Optimal monetary policy in an estimated SIR model

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Abstract

This paper studies the design of Ramsey optimal monetary policy in a Health New Keynesian (HeNK) model with Susceptible, Infected and Recovered (SIR) agents. The nonlinear model is estimated with maximum likelihood techniques on Euro Area data. Our objective is to deconstruct the mechanism by which contagion risk affects the conduct of monetary policy. If monetary policy is the only game in town, we find that the optimal policy features significant deviations from price stability to mitigate the effect of the pandemic. The best outcome is obtained when the optimal Ramsey policy is combined with a lockdown strategy of medium intensity. In this case, monetary policy can concentrate on its price stabilization objective.

Keywords: Covid-19, macroeconomic trade-offs, nonlinear inference, HeNK, Tinbergen principle.

JEL: E52, E32.
Non-technical Summary

This paper analyzes the impact of the COVID-19 crisis on monetary policy. While monetary and lockdown policies do interact through general equilibrium effects, our primary conclusion is that the Tinbergen separation principle applies. Monetary policy should prioritize price stabilization, leaving the mitigation of contagion externalities to lockdown policies.

If monetary policy is the only game in town, we also find that it could be optimal to increase interest rates in response to an epidemic outbreak. This result reflects that, in the absence of government intervention, health considerations considerably alter the conduct of monetary policy, as a reduction in economic activity becomes necessary to slow down the spread of the virus. But since monetary policy is a rather blunt instrument to reduce contagion, this gain in terms of public health comes at a significant economic cost. In contrast, when governments respond to the health crisis by implementing lockdown policies, monetary policy can focus on its price stability objective.

Our results suggest the existence of an optimal confinement level that strikes a balance between curbing infection and minimizing the side effects of lockdown policies on the economy. In our environment, the best possible policy mix is obtained by combining the Ramsey optimal monetary policy with a lockdown of medium intensity. Indeed, whereas necessary to combat the crisis, overly stringent lockdown policies can be counterproductive.

We also introduce a novel decomposition of the channels via which the COVID-19 shock affects inflation and output and then show how the health dimension of the crisis affects inflation, and hence monetary policy. In this new class of models, contagion risk depresses labor supply by acting as a tax on labor supply and consumption. Contagion risk has a direct effect on labor supply, as agents internalize that a significant share of new contagion occurs in the workplace. Furthermore, as contagion risk diminishes agents’ willingness to consume, the necessity to work in order to fund consumption expenses becomes less urgent, resulting in a further reduction in labor supply. The resulting shift in labor supply, in turn, exerts upward pressure on wages, implying higher marginal costs for firms. As a result, given that inflation in these models is determined by marginal costs, the presence of contagion risk leads to elevated inflation rates through its effect on the labor market.

Our main methodological contribution is to study the optimal Ramsey policy in a model with SIR agents that is estimated using Maximum Likelihood techniques. One major advantage of this estimation strategy is that it allows us to capture the highly nonlinear nature of SIR models. Our paper can thus be regarded as a first attempt to take this class of models to the data.
1 Introduction

The economic shock induced by the COVID-19 crisis bears little resemblance with recessions experienced in recent times. The first central difference is the link between economic activity and virus contagion that is specific to epidemic-induced recessions. From the perspective of policy makers, it is therefore necessary to consider the possible effects of policies both on economic activity as well as contagion risk. Moreover, whereas optimal monetary policy analysis typically concentrates on inflation and output stabilization, contagion risk adds an additional dimension. Indeed, in an environment in which catching the virus can lead to fatalities, welfare no longer only depends on consumption and leisure of an average agent but also on the number of infected individuals.

This paper analyzes how the health dimension of the crisis alters the conduct of monetary policy. Our objective is to discuss these channels by considering the smallest possible deviation from the textbook New Keynesian framework (e.g., Galí, 2015). We first provide a novel decomposition of the channels via which the COVID-19 shock affects the main building blocks of the baseline model. In particular, we consider a model in which contagion occurs in the workplace or when making consumption decisions. Relative to the seminal contribution of Eichenbaum, Rebelo, and Trabandt [2021], we study the Ramsey optimal monetary policy in a model in which there is perfect consumption insurance. As we abstract from the inefficiency caused by imperfect risk-sharing, the two remaining distortions are the contagion externality and nominal rigidities. One advantage of this simplifying assumption is that it enables us to analyze how health considerations affect the new IS and Phillips curves within a New Keynesian model that we augment with health block, a Health New Keynesian (HeNK) model. We then show how the health dimension affects the welfare analysis by introducing the notion of a "health wedge". Whereas the business cycle wedge captures the welfare loss due to the standard business cycle effect, the health wedge stems from the effect of contagion risk on welfare.

This decomposition allows us to deconstruct the channels through which contagion risk affects inflation, and hence monetary policy. In this new class of models, contagion risk is akin to a tax on consumption and labor supply. By inducing labor shortages, the first effect of contagion risk is to reduce labor supply, as agents internalize that a significant share of new contagion occurs in the workplace. This reduction in labor supply in turn puts upwards pressure on wages, which implies higher marginal costs of production for firms. As the Phillips curve in turn implies that inflation is determined by the discounted sum of marginal
costs, this effect on wages produces higher inflation rates.

Contagion risk also modifies the New IS curve by introducing two novel terms that work in opposite directions. First, the risk of mortality associated with the disease affects the consumption and saving decision of agents, as an increase in death probability reduces agents’ propensity to save for the future. Second, contagion risk acts as a tax on consumption that encourages agents to postpone current consumption to reduce the risk of catching the virus. We find that this latter effect has a dominating effect and considerably alters the transmission mechanism of New Keynesian models.

Our second main contribution is to study the optimal Ramsey policy in a model with Susceptible, Infected and Recovered (SIR) agents that is estimated using Maximum Likelihood techniques. Our paper can thus be seen as a first attempt to take the class of models developed by Eichenbaum et al. [2021] to the data. Relative to the literature, this estimation procedure implies that our model is able to reproduce the evolution of GDP, as proxied by the OECD weekly tracker, the number of fatalities, as well as the stringency of the lockdown policies implemented in the euro area, which are made available by Wołoszko [2020], the Johns Hopkins coronavirus resource center, and Hale, Angrist, Goldszmidt, Kira, Petherick, Phillips, Webster, Cameron-Blake, Hallas, Majumdar, et al. [2021], respectively.

This is achieved by implementing the extended path solution method initially developed by Fair and Taylor [1983] and more recently refined by Adjemian and Juillard [2014] in the context of a simple SIR model. The main advantage of this estimation strategy is that it allows us to capture the highly nonlinear nature of SIR models, as this estimation method does not require a linearization of the models’ equations, which stands in contrast to what is typically done in the literature (e.g., Smets and Wouters, 2007). While tractable and time-efficient, one limitation of this approach, however, is that uncertainty stemming from future shocks cannot be accounted for.

How do health considerations affect the conduct of monetary policy? In the presence of contagion risk, we find that the optimal monetary policy deviates substantially from its price stability objective. Indeed, during an epidemic outbreak, the planner finds it optimal to address the distortion caused by the contagion externality by curbing economic activity through aggressive interest rate hikes. By slowing down the spread of the virus, the policy has deflationary effects. The planner therefore chooses to deviate from full price stabilization and uses monetary policy to alleviate the welfare loss stemming from the epidemic outbreak. But since monetary policy is a rather blunt instrument to reduce contagion, this gain in terms of public health comes at a significant economic cost. If monetary policy is the only
game in town, there is therefore a strong trade-off between health and the economy. The deviation from full price stabilization is in stark contrast with the usual result in standard Ramsey policy analyses in which full price stability both in the short- and long-term is optimal (Woodford, 2003a, Schmitt-Grohé and Uribe, 2005).

We next study the optimal monetary policy in the case in which the government implements lockdown policies. Whereas interest rates still need to increase on impact, having a tool that addresses the health distortion allows the central bank to reduce rates shortly after the outbreak. By optimizing an operational containment policy rule, our results suggest the existence of an optimal confinement level, which is proportional to the number of COVID cases. Indeed, on the one hand tighter lockdown policies reduce the number of infected agents, an effect which enhances welfare by reducing the loss stemming from the spread of the virus. On the other hand, lockdown policies reduce welfare by amplifying the size of the recession, thereby leading to larger fluctuations in economic activity. As the two effects work in opposite directions, we obtain a concave relationship between the lockdown parameter and welfare. In our environment, the best possible policy mix is obtained by combining the Ramsey optimal policy with a lockdown policy of medium intensity.

In sum, the main takeaway of our analysis is that a concept akin to the Tinbergen separation principle holds. Monetary policy should concentrate on price stabilization, whereas the contagion externality should be addressed by confinement policies. Although the tractability of our framework is obtained at the cost of assuming perfect risk-sharing, it is very reassuring to note that similar conclusions are documented in the work of Lepetit and Fuentes-Albero [2022] and Brzoza-Brzezina, Kolasa, and Makarski [2022]. Overall, a robust message seems to have emerged from the literature that studies monetary policy in the context of SIR models.

Modern models of business cycles are built on the notion that “all business cycles are alike” (e.g., Lucas [1977]). By causing a recession of unprecedented magnitude as well as millions of fatalities globally, the recent COVID-19 outbreak questioned this principle in macroeconomics. Like the 2007-2009 Great Recession, the COVID-19 crisis has also been a major challenge for the economics profession. Except for the seminal paper of Kermack and McKendrick [1927], written many decades ago, the knowledge of the economic implications of epidemics was fairly limited when the shock hit. Given the lack of existing work on this topic, the speed at which the literature has evolved is very striking. Indeed, the economics profession reacted very quickly to the unique nature of this shock, as first drafts of some of the most influential contributions were made publicly available only a few months after the beginning of the pandemic (e.g., Eichenbaum et al., 2021, Glover, Heathcote, Krueger, and
Building on these early contributions, another strand of the literature has also included nominal rigidities. Eichenbaum, Rebelo, and Trabandt [2022] introduce price stickiness into the analysis to capture qualitative features of the COVID recession. They find that COVID acts like a negative demand shock. In contrast, Woodford [2020] argues that the COVID crisis caused an “effective demand failure”. The problem is not only a lack of aggregate demand and a reduction in interest rates is therefore not necessarily warranted.

Since the beginning of the pandemic, considerable progress has also been made in understanding the role of monetary policy during an epidemic outbreak. To our knowledge, one of the first contributions include the work of Brzoza-Brzezina et al. [2022], which studies the interaction between monetary and containment policies. A main conclusion that emerges from their analysis is that monetary and lockdown policies are complementary. In the absence of containment policies, this implies that monetary policy should be contractionary. Relative to their analysis, the main difference is that we derive the Ramsey optimal policy, whereas they focus on the case of simple rules.

Lepetit and Fuentes-Albero [2022] study how an epidemic outbreak affects the transmission mechanism of monetary policy. They show that contagion risk interferes with the standard intertemporal substitution channel of monetary policy. Indeed, since activities that increase consumption, such as purchasing goods in shopping malls, dining in restaurants, or taking a holiday trip increase the risk of catching the disease, the effect of accommodative monetary policy is more muted during an epidemic outbreak. Those authors also study the interaction between monetary and confinement policies by deriving the joint Ramsey optimal policy. Relative to that study, our contribution is to derive the Ramsey optimal policy in a model estimated using full-information methods, and deconstruct the trade-off faced by the monetary authority.

The paper proceeds as follows. Section 2 presents the model. Section 3 provides the estimation strategy. Section 4 shows the optimal policy plan, and compare alternative macroeconomic outcomes by contrasting Taylor versus Ramsey optimal policy, with and without lockdown policy. Section 5 concludes.

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1 The optimal Ramsey policy was added in a subsequent version of their paper but was absent in the initial draft that was made publicly available.
2 The Model

Using the New Keynesian framework as a foundation, this section develops the model used to study the implications of contagion risk for monetary policy. In a nutshell, time is discrete, where \( t \in (0, 1, 2, \ldots, \infty) \), the production sector produces final and intermediate goods using labor. The intermediate good sector is produced by a monopolistic firm that sets prices subject to an adjustment cost. Households consume, offer labor services, and save by holding government bonds. The central bank sets interest rates and the government can decide to impose restrictions on activity to mitigate the effects of an epidemic outbreak. As we abstract from stochastic disturbances, agents populating this economy have perfect knowledge about future states of the economy, we therefore drop the expectation operator.

2.1 Epidemic dynamics

Pandemic dynamics rely on the epidemiologic model of Kermack and McKendrick [1927] referred to as the Susceptible, Infected and Recovered (SIR) model. As Eichenbaum et al. [2021] new infections in the population partially result from social interactions. Susceptible agents catch the virus when they meet infected agents in the workplace or when making consumption decisions.

Let us first discuss the number of infected \( I_t \), whose law of motion is given as follows:

\[
\Delta I_{t+1} = T_t - (\gamma_D + \gamma_R) I_t,
\]

where \( T_t \) is the number of new infections per period and \( \gamma_D \) and \( \gamma_R \) are respectively the fractions of infected individuals that either die or recover from the infection. Recent epidemiological studies (e.g. Odone, Delmonte, Scognamiglio, and Signorelli [2020] or Modi, Böhm, Ferraro, Stein, and Seljak [2021]) have highlighted important heterogeneity in fatality risk across regions of Italy. This heterogeneity is mainly driven by the diversity in regional responses to the emergency, thus showing that mortality risk is endogenous. In particular, the emergence of many cases concentrated within a short period of time stretches hospitals to capacity, resulting in an increased mortality risk.\(^2\) We capture this pattern in fatality

\(^2\)Based on regional data, Odone et al. [2020] find that fatality risk is time-varying and mainly determined by the policy measure implemented to control the outbreak. Veneto opted for strict containment while Lombardy strengthened hospital services to meet a massively increased demand for hospitalisation. As a result, fatality rate in Lombardy (18.3%) was approximately three times higher than that in Veneto (6.4%) through an excess demand of intensive care unit beds.
risk in a similar way as Eichenbaum et al. [2021] by allowing the mortality risk to increase proportionally to the number of cases as follows:

\[ \gamma_{D,t} = \gamma_D + \varpi I_t^2 + \varepsilon_d t. \]  

(2)

In this expression, \( \varpi \) captures the elasticity of mortality risk to the current number of cases pressuring the health services, while \( \varepsilon_d t \) is a fatality risk shock that follows an auto-regressive processes with stochastic structure: \( \varepsilon_d t = \rho \varepsilon_d t_{t-1} + \eta_d t \), where standard Gaussian innovations \( \eta_d t \sim N(0, \sigma_d) \). A positive realization of this shock increases the idiosyncratic fatality of dying from COVID.

The number of susceptible individuals depends on new infections as well as on the exogenous shock:

\[ \Delta S_{t+1} = -T_t. \]  

(3)

The number of recovered, which is denoted by \( R_t \), is given by the cumulative number of recovered individuals:

\[ \Delta R_{t+1} = \gamma_R I_t. \]  

(4)

Finally, since we normalize to one the population, we have that \( S_{t+1} + I_{t+1} + R_{t+1} + D_{t+1} = 1 \), where cumulative deaths are given by \( D_{t+1} = \sum_{s=0}^{t+1} \gamma_D s I_s \).

A key indicator to track the velocity of an epidemic is the effective reproduction number. It is defined as the expected number of infections caused by a single infected individual. It is given by the average infection rate adjusted by the probability to recover or die:

\[ R_t = \frac{T_t}{I_t} + E_t \left\{ (1 - \gamma_D I_t - \gamma_R) (T_{t+1}/I_{t+1}) \right\}. \]  

(5)

Note that \( R_t < 1 \) implies that the inflow of new infections does not compensate the outflows from recovered or deceased agents. In that situation, the epidemic gradually wears out. Following Eichenbaum et al. [2021], and in contrast to the original SIR model of Kermack and McKendrick [1927], the number of new infections \( T_t \) is endogenously determined by the business cycle block of the model and leads to potential trade-offs between business cycle and health stabilization. We discuss this aspect in the next subsections.
2.2 The Household

In each period, the representative household supplies labor to firms, consumes and saves. Relative to Eichenbaum et al. [2021], we simplify the analysis by considering the case of perfect consumption insurance. This simplifying assumption allows us to isolate the effect of health on the dynamics of an otherwise standard textbook New Keynesian model. Indeed, in the original specification, the introduction of different types of agents implies that markets are incomplete, which, relative to the textbook model, adds another source of distortion. Since we are interested in deriving the optimal monetary policy, this assumption also makes the computational aspect more tractable. In our setup, the household is composed of family members of three types: Susceptible, Recovered, and Infected. Perfect consumption insurance within the family implies that the aggregate consumption levels of the different types is equal to the amount consumed by the family times the number of agents of each respective type:

\[
C^s_t + C^i_t + C^h_t = (S_t + I_t + R_t) C_t.
\]

(6)

Note that a similar assumption applies to insure against income risk.

As in Eichenbaum et al. [2021], the number of new cases, which we denote by \( T_t \), is determined as follows:

\[
T_t = \gamma_T I_t S_t + \gamma_C C^i_tC^s_t + \gamma_N N^i_t N^s_t,
\]

(7)

where \( C^i_t \) and \( C^s_t \) (\( N^i_t \) and \( N^s_t \)) denote consumption (labor supply) of infected and susceptible family members, where \( \gamma_C \) and \( \gamma_N \) are two elasticity parameters. Note that if \( \gamma_C = \gamma_N = 0 \), the setup reduces to the standard SIR model. Anticipating symmetry and perfect consumption and income insurance across households members as in Equation 6, the number of new cases can be expressed as follows:

\[
T_t = I_t S_t \left[ \gamma_T + \gamma_C C^2_t + \gamma_N N^2_t \right].
\]

(8)

Since the population is normalize to one, \( T_t \) is also interpreted as the probability that a family member gets infected.

\[\text{Even though we do not explicitly model the insurance mechanism, a microfounded framework in the same vein as in Andolfatto [1996] could be easily introduced, in particular because households here have no financial asset to purchase. Each type of household would subscribe to a perfectly competitive insurance mechanism that equalizes both consumption and labor income across family members.}\]
The preferences of the family are given by:
\[ u(C_t, N_t) = \log(C_t) - \frac{\chi}{1 + \phi} N_t^{1+\phi}, \tag{9} \]
where parameter \( \beta \in (0,1) \) is the discount factor, \( \phi \) is the curvature of the disutility of labor \( N_t \), \( \chi \) is a shift parameter that pins down the number of hours worked.

Thus, the household solves the following maximization problem:
\[ W_t = \max_{(C_t, N_t, B_{t+1}, B^L_{t+52}, T_t)} \mathbb{E} \left[ \exp(\epsilon^l_t) u(C_t, N_t) + \beta (1 - \gamma_{D,t} T_t) W_{t+1} \right], \tag{10} \]
where \( W_t \) denotes the value function of the family that depends on the current flow of utility as well as the continuation value \( W_{t+1} \), adjusted by the discount factor \( \beta \) and the expected survival rate \( 1 - \gamma_{D,t} T_t \) of the family members. As in Eichenbaum et al. [2021], family members internalize how their consumption and labor choices affect the exposure to the virus \( T_t \), but take as given the economy-wide probabilities of being either susceptible or infected. We also introduce a preference shock that follows an auto-regressive processes with stochastic structure:
\[ \epsilon^l_t = \rho \epsilon^l_{t-1} + \eta^l_t, \] with standard Gaussian innovations \( \eta^l_t \sim N(0, \sigma^l). \) A positive realization of this shock increases the marginal utility of consumption and triggers a shift in aggregate demand.

The budget constraint reads as follows:
\[ (1 + \mu_t) C_t + B_{t+1} + B^L_{t+52} = W_t N_t + \frac{r_t}{\Pi_t} B_t + \frac{r^I_{t+52}}{\Pi_t/P_{t-52}} B^L_t + \text{div}_t + tr_t, \tag{11} \]
where labor supply \( N_t \) is remunerated at the real wage \( W_t \). As we assume that government bonds, which are denoted by \( B_t \), are one week bonds, \( r_t \) is the nominal interest received on bonds held and \( \Pi_t \) is the inflation rate between \( t-1 \) and \( t \). We also introduce a one year government bond \( B^L_{t+52} \) which provides an annualized return \( r^I_t \) based on the term structure of interest rates. This variable plays no role in the resource allocation but is useful to describe the interest rate setting of the Ramsey social planner. Finally, imperfect competition in the goods market implies that profits, which are denoted by \( \text{div}_t \), are redistributed to the household. As in Eichenbaum et al. [2021], the exogenous variable \( \mu_t \) is a tax on consumption that mimics lockdown policies. Finally, \( tr_t \) are lump-sum transfers.

The interior solution of this optimization problem (which is described in the appendix) is summarized by the following set of equations. First, marginal utility of consumption \( \lambda_t \) is...
given as follows:

\[
(1 + \mu_t) \lambda_t = \frac{\exp(\epsilon_t^c)}{C_t} - \beta \mathbb{E}_t \{W_{t+1}\} 2 \gamma D_t I_t S_t, \tag{12}
\]

As found in the classic macroeconomic textbook, the first term stems from the concavity of the utility function, which implies that the willingness to consume increases as consumption declines. The second term on the right hand side of equation (12) is new, and reflects the component due to health risk. The probability of infection – and death – increases in consumption. The household internalizes this risk by taking into account that a utility loss from postponing consumption is compensated by a reduction in the risk of infection.

For the sake of notation, let us rewrite equation (12) as follows:

\[
(1 + \mu_t) \lambda_t = \frac{\exp(\epsilon_t^c)}{C_t} (1 - H_t^C), \tag{13}
\]

where the term \(H_t^C\) captures the effect of health risk on the marginal utility of consumption.

Turning to the second condition, the labor supply equation reads as follows:

\[
\exp(\epsilon_t^c) \chi N_t = \lambda_t W_t - \beta \mathbb{E}_t \{W_{t+1}\} 2 \gamma D_t N_t I_t S_t, \tag{14}
\]

Typically, the labor supply implies that the marginal utility loss from supplying labor is compensated by the wage expressed in units of marginal consumption. In a pandemic, this equation is affected by the marginal health risk that increases in the number of hours worked supplied by the agent. During an epidemic outbreak, the household internalizes the effect on contagion risk. Again, to simplify notation, let us rewrite this equation as follows:

\[
\exp(\epsilon_t^c) \chi N_t (1 + H_t^N) = \lambda_t W_t, \tag{15}
\]

where \(H_t^N\) is the additional disutility from supplying labor stemming from health risk.

Finally, the third condition, which is the standard Euler equation that determines the new IS curve, is given as follows:

\[
\beta (1 - \gamma_{D,T}) \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \right\} \frac{r_t}{\mathbb{E}_t \{r_{t+1}\}} = 1. \tag{16}
\]
With respect to the textbook New Keynesian model, the key is that health risk affects the Euler condition. An increase in the probability of catching the disease is akin to a decline in the subjective rate of time preference $\beta$, as households become more impatient to consume. In sum, contagion risk lowers the labor supply but exerts two opposing forces on consumption. These conflicting forces on aggregate consumption are further discussed and deconstructed in the next sections.

2.3 Firms

The final good production is a constant elasticity of substitution (CES) aggregate: 

$$Y_{it}^D = \left( \int_0^1 Y_{it}^{\epsilon/(\epsilon-1)} \, dt \right)^{1/(\epsilon-1)},$$

where $\epsilon > 1$ captures the degree of substitutability between different types of varieties $i$ produced by intermediate firms. The optimal demand for the final firm profit maximization problem is given by: 

$$Y_{it}^D = \left( P_{it}/P_t \right)^{-\epsilon} Y_{it}^D.$$ 

Under perfect competition and free entry, the price of the final good is denoted $P_t$, while the price $P_{it}$ is the price charged by the firm $i$. The aggregate price index reads as: 

$$P_t = \left( \frac{1}{R_0} \sum_{i} P_{it} \alpha Y_{it} \right)^{1/(\epsilon-1)}.$$

The technology of the representative firm is given by:

$$Y_{it} = A N_{it}^\alpha,$$ 

where $N_{it}$ is the labor demand with intensity $\alpha \in [0, 1]$ and $A$ is a fixed economy-wide TFP.

Intermediary firms solve a two-stage problem. In the first stage, they decide the optimal demand of inputs in perfectly competitive input markets to determine their marginal cost of production. The cost-minimization problem, 

$$\min_{Y_{it}, N_{it}} mc_{it} Y_{it} - (1 + \mu_t) N_{it} W_t,$$

yields the marginal cost. In this expression, $\mu_t$ is an additional government tax on labor demand that also mimics a lockdown policy. The first-order condition with respect to the firm’s optimal choice reads as follows:

$$mc_{it} = mc_{it} = (1 + \mu_t) W_t \frac{N_{it}^\alpha}{\alpha Y_{it}},$$ 

where $\mu_t$ is the tax on labor cost that mimics the lockdown policies aimed at reducing the labor demand during the outbreak.

In the second stage of the problem, firms make their optimal pricing decisions by taking into account both nominal rigidities and imperfect substitution in the demand function of final goods producers. Monopolistic firms engage in a price setting à la Rotemberg. Price change is subject to an adjustment cost given by 

$$\Delta_{it} = 0.5 \theta \left( P_{it}/P_{it-1} - 1 \right)^2$$

where $\theta \geq 0$ is the cost parameter that pins down the degree of nominal rigidities. The profit maximization
subject to the demand from final firms reads as follows:

\[
\max_{\{P_t\}} \sum_{t=0}^{\infty} \mathbb{E}_{\tau} \left\{ m_{t+\tau} \left( \frac{P_{t+\tau}}{P_{t+\tau-1}} - mc_{t+\tau} \right) Y_{t+\tau} - \Delta_{t+\tau} Y_{D_{t+\tau}} \right\}
\]

where \( P_t \) is the optimal selling price for firms and \( m_{t+\tau} \) is the stochastic discount factor from Equation 16.

### 2.3.1 Authorities and aggregation

As in the standard macro economic textbook of Gali [2015], the central bank interest rate follows a Taylor Rule:

\[
\frac{r_t}{r_{t-1}} = \left( \frac{\Pi_t}{\Pi_{t-1}} \right)^\phi \left[ \frac{Y_t}{Y_{t-1}} \right]^{\phi_y} \left[ 1 - \phi \right] e^{\epsilon_R t},
\]

where 0 < \( \rho \) ≤ 1 is the smoothing coefficient, \( \phi_x \geq 1 \) is the stance on inflation deviations from inflation target \( \Pi \), while \( \phi_y \) is the output gap stance. Unlike Eichenbaum et al. [2022] and Smets and Wouters [2007], we do not express the output gap in terms of deviations from the efficient output but from the steady state level \( Y \), as done in macro textbooks such as Woodford [2003a] and Gali [2015]. This rule is also compared to a the Ramsey-optimal monetary policy in the result section of the paper.

Concerning the government, the budget constraint is simply given by fiscal revenues from containment policies:

\[
tr_t = \mu_t \left( W_t N_t^D + C_t \right).
\]

while the containment policy has the following form:

\[
\mu_t = \xi \Delta_I t^\phi,
\]

where \( \xi \) is the policy reaction to infections, while \( \Delta_I t^\phi \) is an AR(1) shock process that discretionary part of the lockdown policy.

Since bonds are in zero net supply, in equilibrium:

\[
B_t = B_t^L = 0.
\]

Regarding the market clearing condition in the labor market, the total supply from
households is equal to the demand from firms:

\[(S_t + \phi I_t + R_t) N_t = N_t^D. \quad (24)\]

Note that the labor supply is given by the three types of agents \(S_t, I_t\) and \(R_t\) from the family. As Eichenbaum et al. [2021], labor productivity of infected individuals is lower. The term \(\phi I_t\) can be interpreted as the loss in labor productivity resulting from infections. Thus, aggregate production is given as follows:

\[Y_t = A \left( (S_t + \phi I_t + R_t) N_t^D \right)^\alpha. \quad (25)\]

Recall that price symmetry across firms clears the dispersion term in the demand function such that \(Y_t = Y_t^D\). Therefore, the resource constraint is given by total consumption as defined in Equation 6 as well as the price adjustment cost:

\[Y_t = (S_t + I_t + R_t) C_t + \Delta Y_t. \quad (26)\]

The appendix contains the optimality conditions.

2.4 The Health New Keynesian Model (HeNK)

This section analyzes how the presence of contagion risk affects the baseline New Keynesian model in terms of aggregate demand and supply. We start by deriving the new IS equation, and then show how health variables affect the dynamics of inflation. For clarity purpose, exogenous shocks as well as expectation operators are neglected.

2.4.1 The New IS curve in the presence of contagion risk

The optimality condition with respect to consumption (Equation 27) as well as the Euler condition (Equation 28) can be combined to derive what is often referred to as the new IS curve:

\[\beta (1 - \gamma_D T_t) \frac{\lambda_{t+1}}{\lambda_t} \frac{\Pi_t}{\Pi_{t+1}} = 1, \quad (27)\]

\[\frac{\exp(c_t)}{C_t} = \frac{(1 + \mu_t)}{(1 - H_t)} \lambda_t, \quad (28)\]
where:

\[ H_t^C = \beta E_t \{ W_{t+1} \} 2 \gamma C^{\gamma} I_t S_t \frac{C_t^2}{\exp(\epsilon)}, \]

Combining these two conditions, we obtain the following expression which comprises 4 components driving aggregate demand:

\[
C_t = C_{t+1} \times \frac{\exp(\epsilon)}{\exp(\epsilon)} \frac{\Pi_{t+1}}{\beta r} \times (1 - \gamma D T_t)^{-1} \times \frac{1 - H_t^C}{(1 - H_{t+1}^C)} \times (1 + \mu_{t+1}) \quad (29)
\]

On the right-hand side of Equation 29, expected consumption as well as the first term labelled “Standard term” denote the typical effect from the plain vanilla New Keynesian model (e.g., Galí, 2015) based on the intertemporal rate of substitution of consumption. The term \( C_{t+1} \) captures the effect of future consumption on current, while the second term is the effect of real rates. Note that this is the standard transmission channel of monetary policy that works via the effect intertemporal rate of substitution of households on aggregate demand.

The second term, labelled “Mortality risk”, is a health-related term that affects aggregate demand: since catching the virus can be fatal, the death probability denoted by the term \( \gamma D T_t \) affects agents’ consumption and saving decisions. In particular, a rise in death probability increases agents’ willingness to consume today and reduces the propensity to save. An increase in this probability induces an effect that is therefore akin to an increase in the degree of impatience.

The third term, which we refer to as “Contagion risk”, highlights how the presence of contagion risk alters agents’ consumption and saving decisions. This term is akin to a tax on consumption, as agents in this economy understand that the risk of catching the disease increases with their level of consumption. This explains why an increase in this implicit tax in period \( t \) depresses consumption. Noteworthy is the fact that the consumption tax induced by contagion risk in period \( t+1 \) has a positive impact on present consumption. Indeed, since consumption is essential, agents evaluate the risk posed by the virus by comparing the current situation with what they expect to happen. This forward-looking component of contagion risk explains why we can obtain effects akin to consumption panic in this environment. If agents expect an epidemic to break out in the near future, they will find it optimal to increase consumption before the shock materializes.

Finally, the last term reflects the impact of lockdown risk on consumption decisions.
Whereas the tax induced by contagion risk is endogenously determined, the difference is that lockdown risk depends on the government and is therefore exogenous. The forward-looking component denoted by $\mu_{t+1}$ illustrates that panic effects can also be caused by lockdown policies. According to this environment, an increase in consumption can be expected if the government announces that a lockdown will be imposed in the near future.

Applying logs to Equation 29 and solving the above expression forward, we derive the following reduced form expression:

$$\hat{c}_t \simeq \sum_{i=t}^{\infty} \left( \Delta \epsilon_{t+1} + \hat{\Pi}_{t+1} - \hat{r}_t \right) + \gamma D_{i,t} T_i + \Delta H_{C,t+1} + \Delta H_{L,t+1} ,$$

(30)

where variables with hat are expressed in percentage deviation from pre-epidemic state.

### 2.4.2 How do epidemics affect the Phillips curve?

We next turn to determination of prices during an epidemic outbreak. Solving problem (19) provides the standard expression for the Phillips curve:

$$\theta \Pi_t (\Pi_t - 1) = (1 - \epsilon) + \epsilon mc_t + \beta \lambda Y_{t+1} \Pi_{t+1} (\Pi_{t+1} - 1) .$$

(31)

In contrast to the New IS curve, notice that the price setting equation is not directly affected by the epidemic cycle, but indirectly through the marginal cost. Indeed the main driver of inflation in this equation is the current and future discounted marginal costs. To illustrate how the price setting mechanism is affected by the input cost structure of firms, let us rewrite the expression of the marginal cost in Equation 18 by replacing wages using the labor supply equation in Equation 14. We therefore split the marginal cost into four forces that determine the pricing behavior of firms:

$$\Pi_t (\Pi_t - 1) =$$

$$\sum_{i=0}^{\infty} \beta_{i+1} \left[ \frac{(1 - \epsilon)}{\theta} + \frac{\epsilon N^\gamma C_t N_t}{\alpha Y_t} \times \frac{(1 + H_{C,t})}{(1 - H_{C,t})} \times \left( S_t + \phi I_t + R_t \right) \times \left( 1 + \mu \right)^2 \right] .$$

(32)
The first component, labelled “Standard cost”, captures how the disutility of labor $\chi N_t$ affects the real wage. A higher disutility reduces labor supply, which in turn puts upwards pressure on real wages. In this condition, the wealth effect on labor supply depends on the level of consumption $C_t$. Indeed, everything else equal, an increase in consumption lowers marginal utility of consumption and the resulting wealth effect reduces agents’ willingness to work. Since this latter effect also reduces labor supply, it has a positive impact on real wages. Finally, the last part of this term is the marginal product of labor that mechanically lowers the cost of production.

In this setup, the key is that equilibrium real wages also depend on the implicit tax on consumption and labor induced by contagion risk. The second term, labelled “Contagion cost”, reflects how the household’s labor supply is affected by the outbreak. Since contagion risk deters agents from supplying hours, an increase in $H_t$ reduces labor supply, an effect which in turn puts upward pressure on real wages. Second, since in this environment contagion risk is also akin to an implicit tax on consumption, an increase in $H_t C_t$ has a similar effect on real wages. Indeed, contagion risk depresses agents’ desire to consume. By lowering marginal utility of consumption, a decline in marginal utility in turn generates a positive wealth effect that reduces labor supply, and hence increases real wages.

The third term, labelled “Infection cost” reflects the decline in the effect of infections on the labor supply, both in terms of a productivity slowdown of infected workers, and also the permanent loss in labor supply from fatalities $D_t+1$.

Finally, the last term labelled “Lockdown cost”, denoted by the term $\mu_t$, shows that lockdowns also depress labor supply and raise real wages via the same channel. This term is squared as a result of both the tax on consumption of households (labor supply channel) as well as the tax on the firm hiring decisions (labor demand channel).

### 2.5 HeNK in the AS-AD framework

How can we rationalize an outbreak into the static aggregate demand/supply framework based on the HeNK? Consider Figure 1 that reports the usual representation of aggregate demand and supply from the macro textbook. Aggregate Demand (AD) is decreasing in inflation, as underlined by the standard part of Equation 29. Indeed, a surge in inflation triggers, through the Taylor principle, a rise in real rates that depresses aggregate consumption. In contrast, the relation works in reverse order for Aggregate Supply (AS), as inflation increases with output. This positive relationship is rationalized from the cost of production.
in Equation 32: a rise in production stimulates labor demand, which in turn implies a rise in the equilibrium wage that fuels a surge in inflation.

Figure 1: Static response of HeNK to an increase in the number of infected in the AS-AD framework

Consider the initial pre-epidemic state in equilibrium, which we denote by point A in Figure 1, that characterizes the equilibrium between producers and consumers in normal times. How does the economy adjust in response to an epidemic outbreak? We consider an increase in the number of infected $\Delta I$ as the initial impulse that affects the initial equilibrium. A rise in the number of infected agents affects the aggregate demand through an increase in contagion risk, the term $\Delta H^C$ in our decomposition. An increased risk of catching the virus with consumption is internalized by the households through a reduction of their time for consumption. In response, the AD schedule shifts to the left and exhibits the same quantitative features as a negative demand shock.

The effect on inflation is apriori ambiguous because the AS curve is also affected by the outbreak. As shown in Equation 32, the risk of catching the disease at the workplace reduces labor supply. Labor market clearing in turn implies an increase in real wages, as the shortage of workers induced by contagion risk increases the marginal cost of production. In terms of our analysis, this effect translates into a leftward shift in the AS curve, comparable to a negative supply shock. If the shift in the AS curve is large enough to counterbalance the deflationary effect stemming from the decline in aggregate demand, an epidemic outbreak generates stagflation, as the decline in activity is accompanied by high inflation rates.
Figure 1, we report a situation in which the shift in the AS curve dominates.

3 Estimation of HeNK model

In this section, we estimate our nonlinear model through maximum-likelihood-based methods. This section first describes how the nonlinear model is solved as well as the filtering method. Next, we discuss the data, our estimation strategy and elaborate on the value of inferred parameters.

3.1 Solution and filtering methods

We consider the extended path solution method from Fair and Taylor [1983] and Adjemian and Juillard [2014] to accurately measure the nonlinear effects of economic decisions. In summary, the extended path approach uses a perfect foresight solver to obtain endogenous variables that are path-consistent with the model’s equations. Each period, agents are surprised by the realization of shocks, but still expect that future shocks will be zero on average, consistently with rational expectations. The advantage of this method is that it provides an accurate and time-efficient solution while considering all nonlinearities inherent to SIR models. The drawback of the approach, however, is that the uncertainty stemming from future shocks is neglected as is the case in linearized DSGE models, such as Smets and Wouters [2007]. As the main bulk of fluctuations is driven by SIR-dynamics, the certainty equivalence implied by our method is a relatively fair drawback with respect to all the advantage it brings on quantitative grounds.

Taking nonlinear models to the data is a challenge as nonlinear filters, which are required to form the likelihood function, are computationally expensive. Inversion filters have recently emerged as a computationally cheap alternative (e.g., Kollmann, 2017; Guerrieri and Iacoviello, 2017). Initially pioneered by Fair and Taylor [1983], this filter extracts the sequence of innovations recursively by inverting the observation equation for a given set of initial conditions. Unlike other filters (e.g., Kalman or particle filters), the inversion filter relies on an exact characterization of the likelihood function. For an advanced presentation of the filtering method with extended path, we refer to Jondeau, Levieuge, Sahuc, and Vermandel [2023].
3.2 Data description

The model is estimated on weekly Euro area data to match macro-epidemic dynamics during the first wave of infections in Europe. The sample comprises five time series. First, to measure economic activity during the pandemic, we consider output data from the OECD weekly tracker measured in Wołoszko [2020]. Second, to identify the parameters of the New Keynesian block, we include a market-based measure of inflation that has the advantage to be in high frequency. We use the Refinitiv’s Euro area inflation-linked swap rate as a measure of annual inflation expectations. Third, we include a measure of monetary policy stance, abstracting from the zero lower bound, by using the Wu and Xia [2016] shadow rate. The measure is monthly and converted into a weekly basis based on a spline-based interpolation. Fourth, to obtain a quantitative model consistent with epidemic dynamics, the observable variable matrix also includes the number of fatalities in the Euro Area from the “Johns Hopkins coronavirus resource center”.5 Last, a factor that was key during the outbreak is the government response to the COVID shock. We therefore consider the governments policy stringency index for Euro Area as third observable variable, and which we take from Hale et al. [2021].5 Country-specific indices are summed into an Euro Area-wide index by weighting countries by their relative population size. Overall, our sample covers the period from 2020W8 to 2020W32. It starts one week before the implementation of lockdown policies and ends when these policies were gradually phased out. Figure 2 reports the five times series used as observable variables.

To map the model to the data, the following measurement equations are employed:

\[
\begin{bmatrix}
\text{Real GDP} \\
\text{Inflation Expectations} \\
\text{Shadow Rate} \\
\text{New Fatalities} \\
\text{Containment Policy}
\end{bmatrix}
= \begin{bmatrix}
\eta_0 + \log(Y_t/Y) \\
E_{t} \{ \Pi_{t+1}\}^{52} - 1 + \eta_{\Pi} \\
\frac{1}{r_{t}^{12}} - 1 \\
\Delta D_{t+1} \bar{L} \\
\varepsilon_{t}^{\mu}
\end{bmatrix}.
\]

In this expression, \(\eta_{\Pi} \sim N(0, \sigma_{\Pi}^2)\) is a measurement error that is aimed to capture possible discrepancy between market-based measure of inflation expectations and its corresponding

---

4Note that the infection data are less reliable than data for fatalities: the number of cases is driven by the access to testing facilities, the latter was highly limited during the first wave of covid, and heterogenous across Euro area member states.

5This index is built on indicators of government policies such as school closures, travel bans, etc. This composite measure is a simple additive score of nine indicators constructed to vary from 0 to 1.
non-market based counterpart in the model. Parameter \( \bar{L} \) denotes the size of the Euro Area population over 25 years (251 millions inhabitants), this parameter is necessary to map the normalized size of the population in the model to its empirical counterpart. Finally, because there is a natural discrepancy between the position in the business cycle in the data and the pre-epidemic state, \( y_{0} \) denotes an estimated shift parameter that controls for this gap.\

### 3.3 Calibrated parameters

A subset of structural parameters are calibrated and reported in Table 1. For parameters related to the New Keynesian model, these are taken from the macroeconomic textbook of Galí [2015]. We set the elasticity of substitution as \( \epsilon = 7 \) and the labor intensity \( \alpha = 2/3 \). Note that this calibration is very close to Eichenbaum et al. [2022]. The pre-epidemic gross rate of inflation, \( \Pi \), is set to 1% annually consistently with pre-epidemic inflation rate in the Euro Area.

For the remaining parameters, we mostly build on the calibration strategy of Eichenbaum et al. [2021] adapted to the Euro Area. The per capita annual GDP was about 33,770€ in 2019, which we convert into a weekly basis \( Y = 33,770/52 \), while the number of hours worked per week \( N \) is set to 37 to match the average of the EU. We take the same discount factor than Eichenbaum et al. [2022] with \( \beta = 0.98^{1/52} \). The latter implies a steady state real rate of about 4% in the Taylor model and 2% in the Ramsey steady state.

On epidemic grounds, it takes on average 18 days to either recover or die from the infection, \( \gamma_{R} + \gamma_{D} = 7/18 \), with an initial fatality rate of 0.3 percent such that \( \gamma_{D} = \)

---

Note Smets and Wouters [2007] also include a similar correction for hours worked in their observation equation.
0.003 × $7/18$ and $\gamma_R = 7/18 − \gamma_D$. 

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ Discount factor</td>
<td>$0.98^{1/52}$</td>
</tr>
<tr>
<td>$N$ Weekly hours worked</td>
<td>37</td>
</tr>
<tr>
<td>$Y$ Weekly output</td>
<td>33,770/52</td>
</tr>
<tr>
<td>$\Pi$ Inflation rate</td>
<td>$1.01^{1/52}$</td>
</tr>
<tr>
<td>$\epsilon$ Goods substitution</td>
<td>7</td>
</tr>
<tr>
<td>$\alpha$ Labor intensity</td>
<td>$2/3$</td>
</tr>
<tr>
<td>$\gamma_D$ Fatality rate</td>
<td>$0.003 × 7/18$</td>
</tr>
<tr>
<td>$\gamma_R$ Recovery rate</td>
<td>$7/18 − \gamma_D$</td>
</tr>
<tr>
<td>$\phi_i$ Productivity loss from infections</td>
<td>0.8</td>
</tr>
<tr>
<td>$S_0 + I_0 + R_0 + D_0$ Population mass</td>
<td>1</td>
</tr>
<tr>
<td>$D_0$ Initial mass of death</td>
<td>0</td>
</tr>
<tr>
<td>$R_0$ Initial mass of recovered</td>
<td>0</td>
</tr>
<tr>
<td>$r_0$ Initial shadow interest rate</td>
<td>$(1 − 0.07)^{1/52}$</td>
</tr>
</tbody>
</table>

Table 1 Calibration of the model

We also must set the initial value for state variables that are going to be taken as given by agents and the Ramsey planner. The plain-vanilla new Keynesian model typically embeds no state variable, except for exogenous disturbances and nominal interest rate considering $\rho > 0$. Based on the shadow rate measure in Wu and Xia [2016], we set to -7% annually $r_0 = (1 − 0.07)^{1/52}$ consistently with the observed value at the start of the sample. Finally, the initial state for population variable is normalized to one, $S_0 + I_0 + R_0 + D_0 = 1$, the initial number of death $D_0$ and recovered $R_0$ are 0, while number of people initially infected $I_0$ is estimated.

3.4 Estimated parameters

We estimate the remaining subset of parameters, namely parameters related to shocks, initial values, macro-epidemic parameters, and the policy stance. Unlike Bayesian inference, the frequentist approach does not require prior distributions for structural parameters. However, the frequentist inference also allows for bound restrictions in order to shrink the search space for the optimization algorithm. Table 2 provides the bounds for each control variable of the optimization problem. We next discuss those bounds restrictions in a more detailed manner.

We start with structural parameters for Markovian processes. For three of the standard deviations $\sigma_j$ with $j = \{\epsilon, \mu, r\}$, we simply impose a large positive support [0,10] while we
limit the contribution of fatality stock to be as big as $\gamma_D$ with $\sigma_d \in [0, 0.001]$. For the persistence of Markovian processes, we exclude unit roots $\rho_j \in [0, 1)$ with $j = \{c, d, \mu, r\}$. In contrast for the standard error in the market-based expectations equation, we follow the usual procedure as for particle filtering literature, and impose a bound restriction up to 20% of the standard error of the observed variable. In other words, the contribution of $\phi_j$ must not exceed 20% of the standard error of the observed path of inflation, with $\sigma_j \in [0, 0.001]$. Note that shocks on interest and inflation rates are reported in weekly basis by multiplying them by 52 (weeks).

We next discuss the bounds for epidemic-related structural parameters. First, the initial fraction of infected $I_0$ must lie between 0 and 1% of the population with bound $I_0/100 \in [0, 1]$. This restriction is consistent with the usual calibration for this parameters in Eichenbaum et al. [2022] is 0.1%. To further detail the bound restriction, let consider $C$ and $N$ the pre-epidemic consumption and labor supply, Equation 5 can be rewritten as follows:

$$R = \frac{1}{\gamma_D + \gamma_R} \left( (\gamma_T + \gamma_C)C^2 + \gamma_N N^2 \right)$$

(33)

The lower bound for the reproduction number $R$ (i.e. the value of $R$ when consumption and labor are all zero), denoted $1/(\gamma_D + \gamma_R)$, is assumed to lie between 0 and 1. Note that any value below one for the reproduction number is consistent with a reduction of infections. How do consumption and labor play a respective role in the transmission of the decease? We express the share of consumption, $\nu$, in the transmission of COVID as follows:

$$\frac{(\gamma_D + \gamma_R)R - \gamma_T}{\gamma_C C^2} = \nu$$

and

$$\frac{(\gamma_D + \gamma_R)R - \gamma_T}{\gamma_N N^2} = (1 - \nu),$$

while the remaining fraction $(1 - \nu)$ is for labor transmission at the workplace. The relative share of consumption in COVID transmission is bounded with a support $\nu \in [0, 1]$. Finally, we assume a positive support for the containment policy stance $\xi \in [0, 5]$. Another important health parameter is the elasticity of fatality to infection, denoted $\varpi$, that we impose to lie between 0 and 100. This is rather uninformative: to gauge realistic parameter range values for $\varpi$, consider a baseline scenario with infection peak up to 1% of the population ($I_{\text{max}} \approx 1\%$). A rise of the fatality rate by a factor 3 as measured by Odone et al. [2020], would yield to a value of $\varpi \approx 23.3$. We allow this value to be even larger than for the one observed during specific event in Lombardy.

We next continue with the bound restrictions for economic parameters. The labor dis-
tility curvature coefficient \( \varphi \) is restricted to ensure that the value lies between 0 and 5, which is consistent with usual calibration for real business cycle models. The parameters related to the Taylor rule are given a bound values that are consistent with New Keynesian models. Monetary policy smoothing is set so as the nominal rate follows a stationary process \( \rho \in [0, 1) \). Inflation stance must be above one to ensure that Taylor principle holds, \( \phi_\pi \in [1, 6] \), while we impose a that annualized output gap stance is positive \( 52 \times \phi_\pi \in [0, 1] \).

Regarding the price rigidity parameter, it is usually more convenient to estimate and interpret a price update probability than the Rotemberg price adjustment cost. We therefore convert to convert the Rotemberg cost \( \theta \) into Calvo update lottery \( \theta_C \) as follows:

\[
\frac{\theta}{(e - 1)} = \frac{\theta_C}{((1 - \theta_C)(1 - \theta_C))}.
\]

Because this parameter is hard to interpret in a weekly basis (as \( \theta_C \) would be close to one), we introduce \( Q \), the number of quarters for a price update: \( Q = 1 - 7/(\theta_C 90) \). In New Keynesian models, the average number of quarters between price update typically lies between 3 and 4 quarters. We therefore estimate \( Q \) by imposing that the number of quarters between price update must lie between 2 and 6 quarters to let the data inform about how quick prices changed over the observed time period.

We next discuss the output from the inference procedure. The maximum of the likelihood function is reached based on a simplex optimization scheme. The vector of estimated parameters maximizing the likelihood function as well as the corresponding vector of the approximated standard deviations are reported in Table 2.

Let us first discuss epidemiological parameters. The initial share of infected represents 0.37% of the total population, which is almost 4 times higher than the usual 0.1% calibrated in the literature. The lower bound for the reproduction number (or the constant term in transmission function in Equation 7) denoted \( \gamma_T/(\gamma_B + \gamma_D) \) is estimated to be close to 0.5, which strikingly contrasts with the usual calibration in macro-epidemic literatures. For instance, Eichenbaum et al. [2021] consider that the constant term accounts for 4/6 of transmissions, making the corresponding lower bound to be close to 1. The reproduction number is close to 1.5, which matches the usual calibration in the macro-epi literature. Regarding the elasticity of fatality to infections \( \varpi \), we find a value 18 which actually assumes that at the peak of the outbreak the fatality rate is 2.5 times higher. This value is reasonable and consistent with Odone et al. [2020]. Finally regarding the lockdown policy stance, to gauge quantitatively how the estimated stance \( \xi \) translates into a tax during the outbreak, we
compute the lockdown rule at the top of the outbreak (2% of infected with stringency index at 0.85) and find a tax rate of about 0.05%. This value is much smaller than those reported by Eichenbaum et al. [2021] who find a tax rate that peak up to 80%. This gap can be explained by our estimated value of $\gamma_T$ that is close to zero. In our estimated model, consumption and hours are relatively more important in driving the reproduction number, therefore a tax hike lowering consumption/hours reduce proportionally more the virus transmission.

Regarding macroeconomic parameters. Let us first discuss the labor disutility curvature is 0.63, which is very close to the value of 0.8 found by Smets and Wouters [2003] for the Euro Area. For coefficients related to monetary policy, we find a smoothing coefficient $\rho = 0.98$ that is relatively bigger to the 0.91 estimated value in Smets and Wouters [2003]. In contrast, the stance on inflation estimated here is slightly bigger while the one for output lower with respect to Smets and Wouters. In our estimated model, it takes on average 5 quarters for firms to update prices, this value is fairly in line with microeconomic evidence on the price setting of producers. Note also that our value is much lower than the 11 quarters estimated in Smets and Wouters [2003] for the Euro area. Finally, we estimate that the initial position in the business cycle of the Euro area denoted $y_0$ was about 4.5% percent above its trend.

<table>
<thead>
<tr>
<th>Name</th>
<th>Support</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_c$</td>
<td>[0; 10]</td>
<td>0.20993180</td>
<td>0.075482</td>
</tr>
<tr>
<td>$\sigma_r \times 52$</td>
<td>[0; 10]</td>
<td>0.00091094</td>
<td>0.000053</td>
</tr>
<tr>
<td>$\sigma_d$</td>
<td>[0; 10]</td>
<td>0.00099859</td>
<td>0.000001</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>[0; 10]</td>
<td>0.15392397</td>
<td>0.000706</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>[0; 1)</td>
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<td>0.000001</td>
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<td>$\phi_\ell$</td>
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<td>$Q$</td>
<td>[2; 6]</td>
<td>5.80346264</td>
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</tr>
<tr>
<td>$d_i \times 100$</td>
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<td>$\gamma_T / (\gamma_D + \gamma_A)$</td>
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<td>$v$</td>
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<td>0.000183</td>
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<tr>
<td>$\kappa$</td>
<td>[5; 4]</td>
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</tr>
<tr>
<td>$\psi$</td>
<td>[0; 10]</td>
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<td>0.537163</td>
</tr>
<tr>
<td>$y_0$</td>
<td>[-0.1; 0.1]</td>
<td>0.04575181</td>
<td>0.000109</td>
</tr>
</tbody>
</table>

Table 2
Parameters estimates based on the maximum likelihood estimation
3.5 Propagation mechanism to stochastic innovations

Our quantitative framework includes four sources of fluctuations (abstracting from the observation equation shock). This section investigates the propagation mechanism of the model by analyzing how the system responds to those four stochastic disturbances. Because the model is fully nonlinear, we compute generalized impulse response functions. We compare the HeNK and the NK models in order to assess the relative importance of the epidemic block in driving the propagation of aggregate shocks.

Figure 3: Generalized impulse response functions of the HeNK model and the standard New Keynesian model

Generalized system responses are reported in Figure 3. Each row provides a specific exogenous disturbance, while each column reports a specific endogenous variable.

The demand or preference shock triggers an immediate boost in consumption, which increases by around 20%, as a result of an increased willingness to consume. This large change in consumption strongly reduces the stochastic discount factor, making firms relatively more
impatient. Because expected future profits are relatively less important in the pricing of firms, they adjust downward their selling price. This explains the immediate decline in inflation. Note that because the nominal rigidities are intense, inflation is less reactive than other variables. As both consumption and hours increase, the shock propagates the virus, leading to a peak increase in infection, compared to the case without shocks, of 30%. Relative to the standard New Keynesian model, this epidynamics substantially increases the persistence of the shock. Since the virus circulates faster, consumption decreases a few periods after the realization of the shock, whereas the reduction in hours worked stemming from the increase in the number of new cases generates inflationary pressures.

In contrast, the fatality shock in second row of Figure 3 raises the risk of catching the disease. In response, households reduce consumption and hours. Monetary policy is slightly more accommodative to alleviate the subsequent recession as well as the decline in inflation that occurs shortly after the occurrence of the shock.

For the monetary policy shock, as for the standard New Keynesian model, it leads to a joint decrease in quantities and prices. This in turn reduces the transmission of the virus. However, a rise in consumption is next observed a few weeks after the realization of the shock. This delayed boost in aggregate demand is triggered by the lower risk of contagion, which encourages households to spend more.

Finally, the lockdown shock is reported in the last row of Figure 3. A tightening of containment measures restricts household’s mobility, making it harder for them to purchase goods and services or go to the workplace. It mechanically reduces the transmission of the virus, but also creates a labor shortage that fuels the rise in inflation.

3.6 Decomposing the COVID recession

This model can also be used to provide a description of the unprecedentedly large recession induced by the epidemic outbreak. To do so, consider the HeNK decomposition in Equation 30. The path of consumption during the outbreak is reported in Figure 4 and expressed in percentage deviation from the pre-epidemic state.

During the outbreak, there are two main driving forces at play. The first force is contagion risk, which directly relates to how households’ risk of catching the disease reduces their marginal utility to consume. As the virus spreads within the population, households naturally decrease their time spent consuming in restaurants or shopping malls. In contrast, the accommodative monetary policy represented by the term ‘standard’ force in Figure 4
Notes: This figure decomposes the four forces driving consumption based on Equation 30. These forces are standard IS, mortality risk, contagion risk and lockdown risk. Consumption is expressed in percentage deviation from the initial state.

Figure 4: Decomposition of the euro area recession

provides a boost that partially offsets the recession. This term also includes demand shocks, and plays a stimulating role during the outbreak. Specifically, this term capture exogenous policy measures taken by governments during this time period such as fiscal measures. In contrast, as households internalize the risk of catching the virus, they modify their behavior, which explains why lockdown only reduces consumption by around 2%. Finally, the last term is the infection effect, which only plays a minor role in accounting for the recession.

4 Results

This section compares the effect of an epidemic outbreak across two scenarios. First, we start with a quantitative assessment of how the economy responds to the COVID shock when the nominal interest rate follows a Taylor rule (see Equation 20) and when there is no lockdown policy \( \mu_t = 0 \). We then compare this outcome with that obtained when interest rates are set optimally by deriving the Ramsey optimal monetary policy.\(^7\) Finally,

\(^7\)Note that a deterministic simulation algorithm is employed to simulate the Health-New Keynesian model. Let \( f(y_{t-1}, y_t, y_{t+1}) = 0 \) denote the system of equations in Appendix (A) for a vector of endogenous variables \( y_t \) with same initial and terminal states \( y \), with a finite number of \( T \) periods between these two states. The perfect foresight algorithm aims to calculate the path of endogenous variables between initial and terminal
we examine the outcome when a lockdown policy is introduced.

4.1 Macroeconomic propagation under a Taylor Rule

The propagation following an epidemic outbreak is depicted in Figure 5. After 20 weeks, the peak in the number of newly infected agents stands slightly below 2% of the total population. Whereas a small increase is observed on impact, an epidemic outbreak triggers a dramatic decline in consumption and hours worked. The fall in consumption is consistent with real time estimates of output during the first outbreak. Under a Taylor rule, inflation decreases sharply on impact before increasing gradually.

As illustrated by the first panel on the second row of Figure 5, the central bank eases monetary conditions in response to an epidemic outbreak, as the shock decreases aggregate demand. Regarding the reproduction number, the fall in consumption and hours is not sufficient to slow down the rise in infections, leading the virus to further spread in the early stage of the crisis.

What drives the dynamics of consumption?

The dynamics of consumption is determined by the modified New IS curve shown in Equation 29. Solving this dynamic equation forward, consumption today can be expressed as the product from period \( t \) to \( t + j \) of these 4 terms. Taking logs then allows us to decompose aggregate consumption, which is shown in panel (A) of Figure 6, into a sum of these 3 effects.

Panel (B) of Figure 6, which denotes the standard IS term, represents the effect of current and future real interest rates on consumption. Overall, this term has a positive impact on consumption. The reason is that the recessionary effect of the shock lowers future real rates, an effect which in turn stimulates agents’ willingness to consume. At the same time, the states that is consistent with the model’s equations. Let \( F(Y) \) denote the system of equations \( f(\cdot) \) stacked over \( T \) periods, the goal of the perfect foresight algorithm is to minimize the residual function \( F(Y) \) using as control variables the path of endogenous variables \( Y \). In what follows, we use the relaxation algorithm of Juillard et al. [1996] to minimize \( F(Y) \). The choice of the number of periods \( T \) is arbitrary; we set \( T = 500 \) (about 10 years) to be high enough to allow the epidemic-induced dynamics to converge to their terminal state.

Woloszko [2020] provides an estimate of output in a weekly basis for OECD economies, in particular for major European economies. At a 95% confidence level, output is estimated to have fallen in March 2020 to about \([-34%;-20%]\) for Italy, \([-34%;-22%]\) for France and \([-22%;-15%]\) for Germany.

We do not report the lockdown term from Equation 29 into Figure 6 as lockdown policy is not implemented in this simulation.
Taylor principle implies that the decrease in interest rates is larger than that of inflation. Consequently, on impact, the central bank largely engages into a low interest rate policy to revive aggregate demand and avoid deflation.

Notice also that consumption increases on impact. This effect can be explained by the sharp reduction in interest rates that occurs when the shock hits. Since agents are forward-looking, this current and anticipated reduction in interest rates has a positive, albeit short-lived effect on consumption. Along with the cost of adjusting prices, this increase in consumption on impact then explains the initial response of hours worked when the outbreak starts.

As shown in panel (C), agents perfectly internalize that an increase in the number of cases also raises their death probability. This effect, however, only has a small impact on the dynamics of consumption as illustrated by the negligible contribution of this term to the overall dynamics of consumption.

Finally, panel (D) in Figure 6 shows that the decline in consumption induced by the epidemic is mainly driven by contagion risk. Contagion risk introduces a distortion in the optimality condition with respect to consumption that is akin to tax. Since consuming goods
What drives the dynamics of inflation?

As in the plain-vanilla New Keynesian model, inflation is determined by the discounted sum of current and future marginal costs. The positive response of inflation can thus be better understood by isolating the different components of marginal cost, as done in Equation 32. The contribution of these various components to the dynamics of inflation is in turn illustrated in Figure 7. Panel (A) in Figure 7 shows the inflation (in percentage deviations from the pre-epidemic state), whereas panels (B), (C) and (D) show the contribution of the standard term as well as infection and contagion risk.
On impact, and as illustrated by Panel (A), the monthly inflation is around 0.2 percentage point below its pre-epidemic steady state. The source of this decrease can be analyzed by decomposing the driving forces of inflation. This decomposition reveals that the immediate decrease in the price of goods can mainly be explained by the standard component of the inflation equation shown in Panel (B) as well as in a much lower proportion to new infections, which is depicted in Panel (C).

After 20 periods, inflation is above its pre-epidemic level as a result of contagion risk, which is shown in Panel (D) and Equation 32. Contagion risk works via its effect on both consumption ($H_C t$) and labor supply ($H_N t$). These two effects both increase marginal costs by reducing agents’ willingness to work. The effect of contagion risk on labor supply can be decomposed into a direct as well as an indirect effect. First, since contagion risk reduces marginal utility of consumption through the term $H_C t$, it has an effect on labor supply that is similar to a positive wealth effect. Indeed, since working is a source of disutility, agents can afford to reduce hours worked if they are less willing to consume.

In addition to this indirect effect, contagion risk introduces an additional term in the labor supply equation, $H_N t$, that is akin to a tax on labor. Indeed, since supplying labor increases the risk of catching the disease, an epidemic outbreak deters agents from working. This direct effect reduces labor supply, an effect which in turn raises real wages. Contagion risk therefore introduces two distortions that raise real wages, and hence the marginal cost of production of firms, by causing a reduction in labor supply.

As shown in Panel (B), on impact, the effect stemming from the rise in contagion cost is mostly compensated by the standard term of the marginal cost, as the joint fall in hours worked and consumption reduces the equilibrium real wage, and hence the marginal cost of producing goods.

**The trade-off between stabilization and health**

According to the standard textbook analysis in Woodford [2003a], for instance, the objective of monetary policy is to stabilize the cycle by fostering consumption smoothing and reduce the distortion caused by price fluctuations. In a model with SIR agents, a key difference is that the aggregate utility function of the family, which is given as follows, is also affected by health considerations.

$$U_t = \left( \frac{S_t + I_t + R_t}{S_t + I_t + R_t} \right) \times u(C_t, N_t).$$
This additional term denoted by \((S_t + I_t + R_t)\) can be interpreted as an extensive margin in the sense that it depends on the number as well as the composition of agents. Since this term is an important determinant of total welfare, it naturally interferes with the conduct of monetary policy. In particular, a deterioration in health conditions reduces welfare, and may force the monetary authorities to react. The effect on welfare of an increase in the number of infected agents firstly works by reducing the total number of agents in the economy. Indeed, since a fraction of agents succumbs from the disease, the first effect of an epidemic is to lower the population. Second, an epidemic outbreak also affects the intensive margin of the welfare function by causing business cycle fluctuations.

To gauge the relative importance of these two objectives on the optimal policy, we calculate the welfare cost of fluctuations expressed in terms of contemporaneous percentage of consumption for each objective, which we denote by \(\psi^b_t\), \(\psi^h_t\) and \(\psi_0\), respectively. The conditions determining these wedges are given as follows:\(^\text{10}\)

\[
(S + I + R)\ u\left((1 - \psi^b_t)C, N\right) = (S + I + R)\ u\ (C_t, N_t)
\]
\[
(S + I + R)\ u\left((1 - \psi^h_t)C, N\right) = (S_t + I_t + R_t)\ u\ (C, N)
\]
\[
(S + I + R)\ u\left((1 - \psi_t)C, N\right) = (S_t + I_t + R_t)\ u\ (C_t, N_t),
\]

where \(S, I, R, C\) and \(N\) denote endogenous variables in the pre-epidemic state. The standard cost of business cycle fluctuations term, denoted by \(\psi^b_t\), captures the cost of changes in consumption as well as the resource cost from updating prices stemming from the price rigidity. The new term \(\psi^h_t\) captures the contribution of health to total welfare. In this environment, the total cost of business cycle fluctuations \(\psi_t\) is the sum of these two wedges (abstracting from cross-products between these two objectives).

In Figure 8, we report the total wedge (Panel A) as well as the decomposition between the health (Panel B) and business cycle (Panel C) wedges. The immediate economic recession combined with changes in inflation, explain the initial spike in the stabilization wedge. In sum, households would be willing to abandon up to 10% of their current consumption to avoid the recession caused by the pandemic. The business cycle cost gradually declines back to zero, as the economy recovers. At the same time, the outbreak has scarring effects on the utility of households. Indeed, fatalities permanently reduce the extensive margin of the

---

\(^\text{10}\)\ With respect to the usual practice in the literature of the welfare cost of business cycles (e.g. Lucas 1987), we do not measure the permanent but the contemporaneous cost of fluctuations. Unlike conventional sources of fluctuations characterized by repeated shocks, the covid shock exhibits an unique occurrence that would make the unconditional mean of \(\psi^b_t\) to converge to zero asymptotically.
welfare objective. Whereas the stabilization wedge remains the most important component in the short-term, the health wedge nevertheless accounts for a non-negligible part of the total wedge in the long-run. This illustrates that health considerations have significant welfare implications and therefore affect the design of optimal monetary policy.

4.2 Optimal monetary policy conduct during an outbreak

How should monetary policy react during an outbreak? Should it consider distortions associated to the epidemic or should it focus exclusively on its stabilization objective? To answer those questions, we examine the Ramsey optimal monetary policy in the presence of COVID-19 health risks. In this economy, the social or Ramsey planner seeks to maximize the expected discounted utility of households, given the set of economy constraints. More specifically, the social planner is assumed to commit to the contingent policy rule announced at time 0. This ex-ante commitment provides the social planner with the ability to dynamically adapt the policy to changes in economic conditions. In particular, here we consider the case of a Ramsey planner setting the optimal trajectory for the monetary policy rate. In terms of inefficiencies, our economy exhibits a distortion at the initial state stemming from monopolistic competition, but as in Schmitt-Grohé and Uribe [2004], we do not introduce any subsidies and consider a second-best allocation.

Definition 1 From a timeless perspective, the social planner will maximize household’s lifetime utility, \( \sum_{t=0}^{\infty} \beta^t (S_t + I_t + R_t) u(C_t, N_t) \), subject to the set of constraints (1), (3), (4), (8), (10), (12), (14), (16), (18), (25), (26) and (31).

Let \( \phi_{j,t} \), where \( j \in \{1, \ldots, 12\} \) represents the sequences of Lagrange multipliers correspond-
ing to the set of allocations and prices \( \{I_{t+1}, S_{t+1}, R_{t+1}, T_t, \lambda_t, W_t, C_t, mc_t, N_t, Y_t, \Pi_t, W_t, r_t\} \) defining the sequences of constraints and first-order conditions referenced in the core model, the chosen health policy \( (\mu_t = 0) \), which impacts the infections dynamics, and a given plan for the state variables \( \{I_0, S_0, R_0\} \).

The comparison with the allocation obtained under a Taylor rule is shown in Figure 9.\(^{11}\)

As illustrated by panel (A) in the first row, the first difference is that the decline in consumption is somewhat larger on impact and the decline slightly more persistent under the optimal policy. As panel (C) shows, this decline is caused by a large increase in interest rates, as shown by the forward interest rates \( r^f_L \), engineered by the Ramsey policy. This persistent increase in interest rates translates, via the IS effect depicted on panel (G), into a decline in aggregate consumption. In response to an epidemic outbreak, the planner therefore finds it optimal to exacerbate the recession caused by the outbreak to curb fatalities.

As panel (B) shows, this more persistent policy response triggers a fall in inflation. This difference in inflation dynamics across monetary policy regimes is driven by the contagion cost term shown in panel (K). By further reducing economic activity, the planner reduces the number of cases which partially mutes inflationary pressures stemming from contagion risk. As a result, optimal policy features significant deviations from price stability in response to the COVID shock. This is so since contagion risk generates a health-recession trade-off which induces the monetary authority to strike a balance between reducing the cost of adjusting prices and reducing fatalities. This trade-off suggests that optimality imposes relatively large deviations from full price stabilization. This result strikingly contrasts with the core finding in the New Keynesian literature in which price stability almost always prevails, as summarized in Woodford \( [2003a] \).\(^{12}\)

To understand the forces driving the trade-off faced by the monetary authority, consider the welfare and wedges reported in panels (D), (E) and (F) in Figure 9. Relative to the allocation obtained under the Taylor rule, the key is that the health wedge becomes smaller under the optimal monetary policy. As the third panel on this second row shows, the stabilization wedge is however larger. When monetary policy is set optimally, the planner is therefore willing to accept larger fluctuations in economic activity, and hence a larger stabilization wedge, in order to reduce the number of infected agents. In the presence of large fluctuations, the planner

\(^{11}\)Note that the Ramsey optimal steady state obtained from Definition (1) implies a zero inflation in the initial state, \( \Pi = 1 \) that is achieved by the planner by setting its instrument as follows: \( \bar{r} = 1/\beta \). This result is standard in the literature of ramsey optimal policies such as Schmitt-Grohé and Uribe [2004].

\(^{12}\)This result holds either with optimized or ramsey optimal policies. See for instance Kollmann [2003] or Schmitt-Grohé and Uribe [2004] for cases with optimal simple rules.
Notes: Panel (G), (H) and (I) are taken from the linearized Euler equation in Equation 29, each term being a forward sum. Panels (J), (K) and (L) are taken from the approximated contributions of inflation in Equation 32.

Figure 9: Ramsey versus Taylor monetary policies (with no lockdown)
of contagion risk, preserving public health becomes an additional objective that greatly complicates the conduct of monetary policy. Faced with this difficult trade-off, the planner finds it optimal to slightly amplify the persistence of the recession in order to slow down the propagation of the virus, which explains the smaller health wedge relative to the Taylor rule case in Panel (E). The large difference between the two allocations emphasizes the importance of health considerations for the conduct of optimal monetary policy.

4.3 Lockdown policies

Since the health dimension adds a new objective in the conduct of monetary policy, in line with the Tinbergen principle, we introduce a new policy instrument to address this additional distortion. In what follows, a containment policy, denoted $\mu_t$, is introduced to tackle contagion risk by decreasing consumption and hours worked. We next discuss the difference between the Taylor rule and the Ramsey optimal monetary policy. Optimized lockdown policies are then added to address distortions caused by the presence of contagion.

4.3.1 Optimizing the lockdown policy rule

As shown in Eichenbaum et al. [2021], the best containment policy from a welfare perspective implies that the magnitude of the lockdown should be proportional to the number of cases. To introduce lockdown policies in the most simple way, and in a consistent way with the estimated model, we take our estimated lockdown policy rule (inspired from Brzoza-Brzezina et al. [2022]). In this policy rule, we neglect the policy shock to focus only on the endogenous determinants of lockdown policies. The operational rule reads as:

$$\mu_t = \xi I_t,$$

where $\xi$ is the tightness of the containment policy. Notice that the simulations done in the previous section correspond to the case in which $\xi$ is set to 0. This parameter is optimally determined by maximizing the same objective as the Ramsey planner:

$$\sum_{t=0}^{\infty} \beta^t (S_t + I_t + R_t) u (C_t, N_t).$$

The optimal control problem under containment is slightly modified for the Ramsey social planner, as the planner must internalize in its decision plan the containment policy rule. A new definition for this problem is introduced:

**Definition 2** The new planning problem with respect to Definition (1) is characterized by
an additional constraint from the endogenous health policy (37), as well as a new Lagrangian multiplier $\varrho_{13,t}$ and an additional control variable $\mu_t$.

In Figure 10, each line reports how average welfare is affected by the tightness of the lockdown under either the Taylor or the Ramsey policy. The concave relationship under both policies suggests the existence of an optimal degree of confinement that is reached when $\xi$ is close to 25. Indeed, for low values of $\xi$ health considerations are not taken into account, which is not optimal from a welfare perspective. Conversely, the decline in average welfare that is obtained for very large values of $\xi$ shows that lockdown policies that are too tight can be counterproductive. Indeed, imposing very stringent restrictions leads to reductions in consumption that are too large relative to their benefits.

Figure 10: Welfare, economic and health wedges for various containment policies under Taylor (blue crosses) and Ramsey (green circles) monetary policies

The panel (B) of Figure 10 shows how the health wedge varies with the degree of restrictions imposed by the government. As expected, tighter restrictions reduce infections and hence the contribution of the health wedge. As depicted by the third panel, this reduction in health risk comes at the cost of increasing the economic wedge. Indeed, whereas lockdown policies reduce infections, this gain is achieved by amplifying the size of the recession. The opposite effect of the lockdown parameter $\xi$ on these two wedges in turn explains the concave relationship depicted in the first panel. The optimal lockdown policy is therefore one that
strikes a balance between the costs in terms of economic stabilization and the benefits for public health.

Unsurprisingly, the Ramsey policy performs better than the Taylor policy by producing a higher level of welfare. As the comparison between the second and third panel illustrates, relative to the Taylor rule case, this improvement is obtained by reducing the contributions of both the health and economic wedges.

4.3.2 Ramsey monetary policy under containment

We now examine how an optimized lockdown policy is able to address distortions entailed by the epidemic outbreak. Figure 11 reports the response of the model under Ramsey monetary policy when the government implements a lockdown policy (green line with circles). The lockdown policy \( \mu_t \) implies an implicit rise in the cost of labor and consumption goods by 23% at the peak of the pandemic. The resulting outcome is a flattening of the infection curve, which translates into a lower death toll. As underlined in Odone et al. [2020], the implementation of restrictions – that avoid a surge in infections stretching hospitals to capacity – is able to reduce the number of fatalities.

![Figure 11: HeNK response under a Ramsey monetary policy with (green circles) and without containment policy (red dashed)](image)

The system response under containment and Ramsey monetary policy is reported in Figure 12. The lockdown deepens the recession in panel (A) by amplifying the fall in consumption. By curbing the transmission probability of the disease, the recessionary effects of contagion risk in panel (H) is attenuated, but exacerbated by the effect of the lockdown in Panel (I), making the recession deeper via an aggregate demand effect. On the supply...
side, the lockdown policy reduces the cost of contagion in Panel (K), but creates an intense labor shortage that boosts the real wage, and mechanically increases inflation via its effect on marginal costs.

For monetary policy, the main difference is that the persistent increase in policy rates when monetary is the only game in town is no longer necessary when confinement policies are in place, as shown by the one-year interest rate in Panel (C). Whereas a large increase remains necessary on impact, the policy can be quickly reverted. Indeed, under the optimal Ramsey policy, a rapid reduction of interest rates follows the initial hike. In this case, since the contagion externality is mainly addressed by the government, monetary policy can focus on inflation stabilization. Indeed, as shown by Panel (B), the fall in inflation is much smaller once the Ramsey policy is combined with lockdown policies. Therefore, although monetary and lockdown policies interact through general equilibrium effects, the Tinbergen principle remains valid in our environment.

The different effect of monetary policy in the two cases can be summarized by the contribution of the standard term in the New IS curve, which is shown in Panel (G). When monetary policy is the only game in town, the contribution of monetary policy to the dynamics of consumption is strongly negative. In that case, the Ramsey planner has no other choice than use monetary policy to slow down activity and hence the spread of the virus. In contrast, and as depicted by the green dotted line in Panel (G), once contagion risk is properly addressed by the government, monetary policy can mainly be used to stimulate consumption and thus mitigate the deflationary pressures caused by the lockdown.

From an efficiency perspective, panels (D), (E) and (F) of Figure 12 show that the benefit relative to the case without lockdown policies essentially originates from the contribution of the health wedge, which is considerably reduced once the optimal policy is combined with a lockdown strategy. Once a more efficient instrument to address the contagion externality is introduced, monetary policy can mainly be used to stimulate consumption. Under containment policy, the core policy prescription of the New Keynesian literature is therefore valid: monetary policy should concentrate on its price stability objective.

### 4.4 A counterfactual scenario

What would have been the optimal conduct of monetary policy during the economic outbreak? To answer this question, the estimated model can be used to provide a counterfactual analysis. Note however that optimal policy recommendations from New Keynesian models
Notes: Panel (G), (H) and (I) are taken from the linearized Euler equation in Equation 29, each term being a forward sum. Panels (J), (K) and (L) are taken from the approximated contributions of inflation in Equation 32.

Figure 12: HeNK response under a Ramsey monetary policy with (green circles) and without (red dashed) containment policy
typically exhibit unreasonably large fluctuations in nominal interest rates. This originates in normative analysis from the omission in the model of all the operational constraints faced by central banks when implementing their policies. Woodford [2003b] provides a simple way to address the issue. To do so, consider the optimal policy problem defined in Definition (1) with an objective function augmented with an additional term:

$$E_t \sum_{t=0}^{\infty} \beta^t \left[ \left(S_t + I_t + R_t \right) e^{\gamma} u(C_t, N_t) - \lambda_r r^2_t \right],$$

(38)

where $\lambda_r$ denotes the operational costs for central banks to adjust their interest rates. We therefore look for the value of $\lambda_r$ that allows our model to provide the same variance $E(t^2)$ than the sample variance. The corresponding value is $\lambda_r = 650$.

Figure 13: Counterfactual path of HeNK model under Ramsey versus observed path in Taylor rule

The corresponding time paths are reported in Figure 13. As the Ramsey social planner must strike a balance between health and economic considerations, in the first periods of the outbreak the planner reduces the health wedge. This is achieved by increasing interest rates relatively more than under the Taylor rule model. This policy reduces consumption, and curb the transmission of the virus in the early stage of the pandemic. This restrictive monetary policy also generates deflation, as the planner is committed to contain infections in the future. This aggressive monetary response stems from the fact that the estimated coefficient of containment policy is too small relative to its optimal value. Consequently, and as illustrated on Panel D, implementing the optimal Ramsey policy generates a significant
difference in the dynamics of new infections. Once the peak of the pandemic outbreak is passed, the monetary authority reduces interest rates more aggressively under the Ramsey policy, as alleviating the recession becomes the main priority. This aggressive easing in turn implies a gradual return of expectations to the target.

5 Conclusion

This paper derives the Ramsey optimal monetary policy in an estimated SIR model with sticky prices. In response to a pandemic outbreak, the optimal monetary policy response deviates substantially from that implied by a standard Taylor rule. If monetary policy is the only game in town, we also find that it is optimal to engineer a persistent increase in interest rates in response to an epidemic outbreak. In this case, health considerations considerably alter the conduct of monetary policy, as a reduction in economic activity is necessary to slow down the spread of the virus. In contrast, when governments respond to the health crisis by implementing lockdown policies, monetary policy can focus on its price stability objective. Therefore, although monetary and lockdown policies interact via general equilibrium effects, the Tinbergen principle applies in our environment.

Our results also suggest the existence of an optimal confinement level that strikes a balance between curbing infection and minimizing the side effects of lockdown policies on the economy. In our environment, the best possible policy mix is obtained by combining the Ramsey optimal policy with a lockdown of medium intensity.

6 Bibliography


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A Taylor rule model’s summary

The Taylor rule model comprises a set of 14 equations and endogenous variables \{I_{t+1}, \quad S_{t+1}, \quad R_{t+1}, \quad T_t, \quad \lambda_t, \quad W_t, \quad C_t, \quad N_t, \quad Y_t, \quad m_{c_t}, \quad \Pi_t, \quad r_t, \quad W_t, \quad \mu_t\} given by:

\[
\Delta S_{t+1} = -T_t \\
\Delta I_{t+1} = T_t - (\gamma_D + \omega I_t^2 + \gamma_R + \varepsilon_t^I) I_t \\
\Delta R_{t+1} = \gamma_R I_t \\
T_t = I_t S_t \left[ \gamma_T + \gamma C_t^2 + \gamma_N N_t^2 \right] \\
(1 + \mu_t) \lambda_t = \frac{\varepsilon_t}{C_t} - \beta W_{t+1} 2 \gamma_C (\gamma_D + \omega I_t^2 + \varepsilon_t^I) C_t L_t \\ 
\varepsilon_t N^\gamma = \lambda_t W_t - \beta W_{t+1} 2 \gamma_N (\gamma_D + \omega I_t^2 + \varepsilon_t^I) N_t L_t \\
W_t = \varepsilon_t u (C_t, N_t) + \beta (1 - (\gamma_D + \omega I_t^2 + \varepsilon_t^I) T_t) W_{t+1} \\
\beta (1 - (\gamma_D + \omega I_t^2 + \varepsilon_t^I) T_t) \frac{\lambda_{t+1}}{\lambda_t} \frac{\varepsilon_t}{\Pi_{t+1}} = 1 \\
Y_t = A (S_t + \phi_t I_t + R_t) N_t^n \\
Y_t = (S_t + I_t + R_t) C_t + 0.59 (\Pi_t - 1)^2 Y_t \\
\theta \Pi_t (\Pi_t - 1) = (1 - \varepsilon) + \epsilon_m a_t + \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{Y_{t+1}}{Y_t} \theta \Pi_{t+1} (\Pi_{t+1} - 1) \\
m_{c_t} = \frac{(1 + \mu_t) W_t (S_t + \phi_t I_t + R_t) N_t}{\alpha Y_t} \\
\frac{r_t}{\bar{r}} = \left( \frac{\Pi_t}{\Pi_t^*} \right)^{\delta_{Y} (1 - \rho)} \left( \frac{Y_t}{\bar{Y}} \right)^{\delta_{Y} (1 - \rho)} \varepsilon_t \\
\mu_t = \xi I_t \varepsilon_t^\mu
\]

As well as four exogenous shocks:

\[
\varepsilon_t^I = \rho_v \varepsilon_{t-1}^I + \eta_t^I \quad \text{with} \quad \eta_t^I \sim N(0, \sigma_t^2) \\
\varepsilon_t^I = \rho_v \varepsilon_{t-1}^I + \eta_t^I \quad \text{with} \quad \eta_t^I \sim N(0, \sigma_t^2) \\
\varepsilon_t^I = \rho_v \varepsilon_{t-1}^I + \eta_t^I \quad \text{with} \quad \eta_t^I \sim N(0, \sigma_t^2) \\
\varepsilon_t^I = \rho_v \varepsilon_{t-1}^I + \eta_t^I \quad \text{with} \quad \eta_t^I \sim N(0, \sigma_t^2)
\]
B Household’s problem

Denoting \( \varrho_t \) and \( \lambda_t \) the constraints multipliers, we rewrite the optimization problem as follows:

\[
W_t = \max_{\{C_t, N_t, B_{t+1}, T_t, B_{t+52}\}} \min_{\{\lambda_t \geq 0, \varrho_t \geq 0\}} \left( \frac{e^{\varepsilon}u(C_t, N_t) + \beta (1 - \gamma_{D,t} T_t) W_{t+1}}{C_t} \right) \tag{40}
\]

\[
+ \lambda_t \left[ W_t N_t + \frac{B_{t+52}}{P_t/P_{t-52}} B_t + \frac{T_{t+1}}{P_t} B_t + div_t + tr_t - (1 + \mu_t) C_t - B_{t+1} - B_{t+52} \right]
\]

\[
+ \varrho_t \left[ T_t - I_t S_t (\gamma T + \gamma C_t^2 + \gamma N_t^2) \right]
\]

The first order conditions of our optimization problem read:

\[
C_t : e^{\varepsilon} \frac{\partial u(C_t, N_t)}{\partial C_t} - \varrho_t I_t S_t 2\gamma C_t (1 + \mu_t) \lambda_t = 0
\]

\[
N_t : e^{\varepsilon} \frac{\partial u(C_t, N_t)}{\partial N_t} - \varrho_t I_t S_t 2\gamma N_t - \lambda_t W_t = 0
\]

\[
T_t : - \beta \gamma_{D,t} W_{t+1} + \varrho_t = 0
\]

\[
B_{t+1} : \beta (1 - \gamma_{D,t} T_t) \frac{\partial W_{t+1}}{\partial B_{t+1}} - \lambda_t = 0
\]

\[
B_{t+52} : \prod_{i=1}^{52} \beta([1 - \gamma_{D,t-1+i} T_{t-1+i}]) \frac{\partial W_{t+52}}{\partial B_{t+52}} - \lambda_t = 0
\]

Thus, we can retrieve the following expression for the marginal utility of consumption and labor supply by using the expression obtained through the FOC on \( T_t \):

\[
(1 + \mu_t) \lambda_t = \frac{e^{\varepsilon}}{C_t} - \beta \gamma_{D,t} W_{t+1} 2\gamma C_t I_t S_t
\]

\[
\chi N_t = \varrho_t W_t - \beta \gamma_{D,t} W_{t+1} 2\gamma N_t I_t S_t
\]

Similarly, by using the envelope condition we get the following expression for the FOC of
Thus, by substituting this result in the FOC equation we get:

\[
\beta(1 - \gamma D, t) \frac{r_t}{\Pi_{t+1}} = \frac{\lambda_t}{\lambda_{t+1}} \tag{41}
\]

New IS curve:

\[
\lambda_t = \beta(1 - \gamma D, T) \frac{r_t}{\Pi_{t+1}} \lambda_{t+1} \tag{42}
\]

where

\[
(1 + \mu_t) \lambda_t = \frac{\epsilon_i}{C_t} - \beta W_{t+1} 2 \gamma C D, C_t. \tag{43}
\]

Regarding the annual claim, the first order condition reads as:

\[
\lambda_t = r^L_t \lambda_{t+52} \prod_{i=1}^{52} \frac{[\beta(1 - \gamma D, t+I) \Pi_t]}{\Pi_{t+i}} \tag{44}
\]

The latter can be rewritten using the one-week claim:

\[
r^L_t = \prod_{i=1}^{52} r_{t-i+1} \tag{45}
\]

where \(r^L_t\) denotes the term structure of interest rates over the next 52 weeks.

\section{Imperfect risk sharing}

While our baseline model features perfect consumption insurance, the benchmark model of Eichenbaum et al. [2021] imposes imperfect risk sharing across family members. Consider the problem of the family as in which consumption and hours are different across Susceptible,
Infected and Recovered family members:

\[ W_i = \max_{\{c_t, c_t', n_t, n_t', B_{t+1}, T_i\}} \min_{\{\lambda, \gamma, \nu > 0\}} \left( e^{t_i} \left( S_i (c_t', n_t') + I_i (c_t, n_t') + R_i (c_t', n_t') \right) \right) \]

\[ + \beta \left( 1 - \gamma_D T_i \right) W_{i+1} \]

\[ + \lambda_i \left( W_i \left( S_i n_t' + I_i n_t' + R_i n_t' \right) + \frac{r_{t+1} B_i}{1 + \nu_i + \nu} - (1 + \mu_i) \left( S_i c_t' + I_i c_t' + R_i c_t' \right) - B_{i+1} \right) \]

\[ + \theta_i \left[ T_i - I_i S_i \left( \gamma_r + \gamma_C (S_i c_t') (I_i c_t') + \gamma_N (S_i n_t') (I_i n_t') \right) \right] \]

Note that as Eichenbaum et al. [2022], each type of agent considers probabilities, as well as aggregate variables \( C_t' \) and \( N_t' \) in the virus transmission function.

The aggregation yields:

\[ C_t = S_i C_t' + I_i C_t' + R_i C_t' \]

\[ N_t = S_i N_t' + I_i N_t' + R_i N_t' \]

The first order conditions of our optimization problem reads as follows:

\[ c_t': e^{t_i} \frac{\partial u(c_t', n_t')}{\partial c_t'} - \theta_i \gamma_C I_t c_t' - (1 + \mu_i) \lambda_i = 0 \]

\[ c_t': e^{t_i} \frac{\partial u(c_t', n_t')}{\partial c_t'} - (1 + \mu_i) \lambda_i = 0 \text{ for } j = \{i, r\} \]

\[ n_t': e^{t_i} \frac{\partial u(c_t', n_t')}{\partial n_t'} - \theta_i \gamma_N I_t n_t' - \lambda_i W_t = 0 \]

\[ n_t': e^{t_i} \frac{\partial u(c_t', n_t')}{\partial n_t'} - \lambda_i W_t = 0 \text{ for } j = \{i, r\} \]

\[ T_i: -\beta \gamma_D T_i W_{i+1} + \theta_i = 0 \]

\[ B_{i+1}: \beta \left( 1 - \gamma_D T_i \right) \frac{\partial W_{i+1}}{\partial B_{i+1}} - \lambda_i = 0 \]

We next report in Figure 14 the system response to an epidemic outbreak of the estimated model with imperfect risk sharing across family members. As Eichenbaum et al. [2022], infected as well as recovered exhibit the same first order conditions, and therefore the same consumption and labor supply. Because the risk of catching the virus rises, susceptible households strongly reduce their consumption, as it is the main vector of the disease. As the central bank dampens the recession with low interest rates, other types of households
experiences a boost in their willingness to consume. In contrast, hours dynamics are all quite similar across the three types of households because the estimated share of transmission $v$ is mainly consumption. The rise in the willingness to consume reduces the wealth effects. In response, the households are less willing to offer hours to the producing sector.

Figure 14: System response to an epidemic outbreak under a Taylor rule with imperfect risk sharing

Figure 15 compares the path of consumption with perfect consumption insurance and without. The main major difference concerns aggregate consumption. This difference mainly originates from contagion risk that is drastically different with and without consumption insurance. In presence of consumption insurance, contagion risk is calculated on aggregate consumption, $\varrho_c C_t$. Therefore during an outbreak, aggregate consumption $C_t$ falls which reduces the contribution of contagion risk on consumption. In contrast when susceptible households exhibit imperfect risk sharing, contagion risk is computed based on consumption of infected agents, $\varrho_c I_t c_i t$. During an outbreak, as consumption of infected rises $c_i t$, it increases drastically the contribution of contagion risk, and enlarges the magnitude of the recession. Note that the conduct of monetary policy is not much affected under this scenario, because much of transmission of the COVID is addressed by consumption (as a result of high elasticity $v$). An estimation of this imperfect consumption model would therefore yield different inferred parameter values, with respect to the ones presented earlier. We notably anticipate that the relative contribution of consumption on the reproduction number would substantially lower in order to get more realistic fluctuations in aggregate demand during the outbreak. This change would however implies modest changes on nominal fluctuations, and therefore would yield minor changes on the main message of the paper with respect to the optimal conduct of monetary policy.
Figure 15: System response to an epidemic outbreak under a Taylor rule with and without imperfect risk sharing.
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