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Government loan guarantees, market liquidity, and lending standards

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Abstract

We study third-party loan guarantees in a model in which lenders can screen, learn loan quality over time and can sell loans before maturity when in need of liquidity. Loan guarantees improve market liquidity and reduce lending standards, with a positive overall welfare effect. Guarantees improve the average quality of non-guaranteed loans traded and thus the market liquidity of these loans due to both selection and commitment. Because of this positive pecuniary externality, guarantees are insufficient and should be subsidized. Our results contribute to a debate about reforming government-sponsored mortgage guarantees by Fannie Mae and Freddie Mac.

Keywords: Mortgage guarantees, adverse selection, market liquidity, pecuniary externality, Pigouvian subsidy, Government Sponsored Enterprises.

JEL classifications: G01, G21, G28.
Non-Technical Summary

Risk in credit markets is often assumed at loan origination by third parties in return for a fee. Insurance, various guarantees, and external credit enhancements protect the owner of the loan against borrower default. Governments are often involved in those contracts by subsidizing default guarantees for various types of loans, including mortgages, student loans, and small business loans. The quantitatively most important example are government mortgage guarantees. In 2018, the U.S. government guaranteed 62% of outstanding residential mortgages (equal to 32% of GDP) via institutions such as Fannie Mae and Freddie Mac, which are known as Government Sponsored Enterprises (GSEs).

Mortgage guarantees were traditionally viewed as a way to promote homeownership, but the financial crisis of 2007-09 sparked their criticism and led to calls for reform of GSEs by academics and policymakers. We contribute to this debate by identifying a new benefit of third-party loan repayment guarantees. We show how loan guarantees create a positive externality on the liquidity in secondary markets for non-guaranteed loans, resulting in insufficient guarantees. This externality provides new economic rationale for government loan guarantee subsidies and is consistent with evidence from U.S. mortgage markets.

To illustrate the mechanism, we introduce loan guarantees into a parsimonious model of lending where screening of borrowers at loan origination raises lending standards and liquidity shocked lenders want to sell loans in secondary market subject to adverse selection. The model features a trade-off between productive and allocative efficiency. Loan guarantees improves allocative efficiency by creating information-insensitive and thus liquid guaranteed loans but more importantly also by improving the quality of non-guaranteed loans traded and thus their liquidity. The latter externality arises because of self-selection of high-cost lenders into guarantees and a guarantee-induced commitment to not exploiting future private information about loan quality. Since lenders do not internalize the positive externality of insurance on the liquidity in the market for uninsured loans, there is insufficient loan insurance in
equilibrium. An uninformed regulator can implement planner’s choice of loan guarantees with a loan guarantee subsidy. We interpret this subsidy as government backing and subsidized guarantees in credit markets (e.g., via GSEs).
1 Introduction

Default risk in credit markets is often assumed upon origination by third parties for a fee. Governments are often involved in those contracts by subsidising default guarantees for various types of loans, including student loans, small business loans, export loans, and mortgages. The quantitatively most important example are government mortgage guarantees. In 2018, the U.S. government guaranteed 62% of outstanding residential mortgages (equal to 32% of GDP) via institutions such as Fannie Mae and Freddie Mac, which are known as Government Sponsored Enterprises (GSEs).

Mortgage guarantees were traditionally viewed as a way to promote homeownership, but the financial crisis of 2007-09 sparked their criticism and led to calls for reform of GSEs by academics and policymakers.\textsuperscript{1} We contribute to this debate by identifying a new benefit of third-party loan repayment guarantees—henceforth loan guarantees for short. We show how loan guarantees create a positive externality on the liquidity in secondary markets for non-guaranteed loans, resulting in insufficient guarantees. This externality provides an economic rationale for government loan guarantee subsidies and is consistent with evidence from U.S. mortgage markets.

To illustrate the mechanism, we introduce loan guarantees into a parsimonious model of lending with a tradeoff between productive and allocative efficiency, as described in Section 2. We start with a benchmark model without loan guarantees. All agents are risk-neutral to highlight the effects of guarantees beyond a well-known risk-sharing motive. At origination each lender has access to a pool of borrowers and can screen at a heterogeneous cost. Screening improves the probability of repayment by identifying a borrower with a low default probability, raising lending standards. Lenders who are subject to a liquidity shock want to sell their loans in the secondary market to outside financiers before maturity. But this market is subject to a standard adverse selection problem since lenders privately learn loan quality over time.

In competitive equilibrium, only lenders with screening costs below a threshold (labelled low-cost lenders) choose to screen. The screening choice determines produc-
\textsuperscript{1}Congressional Budget Office (2014) states proposals for GSE reform considered by policymakers.
tive efficiency—the average quality of loans originated net of screening costs. Adverse selection in the secondary loan market reduces the social gains from trade (allocative efficiency) and results in multiple equilibria. For clarity of exposition, we focus in the main text on the liquid equilibrium, which exists when high-quality loans are sold upon a liquidity shock. This equilibrium features a trade-off between allocative and productive efficiency: when more lenders choose to screen, the secondary market price is higher but a higher price, in turn, reduces the incentives of lenders to screen.

Our main innovation is to introduce loan guarantees upon origination and to study its effects on lending standards and market liquidity as well as its normative implications. Loan guarantees pass default risk to financiers for a competitive fee—before lenders privately learn loan quality. Since guarantees are backed by the government, we abstract from default risk of the guarantor. Consistent with our application to government-backed mortgage guarantees, whether a loan is guaranteed is observable and the loan trades together with its guarantee in a market for guaranteed loans. One implication is that guaranteed loans are insensitive to future private information about loan quality. Another implication is the segmentation of secondary markets into guaranteed and non-guaranteed loans, consistent with the existence of separate markets for government-backed securities and private-label securities, such as agency mortgage-backed securities (agency MBS) and private-label MBS.

In equilibrium described in Section 3, only high-cost lenders (who do not screen) buy guarantees. Intuitively, a guarantee prevents the lender from reaping the benefits of costly unobserved screening. Guarantee usage improves allocative efficiency in two dimensions. First, loan guarantees create a safe, information-insensitive and thus always liquid asset. Second, and more importantly, loan guarantees also improve the

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2One interpretation of the choice to buy a guarantee on a loan is that lenders select into different parts of the market where third-party guarantees are available or required. In the United States, eligibility for GSE guarantees requires a maximum size of the mortgage (the conforming limit). Consistent with this timing, a popular business model is to specialize in origination of conforming loans, followed by the immediate sale to GSEs, which provide a non-default guarantee for further trading in secondary markets (Hurst et al., 2016; Buchak et al., 2018).

4This feature is a key difference to contracts like CDS that lack the same positive externality.

5This information-insensitivity is more robust to alternative sources of private learning than in the case of outright sale without a third-party guarantee. In fact, outright loan sales under learning-by-holding exacerbate adverse selection and should therefore be taxed. See Section 6.3 for details.

6Hence, guaranteed loans fetch a higher price than non-guaranteed loans due to adverse selection.
quality of non-guaranteed loans traded. This result is the main positive contribution of the paper. It arises because of self-selection of high-cost lenders into guarantees and a guarantee-induced commitment to not exploiting future private information about loan quality. First, lenders with guarantees do not screen and thus have more lemons on their balance sheets than lenders without guarantees (some of whom choose to screen). Second, by buying a guarantee before privately learning loan quality, lenders forgo the private benefit of selectively selling lemons. The resulting higher average quality of loans traded in the non-guaranteed market increases its price and allocative efficiency. It also reduces screening incentives but overall efficiency improves.

Turning to normative implications in Section 4, we consider a planner who chooses loan guarantees for all lenders. The planner internalizes the positive externality of guarantees on the price of non-guaranteed loans and chooses more guarantees at both the intensive and the extensive margin. A regulator subject to a balanced budget and with no information advantage over financiers can achieve the planner’s allocation via a subsidy on loan guarantees. We interpret this loan guarantee subsidy as government backing and subsidized guarantees in credit markets (e.g., via GSEs).

Our results contribute to a debate about the design of government-backed mortgage guarantees after the Great Financial Crisis. Our model implies that subsidies on mortgage guarantees should occur for loans with low observable default risk (e.g., borrowers with sufficiently high credit scores—consistent with the practices of GSEs), when screening costs are higher, and for loans with lower payoffs. When lower payoffs are interpreted as more competitive lending markets, then subsidies should occur in countries with a less concentrated lending market, e.g. more in the U.S. than in Canada, or more after recent increase of competition from specialized online lenders (e.g., by FinTechs). We discuss further implications of the model in Section 4.3.

In Section 5 we review empirical evidence from the U.S. residential mortgage market that support our key result—the positive impact of loan guarantees on the market of non-guaranteed loans—as well as key ingredients of our mechanism, especially in the latter market. Consistent with this differential pricing implication, agency MBS maintained robust issuance and trading volumes as well as low spreads compared to private-label MBS even during the recent financial crisis (e.g., Vickery and Wright 2013; Loutskina and Strahan 2009).
cially the self-selection of high-cost lenders into guarantees and the substitution of non-guaranteed lending by guaranteed lending in response to GSE subsidies.

We probe the robustness of our results in Section 6. First, we introduce collateral and show that subsidies should be made for loans with high enough collateral, consistent with GSE requirements. Second, we study the illiquid equilibrium. We show that our mechanism naturally extends to the condition for the existence of the liquid equilibrium and implies an additional role for the planner in ‘liquifying’ the secondary market via more loan guarantees. Third, we contrast loan guarantees with loan sales upon origination and show that the former fares better when the sources of private information are uncertain. Fourth, the normative case for loan guarantee subsidies becomes even stronger in the presence of adverse selection in the loan guarantee market. Finally, other extensions in Appendix A study partial guarantee coverage, upfront payment of the guarantee fee, and partial loan sales. Our results are qualitatively unchanged in these alternative setups.

Literature. Our paper is related to three strands of literature. First, it is closely related to a literature on the interaction between productive efficiency and allocative efficiency. The market liquidity of a loan affects a lender’s incentive to screen or monitor borrowers (e.g., Pennacchi 1988; Gorton and Pennacchi 1995; Parlour and Plantin 2008; Parlour and Winton 2013; Chemla and Hennesey 2014; Vanasco 2017; Daley et al. 2020). Our contribution is to examine the implications of loan guarantees for productive and allocative efficiency.

Second, our paper is related to a literature on the role of government in the residential mortgage market, GSEs, and the debate about GSE reform. On the supporting side of this debate, Frame and White (2005) suggest that government subsidy to GSEs could be motivated by positive externalities of homeownership (Green and White, 1997; DiPasquale and Glaeser, 1999),7 willingness to redistribute income to

\footnote{Despite the focus on homeownership accentuated by an requirements for minimum GSE activity in disadvantaged areas, the evidence suggests that GSE activities had no or little impact on homeownership and on access to credit in disadvantaged areas (Boatic and Gabriél, 2006; Grundl and Kim, 2021; Painter and Redfearn, 2002). This is because eligibility for GSE guarantee is very broad-based (Frame and White, 2005) and minimum borrower requirements are in general enforced.}
lower income households, make fixed-rate mortgages with long-maturities more available (Fuster and Vickery, 2015), or maintaining the flow of new mortgage credit during periods of financial stress (Rappaport, 2020). Additionally, subsidy can be motivated by positive information externalities of GSEs lending in areas with low property transaction volume on future (non-guaranteed) lending (Lang and Nakamura, 1990, 1993; Ling and Wachter, 1998; Harrison et al., 2002). On the critical side of this debate, recent papers suggested that GSE subsidies reduce welfare because they may lead to more frequent financial crises (Elenev et al., 2016), are regressive (Jeske et al., 2013), and redistribute across regions (Hurst et al., 2016). We find a complementary and positive effect of loan guarantees via secondary market liquidity.

Third, our paper relates to a literature on guarantee provision in lending under asymmetric information. A part of this literature suggest that government loan guarantees can be welfare improving in the setup of Stiglitz and Weiss (1981) with underinvestment due to adverse selection (e.g. Mankiw 1986) and suggest that guarantees lower non-guaranteed loan issuance (e.g. Gale 1990, 1991).\textsuperscript{8} These studies focus on lending in primary markets subject to credit rationing and redlining where government subsidies crowd in subsidised borrowers but crowds out borrowers that do not receive subsidies (a negative externality). Our focus is instead on how loan guarantees affect the liquidity in secondary markets and we highlight a positive externality of guarantees on the liquidity of non-guaranteed loans. Some other literature focuses on the provision of guarantees by the informed party: in the primary market (via collateral by borrower, e.g. Bester 1985, Besanko and Thakor 1987) or in the secondary market (via credit enhancement by loan sellers, e.g. Pennacchi 1988, DeMarzo 2005). These guarantees represent risk retention by the informed party, can signal high quality, and lead to a separating equilibrium or make loans information insensitive and thus liquid.\textsuperscript{9} The externality of guarantees mostly take the form of credit rationing or higher costs for borrowers without guarantees or lower liquidity of loans without guarantees. In contrast, we study third-party loan guarantees.

\textsuperscript{8}Evidence on the guarantee externality to non-guaranteed lending is mixed (e.g., a negative effect on non-guaranteed lending in Ono et al. 2013 and positive effect in Wilcox and Yasuda 2019).

\textsuperscript{9}The notion of information insensitivity goes back to Gorton and Pennacchi (1990). A recent paper includes Dang et al. (2017), where keeping loan information secret supports market liquidity.
2 Model

There are three dates $t = 0, 1, 2$ and one good for consumption and investment. Two groups of risk-neutral agents—financiers and lenders—are protected by limited liability. Outside financiers are competitive, deep-pocketed at $t = 1, 2$, and require a gross return normalized to one. Each lender has one unit of funds at $t = 0$ to originate a loan (a mortgage) and has access to an individual pool of borrowers.

Without screening, $s_i = 0$, lender $i$ finds an average borrower at $t = 0$ and receives $A$ (repayment) with probability $\mu \in (0, 1)$ or 0 (default) at $t = 2$. The loan payoff $A_i \in \{0, A\}$ is independently and identically distributed across lenders $i \in [0, 1]$ and publicly observable at $t = 2$. A higher payoff $A$ reflects more profitable lending opportunities, a less competitive lending market, or a lower bargaining power of borrowers. The repayment probability $\mu$ reflects any publicly available information about the quality of non-screened loans, such as a borrower credit score or regional labor and housing market characteristics. Screening, $s_i = 1$, improves the repayment probability to $\psi \in (\mu, 1)$, as shown in Figure 1. A heterogeneous non-pecuniary cost of screening $\eta_i$ reflects differences in lender types (e.g., traditional versus online lenders) or in screening ability (e.g., because of pre-existing relationships with a borrower). Its density $f > 0$ has support $[0, \eta_i]$, and $F$ is the cumulative distribution.

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Figure 1: Screening improves the probability of repayment.

Evidence consistent with this assumption includes Berger and Udell (2004), who show a positive association between screening and loan quality in a sample of US banks, and Pierri and Timmer (2020) who find that US banks that had invested more in screening, measured by IT adoption, originated mortgages that performed better in a crisis. Loutskina and Strahan (2011) find that geographically concentrated mortgage lenders invest more in information collection (screening technology), which reduces loan losses and improves bank profits.

If screening costs were homogeneous, all lenders would be indifferent about screening in equilibrium. All of our results qualitatively carry over to this alternative setup as long as lenders share a common pool of borrowers and the screening cost increases (or the probability of finding a good loan decreases) in the aggregate share of lenders who screen (known as the thinning effect of screening).
At $t = 1$, lenders receive two pieces of private information. First, each lender learns the future loan payoff $A_i$. This assumption is consistent with (i) relationship lending and (ii) learning-by-holding an asset (Plantin, 2009). Second, each lender learns the realization of an idiosyncratic liquidity shock $\lambda_i$, whereby the preference for interim consumption is $\lambda_i \in \{1, \lambda\}$ with $\lambda > 1$. Our reduced-form modelling of the gains from a loan sale before maturity captures consumption needs or superior non-contractible investment opportunities (see, e.g. Aghion et al., 2004; Holmstrom and Tirole, 2011; Vanasco, 2017). The liquidity shock $\lambda_i$ is i.i.d. across lenders, independent of the loan payoff, and arises with probability $P\{\lambda_i = \lambda\} \equiv \nu \in (0, 1)$.

The utility of lender $i$ is $u_i = \lambda_i c_{i1} + c_{i2} - \eta_i s_i$, where $c_{it}$ is her consumption at date $t$.

At $t = 0$, each lender chooses whether to buy a guarantee for the loan against default, $\ell_i \in \{0, 1\}$. We focus on a full guarantee without loss of generality. This guarantee ensures the payoff $A$ to the owner of the loan for a fee $k$. Both the guarantee payoff and the fee are charged at $t = 2$, resulting in a safe payoff $\pi = A - k$. Effectively, the guarantee swaps a loan’s risky payoff $A$ for a safe payoff $\pi$.

At $t = 1$, a lender can sell the loan in secondary markets to financiers who are uninformed about the screening cost $\eta_i$ and choice $s_i$, liquidity shock $\lambda_i$, and loan quality $A_i$. Consistent with our focus on mortgage guarantees, financiers observe whether a loan is guaranteed $\ell_i$ and a guaranteed loan is sold together with its guarantee. Thus, segmented markets for guaranteed ($G$) and non-guaranteed ($N$) loans exist with respective prices $p_G$ and $p_N$ and sale choices $q_G^i \in \{0, \ell_i\}$ and $q_N^i \in \{0, 1 - \ell_i\}$.

No guarantees. Consider a benchmark in which loan guarantees are unavailable. We focus on key economic forces and relegate details and proofs to Appendix B.1.

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12 The assumption that banks acquire private information about borrowers during the lending relationship is consistent with evidence in e.g. James (1987), Lummer and McConnell (1989), Slovin et al. (1993), Agarwal and Hauswald (2010), Norden and Weber (2010), Botsch and Vanasco (2019).

13 We show in Appendix A.1 that partial guarantees are neither privately nor socially optimal.

14 This approach parallels the non-pecuniary screening cost in that it does not affect lending volume at $t = 0$. In Appendix A.2, we show that our results are qualitatively unchanged in an alternative setup in which a guarantee fee must be paid up front.

15 We allow for partial sales in Appendix A.3 and show that our results are qualitatively unchanged.
Asymmetric information between lenders and financiers at $t = 1$ implies a standard adverse selection problem and multiplicity of equilibria. All low-quality loans (lemons) are always sold and a defining feature of equilibrium is whether lenders sell high-quality loans upon a liquidity shock, $\lambda p_N \geq A$. If this condition holds, the market for loans is liquid (liquid equilibrium). To illustrate our main mechanism, we focus on the liquid equilibrium throughout the main text. That is, we focus on a range of parameters, $\lambda > \lambda_L$, for which the liquid equilibrium exists.\footnote{The bound $\lambda_L$ is defined in Appendix B.1. We study an illiquid equilibrium with $p_N = 0$ in Section 6.2. Our mechanism and both positive and normative results naturally extend to the condition for the existence of liquid equilibrium. We show that an additional role arises for the planner to ‘liquify’ the secondary market by using more loan guarantees. See Section 6.2 for details.}

Lenders use a threshold strategy for their screening choice: each lender with a screening cost below some threshold $\eta$ screens. This threshold affects productive efficiency—the average quality of loans originated net of screening costs. We refer to lenders with screening costs below (above) the threshold as low-cost (high-cost) lenders. A marginal lender, $\eta_i = \eta$, is indifferent about screening, which results in the threshold\footnote{The expected payoff of screening is $\nu \lambda p_N + (1 - \nu)\psi A + (1 - \psi)p_N - \eta$ and the expected payoff of not screening is $\nu \lambda p_N + (1 - \nu)[\mu A + (1 - \mu)p_N]$. Equating both payoffs yields the threshold cost.}

$$\eta = (1 - \nu)(\psi - \mu)(A - p_N). \quad (1)$$

Intuitively, the screening benefit (the right-hand side of Equation 1) arises in the absence of liquidity shock when low-costs lenders do not sell all loans (with probability $1 - \nu$) and from the higher probability of originating a high-quality loan, $\psi - \mu$, and keeping it to maturity rather than selling a lemon, $A - p_N$. A higher secondary market price lowers screening, at origination $\frac{d\eta}{dp_N} < 0$, due to the option to sell lemons.
The competitive price of loans in the secondary market reflects the average quality of traded loans that is given by the value of high-quality loans sold divided by total loans sold. Liquidity-shocked lenders sell all loans $\nu$ (some of which are high-quality), while lenders without a shock, $1 - \nu$, only sell lemons:

$$p_N = \frac{\nu [\psi F + \mu (1 - F)]}{\nu + (1 - \nu) [\psi (1 - F) + \mu (1 - F) (1 - F)]}, \quad (2)$$

where $F(\eta)$ is the equilibrium share of low-cost lenders. Equation 2 clarifies that screening supports the price, $\frac{dp}{d\eta} > 0$. More screening leads to fewer low-quality loans originated at $t = 0$, which improves the quality of loans traded at $t = 1$.

We define allocative efficiency as the social gains from trade, $\nu (\lambda - 1) p_N$, that arise from the redistribution of funds across financiers and lenders at $t = 1$. These gains are proportional to the difference in interim utilities between lenders with and without liquidity shock, $\lambda - 1$, and the market value $p_N$ of loans sold by liquidity-shocked lenders of quantity $\nu$. Thus, allocative efficiency increases in the price. Liquidity-shocked lenders sell both high- and low-quality loans for a depressed price due to the pooling with lenders without a shock, who sell lemons only. This pooling effectively redistributes resources from liquidity-shocked lenders with high interim utility $\lambda$ to lenders without a shock with lower utility, reducing the gains form trade.

In sum, the equilibrium $(\eta^*, p_N^*)$ is given by Equations (1) and (2). Because of the two-way feedback between the screening threshold and the secondary market price, there is a trade-off between allocative and productive efficiency.

3 Equilibrium

We characterize the equilibrium when loan guarantees are available. The formal definition of equilibrium is in Appendix B.2. Our first result describes under which conditions guarantees upon loan origination are used and by which type of lenders.\footnote{We henceforth focus on a sufficiently productive screening technology, $\psi > \psi$. (This bound is defined in Appendix B.2.) This assumption simplifies the analysis because it rules out a corner solution in which all high-cost lenders buy guarantees, $m = 1$. This case is arguably counterfactual.}
Proposition 1. **Equilibrium.** Low-cost lenders \((\eta_i \leq \eta^*)\) screen and do not buy a guarantee: \(s^*_i = 1\) and \(\ell^*_i = 0\). Thus, the guarantee fee is \(k^* = (1 - \mu)A\), resulting in a safe payoff and a secondary market price of guaranteed loans of \(\pi^* = \mu A = p^*_G\).

The share of high-cost lenders \((\eta_i > \eta^*)\) who buy guarantees is \(m^* < 1\). There exists a unique bound \(\tilde{\mu}_G\). If \(\mu > \tilde{\mu}_G\), some high-cost lenders purchase a guarantee, \(m^* > 0\). Otherwise, loan guarantees are not purchased, \(m^* = 0\).

**Proof.** See Appendix B.2 (which also defines the bound \(\tilde{\mu}_G\), characterizes the equilibrium allocation, and states comparative statics).

In equilibrium, when lenders buy guarantees, they do not screen and when lenders screen, they do not buy guarantees. Guarantees convert the risky loan payoff \(A_i\) into a safe payoff \(\pi\) that is independent of the screening choice. Thus, a lender with a guarantee does not screen because it would not benefit from costly but unobservable screening. Hence, the competitive guarantee fee is the expected cost of guaranteeing the repayment of non-screened loans, \((1 - \mu)A\). For such a high fee, though, no lender who screens chooses to buy a guarantee. Because of this selection by screening cost, buying a loan guarantee reveals that the lender does not screen.

Since a guaranteed loan is sold together with its guarantee at \(t = 1\), guaranteed loans are free from default risk. Thus, there is no adverse selection in the secondary market for guaranteed loans and these loans are always liquid. These results arise because lenders do not yet know loan quality \(A_i\) when buying guarantees at \(t = 0\). The competitive price of guaranteed loans at \(t = 1\) equals its guaranteed payoff net of the guarantee fee at \(t = 2\), \(p^*_G = \pi^* = A - k^* = \mu A\). When loan guarantees are bought, \(m^* > 0\), high-cost lenders are indifferent about them: 19

\[\nu \lambda (p^*_G - p^*_N) = (1 - \nu) (1 - \mu) p^*_N.\]  

(3)

19With a guarantee, a high-cost lender prefers selling the guaranteed loan after the liquidity shock at \(t = 1\) at price \(p^*_G\) and is indifferent about a sale without shock because \(p^*_N = \pi^*\). Thus, the expected payoff from buying guarantees is \(\nu \lambda p^*_G\), where \(\kappa = \nu \lambda + 1 - \nu < \lambda\) is the average interim utility of consumption. Without guarantee, a high-cost lender sells the non-guaranteed loan after a shock at price \(p^*_N\). Without a shock, the loan is also sold if a lemon, else it is kept until maturity. Thus, the expected payoff of a high-cost lender who does not buy a guarantee is \(\nu \lambda p^*_N + (1 - \nu) \mu A + (1 - \mu) p^*_N\). Equating both payoffs yields the indifference condition for loan guarantees.
This indifference condition has an intuitive interpretation. Its left-hand side is the private benefit of guarantees: the gain of selling the loan after a liquidity shock at a higher price $p^*_G > p^*_N$ (because adverse selection only occurs in the market for non-guaranteed loans). Consistent evidence includes that agency MBS had lower spreads than private-label MBS (Vickery and Wright, 2013; Loutskina and Strahan, 2009).

The right-hand side is the private cost of guarantees: losing the option to sell lemons in the non-guaranteed market without a liquidity shock.

Our main positive result in the equilibrium with guarantee usage follows.

**Proposition 2. Pecuniary externality.** Consider $\mu > \bar{\mu}_G$. Loan guarantees increase the price of non-guaranteed loans and lower screening.

**Proof.** See Appendix B.2. ■

A critical mechanism of our paper is how loan guarantees affect the quality of non-guaranteed loans traded and is shown in Figure 3. Panel (a) shows loan sales without guarantees. The loan payoff $A_i$ depends on the screening choice $s_i$. The area shaded in blue horizontal lines shows non-guaranteed loans traded, which depends on the liquidity shock $\lambda_i$ and the private information about $A_i$. Panel (b) shows the impact of loan guarantees. Some high-quality and low-quality loans are removed from the market of non-guaranteed loans—shaded in red grid and green vertical lines—and trade in a separate market for guaranteed loans.

Loan guarantees improve the average quality of non-guaranteed loans traded due to the selection of high-cost lenders into guarantees and the commitment not to selectively sell lemons. First, since high-cost lenders self-select into guarantees, guarantees remove lenders with lower average portfolio quality from the market, compared to remaining lenders without guarantees because some of whom screen. Second, by buying a guarantee at $t = 0$ lenders forgo the option to selectively sell lemons at $t = 1$. That is, buying a guarantee commits a lender to not exploiting future private information about loan quality without a liquidity shock. As a result

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Without self-selection, a randomly allocated guarantee would remove a random fraction of loans from the market for non-guaranteed loans—with no effect on the average quality of loans traded.
Figure 3: Loan guarantees improve the quality of non-guaranteed loans traded. Loan guarantees remove loans from the non-guaranteed market that are of below-average quality.
of loan guarantees, lemons of high-cost lenders without liquidity shock of quantity 
\((1 - \nu)(1 - \mu)(1 - F)m\) — shaded in red grid — and loans of high-cost lenders with shock of quantity \(\nu(1 - F)m\) — shaded in green vertical lines — are removed from the non-guaranteed loans market.

These effects of guarantees and the resulting improvement in the quality of non-guaranteed loans traded is reflected in a modified competitive market price:

\[
p_N = \frac{\nu(\psi F + \mu(1 - F)(1 - m))A}{\nu(1 - (1 - F)m) + (1 - \nu)[(1 - \psi)F + (1 - \mu)(1 - F)(1 - m)]} \quad (2')
\]

While both selection and commitment increase the price, it is their interaction that increases allocative efficiency: the commitment of high-cost lenders not to sell lemons when not facing a liquidity shock reduces the inefficient redistribution from liquidity-shocked lenders to lenders without shock caused by adverse selection.\(^{21}\)

To summarize, guarantees are used for \(\mu > \tilde{\mu}_G\) and the equilibrium allocation \((\eta^*, p_N^*, m^*)\) is defined by Equations (1), (2'), and (3). Condition (3) pins down the price \(p_N^*\) at which high-cost lenders are indifferent about guarantees, which is higher than in the benchmark without guarantees. The screening-indifference condition (1) determines the screening threshold \(\eta^*\), which is lower than in the benchmark. Thus, guarantees lower screening. Finally, the competitive price of non-guaranteed loans (2') determines the share of high-cost lenders who buy guarantees \(m^*\). Since both guarantees and screening increase the price \(p_N^*\), they behave as substitutes in many comparative statics, reviewed in Appendix B.2. For \(\mu \leq \tilde{\mu}_G\), by contrast, guarantees are not used, \(m^* = 0\), and \((p_N^*, \eta^*)\) are determined by (1) and (2), as in the benchmark.

\(\text{To see this, we use (2') to decompose the gains from trade, } \nu(\lambda - 1)[\gamma(1 - (1 - F)m) + p_G(1 - F)m], \text{ which reflect that the market value of sold loans depends on the prices in both secondary markets and the respective loan sale shares:}

\[
(\lambda - 1) \left\{ \frac{\nu(\psi F + \mu(1 - F))A}{\nu(1 - (1 - F)m) + (1 - \nu)[(1 - \psi)F + (1 - \mu)(1 - F)(1 - m)]} \right\},
\]

Fundamental value of sold loans  \quad Funds diverted by sellers of lemons without liquidity shock

where the benefit of guarantees for the gains from trade is highlighted.
4 Welfare and regulation

We turn to normative implications of loan guarantees. We first characterize a welfare benchmark in which a constrained planner internalizes the pecuniary externality of loan guarantees. We then show that an uninformed regulator subject to a balanced budget can achieve this benchmark via a Pigouvian subsidy on loan guarantees. Finally, we describe implications for government-sponsored mortgage guarantees.

4.1 A welfare benchmark

We consider a constrained planner $P$ who maximizes utilitarian welfare $W$. To highlight the effects of loan guarantees, we let the planner choose loan guarantees for all lenders, based on observing lender screening costs $\eta_i$. To keep the focus on the loan guarantee externality, we preserve the friction of asymmetric information and adverse selection in the secondary market for non-guaranteed loans. Thus, the planner is subject to the privately optimal choices of loan sales and screening. In sum, the planner who internalizes the benefit of loan guarantees on secondary market liquidity solves

$$\max_m W \equiv \max_m \left[ \nu(\lambda - 1)\left(p_N + (p_G - p_N)(1 - F(\eta))m\right) + \left[\psi F(\eta) + \mu(1 - F(\eta))\right]A - \int_0^\eta \bar{\eta} dF(\bar{\eta}) \right]$$

s.t. (1), (2'), and $p_G = \mu A$.

Welfare is the sum of expected payoffs of lenders (up to a constant for the expected payoff of financiers) and is derived in Appendix B.3. It comprises terms associated with productive efficiency and allocative efficiency. Productive efficiency refers to the average quality of loans originated (the fundamental value) net of total screening costs. Allocative efficiency refers to the social gains from trade. As for the market value of

Choosing the proportion of high-cost lenders who buy a guarantee, $m$, is equivalent to choosing loan guarantee for each lender, $\{\ell_i\}$. A lender required by the planner to buy guarantee chooses not to screen. Since the benefit of buying a guarantee for market liquidity is the same across lenders but the cost of screening is heterogeneous, the regulator targets guarantees to high-cost lenders.
loans sold by liquidity-shocked lenders, guaranteed loans (a share \(m(1 - F)\)) are a safe and liquid asset and fetch a higher price than non-guaranteed loans, \(p_G > p_N\).

Proposition 3 summarizes the planner’s allocation.

**Proposition 3. Welfare.** There exists a unique bound \(\mu^{PG}\), where \(\mu^{PG} < \tilde{\mu}_G\). For \(\mu > \mu^{PG}\) the planner chooses more loan guarantees, \(m^P > m^*\), to improve allocative efficiency, \(p^G > p^*_N\), at the expense of productive efficiency, \(\eta^P < \eta^*\).

**Proof.** See Appendix B.3 (which also defines the bound \(\mu^{PG}\)).

The planner’s choice of loan guarantees balances the marginal social benefits of higher allocative efficiency with the marginal social costs of lower productive efficiency. Since the planner internalizes the pecuniary externality, it chooses more guarantees to improve allocative efficiency. While productive efficiency is lowered because a higher secondary market price of non-guaranteed loans reduces screening at origination, there is an overall improvement in efficiency. The planner buys more guarantees when lenders also buy guarantees in the unregulated economy \((m^P > m^* > 0\) for \(\tilde{\mu}_G < \mu\), and when lenders do not \((m^P > m^* = 0\) for \(\mu^{PG} < \mu \leq \tilde{\mu}_G\)). For low repayment probabilities of non-screened loans, \(\mu \leq \mu^{PG}\), however, the implications for productive efficiency would be so severe that the planner refrains from guaranteeing any loans.

### 4.2 Regulation

We consider a regulator \(R\) subject to a balanced-budget constraint and with the same information as outside financiers. As a result, a direct implementation of the planner’s allocation by choosing guarantees for each lender is infeasible. Only high-cost lenders should buy guarantees but lender screening costs are unobserved by the regulator.

We show that the regulator can achieve the welfare benchmark with a subsidy \(b_G \geq 0\) at \(t = 2\) to lenders who buy a loan guarantee at \(t = 0\). The regulator has commitment, announces \(b_G\) at the beginning of \(t = 0\), and lenders make their privately optimal choices of loan guarantee and screening at \(t = 0\) and of loan sales at \(t = 1\).
We refer to this arrangement as the regulated economy. The subsidy is funded by a lump-sum tax $T$ on lenders at $t = 2$. To ensure that lenders can always pay the tax (and to avoid technical complications associated with limited liability), we introduce an additional non-pledgeable endowment $n$ received when taxes are due. Thus, these resources can be used to pay taxes or for consumption at $t = 2$. Figure 4 shows the timeline of the regulated economy. Its equilibrium is defined in Appendix B.4.

![Figure 4: Timeline of the regulated economy.](image)

When loan guarantees are used in the regulated economy, the indifference condition about guarantees, Equation (3), generalizes to

$$
\nu \lambda (p_G - p_N) + b_G = (1 - \nu) (1 - \mu) p_N. \tag{3'}
$$

The subsidy $b_G$ increases the incentives to buy guarantees and the share of guaranteed loans $m$, which indirectly increases the price of non-guaranteed loans $p_N$.

Our main result on regulation and loan guarantee subsidies follows.

**Proposition 4. Loan guarantee subsidies.** For $\mu > \mu^*_G$, the regulator implements the welfare benchmark by subsidizing loan guarantees:

$$
b^*_G = \frac{(1 - \nu) (1 - \mu) p^*_N}{\text{Private cost of guarantee}} - \frac{\nu \lambda (p_G - p^*_G)}{\text{Private benefit of guarantee}}. \tag{5}
$$

**Proof.** See Appendix B.4. ■

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23 An endowment $n = (1 - \mu)A$ covers any meaningful regulation. If $b_G = (1 - \mu)A$, all lenders buy guarantees $m = 1$, no lender screens $\eta = 0$, and the maximum required revenue is $T = (1 - \mu)A$.

24 Inducing a strict preference for guarantees reduces screening and welfare and is not desirable.
The optimal guarantee subsidy implements the social choice of guarantees. The size of the subsidy incentivizes lenders who do not buy guarantees in the unregulated equilibrium because their private costs of guarantee exceed the private benefits.

Subsidies to the fee of a loan guarantee can be mapped to the guarantee fee of GSEs that does not fully reflect the total cost of the guarantee. For example, Congressional Budget Office (2014) argues Fannie Mae’s and Freddie Mac’s fees were lower than what competitive firms would charge for the same guarantee. And Passmore (2005) estimates the size of the government implicit subsidy to GSEs.

4.3 Implications for government-backed loan guarantees

The benefit of loan guarantees for market liquidity implies insufficient guarantees in the unregulated economy and motivates guarantee subsidies. The condition for the provision of guarantee subsidies in Proposition 4—a high enough loan repayment probability of non-screened loans, \( \mu > \mu^P_G \)—can also be expressed as a low enough payoff upon repayment, \( A < A^P_G \), or a distribution \( F(\eta) \) shifted sufficiently towards higher screening costs, as proven in Appendix B.3. Thus, our mechanism motivates subsidies in several economic situations.

First, guarantee subsidies should cover loans with a low observable default risk (high \( \mu \)). Recall that the repayment probability of non-screened loans \( \mu \) captures all publicly observable information that determines the repayment probability of a loan for which a lender wants to purchase a guarantee. The guarantor understands that such loans are not screened in equilibrium. In contrast, this hard information is less relevant for loans that were screened and thus not submitted for a guarantee purchase.\(^{25}\) The publicly observable information \( \mu \) can be specific to the borrower, loan or region. Accordingly, we interpret high values of \( \mu \) as borrowers with high credit scores or loans in regions with low predictable default risk (regions with strong labor

\(^{25}\)Hard information being less informative for screened loans is consistent with findings that loans of more concentrated lenders were more risky ex ante based on publicly observable information but recorded lower losses ex post (Loutskina and Strahan, 2011). The rates of Fintech shadow banks are less explained by standard observables relative to other shadow banks (Buchak et al., 2018), suggesting that FinTechs used information unavailable to other lenders as they may have screened.
markets, regions with stability or sustainable growth in house prices). Conditioning mortgage guarantees on high credit scores is consistent with the practices of GSEs in the US. However, government support for mortgage guarantees does not vary across regions despite large regional variation in predictable default risk (Hurst et al., 2016).

Second, guarantee subsidies should arise when loans are less profitable, borrowers have a lot of bargaining power, or lending markets are more competitive (low \( A \)). This implies that the benefits of loan guarantees are higher in countries with a less concentrated lending market and lower profit margin of lenders (e.g., the United States as opposed to Canada). Third, less guarantees is desirable when screening costs are lower (a shift in \( F \)). Recent technological advances and extensive data analysis of borrowers, such as big data or machine learning innovations (e.g., Fuster et al., 2019; Buchak et al., 2018), would reduce the benefits of guarantees.

Our model suggests that the recent competition from Fintech (e.g., specialized online lenders) have an ambiguous impact on subsidies to mortgage guarantees. On the one hand, Fintechs increase lending market competition that supports mortgage guarantee subsidies. On the other hand, technological advances introduced by Fintechs reduce the cost of screening that reduces the benefit of mortgage guarantees.

## 5 Empirical evidence and testable implications

In this section, we review several sets of evidence consistent with our main mechanism. We also state some testable implications of the model and discuss related evidence. Throughout we focus on the mortgage market in the United States. Recall that we interpret the choice to buy guarantees on a loan as a lender’s self-selection in the parts of the market where government-backed guarantees are available. (In the United States mortgages with size above a conforming limit (jumbo loans) cannot qualify for a guarantee even if they satisfy the quality requirements such as high credit score.)

\[26\] Furthermore, we introduce collateral, an additional observable characteristic of loans, in Section 6.1 and we link to the loan-to-value requirements of GSEs.
Our key result—a positive impact of loan guarantees on the market of non-guaranteed loans—is supported by empirical evidence. Naranjo and Toevs (2002) document that GSE activities have a positive spillover on non-conforming loans, i.e. loans that cannot qualify for GSE guarantees. In particular, GSE activities reduce the spread between non-conforming loan rates and Treasury rates. This result is consistent with higher liquidity of non-conforming loans.

Key ingredients of our mechanism are also supported by evidence. Our mechanism suggests that lenders with higher screening costs (or lower screening ability) are attracted by GSE subsidies. Several papers find that lenders with better screening technology rely less on GSE guarantees. Loutskina and Strahan (2011) find that more geographically concentrated lenders accept a higher proportion of ex-ante riskier mortgage applications measured by standard observable characteristics but record lower ex-post loan losses. This implies that these lenders have a better screening technology. Concentrated lenders focus on the information-sensitive jumbo market (non-conforming loans without GSE guarantees). Moreover, Buchak et al. (2018) show that, within the set of shadow banks, Fintech lenders rely less on standard hard information when setting interest rates, suggesting information unavailable to other lenders is used. We interpret this as a better screening technology. Fintech shadow banks are also less likely to sell loans to GSEs compared to other shadow banks.

A second key ingredient of our mechanism is that GSE subsidies induce some substitution of origination from non-guaranteed loans to guaranteed loans. Consistent with this, Gabriel and Rosenthal (2010) show that GSE activity crowds out mortgage purchases in the private secondary market without guarantees. Substitution effects are also consistent with findings that GSE activities had no or little impact on homeownership as “most beneficiaries would have bought [a house] anyways” (Frame and White, 2005, p.172). More generally, our assumption that the volume of lending and origination is constrained is consistent with evidence of GSE activities that crowds out the issuance of commercial mortgages (Fieldhouse, 2019), mortgage refinancing crowding out purchase mortgages (Sharpe and Sherlund, 2016), and housing booms crowding out non-mortgage lending (Chakraborty et al., 2018).

Indeed, despite the political goal of promoting homeownership in disadvantaged neighborhoods (an effect not considered in this paper) and the resulting binding minimum requirement for GSE activity in these neighborhoods, many studies found no effect of GSE activities on overall mort-

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27 More generally, our assumption that the volume of lending and origination is constrained is consistent with evidence of GSE activities that crowds out the issuance of commercial mortgages (Fieldhouse, 2019), mortgage refinancing crowding out purchase mortgages (Sharpe and Sherlund, 2016), and housing booms crowding out non-mortgage lending (Chakraborty et al., 2018).

28 Indeed, despite the political goal of promoting homeownership in disadvantaged neighborhoods (an effect not considered in this paper) and the resulting binding minimum requirement for GSE activity in these neighborhoods, many studies found no effect of GSE activities on overall mort-
The two ingredients above directly generate the main results in our model. The fact that subsidies induce lenders with high screening costs to switch from issuing non-guaranteed loans to issuing guaranteed loans imply the externality. Indeed, because high-cost lenders originate fewer non-guaranteed loans, the average quality of non-guaranteed loans traded increases, which makes its secondary market more liquid.

Finally, we state testable implications about loan guarantees, focusing on \( m^* \in (0, 1) \). Higher loan profitability \( \mathcal{A} \)—which may also proxy for lower lending market competition—reduces the share of guaranteed loans \( m^* \). Thus, the model implies that loan guarantees (i) occur more in countries with more competitive lending markets (such as Canada in contrast to the US) and (ii) may become more prevalent due to the recent competition from Fintech (e.g., specialized online lenders). Interpreted for a cross-section of banks, more profitable banks should screen more and buy guarantees for fewer loans. Consistent with this implication, Loutskina and Strahan (2009) document that banks with lower deposit costs (which we interpret as a more profitable lending margin) originate more non-conforming loans relative to conforming loans.

A first-order stochastic dominance (FOSD) reduction in screening costs \( F(\cdot) \)—which may proxy for a more productive screening technology, e.g., recent technological advances and better data processing and analysis by Fintechs (e.g., Fuster et al. 2019)—reduces guarantees. Consistent evidence includes Buchak et al. (2018), who show that those shadow banks with better screening technology—namely, FinTechs—are also less likely to seek a guarantee from GSEs. Finally, a higher probability of loan repayment without screening \( \mu \) raises the share of guaranteed loans. We encourage future empirical work on this implication.

6 Extensions and Discussion

To derive additional results, we study first the role of collateral and then an illiquid equilibrium. To explore the robustness of our results, we then discuss three alternative
gage credit (Ambrose and Thibodeau, 2004) or homeownership in these neighborhoods (Ilość and Gabriel, 2006; Grundl and Kim, 2021; Painter and Redlern, 2002).
models. We explore the nature of loan guarantees further by contrasting it with another type of credit risk transfer before privately learning loan quality—loan sales upon origination without guarantee. Finally, we introduce asymmetric information in the guarantee market and show that our normative results even become stronger.

6.1 Collateral

Suppose that lenders receive collateral $C$ upon default at $t = 2$. To preserve the model’s linearity in $A$, we define $C \equiv \gamma A$ for $\gamma \in (0, 1)$ without loss of generality.

**Proposition 5. Collateral.** Higher collateral $\gamma$ increases the secondary market prices of loans, lowers screening, and increases guarantees. Higher collateral induces the planner to choose more loan guarantees: $\frac{d\mu^G}{d\gamma} > 0$ for $\mu > \mu^G_L$ and $\frac{d\mu^G}{d\gamma} < 0$.

**Proof.** See Appendix B.5. ■

Collateral provides a lower bound on the loan’s return, so prices for both non-guaranteed and guaranteed loans increase and are less sensitive to screening and guarantees. A higher price of non-guaranteed loans lowers screening incentives. Lower screening, in turn, indirectly increases the incentives to buy guarantees.

Lower screening in the presence of collateral reduces the social cost of guarantees. As a result, the planner (who internalizes the positive externality of guarantees) chooses to use more guarantees on both the intensive and the extensive margins.

The normative result in Proposition 5 has an interesting implication for mortgage guarantees. In particular, the result suggests that guarantee subsidies should be provided only for loans with sufficiently high collateral. To see this, we can rewrite $\mu < \mu^G_L$ as $\gamma > \gamma^P$. Consistent with this lower bound on collateral, GSEs provide guarantees only to mortgages with LTV below 80% (or a similar credit enhancement).
6.2 Illiquid equilibrium and liquifying the market

Here we turn to the illiquid equilibrium, in which lenders with liquidity shock do not sell high-quality loans. Thus, only lemons are traded in the non-guaranteed market.

Proposition 6. Illiquid equilibrium and liquifying the market.

1. There always exists an illiquid equilibrium, $p_N^* = 0$.

2. Guarantees increase the parameter space in which the liquid equilibrium exists: from $\lambda > \lambda_L$ without loan guarantees to $\lambda > \min \left\{ \lambda_L, \lambda_{\tilde{L}} \right\}$ with guarantees, where $\lambda_{\tilde{L}} < \lambda_L$ when guarantees are used (i.e. for $\mu > \tilde{\mu}_G$).

3. Suppose the planner can select the equilibrium type. For $\lambda_P^L < \lambda < \min \left\{ \lambda_L, \lambda_{\tilde{L}} \right\}$, the planner chooses more guarantees, $m^* > \bar{m}$, to create the liquid equilibrium.

4. A regulator implements planners allocation by using (i) the guarantee subsidy $b_G$ to keep the equilibrium liquid, and (ii) a threat of subsidized purchases of non-guaranteed loans to eliminate the welfare-inferior illiquid equilibrium.

Proof. See Appendix B.6 for a proof, a formal characterization of the illiquid equilibrium, a definition of the bound $\lambda_P^L$ and modified definitions of the problems of the planner and the regulator, respectively. The bound $\lambda_{\tilde{L}}$ is defined in Appendix B.2.

The illiquid equilibrium always exists because a zero price and no sales of high-quality loans are mutually consistent. Since lenders do not sell non-guaranteed loans, lenders have higher incentives to screen than in the liquid equilibrium, $\eta^* = (\psi - \mu)\Lambda$. Loan guarantees have no trade-off in the illiquid equilibrium: the benefit is a higher price upon a liquidity shock, $p_G > p_N$, but its cost—forgoing the option to sell a lemon—does not apply because $p_N^* = 0$. Hence, all high-cost lenders buy guarantees, $m^* = 1$. Guarantees still improve allocative efficiency because all liquidity-shocked high-cost lenders can sell in the liquid market for guaranteed loans.

Evidence consistent with this implication includes Mian and Sufi (2009) and Keys et al. (2010).
Guarantees increase the parameter space for which the liquid equilibrium exists, $\lambda_P N \geq A$, since guarantees raise the price of non-guaranteed loans (Proposition 2)—see the shaded area in Figure 5. We refer to this effect of guarantees as an increase in the quantity dimension of allocative efficiency. It complements the guarantee-induced increase in the price dimension of the allocative efficiency described in Section 3.

Next, we consider a planner who not only chooses the loan guarantees as in Section 4 but can also select the equilibrium (liquid or illiquid). A new result is that the planner internalizes the effects of loan guarantees on the quantity dimension of allocative efficiency. The planner liquifies the secondary market for non-guaranteed loans, as shown in Figure 5, while the illiquid equilibrium is unique in the unregulated economy. The planner refrains from liquifying the market, however, when loan guarantees would reduce screening incentives and productive efficiency severely.

Figure 5: Liquid and illiquid equilibrium. A liquid equilibrium exists for $\lambda > \lambda_L$ when loan guarantees are unavailable. When they are available and used, the liquid equilibrium exists for a wider range of parameters, $\lambda_L \leq \lambda < \lambda_P$, adding the shaded area. For $\lambda_P < \lambda < \min\{\lambda_L, \tilde{\lambda}_L\}$, the planner liquifies the market for non-guaranteed loans by buying enough guarantees, creating the liquid equilibrium. For $\lambda \leq \lambda_P$, the illiquid equilibrium is chosen.

The regulator can implement the modified planner’s allocation with (i) the guarantee subsidies $b_G$ (as studied before) and (ii) a subsidy to sellers of non-guaranteed loans $b_N$. The latter subsidy can be interpreted as TARP in the U.S. in the originally envisioned form. Eliminating the illiquid equilibrium when it is inferior can only be achieved via subsidies to sellers of non-guaranteed loans, $b_N = A$. In the liquid equilibrium, both a non-guaranteed loan sale subsidy $b_N$ and a guarantee subsidy $b_G$ can
keep the market liquid. However, the loan guarantee subsidy is superior to the loan sale subsidy because of the pecuniary externality of the former. The sale subsidy does not take advantage of this externality and is therefore more expensive.

6.3 Loan sales upon origination

We study the option for lenders to sell loans to outside financiers at $t = 0$ upon origination. Early loan sales require a couple of small changes in the model. First, financiers are endowed at $t = 0$ as well. Second, for lenders to consume at $t = 1$ (when their expected utility of consumption is high), we introduce a storage technology for the proceeds of loan sales until $t = 1$.

The key difference between guarantee and loan sales without guarantees is that the guarantee creates a risk-free and information-insensitive asset, while a loan sale merely transfers the loan ownership from lender before private learning occurs. Whether this difference matters depends on the learning technology. To show this, we consider two cases of private learning about the loan payoff $A_i$ at $t = 1$: (a) relationship banking, whereby only lenders can learn loan quality but outside financiers receive no private information; and (b) learning-by-holding (Plantin, 2009), whereby any holder of a loan since $t = 0$, including outside financiers, privately learn $A_i$.

Proposition 7. Loan sales upon origination. The implications of loan sales at $t = 0$ depend on whether outside financiers privately learn loan quality at $t = 1$. For relationship lending, loan sales upon origination are equivalent to loan guarantees and all of our positive and normative implications of loan guarantees carry over to loan sales upon origination. For learning-by-holding, loan sales upon origination exacerbate adverse selection in the secondary market for non-guaranteed loans. Because of this negative pecuniary externality, loan sales upon origination are excessively high, so the planner reduces these sales to improve allocative and overall efficiency.

Proof. See Appendix B.7.

If only lenders can learn loan quality at $t = 1$ (e.g., due to relationship lending),
then loan sales upon origination reduce the adverse selection problem, resulting in the same positive pecuniary externality as before. This implies the same normative results for early loan sales and loan guarantee as both instruments are formally equivalent.

If any holder of the loan can privately learn its quality at $t = 1$ (e.g., learning by holding), however, outside financiers selectively sell lemons in the market at $t = 1$. Since financiers are not subject to the liquidity shock, unlike lenders they never sell high-quality loans. Hence, loan sales upon origination increase adverse selection in the market at $t = 1$, a negative pecuniary externality. In contrast to loan guarantee, the option of loan sales upon origination is not a Pareto improvement in the unregulated equilibrium. Moreover, the normative implication of loan sales upon origination in this case are the opposite of loan guarantee. The planner wants to reduce the volume of loan sales upon origination and, accordingly, a regulator wishes to tax it.

Since loan guarantee welfare-dominates loan sales under learning-by-holding, while they are equivalent under relationship lending, we conclude that subsidizing guarantees is preferable in the presence of uncertainty about the learning technology.

6.4 Adverse selection in loan guarantee market

In this extension, we study the possibility of adverse selection in the loan guarantee market. To do so, we modify the screening technology: lenders who screen privately learn loan quality $A_i$ already upon origination at $t = 0$. Lenders who do not screen still privately learn $A_i$ at $t = 1$, as in the main model. A share $1 - \psi$ of lenders who screen privately learn that they financed a lemon and may buy guarantees for them, resulting in adverse selection in loan guarantees at $t = 0$.

**Proposition 8. Adverse selection in loan guarantee market.** In the modified model with asymmetric information at $t = 0$, additional multiple equilibria arise:

1. An equilibrium with an illiquid guarantee market, $k = A$ and $p_G = 0$, always exists. Lemma 1 in Appendix B.1 from the benchmark model applies.
2. For $\lambda \geq \tilde{\lambda}_L$ and $\mu > \tilde{\mu}_G$, where $\tilde{\lambda}_L > \tilde{\lambda}_L$ and $\tilde{\mu}_G > \tilde{\mu}_G$, an equilibrium exists in which both markets for guarantees and non-guaranteed loans are liquid:

(a) Compared to the liquid equilibrium in Proposition 2, the prices of both loan types are lower, the screening is higher, and overall welfare is lower.

(b) There may be multiple equilibria, with different shares $m^*$. Among these, the equilibrium with the highest $m^*$ has the highest welfare.

The regulator subsidizes guarantees to eliminate the welfare-dominated equilibrium with an illiquid guarantee market as well as equilibria with liquid guarantee market but lower $m^*$. The optimal guarantee subsidy in the liquid equilibrium is

$$b^T_G = (1 - \nu) \left[ (1 - \mu) p_N + (\mu A - p_G) \right] - \nu \lambda (p_G - p_N).$$

Proof. See Appendix B.8 (in which $\tilde{\lambda}_L$ and $\tilde{\mu}_G$ are defined).

Due to private learning of loan quality $A_i$ at $t = 0$, low-cost lenders can selectively buy guarantees for lemons. Thus, the additional defining feature of equilibrium is whether high-cost lenders buy guarantees and make the guarantee market liquid because not only lemons are guaranteed, $k < A$ and $p_G > 0$. The guarantee markets can always be illiquid since $p_G = 0$ and low-cost lenders selectively buying guarantees for lemons are mutually consistent. The equilibrium with a liquid guarantee market has a higher price for non-guaranteed loans, lower screening, and higher welfare than the equilibrium with an illiquid guarantee market—similar to the effect of loan guarantees on the liquid equilibrium in Section 3.

Next, we compare the equilibrium with a liquid guarantee market to the main model. Asymmetric information at $t = 0$ reduces the benefits of guarantees. Low-cost lenders privately find out whether, despite screening, they have invested into a lemon and then selective purchase guarantees for their lemons. The resulting adverse selection reduces the price for guaranteed loans, $p_G < \mu A$. Hence, guarantees occur for a smaller parameter range, $\tilde{\mu}_G > \tilde{\mu}_G$, and fewer loans are guaranteed. While
lemons by low-cost lenders are removed from the market for non-guaranteed loans, the effect of fewer guarantees on the price dominates, reducing the price of non-guaranteed loans overall. Screening incentives are higher in the modified model for two reasons. First, screening has an additional benefit of learning loan quality at \( t = 0 \) and selectively buying guarantees for lemons (at an advantageously low fee). Second, the lower price of non-guaranteed loans lowers the payoff from not screening.

Another new feature of this liquid equilibrium is a strategic complementarity in the lenders’ choices to buy guarantees. The benefit of guarantees increases in the proportion of high-cost lenders with guarantees: \( dp_G/dm > 0 \). Since all low-cost lenders buy guarantees for lemons, a higher share of high-cost lenders who buy guarantees spreads the costs of cross-subsidizing guaranteed lemons among more lenders. This lowers the guarantee fee and improves the price of guaranteed loans \( p^*_G \). This strategic complementarity can result in multiple equilibria even when the market for both guarantees and non-guaranteed loans are liquid (multiple \( m^* \)).

As in the main model, guarantees improve the average quality of non-guaranteed loans traded. Moreover, guarantees remove all lemons owned by low-cost lenders from the non-guaranteed market because informed low-cost lenders sell lemons for the highest price, which is in the market for guaranteed loans, \( p^*_G > p^*_N \). However, adverse selection in guarantees lowers the private incentives to buy guarantees. Thus, the equilibrium guarantee benefits are lower in the unregulated equilibrium with liquid guarantees and eliminated in the equilibrium with an illiquid guarantee market.

Adverse selection in the loan guarantee market further strengthens the case for loan guarantee subsidies compared to the main model. It creates a new and additional incentive to liquify the guarantee market and improve allocative efficiency, with an independent welfare benefit. Indeed, the regulator can use guarantee subsidies to eliminate the welfare-dominated equilibrium with illiquid guarantee market and equilibria with a liquid guarantee market but low \( m^* \). In the liquid equilibrium, the optimal subsidy in (6) incentivizes high-cost lenders to buy guarantees and reflects the lower guarantee incentives due to adverse selection \( (p^*_G < \mu A) \), compared to (5).
7 Conclusion

Credit risk is often assumed upon loan origination by third parties for a fee (e.g., mortgage guarantees). We study third-party loan repayment guarantees when lenders can screen, learn loan quality over time, and can sell loans in secondary markets. Since loan guarantee trades with the underlying loan, secondary markets are segmented into markets for guaranteed and non-guaranteed loans. In equilibrium, only lenders with lower screening ability choose to buy guarantees. This is because guarantee passes the benefit of unobserved screening to the guarantor, while its cost remains with the lender. Moreover, a lender who buys a guarantee can no longer exploit private information about loan quality learned after buying a guarantee (and dump lemons in the non-guaranteed loans market). Both the selection of lenders with worse loans into buying guarantee and their commitment to not exploiting future private information improve the average quality of non-guaranteed loans traded. This raises both market liquidity and allocative efficiency. Higher liquidity, in turn, lowers screening and thus reduces productive efficiency, but the overall effect on welfare is positive.

Since lenders do not internalize the full benefit of loan guarantee for allocative efficiency and welfare, guarantees are insufficient in the unregulated economy. We define a welfare benchmark in which the planner chooses guarantees for all lenders to maximize utilitarian welfare. This benchmark can be achieved with loan guarantee subsidies that align the private and social incentives of guaranteeing a loan. Therefore, our results provide an economic rationale for government subsidies to loan guarantees. These results contribute to the debate about the role of government-backed guarantees in lending markets, such as the activities of GSEs in the mortgage market. We also describe under which economic conditions mortgage guarantees should be subsidized.

We wish to discuss potential directions for further work. First, we have assumed that each lender has access to a separate pool of borrowers. If lenders share a common pool instead, then screening has a thinning effect and a lender’s choice of screening reduces the quality of the residual pool, a negative externality. Since lenders who screen never buy guarantees in equilibrium, we expect loan guarantees to mitigate this
negative externality and the social incentives to subsidize loan guarantees would even be higher. Second, we have normalized the rate of return required by outside financiers to zero. If a general required return is considered instead, we expect that a lower required return (e.g. due to a savings glut or stimulative monetary policy) boosts market liquidity, which reduces lending standards and raises guarantee benefits.
References


A  Further extensions

In these extensions we solve for the general case with both the liquid and the illiquid equilibrium. Unless stated otherwise, we focus on the simplification $\psi \to 1$ throughout.

A.1 Partial guarantees

Suppose lenders can choose the share $\omega$ of government coverage. Such a guarantee contract is equivalent to guaranteeing the non-default payment $A$ with a deductible $(1 - \omega)A$, where the owner of the loan pays the guarantee fee at $t = 2$. Since only high-cost lenders buy guarantees, a competitive guarantee fee is actuarially fair, $k = \omega(1 - \mu)A$.

**Proposition 9.** A full guarantee, $\omega^* = 1$, is both privately and socially optimal.

**Proof.** See Appendix B.9. ■

With a partial guarantee, $\omega < 1$, the value of a guaranteed loan of low quality is $\omega A - k = \omega \mu A$, which is below the value of a guaranteed loan of high quality, $A - k = A[1 - (1 - \mu)\omega]$, which contrasts with the main text. There is adverse selection in the market for partially guaranteed loans since lenders without a liquidity shock sell only low-quality loans. Adverse selection redistributes wealth from lenders with a liquidity shock (who always sell) to lenders without liquidity shock (who sell only lemons). If this redistribution is severe, guaranteed loans of high quality are not traded. Since lenders have a higher utility in states with liquidity shock, they choose full coverage, $\omega^* = 1$, to avoid the costs of adverse selection. As for social optimality, a higher guarantee coverage has a positive externality on the price of non-guaranteed loans, so a planner chooses full coverage.

An alternative interpretation of partial guarantee is guarantor default. We have assumed so far that the guarantor has deep pockets, perhaps because of (implicit) government backing. In contrast, suppose the guarantor defaults on its liabilities after the fee is paid at $t = 2$ with exogenous probability $1 - \omega$. The expected value of a guaranteed loan is $\omega A - k$ upon loan default ($-k$ when guarantor defaults and $A - k$ otherwise) and $A - k$ upon loan repayment. The guarantee fee is $k = \omega(1 - \mu)A$. Since the expected payoffs are equal to those for partial guarantee, the problem with guarantor default is identical. Accordingly, Proposition 9 implies that welfare decreases in guarantor default risk.

A.2 Upfront guarantee fee

We consider a guarantee fee $k$ that must be paid at $t = 0$. Thus, a lender who buys guarantee can fund only a share $1 - k$ of the loan, reducing the lending volume. We show that the positive implications are qualitatively the same. Since loan guarantee still has a positive impact on the price of non-guaranteed loans not internalized in the unregulated economy, our normative results are also qualitatively the same.

**Proposition 10.** **Upfront guarantee fee.** The fee paid at $t = 0$ is $k^* = \frac{A(1 - \mu)}{1 + A(1 - \mu)}$. 

1. Guarantees increase the non-guaranteed loan price, lower screening, and increase welfare.

2. For \( \mu > \hat{\mu}_G \) and \( \lambda \geq \hat{\lambda}_L \), the screening threshold is \( \eta^* \equiv \frac{(1 - \nu)(1 - \mu)}{\nu + (1 - \mu)(1 - \rho)} \), the price of non-guaranteed loans is \( q_N^* \equiv \frac{\E(1 - \rho)A}{\nu + (1 - \mu)(1 - \rho)} \), and some loans are guaranteed, \( m^* = 1 - \frac{\eta^*}{1 - (1 - \rho)\eta^*} \) \( (1 - \rho h)\eta^* \equiv (1 - \nu)(1 - \mu)(1 - \rho) \) for \( \eta^* \equiv (0, 1) \), where \( \delta \equiv \frac{\mu A - 1}{\nu + (1 - \mu)(1 - \rho)} \).

3. A planner buys guarantees for more loans, \( m'^0 \geq m^* \).

Proof. See Appendix B.10 (which also defines the bounds \( \hat{\mu}_G \) and \( \hat{\lambda}_L \)). ■

A.3 Partial loan sales

We allow for partial sales of non-guaranteed loans, where \( q^N \equiv [0, 1 - \ell_i] \) is a continuous choice of lenders and retaining default risk \( 1 - \ell_i - q^N \) may signal loan quality to financiers. That is, financiers use \( 1 - \ell_i \) to update their beliefs about loan quality. A continuum of perfect Bayesian equilibria (PBE) may exist but our results are qualitatively unchanged.

Proposition 11. Partial loan sales. All PBE are pooling equilibria characterized by \( q^N \equiv [0, 1] \) and screening \( \eta^*(q^N) \), sustained by out-of-equilibrium beliefs interpreting \( q^N \neq q^N^* \) as a signal of low quality. The quality of non-guaranteed loans remains private information. Except for the corner case of perfect screening by all lenders (\( \psi \to 1 \)), \( \bar{q} < (1 - \mu)A \), \( q^N^* \equiv q^N \), adverse selection remains and:

1. For \( \mu > \hat{\mu}_C(q^N) \), some loans are guaranteed in the liquid equilibrium, \( m^* > 0 \).

2. The planner buys guarantees for more loans in the liquid equilibrium, \( m'^0 > m^* > 0 \) for \( \mu > \hat{\mu}_C(q^N) \), and for more parameters, \( m'^0 > m^* = 0 \) for \( \mu \equiv \hat{\mu}(q^N) \).

Proof. See Appendix B.11 (which defines the equilibrium and bounds \( \hat{\mu}_C(q^N) \) and \( \hat{\lambda}_L(q^N) \)). ■

Since lenders have limited liability, any loan sale \( q^N \) would be mimicked by sellers of low-quality loans (similar to Parlour and Plantin 2008). Thus, guarantees have a positive effect on the reduction of adverse selection and our positive and normative results are qualitatively unchanged. The only exception is the case where all lenders screen (\( \eta^*(q^N) > \bar{\eta} \)) and screening technology is perfect (\( \psi \to 1 \)), and thus it eliminates adverse selection. This arises for \( \bar{\eta} < (1 - \mu)A \) and \( q^N^* > q^N \). In this case, the upper bound on screening costs is low enough so that sufficient default risk retention incentivizes all lenders to screen, so for \( \psi \to 1 \) all loans are of high-quality. When screening is imperfect, however, low-quality loans are originated and at \( t \to 1 \) their holders mimic the risk retention of lenders with high-quality loans, resulting in adverse selection in the market for non-guaranteed loans.

So our results from the main text extend to partial loan sales, whereby loan guarantees reduce such adverse selection and the planner buys guarantees for more loans.
B Proofs

B.1 Benchmark without loan guarantees

We define the equilibrium, state the liquid equilibrium and its comparative statics.

**Definition 1.** An equilibrium comprises screening \( \{ s_i \} \), loan sales \( \{ q_i^N \} \), and a price of loans \( p_N \) such that: (i) at \( t = 1 \), for each \( \lambda_i \) and \( A_i \), each lender \( i \) optimally chooses loan sales \( q_i^N \); (ii) at \( t = 1 \), the price \( p_N \) is set for outside financers to break even in expectation; and (iii) at \( t = 0 \), each lender \( i \) chooses screening \( s_i \) to maximize her expected utility:

\[
\max_{s_i, q_i^N, p_N} \mathbb{E} [\Lambda_{c1} + c_2 - q_i^N s_i] \quad \text{subject to} \quad c_1 = q_i^N p_N, \quad c_2 = (1 - q_i^N)A_i, \quad \Pr (A_i = A) = \psi s_i + \mu (1 - s_i).
\]

**Lemma 1.** Liquid equilibrium when loan guarantees are unavailable. If \( \lambda \geq \lambda^*_N \) and screening costs are heterogeneous enough, \( \eta > \frac{(1 - \psi)(1 - \mu)}{(1 - \psi)\mu} \), then there exists a unique and interior liquid equilibrium. Its cost threshold, \( \eta^* \in (0, \bar{\eta}) \), is implicitly given by

\[
\eta^* = \frac{(1 - \psi)(\psi - \mu) [1 - \mu - (\psi - \mu)F(\eta^*)]}{\psi + (1 - \psi) [1 - \mu - (\psi - \mu)F(\eta^*)]} \quad \text{(7)}
\]

and the price of non-guaranteed loans is \( p_N^* = A - \frac{\eta^*}{(1 - \psi)(\psi - \mu)} \in \left( \frac{A}{\lambda^*_N}, A \right) \). The lower bound on the size of the liquidity shock is \( \lambda^*_N = \frac{\psi^*}{p_N^*} \in (1, \infty) \).

The proof follows. In the liquid equilibrium, screening yields the expected payoff \( \lambda p_N + (1 - \psi)\mu A + (1 - \mu)p_N \) and not screening yields \( \lambda p_N + (1 - \mu)\mu A + (1 - \mu)p_N \), so the cost threshold in (1) follows. Inserting (2) in (1) yields \( \eta^* \) determined by equation (7). Within the class of liquid equilibria, does a unique equilibrium exist? Regarding uniqueness, the left-hand side (LHS) of the equation (7) increases in \( \eta \) and its right-hand side (RHS) decreases in it, so at most one intersection exists. Regarding existence, we evaluate both sides at the bounds, using \( F(0) = 0 < 1 = F(\bar{\eta}) \). Note that \( \text{LHS}(0) < \text{RHS}(0) \) always holds and \( \text{LHS}(\bar{\eta}) > \text{RHS}(\bar{\eta}) \) if \( \bar{\eta} > \frac{(1 - \psi)(\psi - \mu) [1 - \mu - (\psi - \mu)A]}{\psi + (1 - \psi) [1 - \mu - (\psi - \mu)A]} \). For \( \psi \to 1 \), this condition always holds. For \( \psi < 1 \), we assume that the screening cost is heterogeneous enough. Hence, there exists a unique and interior screening threshold \( \eta^* \in (0, \bar{\eta}) \). The price of loans sold at \( t = 1 \) is given by (2) where \( F(\eta^*) \) is the equilibrium share of low-cost lenders and \( \eta^* \) is given in equation (1). To verify the supposed liquid equilibrium, we use conditions (2) and

\[
\lambda p_N \geq A. \quad \text{(8)}
\]

Thus, the condition for the liquid equilibrium is \( \lambda \geq \lambda^*_N = \frac{(1 - \psi)(\psi - \mu)F(\eta^*) + (1 - \mu)F(\eta^*))}{\psi + (1 - \psi)F(\eta^*)} \), where its RHS is independent of \( \lambda \). We conclude with comparative statics.

**Corollary 1.** The threshold \( \eta^* \) increases in \( A \) and decreases in \( \mu, \psi \), and after a first-order stochastic dominance (FOSD) reduction in \( F \). The price \( p_N^* \) increases in \( A \) and after a FOSD reduction. The price can also be non-monotonic in \( \mu \) and \( \psi \). Similarly, the bound \( \Delta \eta \) decreases in \( A \) and after a FOSD reduction and can be non-monotonic in \( \mu \) and \( \psi \).
B.2 Proof of Propositions 1 and 2, definition and characterization of liquid equilibrium, and comparative statics

We first define the equilibrium when loan guarantees upon origination are available.

Definition 2. An equilibrium comprises screening \( \{s_i\} \), loan guarantee \( \{\ell_i\} \), the sales of guaranteed and non-guaranteed loans \( \{q^G_i, q^N_i\} \), prices \( p_G \) and \( p_N \), and a guarantee fee \( k \).

1. At \( t = 1 \), for each shock \( \lambda_i \in \{1, \lambda\} \) and loan quality \( A_i \in \{0, A\} \), each lender \( i \) optimally chooses the sales of guaranteed and non-guaranteed loans, \( q^G_i \) and \( q^N_i \).

2. At \( t = 0 \), each lender \( i \) chooses screening \( s_i \) and loan guarantee \( \ell_i \) to solve

\[
\max_{s_i, \ell_i, c_1, c_2} \mathbb{E}[\lambda_i c_1 + c_2 - \eta_i s_i] \quad \text{subject to}
\]

\[
c_1 = q^G_i p_G + q^N_i p_N,
\]

\[
c_2 = (\ell_i - q^N_i)\pi + (1 - \ell_i - q^N_i)A_i, \quad \Pr\{A_i = A\} = \psi s_i + \mu(1 - s_i).
\]

3. The guarantee fee \( k \) at \( t = 0 \) and the prices of loans \( p_G \) and \( p_N \) at \( t = 1 \) are set for outside financiers to break even in expectation.

The payoff from a guaranteed loan is independent of the screening choice (as financiers cannot observe screening), so a lender who buys a guarantee receives \( \nu\lambda p_G + (1 - \nu)\pi = \kappa p_G \) when not screening and \( \kappa p_G - \eta \) when screening. Thus, she generically prefers not to screen.

For productive enough screening, \( \psi > \psi^* \), high-cost lenders are indifferent about guarantee and \( m^* < 1 \). If \( m = 1 \), no lemons would be sold by high-cost lenders, implying a liquid equilibrium with loan guarantees.

Lemma 2. Liquid equilibrium with loan guarantees. Suppose \( \mu > \bar{\mu}_G \) and \( \lambda \geq \bar{\lambda}_L \).

1. The screening threshold is \( \eta^* = \frac{(1 - \nu)(1 - \mu)(\psi - \mu)}{\nu(\psi + (1 - \psi)\mu)A} \), the price is \( p^*_N = \frac{\kappa m^* \lambda}{\psi + (1 - \psi)\mu} \in \left[ \frac{\kappa}{2}, p^*_G \right] \), and the share of guaranteed loans is \( m^* = 1 - \frac{(1 - \mu)(\psi - \mu)}{\nu(\psi + (1 - \psi)\mu)A} \).

2. The proportion of high-cost lenders who buy guarantees \( m^* \) increases in \( \mu \) and \( \lambda \); decreases in \( A \) and upon a FOSD reduction in \( F \) and can be non-monotonic in \( \nu \). The screening threshold \( \eta^* \) increases in \( A \), decreases in \( \mu, \nu \), and \( \lambda \). The price \( p^*_N \) increases in \( A, \mu, \nu \), and \( \lambda \). The bound \( \bar{\lambda}_L \) decreases in \( \mu \) and \( \nu \). The bound \( \bar{\mu}_G \) decreases in \( \lambda \) and increases in \( A \) and upon a FOSD reduction in \( F \).
The proof follows. The indifference condition for loan guarantee (3) pins down the price of non-guaranteed loans, \( p_N^* = \frac{-\lambda}{\psi + (\frac{\lambda}{\psi} - 1) p_G} \). Since high-cost lenders are indifferent about guarantee, the screening threshold can be obtained by equalizing payoff of screening with payoff of not screening and not buying guarantees in equation (1). Substituting \( p_N^* \) from above into (1), the screening cost threshold stated in the proposition follows.

To ensure a liquid equilibrium, the price \( p_N^* \) must satisfy \( \lambda p_N^* \geq A \). Thus, a liquid equilibrium in which guarantees are used exists if \( \mu \lambda^2 - \nu \lambda - (1 - \mu)(1 - \nu) \geq 0 \). Since only the larger root is positive, this condition reduces to \( \lambda \geq \lambda_L \equiv \frac{\nu}{\psi} - \frac{1}{\nu} + \frac{(1 - \mu)(1 - \nu)}{\psi} \). An equivalent expression is \( \mu \geq \mu_L \equiv \frac{\nu}{\psi} - \frac{1}{\nu} + \frac{(1 - \mu)(1 - \nu)}{\psi} \). When guarantees are available, the liquid equilibrium exists if \( \lambda \geq \min(\lambda_L, \lambda_L) \). The comparative statics of \( \lambda_L \), are defined by \( \frac{d\lambda_L}{d\nu} = -\frac{1}{\psi^2} + \frac{1}{\nu^2} < 0 \) and \( \frac{d\lambda_L}{d\mu} = \frac{1}{\nu^2} \frac{1 - \mu}{\psi^2} < 0 \), where \( \chi \equiv \left( \frac{\psi}{\nu} + \frac{(1 - \mu)(1 - \psi)}{\psi} \right)^{-0.5} > 0 \).

Next, we solve for \( m^* \). Combining the two expressions for \( p_N^* \) from the break-even condition (2') and the guarantee-indifference condition in Lemma 2 yields for \( \rho/\psi \):

\[
\frac{\psi F(\eta) + \mu (1 - F(\eta)) (1 - m)}{\nu (1 - \psi) (1 - m) + (1 - \nu) [1 - \psi F(\eta) + (1 - \mu)(1 - F(\eta))(1 - m)]} = \frac{\nu \lambda^2 + \nu \lambda}{\nu (1 - \nu)} (1 - m) \tag{10}
\]

which yields the share of loans of high-cost lenders with guarantees \( m^* \) in Proposition 2. Since the LHS of (10) increases in \( m \), guarantees are used when \( \frac{\psi}{\nu} \left| \frac{m = 0}{\delta \psi (1 - \nu)} \right. = \frac{\nu \lambda^2}{\nu (1 - \psi)(1 - m)} \supset \frac{\nu \lambda \mu}{\nu (1 - \nu)} (1 - m) \Rightarrow \frac{\psi F(\eta) + \mu (1 - F(\eta)) (1 - m)}{\nu (1 - \psi) (1 - m) + (1 - \nu) [1 - \psi F(\eta) + (1 - \mu)(1 - F(\eta))(1 - m)]} > \frac{\nu \lambda^2 + \nu \lambda}{\nu (1 - \nu)} (1 - m) \tag{11}
\]

The LHS of (11) increases in \( A \) and after a first-order stochastic dominance shift in \( F(\cdot) \) (cheaper screening), and decreases in \( \lambda \). The RHS is independent of both \( A \) and \( F(\cdot) \) and increases in \( \nu \). Hence, the condition for loan guarantees to occur can be expressed as \( A < \hat{A}_G, \lambda > \hat{\lambda}_G, \mu > \hat{\mu}_G \), or high enough screening costs \( F(\cdot) \). The parameter thresholds \( \{\hat{A}_G, \hat{\lambda}_G, \hat{\mu}_G\} \) are defined by \( \frac{\psi}{\nu} \left| \frac{m = 0}{\delta \psi (1 - \nu)} \right. = \frac{\nu \lambda^2}{\nu (1 - \psi)(1 - m)} \) and the threshold \( \hat{A}_G \) solves

\[
\hat{A}_G \equiv \frac{\nu \lambda^2 + \nu \lambda}{\nu (1 - \nu)} (1 - m) = \frac{\nu \lambda^2}{\nu (1 - \psi)(1 - m)} \Rightarrow \frac{\psi F(\eta) + \mu (1 - F(\eta)) (1 - m)}{\nu (1 - \psi) (1 - m) + (1 - \nu) [1 - \psi F(\eta) + (1 - \mu)(1 - F(\eta))(1 - m)]} > \frac{\nu \lambda^2 + \nu \lambda}{\nu (1 - \nu)} (1 - m) \tag{11}
\]

Equivalently, the threshold \( \hat{A}_G \) is implicitly defined by \( \nu \left| \frac{m = 0}{\delta \psi (1 - \nu)} \right. = \frac{\nu \lambda^2}{\nu (1 - \psi)(1 - m)} \), where uniqueness arises when its RHS exceeds its LHS for \( \lambda \to \infty \), for which \( \nu \leq \frac{2 \mu}{\psi + (1 - \nu)} \) is sufficient. Similarly, the threshold \( \hat{\lambda}_G \) is implicitly defined by \( m^* = 0 \), resulting in

\[
\hat{\lambda}_G \equiv \frac{\nu \lambda^2 + \nu \lambda}{\nu (1 - \nu)} (1 - m) = \frac{\psi F(\eta) + \mu (1 - F(\eta)) (1 - m)}{\nu (1 - \psi) (1 - m) + (1 - \nu) [1 - \psi F(\eta) + (1 - \mu)(1 - F(\eta))(1 - m)]} \Rightarrow \frac{\psi F(\eta) + \mu (1 - F(\eta)) (1 - m)}{\nu (1 - \psi) (1 - m) + (1 - \nu) [1 - \psi F(\eta) + (1 - \mu)(1 - F(\eta))(1 - m)]} > \frac{\nu \lambda^2 + \nu \lambda}{\nu (1 - \nu)} (1 - m) \tag{11}
\]

The bound \( \hat{\mu}_G \) is unique since the LHS of (12) increases in \( \mu \) and the RHS decreases in \( \mu \). It is also interior since the following limits do not satisfy equation (12): first, \( \lim_{\mu \to 0} LHS = 0 \) while \( \lim_{\mu \to 0} RHS > 0 \) (guarantee costs outweigh benefits for \( \mu \to 0 \)); second, \( \lim_{\mu \to \infty} LHS = \psi \) while \( \lim_{\mu \to \infty} RHS = 0 \) (guarantee benefits outweigh costs for \( \mu \to \infty \) since no lender screens, \( F = 0 \)). The existence of \( \hat{\mu}_G \) follows from continuity in \( \mu \).
Let $X$ be the difference between the RHS and the LHS of (11). Then, $X = 0$ defines the boundary of the extensive margin of guarantees. The results derived above can be expressed as $\frac{\partial X}{\partial \alpha} < 0$, $\frac{\partial X}{\partial \mu} > 0$, $\frac{\partial X}{\partial \psi} > 0$ and $X$ decreases after a FOSD reduction in $F$. Hence, $\frac{\partial X}{\partial \mu} = -\frac{\partial m}{\partial \psi} > 0$, and $\frac{\partial X}{\partial \psi} = -\frac{\partial m}{\partial \mu} < 0$, and $\mu_G$ increases after a FOSD reduction.

**Comparative statics: screening threshold and price of non-guaranteed loans.** Using (2'), the total derivative of the price w.r.t. loan guarantees is:

$$\frac{dp_N}{dm} = \frac{d}{dm} \left( \frac{\partial p_N}{\partial m} \right) + \frac{d}{dm} \left( \frac{\partial p_N}{\partial \eta} \right) \frac{d\eta}{d\psi} + \frac{d}{dm} \left( \frac{\partial p_N}{\partial \mu} \right) \frac{d\mu}{dN} > 0.$$

Using (2'), the total derivative of the price w.r.t. loan guarantees is:

$$\frac{dp_N}{dm} = \frac{\partial p_N}{\partial m} + \frac{\partial p_N}{\partial \eta} \frac{d\eta}{d\psi} + \frac{\partial p_N}{\partial \mu} \frac{d\mu}{dN} > 0.$$

Since the threshold $\lambda_L$ decreases in the price $p_{N*}$, it decreases in $m^*$: $\frac{d\lambda_L}{dm} = \frac{dN}{d\psi} \frac{d\mu}{dN} < 0$. As a result, when guarantees are used, $m^* > 0$, the threshold for the existence of a liquid equilibrium is lower compared to the case when guarantees are unavailable, $\lambda_L < \lambda_L^*.$

Using the screening threshold stated and $D' \equiv \nu \lambda + (1 - \nu)(1 - \mu)$, we get

$$\frac{dp_N}{dm} = \frac{(1 - \nu)(1 - \mu)(\psi - \mu)\kappa}{D'} > 0, \quad \frac{dp_N}{d\eta} = -\mu(1 - \nu)(1 - \mu)(\psi - \mu)A \frac{D''}{D^2} < 0,$$

$$\frac{dp_N}{d\mu} = \frac{(1 - \nu)\kappa A \mu(1 - \nu)(1 - \mu)^2}{D^2} < 0,$$

$$\frac{dp_N}{d\psi} = \frac{(1 - \mu)(\psi - \mu)A \mu(1 - \nu)^2(\lambda - 1)}{D^2} < 0, \quad \frac{dp_N}{d\psi} = \frac{(1 - \nu)(1 - \mu)\kappa A}{D'} > 0.$$

For the effect on the price, we use $p_{N*}$ given in Proposition 2 to obtain:

$$\frac{dp_N}{dX} = \frac{\nu \lambda \kappa}{D'} > 0, \quad \frac{dp_N}{d\alpha} = \frac{\nu \lambda A \kappa}{D'^2} > 0, \quad \frac{dp_N}{d\psi} = 0, \quad \frac{dp_N}{d\mu} = \frac{\lambda \mu(1 - \mu)A}{D'^2} > 0.$$

**Comparative statics: share of high-cost lenders who buy a loan guarantee.**

Since $m^*$ is given in Proposition 2 as a function of $\eta^*$, the total effect of parameters $\alpha \in \{\nu, \lambda, \mu\}$ on $m^*$ consists of a direct and indirect effect via screening. $\frac{dm^*}{dm} = \frac{dm^*}{d\alpha} + \frac{dm^*}{d\mu} \frac{d\mu}{d\alpha}.$

$$\frac{dm^*}{d\eta} = \frac{\nu (1 - \mu) \psi - (1 - \nu) \lambda \mu f}{\mu(1 - \mu)(\lambda - 1)(1 - \nu)(1 - F)} < 0, \quad \frac{dm^*}{d\mu} = \frac{(1 - \mu) \psi - (1 - \nu) \lambda \mu F}{\mu(1 - \mu)(1 - \nu)(\lambda - 1)^2(1 - F)} > 0,$$

$$\frac{dm^*}{d\psi} = \frac{\mu^2(1 - \mu)^2(\lambda - 1)(1 - \nu)(1 - F(\eta^*))}{\mu^2(1 - \mu)^2(\lambda - 1)(1 - \nu)(1 - F(\eta^*))} F(\eta^*) > 0, \quad \frac{dm^*}{d\psi} = 0,$$

$$\frac{dm^*}{d\alpha} = \frac{\mu^2(1 - \mu)^2(\lambda - 1)(1 - \nu)(1 - F(\eta^*))}{\mu(1 - \mu)(\lambda - 1)(1 - \nu)(1 - F(\eta^*))} \lambda F(\eta^*) < 0.$$

$$\frac{dm^*}{d\mu} = \frac{\kappa(1 - \mu) \psi - (1 - \nu) \lambda \mu}{\mu(1 - \mu)(\lambda - 1)(1 - \nu)(1 - F)} < 0, \quad \frac{dm^*}{d\psi} = \frac{\kappa(1 - \mu) + \lambda \mu}{\mu(1 - \mu)(\lambda - 1)(1 - \nu)(1 - F)} < 0.
The following total derivatives are unambiguous, \( \frac{\partial W}{\partial m} > 0, \frac{\partial W}{\partial \eta} < 0, \frac{\partial W}{\partial \psi} < 0 \), and the FOSD shift. The total effect of \( \nu \) on \( m^* \) can be ambiguous since the direct effect is negative and the indirect one is positive. A sufficient condition for non-monotonicity is the opposite sign of derivatives at both limits, \( \nu \to \{0, 1\} \), where \( \lim_{\nu \to 1} \frac{\partial W}{\partial m} = -\infty \) and

\[
\lim_{\nu \to 0} \frac{\partial W}{\partial m} = -\frac{(1-\mu)\psi - (1-\psi)\mu}{\mu(1-\rho)(\lambda - 1)(1-F)} \lambda F + A(1+(\lambda - 1)\mu)\psi - (1-\psi)\lambda \mu \frac{1}{1-\mu(1-\rho)(\lambda - 1)(1-F)} f.
\]

The sufficient condition for non-monotonicity is \( \frac{\partial W}{\partial m} \bigg|_{\nu \to 0} < A(1+(\lambda - 1)\mu)\psi - (1-\psi)\lambda \mu \bigg|_{\nu \to 0} \).

### B.3 Proof of Proposition 3

**Utilitarian welfare in the liquid equilibrium.** Welfare is the sum of expected payoffs to lenders and financiers. Up to a constant for financiers who expect to break even, welfare \( W \) is the expected payoffs to lenders. In a liquid equilibrium, low-cost lenders, \( \eta_i \leq \eta^* \), of mass \( F(\eta^*) \) receive \( \nu A p_F^* + (1-\nu)[\psi A + (1-\psi)p_F^*] - \eta_i \), non-guaranteed high-cost lenders of mass \( (1-F(\eta^*)) \) receive \( \nu A p_F^* + (1-\nu)[\psi A + (1-\psi)p_F^*] \), and high-cost lenders with guarantees of mass \( (1-F(\eta^*))m^* \) receive \( \nu A p_F^* \). Adding up all lenders yields

\[
W = \nu \lambda \left\{ p_F^* \left[ F(1-m^*) + p_F^* (1-F)m^* \right] + (1-\nu) \left[ \psi A + (1-\nu)p_F^* \right] \right\} - \int_0^1 \nu dF(\eta),
\]

where we used the short-hand \( F = F(\eta^*) \) unless stated otherwise. Substituting the price of non-guaranteed loans, \( p_F^* (1-F(1-m)) + (1-\psi) \left[ \psi F + (1-\mu)(1-F)(1-m) \right] = \nu A \psi F + (1-\mu)(1-F)(1-m) \), results in the simplified expression of welfare in Equation (4).

**Planner’s choice in liquid equilibrium.** We prove existence of \( m^* \) and that the planner buys guarantees for more loans by showing that welfare increases in \( m \) on the interval \( m \in [0, m^*] \). Welfare is continuous and defined everywhere, so the planner’s choice \( m^* \) exceeds the unregulated level \( m^* \). To see this, the total derivative of welfare in (14), \( \frac{\partial W}{\partial m} = \frac{\partial W}{\partial m^*} + \frac{\partial W}{\partial \eta} \frac{\partial \eta}{\partial m} + \frac{\partial W}{\partial \psi} \frac{\partial \psi}{\partial m} + \frac{\partial W}{\partial \mu} \frac{\partial \mu}{\partial m} \), is evaluated using

\[
\frac{\partial W}{\partial m} = (1-\mu)(\nu A p_F^* - \nu A p_F^* - (1-\nu)[\psi A + (1-\nu)p_F^*]) > 0,
\]

\[
\frac{\partial W}{\partial \eta} = \nu A (1-F)(1-m) + (1-\psi) (1-F)(1-m) > 0,
\]

\[
\frac{\partial W}{\partial \psi} = \int (1-\nu) \left\{ \psi A + (1-\nu)p_F^* \right\} - \eta^* + m^* \left\{ \nu A p_F^* + (1-\nu)[\psi A + (1-\nu)p_F^*] - m^* p_F^* \right\} > 0.
\]

At the level of the unregulated equilibrium \( m = m^* \), the total derivative \( \frac{\partial W}{\partial m} \) is positive due to the positive pecuniary externality, \( \frac{\partial W}{\partial m} \bigg|_{m=m^*} = \frac{\partial W}{\partial m^*} > 0 \), where \( \frac{\partial W}{\partial m} > 0 \) reflects the positive pecuniary externality. By an envelope-type-argument, the direct effect of guarantees (except in the corner of \( m = 1 \)) and screening on welfare is zero in the
unregulated economy, \( \frac{\partial W}{\partial m} = 0 = \frac{\partial W}{\partial s} \), as lenders choose guarantee and screening privately optimally. Moreover, for any \( m < m^* \), the total derivative is also positive because \( \frac{\partial W}{\partial m} \big|_{m>0} > 0 \), \( \frac{\partial W}{\partial m} \big|_{m<0} < 0 \), and \( \frac{\partial W}{\partial s} < 0 \), whereby loan guarantees reduce screening incentives. Once guarantees increase above their unregulated level, \( m > m^* \), the positive welfare effect of higher social gains from trade, \( \frac{\partial W}{\partial m} > 0 \), are mitigated by the negative effect of lower screening, \( \frac{\partial W}{\partial s} \big|_{m=m^*} > 0 \). That is, lenders who in the unregulated economy do not buy guarantees (and some of whom screen) are required to buy them and, therefore, do not screen. These lenders are individually worse off, while other lenders are better off due to higher gains from trade upon a liquidity shock and overall welfare increases. Hence, we conclude that the price of non-guaranteed loans is higher and screening lower, \( p^G > p^N \) and \( \eta^G < \eta^N \).

To prove a positive share of guaranteed loans for a larger set of parameters, we compare the thresholds at which guarantees are zero in the unregulated equilibrium, \( \{ \tilde{A}_G, \tilde{\mu}_G, \tilde{\lambda}_G \} \), and in the planner’s choice, \( \{ A^*_G, \mu^*_G, \lambda^*_G \} \), and the screening cost distributions \( F(\cdot) \). First, \( \{ \tilde{A}_G, \tilde{\mu}_G, \tilde{\lambda}_G \} \) satisfy \( m^* = 0 \) and \( \frac{\partial W}{\partial m} = (1 - F)[\mu A - \nu \lambda \tilde{p} - (1 - \nu)\tilde{p} + (1 - \mu)\tilde{p}^N] = 0 \) (the indifference condition for guarantee). Second, \( \{ A^*_G, \mu^*_G, \lambda^*_G \} \) satisfy \( m^* = 0 \) and \( \frac{\partial W}{\partial m} = 0 \). We substitute \( p^*_N \) to obtain for \( \nu / \lambda \):

\[
\frac{\nu [\psi F + \mu (1 - F)]}{\psi + (1 - \nu)[(1 - \psi)F + (1 - \mu)(1 - F)]} = \frac{\nu \lambda A}{(1 - \nu)(1 - \psi)F + (1 - \mu)(1 - F)} + \frac{\frac{\partial W}{\partial p} dp}{\frac{\partial W}{\partial m} dm} = 0,
\]

Since the pecuniary externality term \( \frac{\partial W}{\partial m} \) is positive, the LHS of (16) exceeds the LHS of (11). The LHS of (11) and (16) have the same functional form, increase in \( A \) and after a FOSD reduction in \( F \), and decrease in \( \lambda \). Hence, the planner uses guarantees for larger parameter space \( A^*_G > \tilde{A}_G \), \( \lambda^*_G < \tilde{\lambda}_G \), and cheaper screening. A sufficient condition for \( \lambda^*_G \) to exist is \( \nu \leq \frac{2\psi \lambda}{(1 - \lambda)} \). Rewriting \( \frac{\partial W}{\partial m} = 0 \) also yields an implicitly defined lower bound \( \mu^*_G \):

\[
\mu^*_G \equiv \frac{\nu \psi (1 - \psi) \lambda A}{(1 - 1)(1 - \nu)(1 - F)} F - \frac{\nu (1 - \nu) [1 - \psi F + (1 - \mu)(1 - F)] \frac{\partial W}{\partial p} dp}{(1 - 1)(1 - \nu)(1 - \mu)(1 - F) \psi A} \frac{\partial W}{\partial m} dm.
\]

A direct comparison of (12) and (17) implies that \( \mu^*_G < \tilde{\mu}_G \).

B.4 Definition of equilibrium and proof of Proposition 4

**Definition 3.** A regulated equilibrium comprises the choices of screening \( s_i \), loan guarantees \( (\ell_i) \), guaranteed and non-guaranteed loan sales \( \{ q^G_i, q^N_i \} \), a guarantee subsidy \( h_G \), lump-sum taxes \( T \), prices \( p_G \) and \( p_N \), and a guarantee fee \( k \) such that:

1. At \( t = 1 \), for each \( \lambda_i \) and \( A_i \), each lender \( i \) optimally chooses sales \( q^G_i \) and \( q^N_i \).

2. At \( t = 0 \), each lender \( i \) chooses screening \( s_i \) and loan guarantee \( \ell_i \) to solve

\[
\max_{s_i, \ell_i, c_i, b_i} E[s_i, \ell_i, c_i, b_i] \quad \text{subject to} \quad \begin{align*}
c_{i1} &= q^G_i p_G + q^N_i p_N, \\
c_{i2} &= (\ell_i - q^G_i) \pi + (1 - \ell_i - q^N_i) A_i + \ell_i b_G + n - T, \quad \Pr(A_i = A) = \psi s_i + \mu (1 - s_i).
\end{align*}
\]
3. The guarantee fee $k$ at $t = 0$ and the prices of loans $p_G$ and $p_N$ at $t = 1$ are set for outside financiers to break even in expectation.

4. At $t = 0$, the regulator chooses the guarantee subsidy $b_G$ to maximize welfare subject to a balanced budget, $T = b_G \int l \, dt$.

WLOG we focus on the interval $b_G \leq (1 - \mu)A$. Higher subsidies have no effect on welfare, as the payoff of guaranteed loans $\mu A + b_G$ would exceed the payoff from high-quality loans, so all lenders buy guarantees and do not screen. Thus, the regulator solves:

$$W_R^b \equiv \max_{b_G} \nu(A - 1)[p_N + (p_G - p_N)(1 - F(q))] m$$

$$+ \left[\psi F(q) + \mu(1 - F(q))\right] A - \int_0^{\eta} \hat{q} d\hat{q} + n - T + b_G (1 - F(q)) m \quad (18)$$

s.t. (1), (2'), (3'), and $p_G = \mu A$.

**Liquid equilibrium: guarantee subsidy attains welfare benchmark.** Given the balance budget constraint of the planner, the objective functions of the planner in (4) and the regulator in (18) are identical (up to a constant for final date endowment), and so are the screening threshold and the non-guaranteed loan price. Hence, the subsidy is set to achieve the non-guaranteed loan price in welfare benchmark. Solving equation (3') and evaluating at $p_N(b_G) = p_N^b$ yields the value of $b_G^*$ stated in the proposition.

**B.5 Proof of Proposition 5**

Collateral does not affect the screening incentives in the liquid equilibrium as $\eta^* = (1 - \nu) (\psi - \mu) (A - p_N)$, but lower screening indirectly because of higher price in secondary market:

$$p_N = \frac{\nu (\psi + (1 - \psi) \gamma) + (1 - \nu) (1 - \psi) \gamma)}{\nu (1 - (1 - m)F + (1 - \nu) (1 - \psi) (1 - \mu) \gamma) (1 - m)} A$$

$$= \gamma A + (1 - \gamma) \frac{\nu (\psi F + \mu (1 - m)) A}{\nu (1 - (1 - F) m) + (1 - \nu) [(1 - \psi) F + (1 - \mu) (1 - F) (1 - m)]}.$$  \hspace{1cm} (19)

Collateral increases the price of non-guaranteed loans, $\frac{\partial p_N}{\partial \nu} = \bar{A} - \bar{\nu} > 0$, and decreases its sensitivity to screening, $\frac{\partial p_N}{\partial \gamma} = [1 - \gamma] \frac{\partial \nu}{\partial \nu}$. The guarantee indifference condition for the equilibrium with guarantees, $\kappa \frac{\nu + (1 - \mu) \gamma}{\nu} = \nu \lambda + (1 - \nu) [\mu A + (1 - \mu) p_N]$, implies that the price when guarantees are used in equilibrium, $p_N = \gamma A + (1 - \gamma) \frac{\nu \lambda b_G}{\nu \lambda (1 - m) + (1 - \nu) \nu}$, is also increasing in collateral. Higher price in turn lowers again screening incentives. Because collateral increases prices of both guaranteed and non-guarantees loans, collateral has no direct effect on guarantees (formula for $m^*$ is unchanged). But collateral-induced lower screening increases $m^*$ at both the intensive, $\frac{\partial m^*}{\partial \bar{\nu}} = \frac{\partial \bar{m}^*}{\partial \bar{\nu}} \frac{\partial \bar{m}^*}{\partial \bar{\nu}} > 0$ (because $\frac{\partial \bar{m}^*}{\partial \bar{\nu}} < 0$), and the
extensive margins, \( \tilde{\lambda}_G = \tilde{\lambda}_G/(1 - \gamma) \). The expression for the threshold for guarantee usage in terms of \( \mu \) has the same functional form as (12) but the threshold is lower, \( \tilde{\mu}_G < \tilde{\mu}_G \), because screening is lower. The planner chooses to use loan guarantees in the modified model for \( A < \tilde{\mu}_G \), where \( \tilde{\mu}_G \) is implicitly given by

\[
\frac{\psi F + \mu (1 - F)}{v + (1 - \nu) [(1 - \psi) F + (1 - \mu) (1 - F)]} = \frac{\nu \lambda \mu}{\nu \lambda + (1 - \nu) F (1 - \gamma) A (1 - \gamma)} 
\]

This equation differs from the equation (2) determining \( \tilde{\mu}_G \) in the term \( \frac{\partial W}{\partial N} \gamma/(1 - \gamma) \) on the RHS, which is increasing in collateral \( \gamma \). This implies that \( A^G > \tilde{\mu}_G \). Since the RHS of (20) increases in \( \gamma \) and the LHS decreases in \( \gamma \) as collateral lowers screening, the equation (20) defines threshold \( \gamma^F \) such that the planner chooses more guarantees for \( \gamma > \gamma^F \). Alternatively, the condition \( A < \tilde{\mu}_G \) can be expressed as \( \mu > \mu_G \), where a modified (17) implicitly defines \( \mu_G \)

\[
\mu_G = \frac{\kappa \psi - (1 - \psi) \lambda A}{\lambda (1 - \lambda)(1 - \mu)(1 - F)} F - \frac{\nu + (1 - \nu) [(1 - \psi) F + (1 - \mu) (1 - F)]}{\lambda (1 - \lambda)(1 - \mu)(1 - F)} \frac{\partial W}{\partial p_N} \partial p_N \partial m. 
\]

**B.6 Proof of Proposition 6**

The illiquid equilibrium always exists. If the price of non-guaranteed loans is zero, only lemons are sold in this market, which justifies \( p_N = 0 \). The screening threshold is given by the indiffERENCE of the marginal lender who compares payoffs from screening, \( \psi A - \eta \), and not screening but buying guarantees, \( \kappa \eta A \). Equating those yields the stated threshold, which is below the threshold in the illiquid equilibrium without guarantees, \( (\psi - \mu) A \).

**Modified planner’s problem.** The planner can also select the equilibrium (liquid or illiquid), thus solving \( \max \{W_L, W^IL\} \), where welfare in the liquid \( W_L \) solves (4) subject to (1), \( \lambda p_N \geq A \), and \( p_C = \mu A \), and the welfare in the illiquid (IL) equilibrium solves

\[
W^IL = \max_m W \text{ s.t. } \eta = (\psi - \kappa) A, \quad \lambda p_N = 0 < A, \quad \text{and } p_C = \mu A. 
\]

**Planner compares illiquid and liquid equilibrium.** Next, we define the threshold \( \lambda^*_G \) and show that it exists and is unique. Welfare in the liquid equilibrium is

\[
W^L = \nu(\lambda - 1) \left[ p_N + (\mu A - p_N) (1 - F(\eta^L)) m \right] + \left[ \mu + (\psi - \mu) F(\eta^L) \right] A - \int_0^{\eta^L} \eta d F, 
\]

subject to \( \eta^L \) in (1), \( p_N \) in (2'), and \( \lambda p_N \geq A \). Welfare in the illiquid equilibrium is

\[
W^IL = \nu(\lambda - 1) \mu A \left[ 1 - F(\eta^IL) \right] + \left[ \psi F(\eta^IL) + \mu (1 - F(\eta^IL)) \right] A - \int_0^{\eta^IL} \eta d F. \]

where \( \eta^IL = (\psi - \kappa) A \). At some \( \lambda^*_G \) given in equation (20), the planner is indifferent between both equilibria, \( W^IL \equiv W^L \). This equation implicitly and uniquely defines a \( \lambda^*_G \in (1, \infty) \). For existence, the gains from trade term dominates for \( \lambda \to \infty \), so \( \lambda^*_G \to \infty \), while this term vanishes for \( \lambda \to 1 \). The existence of \( \lambda^*_G \) follows from continuity.
For uniqueness, we show that the welfare difference \( W^L - W^IL \) increases in \( \lambda \). But first we need to characterize the liquid equilibrium at \( \lambda = \lambda^L \), which does not exist for in the unregulated economy. The liquid equilibrium can only be sustained with a high enough share of guaranteed loans that satisfy \( A/\lambda \geq A \). Hence, the FOC for \( m^L \) is \( \frac{dW^L}{dm^L} + \gamma \frac{dp^L}{dm^L} = 0 \), where \( \gamma \) is the Lagrange multiplier on \( \lambda p_N \geq A \). When the planner values the price dimension of allocative efficiency, \( p^L > A/\lambda \), the liquid equilibrium is preferred \( (\lambda > \lambda^L) \). At \( \lambda = \lambda^L \), by construction the planner is indifferent between the illiquid equilibrium and the liquid equilibrium with the highest possible screening consistent with \( p_N = A/\lambda \), so \( \gamma > 0 \). In fact, the Lagrange multiplier is positive for \( \lambda^L \leq \lambda < \lambda^P \), where \( \lambda^P \) is implicitly defined by \( dW^L/dm^L = 0 \) and \( p^L = A/\lambda \). Note that \( \lambda^L \) is defined on the interval \( \mu \geq \tilde{\mu} \), where \( \tilde{\mu} < 1 \) solves \( \lambda^L(\tilde{\mu}) = \lambda^L(\mu) \). That is, \( \tilde{\mu} \) satisfies \( dW^L/dm^L = 0 \), \( p^L = A/\lambda \), and \( m = 0 \). Next, \( \frac{dp^L}{dm^L} < 0 \) and \( \frac{dp^L}{dm^L} > 0 \) imply \( \frac{dp^L}{dm^L} < 0 \) at \( \lambda = \lambda^L \), so the planner would buy guarantees for fewer loans without the binding constraint for a liquid equilibrium.

The total derivative of the difference \( W^L |_{p_N=A/\lambda} - W^IL \) with respect to \( \lambda \) is:

\[
\frac{dW^L}{dm^L} \left|_{p_N=A/\lambda} \right. \frac{dp^L}{dm^L} > 0
\]

\[
\frac{d}{d\lambda} \left[ \lambda^L \right] = \frac{d}{d\lambda} \left[ \lambda^L \right] + \left[ \frac{d}{d\lambda} \left( \lambda^L \right) \right] = \left[ \frac{d}{d\lambda} \left( \lambda^L \right) \right] = 0
\]

This derivative is positive because both effects of a higher size of the liquidity shock \( \lambda \) are positive: the indirect effect through a lower level of guaranteed loans and the direct effect of higher gains from trade. For \( \gamma > 0 \), guarantees \( m^L \) target \( \lambda^L = A/\lambda \), thus a higher \( \lambda \) means that less guarantees are needed to achieve the reduced price necessary to liquify the market (recall that \( \frac{dp^L}{dm^L} > 0 \)). Equation (26) has already established that the gains \( \gamma = 0 \) at \( \lambda = \lambda^L \), so the total effect is still positive. In sum, the welfare difference between a liquid and illiquid equilibrium monotonically increases in \( \lambda \), so equation (26) defines \( \lambda^L \) uniquely.

To summarize, at \( \lambda = \lambda^L \), the planner is indifferent between the two equilibria. Screening incentives and productive efficiency is higher in the illiquid equilibrium, while the social gains from trade (allocative efficiency) are higher in the liquid equilibrium. These forces transparently show up in the definition of \( \lambda^L \):

\[
\text{Higher gains from trade in liquid equilibrium} \quad \text{Higher net benefits of screening in illiquid equilibrium}
\]

\[
\eta^L \left( \eta^L \right) - \left[ \psi - \mu A \left( F(q^L) - F(q^*) \right) - \int_{\xi^L}^{\eta^L} \psi \, d\xi \right] \equiv \left[ \psi - \mu A \left( F(q^L) - F(q^*) \right) - \int_{\xi^L}^{\eta^L} \psi \, d\xi \right]
\]

where \( \eta^L \) and \( \eta^* \) are the screening thresholds in the illiquid and liquid equilibrium.

For \( \lambda \leq \lambda^L \), the social gains from trade have a lower impact on welfare, so the planner prefers the illiquid equilibrium and does not alter the guarantee choice of high-cost lenders \( (m^L = m^* = 1) \). For \( \lambda^L < \lambda < \min \{ \Delta L, \Delta I \} \), the planner buys enough guarantees \( m^L \) to create the liquid equilibrium, \( p^L \geq A/\lambda \), that did not exist in unregulated economy. Thus, the planner improves the quantity dimension of allocative efficiency. For \( \lambda \geq \min \{ \Delta L, \Delta I \} \) and \( \mu > \tilde{\mu}^L \), the planner improves allocative efficiency on the price dimension only.

Modified regulator’s problem.
Definition 4. A modified regulated equilibrium comprises screening \( \{ s_i \} \), guarantees \( \{ \ell_i \} \), loan sales \( \{ q_i^G, q_i^N \} \), a guarantee subsidy \( b_G \), a subsidy to sellers of non-guaranteed loans \( b_N \), lump-sum taxes \( T \), prices \( p_G \) and \( p_N \), and a guarantee fee \( k \) such that:

1. At \( t = 1 \), for each \( \lambda_i \) and \( A_i \), each lender \( i \) optimally chooses sales \( q_i^G \) and \( q_i^N \).

2. At \( t = 0 \), each lender \( i \) chooses screening \( s_i \) and loan guarantee \( \ell_i \) to solve:
   \[
   \max_{s_i, \ell_i, c_i, \ell_i, s_i} \mathbb{E}[\lambda_i c_i + c_2 - \eta_i s_i] \quad \text{subject to}
   \]
   \[
   c_1 = q_i^G p_G + q_i^N p_N, \quad c_2 = (\ell_i - q_i^N) \pi + (1 - \ell_i - q_i^G) A_i + \ell_i b_G + q_i^N b_N + n - T, \quad \Pr\{ A_i = A \} = \psi s_i + \mu(1 - s_i).
   \]

3. The guarantee fee \( k \) at \( t = 0 \) and the prices of loans \( p_G \) and \( p_N \) at \( t = 1 \) are set for outside financiers to break even in expectation.

4. At \( t = 0 \), the regulator chooses the subsidies \( b_G \) and \( b_N \) to maximize welfare subject to a balanced budget, \( T = b_G \int \ell_i \, ds_i + b_N \int q_i^N \, ds_i \).

The non-guaranteed loan sale subsidy changes the screening threshold from Equation (1) to

\[
\phi = (1 - \nu)(\psi - \mu)[A - (p_N + b_N)],
\]

where either subsidy reduces the incentives to screen (directly or via the price \( p_N \)). Similarly we obtain a generalized version of the guarantee indifference condition (3):

\[
m(\nu \lambda (p_G - p_N) + b_G - \nu b_N - (1 - \nu)(1 - \mu)(p_N + b_N)) = 0
\]

with complementary slackness. Thus, loan guarantees are used, \( m > 0 \), whenever high-cost lenders are indifferent about guarantee.

Without loss of generality, we focus in the liquid equilibrium\(^{30}\) on the interval \( b_G \leq (1 - \mu)A \) and \( b_N \leq (1 - \mu)A \). Higher subsidies have no effect on welfare, as the payoff of guaranteed loans \( \mu A + b_G \) (sold loans \( p_N + b_N \)) would exceed the payoff from high-quality loans, so all lenders buy guarantees (sell all high-quality loans irrespective of liquidity shock) and do not screen. Thus, the regulator solves \( \max\{W_R^L, W_R^{LQ}\} \), where welfare in the illiquid equilibrium is defined below and welfare in the liquid equilibrium solves:

\[
W_R^L \equiv \max_{b_G, b_N} \text{Gains from trade} \quad \text{s.t. (2'), (27), (28), } p_G = \mu A, \text{ and } \lambda p_N \geq A.
\]

\(^{30}\) If regulator wants to eliminate the illiquid equilibrium, she needs to be able to credibly promise \( b_N = A \). Note that this does not increase the requirement on \( n \) because for \( b_N = A \) all lenders weakly prefer to sell. Lenders without shock and with high-quality loan are indifferent about sales. Both the former and the latter will have enough resources to pay the tax \( T \leq A \) without any endowment.
Thus, a non-guaranteed loan sale subsidy can eliminate the illiquid equilibrium. In the liquid equilibrium, however, the sale subsidy does worse than the guarantee subsidy.

**Illiquid equilibrium: no intervention.** Conditional on the liquid equilibrium, the regulator solves:

\[
W_*^{L} = \max_{b_N, \nu, \psi} W + \nu s.t. \eta = \psi A + (1 - \psi)b_N - \max(b_N, \mu A + (1 - \mu)b_N), \ p_N = 0,
\]

and subject to the privately optimal guarantee choice. Lenders buy guarantees if its payoff, \(\eta\), exceeds the payoff from not buying guarantees, \(\mu A + (1 - \mu)b_N\). In the unregulated equilibrium, all high-cost lenders buy guarantees, \(m^* = 1\), and the screening threshold is \(\eta^* = (\psi - \mu)A\). This allocation corresponds to the planner’s choice since both subsidies \(b_N\) and \(b_N\) only reduce the screening threshold with no positive effect on gains from trade. This level of screening also maximizes welfare: \(\frac{\partial^2 W_*^{L}}{\partial \psi^2} = 0\) yields \(\eta^*\). Thus, a non-guaranteed loan sale subsidy can affect screening and lower guarantee but is undesirable. Hence, \(b_N^L = 0 = b_N^L\).

**Subsidies for sales of non-guaranteed loans.** It is immediate that an illiquid equilibrium, \(p_N^L = 0\), can be eliminated with a subsidy \(b_N^L = A\) because \(\lambda p_N^L + b_N^L > A\).

Next, we compare welfare when the same target cash-flow from sale of non-guaranteed loans \(p_N^L \equiv TA < A\) is achieved (i) with a guarantee subsidy, \(p_N^L = p_N(b_N)\), and (ii) with a non-guaranteed loan sale subsidy, \(p_N^L = p_N + b_N\). The equal cash-flows in the two cases imply that the screening threshold is the same in both cases. Using the indifference in (28), welfare with a guarantee subsidy in (18) equals

\[
W_*^{G} = \max_{b_N, \nu, \psi} W + \nu s.t. \eta = \psi A + (1 - \psi)b_N - \max(b_N, \mu A + (1 - \mu)b_N), \ p_N = 0,
\]

where \(p_N = p_N^G\) and \(b_N(p_N^G), \eta(p_N^G)\), and \(m(\eta(p_N^G))\) are given by (28), (1), and (2').

In contrast, welfare with subsidized sales of non-guaranteed loans, \(p_N^G > p_N^L\), is

\[
W_*^{F} = \max_{b_N, \nu, \psi} W + \nu s.t. \eta = \psi A + (1 - \psi)b_N - \max(b_N, \mu A + (1 - \mu)b_N), \ p_N = 0,
\]

where \(p_N\) is given by (2), \(b_N = p_N^G - p_N\), \(\eta = (1 - \nu)(\psi - \mu)(A - p_N^G)\), and the quantity of non-guaranteed loans sold \(\int q_N^G dt = (1 - \nu)A + (1 - \nu)(1 - \psi)F + (1 - \mu)(1 - F)\).

Since screening threshold is the same in both cases, welfare differs in the policy costs and in the gains from trade.\(^{31}\) A guarantee subsidy has higher welfare than subsidized loan

\(^{31}\)In an earlier version of the paper, (Alnert and Kuncl, 2020), subsidies and taxes were transferred
expected payoff of keeping a good loan until maturity with probability $\mu$, the price of non-guaranteed loans at $\mu A + (1 - \mu)p_N$, which always holds. To see this, substitute $b_C$ from (28), $m(1 - F) = \frac{p_S(\nu + (1 - \nu)(1 - \mu)(1 - F) + \nu(1 - \mu)(1 - F)\lambda)}{p_S(1 - \nu)(1 - \mu)(1 - F)}$ from (2'), and for $p_N$ $q_N = \nu(\psi F + \mu(1 - F))A$ from (2'), we can rewrite the required inequality as $\frac{(1 - \omega)(1 - \mu)(1 - F)\lambda}{(1 - \nu)(1 - \mu)(1 - F)} \leq 1 + \frac{\nu(1 - \mu)}{1 + \nu(1 - \mu)}, \quad \text{for guarantee purchase},$ where $\omega = \frac{1}{\mu}$. We can show that the inequality holds for $T = 1$ and since the LHS (RHS) increases (decreases) in $T$, the inequality holds for any meaningful target cash-flow $p^T$.

B.7 Proof of Proposition 7

In this extension lenders can sell non-guaranteed loans in secondary markets at $t = 0$ and at $t = 1$. We denote the prices on these markets as $p_0$ and $p_1$, respectively.

B.7.1 Relationship lending case

It is easy to show that loan sales upon origination are equivalent to loan guarantees in this case. Both choices result in the same expected payoffs for lenders: $\kappa p_0$ for sale upon origination and $\kappa p_1$ for guarantee purchase, where $p_0 = p_C = \mu A$ in equilibrium. Also both loan sales upon origination and guarantee serve as a commitment for lenders not to act on private information at $t = 1$. There is similar self-selection of high-cost lenders into sale upon origination. Therefore, both choices have the same positive pecuniary externality on the price of non-guaranteed loans at $t = 1$, $p_1$, which is given by:

$$p_1 = \frac{\nu(\psi F + \mu(1 - F)(1 - m^f - m^g))A}{\nu(\psi F + (1 - F)(1 - m^f - m^g)) + (1 - \omega)(\psi F + (1 - \mu)(1 - F)(1 - m^f - m^g))},$$

(30)

where $m^f$ ($m^g$) is the fraction of high-cost lenders buying guarantees (selling upon origination). Therefore, all positive and normative result regarding loan guarantee in the main text extend to loan sales upon origination under relationship learning assumption.

B.7.2 Learning-by-holding case

Positive analysis. There are two equilibria (liquid and illiquid) depending on whether high-quality loans are sold at $t = 1$. The illiquid equilibrium equals the illiquid equilibrium with guarantees. All high-cost lenders sell upon origination at price $p_0 = \mu A$, and low-cost lenders keep all loans until maturity. The screening threshold is $q^L = (1 - \mu)A$.

In the liquid equilibrium, all high-cost lenders sell upon origination for a price $p_0 = \mu A + (1 - \mu)p_1$ (outside financiers break-even in equilibrium and, thus, the price equals the expected payoff of keeping a good loan until maturity with probability $\mu$ and selling lemons at $t = 1$ with probability $1 - \mu$). This is because the expected payoff from selling upon origination, $\kappa p_0$, strictly dominates the expected payoff of not selling upon origination and not buying guarantees, $\nu A + (1 - \nu)(\mu A + (1 - \mu)p_1)$, as well as the expected payoff of buying guarantees, $\kappa A$. Hence, no lender buys guarantee.

at the end of $t = 1$. As a result, welfare under the two policy tools differed only in the policy cost term, but the result is unchanged: guarantee subsidies are preferred.
The option to sell upon origination increases adverse selection in the market at \( t = 1 \) because high-quality loans previously owned by high-cost lenders are never sold in this market:

\[
p_{1} = \frac{\nu \psi A}{F(\nu + (1 - \nu)(1 - \psi)) + (1 - F)(1 - \mu)} \tag{31}
\]

The loan sale upon origination has two opposing effects on the screening threshold. First, higher payoff for high-cost lenders from sales upon origination lowers screening incentives. Second, lower price \( p_{1} \) tends to increase them. The screening threshold equates the payoff from screening, \( \nu \lambda p_{1} + (1 - \nu)(\psi A + (1 - \psi)p_{1}) - \eta \), with the payoff from not screening, \( \eta \psi A \):

\[
\eta = \max \{0, [(1 - \nu)(\psi - \mu) - \nu \mu \lambda](A - p_{1})\}. \tag{32}
\]

The liquid equilibrium exists when \( p_{1} \geq A/\lambda \). A threshold that satisfies the liquidity condition must satisfy

\[
\frac{\nu \psi F(q)A}{F(q)[\nu + (1 - \nu)(1 - \psi)) + (1 - F(q))(1 - \mu)] - A}{\lambda} \tag{33}
\]

where \( \eta = \max \{0, [(1 - \nu)(\psi - \mu) - \nu \mu \lambda](A - A/\lambda)\} \). The RHS of equation (33) decreases in \( \lambda \). The LHS of equation (33) is non-monotonic in \( \lambda \), because \( F \) increases in \( \eta \), which is non-monotonic in \( \lambda \). This implies that both for \( \lambda = 1 \) and for \( \lambda \geq (1 - \nu)(\psi - \mu)/(\nu \mu) \), the liquid equilibrium is not sustainable because of the implied variables \( \eta = p_{1} = 0 \). Therefore, there may exist an interval \( \lambda \in [\lambda_{1}, \lambda_{2}] \), where liquid equilibrium exists. Thresholds \( \lambda_{1}^{S} \) and \( \lambda_{2}^{S} \) are the two roots of (33) that lie on the interval \( (1, (1 - \nu)(\psi - \mu)/(\nu \mu)) \). These thresholds exist for \( A > \lambda_{1}^{S} \), where \( \lambda_{1}^{S} \) is implicitly given by

\[
\frac{\nu \psi F(A_{1}^{S})}{F(A_{1}^{S})[\nu + (1 - \nu)(1 - \psi)) + (1 - F(A_{1}^{S}))(1 - \mu)]} = \frac{\nu \mu}{(1 - \nu)(1 - \mu)}. \tag{34}
\]

Due to negative effects of loan sales upon origination on the price \( p_{1} \) (both direct and indirect through screening incentives), the lower threshold for the existence of liquid equilibrium is higher than in the benchmark model (with or without guarantee), that is \( \lambda_{1}^{S} > \lambda_{L} > \lambda_{L} \).

**Normative analysis.** A planner who controls loan sales upon origination reduces their amount. Since it is not efficient for low-cost lenders to sell upon origination, this is equivalent to choosing the fraction of high-cost lenders who sell upon origination, \( m^{S} \).

First, we study the case where lower \( m^{S} \) could increase \( p_{1} \) to sustain the liquid equilibrium. Using similar steps as in the Appendix B.3, we can show that the liquid equilibrium is socially preferred for large enough \( \lambda > \lambda_{1}^{S} \), where \( \lambda_{1}^{S} < \lambda_{1}^{S} \). Note that for \( \lambda > \lambda_{1}^{S} \) all lenders prefer the liquid equilibrium, because it gives an additional option to sell at positive price at \( t = 1 \). Even the lenders who are constrained not to sell upon origination, (weakly) prefer the liquid equilibrium as they have an option to buy guarantees and achieve a higher payoff, \( \max\{\nu \mu A, \nu \lambda p_{1} + (1 - \nu)(\mu A + (1 - \mu)p_{1})\} \), than in the illiquid equilibrium (\( \nu \mu A \)).

Second, we study whether the planner would like to reduce sales upon origination in the liquid equilibrium, when \( \lambda \in [\lambda_{1}, \lambda_{2}] \). We express the welfare as:
\[ W = F[\nu\lambda p_1 + (1 - \nu)(\psi A + (1 - \psi)p_1)] - \int_0^\eta \tilde{\eta} dF(\tilde{\eta}) \\
+ (1 - F) \left\{ m^S \kappa (\mu A + (1 - \mu)p_1) + (1 - m^S) \max\{ \kappa \mu A, \nu\lambda p_1 + (1 - \nu)(\mu A + (1 - \mu)p_1) \} \right\}. \]

It can be shown that a necessary condition for negative effects of loan sales upon origination on welfare \( dW/dm^S < 0 \) collapses to:

\[ [F(\nu + (1 - \nu)(1 - \psi)) + (1 - F)(1 - \mu)] \left( \lambda(1 - F) + (\lambda - 1)(1 - \nu) \frac{1 - \psi}{1 - \mu} \right) - \nu F A \kappa A \psi \\
< \int \nu \lambda A \left[ \nu \lambda A (1 - \nu - \mu) + \frac{F}{1 - F}(\psi A + (1 - \nu)(1 - \psi)) \right] \\
+ (1 - \mu) \left( \psi + (1 - \nu)(1 - \psi) + (1 - F)(1 - \mu) \right). \]  \( (34) \)

For \( A \to \infty \) the above condition \( (34) \) is satisfied, while for \( A \to 0 \) it is not. By continuity there is a threshold \( A^S \) implicitly defined by \( (34) \) with equality, such that for \( A > A^S \) the planner wants to lower loan sales upon origination in the liquid equilibrium.

A regulator can implement fewer loan sales \( m^S \) by imposing taxes \( T^S \) on sellers of loans at \( t = 0 \) and redistributing the proceeds to all lenders. Optimal tax makes high-cost lenders indifferent about loan sale upon origination at the price \( p^T = p^T_N \), so:

\[ T^S = (1 - \mu)p^T_N - \frac{\max\{ 0, \nu \lambda p^T_N - \mu A + (1 - \nu)(1 - \mu)p^T_N \} \kappa}{\kappa}. \]

B.8 Proof of Proposition 8

We focus on the equilibrium in which the market for loan guarantees at \( t = 0 \) is liquid. We exclude the equilibrium in which the market for non-guaranteed loans is illiquid, \( p_N = 0 \), but the loan guarantee market is liquid, \( p_G > 0 \), based on its instability. This equilibrium requires that high-cost lenders are indifferent about guarantee, \( 0 < m < 1 \). Guarantee purchase by all high-cost lenders, \( m = 1 \), is not an equilibrium: since all lemons by low-cost lenders are also guaranteed, no lemons are traded at \( t = 1 \), implying \( p_N = A \) and violating the supposed \( p_N = 0 \). Guarantee purchase by no high-cost lenders, \( m = 0 \), is not an equilibrium either, because the price of guaranteed loans would be \( p_G = 0 \), violating the supposed \( p_G > 0 \). Since the price of guaranteed loans increases in \( m \), any deviation from the equilibrium level of \( m \) leads to the equilibrium in which all markets are illiquid or to the one in which all markets are liquid—hence the instability.

**Positive analysis.** Consider the equilibrium with liquid markets for loan guarantee at \( t = 0 \) and sales of non-guaranteed loans at \( t = 1 \). For the former market to be liquid, some high-cost lenders must buy guarantees, \( m > 0 \), so the payoff from buying guarantees exceeds that from not buying guarantees:

\[ \kappa p_G \geq \nu \lambda p_N + (1 - \nu) \mu A + (1 - \mu)p_N. \]  \( (35) \)

Some high-cost lenders buy guarantees for loans worth \( \mu A \) and low-cost lenders may buy...
guarantees for lemons worth 0, so \( p_G > p_N \). Combining this with equation (35), we find that \( p_G > p_N \). Hence, all low-cost lenders buy guarantees for their lemons and \( p_G < \mu A \). Moreover, only some high-cost lenders buy guarantees in equilibrium, \( m < 1 \), because \( m = 1 \) would imply that a price \( p_N = A > p_G \), a contradiction. Hence, equation (35) holds with equality. The screening threshold equals the payoff from screening, \( \psi[p\lambda p_N + (1 - \psi)]A + (1 - \psi)p_G - \eta \), and from not screening, \( \nu p_N + (1 - \nu)[\mu A + (1 - \mu)p_N] \). Using equation (35) with equality to simplify yields

\[
\eta = \psi(1 - \nu)(1 - \mu)(A - p_N),
\]

which is a higher schedule than in the main model because of the additional benefit of screening—the option to buy guarantees for lemons. The competitive prices of loans are:

\[
p_G = \frac{\mu A(1 - F)m}{(1 - F)m + (1 - \psi)}F = \frac{\mu A - \mu A}{(1 - F)m + (1 - \psi)}F, \tag{37}
\]

\[
p_N = \frac{\nu [\psi F + \mu(1 - F)(1 - m)] A}{\nu [\psi F + (1 - F)(1 - m)] + (1 - \nu)(1 - \mu)(1 - F)(1 - m)} \tag{38}
\]

The adverse selection discount in the loan guarantee market vanishes for \( \psi \to 1 \) as no more lemons are guaranteed. Loan guarantee purchase by high-cost lenders \( m \) increases both prices \( p_N \) and \( p_G \). As in the main model in Equation (14), we have \( \delta p_G/\delta m > 0 \). Moreover, \( \frac{\partial m}{\partial \lambda} = \frac{2\mu}{\mu A} + \frac{2\mu}{\mu A} \frac{\partial m}{\partial \mu} > 0 \), since \( \frac{2\mu}{\mu A} > 0 \), \( \frac{\partial m}{\partial \mu} < 0 \), and \( \frac{\partial m}{\partial \mu} > 0 \). Using Equation (35) with equality yields:

\[
p_N = \mu A - \frac{\nu A}{(1 - \psi)}F \frac{(1 - F)m + (1 - \psi)}{(1 - F)m + (1 - \psi)}F, \tag{39}
\]

Equations (37) and (39) state that adverse selection in the loan guarantee market reduces the prices of both guaranteed and non-guaranteed loans relative to the main model (adverse selection discount). A lower price together with a higher screening schedule implies that the screening threshold \( \eta \) is higher than in the main model. Combining (38) and (39) gives

\[
\psi F + \mu(1 - F)(1 - m) \tag{40}
\]

\[
\nu [\psi F + (1 - F)(1 - m)] + (1 - \nu)(1 - \mu)(1 - F)(1 - m)
\]

\[
\eta = \psi(1 - \nu)(1 - \mu)(A - p_N),
\]

where \( \eta \) is given by (36). The RHS of (40) is the benefit of a loan guarantee and the LHS its opportunity costs. In the limit of \( \mu \to 1 \), \( F(m) = 0 \) and the RHS of (40) collapses to 1 and exceeds the LHS that collapses to \( \nu \). Therefore, guarantees are strictly preferred in this limit. In contrast, for \( \mu = \bar{\mu}_G \), which satisfies Equation (11), the guarantee costs exceed guarantee benefits. The guarantee benefits are lower (RHS of (40) is smaller than the RHS of (11)) and costs larger (LHS of (40) is larger than the LHS of (11)). By continuity, there exist a \( \bar{\mu}_G \) such that guarantees are used, \( m > 0 \), for \( \mu > \bar{\mu}_G \).

Both RHS and LHS of (40) increase in \( m \), so multiple equilibria with a liquid guarantee market may exist (see Figure 6). Higher \( \lambda \) increases \( p_N \) directly (Equation 39) and indirectly via lower screening (Equation 36) and thus fewer lemons of low-cost lenders with a guarantee. Hence, there is a threshold \( \lambda_G^{LS} \) such that for \( \lambda \geq \lambda_G^{LS} \) the liquid equilibrium...
exists (Condition 8 holds) conditional on guarantees being used. Due to the negative effect of adverse selection (see 39), this threshold is higher than in the main model, $\hat{\lambda}_L < \tilde{\lambda}_L^{AS}$.

**Figure 6:** Multiple equilibria for liquid markets of both guarantees and non-guaranteed loan sales. The red solid line plots the price of non-guaranteed loans $p_N$ and the blue dashed line shows the price $p_N$ at which high-cost lenders are indifferent about guarantees. There are two equilibria with positive guarantees ($m^*_1, m^*_2$) but only $m^*_2$ is stable.

**Normative analysis.** The equilibrium with guarantees welfare-dominates equilibria with an illiquid guarantee market. The screening and guarantee choice are privately optimal and all externalities are pecuniary. And since the equilibrium with guarantees has higher prices in secondary markets, both $p_G$ (by definition because $p_G = 0$ in the illiquid guarantees equilibrium) and $p_N$ (see above for $dp_N/dm$), welfare in the equilibrium with guarantees welfare-dominates the equilibria without this option. As in the main model, we express the welfare in the equilibrium with a liquid guarantee market as the sum of lender payoffs,

$$W = \psi \left[ F(\nu \lambda p_N + (1 - \nu)A^* + (1 - F)^* \nu \mu A + (1 - \nu)(1 - F) p_N) (1 - m) \right] + \nu (1 - \psi) A - R \eta_i \int_0^\infty dF,$$

where $p_N$ and $\eta$ are given by generalized (38) and (36):

$$p_N = \frac{\nu \psi F + \mu (1 - F)(1 - m)}{\nu \psi F + (1 - \nu)(1 - F)(1 - m) + (1 - F)(1 - m)} A - \eta (1 - \nu)(1 - m),$$

and $m^l$ is the share of low-cost lenders who buy guarantees for lemons ($m^l = 1$ in unregulated equilibrium). As in the unregulated equilibrium, more guarantees purchased by high-cost lenders increases welfare by raising the price of non-guaranteed loans. Moreover, more guarantees improve the price of guaranteed loans (as in the unregulated equilibrium) given by the final term:

$$\frac{dW}{dm} = \frac{\partial W}{\partial m} + \frac{\partial W}{\partial p_G} \frac{dp_G}{dm} + \frac{\partial W}{\partial p_N} \frac{dp_N}{dm} + \frac{\partial W}{\partial \eta} \frac{d\eta}{dm} > 0.$$
high-cost lenders without direct redistribution due to adverse selection:

$$W' = F[\psi(\nu A) + (1 - \nu)A] - \int_{0}^{\eta} \gamma dF + (1 - F)[\kappa \mu A + \nu \rho N + (1 - \nu)(\mu A + (1 - \mu)p_N)(1 - m)].$$

Thus, \( m' \) does not affect welfare directly but only via the price of non-guaranteed loans, the screening threshold, and guarantee purchases by low-cost lenders. These are given (implicitly) by (41) and generalizations of (39) and (40): \( p_N = \frac{\nu \lambda - (1 - \psi)F \nu \lambda}{(1 - F)m + (1 - \psi)F m} \) and

$$\nu [\psi F + \mu(1 - F)(1 - m)] + (1 - \nu)[\mu(1 - F)(1 - m) + (1 - \psi)F(1 - m')] < 0, \text{ with } \frac{m'}{m} = -\psi(1 - \nu)(1 - \mu) \psi F + \mu(1 - F)(1 - m),$$

$$\nu [\psi F + (1 - F)(1 - m)] + (1 - \nu)[\mu(1 - F)(1 - m) + (1 - \psi)F(1 - m')] < 0.$$

Hence,

$$\frac{\partial W'}{\partial m} = \frac{\partial W'}{\partial \mu} \frac{\partial \mu}{\partial m} + \frac{\partial W'}{\partial \psi} \frac{\partial \psi}{\partial m} + \frac{\partial W'}{\partial \lambda} \frac{\partial \lambda}{\partial m} + \frac{\partial W'}{\partial \kappa} \frac{\partial \kappa}{\partial m} < 0, \text{ with } \frac{\partial \psi}{\partial m} = -\psi(1 - \nu)(1 - \mu) \psi F + \mu(1 - F)(1 - m),$$

$$\mu \frac{\partial \psi}{\partial m} F + (p_N' - p_N^*) > 0, \text{ with } \frac{\partial \psi}{\partial m} = \mu \frac{\partial \psi}{\partial m} F + (p_N' - p_N^*) > 0.$$

$$\frac{\partial W'}{\partial \kappa} = \mu \lambda \psi F + [\nu \lambda(1 - \nu)[(1 - \mu)(1 - m') > 0],$$

$$\frac{\partial W'}{\partial \psi} = (1 - F)[\mu A - \nu \lambda p_N^* + (1 - \nu)(\mu A + (1 - \mu)p_N^*)] > 0,$$

$$\frac{\partial W'}{\partial \psi} = \left[ \psi(\nu A p_N^* + (1 - \nu)A) - \eta - [\mu A m^* + \nu \lambda p_N^* + (1 - \nu)(\mu A + (1 - \mu)p_N^*)](1 - m')] \right]$$

$$= -\kappa (\mu A - p_N^*) m^* + (1 - \psi)p_N^* < 0. \quad (42)$$

Guarantee purchases by low-cost lenders unambiguously lowers welfare. The direct effect of higher \( m' \) is higher adverse selection in guarantee market and lower adverse selection in the secondary market for non-guaranteed loans. For the price dimension of the allocative efficiency, the former adverse selection redistributes resources from low-cost lenders to high-cost lenders that both have the same expected utility of consumption, and thus there in no direct impact on the social gains from trade. In contrast, the adverse selection in the market for non-guaranteed loans redistributes resources from liquidity shocked lenders to lenders without liquidity shock, reducing the social gains from trade and allocative efficiency.

The key negative effect of guarantee purchases by low-cost lenders is that it reduces guarantee purchases by high-cost lenders. Thus, the overall adverse selection in both the market for guaranteed and the market for non-guaranteed loans increases with a negative effect on allocative efficiency (lower \( p_N^* \) and \( p_N' \)). The guarantee purchases by low-cost lenders also increases screening incentives, but this has negative effects on welfare \( \partial W'/\partial m' < 0 \) because higher screening is due to two factors: the option to selectively buy guarantees for lemons by low-cost lenders (marginal benefits of \( \kappa(1 - \psi)p_N^* \)) and lower benefits of guarantees for high-cost lenders due to adverse selection in guarantee market (benefits lowered by \( \kappa(\mu A - p_N^*) \), see equation (42)). Both factors reduce welfare.

The planner who observes screening costs and makes guarantee choice on behalf of lenders eliminates guarantee purchases by low cost lenders, \( m' = 0 \), and chooses the welfare
benchmark of the main model, \( m = m^P \). Next, a regulator who does not observe screening costs cannot directly eliminate adverse selection in the loan guarantee market. However, the regulator can improve welfare by subsidizing loan guarantees. The optimal subsidy in the liquid equilibrium balances the social benefits, which includes a higher price \( p_G \), and the guarantee costs. Moreover, a guarantee subsidy eliminates the welfare-dominated equilibrium with an illiquid guarantee market.

### B.9 Proof of Proposition 9

We derive the privately optimal guarantee coverage \( \omega^* \). The price for guaranteed loans is

\[
p_C = \frac{\nu \lambda p_N + (1 - \nu)(1 - \mu)p_N}{\nu + (1 - \nu)(1 - \mu)} = \frac{\nu + (1 - \nu)(1 - \mu) \mu A}{\nu + (1 - \nu)(1 - \mu)} \mu A,
\]

which implies that \( p_C \) monotonically increases in guarantee coverage, \( \frac{dp_C}{\omega} > 0 \). If \( \omega < \frac{(1 - \nu)(1 - \mu) \mu A}{1 - \nu} \), then high-quality guaranteed loans are not sold in the market as \( p_C \lambda < A - k \) and the price for guaranteed loans drops further to \( p_C(\omega) = \omega \mu A \). Lenders who buy a guarantee again do not screen, so they solve

\[
\max \nu \lambda p_N + (1 - \nu)[\mu(A - k) + (1 - \mu)p_N] = \frac{\nu + (1 - \nu)(1 - \mu)[\nu + \nu(\omega - 1)(\lambda - 1)]}{\nu + (1 - \nu)(1 - \mu)} \mu A.
\]

Since this expected payoff increases in \( \omega \), the corner solution \( \omega^* = 1 \) is privately optimal.

Next, consider the planner’s choice of guarantee coverage. The payoff of non-guaranteed low-cost lenders, \( \nu \lambda p_N + (1 - \nu)A - \eta_i \), and high-cost lenders, \( \nu \lambda p_N + (1 - \nu)[\mu A + (1 - \mu)p_N] - \eta_i \), also increases in \( \omega \) since guarantee coverage raises the price of non-guaranteed loans,

\[
\frac{dp_N}{\omega} > 0.
\]

Hence, the planner also chooses full coverage, \( \omega^{SP} = 1 \):

\[
\omega^{SP} = \arg\max_\omega \nu \lambda p_N (A - k) + \frac{\nu + (1 - \nu)(1 - \mu)[\nu + \nu(\omega - 1)(\lambda - 1)]}{\nu + (1 - \nu)(1 - \mu)} \mu A - \nu \lambda p_N (1 - F) (1 - m) + \int \delta dF(\tilde{\eta})
\]

\[
+ \frac{\nu \lambda p_N (1 - \nu)(1 - \mu)p_N}{\nu + (1 - \nu)(1 - \mu)} \mu A - \nu \lambda p_N (1 - F) (1 - m) - \nu \lambda p_N (1 - F) (1 - m) - \nu \lambda p_N (1 - F) (1 - m)
\]

s.t. (1) and (2'), where equation (43) is obtained after substituting the indifference condition (3). The solution, \( \omega^{SP} = 1 \), follows from

\[
\frac{\partial W}{\partial \omega} > 0, \quad \frac{\partial W}{\partial \eta} > 0, \quad \frac{\partial W}{\partial \eta} = 0, \quad \frac{\partial \omega}{\partial \omega} > 0.
\]

### B.10 Proof of Proposition 10

In the equilibrium with guarantees, the competitive guarantee fee solves \( k = (1 - k)A(1 - \mu) \), yielding the \( k^* \) stated. High-cost lenders are indifferent between guarantee purchase, \( (1 - k) \mu A = \kappa A(\mu - \delta(1 - \mu)) \), and no guarantee, \( \nu \lambda p_N + (1 - \nu)[\mu A + (1 - \mu)p_N] \), which yields the stated expressions for \( p_N^* \). Combining this equation with the competitive price
in equation (2') gives \( m^* \) stated in the proposition. Finally, substituting \( p_N^U \) into equation (1) gives the screening cost threshold \( \eta^* \) stated in the proposition. Since the price \( p_N^U \) must satisfy condition (8), a necessary condition for a liquid equilibrium when guarantees are used is \( \nu [\mu - \delta (1 - \mu)] \lambda^2 - [\nu + \delta(1 - \mu)(1 - \nu)] \lambda - (1 - \nu)(1 - \mu) \geq 0 \). Since only the larger root of this quadratic condition is positive, the condition collapses to \( \lambda \geq \lambda^* \) if

\[
\frac{\nu + \delta(1 - \mu)(1 - \nu)\lambda^2}{2 \nu [\mu - \delta (1 - \mu)]} + \frac{1 - \nu}{\nu \mu - \delta(1 - \mu)} \geq 0.
\]

Guarantees take place on the subset \( A < \lambda^* \) or alternatively when \( \mu > \tilde{\nu}_G \), where thresholds \( \lambda^* \) and \( \tilde{\nu}_G \) are implicitly defined by combining \( p_N^U \) and equation (2') evaluated at \( m^* = 0 \):

\[
\kappa (1 - \delta) F(\tilde{\nu}) = (1 - F(\tilde{\nu}))[\mu (1 - \nu) (\lambda - 1) - \frac{\kappa}{\nu} \tilde{\nu} (1 - \nu) (1 - \mu)] .
\]

Regarding normative implications, it is easy to show that the planner’s choice of guarantees exceeds the unregulated level, following the same steps as in Appendix B.3.

**B.11 Proof of Proposition 11**

This proof proceeds as follows. First, we define the equilibrium in which lenders can sell a share of the loan and financiers update their beliefs based on it. Second, we show that risk retention via partial loan sales cannot result in a separating equilibrium in which loan quality is revealed at \( t = 1 \). The only exception is the corner solution with full screening and effectively just one type of lenders. Finally, we show that our main results carry over to the continuum of pooling equilibria sustained by particular out-of-equilibrium beliefs.

**Definition 5.** An equilibrium comprises screening \( \{s_i \} \), guarantee choice \( \{\ell_i \} \), loan sales \( \{q^G_i, q^N_i \} \), beliefs about loan quality \( \phi_{t,k} \), prices \( p_G \) and \( p_N \), and a fee \( k \) such that:

1. At \( t = 1 \), for each \( \lambda \) and \( A_i \), each lender i optimally chooses loan sales, \( q^G_i \) and \( q^N_i \).

2. At \( t = 0 \), each lender i chooses screening \( s_i \) and loan guarantee \( \ell_i \) to solve

\[
\max_{s_i, \ell_i, k_1, k_2} E[\lambda r_{c_1} + c_2 - n s_i] \quad \text{subject to}
\]

\[
c_1 = q^G_i p_G + q^N_i p_N, \quad c_2 = (\ell_i - q^G_i)\pi + (1 - \ell_i - q^N_i)A_i, \quad \Pr\{A_i = A\} = \psi s_i + \mu (1 - s_i).
\]

3. At \( t = 1 \), financiers use Bayes’ rule to update their beliefs \( \phi_{t,1}(q^G_i, q^N_i, \ell_i) \) on the equilibrium path, and prices \( p_G \) and \( p_N \) are set for financiers to expect to break even.

4. At \( t = 0 \), financiers use Bayes’ rule to update their beliefs \( \phi_{t,0}(\ell_i) \) on the equilibrium path, and the fee \( k \) is set for financiers to expect to break even.

**Risk retention as signal of loan type.** Suppose there is a separating equilibrium with both high-cost and low-cost lenders. In such an equilibrium, lenders with different loan qualities choose different risk retention and thus financiers learn the loan quality reflected in the price. Sellers of high-quality loans choose \( q^N = 0 \) (since \( \ell_i \in \{0, 1\} \)), and sellers of low-quality loans choose \( q^N = 1 \), and thus \( p_N(q^N) = A \) and \( p_N(q^N') = 0 \). For this
equilibrium to exist, the sellers of low-quality loans must find it optimal to signal their type and receive the zero price rather than mimic the risk-retention of sellers with high-quality loans. Since lenders cannot commit to negative consumption, high-cost lenders with lemons always want to mimic sellers with high-quality loans since \( q^N p_N (q^N) = q^N A = q^N p_N (q^N) = 0 \). Hence, there exists no separating equilibrium with partial screening, \( q^s < \eta \).

However, there could exist an equilibrium with \( q^N < 1 \), where all lenders screen and, thus, for \( \psi \rightarrow 1 \) the quantity of low-quality loans originated vanishes and so does uncertainty about the quality of loans traded. This equilibrium is pooling as the vanishing amount of lenders with low-quality loans mimic the risk retention of lenders with high-quality loans, but the adverse selection in the market for non-guaranteed loans vanishes. We derive the threshold screening cost by equating the payoff from screening, \( \nu \lambda p_N q^N + \nu (1 - q^N) A + (1 - \nu) A - \eta \), and payoff when not screening, \( \nu \lambda p_N q^N + (1 - q^N) \mu A + (1 - \nu) (\mu A + (1 - \mu) p_N q^N) \):

\[
\eta = (1 - \mu) \nu (1 - q^N) A + (1 - \nu) (A - p_N q^N) = (1 - \mu) (1 - q^N) A,
\]

where the second equality comes from \( p_N = A \) (under screening by all lenders and \( \psi \rightarrow 1 \)). Equation (45) implies that there are no high-cost lenders, \( \eta \geq \tilde{\eta} \), if retention is large enough, \( (1 - q^N) \geq \frac{\eta}{1 - p_N A} \). Thus, a sufficient condition for ruling out this equilibrium is \( \eta \geq (1 - \mu) A \).

**Pooling equilibria with partial sales.** The remainder of the proof focuses on pooling equilibria with adverse selection and shows that our main results are qualitatively unchanged. Let the maximum loan sales consistent with full screening, \( q^s \geq \tilde{\eta} \), be \( q^N \equiv \min \{0, 1 - \frac{\eta}{1 - p_N A} \} \). Then there exists a continuum of PBE with adverse selection, where \( q^N \in (q^N, 1] \) in the appropriately generalized liquid equilibrium, \( \lambda > \lambda_G (q^N) \), and out-of-equilibrium beliefs \( \phi_{i,1} = 0 \) if \( q^N \neq q^N \).

If guarantees are used in this equilibrium, high-cost lenders have to be indifferent between payoff when not buying guarantee, \( \nu \lambda p_N q^N + \nu (1 - q^N) \mu A + (1 - \nu) (\mu A + (1 - \mu) p_N q^N) \), and when buying guarantee, \( s_k A \). Equating those payoffs determines the price of non-guaranteed loans:

\[
p_N = \frac{\nu \lambda A \left[ \lambda + \frac{(1 - \mu)(1 - q^N)}{\eta} \right]}{\nu \lambda + (1 - \nu)(1 - \mu)},
\]

which is a generalization of \( p_N^* \) in Proposition 2. It decreases in \( q^N \), \( dp_N / dq^N < 0 \), because higher sales of non-guaranteed loans make guarantees relatively less attractive, and a lower price of non-guaranteed loans satisfies the indifference about guarantee. Using (45), the screening cost threshold is

\[
q^s = \frac{(1 - \mu) \nu A}{\nu \lambda + (1 - \nu)(1 - \mu)} \left[ (1 - \nu)(1 - \mu) + \nu (1 - q^N) \right],
\]

which is a generalization of the threshold in Proposition 2. It decreases in \( q^N \), \( dq_N / dq^N < 0 \), since a higher \( q^N \) lowers the net benefits of screening from loans held to maturity upon a liquidity shock, \( (1 - \mu) \nu (1 - q^N) A \), and increases the payoff from the sale of lemons when not screening, \( (1 - \nu) p_N q^N \), where \( dp_N q^N / dq^N > 0 \). Combining equation (46) with equation (2') yields

\[
m^* = 1 - \frac{[\nu q^N (1 - \mu) - \mu (\lambda - 1)(1 - q^N)] F(q^*)}{\mu (\lambda - 1)(1 - \nu)(1 - \mu) + (1 - q^N) \nu (1 - F(q^*))}.
\]
which generalizes the expression for $m^*$ stated in Proposition 2. Hence, $m^* > 0$ whenever

$$A < \tilde{A}_G(q^N) \equiv \frac{\nu \lambda + (1 - \nu)(1 - \mu)}{(1 - \mu)\nu(N(1 - \nu)(1 - \nu) + (1 - q^N)\nu)} \tilde{F}^{-1} \left( \frac{\mu(\lambda - 1)(1 - \nu)(1 - \mu) + (1 - q^N)\nu}{\nu(1 - \mu)q^N + \mu(\lambda - 1)(1 - \nu)(q^N - \mu)} \right).$$

The equilibrium guarantee condition can also be expressed as $\mu > \tilde{\mu}_G(q^N)$, where $\tilde{\mu}_G(q^N)$ is implicitly defined by (48) after substituting $m^* = 0$, similarly as in the main model. It is easy to show that the planner again buys guarantees for more loans than in the unregulated economy, using the same steps as in Appendix B.3.
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