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Stefan Girstmair  The effect of new housing supply in structural models: a forecasting performance evaluation

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Abstract

This paper investigates the importance of including data on new housing supply in Dynamic Stochastic General Equilibrium (DSGE) models in forecasting the Great Financial Crisis (GFC), focusing on the U.S. While existing models have added a financial sector and real estate sector, they have largely overlooked housing supply. I develop an extended DSGE model that includes both the financial sector and endogenous housing supply and show that forecasting accuracy significantly improves when data on new houses is included. Robustness checks confirm the importance of these additions to the model. The findings highlight the necessity of combining model extension and housing supply data for accurate forecasting during economic crises. I identify negative housing demand shocks and escalating adjustment costs as primary drivers of the GFC, propagating into the real economy and accelerating through the financial sector. Additionally, this paper addresses the zero lower bound challenge in modeling forward guidance using a regime change approach.

Key words: DSGE, Housing, Model Projection, Bayesian Estimation

JEL Classification: E17, E32, E37, R21, R31
Non-technical summary

Over the past decade, many new additions have been made to structural macroeconomic models such as financial frictions and the real estate sector, to overcome the flaws exposed by the Great Financial Crisis (GFC). This paper focuses on the importance of new housing supply as a key driver of the business cycle. A model with endogenous housing supply is estimated and forecasted, taking into account the emergence of the zero lower bound, on real-time U.S. data to analyze the importance of this new sector during the GFC. I find that the combination of the model setup and housing supply data is crucial to increase forecasting performance. This result is tested via an extensive robustness check in which I analyze different versions of the model and exclusions of crucial housing supply data to show that the combination is indeed cardinal.

To achieve this, I take a model that already includes a rich financial sector from existing literature and extend it with an endogenous housing supply sector in which hand-to-mouth households work. In each period, two types of households - savers and borrowers - demand this newly produced housing supply to fulfill utility and saving needs and sell the not depreciated part. In this setup, variation in new housing supply has sizable effects on the business cycle as what matters is the margin of the housing sector.

The model is estimated on real-time U.S. data for six quarters during the GFC around 2008. To account for the zero lower bound periods and the resulting nonlinearities, I follow and extend the approach of Kulish et al. (2017) during the solution and estimation step. The usage of real-time data enables me to set side by side the forecasts produced by the model and the Survey of Professional Forecasters (SPF), as the data used was not revised and thus contains the same information that was available at the time. I show that the model extended with an endogenous housing supply sector and estimated with data on new houses fares very well in forecasting the GFC.

In order to provide a comprehensive analysis of the importance of both features combined, I conducted an extensive and rigorous robustness check. Specifically, I compare the newly built model, which incorporates the housing supply data and features construction workers, to various versions of itself and a baseline model similar to the standard model of Smets and Wouters (2007). The results of this comparison indicate that both robustness check versions perform very similarly to the benchmark model when it comes to forecasting the GFC. Therefore, I am confident in the finding that the combination of the extended model and the housing supply data is necessary to enhance the forecasting accuracy of the model during periods of economic crisis.

Analyzing the estimation results shows two main drivers behind the GFC and its propagation. First, negative housing demand shocks are crucial in understanding the dynamics. They immediately lead to lower housing demand and thus lower housing supply, which propagates through workers in the construction sector lowering their consumption - into the real economy and lowers consumption and thus output. Households also reduce their demand for loans, increasing stress in the financial markets that again accelerate the effects on the real economy. The inclusion of housing supply helps identify the true size of these (negative) housing demand shocks that the baseline version is not able to do. The second aspect comes from an increase in the cost of adjusting prices for intermediate goods and especially in the cost of adjusting wages. Since there is no employment in this model, adjusting wages is the only way to react to macroeconomic deviations. Making this harder not only increases the severity of shocks but also the propagation through the economy, which takes considerably longer.
"Housing IS the business cycle."

Leamer (2007)

1 Introduction

The Great Financial Crisis (GFC) exposed major flaws in most structural macroeconomic models, which were unable to predict the crisis or its severity. As a result, researchers have made two major additions to the standard model setups. First, the financial sector has been given more attention and has been included in most models through some form of financial frictions. This is crucial, as problems in the financial sector can have serious effects on the real economy. Second, the real estate sector has also been added, as it played a key role in the formation of the crisis. Since most households’ wealth was built either directly from real estate or by borrowing against its value, the significant drop in house prices had a ripple effect that impacted the financial sector. Therefore, both of these features played a crucial role in the development and transition of the crisis, not only in the U.S. but also in most European countries.

However, one aspect of the real estate sector that has received less attention in Dynamic Stochastic General Equilibrium (DSGE) models is the housing supply. Most models that include a housing sector (Iacoviello (2005), Gerali et al. (2010)) assume a fixed housing supply, leaving out a crucial component of the story. While some models have included the supply side (Davis and Heathcote (2005), Paries and Notarpietro (2008), Iacoviello and Neri (2010), Sun and Tsang (2017)), they do not match this variable with data during the estimation step. This paper focuses on the housing supply sector and shows that the trajectory and severity of the GFC in the U.S. can be accurately forecasted with the inclusion of data on new houses during estimation. In other words, it is not enough to simply extend a model to include the housing supply sector, but data on new houses is also necessary for accurate forecasting. By including both aspects, the gains in forecasting accuracy are significant.

In order to achieve accurate forecasting, I develop a DSGE model that extends the work of Giri (2018). The model includes a sophisticated financial sector with an interbank market, where banks can choose to forward household savings into the market or invest them in government bonds. However, banks have limited enforcement over the health of deficit banks that may optimally default on interbank loans. Deficit banks forward these loans to households and entrepreneurs who are limited in the amount they can borrow. When the value of real estate decreases, the collateral value also falls, leading to decreased borrowing capacity for households. This can lead to trouble for banks that issued these credits, and they may choose to default on some of their interbank loans. In addition to the real estate sector of Gerali et al. (2010) with fixed housing supply, I add an endogenous housing supply sector to the model. This new housing supply is created by hand-to-mouth consumers, which amplifies responses on the real side of the economy stemming from financial and/or real shocks. As these construction workers consume all their income in each period, troubles in the housing market can quickly amplify through their consumption demand into the real sector. In this model setup, two types of households, savers and borrowers, demand this newly produced housing supply for utility and saving needs in each period and also sell the non-depreciated housing. Thus, the inclusion of an endogenous housing supply sector allows for a more accurate representation of the dynamics of the housing market, its interactions with the broader economy, and leads to stronger effects on the business cycle.

In order to provide a comprehensive analysis of the importance of both features combined, I conduct an extensive and rigorous robustness check. Specifically, I compare the newly built
extended model, which incorporates the housing supply data and features construction workers, to various versions of itself and a baseline model. The baseline model, which was developed by Giri (2018), serves as a comparison point for two reasons. Firstly, it forms the foundation of the extended model, and therefore a comparison between the two ensures that any differences arise solely from the extension and the additional information provided by more data. Secondly, the baseline model can be regarded as a representative model for the class of medium-scale DSGE models frequently used in central banks and policy institutions, as it builds on the work of Smets and Wouters (2007) and features a more sophisticated financial sector and a fixed housing supply.

Moreover, the comparison between the extended model and the ”enhanced” version of the Smets and Wouters (2007) model, which was introduced by the baseline model, reinforces the main argument of this paper. Namely, that the additions of a more sophisticated financial sector and an endogenous housing supply significantly enhance the forecasting accuracy of the model during the GFC. The fact that the extended model outperforms the ”enhanced” version of the Smets and Wouters (2007) model, that includes the same financial sector and ”only” a fixed housing supply, implies that the former is compared to a higher benchmark.

The main robustness check is conducted by comparing two versions of the extended model to the benchmark model. The first version uses the same data for estimation and forecasting as the benchmark model, which does not include data on housing supply and only features one series on fixed private investment (FPI). Therefore, any differences in performance between the two versions could only arise from differences in the model itself. The second version splits FPI into residential and nonresidential investment, potentially including more information. The results of this comparison indicate that both robustness check versions perform very similarly to the benchmark model when it comes to forecasting the GFC. Therefore, I am confident in the finding that the combination of the extended model and the housing supply data is necessary to enhance the forecasting accuracy of the model during periods of economic crisis.

Analyzing the estimation results, through IRF’s and posterior density analysis, shows two main drivers behind the GFC and its propagation. First, negative housing demand shocks are crucial in understanding the dynamics. They immediately lead to lower housing demand and thus lower housing supply, which propagates - through workers in the construction sector lowering their consumption - into the real economy and lowers consumption and thus output. Households also reduce their demand for loans, increasing stress in the financial markets that again accelerate the effects on the real economy. The inclusion of housing supply helps identify the true size of these (negative) housing demand shocks that the baseline version is not able to do. The second aspect comes from an increase in the cost of adjusting prices for intermediate goods and especially in the cost of adjusting wages. Since there is no employment in this model, adjusting wages is the only way to react to macroeconomic deviations. Making this harder not only increases the severity of shocks but also the propagation through the economy, which takes considerably longer. These two aspects combined really help forecast and understand the GFC of 2008.

In response to the financial crisis and the accompanying economic downturns, many central banks around the world, including the Federal Reserve and the European Central Bank, implemented unconventional monetary policy known as forward guidance, which involves clear communication of the central bank’s plans for adjusting policy rates in the future. By promising to maintain a lower nominal policy rate for an extended period, central banks can stimulate aggregate demand and mitigate the effects of the crisis. However, the zero lower bound presents a significant challenge to modeling this policy, as linear functions cannot approximate the non-
linear kink. To address this issue, I use the novel method introduced by Kulish et al. (2017), which uses regime changes to approximate the full non-linear solution with a piecewise linear one, enabling likelihood estimation. I extend this method to a more complex model with financial frictions, using the code from Holden (2012) to log-linearize the equations. Additionally, I simplify the Metropolis-Hastings algorithm used for estimation to reduce the required time by half. My analysis focuses on calendar-based forward guidance, where agents expect the fixed rate regime to continue for a specific number of periods before a Taylor-style rule is reintroduced. Campbell et al. (2012) call this approach Odyssean forward guidance as compared to Delphic forward guidance. This approach has limitations, as it may prevent central banks from reacting to urgent issues, but it can still reduce uncertainty and have macroeconomic effects.

This paper falls into various strands of literature in the macroeconomic field of research. On the modeling side these contain the inclusion of financial frictions as well as an endogenous housing supply sector into DSGE models. Another one is on the solution method and inclusion of unconventional monetary policy that is enabled by this method. The last strand covers (real-time) forecasting exercises and there particularly for the Great Financial Crisis.

Regarding the inclusion of financial frictions, the main building blocks have already been around prior to the GFC but were not considered to be important and thus not part of the main models. This, of course, changed quickly with the transition of shocks stemming from the financial sector into the real economy. The role of financial frictions is to introduce some form of difficulty in firms financing - a friction - through which aggregate disturbances become amplified and propagated across the economy. Two of the main contributions in this field are the financial accelerator by Bernanke et al. (1999) and the borrowing constraints by Kiyotaki and Moore (1997). The latter was developed further and used by Iacoviello (2005) in the context of housing, a sector that was also neglected as a primary source of the business cycle. Thus, in this model, entrepreneurs’ level of borrowing is constrained by their level of capital and impatient households’ borrowing is constrained by their level of housing. This gives rise to housing playing a key role during the business cycle in enabling agents to borrow more or less depending on its value. Other important work in that field came from Gerali et al. (2010), Paries and Notarpietro (2008), Iacoviello and Neri (2010), and Sun and Tsang (2017) where the last three do feature endogenous housing. Around that time, after the GFC, including a more sophisticated financial and banking sector in DSGE models became a much larger priority and thus many more models evolved. An incomplete list consists of Dib (2010a,b), Gertler and Kiyotaki (2010), and Gertler and Karadi (2011), many of which were also studying the interaction of the banking sector with conventional and unconventional monetary policy, similar to this paper.

The second strand of literature is about the inclusion of unconventional monetary policy and there specifically forward guidance into DSGE models, which became necessary since the nominal rates started being binding at the zero lower bound in early 2009. One of the earlier contributions which showed that the adjustment of expectations on future rates, through credible commitment of the central bank, can have sizable impacts on current macroeconomic outcomes was Eggertsson and Woodford (2003). It is also well documented that in forward looking models with rational expectations, such as the present one, forward guidance can have much larger than expected responses of aggregate variables, Del Negro et al. (2012) call this the forward guidance puzzle. This phenomenon is being used here to pin down the sequence of expected durations in estimation. Other varieties of unconventional monetary policy such as quantitative easing (QE) are not covered, as it would be beyond the scope of this paper.

The last important strand of literature this paper touches on is the one that covers forecasting comparisons of DSGE models. Christoffel et al. (2011) and Del Negro and Schorfheide (2013)
are not only the standard work regarding the current theory of forecasting but also features a forecasting comparison as well as a summary of papers that do similarly. Within this field there is a subfield that works on real-time data vintages. A main motivation for the current paper is the work by Kolasa et al. (2012) that puts the standard medium-scale Smets and Wouters (2007) model to a real-time forecasting test versus the Survey of Professional Forecasts (SPF) and some DSGE-VARs. They find that this set of models does relatively well, especially once the forecasts are conditional on SPF nowcasts. Kolasa and Rubaszek (2015) perform a similar exercise, but now this time they focus on various implementations of financial frictions. They find that including a housing market helps outperforming frictionless and other financial frictions models. The way they include housing follows the setups of Iacoviello (2005) and Gerali et al. (2010). A very different approach is chosen by Gelfer (2019) where relatively standard models are estimated in a data-rich environment, which incorporates a dynamic factor model setup and enables the use of a large data set. This extension does yield very good forecasting performance improvements that outperform SPF and DSGE models that were not estimated in this way. This paper, unfortunately, does not use real-time data sources, which makes the comparison to SPF vintages rather difficult. Two other important papers in this literature are Edge and Gürkaynak (2010) and Cai et al. (2019) that also check the importance and quality of (real-time) forecasting performance of modern DSGE models.

Recently, there have also emerged other methods to estimate DSGE models that include the zero lower bound, some of which are closely related to the one used in this analysis. Using the full non-linear solution for estimation is one of them which is computationally very demanding and involved. Another, quickly growing strand of this literature, is the one that incorporates occasionally binding borrowing constraints. The two main methods there were developed by Guerrieri and Iacoviello (2015) and Holden (2016), and both use some form of piecewise linear solutions to approximate the full non-linear one. In that sense they are comparable to the present one from Kulish et al. (2017) as they use a guess and verify approach to find the expected duration which then gives the same reduced form matrices. Since they determine their durations through specifying a sequence of shocks, to keep the interest rate at the zero lower bound, they are thus not able to let the data determine them and estimate the expected durations which is done here. The occasionally binding constraints has also been used recently by Böhl (2021) with a much more sophisticated approach that made it also possible to estimate expected durations. A third strand is where Markov switching methods are implemented to account for zero lower bound periods such as Binning and Maih (2016). The drawback there is that the zero lower bound is binding at each period with the exact same probability, thus making transitions to and from it equally likely at each point in time which makes it impossible to estimate expected durations.

The rest of the paper is organized as follows, Section 2 describes the model in detail, Section 3 discusses the solution method, Section 4 shows the estimation and forecasting methodology, then there is an analysis of these forecasts in Section 5, and Section 6 concludes.

2 The Model

The models used for this analysis are the one of Giri (2018) and an extension thereof, which is derived and explained below. The main reason for this choice is that the original model is based on the widely used work of Gerali et al. (2010) that already introduces a real estate and financial sector into an otherwise relatively standard DSGE model and further enhances the financial sector. Its central question is the evaluation of default risk on the banking market and the transmission into the real economy. For this purpose Giri (2018) adds to the framework laid out
by Gerali et al. (2010) some features proposed in Dib (2010a). In the resulting interbank market, banks can either invest their liquidity into risky interbank lending or into safe government bonds. The riskiness comes from the fact that banks can endogenously default on their borrowing. In its general transmission characteristics, not of the financial side however, it closely follows the seminal work of Smets and Wouters (2007). This is widely regarded as the workhorse model in modern macroeconomics and does - supposedly - quite well in forecasting important endogenous variables over the business cycle. They all belong to the class of medium sized DSGE-models and for that reason, the model of Giri (2018) is from now on referred to as the baseline model to which the extended model that is derived below is compared to.\footnote{For a detailed description of the baseline model, the interested reader is referred to the original work of Giri (2018).}

The model developed on the baseline model and described below is referred to as extended model and features an endogenous real estate sector in which hand-to-mouth consumers work to build new housing. There are three kinds of households populating the economy. These are patient, impatient, and hand-to-mouth consumers. The first two are standard in the literature with housing first introduced by Iacoviello (2005). The goal there is that the (representative) patient household has a higher intertemporal discount factor than the (representative) impatient household. This makes them net savers and thus they decide on how much to consume, to work, to purchase new housing, and to save - as deposits - at the surplus bank. Similarly, this setup makes impatient households net borrowers. Hence, they decide on how much to consume, to work, to purchase new housing, and to borrow from the deficit bank to finance a share of their consumption. This saving is done against collateral, which in the case of the impatient households is the present expected value of their housing assets (Kiyotaki and Moore (1997)). This means the more real estate they own in general, the more they are able to borrow against that using it as collateral. The amount of housing is not the only thing that determines the level of collateral. Its price is equally as important as it interacts in setting the value of the asset class housing. Thus, a drop in house prices lowers the value of collateral, which in turn makes it harder to borrow for impatient households. Hand-to-mouth households are the only ones working in the housing market and consume their income each period in its entirety. These construction workers thus, through their supply of labor and correlated consumption levels, transmit the housing cycle into the real economy. The entrepreneurs hire three kinds of labor to produce the intermediate good as well as new houses, which they then sell to final good producers and households respectively. What matters in this model is new housing services for utility and borrowing reasons. Households’ demand for new housing equals new houses produced by the entrepreneur in every period. The surplus bank receives saving from the patient households, which they can either forward into the interbank market or save in government bonds. The deficit bank takes loans on the interbank market and hands out funds to impatient households and entrepreneurs. It can also decide to default on its interbank loans. The monetary authority follows a standard Taylor type rule on how to set the nominal interest rate.

2.1 Households and Entrepreneurs

2.1.1 Patient Households

In every period, a continuum of patient households choose their preferred levels of consumption $c^P_t(i)$, housing $h^P_t(i)$, and deposits $d^P_t(i)$ with the intention to maximize their utility function:
\[
E_0 \sum_{t=0}^{\infty} \beta_P^t \left[ (1 - a^P) \varepsilon^c_t \log(c^P_t(i) - a^P c^P_{t-1}) + \varepsilon^h_t \log(h^P_t(i)) - \frac{l^P_t(i)^{1+\phi}}{1 + \phi} \right]
\] (1)

They receive utility from consumption, which they are smoothing across time through the parameter \( a^P \), new housing services, and negative utility from working \( l^P_t(i) \). In the equation above, \( \beta_P \) denotes the intertemporal discount factor of these patient households and \( a^P \) the consumption habit formation term. As in Gerali et al. (2010), one minus this habit term is premultiplied to offset its effect on the steady state marginal utility of consumption. \( \varepsilon^c_t \) represents a consumption preference shock and \( \varepsilon^h_t \) is a similar shock to demand of new housing. The process for consumption preferences holds across all three types of households whereas the housing demand shock is only incorporated in the patient and impatient households utility functions. \( \phi \) measures the disutility of labor and the budget constraint is given by:

\[
c^P_t(i) + q^h_t h^P_t(i) + d^P_t(i) = w^P_t(i) l^P_t(i) + q^h_t (1 - \delta^h) h^P_{t-1}(i) + \frac{(1 + r^d_t)}{\pi_t} d^P_{t-1}(i) + J^t_t + J^{sb}_t + T_t
\] (2)

On the expenditure side, the patient household consumes, purchases new housing services at the real price \( q^h_t \) and allocates the amount \( d^P_t(i) \) as savings at the surplus bank. The income side features hourly wages earned for working in the production sector \( w^P_t(i) \), returns on savings with the deposit interest rate \( r^d_t \), income from selling not depreciated, at rate \( \delta^h \), housing, and \( \pi_t \) denoting net inflation. Furthermore, patient households own the final good producing firm, as well as the surplus bank, and receive all the profits earned in those sectors. \( T_t \) denotes net (lump-sum) transfers from the government. This setup makes patient households net savers such that the they are placing a fraction of their income as deposits in the banking sector. All variables are expressed in real terms.

### 2.1.2 Impatient Households

Similar to the patient households described above, a continuum of impatient households also choose their preferred levels of consumption \( c^I_t(i) \) and new housing \( h^I_t(i) \) to maximize utility. Since they value consumption in the current period more, compared to the patient households, they are net borrowers and thus obtain loans \( b^I_t(i) \) from the deficit bank to partially finance some of their spending.

\[
E_0 \sum_{t=0}^{\infty} \beta_I^t \left[ (1 - a^I) \varepsilon^c_t \log(c^I_t(i) - a^I c^I_{t-1}) + \varepsilon^h_t \log(h^I_t(i)) - \frac{l^I_t(i)^{1+\phi}}{1 + \phi} \right]
\] (3)

The two shock processes \( \varepsilon^c_t \) and \( \varepsilon^h_t \) are the same as in the patient households utility function and \( \beta_I \) is smaller, in absolute terms, than \( \beta_P \). \( a^I \) again measures habit formation and \( \phi \) the disutility of hours worked \( l^I_t(i) \). The budget constraint for impatient households is given by the following expression.
\[ c_t^I(i) + q_t^b h_t^I(i) + \frac{(1 + r_t^{bl})}{\pi_t} b_{t-1}^I(i) = w_t^I(i) h_t^I(i) + q_t^b (1 - \delta^h) h_{t-1}^I(i) + b_t^I(i) + (1 - \Omega) J_t^{db} \]  

\( r_t^{bl} \) denotes the interest rate on loans from the deficit bank and \( (1 - \Omega) J_t^{db} \) the fraction of profits that flow to the impatient households. They also work in the production sector, earn hourly wages \( w_t^I(i) \), and purchase and sell new housing at real price \( q_t^h \).

Following Kiyotaki and Moore (1997), the impatient households face a constraint on how much they can borrow from the banking system. They use their housing stock as collateral while borrowing. The expected value of this collateral must guarantee repayment of the loans and interest.

\[ (1 + r_t^{bl}) b_t^I(i) \leq m_t^E E_t \left[ q_{t+1}^h (1 - \delta^h) h_t^I(i) \pi_{t+1} \right] \]  

Borrowing constraints of the form seen in equation (5) are common in this literature and can be found - among others - in Iacoviello and Neri (2010). \( m_t^E \) denotes the stochastic loan-to-value (LTV) ratio for mortgages that follows an exogenous AR(1) process and gives the level of credits banks can offer to impatient households for the given discounted level of collateral. The shocks are assumed to be small enough for the borrowing constraint to always be binding.\(^2\)

2.1.3 Hand-to-Mouth Households

A new feature to this model is the inclusion of hand-to-mouth (HTM) households similar to Boscá et al. (2020). They are the only ones that work in the housing production sector and consume the entirety of their income in every period and thus amplify business cycle movements coming from this sector. Hence, the utility function depends on their level of consumption \( c_t^M(i) \) and hours worked \( l_t^M(i) \). They do not save, borrow, or spend resources on purchasing housing services.

\[ E_0 \sum_{t=0}^{\infty} \beta_t^M \left[ (1 - a^M) \varepsilon_t^M \log(c_t^M(i) - a^M c_{t-1}^M) - \frac{l_t^M(i)^{1+\phi}}{1+\phi} \right] \]

Subject to the budget constraint given by.

\[ c_t^M(i) = w_t^M(i) l_t^M(i) \]

2.1.4 Entrepreneurs

The economy is populated by a continuum of self employed entrepreneurs that do not only produce the intermediate good but also the real estate newly supplied, that is thus endogenously determined in this model. Each entrepreneur chooses consumption \( c_t^E \), capital used for production of intermediate goods and housing supply, \( k_t^E(i) \), \( k_t^{Eh}(i) \), labor from patient, impatient and hand-to-mouth households, \( l_t^{E,P}(i) \), \( l_t^{E,I}(i) \), \( l_t^{E,M}(i) \), the level of loans obtained

\(^2\)From this assumption, as in Iacoviello (2005), it follows that \( (1 + r_t^b) b_t^I(i) = m_t^E E_t [q_{t+1}^b (1 - \delta^h) h_t^I(i) \pi_{t+1}^I] \) holds in every period.
from the banking system, $b_F^t(i)$, and the degrees of capital utilization in intermediate goods and housing production sectors, $u_t(i), u^h_t(i)$. Similarly to impatient households described above, entrepreneurs are also net debtors and are thus constrained as well. Other than impatient households, however, they do not use the expected value of their housing as collateral but their

entrepreneurs are also net debtors and are thus constrained as well. Other than impatient

level of total factor productivity, $\mu$ measures the labor income share of patient households. As emphasized earlier, entrepreneurs are also subject to a borrowing constraint which states that the amount the bank is willing to lend is constrained by the expected value of the entrepreneurs physical capital in both sectors.

The entrepreneurs’ utility function only depends on consumption, as has been done several times in similar models like e.g. Sun and Tsang (2017), and is not affected by the consumption preference shock seen for households.

$$E_0 \sum_{i=0}^{\infty} \beta_E^t \left[ (1 - a^E) \log(c^E_t(i) - a^E c^E_{t-1}) \right]$$

(8)

The associated budget constraint of each entrepreneur is expressed below.

$$c^E_t(i) + w^P t^P E^P_t(i) + w^E t^E I(i) + w^M t^E M_t(i) + \left( \frac{1 + r^E_t(i)}{\pi_t} - b^E_t(i) + \frac{q^k_t k^E_t(i) + f(u_t(i)) k^E_{t-1} + \bar{h}_t}{\pi_t} + \bar{l}_t (l_t(i) - l_{t-1}(i)) + q^k_t h^E_t(i) + g(u^h_t(i)) k^E^h_{t-1} = \frac{\xi^E_{i}}{\pi_t} + \frac{q^k_t}{(1 - \delta) k^E_{t-1} + \frac{q^h_t(1 - \delta^h)}{k^E^h_{t-1}}}$$

(9)

The prices for capital in terms of consumption are $q^k_t$ and $q^h_t$. $\delta$ and $\delta^h$ represent the rates of depreciation of capital used in producing intermediate goods and housing supply respectively. $k^t$ denotes intermediate inputs for housing production and $\bar{l}_t$ is the land used which has a price $\bar{p}_t$ and is set to unity. $f(u_t(i)) k^E_{t-1}$ and $g(u^h_t(i)) k^E^h_{t-1}$ are the real costs of setting utilization rates $(u_t(i)$ and $u^h_t(i))$ in the two sector and $1/\pi_t$ is the relative competitive price of the intermediate good produced.

$$f(u_t(i)) = \xi_1(u_t(i) - 1) + \frac{\xi_2}{2} (u_t(i) - 1)^2$$

(10)

$$g(u^h_t(i)) = \xi_1(u^h_t(i) - 1) + \frac{\xi_2}{2} (u^h_t(i) - 1)^2$$

(11)

The entrepreneurs utilize the following two Cobb-Douglas production functions to produce the intermediate good and new housing supply. Both feature distinct shocks to the respective level of total factor productivity, $a_t^E$ and $a_t^H$. It is important to mention that house prices are flexible and are not constrained by any frictions.

$$g^E_t(i) = a_t^E \left( k^E_{t-1}(i) u_t(i) \right)^{\alpha} \left( l^E P_t(i) l^E I_t(i) i^{\delta} \right)^{1-\alpha}$$

(12)

$$I H_t(i) = a_t^H \left( k^E^h_{t-1}(i) u^h_t(i) \right)^{\beta} \left( \bar{w}_t \bar{p}_t l^E M_t(i) \right)^{1-\beta}$$

(13)

As can be seen from equation (13), HTM-households are the only ones employed in the house production sector, whereas patient and impatient households work in the intermediate goods sector, (12), where $\mu$ measures the labor income share of patient households. As emphasized earlier, entrepreneurs are also subject to a borrowing constraint which states that the amount the bank is willing to lend is constrained by the expected value of the entrepreneurs physical capital in both sectors.

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The functional forms of equations (10) and (11) follow Schmitt-Grohé and Uribe (2005).
\[ (1 + \delta^E)k_t^E(i) \leq m_t^E E_t \left[ \left( \alpha_{t+1} (1 - \delta) k_{t+1}^E(i) + \delta^{kh} k_t^{Eh}(i) \right) \pi_{t+1} \right] \]  

Similarly to the impatient households, \( m_t^E \) is the stochastic LTV ratio. And the assumption is again that the size of the shock is sufficiently small, such that the constraint always binds in a neighborhood of the steady state.

### 2.2 Labor Market

Each kind of worker offers a differentiated type of labor to labor unions. The market has three types of unions for each labor type, one for patient households, one for impatient households, and one for HTM-households. The role of the union is to set optimal nominal wages for all of their members. To achieve this they are constrained by downward sloping demand and quadratic adjustment costs (parameterized by \( \kappa_w \)) that feature indexation to a weighted average of lagged inflation, with weight \( \iota_w \), and steady state inflation, with weight \( (1 - \iota_w) \). The demand comes from labor packers which the unions sell the labor to, that combine them via a CES aggregator, incorporating the stochastic wage markup shock \( \epsilon^l_t \), to homogeneous goods. Therefore, there are three homogeneous labor goods the packers then further sell to entrepreneurs used in production.

For \( x \in (P, I, M) \) and each labor type \( m \) the unions maximize:

\[
\max_{W_x^E(m)} E_t \sum_{t=0}^{\infty} \beta^t \Lambda_{E}^t \left[ \frac{W_x^E(m)}{P_t} l_t^x(i, m) - \frac{\kappa_w}{2} \left( \frac{W_x^E(m)}{W_x^E(m-1)} - \pi_{t-1}^{aw} (1 - \tau_w) \right)^2 \frac{W_x^E(m)}{P_t} - \frac{l_t^x(i, m)^{1+\phi}}{1 + \phi} \right] \\
\text{subject to } l_t^x(i, m) = \left( \frac{W_x^E(m)}{W_x^E(m-1)} \right)^{-\epsilon^l_t} l_t^x
\]

### 2.3 Capital Market

The capital market is segmented into two parts, one for the physical capital used in the production of the intermediate good, and one for the physical capital used in the production of housing supply. In each competitive sector there is a continuum of capital producers that buy last periods undepreciated capital \( (1 - \delta) k \) from the entrepreneurs at real price \( q_t \) and \( i_t \) units of the final goods from the retailers. With these inputs they produce new physical capital that is then sold to entrepreneurs who use it for production. The transformation of final good to capital, however, is costly and thus capital producers have to pay quadratic adjustment costs that are parameterized by \( \kappa_i \) and \( \kappa_{ih} \) respectively.

The capital producers maximize:

\[
\max_{i_t} E_t \sum_{t=0}^{\infty} \beta^t \Lambda_{E}^t \left[ q_t^k (k_t - (1 - \delta) k_{t-1} - i_t) \right] \\
\text{subject to } k_t = (1 - \delta) k_{t-1} + \left[ 1 - \frac{\kappa_i}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 \right]
\]
\[
\max E_t \sum_{t=0}^{\infty} \beta^t E_t \left[ q_t^{kh} (k_t^{kh} - (1 - \delta^h) k_{t-1}^{kh}) - \delta^h k_t^{h} \right] 
\]
(19)

s.t. \[ k_t^{h} = (1 - \delta^h) k_{t-1}^{h} + \left[ 1 - \frac{\kappa^p}{2} \left( \frac{\epsilon_t^{yk} \sqrt{y_t}}{y_{t-1}^{h}} - 1 \right) \right]^2 \]
(20)

In the resulting equations of the Tobin’s Q form, \( \epsilon_t^{yk} \) denotes the shock to the efficiency of investment in both sectors.

### 2.4 Final Goods Market

The retailers in the final goods market face monopolistic competition as is standard in the literature. They buy intermediate goods from entrepreneurs and combine them costlessly into a homogeneous final good. The prices they set are sticky and indexed to past and steady state inflation with weight \( \iota^p \). The profit maximization problem is given by:

\[
\max_{P_t} E_t \sum_{t=0}^{\infty} \beta^t P_t \left[ P_t^{y} y_t(j) - P_t^{W} y_t(j) - \frac{\kappa^p}{2} \left( \frac{P_t(j)}{P_{t-1}^{y}(j)} - \pi^w_{t-1} \bar{\pi}^{1-\iota^p} \right)^2 P_t y_t \right] 
\]
(21)

\( \kappa^p \) determines the level of adjustment costs, the retailers have to pay when changing prices. Final goods producers are constrained by the demand for their goods, which comes from consumers’ maximization, where \( \epsilon_t^{y} \) is a stochastic markup shock.

\[
y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_t^y} y_t 
\]
(22)

### 2.5 Banking Market

The banking system of the model is taken from Giri (2018), who combines the setups of Gerali et al. (2010) and Dib (2010a) into one. Overall, there are two sides to the interbank market, the surplus banks that take deposits from patient households and deficit banks that hand out loans to impatient households and entrepreneurs. Both banks are further divided into a wholesale branch and a retail branch. The deficit banks’ retail branches operate under monopolistic competition and set the interest rates on loans given. The wholesale branch chooses the optimal balance sheet of the bank. To introduce default risk in the banking sector, the deficit bank can decide to default over its interbank market borrowing. The retail branches of the surplus bank gather savings from patient households while the wholesale branch decides how these are further used. They can thus decide either to buy government bonds or forward them into the interbank market towards the deficit bank. As the deficit bank can decide to default on loans the surplus bank has to pay such that it can get information on the market status. These monitoring costs also increase with the amount of interbank lending.
2.5.1 Deficit Bank: Wholesale Branch

As described above, the wholesale branch of the deficit bank is responsible for the optimization of the balance sheet. The bank is assumed to be owned by impatient households which yields that the stochastic discount factor of the bank is equal to the households marginal utility of consumption. Each branch operates under perfect competition and combines bank capital $K_{tb}$ and interbank loans $IB_t$ to issue loans $B_t$, which gives the balance sheet with a stochastic balance sheet shock $\varepsilon_{tb}^b$.

$$B_t = IB_t + K_{tb} + \varepsilon_{tb}^b$$

(23)

Bank capital evolves according to the following law of motion:

$$K_{tb}^{\pi_t} = (1 - \delta_b)K_{t-1}^{tb} + \Omega J_{tb}^{db}$$

(24)

$\delta_b$ denotes the rate of depreciation of bank capital between periods and $\Omega$ is the share of profits used for bank capital accumulation. The problem for the wholesale branch of the deficit bank is to choose optimal levels of loans to impatient households and entrepreneurs $B_t$, borrowing from the surplus bank $IB_t$, and interbank borrowing defaults $\delta_d^t$ to maximize the discounted sum of cash flows:

$$\max_{B_t, IB_t, \delta_d^t} E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left[ (1 + R_{tb}^b)B_t - B_{t+1} \pi_{t+1} - (1 + r_{tb}^i)(1 - \delta_d^t)IB_t + IB_{t+1} \pi_{t+1} \right.$$ 

$$\left. + \left( K_{t+1}^{\pi_{t+1} - K_{tb}^b} - AC_{tkb} - AC_{td} \right) \right]$$

(25)

$R_{tb}^b$ describes the net wholesale loan rate and $r_{tb}^i$ is the net interest rate on interbank loans obtained from the surplus bank. $AC_{tkb}$, and $AC_{td}$ stand for the capital requirement of the deficit bank and the penalty cost the bank has to pay the period after defaulting.

$$AC_{tkb} = \frac{\kappa_{kb}}{2} \left( \frac{K_{tb}}{B_t} - \nu_b \right)^2 K_{tb}$$

(26)

$$AC_{td} = \frac{\chi_{db}}{2} \left( \frac{IB_{t-1} \delta_{t-1}^d}{\pi_t} \right)^2$$

(27)

Using the balance sheet constraint twice, at time $t$ and $t+1$, the objective can be simplified to:

$$\max_{B_t, IB_t, \delta_d^t} E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left[ R_{tb}^b B_t - (1 + r_{tb}^i)\delta_d^t IB_t - r_{tb}^i IB_t - AC_{tkb} - AC_{td} \right]$$

(28)

Combining the optimality conditions with respect to $B_t$ and $IB_t$ yields an expression for the wholesale loan rate that links it not only to the degree of leverage - that is given by $B_t/K_{tb}^b$.
- but also to the adjustment costs the deficit bank has to face in case of default. This means that the wholesale rate depends positively on the expected value of defaults such that more interbank defaults will increase the rate. Similarly, undercapitalization increases the wholesale rate as well.

\[
R^b_t = r^b_t - \delta^d_t (1 + r^b_t) - \kappa_{kb} \left( \frac{K^h_t}{B_t} - \nu^b \right) \left( \frac{K^h_t}{B_t} \right)^2 + \beta_t \lambda_{db} E_t \left\{ \left( \frac{\delta^d_t}{\pi_{t+1}} \right)^2 IB_t \lambda_{t+1}^I \right\} \quad (29)
\]

The remaining optimality condition states the amount of interbank defaults.

\[
\delta^d_t = E_t \left( \frac{\lambda_t^I (1 + r^b_t) \pi_{t+1}^2}{\beta_t \lambda_{t+1}^I \lambda_{db} IB_t} \right) \quad (30)
\]

Equation (30) shows that interbank defaults increase with the higher interest rate and decrease with the amount of interbank borrowing. Giri (2018) added an interbank default shock in that equation, which is removed for this analysis to keep the shock dimension slightly smaller. It was, however, the main aspect of that work and its transition dynamics through the model are documented there. The last remaining equation governing the wholesale branch of the deficit bank is the definition of its profits that are given by subtracting costs from revenues.

\[
J^d_t = r^b_t b_t^I + r^b E_t^I - (1 + r^b_t) \delta^d_t IB_t - AC_{kb}^t - AC^\delta_t \quad (31)
\]

### 2.5.2 Deficit Bank: Retail Branches

The under monopolistic competition operating retail branches of the deficit bank supply the loans to the two constraint sets of agents, impatient households and entrepreneurs, for which they can also set the interest rates. The retail branch receives real loans \(B_t\) from the wholesale branch at interest rate \(R^b_t\). They then costlessly differentiate and forward them asking two different markups. Adjusting the two respective interest rates is costly for the retail branch, that has to pay quadratic adjustment costs with shift parameters \(\kappa_{bl}\) and \(\kappa_{bE}\). Each retail branch \(j\) maximizes its objective subjective to loan demand of impatient households and entrepreneurs\(^4\) and its maximization problem reads as follows:

\(^4\)Loan demands, as well as deposit demand, are modelled following Gerali et al. (2010). They follow a Dixit-Stiglitz framework and thus are assumed to be a constant elasticity of substitution combination of slightly differentiated products. The important components are the stochastic elasticity terms \(\varepsilon^*_t\)
\[
\max_{r_t^b(j), r_t^E(j)} E_0 \sum_{t=0}^{\infty} \beta_t^I \lambda_t^I \left[ r_t^b(j)b_t^I(i) + r_t^E(j)b_t^E(i) - R_t^b B_t(j) - AC_t^{m.b} - AC_t^{m.E} \right]
\] (32)

\[
b_t^I(i) = \left( \frac{r_t^b(j)}{r_t^b(j)} \right)^{-\epsilon_t^I} b_t^I
\] (33)

\[
b_t^E(i) = \left( \frac{r_t^E(j)}{r_t^E(j)} \right)^{-\epsilon_t^E} b_t^E
\] (34)

\[
AC_t^{m.b} = \frac{\kappa_{bE}}{2} \left( \frac{r_t^b(j)}{r_{t-1}^b(j)} - 1 \right)^2 r_t^b b_t^I
\] (35)

\[
AC_t^{m.E} = \frac{\kappa_{bE}}{2} \left( \frac{r_t^E(j)}{r_{t-1}^E(j)} - 1 \right)^2 r_t^E b_t^E
\] (36)

Using the fact that \( B_t(j) = b_t^I(j) + b_t^E(j) \), imposing a symmetric equilibrium in both sectors, and taking first order conditions yields the optimal loan demand interest rates where \( \Lambda_t \) is the markup defined from \( \epsilon_t = \frac{\Lambda_t}{\Lambda_{t-1}} \).

\[
1 - \frac{\Lambda_t^b}{\Lambda_t^b - 1} + \frac{R_t^b}{r_t^b} \frac{\Lambda_t^b}{\Lambda_{t-1}^b - 1} - \kappa_{bI} \left( \frac{r_t^b}{r_{t-1}^b} - 1 \right) \frac{r_t^b}{r_{t-1}^b} + \beta_t^I \lambda_t^I \frac{\lambda_t^I}{\lambda_{t-1}^I} \kappa_{bI} \left( \frac{r_{t+1}^b}{r_t^b} - 1 \right) \left( \frac{r_{t+1}^b}{r_t^b} \right)^2 \frac{b_{t+1}^I}{b_t^I} = 0
\] (37)

\[
1 - \frac{\Lambda_t^E}{\Lambda_t^E - 1} + \frac{R_t^E}{r_t^E} \frac{\Lambda_t^E}{\Lambda_{t-1}^E - 1} - \kappa_{bE} \left( \frac{r_t^E}{r_{t-1}^E} - 1 \right) \frac{r_t^E}{r_{t-1}^E} + \beta_t^E \lambda_t^E \frac{\lambda_t^E}{\lambda_{t-1}^E} \kappa_{bE} \left( \frac{r_{t+1}^E}{r_t^E} - 1 \right) \left( \frac{r_{t+1}^E}{r_t^E} \right)^2 \frac{b_{t+1}^E}{b_t^E} = 0
\] (38)

### 2.5.3 Surplus Bank: Wholesale Branch

The surplus bank is modeled as in Dib (2010a) such that it collects deposits from the only savers in the economy, the patient households, and either forwards them into the interbank market or decides to purchase government bonds instead. It is, in contrast to the deficit bank, owned by the patient households and thus transfers its profits to them. The wholesale branch maximizes its objective subject to the balance sheet:
\[
\begin{align*}
\max_{s_t} E_t \sum_{t=0}^{\infty} \beta_t \lambda_i^P & \left[ (1 + r_t^{ib}) s_t D_t (1 - \delta_t^d) - s_{t+1} D_{t+1} (1 + r_t)(1 - s_t) D_t ight. \\
& - (1 - s_{t+1}) D_t \pi_{t+1} - (1 + r_t) D_t + D_{t+1} \pi_{t+1} - AC_t^{m} \bigg] \\
\text{s.t. } & IB_t + GB_t = D_t \\
AC_t^{m} & = \frac{\Theta}{2} \left[ (s_t - \bar{s}) D_t \right]^2
\end{align*}
\]

(39)

(40)

(41)

\(s_t\) denotes the share the wholesale branch invests into the interbank market, thus \(s_t D_t\) is the share of deposits in the interbank market and \((1 - s_t) D_t\) the opposite share of deposits held as government bonds. The surplus bank has to pay monitoring costs on the interbank market that increase with the size of interbank lending. To receive the optimal supply of funds going to the deficit bank \(s_t\), equation (40) is substituted twice into the objective and then solved.

\[
s_t = \bar{s} + \frac{r_t^{ib} - \delta_t (1 + r_t^{ib}) - r_t}{\Theta D_t}
\]

(42)

Equation (42) shows on what the wholesale branch’s decision for distributing its funds depends. Higher interbank interest rates makes the bank want to put a higher share of its collected deposits in the interbank market. This comes from the fact that the surplus bank is risk neutral. More defaults and a higher monetary policy rate, however, reduce funds in the interbank market as it becomes more attractive to buy government debt. The profits of the wholesale branch, that are transferred to the patient households, are given by:

\[
J_t^{ib} = r_t^{ib} IB_t + r_t GB_t - (1 + r_t^{ib}) \delta_t IB_t - r_t^d - AC_t^{m}
\]

(43)

Here, I already set the short-term interest rate the surplus bank receives for holding government bonds equal to the monetary policy rate set by the central bank.

### 2.5.4 Surplus Bank: Retail Branches

The retail branches of the surplus bank have a similar role to those of the deficit bank. They search the optimal interest rate on deposits from patient households facing quadratic adjustment costs (parameterized by \(\kappa_d\)) and downward sloping deposit demand.

\[
\max_{r_t^d(j)} E_t \sum_{t=0}^{\infty} \beta_t \lambda_i^P \left[ R_t^d d_t^P - r_t^d d_t^P(j) - \frac{\kappa_d}{2} \left( \frac{r_t^d(j)}{r_t^d(j-1)} - 1 \right)^2 r_t^d d_t^P \right] \\
d_t^P(j) = \left( \frac{r_t^d(j)}{r_t^d(j)} \right)^{-\frac{\kappa_d}{2}} b_t^P
\]

(44)

(45)

The resulting optimal deposit interest rate, in similar style to the ones of the deficit bank, is given by:
\[
1 - \frac{\Lambda^d_t}{\Lambda^d_t - 1} + \frac{r^b_t}{\Lambda^d_t - 1} - \kappa_d \left( \frac{r^d_t}{r^d_{t-1}} - 1 \right) \left( \frac{r^d_t}{r^d_{t-1}} - 1 \right) + \beta_t P_t \left[ \frac{d^{d+1}_t}{d^{d}_t} \right] = 0 \quad (46)
\]

### 2.6 Monetary and Fiscal Policy

The central bank follows a standard (non-linear) inflation and output targeting Taylor rule to set the short term interest rate:

\[
(1 + r_t) = (1 + \bar{r})(1 + \phi_R)(1 + r_{t-1})^{\phi_R} \left( \frac{\pi_t}{\pi_{t-1}} \right)^{\phi_y} \left( \frac{y_t}{y_{t-1}} \right) \varepsilon_t \quad (47)
\]

Hence, it also takes into account last period’s interest rate and \( \bar{r} \) denotes the steady state level of the monetary policy rate. \( \phi_R, \phi_y, \) and \( \phi_y \) are the parameters the central bank sets to express how much it cares about the past interest rate, the inflation target, and GDP growth denoted at steady state prices.

The government faces an intertemporal budget constraint that has to hold in every period. The funds from the collected taxes \( T_t \) and offered government bonds \( GB_t \) can either be used for government expenditure \( G_t \) - which follows a simple rule that includes a government spending shock \( g_t \) - or to purchase back last periods bonds the surplus bank bought. The amount of government bonds is fixed to unity.

\[
G_t + GB_{t-1} \frac{(1 + r_{t-1})}{\pi_t} = GB_t + T_t \quad (48)
\]

\[
G_t = g_t y_t \quad (49)
\]

\[
GB_t = 1 \quad (50)
\]

### 2.7 Market Clearing

The model is closed with a set of market clearing conditions:

\[
y_t = c_t + q^k_t [k_t - (1 - \delta)k_{t-1}] + k_{t-1}(\xi_1(u_t(i)) - 1) + \frac{\xi_2}{2}(u_t(i) - 1)^2 + q^{kh}_t \left[ k^h_t - (1 - \delta^{kh})k^h_{t-1} \right]
\]

\[
+ k^h_{t-1}(\xi_1(u^h_t(i)) - 1) + \frac{\xi_2}{2}(u^h_t(i) - 1)^2 + G_t + \frac{\delta_t K^b_t}{\pi_t} - q^{h}_t(1 - \delta_h)IH_{t-1} + \sum_j AC^j_t
\]

\[
c_t = c^P_t + c^I_t + c^M_t + c^E_t \quad (51)
\]

\[
IH_t = h^P_t + h^I_t \quad (52)
\]

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where $\sum AC_t$ includes all adjustment costs in the model and $D_t = d^P_t, B_t = b^I_t + b^E_t$. As can be seen from equations (51) and (53), households’ housing demand equals the entrepreneur’s housing supply in each period. The resource constraint also accounts for the non-depreciated level of housing services. Hence, in this setup households always desire and purchase new housing services for utility and borrowing reasons. But since it is also an investment, they sell the non-depreciated one before purchasing newly produced housing supply. Therefore, what matters here is the margin of the housing market that is newly produced, which makes housing services matter during the business cycle. This is different from the standard approach in which housing is modelled as a durable good and changes in housing supply margin do not have strong macroeconomic effects as they are not altering utility and borrowing limits much.

The set of 15 exogenous shocks is further given by:

$$\{\varepsilon^z_t, \varepsilon^h_t, m^I_t, m^E_t, a^E_t, a^H_t, \varepsilon^{kq}_t, \varepsilon^y_t, \varepsilon^k_t, \varepsilon^y_t, \varepsilon^{kb}_t, \varepsilon^{bl}_t, \varepsilon^{be}_t, \varepsilon^R_t, g_t\},$$

where each follows the same, in log-linearized form, AR(1) structure:

$$\varepsilon^x_t = \rho_x \varepsilon^x_{t-1} + \varepsilon^x_t$$

(54)

Except for the monetary policy shock that is governed only by its innovations $\varepsilon^R_t = \varepsilon^R_t$. The autoregressive coefficients are given by $\rho_x$ and $\varepsilon^x_t$ follow normal i.i.d. processes with mean zero and standard deviation $\sigma_x$.

3 Solution

The model equations are log-linearized around the non-stochastic steady state as the solution method described below uses log-linear equations. There is no analytical solution, thus numerical methods are used. I build on the code offered by Holden (2012) and extend it to log-linearize the equations.5

The periods covered in this analysis fall into two distinct regimes of monetary policy where the central bank was either able to adjust its main policy instrument, the nominal interest rate, to account for changes in the economy during the business cycle, or where it already lowered it to the extent for that it was constrained by the effective lower bound and thus was no longer able to adjust downwards. The solution method applied to the model, in order to incorporate the fixed-rate regime of the zero lower bound, follows the work of Kulish et al. (2017) (KMR). What they did is to use the methods developed in Kulish and Pagan (2017), that are explained in more detail below, and apply them to the DSGE workhorse model of Smets and Wouters (2007). These methods enable one to solve and estimate models that have structural changes in them. This project uses one special case for anticipated structural changes, here the implementation of the ZLB regime. To be clear, during the fixed rate regime the solution is highly non-linear and thus, even though it uses linear equations can be interpreted as an approximation of the full non-linear solution, Guerrieri and Iacoviello (2015) show that this is indeed the case.

For the following description of the solution method I follow the notation used in KMR.

5For this, the steady state for as many variables is calculated analytically, and the rest numerically. To ensure the model works and finds a steady state it is then implemented into Dynare, a software preprocessor for solving, simulating, and estimating DSGE models in Matlab, Adjemian et al. (2022) which also solves for the steady state. Using these values, I proceed in numerically log-linearizing the endogenous variables around this non-stochastic steady state with the help of Holden (2012). Afterwards my extension cleans the code to make it usable/readable for the solution method explained above.
Consider a sample of duration $T$ that has normal times where the central bank follows its Taylor rule but also at least some periods during which this is no longer possible and the nominal policy rate is constrained at the zero lower bound. The system of log-linearized equations from the model described in part two can be expressed in the form of Binder and Pesaran (1995), which is needed to solve for the time varying coefficient outcomes.

\begin{equation}
y_t = J + Ay_{t-1} + BE_t [y_{t+1}] + D\varepsilon_t
\end{equation}

$y_t$ is a vector containing all state and jump variables of size $n \times 1$, $\varepsilon_t$ is a vector of size $l \times 1$ that contains all white noise shocks, and the matrices $J, A, B$, and $D$ contain the model information are of according sizes. It is assumed that prior to the constrained periods the economy follows equation (55) and has a unique, stable solution of the VAR process:

\begin{equation}
y_t = C + Qy_{t-1} + G\varepsilon_t
\end{equation}

For clarification, I assume that the zero lower bound constrained periods start at $t = 1$, from then on the central bank sets a monetary policy rate of zero - which in a log-linearized model is equal to minus its steady state value - and starts to communicate via forward guidance how long it would stay at the lower bound. After the announced time, $t = d^e + 1$, it returns to conventional monetary policy following its pre-ZLB Taylor rule. Under a credible communication strategy, the expected duration of the constrained regime in period $t = 1$ is denoted by $d^e$, during which the log-linearized model equations follow a different structure:

\begin{equation}
y_t = \bar{J} + \bar{A}y_{t-1} + \bar{B}E_t [y_{t+1}] + \bar{D}\varepsilon_t
\end{equation}

In such a setup that includes the zero lower bound, indeterminacy does arise for these periods, but Caglia
tini and Kulish (2011) have shown that if expected monetary policy returns to a unique equilibrium outcome after $d^e$, a fixed rate regime like the one explained above can be temporarily consistent with such a unique equilibrium. The way to solve for these outcomes is to use the method of undetermined coefficients, under the assumption that conventional policy is indeed returning - as communicated - at $t = d^e$. Then, for the periods $t = 1, 2, 3, ..., d$ the time varying solution follows the VAR process:

\begin{equation}
y_t = C_t + Q ty_{t-1} + G_t\varepsilon_t
\end{equation}

Using the techniques of Kulish and Pagan (2017), which are possible through the Binder and Pesaran (1995) setup, I iterate the above equation forward by one period and take expectations. The resulting equation 59, together with the system given in 57 and after some adjusting, yield
the expressions for equations 60-62:

\[ E_t[y_{t+1}] = C_{t+1} + Q_{t+1}y_t \]  
\[ C_t = (I - BQ_{t+1})^{-1}(\tilde{J} + BC_{t+1}) \]  
\[ Q_t = (I - BQ_{t+1})^{-1}A \]  
\[ G_t = (I - BQ_{t+1})^{-1}D \]

They way this solution method works is to apply backwards recursion starting from the final solution - that is again the unconstrained unique solution that held before the zero lower bound regime - and step-by-step move through the fixed rate regime. Thus, in the final outcome it holds that \( C_{d+1} = C \) and \( Q_{d+1} = Q \), which can then be used to move backwards via equations (60), (61), and (62) to get the complete sequences across the expected duration of the regime \( \{C_t\}_{t=1}^d, \{Q_t\}_{t=1}^d, \) and \( \{G_t\}_{t=1}^d \). The matrices with subscript 1 (\( C_1, Q_1, \) and \( G_1 \)) refer to a solution with expected duration of \( d \) periods, with subscript 2 for a duration of \( d - 1 \), and so forth.

### 4 Estimation and Forecasting Methodology

Before being able to forecast key macroeconomic variables, the model has to be brought to the data and for that Bayesian estimation techniques à la An and Schorfheide (2007) are used. Since in some periods this is not standard estimation for a fixed structure, but it includes structural changes in the presence of fixed nominal policy periods, the approach has to be modified in some important directions. For those periods featuring the ZLB the monetary policy rate is removed from the list of observables. This comes from the fact that it is constant and thus, does not feature any variance, which would effectively make the variance-covariance matrix of the Kalman filter’s prediction step singular. This setup features two sets of parameters that have to be jointly estimated, however over different supports on which their prior and posterior distributions are established on. The first is the vector of structural parameters normal in DSGE estimation denoted by \( \theta \) with continuous support. An example of this could be \( \phi_\pi \), which is the parameter stating the central banks response towards inflation in the economy. The second is the sequence of expected durations \( \{d_t^e\} \) that can only take integer values, here measured in quarters. For this an example could be the first quarter of the fixed-rate regime in \( Q1 \) of 2009. Thus each quarter of the zero lower bound is considered a parameter to be estimated where a value of 1 would mean that the agents expect the nominal interest rate to be constrained at zero for one quarter.

The first forecasting exercise covers the Great Financial Crisis (GFC) and there six specific quarters are investigated starting in (real-time) \( Q1 \) of 2008 for which data up to and including \( Q4 \) of 2007 was available and ending in (real-time) \( Q2 \) of 2009 for which data up to and including \( Q1 \) of 2009 was available. These six quarters are popular for forecasting investigations of the GFC as they cover the whole start and most severe downturn periods and have been used, among others, by Gelfer (2019). Hence, the models described above are recursively estimated for all of these periods. In the last quarter, however, each model is estimated with the solution and estimation method described above that takes into account the zero lower bound.
4.1 State-Space Representation and Sampler

The state-space representation that holds in "normal" times, i.e. when there are no zero lower bound periods, adds to the model solution in (56) the measurement equation given below that connects variables to observables.

\[ z_t = H y_t \]  
(63)

Equation (63) does not feature an error term as the assumption is that there is no measurement error and \( H \) is an \( n_z \times n \) matrix that maps the observables to model variables. During the fixed-rate regime of the zero lower bound \( z_t \) and \( H \) are premultiplied by a selector matrix (\( W \)) that removes the monetary policy rate from the list of observables and the model solution follows equation (58) to incorporate the introduced sequence of expectations under calendar based forward guidance. The resulting measurement equation is given by:

\[ \tilde{z}_t = \tilde{H} y_t \]  
(64)

To achieve the joint estimation of the structural parameters and the sequence of expected durations at the zero lower bound, a randomized blocking Metropolis-Hastings algorithm following Chib and Ramamurthy (2010) is used. As in Kulish et al. (2017), there are two blocks from which a randomized sample is updated at each iteration step and draw from the joint posterior \( p(\theta, d|Z_{1:T}) \propto L(Z_{1:T}|\theta, d) p(\theta, d) \) with \( \theta \) being the set of structural parameters, \( d \) the set of expected durations at the zero lower bound, and \( Z_{1:T} \) the data series’ with length \( T \) described below. The priors on the durations and parameters are assumed to be independent, such that \( p(\theta, d) = p(\theta) p(d) \).

Compared to the sampler used in Kulish et al. (2017) I simplify the algorithm in order to gain more speed during estimation. Where they strictly follow Chib and Ramamurthy (2010) in using two separate blocks at each iteration step, I combine the two blocks to one. The main advantage over the other version is that I only need to evaluate the likelihood half as often, which results in more than double the speed. Since this method of estimation is very time-consuming to begin with I consider this an important contribution.

Thus, I update and evaluate the joint posterior likelihood of both the sequence of expected durations and the vector of structural parameters at the same time in each iteration step \( j \). Since the support for the durations is only covering integer values, the draws come from a uniform distribution. The resulting Metropolis-Hastings algorithm uses the following steps:

1. (a) The number of quarters to be updated is randomly sampled from a discrete uniform distribution \([0, d^*]\), with \( d^* = 1 \) being the maximum number possible.
   (b) Randomly sample without replacement the exact quarters to be updated from a discrete uniform distribution \([1, L]\), with \( L \) being the sample length of the zero lower bound. Thus, in \( Q1 \) of 2009 \( L = 1 \), since it is the first quarter where the ZLB was binding.
   (c) For each quarter to be updated, a random step-size is drawn from the set \([-1, 0, 1]\), while all other quarters are set to their \( d_{j-1} \) values.
2. (a) The number of parameters to be updated is randomly sampled from a discrete uniform distribution \([l, u]\), with \( l = 30 \) (\( u = 46 \)) being the minimum (maximum) number of
parameters to be updated.

(b) Randomly sample without replacement the exact parameters to be updated from a discrete uniform distribution \([1, u]\).

(c) For the randomly chosen parameters to be updated, a proposal is calculated using a multivariate Student t distribution with \(\nu = 12\) degrees of freedom. The scale matrix comes from the negative inverse Hessian derived at the posterior mode multiplied by a tuning parameter \(\kappa\) to achieve an acceptance rate between 20% and 30%. Given these new draws of parameters, \(\theta^{\text{draw}}_j\), and if needed ZLB-quarters, \(d^{\text{draw}}_j\), the posterior distribution, \(p(\theta^{\text{draw}}_j, d^{\text{draw}}_j | Z)\), is calculated using the Kalman Filter and the prior distributions.

3. The acceptance ratio is then given by

\[
\alpha_j = \frac{p(\theta^{\text{draw}}_j, d^{\text{draw}}_j | Z)}{p(\theta_{j-1}, d_{j-1} | Z)}.
\]

4. The proposed draw is accepted with probability \(\min\{\alpha_j, 1\}\), thus setting \(d_j = d^{\text{draw}}_j\) and \(\theta_j = \theta^{\text{draw}}_j\). Otherwise \(d_j = d_{j-1}\) and \(\theta_j = \theta_{j-1}\) are set.

The algorithm above is being initialized with \(\theta_0\) and \(d_0\). The values of the parameter-vector \(\theta_0\) come from the mode of the model of which the inverse Hessian is also used as the scale matrix in the updating step. This two-block algorithm is used to calculate a chain with 500,000 draws of each model considered at each point of interest. These draws do come from the joint posterior distribution \(p(\theta, d | Z)\), whereas the first half is discarded as burn-in and the mean of the remaining draws is derived.

4.2 Data and Priors

In order to estimate the models within the most realistic environment, real-time data vintages are used. Thus, original data that was available to economic agents at the time, and not the one that went through several rounds of revision, is taken into account to paint a clear picture.\(^6\) For each given forecast the models are estimated with the most up-to-date data available at that time. For each quarter considered in the forecasting exercise, data vintages published closely to the middle of the previous quarter are chosen. Hence, these forecasts also contain a nowcast of the current quarter, for which no official (quarterly) data was available yet. Data vintages published in February are used for Q1 estimations and forecasts. In this case, the last observable and fully published period is Q4 of the previous year. For Q2, data vintages published in May of the year are being used. Q3 and Q4 than implement those of August and November respectively. In each case the last observable quarter is always the previous one, this way the first quarter in each forecast actually can be considered a nowcast.

The data series used to estimate the model explained above are the federal funds rate, residential investment, non-residential investment, output growth, house prices, wages, housing supply, household loans, firm loans, and deposits. All series but the interest rates, output, and inflation are expressed in real per capita terms and made stationary by taking out the trend using the one sided HP-Filter and the code by Meyer-Gohde (2010). The nominal interest rate is demeaned and it is transformed to a quarterly structure. First differences are

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\(^6\)Some data series do not go all the way back, hence only the available data vintages can be used. The exact vintages are explained in more details in Appendix B.
For all data series’ except the nominal interest rate, the logarithm is taken to match them with log-linearized model variables. A more detailed description of the data and its vintages can be found in Appendix B. The sample includes data between Q1 of 1986 and Q1 of 2009 and thus covers the most important periods of the GFC. To keep the data and results comparable to Giri (2018) and other similar work, I refer from adding too many new data series. In general, it is a similar set of data with one difference being that there is now data on residential investment and housing supply. This is possible because of the model setup explained above as there is an endogenous housing supply sector.

The prior distribution for the standard deviations of the structural shocks is an Inverse Gamma with prior mean 0.01 and prior standard deviation of 0.05. Thus, I remain very agnostic on which shocks are crucial in driving the business cycle. For the associated autoregressive components a Beta distribution with prior mean 0.8 and prior standard deviation 0.1 is used. These can be seen in the appendix Tables (A1) and (A2) respectively. The rest of the priors, because of obvious similarities, mainly follow Giri (2018) and Gerali et al. (2010). Table (2) shows the specific prior distributions as well as estimation results for the extended model given the data available up to Q4 of 2008. These results include the posterior mean and mode, as well as the 90% highest posterior density (HPD) intervals. Here, only the structural parameters are shown that are neither shock components, nor part of the AR(1) processes.

As is usual in the (Bayesian) estimation literature, not all parameters are fully estimated. A subset, that mostly focuses on steady state outcomes and not dynamics, is being calibrated. This step’s main goal is to capture important macroeconomic features of the U.S. economy, Table (1) displays these calibrated parameters. I set the discount rate of the savers ($\beta_P$) to 0.9925 in

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7The reason output is used as a growth rate, log of the first differences, and not like the other trending variables as HP-filtered is that in the later part the goal is to forecast output growth. Using HP-filtered output yields similar results, it simply makes the matching to the real data and the SPF-forecasts more difficult.
order to match the interest rate in the sample, a standard choice. Following Iacoviello and Neri (2010) and Gerali et al. (2010), the discount factors of borrowers (\(\beta_I\)) and entrepreneurs (\(\beta_E\)) are set to 0.975 and 0.975 respectively to guarantee that both borrowing constraints are binding. The discount rate of the construction workers (\(\beta_M\)) is also set to 0.975, even though it does not matter much since they consume their whole income in each period. The capital share in the entrepreneur’s production function is set to be 0.33, a usual value widely applied. This choice comes at the cost of having a slight mismatch for the steady state shares of residential and non-residential investment, which are too low compared to the data. This is a common problem and the only solution would be to dramatically increasing capital shares in production of intermediate goods and new houses, which also introduces a mismatch between model and reality based on micro data. As a check, I have done the same exercise with a different calibration - high capital shares - and can achieve similar results. The choice of depreciation rates for physical capital in production of goods and houses falls into the same category as capital shares. Thus, I set the depreciation rate of physical capital in goods production to 0.025 and the one for capital in housing production to 0.03. This means that capital used in housing production depreciates faster, an observation already used in Iacoviello and Neri (2010). Compared to that work I use a slightly higher value for housing depreciation of 0.012 that comes from Sun and Tsang (2017). They introduce a distinction between owned and rented housing and have two separate depreciation rates of which I take the mean. The resulting ratios of residential investment and non-residential investment to total output are around 1.8% and 7.2% respectively.

The Basel II steady state ratio of capital requirements for banks (bank capital to total loans) is set to 11%. Hollander and Liu (2016) argue that this can reflect the higher value of US banks holdings in the data compared to the officially required 9%. In order to match this ratio, the depreciation rate of bank capital is set at 13.6%. This is slightly higher than what Gerali et al. (2010) used for the Euro Area, however under the assumption of a 9% requirement. Dib (2010a) uses the value of 1% yearly rate of interbank defaults, which I mimic by calibrating \(\delta_{db} = 81.3\). This yields a steady state value of defaults \(\delta_d = 0.0025\) which, as this is a quarterly model, gives the same yearly rates. The remaining parameters governing the housing supply sector follow Sun and Tsang (2017). One parameter that usually is estimated in models built on Gerali et al. (2010) is the one measuring the capital requirement adjustment costs \(\kappa_{kb}\). I find that variations in this parameter doe in fact alter steady state outcomes and I thus calibrate it. The value comes from Gallegati et al. (2019). From this work, I also take the values for the household and entrepreneur LTV rations, \(m_I\) and \(m_E\), that should match micro data better compared to the slightly lower ones used in Gerali et al. (2010). To match data averages of deposit and the two loan rates I calibrate the elasticities of substitution to \(-1.6725, 2.3969,\) and \(2.6091\). This data comes from mortgage and BAA rates and captures the fact that corporate lending rates are adjusted more frequently.\(^8\)

\(^8\)The calibrated parameters for the baseline model as well as the prior and posterior distributions for all estimated parameters can be seen in Appendix A.
Table 1. Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^P$</td>
<td>Patient HH Discount Factor</td>
<td>0.9925</td>
</tr>
<tr>
<td>$\beta^I$</td>
<td>Impatient HH Discount Factor</td>
<td>0.975</td>
</tr>
<tr>
<td>$\beta^M$</td>
<td>HTM HH Discount Factor</td>
<td>0.975</td>
</tr>
<tr>
<td>$\beta^E$</td>
<td>Entr. Discount Factor</td>
<td>0.975</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation Rate of Physical Capital</td>
<td>0.025</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital Share</td>
<td>0.33</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Share of Patient HH’s in Production</td>
<td>0.8</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Inverse of Frisch Elasticity</td>
<td>1</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Steady State Inflation</td>
<td>1</td>
</tr>
<tr>
<td>$\psi_1$</td>
<td>Degree of Capital Utilization</td>
<td>0.0483</td>
</tr>
<tr>
<td>$\psi_2$</td>
<td>Degree of Capital Utilization</td>
<td>0.00483</td>
</tr>
<tr>
<td>$\nu_b$</td>
<td>Basel II Capital Requirement</td>
<td>0.11</td>
</tr>
<tr>
<td>$\delta_b$</td>
<td>Depreciation Rate of Bank Capital</td>
<td>0.136</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Share of Profits Invested into New Bank Capital</td>
<td>1</td>
</tr>
<tr>
<td>$\varepsilon^y$</td>
<td>Markup in Goods Market</td>
<td>6</td>
</tr>
<tr>
<td>$\varepsilon^l$</td>
<td>Markup Labor Market</td>
<td>5</td>
</tr>
<tr>
<td>$\varepsilon^d$</td>
<td>Elasticity of Substitution of Deposits</td>
<td>-1.6725</td>
</tr>
<tr>
<td>$\varepsilon^{bh}$</td>
<td>Elasticity of Substitution of HH Loans</td>
<td>2.3969</td>
</tr>
<tr>
<td>$\varepsilon^{be}$</td>
<td>Elasticity of Substitution of Entr. Loans</td>
<td>2.6091</td>
</tr>
<tr>
<td>$\nu^I$</td>
<td>HH LTV Ratio</td>
<td>0.75</td>
</tr>
<tr>
<td>$\nu^E$</td>
<td>Entr. LTV Ratio</td>
<td>0.5</td>
</tr>
<tr>
<td>$\chi_{db}$</td>
<td>Deficit Bank Default Cost</td>
<td>81.3</td>
</tr>
<tr>
<td>$\bar{\gamma}$</td>
<td>Government Expenditure Steady State Share</td>
<td>0.2</td>
</tr>
<tr>
<td>$\kappa_{bb}$</td>
<td>Capital Requ. Adj. Cost</td>
<td>1.465</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>Depreciation Rate of Housing</td>
<td>0.012</td>
</tr>
<tr>
<td>$\mu_k$</td>
<td>Capital Share in Housing Production</td>
<td>0.1</td>
</tr>
<tr>
<td>$\mu_b$</td>
<td>Intermediate Input Share in Housing Production</td>
<td>0.1</td>
</tr>
<tr>
<td>$\mu_l$</td>
<td>Land Share in Housing Production</td>
<td>0.1</td>
</tr>
<tr>
<td>$\delta_{kh}$</td>
<td>Depreciation Rate of Housing Capital</td>
<td>0.03</td>
</tr>
</tbody>
</table>

For the priors on the expected zero lower bound duration, I follow the data Kulish et al. (2017) used for their estimation and setup an informative prior based on two sources. The Blue Chip Financial Forecasts are used to construct the priors for March 2009 until December 2010 as the main source, the Federal Reserve Bank of New York’s Survey of Primary Dealers, which is used for the rest of the zero lower bound periods is only available starting in 2011. In the Blue Chip Financial Forecasts - a monthly survey with only the last month of each quarter used - the respondents were asked to forecast the federal funds rate over the next six quarters. The cross-section of these point forecasts are combined to a proxy of the probability distribution. The availability of open-ended responses necessitates a special treatment for those answers. Half of the probability is spread equally over the next half year and the other half equally over the next year. From 2011 onward the Survey of Primary Dealers is used to construct the probability distribution, where again the last month of each quarter is utilized. The relevant question in the survey is “Of the possible outcomes below, please indicate the percent chance you attach to the timing of the first federal funds rate increase.” The open-ended responses are distributed as explained above. The final distribution for the zero lower bound period covered compared to its posterior distribution can be seen in Figure A1 as part of the Appendix A.
Parameter identification is checked within Dynare (Adjemian et al. (2022)), that implements various identification checks following for example Iskrev (2010) and Qu and Tkachenko (2012). According to these, I find that all parameters within all models analyzed in this paper, given the data used, are fully identified. Hence, the results can be interpreted and do not come from unidentified structural parameters.

4.3 Generating Forecasts

The randomized blocking Metropolis-Hastings algorithm used for estimation delivers serially correlated sequences of the parameter draws \( \{ \theta_j \}^\text{#sim}_{j=1} \), where \( \text{#sim} \) is the number of draws from the posterior density \( p(\theta|Z_{1:T}) \). This density, however, is not the main objective when it comes to the goal of forecasting. The goal is to generate predictive distributions that can be analyzed and are of the form: \( p(Z_{T+1:T+h}|Z_{1:T}) = \int p(Z_{T+1:T+h}|\theta, Z_{1:T})p(\theta|Z_{1:T})d\theta \). The expression before shows that draws from the predictive distribution can be obtained by simulating the model in question on parameter draws \( \theta^j \) from the posterior density and the observational data \( Z_{1:T} \). Having a large number of these draws enables one to then approximate moments and quantiles.

Following this setup, each forecast is thus built on an estimation of the models with the data available at the time and generated using \( M_1M_2 = 500,000 \) simulations. In a first step after the estimation, \( M_1 = 5000 \) draws from the posterior distribution are taken. Then each forecast is simulated using \( M_2 = 100 \) draws of future shocks for 8 quarters. These simulations build the Bayesian forecasts over the predictive distribution. This setup follows the so-called sampling the future algorithm adapted for state-space models of the form explained above in Adolfsson et al. (2007), where a detailed explanation can be found in Christoffel et al. (2011) and Del Negro and Schorfheide (2013).
The algorithm for DSGE models expressed in state-space form is given by:

1. Draw $\theta_j$ from $p(\theta|Z_{1:T})$

2. Draw the current state vector from $y_T \sim N(y_{T|T}, P_{T|T})$, where $y_{T|T}$ and $P_{T|T}$ are the posterior mean and covariance in the final step of the Kalman filter

3. Simulate a sequence of future state vectors $y_{T+1:T+h}$ from the transition equation either in (56) or (58) using the $y_T$ from step 2 and a sequence of structural shocks $\varepsilon_{T+1:T+h}$ drawn from $N(0, I)$

4. Repeat steps 2 − 3 $M_2$ times for the same $\theta_j$

5. Repeat the steps 1 − 4 $M_1$ times

The 68% forecast posterior density intervals for each forecast are calculated and shown, whereas the median is depicted as the solid line in each figure. Actual data is also included in each plot. Since real-time data is used and I do not want to compare the model forecasts to revised data, a string of actual data is build following a simple structure. For each quarter in the forecasted horizon the actual value of the data vintage published two quarters afterwards is used. This way, an initial revision is included but the big multi-period revisions that follow are excluded. This is why in the analysis below, the trajectory of the actual data changes over the 6 quarters observed. To ensure that the information set used for the estimation and forecast creation coincide with what professional forecasters had available at the time, the data vintages used are close to the deadline for these professional forecasters. These are for the quarters in February, May, August, and November of each year. This guarantees that neither the two models nor the professional forecasters have any informational advantage over each other. As an example, the forecasts from the first quarter of 2008 have the data vintage from February 2008 available. Data on this first quarter is not yet released so the last quarterly data point is Q4 of 2007. Hence, the forecast for Q1 of 2008 can indeed be interpreted as a nowcast.

5 Forecasting Analysis

The first forecasting exercise covers six quarters during the Great Financial Crisis. The first quarter is Q1 of 2008 and the last Q2 of 2009, as explained above real-time data vintages that were available up to but not including the "current" period are used for estimating the model, which then is the basis for forecasting. Three endogenous variables are analyzed, quarter to quarter real GDP growth, the nominal interest rate, and quarter to quarter inflation. All of these follow the structure explained above.

5.1 Quarter to Quarter GDP Growth Forecasts

Figure 2 shows quarter to quarter real GDP growth forecasts with a horizon of 8 quarters of the baseline, as well as the extended model. Since the model described above only features per capita output growth forecasts, while Blue Chip and Greenbook forecasts are with respect to the total growth rates of real GDP, some transformations of the forecasts are performed.
in order to make them comparable. The first thing that becomes clear is that the extended model outperforms the baseline model in terms of forecast accuracy during the GFC. As the main difference between the two is the inclusion of a housing supply sector it must add crucial informational detail to the model. Housing supply started to decrease already prior to the crisis, which can be seen in Figure 1, as did the prices for houses. These two effects combined lowered the availability of collateral against which households and firms can borrow, but also its quality and value. The banking sector propagated this effect through the financial side back into the real economy, amplifying the downturn. Another thing that can clearly be seen is the strong mean reversion tendencies of the baseline model after an initial overshoot. The extended model on the other hand features various forms of forecasts that go beyond the standard mean reversion. It is also much more capable in capturing the severity and prolonged slack in output growth. The plot for quarter to quarter real GDP growth forecasts also features the Survey of Professional Forecasters median mentions, so I am furthermore able to compare the two models to those experts. The picture is very similar, the extended model is able to outperform the SPF forecasts in some of the quarters, especially when it comes to the severity of the downturn.

Figure 2. Forecasts for Quarter to Quarter Real GDP Growth

The fact that the housing sector already declined prior to the crisis is the reason why the extended model prematurely forecasts a severe downturn in the first quarter of 2008 in which only data of 2007 – Q4 is available. Output growth in the following quarter does show a slight recovery of one quarter before the sharp decrease happens. Both models are able to forecast the first quarters increase in real GDP growth, but the extended model than also predicts the beginning of a decline and negative rates. It has to be remembered that the comparison is with

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9 Per capita rates are thus transformed using the final growth rates of population that was also used to transform them to per capita terms in the first place (POPTHM). This is also the reason why POPTHM is used and not the more commonly applied series of population CNP16OV. The first of the two is more smooth and thus is better suited for a use case like this one. Del Negro and Schorfheide (2013) also talk about the importance of using a smooth population series in such a scenario.

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a revised set of data, which could potentially alter the results slightly. For the forecasts starting from 2008 – Q3 the extended model does surprisingly well in capturing the movement of output growth over the observed horizon of two years both in size and magnitude. There, the actual realization is almost always within the 68% forecast posterior density intervals. This means that this model, estimated on this set of data, would have been able to forecast the Great Financial Crisis of 2008. Here again, it can be seen that the baseline model, after an initial increase, reverts back to its mean. This is also the first quarter in which the professional forecasters foresee a slight decrease in output growth. But they are not able to match the severity of the decline. The same thing can be observed for the fourth plot that shows forecasts from Q4 of 2008. The baseline model quickly moves back to its mean and the SPF 'only' forecast a minor decline in output growth, while the extended model matches the actual data remarkably well. One thing that stands out in which the extended model struggles is the forecasts starting in the second quarter of 2009. There it is not able to match the recovery perfectly as in the periods before. It first forecasts a recovery that matches actual outcomes but than drops off again for one quarter before moving back. This probably comes from the fact that the data on new housing, after starting to recover in the previous quarter, shows a large drop again. This phenomenon is much less visible in the final revised data so it could potentially come from measurement errors. The baseline model thus is well suited when it comes to the recovery phase of the crisis, but other than that it mostly just forecasts a slight overshoot and a reversion back to its zero mean.

Another observation that can be made is that the calculated 68% forecast posterior density intervals, i.e. the shaded areas, are considerably larger for the extended model compared to the baseline one. The prior on the shocks during estimation and the number of shocks drawn to create the forecasted paths are the same for both models analyzed. This means that the inclusion of the real estate sector not only improves forecasting accuracy but also severely increases the variance of the simulated paths. The shocks taken from the posterior estimates, thus are larger and increase the available outcomes. This phenomenon is analyzed in Section 5.7 where the parameters - including shocks - that drive the results are investigated in greater detail.

5.2 Nominal Interest Rate Forecasts

Figure 3 shows the forecasts regarding the annualized nominal interest rate set by the central bank. Again, the two models in competition are the baseline model as well as the extended model. The results are similar to the ones obtained through the output growth exercise. Overall, the extended model outperforms the baseline one in terms of matching the actual trajectory of the federal funds rate. It is able to forecast a long and severe downwards movement in the monetary policy rate, that stems from a slack in output growth, as shown above, and low levels of inflation, as shown in the following part. From the first period onward, the extended model projects that there is pressure on the central bank to ease monetary policy to counteract the problems in the economy. In the early periods, the baseline model predicts a fast reversion to the mean of the federal funds rate. It can furthermore be seen that in multiple forecasted periods, the extended model projects a negative rate that the baseline model is not capable of. In the last quarter - which is the first in which the actual rate was constrained by the ZLB - the agents in the extended model also do forecast a longer binding rate and a slower recovery back to positive policy rates. The models have been calibrated such that the steady state annual interest rate is around 4.8%. Thus, both models will eventually converge back to this value. The baseline model again has this strong mean reversion that it already showed in the output growth analysis. In almost all observed quarters it quickly and steadily converges back to its long run mean. Only in the forecasts beginning in 2009 – Q1 it actually shows a little downwards movement in the first
quarter. Remember that the first quarter of each forecast can be interpreted as a nowcast. The
deadline for data availability is the middle of each quarter in which the newest information is
not yet available. As already mentioned, the extended model does not show this strong behavior
of mean reversion, it of course moves back to it eventually, but forecasts in each observed period
a significant downturn for the nominal interest rate that even turn negative. Importantly, the
last observed period is the first in which the ZLB was binding and so there is an additional
parameter during estimation that for the expected duration of the ZLB.

Figure 3. Forecasts for the Nominal Interest Rate

Again, the 68% forecast posterior density intervals for the forecasts of the nominal interest
rate are considerably larger in the case of the extended model, highlighting increased variance
of the forecasts during the observed periods. The difference, however, is not as big as before in
the case of output growth.

5.3 Quarter to Quarter Inflation Rate Forecasts

In a last step, I investigate the forecasting performance of the two models regarding inflation.
Similarly to before, the models here use quarter to quarter inflation rates which tend to be much
more volatile and thus more difficult to capture by the model forecasts. Hence, I smooth the
actual data shown in Figure 4 to make the comparison easier.\textsuperscript{10} In Appendix A, I display the
same plot without the smoothed data to show that it does not change the interpretation of the
results (Figure A5). As before for output growth and the nominal interest rate, the extended
models forecasts are also closer to the (smoothed) actual data compared to the baseline model.
The baseline model, again, has a strong mean reversion behavior. Starting in each period, no
matter if above or below zero, its forecasts always tend to predict a fast convergence back to its

\textsuperscript{10}To achieve this, the built-in Matlab function smoothdata.m is used. This calculates a moving average over
a given window length, which in this case is equal to the forecasting horizon 8. The goal is to better show the
trend of inflation.
mean. In opposition to that the extended model performs much better. It is able to predict a strong and prolonged downturn in inflation starting in the first observed period, which match the actual data realizations much better. These low levels of inflation are also responsible for the more accurate forecasts of the nominal interest rate in the previous parts. They were one reason, among others such as of course low output growth, that the central bank kept interest rates low for a very long time. The only forecast that does not perform as good as the others, again, is the one starting in Q2 of 2009. The extended model predicts a prolonged and severe downturn in the inflation rate, which is also the reason the monetary policy rate forecasts were negative for such an extended period of time.

Figure 4. Forecasts for the Inflation Rate

5.4 Numerical Accuracy of Point Forecasts

The previous sections have shown graphically that the extended model was able to outperform the baseline model and the SPF during most observed periods and for the three variables of interest. In this section I conduct an empirical comparison by computing Root Mean Squared Errors (RMSEs) of the out-of-sample forecasts. This is done over the six quarters analyzed previously. Hence, each quarter has a relatively strong share during this calculation and can hence drive numerical results. The definition of the RMSE follows Wieland and Wolters (2011). Thus, the RMSE for model $m$ at forecasting horizon $h$ is given by:

$$\text{RMSE}_{m, h} = \sqrt{\frac{\sum_{t = t_{\text{start}} + h - 1}^{t_{\text{end}}} (E[x_{m, t+h}|I_t] - x_{t+h})^2}{t_{\text{end}} - t_{\text{start}} - h + 1}},$$

where $E[x_{m, t+h}|I_t]$ denotes the forecast of variable $x$ from model $m$ estimated conditional on the information set $I_t$. This includes the model equations and the data vintage up to period $t$ and thus accounts for the learning with each new data point becoming available. $x_{t+h}$ is the
data realization \( h \) periods ahead. As explained above, the actual data realization used is the one from a vintage released two quarters after the point of interest. \( t_{\text{start}} \) and \( t_{\text{end}} \) give the start (2008 – Q1 + \( h \)) and end (2009 – Q2 + \( h \)) of the evaluation periods respectively.

Table 3 shows the RMSE’s for real GDP growth, the nominal interest rate, and the inflation rate. The columns depict the different forecasting horizons ranging from \( h = 1 \) to \( h = 8 \), while the rows show the two models. For the case of GDP growth, I also consider the SPF forecasts that historically provide a nowcast and four quarters of forecasts. In this scenario, \( h = 1 \) can be interpreted as a nowcast, which in some studies is labeled \( h = 0 \). A RMSE that is lower (higher) for one model relative to another model indicates that this models forecast is more (less) accurate than the other ones for this specific forecasting horizon and information set.

Table 3. Root Mean Squared Errors

<table>
<thead>
<tr>
<th></th>
<th>( h = 1 )</th>
<th>( h = 2 )</th>
<th>( h = 3 )</th>
<th>( h = 4 )</th>
<th>( h = 5 )</th>
<th>( h = 6 )</th>
<th>( h = 7 )</th>
<th>( h = 8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Output</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extended Model</td>
<td>3.0358</td>
<td>3.9710</td>
<td>4.3632</td>
<td>4.9640</td>
<td>4.3217</td>
<td>2.0739</td>
<td>1.2817</td>
<td>1.6634</td>
</tr>
<tr>
<td>Baseline Model</td>
<td>3.8950</td>
<td>4.7169</td>
<td>4.8255</td>
<td>4.6650</td>
<td>3.7492</td>
<td>1.7899</td>
<td>1.5664</td>
<td>1.7262</td>
</tr>
<tr>
<td>SPF</td>
<td>1.7723</td>
<td>3.6678</td>
<td>4.9399</td>
<td>5.3360</td>
<td>4.3956</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Interest Rate</strong></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extended Model</td>
<td>1.4929</td>
<td>0.9480</td>
<td>1.8717</td>
<td>3.6420</td>
<td>4.0379</td>
<td>1.8314</td>
<td>1.1709</td>
<td>1.4419</td>
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<tr>
<td>Baseline Model</td>
<td>1.9268</td>
<td>2.1161</td>
<td>3.3812</td>
<td>5.1178</td>
<td>5.6513</td>
<td>4.1804</td>
<td>3.8889</td>
<td>3.9186</td>
</tr>
<tr>
<td><strong>Inflation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extended Model</td>
<td>0.5827</td>
<td>1.2591</td>
<td>0.1635</td>
<td>2.1181</td>
<td>2.5728</td>
<td>0.3154</td>
<td>1.0836</td>
<td>2.1434</td>
</tr>
<tr>
<td>Baseline Model</td>
<td>0.5403</td>
<td>1.1914</td>
<td>0.4234</td>
<td>2.2773</td>
<td>2.7250</td>
<td>0.4717</td>
<td>0.9726</td>
<td>2.0223</td>
</tr>
</tbody>
</table>

For each variable, i.e. Output, Interest Rate, and Inflation, the forecast with the smallest RMSE according to every forecasting horizon is highlighted green in Table 3. Some clear patterns emerge between the models over different time frames.

For real quarter to quarter GDP growth, the SPF does outperform both models considered in the first two periods with significantly lower RMSEs, especially in the nowcast. This can be attributed to the fact that professional forecasters do not only have the (small) set of quarterly data available when performing their forecasts but a much wider range of high frequency data, which leads to better short-term forecasts. SPF forecasts fall behind both models for medium run forecasting horizons. Between the two models, the extended model does a better job in the short horizon forecasts with forecast horizons \( h = 1 \) to \( h = 3 \). Given that the first couple of quarters are usually more important, this is a success for the extended model. Especially during such unsteady times as during the GFC. From period 4 onward up until \( h = 6 \), the baseline model has the smallest RMSEs and thus outperforms both the extended model as well as the SPF. It is known that this class of models do quite well in providing accurate medium run forecasts because of their strong mean reversion tendencies. As I explained above, each RMSE is based only on the six quarters considered in this forecasting exercise. Hence, each period has a strong impact on the results. In the graphical analysis before the forecasts of output growth starting in Q1 of 2008 were quite off for the extended model that already predicted a severe recession. Removing these forecasts from the calculation, I can show that the extended
model indeed outperforms the baseline model and the SPF from period $h = 1$ onward. This can be seen in Table A7 in Appendix A. The RMSEs of the two models show that the forecasting performance of output growth is worse in the short-term compared to the long-term horizon. The nowcast RMSEs are 3.0358 and 3.8950 for the extended and baseline models respectively. While the 2 year into the future forecasts are 1.6634 and 1.7262, which are considerably smaller values and thus (relatively) more accurate forecasts. This makes sense for the observed period of the GFC in which most short-term forecasts are compared to data coming from tremendous crisis outcomes, while long-term forecasts are compared to the recovery periods and the time after the crisis.

The picture of which models’ RMSEs are smaller changes completely when the variable considered is the nominal interest rate, the second part in Table 3. The extended model outperforms the baseline model, i.e. has lower RMSEs, for all horizons considered in this forecasting exercise. Hence, the models’ ability to predict movements in the interest rate increases significantly through the inclusion of the endogenous housing sector.

Both models’ RMSEs of the inflation rate for the first two periods are very similar. Nonetheless, the baseline model has lower values and thus does better in forecasting the short-term movements of inflation. The extended model outperforms the baseline model in the medium-term. In the graphical analysis of inflation forecasts (Figure 4) I found that the extended model was able to project some downturns that the baseline model was not capable of. This is the reason for this result in the numerical analysis. The baseline model projected very little movement in the inflation rate and let it move back steady state relatively quickly, which is why the long term forecasts $h = 7$ and $h = 8$ again are closer to actual data since the extended model had longer lasting effects on prices.

5.5 Robustness Check

This section of the paper can be seen as a robustness check of the results presented before. The claim that has been made is that there is a combination of two important inclusions necessary in order to gain improved forecasting accuracy during the GFC. The first is an endogenous housing supply sector in the form of a model extension, explained in Section 2. The second inclusion is to add data for this sector as an observable variable during the estimation and forecasting exercise to add crucial informational detail. One argument in this favor is that models that do indeed ‘only’ include this sector but no additional data during estimation are not able to forecast the crisis (Iacoviello and Neri (2010), Sun and Tsang (2017)) even though the fact that they have this sector can potentially increase their accuracy as was shown by Kolasa and Rubaszek (2015).

Therefore, in this part I introduce two robustness check versions of the extended model and compare them with the baseline version as done before. There are potentially two sources of information that drove the previous results, the model structure as well as the additional data. In the first version, called extended model robustness check 1, I drop the additional data on housing supply and combine the two data series’ on residential and non-residential investment into one. The observables used for both models are the nominal interest rate, investment, output growth, house prices, inflation, wages, loans to households, loans to firms, and deposits. Therefore, the data used during estimation is exactly the same as what is available in the baseline model and two series’ less compared to the extended model before. Hence, all differences that arise now come from the model structure itself and not from additional information through the data. In the second option, called extended model robustness check 2, I reintroduce the division in investment data into residential and non-residential investment to see whether this aspect introduces notable benefits to the forecasting capability of the model. This section can also be
cross-referenced with the previous one in the sense that the difference between the forecasts of the two robustness check versions of the extended model and the model explained before solely come from the extra data on housing supply. So that than there are three versions of the model in total, each including some more information during estimation. This makes it possible to interpret the two aspects and investigate what really drives the results in the previous section. In the following analysis these two versions of the extended model - given different data during estimation - are shown and compared to the baseline case.

Figure 5. Forecasts for Quarter to Quarter Real GDP Growth

Figure 5 shows the two model forecasts side by side, again starting with quarter to quarter real GDP growth. It is clear that even the extended model that did so well in the previous sections is no longer able to forecast the GFC as well as before when it had additional data available. And the differences between the two robustness check versions of the extended model are also small. Most forecasts show almost identical or very similar paths between the two models. Only the two forecasts from 2009 − Q1 and 2009 − Q2 display some minor differences where it even seems that the baseline model very slightly outperforms the extended model. Overall, it can be said that the extended model structure itself does not add to the forecasting performance of quarter to quarter real GDP growth. From there it follows that the extra information came from the data on housing supply and thus the the combination of model and data yields the superior forecasting performance. But what is also important is that the sole inclusion of construction workers and the housing supply sector does not worsen the model as it has been shown that larger model with financial frictions sometimes tend to perform worse in forecasting or at least not better, especially during non-crisis times. Kolasa and Rubaszek (2015) also show that the addition of a housing market can be relatively successful in this regard.
A similar conclusion can be made for the forecasts of the nominal interest rate. Figure 7 shows that in most periods especially where the three models had very similar forecasts of GDP growth, most notably quarters 2 to 4 of 2008, they project comparable trajectories of the interest rate. Only in the last two periods, starting in Q1 of 2009 and Q2 of 2009, do they have slightly different forecasts. But even here the overall directions are very similar and no major macroeconomic differences from monetary policy can be made out.
Based on the fact that the forecasts of the three models for output growth and the interest rate are very similar, it is not surprising that the ones for inflation are also close. The same can be said, in most quarters the three models produce almost identical forecasts of the inflation rate. Only in the last two periods they do look slightly different. This makes sense as what drives the nominal interest rate response of the central bank is the output gap as well as inflation. So if these two look similar across models, they also produce matching interest rate responses.

Thus, from this robustness check exercise I can conclude that the extended model by itself, with the same or similar data to the baseline model, does forecast key macroeconomic variables in the same way as the baseline model. The model structure itself does not necessarily increase nor decrease forecasting ability. What really drives the results in the main part is the combination of the model extension and the additional information coming from housing supply data.

5.6 Properties of the Estimated Models

In this section, I examine how shocks are transmitted through the economy in the various versions of the model that either include endogenous housing and data on it or not. The forecasts of the previous section differ strongly, which hints at the fact that the models can have different policy implications as well. The main question is to see the propagation of shocks of the extended model compared to its baseline counterpart. For completeness, a robustness check version of the model, as explained above, is also included in the analysis.

Figure 8. Monetary Policy Shock

The first shock analyzed is an unanticipated 50 basis points increase in the monetary policy rate and can be seen in Figure 8. The three models analyzed are calibrated to the estimated posterior mode from each respective estimation, as explained above, with data available up to
2008 – Q4. Overall, the responses follow standard models in that output and inflation fall, which also lowers consumption. Higher interest rates for households and entrepreneurs to borrow come from higher bank interest rates and overall lower present discounted value of collateral as house prices decrease. This makes loans to households and entrepreneurs decline. The increase in interest rates also makes overall investment decline. On the household side, patient households lower their level of deposits put into the banking sector while also lowering their demand for housing. In the models that feature a housing supply sector, it decreases as well following higher costs of borrowing, lower values of collateral, and the decrease in housing demand.

The introduction of an endogenous housing supply sector exacerbates the effect of the observed policy tightening. Output and consumption decrease considerably more in the extended model compared to the other two. Simultaneously, patient households deposit less at surplus banks while impatient households dramatically decrease their borrowing from the deficit banks. Figure 8 furthermore shows that firms reduce their borrowing, on impact, by about the same amount but keep it contracted for much longer. Total investment reacts in a related way. Hence, firms reduce their production of intermediate goods and new houses. Overall housing demand also decreases much more in the extended model compared to the robustness check version, while the baseline model also features strong negative demand. This decrease comes mostly from impatient households that face stronger declines in their present discounted value of collateral. These effects combine to a stronger and more prolonged business cycle.

What can be seen is that the robustness check version of the model much more closely follows the baseline model than the extended one. Since the only difference between the robustness check version and the extended model is housing supply data during estimation the differences must come from there. This result matches the one from the forecasting analysis.

Figure 9. Productivity Shock
The second shock I analyze is a standard productivity shock shown in Figure 9. Again, it shows the standard dynamics. As output and hence consumption increase, inflation falls, which makes the central bank lower its policy rate. As it becomes relatively cheaper to borrow, households and firms increase their level of borrowing from the deficit bank and total investment surges. Simultaneously, because of increased house prices, borrowers present discounted value of collateral increases, leading to a prolonged increase in housing demand.

The difference between the extended model and the other two models are less severe in the case of the productivity shock. Output and consumption behave very comparably, while the reaction of inflation is attenuated, leading also to a smaller response of the monetary authority and thus lending rates. The most interesting difference is in the housing supply, which equals demand, between the extended model and the robustness check version of it. While it increases in the extended model, a reaction that would be expected following a technology shock and an increase in economic activity, it falls on impact in the robustness check version of the model and then only turns positive after around a year. The reason for this could be that in the robustness check version of the model, house prices do increase substantially more, enabling borrowers to even lower their demand for new houses in order to fund the desired level of consumption. And they only increase their housing demand with the following upswing in real economic activity. As can be seen in the responses of the baseline models housing sector, large changes in demand and prices are not sufficient to alter the propagation in the way the extended model is capable of.

Figure 10. Housing Demand Shock

The last shock analyzed here is the housing demand preference shock of households that turns out to be crucial during the GFC, as can be seen in a historical shock decomposition in Figure A8 in the appendix. A negative housing demand shocks leads to a big reduction in housing supply and a persistent decrease in house prices. This, through the employment of
workers in the construction sector and their consumption, gets propagated into the real economy leading to a decrease in consumption and thus also output. Patient households also lower their level of deposits at the surplus bank, which tightens conditions in the financial sector. At the same time impatient households dramatically lower their loan demand, as it is used for housing consumption. Overall, the drop in loan demand is much more pronounced and can be seen as the main driver of friction in the financial market. Seeing this, the central bank lowers its monetary policy rate to induce crowding-in effects and inflation drops slightly.

The difference between the extended model and the other two models is striking, showing that the introduction of housing supply data during estimation helps properly identifying the size of housing demand responses so crucial during the GFC. The same sized shock induces a much stronger macroeconomic reaction in the extended model. Lower demand for housing now leads to even stronger reduction in its supply, a variable that is observed and used during estimation, which amplifies the responses just explained. Hence, HtM households work even less, reducing their level of consumption and thus the overall stance of the economy, which gets supplemented by frictions in the financial sector both from supply and demand side. Overall, the business cycle movements, again, are exacerbated with bigger movements in output and consumption. This also makes the central bank react stronger in lowering the policy rate. These are all things, that could be seen in the forecasts presented before. Also, it can be seen that this shock affects the demand side of the financial sector for households considerably more than the supply side. In the sense that the size of changes in deposits is much smaller compared to household loans, stemming in larger parts from persistent lower house prices and thus tighter borrowing constraints.

5.7 Parameters Driving the Results

The previous sections have established that a model with an explicit and endogenous housing supply sector combined with data on it during estimation is able, using real-time data, to better forecast the output, inflation, and nominal interest rate dynamics during the Great Financial Crisis. Furthermore, absent the additional information that housing supply data gives while estimating structural parameters, the extended model does mostly similar and sometimes even a bit worse compared to its baseline counterpart. Furthermore, the extended model does produce IRF’s that show exacerbated and prolonged business cycles. I showed in the previous section that negative housing demand shocks can explain the GFC very well. This is a drop in housing demand translates into the real economy through the consumption of HtM households, while impatient households at the same time dramatically lower their loan demand, creating frictions in the financial sector. Since the additional information translates into different posterior distributions for all parameter estimates, I compare these in this section to further get insights into what drives these results. As the robustness check versions of the extended model do have the same set of parameters in estimation but perform very similar to the baseline model, this enables me to deduct crucial information. Figure 11 plots a subset of the posterior distributions of the extended model, the baseline model, the two robustness check versions of the extended model, and the the specific prior distribution for each parameter using for illustrative purposes the sample with data up to 2008 – Q4. This gives me the opportunity to see where the extended model differs from both, the baseline model as well as the robustness check version of the extended model. Discrepancies can be seen and interpreted as a source of the drivers behind the results. Here, only a subset of all structural parameters is analyzed. A complete list, including plots, can be found in Appendix C.
In a first step I analyze structural parameters that neither are shocks nor part of the AR(1) processes of the shocks. Since the results of the forecasting exercise are strikingly different, I expect to see that being visible in at least a subset of parameter posterior distributions. The biggest and most striking difference can be seen in $\kappa_w$, the parameter measuring the quadratic adjustment costs of the labor unions that set optimal wages. The extended model thus has wages that are much more expensive to adjust and as a result get adjusted much less frequently, translating into stronger business cycles as the economy takes longer to adapt to shocks. It can be seen that the baseline model and the robustness check versions of the extended model all have significantly smaller posterior distributions. The same, but less severely, holds true for the parameter governing the cost of adjusting prices of the final goods, $\kappa_p$. It too is larger in the extended model and covers a wider spectrum. Compared to the other three that are more similar in size. This means that the additional data in the housing supply sector in the model introduces adjustment costs that make it much more difficult to adjust wages and prices. As a result, firms adjust their prices and primarily wages much more infrequently in the extended model compared to the other ones. Furthermore, there are interesting aspects about the parameters governing the habit formation of different agents in the models. Patient households show a lower degree of consumption habit formation in the extended model, $\alpha^P$. This does not only affect their consumption behavior, which is more volatile and therefore less persistent in the extended model, but also the supply of deposits for the banking sector. The decreased habit persistence can come from the fact that the financial side of the model demands more volatile inflows of deposits coming from more uncertainty on the lending side as the supply of loans moves more as a result of stronger business cycle fluctuations. Surplus banks want to decrease their lending towards deficit banks observing their default probability increasing. This is done by moving more funds into save government bonds but also by reducing their asked deposits from savers. This effect is combined with the amplified cycles that requires savers to change their consumption and thus reduce persistence. The two robustness check versions of the extended model follow the prior distribution much more closely and the habit parameter of
patient households is higher compared to the baseline and extended model, making savers habit formation stronger. Two more crucial differences can be seen when observing habit formation of impatient households and entrepreneurs, $a^I$ and $a^E$ respectively. For those two parameters the baseline model and the robustness check versions of the extended model react almost identical in the case of entrepreneurs’ habit formation and very similar for borrowers, while the extended model puts a much higher value on habit formation for impatient households and a lower one for entrepreneurs. Both of these agents act as borrowers in the economy as they obtain funding from the banking sector. Since entrepreneurs are also building new real estate, a lower habit formation can mean that since its supply and prices are volatile they are less certain about future paths of their consumption and thus have to change it more often. This finding goes well with the explanation of the lower habit formation parameter of the patient household supplying deposits. Interestingly, wage and price indexation to their previous period’s level, $\iota_w$ and $\iota_p$, are relatively smaller compared to the other models. This means that the more prolonged business cycles observed in the previous section do not come from large persistence in wages and prices. The stronger business cycles do, however, stem from these pricing mechanisms. One more observation is that surplus banks have it easier in adjusting deposit interest rates in the extended model, compared to the other three. Since all four versions of the models use the same data on deposits during their estimation this can mean that due to the lower adjustment costs $\kappa_d$, the supply of deposits and thus of funds within the banking market has to be faster to adapt to real macroeconomic changes. Or put differently, the surplus banks are only willing to pay lower interest rates on deposits since they are less willing to forward it into the financial system and for this to happen even accept lower markups. This creates an amplified negative effects for borrowers. The central bank reacts very similarly across all models but there is one slight difference. Its indexation to the previous periods’ nominal interest rate set, $\phi_R$, is higher. There are two possible explanations for this higher value. One is that the central bank in general keeps its policy rate stable because it wants to and there are less striking deviations in prices and output. Or second, that it is forced to keep them steady because of prolonged macroeconomic conditions, in this case because of the GFC and the ZLB.
In the previous analysis I showed that the extended model is not only more capable of forecasting the trajectories of important macroeconomic variables during the GFC but it also increased its variance and thus the size of the projected paths in these forecasts. There are two possible sources for this, one is the variability and size of the structural parameters that I analyze above. The other is the size of the shocks that can potentially lead to this increased uncertainty in the forecasts additionally to their trajectories.

As before, Figure 12 shows a subset of shocks that feature big differences in their posterior distributions. The consumption preference shock in the utility functions of all three households, $\sigma_z$, is noticeably larger for the extended model. This can drive the greater deviations in consumption and thus output that were observed in the extended model forecasts of the GFC. In the previous section I established the fact that patient households have to adjust their consumption and savings behavior much more aggressively in the extended model, which now can also be seen in this shock size. Another shock that is larger in the extended model is for the LTV-ratio of the impatient household, $\sigma_{mI}$. This ratio gives the level of credits the borrowers can get from banks given their discounted level of collateral. Thus, a larger overall shock means that this transmission channel of funding from the financial sector is more volatile and impatient households do potentially find it more difficult to borrow based on unexpected changes in their ability to take loans, which translates into their demand for housing and consumption. At the same time impatient households want to keep their consumption levels comparatively more stable in the extended model relative to the other models. Hence, greater variation and thus uncertainty in their ability to finance the level of consumption while simultaneously being less willing to adjust it leads to an overall reduction in their dependency from loans and thus a contraction in overall consumption. As was seen with the much higher adjustment cost parameter governing the cost of changing wages, $\kappa_w$, firms find it more difficult to adjust their labor demand in the face of macroeconomic fluctuations. This translates into a much higher wage markup shock in the extended model, $\sigma_l$. The last remaining shock is the housing demand shock for households
that determines their preference for housing, $\sigma_h$. The previous section has shown that negative housing demand shocks have severe effects on the economy and are partially responsible for the huge downturn during the GFC. This is enhanced by the fact that the posterior distribution of the housing demand shock is larger compared to the other ones. It thus, does not only imply stronger macroeconomic reactions, especially on output and the financial system through deposits, but also is increased in size. The remaining shocks are either very similar across all models, such as the markup shock, $\sigma_y$, the markup shock for loans to impatient households, $\sigma_{bI}$, and the monetary policy shock, $\sigma_R$. Or they are similar between the extended model and either one of the robustness check versions. However, to see the main drivers behind the forecasting power of the extended model I do not consider those shocks. Even if they differ to the baseline model, I have established before that it is a combination of model and additional information through the data as the forecasts of these models were much closer to the baseline model. Thus, when the two robustness check versions of the extended model have similar posterior densities for some shocks, they are not crucial in the sense that they drive the results. These shocks include the markup shock for loans to entrepreneurs, $\sigma_{bE}$, the balance sheet shock of the deficit bank, $\sigma_{kb}$, and the productivity shock in the housing supply sector, $\sigma_{ah}$.

Overall, there are two crucial aspects that can be identified as main drivers of the results. The first is the difficulty of firms to adjust prices and especially wages. Thus, the baseline model clearly underestimates the persistence in prices and wages and how crucial they are in transmitting shocks through the economy. Prices are only slightly more expensive to adjust but wages show a dramatic increase in adjustment costs. In these kind of models, adjustment costs are a proxy for how frequent prices and wages can be changed and this result thus shows that firms had trouble adjusting prices but more importantly wages over the observed sample. It is well known that wages tend to be much more downward rigid and hence are more difficult to lower during times of an economic downturn as analyzed here. The other aspect comes from housing demand as the last two sections have shown. The extended model identifies overall larger housing demand shocks that crucially transmit into the real and financial side of the economy. Thus, with more volatile consumption because of negative housing demand shocks, savers reduce their habit formation parameter and thus make their supply of deposits more flexible. The same reason makes impatient households reduce their demand for loans, even in a stronger way, which increases troubles in the financial market. Lower demand for housing also reduces housing supply, that again propagates quickly into the real economy through the consumption of HtM households.

5.8 Forecasting the COVID-19 Pandemic

Recent events surrounding the COVID-19 pandemic and its effect on the economy have led to a new crisis around the world that macroeconomists try to understand through the lenses of various models. The extended model described above does not feature aspects regarding the transmission of a pandemic on the economy such as for instance Eichenbaum et al. (2021). Thus, I do not expect that neither the baseline nor the extended model are able to forecast the beginning and the transmission of the 2020 crisis. They can nonetheless be used to check what it would have forecasted, especially during the recovery and the following quarters where lockdowns and supply chain issues led to tremendous uncertainty. Hence, in this section I do a short but similar comparison to the main part of the paper, comparing the extended and baseline model forecasts.
during the beginning of the COVID-19 pandemic and its follow-up.\footnote{In appendix D, I also show a forecast of the extended model with the most up-to-date data available at the time of writing (Q1 of 2023).} This can furthermore be seen as a check of how the extended model fares in forecasting economic behavior that is not driven by the housing sector, as the GFC - to some extent - clearly was. It has also been established in the literature that larger models, estimated on more data, tend to perform worse in tranquil times. Hence, similar performance in this exercise would mean that the extended model does at least as well as the smaller baseline model.

Figure 13 shows the quarter to quarter real GDP growth between Q1 of 2020 and Q4 of 2020, the occurrence of COVID-19. The corresponding plots for the nominal interest rate and the inflation rate can be found in Appendix A as Figures A6 and A7 respectively. To achieve this, I extend the data set used before to include the required periods and also extend the zero lower bound expectations data used by Kulish et al. (2017) and explained in Section 4.2. There is a second ZLB period starting in Q2 of 2020 that has to be taken into account while estimating and forecasting the models. Thus, there are now two sets of zero lower bound periods in the data set. Appendix A also includes plots on the difference between prior and posterior distributions of the expected zero lower bound expectations from the estimation with data up to Q4 of 2019.

Looking at Figure 13 it can be seen that the two models forecast very similar paths for output growth during the large movements of the pandemic. Both of these models are of course not able to forecast the large drop in output as the COVID-19 shock could not have been anticipated. What they can be used for is how they anticipate the recovery of the huge shock, which can be seen in the forecasts from Q3 of 2020. They do again predict a very similar movement of real GDP growth where the extended model is only slightly lower. Interestingly, the variation in both forecasts is much more similar, indicating that both models have comparable accuracy during this time. This result evidently changes in Q4 of 2020. The variation of the extended models’ forecasts are considerably larger compared to the ones of the baseline model. Here it also begins that the two models show different trajectories. As I showed for the case of the GFC, the extended model has much larger deviations while the baseline model always shows strong and consistent mean reversion. The extended model overshoots the actual outcome before moving back to mean.

What can be said is that the two models perform very similarly during tranquil times. A success as it has been shown that larger models with financial frictions tend to perform worse when there is no crisis. The extended model, even while overshooting in some cases, is still able to capture some of the effects that started to occur toward the end of 2020 when lockdowns and supply chain issues began to increase uncertainty.
6 Conclusion

In summary, the Great Financial Crisis (GFC) exposed flaws in prevailing macroeconomic models, spurring crucial enhancements. Notably, the financial sector and real estate sector gained prominence. However, housing supply in DSGE models received limited attention. This study emphasizes the importance of incorporating housing supply data for accurate forecasting during economic crises. A sophisticated DSGE model integrating financial frictions and an endogenous housing supply sector is constructed, demonstrating its superiority in forecasting the GFC compared to baseline models. Analysis of estimation results underscores the role of negative housing demand shocks and increased adjustment costs in understanding the crisis dynamics. Additionally, the paper explores unconventional monetary policies like forward guidance and contributes to forecasting comparisons in macroeconomic research. The integration of financial frictions and an endogenous housing supply sector represents a significant advancement in macroeconomic modeling, offering a more comprehensive understanding of economic dynamics during crises.
References


Paries, Matthieu Darraaq and Alessandro Notarpietro, Monetary policy and housing prices in an estimated DSGE model for the US and the euro area, European Central Bank Frankfurt, 2008.


## A Appendix: Tables and Figures

Appendix Table A1. Prior and Posterior Distribution - Standard Deviations of Innovations of Shocks

*Estimation of the Extended Model on Data available up to 2008-Q4*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Prior Shape</th>
<th>Post Mean</th>
<th>Post Mode</th>
<th>90 % HPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^z$</td>
<td>Consumption Preference</td>
<td>$\Gamma^{-1}[0.01,0.05]$</td>
<td>0.168</td>
<td>0.161</td>
<td>[0.134, 0.201]</td>
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<tr>
<td>$\sigma^h$</td>
<td>Housing Preference</td>
<td>$\Gamma^{-1}[0.01,0.05]$</td>
<td>0.034</td>
<td>0.032</td>
<td>[0.023, 0.044]</td>
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<td>$\sigma^{mI}$</td>
<td>Impatient HH - LTV</td>
<td>$\Gamma^{-1}[0.01,0.05]$</td>
<td>0.030</td>
<td>0.030</td>
<td>[0.026, 0.034]</td>
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<td>Entr. - LTV</td>
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<td>0.069</td>
<td>[0.052, 0.097]</td>
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<tr>
<td>$\sigma^{aE}$</td>
<td>Entr. TFP</td>
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<td>0.006</td>
<td>[0.004, 0.009]</td>
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<td>0.227</td>
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<td>0.021</td>
<td>[0.018, 0.024]</td>
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<td>[0.004, 0.017]</td>
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<td>HH Interest Rate Markup</td>
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<td>0.007</td>
<td>[0.004, 0.016]</td>
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<td>Entr. Interest Rate Markup</td>
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<td>[0.004, 0.019]</td>
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<td>0.002</td>
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<td>Government Spending</td>
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<td>0.007</td>
<td>[0.004, 0.022]</td>
</tr>
<tr>
<td>$\sigma^{kb}$</td>
<td>Balance Sheet</td>
<td>$\Gamma^{-1}[0.01,0.05]$</td>
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<td>0.085</td>
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<tr>
<td>$\sigma^a^H$</td>
<td>Entr. Housing TFP</td>
<td>$\Gamma^{-1}[0.01,0.05]$</td>
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<td>0.009</td>
<td>[0.009, 0.010]</td>
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Appendix Table A2. Prior and Posterior Distribution - AR(1) Parameters

*Estimation of the Extended Model on Data available up to 2008-Q4*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Prior Shape</th>
<th>Post Mean</th>
<th>Post Mode</th>
<th>90 % HPD</th>
</tr>
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<tbody>
<tr>
<td>$\rho^z$</td>
<td>Consumption Preference</td>
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<td>0.65</td>
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<td>0.96</td>
<td>[0.94, 0.98]</td>
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<tr>
<td>$\rho^{mI}$</td>
<td>Impatient HH - LTV</td>
<td>$B[0.8,0.1]$</td>
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<td>0.91</td>
<td>[0.85, 0.96]</td>
</tr>
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<td>$\rho^{mE}$</td>
<td>Entr. HH - LTV</td>
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<td>0.47</td>
<td>[0.37, 0.57]</td>
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<td>0.85</td>
<td>[0.75, 0.92]</td>
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<tr>
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<td>Wage Markup</td>
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<td>0.92</td>
<td>[0.88, 0.96]</td>
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<td>0.93</td>
<td>[0.90, 0.95]</td>
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Appendix Table A3. Calibrated Parameters

Baseline Model

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<th>Parameter</th>
<th>Description</th>
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<td>$\beta^P$</td>
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<tr>
<td>$\beta^I$</td>
<td>Impatient HH Discount Factor</td>
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<td>$\beta^E$</td>
<td>Entr. Discount Factor</td>
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<td>$\delta$</td>
<td>Depreciation Rate of Physical Capital</td>
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<tr>
<td>$\alpha$</td>
<td>Capital Share</td>
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</tr>
<tr>
<td>$\mu$</td>
<td>Share of Patient HH’s in Production</td>
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</tr>
<tr>
<td>$\phi$</td>
<td>Inverse of Frisch Elasticity</td>
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<td>Steady State Inflation</td>
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<td>$\psi_1$</td>
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<td>$\nu_b$</td>
<td>Basel II Capital Requirement</td>
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<td>Share of Profits Invested into New Bank Capital</td>
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<tr>
<td>$\bar{\varepsilon}^y$</td>
<td>Markup in Goods Market</td>
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<td>$\bar{\varepsilon}^l$</td>
<td>Markup Labor Market</td>
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<td>$\varepsilon^d$</td>
<td>Elasticity of Substitution of Deposits</td>
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<tr>
<td>$\varepsilon^{bh}$</td>
<td>Elasticity of Substitution of HH Loans</td>
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<td>$\varepsilon^{be}$</td>
<td>Elasticity of Substitution of Entr. Loans</td>
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<td>$m^I$</td>
<td>HH LTV Ratio</td>
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<tr>
<td>$m^E$</td>
<td>Entr. LTV Ratio</td>
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<td>$\chi_{db}$</td>
<td>Deficit Bank Default Cost</td>
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<tr>
<td>$\bar{g}$</td>
<td>Government Expenditure Steady State Share</td>
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<tr>
<td>$\kappa_{kb}$</td>
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### Appendix Table A4. Prior and Posterior Distribution - Structural Parameters

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<th>Parameter</th>
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<th>Post. Mode</th>
<th>90 % HPD</th>
</tr>
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<tr>
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<td>Habit Patient HH</td>
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<td>0.3749</td>
<td>0.2546</td>
<td>$[0.2517, 0.4912]$</td>
</tr>
<tr>
<td>$a^I$</td>
<td>Habit Impatient HH</td>
<td>$B[0.5, 0.1]$</td>
<td>0.4711</td>
<td>0.4570</td>
<td>$[0.2974, 0.6554]$</td>
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<tr>
<td>$d^E$</td>
<td>Habit Entr.</td>
<td>$B[0.5, 0.1]$</td>
<td>0.9033</td>
<td>0.9257</td>
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<td>$\nu_w$</td>
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<td>0.4507</td>
<td>$[0.2043, 0.4853]$</td>
</tr>
<tr>
<td>$\nu_p$</td>
<td>Price Indexation</td>
<td>$B[0.5, 0.15]$</td>
<td>0.2070</td>
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<td>$[0.0589, 0.3503]$</td>
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<tr>
<td>$\phi_y$</td>
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<td>0.5778</td>
<td>0.6998</td>
<td>$[0.3860, 0.7720]$</td>
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### Appendix Table A5. Prior and Posterior Distribution - Standard Deviations of Innovations of Shocks

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<tr>
<th>Parameter</th>
<th>Description</th>
<th>Prior Shape</th>
<th>Post. Mean</th>
<th>Post. Mode</th>
<th>90 % HPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^z$</td>
<td>Consumption Preference</td>
<td>$\Gamma^{-1}[0.01, 0.05]$</td>
<td>0.0111</td>
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<td>$\sigma^h$</td>
<td>Housing Preference</td>
<td>$\Gamma^{-1}[0.01, 0.05]$</td>
<td>0.0151</td>
<td>0.0157</td>
<td>$[0.0083, 0.0219]$</td>
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<tr>
<td>$\sigma^m^l$</td>
<td>Impatient HH - LTV</td>
<td>$\Gamma^{-1}[0.01, 0.05]$</td>
<td>0.0106</td>
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<tr>
<td>$\sigma^m^E$</td>
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<td>0.0143</td>
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<td>$\sigma^{Q^k}$</td>
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<td>$[0.0154, 0.0219]$</td>
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<td>$\sigma^y$</td>
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<td>0.0046</td>
<td>0.0053</td>
<td>$[0.0024, 0.0067]$</td>
</tr>
<tr>
<td>$\sigma^{bl}$</td>
<td>HH Interest Rate Markup</td>
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<td>$[0.0037, 0.0202]$</td>
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<td>Entr. Interest Rate Markup</td>
<td>$\Gamma^{-1}[0.01, 0.05]$</td>
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<td>0.0084</td>
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<tr>
<td>$\sigma^d$</td>
<td>Deposit Markdown</td>
<td>$\Gamma^{-1}[0.01, 0.05]$</td>
<td>0.0105</td>
<td>0.0125</td>
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<tr>
<td>$\sigma^r$</td>
<td>Monetary Policy</td>
<td>$\Gamma^{-1}[0.01, 0.05]$</td>
<td>0.0021</td>
<td>0.0022</td>
<td>$[0.0019, 0.0026]$</td>
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<td>$\sigma^G$</td>
<td>Government Spending</td>
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<td>$\sigma^{kb}$</td>
<td>Balance Sheet</td>
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<td>0.0521</td>
<td>0.0531</td>
<td>$[0.0455, 0.0585]$</td>
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### Appendix Table A6. Prior and Posterior Distribution - AR(1) Parameters

Estimation of the Baseline Model on Data available up to 2008-Q4

<table>
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<tr>
<th>Parameter</th>
<th>Description</th>
<th>Prior Shape</th>
<th>Post Mean</th>
<th>Post Mode</th>
<th>90 % HPD</th>
</tr>
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<tr>
<td>$\rho^z$</td>
<td>Consumption Preference</td>
<td>$B [0.8, 0.1]$</td>
<td>0.8493</td>
<td>0.8072</td>
<td>[0.7753, 0.9228]</td>
</tr>
<tr>
<td>$\rho^h$</td>
<td>Housing Preference</td>
<td>$B [0.8, 0.1]$</td>
<td>0.9878</td>
<td>0.9936</td>
<td>[0.9796, 0.9966]</td>
</tr>
<tr>
<td>$\rho^{m_E}$</td>
<td>Impatient HH - LTV</td>
<td>$B [0.8, 0.1]$</td>
<td>0.8863</td>
<td>0.9226</td>
<td>[0.7912, 0.9792]</td>
</tr>
<tr>
<td>$\rho^{m_E}$</td>
<td>Entr. HH - LTV</td>
<td>$B [0.8, 0.1]$</td>
<td>0.9127</td>
<td>0.9565</td>
<td>[0.8654, 0.9634]</td>
</tr>
<tr>
<td>$\rho^{E_E}$</td>
<td>Entr. TFP</td>
<td>$B [0.8, 0.1]$</td>
<td>0.8790</td>
<td>0.9045</td>
<td>[0.8153, 0.9394]</td>
</tr>
<tr>
<td>$\rho^l$</td>
<td>Wage Markup</td>
<td>$B [0.8, 0.1]$</td>
<td>0.9243</td>
<td>0.9923</td>
<td>[0.8748, 0.9750]</td>
</tr>
<tr>
<td>$\rho^{q_k}$</td>
<td>Investment Efficiency</td>
<td>$B [0.8, 0.1]$</td>
<td>0.4331</td>
<td>0.3556</td>
<td>[0.3187, 0.5520]</td>
</tr>
<tr>
<td>$\rho^g$</td>
<td>Price Markup</td>
<td>$B [0.8, 0.1]$</td>
<td>0.7997</td>
<td>0.9338</td>
<td>[0.6474, 0.9589]</td>
</tr>
<tr>
<td>$\rho^{b_I}$</td>
<td>HH Interest Rate Markup</td>
<td>$B [0.8, 0.1]$</td>
<td>0.8042</td>
<td>0.6924</td>
<td>[0.6571, 0.9503]</td>
</tr>
<tr>
<td>$\rho^{b_E}$</td>
<td>Entr. Interest Rate Markup</td>
<td>$B [0.8, 0.1]$</td>
<td>0.8539</td>
<td>0.8556</td>
<td>[0.7931, 0.9203]</td>
</tr>
<tr>
<td>$\rho^d$</td>
<td>Deposit Markup</td>
<td>$B [0.8, 0.1]$</td>
<td>0.7957</td>
<td>0.6831</td>
<td>[0.6595, 0.9586]</td>
</tr>
<tr>
<td>$\rho^G$</td>
<td>Government Spending</td>
<td>$B [0.8, 0.1]$</td>
<td>0.5789</td>
<td>0.6945</td>
<td>[0.4528, 0.6982]</td>
</tr>
<tr>
<td>$\rho^{kb}$</td>
<td>Balance Sheet</td>
<td>$B [0.8, 0.1]$</td>
<td>0.9459</td>
<td>0.9528</td>
<td>[0.9119, 0.9810]</td>
</tr>
</tbody>
</table>

Appendix Figure A1. Prior vs. Posterior of Expected Zero Lower Bound Durations 2009-Q1
Appendix Figure A2. Prior vs. Posterior of Expected Zero Lower Bound from Q4-2019

Appendix Figure A3. Prior vs. Posterior of Expected Zero Lower Bound from Q4-2019
Appendix Figure A4. Prior vs. Posterior of Expected Zero Lower Bound from Q4-2019

Appendix Figure A5. Forecast of the Inflation Rate - No Smoothing
Appendix Table A7. Root Mean Squared Errors Without Forecasts from Q1-2008

<table>
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<tr>
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<th>h = 1</th>
<th>h = 2</th>
<th>h = 3</th>
<th>h = 4</th>
<th>h = 5</th>
<th>h = 6</th>
<th>h = 7</th>
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<tbody>
<tr>
<td><strong>Output</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Extended Model</td>
<td>1.1338</td>
<td>3.1966</td>
<td>4.3533</td>
<td>3.8626</td>
<td>1.6467</td>
<td>1.0608</td>
<td>1.1120</td>
<td>1.5923</td>
</tr>
<tr>
<td>Baseline Model</td>
<td>3.8878</td>
<td>4.6498</td>
<td>4.7756</td>
<td>3.7148</td>
<td>1.7201</td>
<td>1.5154</td>
<td>1.5491</td>
<td>1.1206</td>
</tr>
<tr>
<td>SPF</td>
<td>1.7723</td>
<td>3.6678</td>
<td>4.9399</td>
<td>5.3360</td>
<td>4.3956</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

|               |       |       |       |       |       |       |       |       |
| **Interest Rate** |       |       |       |       |       |       |       |       |
| Extended Model  | 0.8024| 0.9048| 1.1170| 1.0966| 1.0627| 1.0805| 1.1348| 1.2148|
| Baseline Model  | 1.2315| 1.9976| 2.6956| 3.0844| 3.3624| 3.5938| 3.7741| 3.9159|

|               |       |       |       |       |       |       |       |       |
| **Inflation**  |       |       |       |       |       |       |       |       |
| Extended Model  | 0.4363| 0.2753| 0.1422| 0.1873| 0.1914| 0.2707| 0.3316| 0.3649|
| Baseline Model  | 0.4151| 0.3451| 0.3742| 0.4123| 0.3937| 0.3748| 0.3231| 0.2236|

Appendix Figure A6. Forecasts for the Nominal Interest Rate during COVID-19
Appendix Figure A7. Forecasts for the Inflation Rate during COVID-19
Figure A8 shows the main shocks driving the deviations of real GDP in the extended model. Some aspects to consider are that the solid line depicts real GDP as created from the Kalman smoother based on the estimation with data up to Q1 of 2009. It furthermore depicts only those shocks that contributed in a significant way to the change of real GDP. What can be seen is that the main three shocks that drove GDP down during the GFC are the housing preference shock, the markup shock on entrepreneur loans, and the housing productivity shock. At the same time, it can be seen that the monetary policy shock counteracts by increasing real GDP, as do the consumption preference shock and the markup shock on wages.
All data series’ and vintages are taken from ALFRED and explained below. Their respective handles are in the brackets.

**Population Level**: (POPTHM), Thousands of Persons, Not Seasonally Adjusted
**Federal Funds Effective Rate**: (DFF), Percent, Not Seasonally Adjusted
**Fixed Private Investment**: (FPI), Billions of Dollars, Seasonally Adjusted Annual Rate
**Private Residential Fixed Investment**: Private Residential Fixed Investment (PRFI), Billions of Dollars, Seasonally Adjusted Annual Rate
**Private Nonresidential Fixed Investment**: (PNFI), Billions of Dollars, Seasonally Adjusted Annual Rate
**Real Gross Domestic Product**: (GDPC1), Billions of Chained 2012 Dollars, Seasonally Adjusted Annual Rate
**All-Transactions House Price Index for the United States**: (USSTHPI), Index 1980:Q1=100, Not Seasonally Adjusted
**Gross Domestic Product: Implicit Price Deflator**: (GDPDEF), Index 2012=100, Seasonally Adjusted
**Average Hourly Earnings of Production and Nonsupervisory Employees, Total Private**: (AHETPI), Dollars per Hour, Seasonally Adjusted
**Households and Nonprofit Organizations; Loans; Liability, Level**: (HNOLL), Millions of Dollars, Not Seasonally Adjusted
**Nonfinancial Corporate Business; Loans; Liability, Level**: (NCBLL), Millions of Dollars, Not Seasonally Adjusted
**Households and Nonprofit Organizations; Total Currency and Deposits Including Money Market Fund Shares; Asset, Level**: (DABSHNO), Billions of Dollars, Not Seasonally Adjusted
**New Privately-Owned Housing Units Completed; Total Units**: (COMPUTSA), Thousands of Units, Seasonally Adjusted Annual Rate

Appendix Table B1. Data Vintages - ALFRED

<table>
<thead>
<tr>
<th>Name</th>
<th>07-Q4</th>
<th>08-Q1</th>
<th>08-Q2</th>
<th>08-Q3</th>
<th>08-Q4</th>
<th>09-Q1</th>
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</thead>
<tbody>
<tr>
<td>POPTHM</td>
<td>28.02.08</td>
<td>29.05.08</td>
<td>28.08.08</td>
<td>25.11.08</td>
<td>27.02.09</td>
<td>29.05.09</td>
</tr>
<tr>
<td>DFF</td>
<td>20.02.08</td>
<td>09.05.08</td>
<td>07.08.08</td>
<td>07.11.08</td>
<td>17.02.09</td>
<td>15.05.09</td>
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<tr>
<td>FPI</td>
<td>28.02.08</td>
<td>29.05.08</td>
<td>28.08.08</td>
<td>25.11.08</td>
<td>27.02.09</td>
<td>29.05.09</td>
</tr>
<tr>
<td>PRFI</td>
<td>28.02.08</td>
<td>29.05.08</td>
<td>28.08.08</td>
<td>25.11.08</td>
<td>27.02.09</td>
<td>29.05.09</td>
</tr>
<tr>
<td>PNFI</td>
<td>28.02.08</td>
<td>29.05.08</td>
<td>28.08.08</td>
<td>25.11.08</td>
<td>27.02.09</td>
<td>29.05.09</td>
</tr>
<tr>
<td>GDPC1</td>
<td>28.02.08</td>
<td>29.05.08</td>
<td>28.08.08</td>
<td>25.11.08</td>
<td>27.02.09</td>
<td>29.05.09</td>
</tr>
<tr>
<td>USSTHPI</td>
<td>25.08.10</td>
<td>25.08.10</td>
<td>25.08.10</td>
<td>25.08.10</td>
<td>25.08.10</td>
<td>25.08.10</td>
</tr>
<tr>
<td>GDPDEF</td>
<td>28.02.08</td>
<td>29.05.08</td>
<td>28.08.08</td>
<td>25.11.08</td>
<td>27.02.09</td>
<td>29.05.09</td>
</tr>
<tr>
<td>AHETPI</td>
<td>01.02.08</td>
<td>02.05.08</td>
<td>01.08.08</td>
<td>07.11.08</td>
<td>06.02.09</td>
<td>08.05.09</td>
</tr>
<tr>
<td>HNOLL</td>
<td>18.09.15</td>
<td>18.09.15</td>
<td>18.09.15</td>
<td>18.09.15</td>
<td>18.09.15</td>
<td>18.09.15</td>
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<tr>
<td>NCBLL</td>
<td>18.09.15</td>
<td>18.09.15</td>
<td>18.09.15</td>
<td>18.09.15</td>
<td>18.09.15</td>
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</tr>
<tr>
<td>DABSHNO</td>
<td>10.06.10</td>
<td>10.06.10</td>
<td>10.06.10</td>
<td>10.06.10</td>
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<tr>
<td>COMPUTSA</td>
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<td>25.08.10</td>
<td>25.08.10</td>
<td>25.08.10</td>
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</tbody>
</table>
As can be seen from Table B1, there are five data series that are not used with real-time vintages but later releases and thus already include some form of revision. One reason for these data choices is that they are also used in similar work and thus I rather keep it comparable in this regard compared to having real-time data everywhere, which in some cases is not even available. For some data this is more crucial than for others. USSTHPI, the price index for houses in the U.S. and COMPUTSA, the new houses completed, are a price index and a fixed number (level) that is easy to count. Hence, those two do not get revised post release date. This leaves the household and business loans, HNOLL and NCBLL, as well as deposits, DABSHNO, that do run through some sort of revision process and could potentially differ.

Appendix Figure B1. Comparison of HNOLL, NCBLL, and DABSHNO

Figure B1 shows the three data vintages available for HNOLL, NCBLL, and DABSHNO that are most widely apart. The first is the one used in the first forecasting exercise from 18.09.15 or 10.06.10 and the second is the last one available from 09.12.22 or 09.09.2022. This last one is chosen to show the biggest possible difference over time and through revisions. The data has been treated the same way as for the estimation/forecasting, i.e. it is treated with the GDP deflator, the population, and made stationary with the one-sided HP-filter. The period shown here is the one used up to Q1 of 2009. It can be seen that even with the biggest possible difference, these two series’ on loans do not change dramatically. Hence, I can be relatively confident that this choice does not alter the results in a meaningful way. Of course, it is not possible to systematically show the differences a vintage from 2009 would have, so this can only be seen as an approximation. During a robustness check analysis I also used different measures of household and firm lending, which gave similar results in the estimation. Household deposits show the biggest variation but it also starts already in 2010 and not 2015 like the other two series’. It is thus a much longer horizon between the two points to revise the data. Another point is that it is relatively close to the time period used in the estimation and so can be seen as a better approximation. Also the revised data from 2022 shows stronger reactions, which would only, if at all, increase and strengthen the results shown in the paper.
C Appendix: Robustness Check Continued

Here, I continue the robustness check analysis from the main text and plot all parameters, including those not shown before.

Appendix Figure B2. Prior and Posterior Distributions of the Baseline and Extended Model - Parameter

Appendix Figure B3. Prior and Posterior Distributions of the Baseline and Extended Model - Standard Deviation of Shocks
Appendix Figure B4. Prior and Posterior Distributions of the Baseline and Extended Model - AR
D Appendix: Most Recent Forecast

The last forecast that I analyse here is with data up to Q1 of 2023, i.e. the most recent observations available in the time of writing. Since the extended model has fared well during the GFC, and one big reoccurring topic in 2022 and 2023 has been the housing sector within the inflation surge, I want to see how it forecasts the three macroeconomic variables examined before. What can be seen in Figure B5 is that according to the extended model, the U.S. economy goes into a very mild recession before seeing positive growth again toward the end of 2023. This coincides with a halt in rate increases as inflation is coming down.

Appendix Figure B5. Forecasts from Q2 of 2023
Acknowledgements

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