi^i_{t-1,t} + \\
&\quad + \chi \mathbb{E}_t \left[\left(\frac{\tilde{P}^i_t(j)}{P^i_{t+1}} \right)^{-\epsilon} \left(\frac{P^i_{t+1}}{\tilde{P}^i_{t+1}(j)} \right)^{-\epsilon} \left(\frac{\tilde{P}^i_{t+1}(j)}{P^i_{t+1}} \right)^{-\epsilon} \lambda^{R,i}_{t+1} Z^i_{t+1} \mu^i_{t+1} \Pi^i_{t-1,t+1} + \right. \\
&\quad \left. + \chi \left(\frac{\tilde{P}^i_t(j)}{P^i_{t+2}} \right)^{-\epsilon} \left(\frac{P^i_{t+2}}{\tilde{P}^i_{t+1}(j)} \right)^{-\epsilon} \left(\frac{\tilde{P}^i_{t+1}(j)}{P^i_{t+2}} \right)^{-\epsilon} \lambda^{R,i}_{t+2} Z^i_{t+2} \mu^i_{t+2} \Pi^i_{t-1,t+2} + \dots \right] \\
&= \left(\frac{\tilde{P}^i_t(j)}{P^i_t} \right)^{-\epsilon} \lambda^{R,i}_t Z^i_t \mu^i_t \Pi^i_{t-1,t} + \chi \mathbb{E}_t \left[\left(\frac{\tilde{P}^i_t(j)}{\tilde{P}^i_{t+1}(j)} \right)^{-\epsilon} \Gamma^i_{1,t+1} \right]
\end{aligned} \tag{93}$$

with

$$
\begin{aligned}
\Gamma^i_{1,t+1} &= \left(\frac{\tilde{P}^i_{t+1}(j)}{P^i_{t+1}}\right)^{-\epsilon} \lambda^{R,i}_t Z^i_{t+1}\mu^i_{t+1}\Pi^i_{t-1,t+1} + \\
&+\chi\mathbb{E}_{t+1}\left(\left(\frac{\tilde{P}^i_{t+1}(j)}{P^i_{t+2}}\right)^{-\epsilon} \lambda^{R,i}_{t+2} Z^i_{t+2}\mu^i_{t+2}\Pi^i_{t-1,t+2}\right) + \\
&+\chi^2\mathbb{E}_{t+1}\left(\left(\frac{\tilde{P}^i_{t+1}(j)}{P^i_{t+3}}\right)^{-\epsilon} \lambda^{R,i}_{t+3} Z^i_{t+3}\mu^i_{t+3}\Pi^i_{t-1,t+3}\right) + \ldots \qquad (94)
\end{aligned}
$$

In a similar way, we can write Γ^i_{2t} in a recursive way:

$$
\begin{aligned}
\Gamma^i_{2t} &= \sum_{k=0}^{\infty}\chi^k\mathbb{E}_t\left(\lambda^{R,i}_{t+k}\left(\frac{\tilde{P}^i_t(j)}{P^i_{t+k}}\right)^{-\epsilon} Z^i_{t+k}\right) = \\
&= \left(\frac{\tilde{P}^i_t(j)}{P^i_t}\right)^{-\epsilon}\lambda^{R,i}_t Z^i_t + \chi\mathbb{E}_t\left(\left(\frac{\tilde{P}^i_t(j)}{P^i_{t+1}}\right)^{-\epsilon}\lambda^{R,i}_{t+1}Z^i_{t+1}\right) + \\
&+\chi^2\mathbb{E}_t\left(\left(\frac{\tilde{P}^i_t(j)}{P^i_{t+2}}\right)^{-\epsilon}\lambda^{R,i}_{t+2}Z^i_{t+2}\right) + \ldots \\
&= \left(\frac{\tilde{P}^i_t(j)}{P^i_t}\right)^{-\epsilon}\lambda^{R,i}_t Z^i_t + \\
&+\chi\mathbb{E}_t\left[\left(\frac{\tilde{P}^i_t(j)}{P^i_{t+1}}\right)^{-\epsilon}\left(\frac{P^i_{t+1}}{\tilde{P}^i_{t+1}(j)}\right)^{-\epsilon}\left(\frac{\tilde{P}^i_{t+1}(j)}{P^i_{t+1}}\right)^{-\epsilon}\lambda^{R,i}_{t+1}Z^i_{t+1} + \right. \\
&\left. + \chi\left(\frac{\tilde{P}^i_t(j)}{P^i_{t+2}}\right)^{-\epsilon}\left(\frac{P_{t+2}}{\tilde{P}^i_{t+1}(j)}\right)^{-\epsilon}\left(\frac{\tilde{P}^i_{t+1}(j)}{P^i_{t+2}}\right)^{-\epsilon}\lambda^{R,i}_{t+2}Z^i_{t+2} + \ldots\right] \\
&= \left(\frac{\tilde{P}^i_t(j)}{P^i_t}\right)^{-\epsilon}\lambda^{R,i}_t Z^i_t + \chi\mathbb{E}_t\left[\left(\frac{\tilde{P}^i_t(j)}{\tilde{P}^i_{t+1}(j)}\right)^{-\epsilon}\Gamma^i_{2,t+1}\right] \qquad (95)
\end{aligned}
$$

In the above equations, $\lambda^{R,i}_{t+k}$ represents the nominal discounted dividend distribution rate for period $t+k$. Given that the entire profit made by retailers is distributed as dividends to households (there is no net worth for retailers), the nominal dividend distribution rate is equal to one. The adequate discount factor for this rate, *i.e.* a discount factor based on the risk free rate of the economy, should be obtained from the households' optimization program, given that the later are the owners of retailers. We therefore use eq. (39) and (40) to obtain $\frac{1}{1+R_t}$:

$$\lambda_t^{R,i} = (1+\pi_{t+1})\beta^H \mathbb{E}_t \left[\frac{P_{t+1}}{P_t} \frac{P_t^i}{P_{t+1}^i} \left(\frac{C_{t+1}^H}{C_t^H} \right)^{\frac{1}{\varepsilon}-\gamma} \left(\frac{C_{t+1}^{H,i}}{C_t^{H,i}} \right)^{-\frac{1}{\varepsilon}} \right] \tag{96}$$

At the steady state, we obtain:

$$\lambda^{R,i} = \beta^H \tag{97}$$

From the FOC at steady state, we get:

$$\frac{\Gamma_1^i}{\Gamma_2^i} = \frac{\epsilon - 1}{\epsilon} \tag{98}$$

with

$$\Gamma_1^i = \frac{1}{1-\chi} \beta^H Z^i \mu^i \tag{99}$$

$$\Gamma_2^i = \frac{1}{1-\chi} \beta^H Z^i \tag{100}$$

Substituting eq. 99 and 100 in 98, we obtain the inverse of the markup in steady state:

$$\mu^i = \frac{\epsilon - 1}{\epsilon} \tag{101}$$

A.5 Commercial lending branches

$$\frac{\left(1+R_{t-1}^{E}\right)B_{t-1}^{E,k}P_t}{1+\pi_t} - \int_0^{\overline{\omega}_t^E}(1-\delta)Q_t^E K_{t-1}^E \omega_t^E dF(\omega_t^E) -$$

$$\frac{\left(1+R_{t-1}^{F,E}\right)B_{t-1}^{E,k}P_t}{1+\pi_t} + (1-\rho)\int_0^{\overline{\omega}_t^E}(1-\delta)Q_t^E K_{t-1}^E \omega_t^E dF(\omega_t^E) = 0 \tag{102}$$

$$\leftrightarrow \quad \frac{\left(1+R_{t-1}^{E}\right)B_{t-1}^{E}P_t}{1+\pi_t} - \frac{\left(1+R_{t-1}^{F,E}\right)B_{t-1}^{E,k}P_t}{1+\pi_t} = \left[H(\overline{\omega}_t^E) - G(\overline{\omega}_t^E)\right](1-\delta)Q_t^E K_{t-1}^E \tag{103}$$

$$\leftrightarrow \quad R_{t-1}^{E} = R_{t-1}^{F,E} + \frac{1+\pi_t}{B_{t-1}^{E}P_t}\left[H(\overline{\omega}_t^E) - G(\overline{\omega}_t^E)\right](1-\delta)Q_t^E K_{t-1}^E \tag{104}$$

A.6 Steady state assuming two types of entrepreneurs

Let us consider 2 types of entrepreneurs G and B. Assumptions for the steady state

$$L^G + L^B = 1 \tag{105}$$

$$C^{E,B,B} = C^{E,G,B} = cssC^{H,B} \tag{106}$$

where css is a very small parameter ensuring that entrepreneurs consumption does not drive aggregate demand. From eq. 21 , we obtain

$$Z^B \quad = \quad C^{H,B}(1 + 2css) + \delta K^B + \eta^B \left(Z^B\right)^\zeta \tag{107}$$

$$Z^G \quad = \quad C^{H,G} + 2\left(\frac{\theta^G}{\theta^B}\right)\left(\frac{P^B}{P^G}\right)^\varepsilon cssC^{H,B} + \delta K^G + \eta^G \left(Z^G\right)^\zeta \tag{108}$$

From eq. 70 and $u^i = 1$, we know that

$$\frac{K^i}{L^i} = \frac{aw^i}{(1-a)r^{SS}} \tag{109}$$

which implies

$$\frac{Z^i}{L^i} = \left(\frac{aw^i}{(1-a)r^{SS}}\right)^a \tag{110}$$

substituting in eq. 107, we obtain

$$\begin{aligned} C^{H,B} \quad &= \quad -\frac{\delta}{(1+2css)}\frac{aw^B}{(1-a)r^{B,SS}}L^B + \frac{1}{(1+2css)}\left(\frac{aw^B}{(1-a)r^{B,SS}}\right)^a L^B - \\ &\quad \frac{\eta^B}{(1+2css)}\left(\frac{aw^B}{(1-a)r^{B,SS}}\right)^{\alpha\zeta} L^{B^\zeta} \\ &= \quad AA\ L^B - BB\ L^{B^\zeta} \end{aligned} \tag{111}$$

Then for the G sector using also eq. 43 we have

$$\left(\frac{P^B}{P^G}\right)^{\varepsilon}\left[\frac{\kappa^G}{\kappa^G}+2css\left(\frac{\theta^G}{\theta^B}\right)\right]C^{H,B} = -\delta\frac{aw^G}{(1-a)r^{G,SS}}(1-L^B)+ \left(\frac{aw^G}{(1-a)r^{SS}}\right)^{\alpha}(1-L^B)- \eta^G\left(\frac{aw^G}{(1-a)r^{G,SS}}\right)^{\alpha\zeta}(1-L^B)^{\zeta}$$

$$= DD(1-L^B)-EE(1-L^B)^{\zeta} \quad (112)$$

$$\Leftrightarrow\ CC\ C^{H,B} = DD(1-L^B)-EE(1-L^B)^{\zeta} \quad (113)$$

where

$$AA \equiv \frac{1}{(1+2css)}\left[-\delta\frac{aw^B}{(1-a)r^{SS}}+\left(\frac{aw^B}{(1-a)r^{SS}}\right)^{a}\right]$$

$$= \frac{-\delta KLSSB+ZLSSB}{(1+2css)}>0 \quad (114)$$

$$BB = \frac{\eta^B}{(1+2css)}\left(\frac{aw^B}{(1-a)r^{B,SS}}\right)^{a\zeta} = \frac{\eta^B}{(1+2css)}ZLSSB^{\zeta}>0 \quad (115)$$

$$CC \equiv \left(\frac{P^B}{P^G}\right)^{\varepsilon}\left[\frac{\kappa^G}{\kappa^B}+2css\left(\frac{\theta^G}{\theta^B}\right)\right]>0 \quad (116)$$

$$DD \equiv -\delta\frac{aw^G}{(1-a)r^{SS}}+\left(\frac{aw^G}{(1-a)r^{SS}}\right)^{a} = -\delta KLSSG+ZLSSG>0 \quad (117)$$

$$EE \equiv \eta^G\left(\frac{aw^G}{(1-a)r^{G,SS}}\right)^{a\zeta} = \eta^G ZLSSG^{\zeta}>0 \quad (118)$$

Without loss of generality, considering the assumed value for $\eta^G=0$, the steady state value of L^B is the solution of

$$-(CC*BB)L^{B^{\zeta}}+(CC*AA+DD)L^B-DD=0 \quad (119)$$

Then assuming $\zeta = 1$, we obtain:

$$L^B = \frac{DD}{CC(AA - BB) + DD} \tag{120}$$

A.7 Steady state assuming 3 types of entrepreneurs

Let us consider 3 types of entrepreneurs 'LCI', 'MCI', 'HCI'. Assumptions for the steady state:

$$L^{LCI} + L^{MCI} + L^{HCI} = 1 \tag{121}$$

$$C^{E,LCI,HCI} = C^{E,MCI,HCI} = C^{E,HCI,HCI} = cssC^{H,LCI} \tag{122}$$

We know also that:

$$\frac{C^{E,i,LCI}}{C^{E,i,HCI}} = \frac{\theta^{LCI}}{\theta^{HCI}} \left(\frac{P^{HCI}}{P^{LCI}} \right)^{\varepsilon} \tag{123}$$

$$\begin{aligned}
Z^{HCI} &= C^{H,HCI} + C^{E,LCI,HCI} + C^{E,MCI,HCI} + C^{E,HCI,HCI} + \\
&\quad I^{HCI} + \Phi_{HCI}(u^H CI_t) K^{HCI} + T^{HCI} && (124) \\
&= C^{H,HCI} + C^{E,LCI,HCI} + C^{E,MCI,HCI} + C^{E,HCI,HCI} + \\
&\quad \delta K^{HCI} + \eta^{HCI} Z^{HCI} \\
&= (1 + 3css) C^{H,HCI} + \delta K^{HCI} + \eta^{HCI} Z^{HCI} && (125)
\end{aligned}$$

Which implies:

$$\begin{aligned}
C^{H,HCI} &= \frac{((1 - \eta^{HCI})ZLHCISS - \delta KLHCISS)}{1 + 3css} L^{HCI} \\
&= AA * L^{HCI} && (126)
\end{aligned}$$

Considering the steady state for the good 'MCI', we have:

$$\begin{aligned}
Z^{MCI} &= C^{H,MCI} + C^{E,LCI,MCI} + C^{E,MCI,MCI} + C^{E,HCI,MCI} + \\
& I^{MCI} + \Phi_{MCI}(u^M CI_t)K^{MCI} + T^{MCI} \qquad (127) \\
&= C^{H,MCI} + C^{E,LCI,MCI} + C^{E,MCI,MCI} + C^{E,HCI,MCI} + \\
& \delta K^{MCI} + \eta^{MCI} Z^{MCI} \\
&= C^{H,HCI}\frac{\kappa^{MCI}}{\kappa^{HCI}}\left(\frac{P^{HCI}}{P^{MCI}}\right)^{\varepsilon} + C^{H,LCI,HCI}\frac{\theta^{MCI}}{\theta^{HCI}}\left(\frac{P^{HCI}}{P^{MCI}}\right)^{\varepsilon} + \\
& C^{H,MCI,HCI}\frac{\theta^{MCI}}{\theta^{HCI}}\left(\frac{P^{HCI}}{P^{MCI}}\right)^{\varepsilon} + C^{H,HCI,HCI}\frac{\theta^{MCI}}{\theta^{HCI}}\left(\frac{P^{HCI}}{P^{MCI}}\right)^{\varepsilon} + \\
& \delta K^{MCI} + \eta^{MCI} Z^{MCI} \\
&= C^{H,HCI}\left(\frac{P^{HCI}}{P^{MCI}}\right)^{\varepsilon}\left(\frac{\kappa^{MCI}}{\kappa^{HCI}} + 3css\frac{\theta^{MCI}}{\theta^{HCI}}\right) + \\
\delta K^{MCI} &+ \eta^{MCI} Z^{MCI} \qquad (128)
\end{aligned}$$

which implies:

$$\begin{aligned}
C^{H,HCI} &= \left(\frac{P^{MCI}}{P^{HCI}}\right)^{\varepsilon}\frac{((1-\eta^{MCI})ZLMCISS - \delta KLMCISS)}{\frac{\kappa^{MCI}}{\kappa^{HCI}} + 3css\frac{\theta^{MCI}}{\theta^{HCI}}}L^{MCI} \\
&= BB * L^{MCI} \qquad (129)
\end{aligned}$$

Therefore:

$$L^{MCI} = (AA/BB) * L^{HCI} \qquad (130)$$

Similarly with 'LCI':

$$\begin{aligned} C^{H,HCI} &= \left(\frac{P^{LCI}}{P^{HCI}}\right)^{\varepsilon} \frac{((1-\eta^{LCI})ZLLCISS - \delta KLLCISS)}{\frac{\kappa^{LCI}}{\kappa^{HCI}} + 3css\frac{\theta^{LCI}}{\theta^{HCI}}} L^{LCI} \\ &= CC * L^{LCI} \end{aligned} \tag{131}$$

Therefore:

$$L^{LCI} = (AA/CC) * L^{HCI} \tag{132}$$

Then:

$$L^{HCI} + L^{MCI} + L^{LCI} = 1 \tag{133}$$

$$L^{HCI} + (AA/BB) * L^{HCI}(AA/CC) * L^{HCI} = 1 \tag{134}$$

Which implies:

$$L^{HCI} = 1/(1 + (AA/BB) + (AA/CC)) \tag{135}$$

A.8 Calibration of the model

Table 1: Calibration of the DSGE model

Parameter	Description	Calibrated value
a	Share of capital in the production function	0.3
α^{PD}	Autoregressive coefficient of the PD process	0.95
β^E	Discount rate of entrepreneurs	0.96
β^H	Discount rate of households	0.995
χ	Share of retailers that do not reoptimize prices	0.834
CR	Target value for the capital ratio	0.13
css	Part of C^H consumed by entrepreneurs	0.1
δ	Capital depreciation rate	0.1
ϵ	Price elasticity (retailers)	4.33
η	Tax parameter	0.02
γ	CRRA function parameter for consumption	1.1
κ^i	Share of goods with carbon intensity i in the CES consumption of households	0.1
Ξ	Parameter for the adjustment cost function of banks	18
LGD^{HCI}	Loss given default of HCI firms	0.15
LGD^{LCI}	Loss given default of LCI firms	0.15
markup	Markup in the goods market	1.3
n_1^E	Basel parameter	0.12
n_2^E	Basel parameter	0.24
n_3^E	Basel parameter	50
ν	Share of the bank's profits kept as retained earnings	0.9
ϕ^{HCI}	Parameter in the capacity utilization cost adjustment function	0.2755
ϕ^{LCI}	Parameter in the capacity utilization cost adjustment function	0.3755
ρ	Monitoring costs parameter	0.2
rw^{HCI}	Risk-weight on HCI firms	0.5
rw^{LCI}	Risk-weight on LCI firms	0.5
s	Investment function parameter	7.7
σ^{LCI}	CRRA function parameter for labor	1.4
σ^{HCI}	CRRA function parameter for labor	1.4
$\sigma_{\omega^{HCI}}$	Variance of ω^{HCI}	0.254
$\sigma_{\omega^{LCI}}$	Variance of ω^{LCI}	0.254
θ^j	Share of goods with carbon intensity j in the CES consumption of entrepreneurs	0.1
ε	CES concavity parameter	0.999
ζ	Exponent of CO_2 tax	1

Acknowledgements

We thank participants at the Directorate General-Macroprudential Policy and Financial Stability Seminar Series for their useful comments and suggestions. We also thank participants at the Workshop on Global changes, international trade and health held in Bordeaux, France. All remaining errors are our responsibility.

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PDF ISBN 978-92-899-6390-9 ISSN 1725-2806 doi:10.2866/224520 QB-AR-24-027-EN-N