EUROPEAN CENTRAL BANK

EUROSYSTEM

Working Paper Series

Dimitris Christopoulos, Peter McAdam, Elias Tzavalis

Exploring Okun's law asymmetry: an endogenous threshold LSTR approach

No 2345 / December 2019

Abstract

Using a novel methodology, we offer new evidence that a threshold relationship exists for Okun's law. We use a logistic smoothed transition regression (LSTR) model where possible threshold endogeneity is addressed based on copula theory. We also suggest a new test of the linearity hypothesis against the LSTR model. A combination of structural and policy-related variables accounts for changes (rises) in the Okun's parameter in the US in recent decades. Accordingly, the unemployment gap is increasingly associated with a smaller output gap. Whilst the Great Recession accelerated that rise, the bulk of the change occurred beforehand.

Keywords: unemployment, output, asymmetries, logistic transition, endogeneity, copula, Monte Carlo, test for linearity.

JEL: C24, C46, E23, E24.

Non-Technical Summary

Okun's law (OL) refers to a reduced-form relationship between cyclical unemployment and output. It addresses the issue of how much a country's output is "lost" when unemployment exceeds its natural or trend rate. The relationship provides a link between the labor and goods market over the business cycle, and is often considered a key empirical regularity. It is moreover a core part of many macroeconomic models, where the aggregate supply function is derived from combining OL with the Phillips curve which further links to policy trade offs. The relationship also bears implications for macroeconomic policy: it records what rate of growth leads to a reduction in unemployment; plus, it demonstrates that the effectiveness of disinflation policy depends on the responsiveness of unemployment on the rate of output growth.

One common and compelling criticism of the Okun's relationship in the literature, though, is the assumption of linearity. Many studies instead suggest that the relationship is characterized by nonlinearities and asymmetries. A nonlinear asymmetric OL would be an important finding. For instance, it would suggest that the effectiveness (and required 'size') of stabilization policy on the real economy would depend in which 'regime' Okun's relationship then lies. Any non linearity in the relationship would also have implications for macroeconomic projections. Moreover, it may affect other recognised economic relationships such as the price and wage Philips curves.

We use a novel methodology to specifically assess the case for asymmetry. Our econometric framework is an augmented version of the logistic smooth transition regression (LSTR) model. The LSTR model nests the linear and standard threshold specification for "low" and "high" values of its identified regimes. It captures smooth transition across regimes, which may be more reasonable in macroeconomics (compared to abrupt transitions), due to various adjustment mechanisms and frictions.

Notably, though, we depart from the bulk of the literature in that we do not consider the threshold variable to be exogenous. Indeed, in as fundamental and deep-seated a relationship as OL, this is intuitive; if there are asymmetries in OL, then it is likely that they should arise from the workings of the economy itself – either from the operation of booms and busts, or from policy shifts or more persistent structural changes. In so far as the threshold is endogenous (i.e., contemporaneously correlated with the disturbance term of the regression), our empirical methodology explicitly seeks to capture that endogeneity. Failure to do so leads to biased and inconsistent estimates. To deal with threshold endogeneity, we adjust the mean of the model conditional on the regimes identified by the threshold variable for the endogenous effects based on copula approach.

A combination of structural and policy-related variables accounts for changes (rises) in the Okun's parameter in the US in recent decades. Accordingly, the unemployment gap is increasingly associated with a smaller output gap. Whilst the Great Recession accelerated that rise, the bulk of the change occurred beforehand.

1 Introduction

Okun's law (OL) refers to a reduced-form relationship between cyclical unemployment and output (Okun, 1962). It addresses the issue of how much a country's output is "lost" when unemployment exceeds its natural or trend rate. The relationship provides a link between the labor and goods market over the business cycle, and is often considered a key empirical regularity. It is moreover a core part of many macroeconomic models, where the aggregate supply function is derived from combining OL with the Phillips curve which further links to policy trade offs, Mankiw (2015).

One common and compelling criticism of the Okun's relationship in the literature, though, is the assumption of linearity. Many studies instead suggest that the relationship is characterized by nonlinearities and asymmetries (e.g., Virén, 2001; Cuaresma, 2003; Silvapulle et al., 2004).[1] A nonlinear asymmetric OL would be an important finding. For instance, it would suggest that the effectiveness (and required 'size') of stabilization policy on the real economy would depend in which 'regime' Okun's relationship then lies. Any non linearity in the relationship would also have implications for macroeconomic projections. Moreover, it may affect other recognised economic relationships such as the price and wage Philips curves.

We use a novel methodology to specifically assess the case for asymmetry. Our econometric framework is an augmented version of the logistic smooth transition regression (LSTR) model.[2] The LSTR model nests the linear and standard threshold specification for "low" and "high" values of its identified regimes. It captures smooth transition across regimes, which may be more reasonable in macroeconomics (compared to abrupt transitions), due to various adjustment mechanisms and frictions.

Notably, though, we depart from the bulk of the literature in that we do not

[1] The stability and usefulness of OL has also been discussed in Knotek (2007) amongst others, and Perman et al. (2015) provides a meta study. Cuaresma (2003) attributes the instability of this relationship to threshold effects: the effect of growth on unemployment being more pronounced in recessions than in expansions.

[2] See van Dijk et al. (2002) for a survey.

consider the threshold variable to be exogenous.[3] Indeed, in as fundamental and deep-seated a relationship as OL, this is intuitive; if there are asymmetries in OL, then it is likely that they should arise from the workings of the economy itself – either from the operation of booms and busts, or from policy shifts or more persistent structural changes. In so far as the threshold is endogenous (i.e., contemporaneously correlated with the disturbance term of the regression), our empirical methodology explicitly seeks to capture that endogeneity. Failure to do so leads to biased and inconsistent estimates. To deal with threshold endogeneity, we draw on the Christopoulos et al. (2019) approach by adjusting the mean of the model conditional on the regimes identified by the threshold variable for the endogenous effects based on copula approach.

Our approach builds up on the work of Kourtellos, Stengos and Tan (2016) (KST), which include bias correction terms conditional on each regime of the model to account for the endogeneity. Our approach, however, enables us to relax the assumption that the threshold variable is normally distributed by relying on copula theory.[4] In addition, we also allow for different variances of the LSTR regression disturbance term and its correlation structure with the threshold variable across the two regimes. We thus control for threshold variable endogeneity effects on the parameter estimates of the LSTR model, by including copula based transformations of the threshold variable truncated at the location parameter value. The marginal distributions of these transformed variables can be estimated based on a non-parametric or density estimation procedure.

This makes our approach of dealing with threshold endogeneity quite general and flexible. Indeed, we avoid the problem of availability of instruments (or 'weak' instruments), in the case where one would like to estimate the parameters of the model based on an instrumental variables method (e.g., as first pointed out by Park

[3] As Kourtellos, Stengos and Tan (2016) discuss, the assumption of threshold exogeneity undermines the practical usefulness of threshold regression models, since many plausible threshold variables (their examples include trade shares, political risk) are very likely to be endogenous to the process under consideration.

[4] See Patton (2006) for a discussion of, and an economic application of, copula methods.

and Gupta, 2012). Furthermore, to test for smooth transition effects in the data, we enhance our approach by suggesting a new likelihood ratio test to detect linearity against LSTR effects under threshold endogeneity. Both the estimation methodology of our approach and the power performance of the suggested LR test are evaluated through a Monte Carlo (MC) study.

The paper is organized as follows. Section 2 provides a basic discussion and estimation of OL. Section 3 formally introduces the LSTR model. We demonstrate that, if the threshold variable and disturbance terms in the transition regression are correlated, then estimates of the threshold will be biased (or inconsistent). We then present our approach to adjust the model for the endogeneity of the threshold variables, based on copula theory. The adjusted model is estimated using a two-step concentrated nonlinear least squares method. The method is assessed in a MC study in the appendix. We consider the cases that the threshold variable and disturbance term follow a normal as well as a Student-t distribution. The former allows us to make comparisons to KST's approach (as we have adapted it to the LSTR framework). Generating data from the Student t distribution will show the robustness of the method to fat tails of threshold variables often met in practice.

Section 4 describes and motivates our new linearity testing procedure to test for smooth transition threshold effects against linearity under threshold endogeneity. Section 5 explores threshold choices within the OL framework. We test three broad categories drawn from the literature, classified as (i) *Demand & Cyclical* pressures, (ii) *Structural* features of the economy, and finally, (iii) *Policy & Financial* variables.

Given this background, Section 6 estimates OL assessing the selection of the threshold candidates that indicated nonlinearity in the results of the previous section. These are mostly to be found in the last two categories. We provide estimates based on the standard linear approach, and the LSTR model controlling for endogeneity of the threshold variable based on our suggested approach. We also plot the resulting transition probabilities alongside the threshold as a visual plausibility test. In Section 7, we then combine the most relevant threshold candidates into a composite index.

Section 8 concludes.

2 Okun's Law

OL refers to an inverse reduced-form business-cycle relationship between the unemployment rate (u) and real output (Y). The variables may be in (log or level) first differences (the 'first-differences' model) or expressed in terms of trend-deviations (the 'gap' model). These are respectively, given by,

$$\Delta y_t = \beta_0 + \beta_1 \Delta u_t + \eta_t$$

where $\Delta y_t = \log(Y_t) - \log(Y_{t-1})$, $\Delta u_t = u_t - u_{t-1}$, and,

$$y_t^c = \beta_1 u_t^c + \eta_t$$

where $y_t^c = y_t - y_t^*$ is the deviation of log real GDP from log potential real GDP (i.e., the cyclical component of real GDP: y_t^c). Likewise, $u_t^c = u_t - u_t^*$ is the deviation of the unemployment rate from its 'trend'. Parameter β_1 is the Okun coefficient in each case (expected to be negative reflecting the trade off), and η_t denotes a stochastic disturbance. The higher is $|\beta_1|$, the *steeper* is Okun's relationship and thus the higher the output costs of a rise in cyclical unemployment.

The gap form requires us to capture latent trends and is related more to understanding business-cycle trade offs. It has the advantage of taking into account the state of the economy relative to its trend or natural rate. The difference form posits a linear relationship between the first difference of the log of output and the first difference of the unemployment rate. The two versions are equivalent if potential growth and the natural rate of unemployment are constant. Since this is unlikely to hold, the gap version appears preferable (and, accordingly, is the one we emphasize).[5]

[5] Okun's relationship may also be augmented by 'controls': e.g., factor utilization measures, movements in hours worked, capital stocks, the labor force etc – i.e., the 'production function' approach of Pra-

To derive the cyclical component of the series, we explored several different filters common in the literature. In many cases, they gave a relatively similar picture but we chose the Christiano-Fitzgerald one since, for our series of interest, it identified the appropriate frequencies relatively well (i.e., when approximating the ideal band-pass filter over the standard Burns-Mitchell business-cycle frequency of $6 - 32$ quarters). Figure 1 shows both filtered series, the Okun scatter plot, the rolling regression estimates (based on a rolling window of 75 quarters), over 1950q1-2018q4, plus the Andrews (1993), and Andrews and Ploberger (1994) F^{sup} test statistic searching for a break in the above coefficient at unknown date (over the whole sample). Together with the rolling regression estimates, this test statistic can indicate if the Okun relationship is stable and linear relationship, over the sample. If it is not, it can indicate the most likely date after which a break occurs.

Some basic takeaways of asymmetry are revealed by the plots. For example negative output gaps are generally deeper and more abrupt than positive ones (see also Rothman, 1998). The gaps can also be quite changeable: in the first half (or at least second third) of the sample output volatility far exceeded that of unemployment, but became closer in the subsequent decades. Finally, looking at the closeness of the scatter points, we see that the Okun co-movement has been strong at some times and weak at others (suggesting some deviations from linearity). Rolling regression analysis, moreover, suggests that the (unconditional) Okun coefficient has been rising over time. Moreover, that OL may not be stable and linear can also be justified by the F^{sup} test statistic indicating a shift in this coefficient after 2000 (i.e., 2000q4). The 95%

chowny (1993) (see also Gordon, 1984; Freeman, 2001). We choose to work with the simpler form, since it is the most common representation in the literature (and thus admits easy benchmarking). Moreover, although the appeal of the (production-function) approach is due to its inclusion of these additional margins, many of the proposed controls (in cyclical form) such as hours worked are so heavily correlated with unemployment as to potentially dilute any intended structural interpretation of the parameters. Moreover, the Prachowny derivation is a special case assuming that aggregate production is Cobb Douglas and that the capital stock and a disembodied technology factor are always at their long-run levels; these are counter factual in the US aggregate data, see Klump et al. (2007).

confidence interval of this date is 1998q1:2006q1.[6]

Indeed, over our sample the economy has undergone several major events: the productivity slowdown of the early 1970s, the major oil shocks in that decade, and the "Great Moderation" of reduced macroeconomic volatility from the mid 1980s onwards (variously attributed to 'better' monetary-policy stabilization, structural change, greater labor flexibility). Regarding the latter, a period of prolonged expansion may have affected the Okun trade off by enticing more marginal, potentially more flexible workers into the labor force. Moreover, such enhanced economic predictability and perceived economic prospects may have encouraged more financial leverage, as reflected in the build up of public and private debt. Finally, there was also the financial crisis from the late 2000s onwards (the "Great Recession"), followed in turn by extraordinary monetary (and to a lesser extent fiscal) policy accommodation.

There were also, though, major shifts in cyclicality patterns from the mid 1980s onwards, see Fernald and Wang (2016): labor productivity turned from pro to counter cyclical – largely reflecting the weakening pro cyclicality of factor utilization and hence of raw TFP growth.[7] Weakening pro-cyclicality in utilization points to reduced factor hoarding. This may be due to increased flexibility in the economy, as reflected by the expansion of female labor participation and perhaps declining labor power; it may also reflect the decline of manufacturing and manufacturing employment (where utilization was traditionally a more important margin of adjustment).

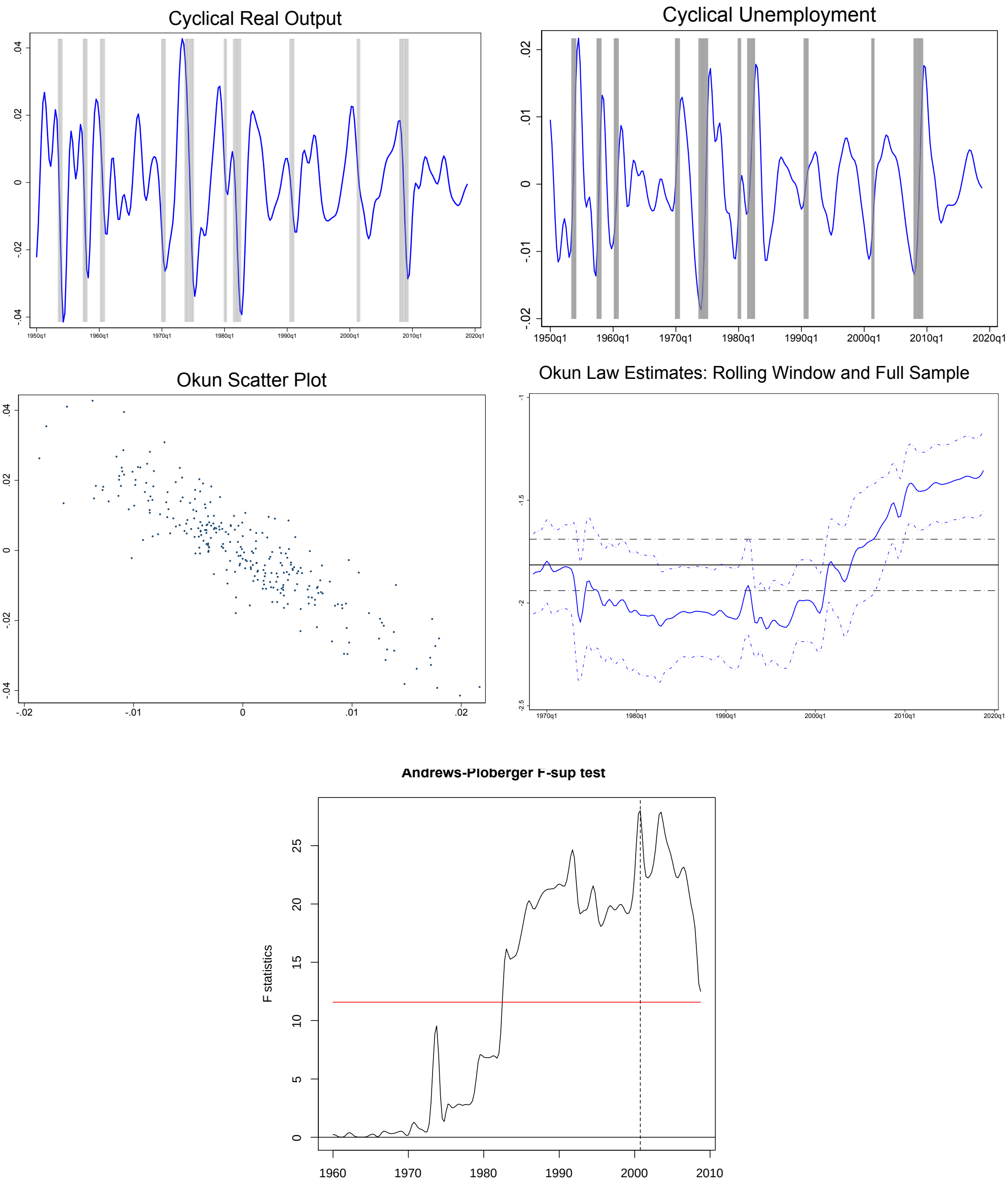
Estimates of OL (both variants) are shown in Table 1. Results corroborate those of the simple scatter plot. They suggest that a unit increase of cyclical unemployment is associated with a decline of just under 2% in output (in cyclical or difference terms). Indeed in line with Okun's original work, most US studies locate the (absolute) coef-

---

[6] The critical values of the F^{sup} test statistic are based on Hansen's (1997) algorithm.

[7] To see why, we can decompose labor productivity growth into growth in TFP, capital deepening and labor quality. Since the latter two are robustly counter-cyclical, the change in labor productivity essentially comes from the weakening pro cyclicality of raw TFP growth. Indeed in recent decades TFP growth (net of its 'pure' component) has become so much less pro-cyclical that overall labor productivity is now judged net counter cyclical.

Figure 1: Okun Law Variables, Correlations and Stability



Notes: Shaded gray areas represents NBER recession dates at quarterly frequency. In the rolling window coefficients graphs (for the gap version of the Okun's Law) the 95% standard errors are given by the blue dashed lines (the window size is 75 quarters).

Table 1: Okun Law Coefficients

Parameters	Δy_t †	y_t^c ‡
β_0	0.007 (0.004)	
β_1	-1.661 (0.107)	-1.815 (0.068)
AIC	-993.318	-986.366

Notes:

† First-differences form: $\Delta y_t = \beta_0 + \beta_1 \Delta u_t + \eta_t$ where $\Delta y_t = \log(Y_t) - \log(Y_{t-1})$, and $\Delta u_t = u_t - u_{t-1}$. Calculations based on year-on-year changes generate similar results. The raw data are taken from Federal Reserve Bank of St. Louis (FRED): the unemployment Rate (*UNRATE*) and Real GDP (*GDPC1*).

‡ The gap form is given by, $y_t^c = \beta_1 u_t^c + \eta_t$ where $y_t^c = y_t - y_t^*$ is the deviation of log real GDP from log potential real GDP (i.e., the cyclical component of real GDP: y_t^c). Likewise, $u_t^c = u_t - u_t^*$ is the deviation of the unemployment rate from its 'trend'.

Numbers in brackets below the coefficients represent bootstrapped standard errors. AIC denotes the Akaike Information criterion (for a given model size, the lowest AIC score is preferred).

ficient in a $(1, 3]$ interval. That the coefficient typically exceeds one in absolute value (i.e., cyclical output drops *by more* than the increase in cyclical unemployment) reflects different adjustment margins that amplify movements in unemployment on output. For example some fraction of the unemployed may cease job search thus reducing the labor force, labor productivity may fall (reflecting labor hoarding), hours worked may fall, and the economy may weaken through the normal Keynesian spending multiplier.

3 A LSTR Model with Threshold Endogeneity

A type of structural threshold model that we can employ to investigate threshold effects is the following two-regime logistic smooth transition regression (LSTR) model:[8]

$$y_t = x_t'\beta_1(1 - \boldsymbol{g}(z_t;\gamma,\delta)) + x_t'\beta_2\boldsymbol{g}(z_t;\gamma,\delta) + \varepsilon_t, \tag{1}$$

where $y_t = y_t^c$ and $x_t = (1, u_t^c)'$ is a $(M \times 1)$ vector of independent variables, with $M = 2$, $\beta_h = (\beta_1, \beta_2)'$ denotes the vector of the slope coefficients of the elements of vector x_t in two distinct regimes, where $h = \{1, 2\}$ denotes the two regimes "**1**" and "**2**", and $\varepsilon_t \sim IID(0, \sigma_\varepsilon^2)$ is the disturbance term. Function $\boldsymbol{g}(z_t;\gamma,\delta)$ is a continuous logistic function of the observable variable z_t, known as the threshold variable, which governs the transition between the two regimes:

$$\boldsymbol{g}(z_t;\gamma,\delta) = \frac{1}{1 + \exp(-\gamma(z_t - \delta))}. \tag{2}$$

The value of $\delta \in \mathbb{R}$ is known as the location or threshold parameter, which defines the two regimes. Parameter $\gamma > 0$, the speed-of-transition parameter, determines the smoothness of the transition from one regime to the other. When $\gamma \to \infty$, $\boldsymbol{g}(z_t;\gamma,\delta)$ tends to indicator function $\mathcal{I}(z_t > \delta)$, for all i. In this case, the LSTR model can be approximated by the standard threshold model and thus, the transmission between regimes is abrupt: the shift from regime "**1**" to "**2**" becomes instantaneous at $z_t = \delta$. On the other hand, when $\gamma \to 0$, then $\boldsymbol{g}(z_t;\gamma,\delta) \to \frac{1}{2}$. In this case, the LSTR model reduces to a linear model, with vector of parameters $\boldsymbol{\beta} = \frac{1}{2}(\beta_1 + \beta_2)$.

Through transition function $\boldsymbol{g}(z_t;\gamma,\delta)$, the slope coefficients of model (1) are time-varying, depending on the value of z_t. One econometric problem often encountered in practice, though, when estimating model (1) is the endogeneity of the threshold

[8] See Teräsvirta (1994). Note that the inclusion of a (potentially regime-specific) intercept in the LSTR model algebra is for generality.

variable z_t with the disturbance term ε_t. Let us therefore split the sample of z_t across the two regimes as follows: $\mathcal{Z}_1 = (-\infty, \delta]$ and $\mathcal{Z}_2 = (\delta, \infty)$, based on a value of location parameter δ, and assume that the disturbance term ε_t is distributed differently across the two regimes, i.e., $\varepsilon_{h,t} \sim IID(0, \sigma_h^2)$, $h = \{1, 2\}$. Endogeneity between z_t and $\varepsilon_{h,t}$ means that $\mathbb{E}(\varepsilon_{1,t}|\mathcal{Z}_1) \neq 0$ and $\mathbb{E}(\varepsilon_{2,t}|\mathcal{Z}_2) \neq 0$, which implies that estimates of the location parameter δ will be biased (or inconsistent). This in turn implies that the estimates of all slope coefficients of model (1) will also be biased.

There are different estimation methods to tackle this endogeneity problem in econometrics (Antonakis et al. (2014) offer a survey). The method chosen here draws on that of Christopoulos et al. (2018) which employs copulas to capture the dependence between the disturbance term ε_t and the threshold variable z_t.[9] This method allows for the level of dependence between $\varepsilon_{h,t}$ and z_t, as well as the variance of $\varepsilon_{h,t}$ to be different across the two regimes. Copulas are functions which can express joint probability distributions (or densities) of random variables $\varepsilon_{h,t}$ and z_t in terms of their marginal probability density functions (PDFs) and a copula function capturing the dependence between $\varepsilon_{h,t}$ and z_t. Based on the copula function, we can derive a single-correlation structure of $\varepsilon_{h,t}$, $h = 1, 2$, which can be used to control the endogeneity of z_t in model (1). To this end, consider the following definition:

Definition: Let $p_\delta \in (0, 1) = P(z_t \leq \delta) = F_z(\delta)$, where F_z is the distribution function of z_t. The joint distribution of the pair of random variables ($\varepsilon_{h,t}$,$z_t \in \mathcal{Z}_h$), $h = \{1, 2\}$, can be written based on copulas as follows:

$$F_{\varepsilon_h z}(\varepsilon_{h,t}, z_t|\mathcal{Z}_h) = C_h \left(F_{\varepsilon_h}(\varepsilon_{h,t}), F_{z|\mathcal{Z}_h}(z_t|\mathcal{Z}_h) \right),$$

where C_h is a bivariate appropriately scaled copula, with $C_h : [0, 1]^2 \rightarrow [0, 1]$, F_{ε_h} is

[9] Although in their study they considered the simpler TAR model.

the marginal distribution function of $\varepsilon_{h,t}$, and

$$F_{z|\mathcal{Z}_1}(z_t|\mathcal{Z}_1) = \frac{F_z(z_t)}{p_\delta} \qquad \text{if} \quad 0 \leq F_z(z) \leq p_\delta$$

$$F_{z|\mathcal{Z}_2}(z_t|\mathcal{Z}_2) = \frac{F_z(z_t)-p_\delta}{1-p_\delta} \quad \text{if} \quad p_\delta \leq F_z(z) \leq 1,$$

are the truncated from above and below location parameter value δ distribution functions of z_t, respectively.

From the above definition, it is clear that the truncated joint distribution of the pair of variables ($\varepsilon_{h,t}$,z_t), $F_{\varepsilon_h z}(\varepsilon_{h,t}, z_t|\mathcal{Z}_h)$, for $h = \{1, 2\}$, constitutes a copula C_h on $[0, 1]^2$ with uniformly distributed on $[0, 1]$ margins, since $F_{z|\mathcal{Z}_h}(z_t|\mathcal{Z}_h)$ is appropriately scaled to integrate to unity. Based on copula theory, the conditional distribution function of $\varepsilon_{h,t}$ on $z_t \in \mathcal{Z}_h$ can be derived from C_h as follows:

$$F_{\varepsilon_h|z}(\varepsilon_{h,t}|\mathcal{Z}_h) = \frac{\partial}{\partial F_{z|Z_h}} C_h \left(F_{\varepsilon_h}(\varepsilon_{h,t}), F_{z|Z_h}(z_t|\mathcal{Z}_h) \right), \tag{3}$$

while the conditional probability density function related to this distribution is given as,

$$f_{\varepsilon_h|z}(\varepsilon_{h,t}|\mathcal{Z}_h) = \frac{\partial^2}{\partial \varepsilon_h \partial F_{z|Z_h}} C_h \left(F_{\varepsilon_h}(\varepsilon_t), F_{z|\mathcal{Z}_h}(z_t|\mathcal{Z}_h) \right) \tag{4}$$

$$= c_h \left(F_{\varepsilon_h}(\varepsilon_t), F_{z|Z_h}(z_t|\mathcal{Z}_h) \right) f_{\varepsilon_h}(\varepsilon_{h,t}),$$

where $c_h \left(F_{\varepsilon_h}(\varepsilon_t), F_{z|\mathcal{Z}_h}(z_t|\mathcal{Z}_h) \right) = \frac{\partial^2}{\partial F_{\varepsilon_h} \partial F_{z|\mathcal{Z}_h}} C_h \left(F_{\varepsilon_h}(\varepsilon_t), F_{z|\mathcal{Z}_h}(z_t|\mathcal{Z}_h) \right)$ is the copula density function corresponding to C_h and $f_{\varepsilon_h}(\varepsilon_t) = \frac{\partial}{\partial \varepsilon_h} F_{\varepsilon_h}(\varepsilon_t)$ is the probability density of $\varepsilon_{h,t}$.[10] Based on the above relationships for $f_{\varepsilon_h|z}(\varepsilon_{h,t}|Z_h)$ and $F_{\varepsilon_h|z}(\varepsilon_{h,t}|Z_h)$,

[10] Note that the joint probability density corresponding to $F_{\varepsilon z}(\varepsilon_t, z_t|\mathcal{Z}_h)$ is given as follows:

$$f_{\varepsilon z}(\varepsilon_t, z_t|\mathcal{Z}_h) = c_h \left(F_\varepsilon(\varepsilon_t), F_{z|\mathcal{Z}_h}(z_t|\mathcal{Z}_h) \right) f_\varepsilon(\varepsilon_t) f_{z|\mathcal{Z}_h}(z_t|\mathcal{Z}_h)$$

it can be shown that, in the case where C_h is Gaussian and $\varepsilon_{h,t} \sim \mathcal{N}(0, \sigma_h^2)$, which is often assumed in regression models, $\varepsilon_{h,t}$, has the following single-factor correlation structure:[11]

$$\varepsilon_{h,t} = \omega_h z_{h,t}^* + Var(\varepsilon_{h,t}|z_{h,t}^*)^{1/2} u_{h,t}, \quad h = \{1, 2\} \tag{5}$$

where $z_{h,t}^*$ constitutes a transformation of z_t which is distributed as $\mathcal{N}(0, 1)$ and defined as,

$$z_{h,t}^* = \Phi^{-1}\left(F_{z|\mathcal{Z}_h}(z_t|\mathcal{Z}_h)\right)$$

where Φ^{-1} is the quantile function of the standard normal distribution. Moreover,

$$\omega_h = \sigma_h \rho_{u_h, z_h^*}$$

where ρ_{u,z^*} is the Pearson correlation coefficient between $\varepsilon_{h,t}$ and $z_{h,t}^*$, $u_{h,t}$ is an $IID(0, 1)$ disturbance is independent of z_t^* and

$$Var(\varepsilon_{h,t}|z_{h,t}^*) = \sigma_h^2(1 - \rho_{u,z^*}^2).$$

Using the above representation of relationship of $\varepsilon_{h,t}$ enables us to write model (1) as,

$$y_t = \begin{cases} x_t'\beta_1(1 - \boldsymbol{g}(z_t; \gamma, \delta)) & + \quad \omega_1 z_{1,t}^* + e_{1,t} \quad \text{if} \quad z_t \in \mathcal{Z}_1 : \text{Regime “}\mathbf{1}\text{”} \\ \\ x_t'\beta_2 \boldsymbol{g}(z_t; \gamma, \delta) & + \quad \omega_2 z_{2,t}^* + e_{2,t} \quad \text{if} \quad z_t \in \mathcal{Z}_2 : \text{Regime “}\mathbf{2}\text{”} \end{cases} \tag{6}$$

where $e_{h,t} = -\omega_h z_{h,t}^* + \varepsilon_{h,t}$ is a disturbance term with conditional mean $\mathbb{E}(e_t|z_{h,t}^*) = 0$ and conditional variance $Var(e_{h,t}|z_{h,t}^*) = \sigma_\varepsilon^2(1 - \rho^2)$, since $\mathbb{E}(\varepsilon_{h,t}|z_{h,t}^*) = \omega_h z_{h,t}^*$ from

where $f_{z|\mathcal{Z}_1}(z_t|\mathcal{Z}_1) = \frac{f_z(z_t)}{p_\delta}$, $-\infty < z_t \leq \delta$, and $f_{z|\mathcal{Z}_2}(z_t|\mathcal{Z}_2) = \frac{f_z(z_t)}{1-p_\delta}$, with $\delta < z_t \leq \infty$, are the truncated from above and below p_δ probability densities $f_z(z_t)$.

[11] See, for example, Joe (2014), Christopoulos et al. (2018).

(5). The last, augmented with random variables $z^*_{h,t}$ version of model (1) can be employed to control for the endogeneity problem of z_t. The random variables $z^*_{h,t}$, added to the rhs of (6), correct the conditional mean of y_t on Z_h, for $h = \{1, 2\}$, for the contemporaneous correlation between $\varepsilon_{h,t}$ and z_t (implying $\mathbb{E}(\varepsilon_{h,t}|Z_h) \neq 0$). This is done, note, without making any assumption about the distribution of z_t. Furthermore, as already highlighted, it allows the distribution of $\varepsilon_{h,t}$ and its correlation structure with z_t to change across the two regimes.

3.1 Estimation Aspects and Monte Carlo Results

Model (6) can be employed to estimate the location parameter δ and its remaining parameters collected in vector $\theta(\delta) = (\gamma,\beta_h, \omega_1, \omega_2)'$ based on a two-step nonlinear least squares (NLLS) method, since the disturbance term $u_{h,t}$ and, hence, $e_{h,t}$ is independent of the transformed variable $z^*_{h,t}$, for $h = \{1, 2\}$.

In particular, δ can be estimated, in a first step, by solving the following NLLS optimization problem:

$$\hat{\delta} = \underset{\delta \in Q_z}{\arg\min} RSS(\delta),$$

where $RSS(\delta) = \sum_{t=1}^{T} \hat{e}^2_{h,t}$, is the residual sum of squares of (6), δ is an interior point of Q_z, since we assume $p_\delta = P(z_t \leq \delta) \in (0, 1)$. To estimate δ, note that we require values of the transformed variables $z^*_{h,t}$, given by $\Phi^{-1}\left(F_{z|\mathcal{Z}_h}(z_t|\mathcal{Z}_h)\right)$. This can be done based on non-parametric estimates of the marginal distribution $F_{z|\mathcal{Z}_h}(z_t|\mathcal{Z}_h)$ (see Silverman, 1986), or based on the empirical cumulative distribution function, denoted ECDF. Given the optimal estimate of $\hat{\delta}$, the slope parameters of model (1) and the speed-of-transition coefficient γ collected in vector $\theta(\hat{\delta})$ can be estimated, in a second step. Following the literature on threshold models (see, e.g., Chan, 1993; Samia and Chan, 2011), the estimator $\widehat{z}_\delta$ is T-consistent and the estimates of vector $\theta(\hat{\delta})$, which correspond to $\hat{\delta}$, are $\sqrt{T}$ asymptotically normal.

As a remark on the above estimation procedure note that, instead of the one dimensional grid search over δ, we can carry out a two dimensional grid search over δ

and γ (see, e.g., Leybourn et al., 1986; and Franses and van Dijk, 2000). This procedure may mitigate optimization problems in estimating γ, due to the nonlinear nature of function $\boldsymbol{g}(z_t; \gamma, \delta)$. Furthermore, it can have better small sample properties, due to grid-search process in estimating δ and γ. Given the estimates of δ and γ, then we can estimate the remaining slope parameters, collected in vector β_h, in a second step.

To evaluate the performance of our estimation approach to successfully control for the endogeneity of threshold variable, we carry out a small MC study, see Appendix A. We consider cases in which the disturbance term $\varepsilon_{h,t}$ and threshold variable z_t are jointly normally and then Student-t distributed, with different values of the speed-of-transition parameter γ and sample sizes T. Generating data from the Student's t distribution, which allows for tail dependence between $u_{h,t}$ and z_t, can show the robustness of our method to such features in the data.

The results of our MC clearly supports the view that our method can successfully control for the endogeneity of z_t on the estimates of location parameter δ. They also show that ignoring this endogeneity leads to series biases in the estimates of δ. These results hold for both the distributions of $\varepsilon_{h,t}$ and z_t considered, meaning that our method is robust to misspecification of these distributions. For the case that $\varepsilon_{h,t}$ and z_t are normally distributed, our method compares favorably to that of Kourtellos et al. (2016) based on the inverse Mills ratios, modified appropriately for the LSTR model (1). Also, our method is robust to the case that both $\varepsilon_{h,t}$ and z_t follow the Student-t distributed. Finally, another interesting result of our MC exercise is that our method can be implemented without any concern for bias or inefficiency of the estimates of δ in the case where the threshold variable z_t is exogenous.

4 A New Test for Linearity Controlling for Threshold Endogeneity

Before estimating model (1), or its extended version (6), a critical prior testing procedure is to diagnose if the data supports our threshold model compared to its linear specification. To this end, we suggest a suitable testing procedure.

We follow recent work in the literature on threshold or LSTR models (see, e.g., Hansen (1996) and KılıÇ, 2016) which is focused on testing H_0: $\beta_1 = \beta_2$ (implying $\gamma \to 0$) against H_a: $\beta_1 \neq \beta_2$ (implying $\gamma > 0$). As noted by KılıÇ (ibid), compared to inference procedures testing for the exclusion restrictions on the threshold variable z_t or its product terms with regressors collected in vector x_t based on an approximation of model (1) under H_0, our suggested procedure may be proved more powerful for γ values far away from the $\gamma = 0$ neighborhood.[12] That is to say where the approximation of (1) is not accurate and depends on the value of the location parameter δ. Furthermore, in our simulations we found that endogeneity of threshold variable z_t makes the power performance of the inference procedures based on the above approximation of (1) even worse, due the estimation bias of the slope coefficients of the auxiliary regression.

More specifically, to carry out a test of H_0: $\beta_1 = \beta_2$ against H_a: $\beta_1 \neq \beta_2$, we rely on the likelihood ratio (LR) test statistic

$$LR(\gamma, \delta) = 2(\log L(\theta(\delta)) - \log L(\beta))$$

where $\log L(\theta(\delta))$ and $\log L(\beta)$ constitute the maximum log-likelihood function of model (1) under the alternative and null hypotheses, respectively. Note that, in order to estimate the model under the alternative hypothesis, we will use the auxiliary regression (6), controlling for the endogeneity of the threshold variable z_t. Since the nuisance parameters δ and γ are not identified under the null, we next suggest a sup-version of statistic $LR(\gamma,\delta)$ (see Andrews and Ploberger, 1994), defined as follows:

$$LR^{\text{sup}} \equiv \sup_{(\gamma,\delta)\in Q_\gamma \times Q_z} LR(\gamma, \delta),$$

[12] The initial literature of tests for linearity against threshold specifications (such as Luukonen et al., 1998) replaced the transition function by a Taylor series expansion which is estimable under the null: with the test amounting to testing for the significance of the interaction of the linear regressors with the polynomial terms. See also Escribano and Jordà (1999), van Dijk et al. (2002).

where Q_γ denotes a compact subspace on the real line, searching for an optimal value of γ and Q_z is defined as before.[13] Since the distribution of $LR^{\sup}$ is non standard under the null, its critical values can be obtained based on a parametric bootstrap procedure, generating data under the null hypothesis of linearity (i.e., H_0: $\beta_1 = \beta_2$).

To evaluate the power performance of statistic $LR^{\sup}$, we carry out a MC exercise in Appendix B. In this exercise, we also consider the case that we ignore the endogeneity of threshold variable z_t. The results of this exercise indicate that $LR^{\sup}$ has satisfactory power. This is true under alternative copula functions and marginal distributions of ε_t and z_t considered in the MC exercise. Another interesting finding is that our test does not lose significant power in the case where z_t is exogenous.

5 Data and Threshold Candidates

Which variables might be associated with threshold effects in OL?

So far we have discussed the possibility of (endogenous) threshold variables leading to threshold effects in Okun's law, without discussing which would be those variable(s). We take an agnostic approach; rather than imposing a particular favored threshold variable, we explore a number of candidates reflecting discussions in the literature. A common feature to all, consistent with our methodology, is their likely endogeneity to the Okun trade off.

Indeed, as already noted, OL is a reduced form relationship, its strength reflects several influences. In that vein, Fernald et al. (2017) and Daly et al. (2018) decompose Okun's coefficient emphasizing the different margins and adjustment channels that firms and households use to respond to different shocks.[14] Although these authors do not explore nonlinearity, their framework is nonetheless suggestive. For instance, labor hoarding (an empirically well-established phenomenon) cushions the unemploy-

[13] Note that, in practice, Q_γ can be set to $Q_\gamma = \left[\frac{1}{10\sigma_z}, \frac{1000}{\sigma_z}\right]$, where σ_z is the standard deviation of the threshold variable z_t, see KılıÇ (2011).

[14] To illustrate, respectively, these are the extensive (e.g., labor force participation and migration etc.) and intensive margins (e.g., hours per worker, factor utilization etc.)

ment consequences of downturns perceived to be temporary or "small". However, a sufficiently large downturn (i.e., in our context, beyond some estimated threshold) may counter labor hoarding incentives, thus changing the output-unemployment nexus. To take another example, a deteriorating sovereign debt outlook may, beyond some point, crowd out activity, impair financial markets and heighten uncertainty, with implications for labor participation and factor intensity.[15] That said, there may be *many* feasible threshold candidates rather than merely a sparse outcome, in effect candidates which may reinforce or counteract one another's effects.[16] Accordingly, we also examine the performance of a composite indicator.

We classify these in three broad (and not necessarily mutually exclusive) categories: (i) *Demand & Cyclical pressures*; (ii) *Structural*; and (iii) *Policy & Financial* (see Table 2).[17] The main data source is the Federal Reserve Bank of St. Louis (FRED) and the BLS (listed are symbol names, plus, where relevant their cyclical equivalent). As before, the sample spans 1950q1 to 2018q4. In the final column, we show the corresponding LR^{sup} test statistic and probability values for the null of linearity against the LSTR specification (performed for the Gaussian copula).

Consider the first category of threshold candidates, *Demand & Cyclical*. Many studies suggest Okun's coefficient moves over the phases of the business cycle, and in a nonlinear, asymmetric fashion (e.g., Lee, 2000; Harris and Silverstone, 2001; Mayes and Virén, 2001; Cuaresma, 2003; Silvapulle et al. 2004). Common rationales for this cyclical asymmetry include: labor hoarding; downward wage rigidity; employment regulations; cyclical firm growth; non-constant factor substitution (Courtney, 1991) etc. In line with this business-cycle asymmetry literature, we consider as possible threshold variables the unemployment and 'natural' unemployment rate, the output

[15] Reinhart et al. (2012) discuss threshold behaviour in the growth-debt nexus.

[16] For an analysis of sparse and dense models, see Giannone et al. (2019).

[17] Many of these are quite representative of macro US series commonly used for econometric studies, and most are included, for instance, in the large dat set of Marcellino et al. (2006) (their economy-wide variables anyway).

gap, output, and inflation rates (which similarly reflect cyclical demand pressures).[18]

Our second category is dubbed *Structural*. Again this reflects various strands of the Okun literature as well as more general themes of structural change in the post-war US economy: namely, that various secular (though not necessarily irreversible) trends may have imparted an effect on Okun's coefficient. For example female labor-market participation has risen, the employment share of manufacturing has declined as has the labor income share, the economy experienced a 'Great Moderation' of reduced volatility etc. It is not unreasonable to suppose that such changes affect the Okun trade off.

The final category assesses *Policy & Financial* variables as threshold candidates in the Okun specification. We include a variety of interest rates,[19] private and public debt ratios. Policy and financial measures, by affecting the different margins underlying Okun's relationship, may thus have an influence on the Okun Law parameters.

Table 2 shows around half the threshold candidates reject linearity (often at or below 1%). Among the first category, demand or cyclical factors, only inflation and capacity utilization come to close to significance (at 10%). This finding is interesting since these have often been the favored candidates of past Okun threshold studies (e.g., Virén, 2001). This, however, need not imply that cyclical pressures do not impact Okun law parameters. But it does suggests that downturns may have long

[18] Note that dichotomous (Heaviside) threshold variables (of the expansion/contraction variety) which are often used in such analyses, such as

$$z_t = \begin{cases} 1 : \text{Expansion} & \text{if } u^c < 0, \text{or } y^c > 0 \\ 0 : \text{Contraction} & \text{otherwise} \end{cases}$$

are not feasible threshold variable types in our framework since they are not continuous.

[19] We also considered the Corporate Bond Credit Spread of Gilchrist and Zakrajšek (2012), and the National Financial Conditions Index (from the FRB Chicago). But their short sample (available from 1973 and 1971, respectively) precluded their use. Although the credit spreads share similar information with the interest rates used since, for instance, they reflect similar monetary or fiscal shocks. Likewise, many variables which might have been of interest, for instance inequality measures, tax burden, 'economic and policy uncertainty' metrics etc were excluded since they tend to be of short sample and/or not available at a quarterly frequency. Further details of these data choices and restrictions are available on request.

and persistent effects; they may induce protracted adjustments in Okun margins that extend well beyond the defined periods of recession.

The second (structural) category, also has some valid threshold candidates. The rolling volatility measures, indicative of the Great Moderation and perhaps reduced macroeconomic risk, can reject linearity. The labor share[20], manufacturing employment share, long term unemployed (percentage unemployed for ≥ 27 weeks), and female participation rates, illustrating important shifts in labor-market features, are also detected.

Finally, policy and financial measures, yield a number of valid threshold candidates. Both long and short policy rates, the shadow short rate (Wu and Xia, 2016) as well as debt ratios, reject the linear specification and are thus viable threshold candidates.

[20] Note, the variant of the labor share that we use is the quarterly indexed one provided by FRED where $2012 = 100$. For comparison purposes we annualized this series and compared it to the Share of Labor Compensation series provided by University of Groningen (also available from FRED), see supplementary material Figure C.1.

TABLE 2: Data: Definitions, Sources and Linearity Testing

Data Series	Symbol	Source (mnemonic)	$LR^{\sup}$
Demand			
Cyclical Unemployment Rate	u^c	FRED (UNRATE)	2.082 [0.266]
Cyclical Real GDP	y^c	FRED (GDPC1)	6.145 [0.189]
Output Gap (% of potential GDP)	og	CBO	5.691 [0.360]
Capacity Utilization	cu	FRED (TCU)	8.723 [0.060]
Consumer Price Inflation (%Δ yoy)	π	FRED (CPIAUCSL)	6.422 [0.058]
Commercial Real Estate Price Inflation	π_H	FRED/Haver Analytics	6.145 [0.180]
Structural			
Rolling 10-year St. Dev. of Cyclical Output (Risk)	$\sigma_{y^c_{t(10)}}$		16.490 [0.000]
Utilization-Adjusted TFP Growth	$gTFP$	Fernald (2018)	5.989 [0.141]
Non-farm Business Sector: Labor Share	$labsh$	FRED (PRS85006173)	39.669 [0.000]
Employment Share: Manufacturing	$lmanu$	FRED (MANEMP/PAYEMS)	24.739 [0.000]
Labor Force Participation Rate (Total; Female)	$pr; pr_f$	FRED (CIVPART; LNS11300002)	5.867 [0.112]; 19.036 [0.001]
Natural Rate of Unemployment	u_n	CBO	19.518 [0.000]
Long-Term Unemployed*	u_l	Bureau of Labor Statistics	31.459 [0.000]
Policy & Financial			
Federal Funds Rate; Long Term Interest Rates (10y Gov. Bonds)**	i_s; i_l	FRED (FEDFUNDS; IRLTLT01USM156N)	30.420 [0.000]; 29.029 [0.000]
Shadow Policy Rate**	$i_{s,wx}$	Wu and Xia (2016)	31.184 [0.000]
Corporate Bond Spread***	i_{Baa}	FRED (BAAFFM)	4.188 [0.166]
Total Debt to GDP (Private NFC; level, cycle)	$dp; dp^c$	FRED and Haver Analytics	16.664 [0.000]; 1.114 [0.421]
Total Debt to GDP (Federal Government; level, cycle)	$d; d^c$	FRED/Haver Analytics	19.472 [0.000]; 1.746 [0.316]

Notes: All series are quarterly and, where relevant, seasonally adjusted and span 1950q1-2018q4 except $i_s, i_{s,wx}$ which start in 1954q3, i_l (1960q1), d_p (1951q4), cu (1967q1).
* Unemployed for $\geq$ 27 weeks (% of civilians unemployed).
** Calculated as end of period. *** Moody's Baa Corporate Bond minus Federal Funds Rate
Numbers in brackets in the final column are bootstrapped probability-values of the threshold linearity tests.

6 Estimation of the LSTR Model for Okun's Law

Having presented our empirical methodology to estimate a LSTR model under endogeneity of the threshold variable, and pretested for linearity, we now estimate the model. To measure directly the magnitude of the shift in the slope coefficient of unemployment, we estimate an augmented version of (1) with the regressors z_1^* and z_2^*:

$$y_t^c = \beta_1 u_t^c + \beta_2 u_t^c \left[1 + e^{-\gamma(z_t - \delta)}\right]^{-1} + \omega_1 z_1^* \left(\mathcal{I}\left(z_t \in \mathcal{Z}_1\right)\right) + \omega_2 z_2^* \left(\mathcal{I}\left(z_t \in \mathcal{Z}_2\right)\right) + e_t. \quad (7)$$

This version nests the no-threshold case, $\beta_2 = 0$, wherein the Okun's coefficient is given solely by β_1. Otherwise, it is given by β_1 in the first regime and $\boldsymbol{\beta} = \beta_1 + \beta_2$ in the second. Thus, β_2 captures the magnitude of the shift of the slope coefficient of unemployment from the first regime to the second, relative to the first.

Table 3 presents model estimates for two cases: Panel A ignores endogeneity of the threshold variable and Panel B allows for it. The latter is based on our preferred estimation approach already presented, including the copula-transformed variables $z_{1,t}^*$ and $z_{2,t}^*$ into the rhs of (7) with the attendant parameters ω_1 and ω_2. For better small sample estimation properties, we present bootstrapped standard errors (in parentheses). These are calculated based on a wild parametric bootstrap method (see Davidson and MacKinnon, 2007). For γ and δ, confidence intervals are shown in braces, based on that bootstrap method. We also present values of information criteria in order to compare the model with its earlier linear (no-regime) specification.[21]

6.1 Results

Table 3 shows the estimated β Okun parameters (again mostly within the $1, 3$ range), and the threshold and speed of adjustment parameters, δ and γ respectively. In the endogenous case, we also show the ω parameters (we scale up by 100 for legibility).

[21] Supplementary Table C.1 presents summary statistics of the threshold variables.

Finally we present the AIC and the sample percentage residing in the second regime, denoted $\mathcal{D}_{\mathbf{2}}$.

To aid interpretation consider the case,

$$\beta_1 \;<\; \boldsymbol{\beta} \;< 0, \tag{8}$$

where, to recall $\boldsymbol{\beta} = \beta_1 + \beta_2$. If condition (8) holds, this implies that a given unemployment gap is associated with a larger output gap in the 1^{st} regime, relative to the 2^{nd}. Accordingly, for a given unemployment gap, we can think of regime "1" with the larger in absolute magnitude in β as the 'steeper' regime, while regime "2" with the flatter.

For instance, when public debt is below its threshold of around 40% of GDP, $\delta_d = 0.404$ in panel B, the output gap is about three times as large as the unemployment gap ($\beta_1 = -3.217$). Otherwise, when debt exceeds its threshold, they move around one-to-one ($\boldsymbol{\beta} = -0.974$). Consider also female labor force participation. This has been rising through most of the sample, boosting growth and countering the decline in male participation. Accordingly, any unemployment falls over this period would have had a more expansionary effect on cyclical output relative to the period (from the mid-1990s onwards) when that increase abated and reversed: i.e., $\beta_1 = -1.967$ vs. $\boldsymbol{\beta} = -1.417$. Indeed, several studies have found Okun's coefficient to be sensitive to female participation rates (e.g., Lee (2000)).

Results can be supplemented by examining the transition probabilities. Figures 2-3 plot the threshold time series, z, overlaid with threshold scalar δ, and the transition probabilities $\boldsymbol{g}(z_t; \gamma, \delta)$.[22] Note we see the graphical analogue of the speed of adjustment parameter values. The γ estimates are significantly different from zero but quite heterogeneous in value. The higher is γ the more rapid is the transition

[22] The equivalent plots with the NBER recession dates overlaid are provided for reference in supplementary Figure C.2-Figure C.3.

between regimes.[23] High values, i.e., $\gamma \geq 10$, as in u_n, pr_f and i_l often have volatile and rapid adjustment transition probabilities. The labor share and the debt ratios exhibit intermediate transitions, i.e., $\gamma \in [1, 10)$. Finally, the lowest speeds adjustment speeds, i.e., $\gamma \in (0, 1)$, are given by long-term unemployment, u_l, macroeconomic risk, the short rates, and manufacturing labor share.

Several interesting conclusions can be drawn from the transition probabilities. For instance, they imply both highly persistent (long lived) and less persistent (temporary) regime shifts of the OL. In the first category of shifts, we can include economic volatility and the debt series (whose transition occurs around mid to late 1980s), and the labor income share, manufacturing employment share and female participation rates (whose transition occurs in the mid to late 1990s).[24]

Second, the category of variables that imply short-term shifts of the Okun law consists of most of the policy/finance threshold variables (such as the interest rates, and perhaps also with the long term unemployed). These shifts and the transition probabilities are, like the series themselves, quite volatile with a cyclical (oscillatory) pattern and they do not impact immediately between the regimes. Regarding interest rates, the three rates point in the same direction. In the aftermath of the Great Recession, they identified a regime of substantially less steep Okun coefficients ($\beta \approx -0.8, -0.9$). In other words, the unemployment and output gap mirrored one another almost one to one. The behavior of unemployment at the onset of the Great Recession is testament to that: unemployment rose more sharply after the financial crisis than would otherwise have been predicted.[25]

Firms may have understood early on the severity of the Great Recession and choose

[23] This recalls the distinction made by Bernanke et al. (2005) who discuss 'slow' and 'fast-moving' variables.

[24] Regime wise, the natural rate is a little more opaque. The natural rate peaked in the late 1970s and fell monotonically thereafter (barring the period of the Great Recession). The start of the protracted fall from 1980s, though, traverses the (bootstrapped) upper confidence interval of the threshold.

[25] The rate of unemployment has also fallen more than the law would have predicted since the Great Recession.

to engage in less labor hoarding than otherwise (implying rising labor productivity), and perhaps substituted towards more capital-intensive production. Thus, this regime of more accommodative monetary policy was associated with substantially flatter Okun's trade offs. These results would speak to the possible state dependence of monetary and fiscal policy.[26]

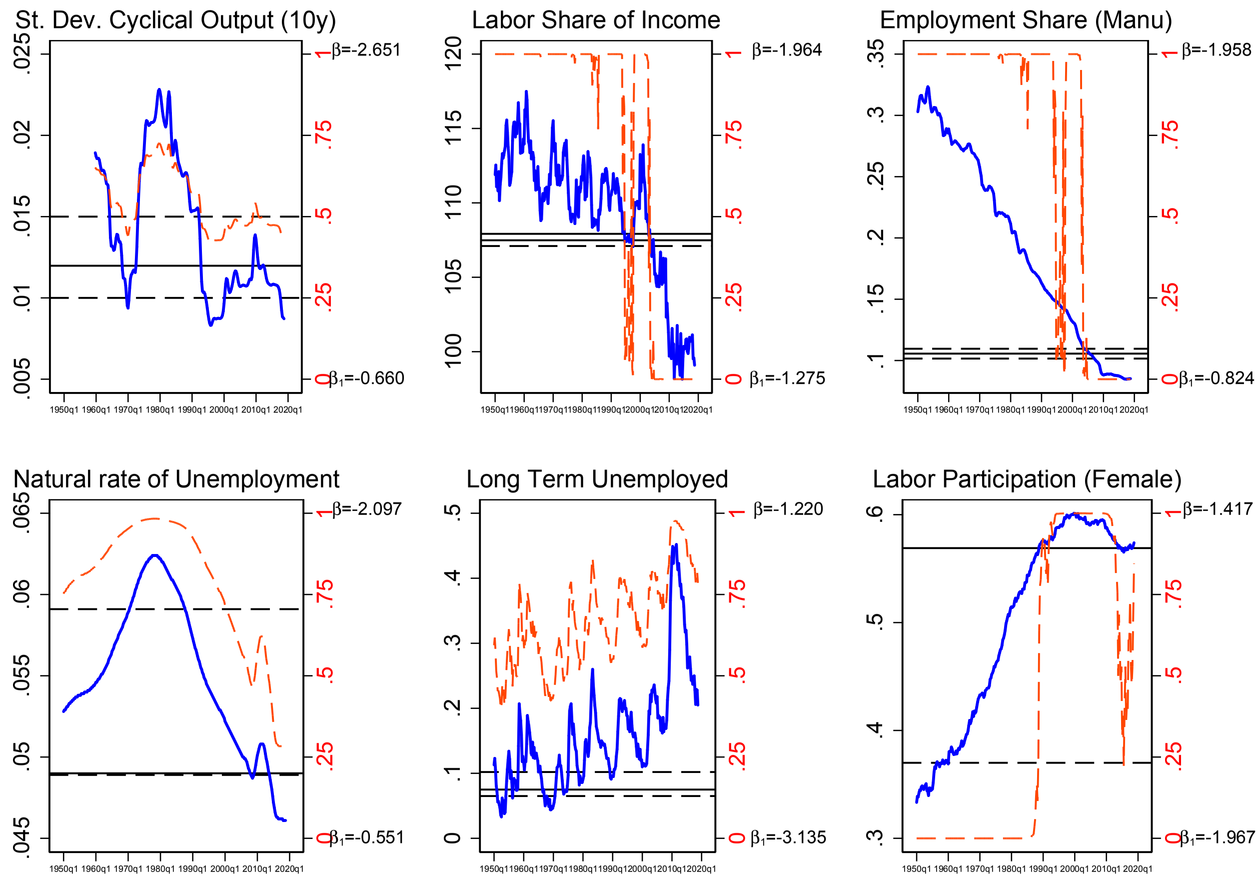
[26] More generally, Auerbach and Gorodnichenko (2012) found that for the US the size of the fiscal multiplier appeared to be state dependent; thus the state of the economy and policy effectiveness are innately liked. Moreover there is a growing literature on state dependency in monetary policy, particularly around the effective lower bound (see Woodford, 2012).

TABLE 3: Threshold Okun Law Results

	$labsh^{\dagger}$	u_n	pr_f	u_l	d	$\sigma_{y^c_{t(10)}}$	i_s	$i_{s,xw}$	dp	i_l	$lmanu$
					Ignoring Endogeneity						
β_1	-1.246	-0.760	-1.967	-2.194	-2.562	-1.593	-1.413	-1.414	-2.024	-0.876	-0.926
	(0.097)	(0.206)	(0.076)	(0.117)	(0.251)	(0.107)	(0.091)	(0.091)	(0.081)	(0.185)	(0.165)
β_2	-0.714	-1.355	0.519	1.339	1.659	-0.923	-0.651	-0.651	0.575	-1.243	-1.035
	(0.122)	(0.283)	(0.112)	(0.277)	(0.504)	(0.251)	(0.147)	(0.147)	(0.126)	(0.256)	(0.197)
$\boldsymbol{\beta}$	-1.960	-2.115	-1.448	-0.855	-0.903	-2.516	-2.064	-2.065	-1.449	-2.119	-1.961
δ	107.519	0.050	0.059	26.100	0.625	2.078	4.310	4.340	34.160	0.004	10.562
	{107.500,107.8210}	{0.0491,0.0591}	{0.037,0.591}	{16.100,25.200}	{0.402,0.932}	{1.482,2.062}	{4.060,4.460}	{4.001,4.560}	{34.100,32.200}	{0.0025,0.084}	{10.157,10.958}
γ	5.5	303.9	340.0	0.1	3.5	3.3	2.0	2.0	2.9	70.0	0.6
	{4.10,7.30}	{301.20,307.26}	{300.00,341.00}	{0.09,0.30}	{0.10,4,00}	{1.50,5.00}	{1.01,3.452}	{1.30,3.50}	{1.50,3.40}	{68.50,71.50}	{0.30,2.60}
$\mathcal{D}_2$	75.00	83.69	40.94	10.51	34.78	11.79	52.14	51.75	52.04	75.32	80.14
AIC	-1006.02	-1004.51	-1002.56	-1002.96	-1000.19	-860.86	-934.79	-934.88	-976.13	-863.40	-1002.91
					Controlling for Endogeneity						
β_1	-1.275	-0.551	-1.967	-3.135	-3.217	-0.660	-0.825	-0.872	-1.992	-0.878	-0.824
	(0.124)	(0.275)	(0.974)	(1.544)	(1.596)	(0.344)	(0.389)	(0.338)	(0.894)	(0.170)	(0.145)
β_2	-0.689	-1.546	0.550	1.915	2.243	-1.991	-1.391	-1.336	0.551	-1.289	-1.134
	(0.162)	(0.475)	(0.278)	(1.052)	(1.180)	(0.866)	(0.543)	(0.495)	(0.279)	(0.650)	(0.185)
$\boldsymbol{\beta}$	-1.964	-2.097	-1.417	-1.220	-0.974	-2.651	-2.216	-2.208	-1.441	-2.167	-1.958
δ	107.919	0.049	0.569	7.500	0.404	0.012	2.160	2.140	34.730	0.044	10.173
	{107.10,107.921}	{0.0489,0.0591}	{0.369,0.591}	{6.500,10.200}	{0.402,0.822}	{0.010,0.015}	{0.160,2.960}	{1.010,2.910}	{34.700,34.800}	{0.035,0.075}	{9.657,10.557}
γ	5.1	309.1	340.0	0.1	3.0	0.9	0.4	0.4	2.2	61.7	0.6
	{4.60,70.00}	{307.50,310.10}	{300.00,341.00}	{0.09,0.301}	{0.100,4.000}	{0.872,1.420}	{0.10,1.90}	{0.10,1.90}	{0.9,3.40}	{60.0,63,01}	{0.30,2.30}
$100\omega_1$	0.020	0.080	-0.004	0.300	0.100	0.100	0.070	0.050	-0.010	0.060	0.040
	(0.019)	(0.043)	(0.045)	(0.207)	(0.043)	(0.039)	(0.039)	(0.026)	(0.005)	(0.051)	(0.031)
$100\omega_2$	-1.280	-0.009	1.260	0.200	-0.010	-1.000	-0.700	-0.700	-0.700	-0.900	0.020
	(0.676)	(0.005)	(3.500)	(0.168)	(0.010)	(0.586)	(0.330)	(0.325)	(0.400)	(0.473)	(0.159)
$\mathcal{D}_2$	73.18	89.85	40.94	86.96	90.22	58.96	75.48	75.09	51.67	67.96	80.14
AIC	-1018.72	-1003.24	-1000.41	-1008.10	-1000.01	-862.43	-936.05	-936.41	-974.22	-865.16	-1004.67

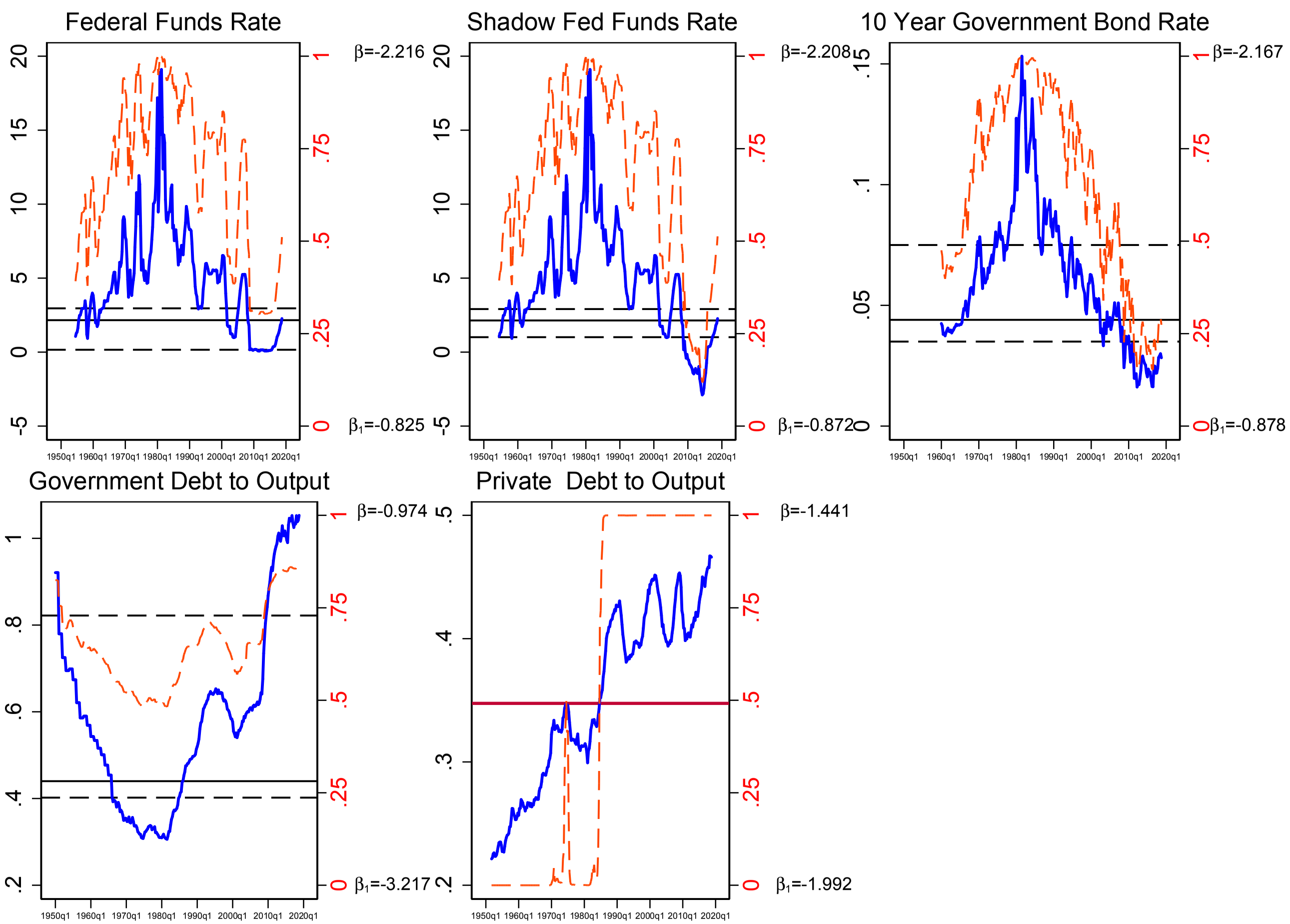
Notes: † Recall that the variant of the labor share that we use is the quarterly indexed one provided by FRED where $2012 = 100$.

Figure 2: Threshold Variables, Threshold and Transition Probabilities (Structural Variables)



Notes: Blue solid lines represent the threshold variable, z, red dashed lines represent the $\{0,1\}$ transition probabilities and the solid black horizontal line represents the threshold, δ (with black dashed confidence intervals). The β values on the rhs axis are the Okun parameters in the respective regimes.

Figure 3: Threshold Variables, Threshold and Transition Probabilities (Policy and Financial)



Notes: See notes to Figure 1.

Finally, in terms of comparing exogenous-endogenous threshold results, female labor force participation and employment in manufacturing are revealed as exogenous threshold types (ω coefficients in these equations are insignificant). This is perhaps not so surprising. Increasing trend female labor participation, though doubtless influenced by economic incentives in many ways, is generally regarded more as a cultural and demographic phenomenon, Bullard (2014). Likewise, the declining employment and value added share of manufacturing is a long standing trend, reflecting changes in global trading relations and shifting comparative advantages etc.

7 A Composite Threshold Variable

Our analysis suggests that many of the regime shifts triggered by the threshold variables examined co-move. This is true for both long-lived and more temporary shifts. Table 4 corroborates this showing the extent of the correlations among the $\{0, 1\}$ transition probabilities across regimes for all z candidates (all cross-correlations turn out to be significant at 1%). This co-movement can be attributed to common, or related, sources of information and economic events underlying the threshold variables triggering the threshold effects in OL. As we saw, these appear mostly related to structural and/or financial policy changes.

Accordingly, we now combine these different information sources into a single common factor and, then, (re-)estimate threshold model (7). The threshold factor effects considered can summarize and smooth out all the alternative sources of the OL regime shifts, which are related, and can potentially better describe and account for the total effects of such shifts over time. Consider the K-dimension column vector of the significant threshold variables considered, defined as $Z = [z_j]$, $j = 1, 2, ..., K$. For notational convenience, we drop time-subscripts from z_{jt}. This then implies that the PCs, or factors, collected in column vector p, can be written as

$$C_i = p_i' Z, \quad i = 1, 2, ..., K$$

Table 4: Correlation of the Transition Probabilities

	$labsh$	u_n	pr_f	u_l	d	σ_y^2	i_s	$i_{s,wx}$	dp	i_l	$lmanu$
$labsh$	1										
u_n	0.84	1									
pr_f	−0.57	−0.59	1								
u_l	−0.71	−0.66	−0.49	1							
d	−0.75	−0.81	0.58	0.74	1						
σ_y^2	0.54	0.62	−0.68	−0.23	−0.55	1					
i_s	0.66	0.78	−0.42	−0.72	−0.76	0.49	1				
$i_{s,wx}$	0.67	0.77	−0.43	−0.74	−0.78	0.47	0.99	1			
dp	−0.59	−0.63	0.84	0.54	0.71	−0.58	−0.42	−0.41	1		
i_l	0.67	0.82	−0.36	−0.56	−0.73	0.53	0.90	0.91	−0.32	1	
$lmanu$	0.88	0.94	−0.53	−0.75	−0.81	0.50	0.81	0.79	−0.60	0.80	1

Notes: The table presents estimates of the correlation coefficients of the transition probabilities from regime **1** to "**2**" for variables z_j which trigger significant threshold effects.

where p_i' is the i^{th}-row of an orthogonal matrix $P = [p_{ij}]$ (normalized, i.e., $P'P = I$) such that $P'\Sigma_z P = \Lambda = diag[\lambda_i]$, with $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_K \geq 0$, where Σ_z is the covariance matrix of vector Z and Λ is the matrix of the eigenvalues of Σ_z, denoted λ_i, by the spectral decomposition theorem. Note that the first factor C_1 (which, to recall, accounts for the single largest part of the data variation) corresponds to the largest eigenvalue λ_1. The covariance and correlation loading coefficients between C_i and a threshold variable z_j, $j = 1, 2, .., K$, collected in Z, are respectively given by

$$cov(C_i, z_j) = p_{ij}\sqrt{\lambda_i} \text{ and } \rho_{ij} = \frac{p_{ij}\sqrt{\lambda_i}}{\sigma_{z_j}}$$

Note, the correlation loading coefficients are net of the effects of differences in the variances of series z_j. The proportion of the variation of the data accounted for by the first v-PCs is given as $\rho_v^2 = \frac{\sum_{i=1}^{v} \lambda_i}{\sum_{i=1}^{K} \lambda_i}$. To formally examine if the first v PCs are non trivial (i.e., reflect a systematic pattern of variation of the data providing a meaningful interpretation of the data) we apply some testing procedures suggested in the literature (see Peres-Neto et al. (2005)).

7.1 Results

Table 5 reports estimates of model (7) based on a PC factor as the threshold variable.[27] Bootstrap standard errors are in parentheses and confidence intervals in braces. Panel A is based on standardized deviations of the variables of vector Z from their mean (i.e., $\tilde{z}_j = \frac{z_j - \mu_{z_j}}{\sigma_{z_j}}$), while Panel B uses transformation $\ddot{z}_j = \frac{z_j - z_j^{\min}}{z_j^{\max} - z_j^{\min}}$. Both transformations re-scale the original variables z_j to avoid the influence of differences in the measurement units and quantities of z_j's.

The PC analysis based on standardized variables $\tilde{z}_j$ has the following useful property. It weights equally the contribution of the variance of each series z_j into the variation C_i, otherwise variables with a larger variance will have larger $cov(C_i, z_j)$ and ρ_{ij} on the first PC C_1 (see, e.g., Timm (2002)). Yet, most of the tests determining the number of the non-trivial PC rely on standardized data.

The second rescaling method, based on $\ddot{z}_j$, has also two main interesting and novel features: (1) it also makes results independent of the measurement units; (2) it provides an indexed-factor between 0 and 1, which may aid interpretability. Note that, in contrast to standardized variables $\tilde{z}_j$, $\ddot{z}_j$ will not have the same variance for all j. Although the variance of $\ddot{z}_j$ will be less than unity, it can differ across j. This is a useful property of transformation $\ddot{z}_j$, if it is meaningful economically to weight more the variables of the PC C_1 with higher variability. This transformation can achieve this, without depending on differences in the scale across the original variables z_j.

Regarding the AIC, the versions of the threshold model with these two scaled threshold variables constitute superior specifications of the data compared to those of Table 3. Results suggest the following. First, the values of the LR^{sup} validate that there are significant threshold effects in OL. Second, the coefficients β_1 and $\boldsymbol{\beta}$ demonstrate that there are substantial differences in the OL slope between its two regimes and that

[27] To check robustness with respect to the frequency of the data, we re-ran our exercises on annual data. Results were qualitatively similar. The longer regime has an OL coefficient of around -2 and -1 otherwise. The estimates are shown in supplementary material Table D.1. The corresponding PCs and transition functions are plotted in supplementary Figure D.1.

the speed of transition between regimes is quite high: $\gamma \approx 10$.

For both scaling methods of z, we present estimates of the eigenvalues λ_i and the proportion of variation ρ_v^2, for $v = 1, 2, ..., K$, the correlation loading coefficients ρ_{ij} for C_1 and the model estimates considering C_1 as the threshold variable. This first factor is found to be the only non-trivial factor spanning the data, based on the Broken-Stick and Auer-Gervini methods determining the number of factors to retain.[28] Further support that C_1 can sufficiently summarize the related information of the data can be obtained by the much higher value estimates of λ_i for C_1, compared to the remaining PC factors reported in the table. The $\rho_{v=1}^2$ estimates indicate that C_1 explains roughly 70% of the data variability, while the estimates of ρ_{ij} imply that all variables z_j contribute to the sample variation of C_1 and are substantially correlated with it. The estimates of ρ_{ij} for the indexed-factor C_1 (see Panel B), which weights more the series z_j with the higher variation indicate that, among all variables z_j, those having the higher correlation and contribution to the variation of C_1 are the following: $lmanu, u_n, d$, dp and pr_f (cells marked in gray).

Figure 4 demonstrates that both scaling methods provide smooth and broadly consistent patterns of movements of factor C_1 and the accompanying transition probabilities. In fact, they indicate that there essentially exists only one regime transition from a steeper to a flatter OL. The steeper slope lasts until the end of 80s (early 90s), with high probability (almost 90%). After this, there is a tendency to move to a flatter Okun relationship, which is noticeably more likely after the financial crisis. The threshold is less precisely estimated under the first scaling method, although the fact that the transition to a flatter OL traverses the upper confidence limit for δ is also interesting and informative.

Third, the positive sign of the estimates of loading coefficients ρ_{1j}, indicate that important sources of the steeper OL relationship (more likely in 70s and 80s) are the

---

[28] These methods are found to work satisfactorily in a number of simulation studies (see Peres-Neto et al (2005), for a survey). The Broken-Stick test statistic compares the variance of factors C_i with the values expected from the broken-stick distribution, while the Auer-Gervini method is Bayesian and relies on the model selection criterion (see Auer and Gervini (2008)).

higher levels of labor income share, manufacturing employment share, the natural rate of unemployment and interest rates. On the other hand, important sources of occurrence of a more flat Okun relationship and a transition to such a regime are the following: the further increase of the public and private debts since 90's and middle 80's, respectively, the decline of policy interest rates, especially after 2000, and the slight positive trend of long run unemployment since the 80s (plus female labor participation).

These can be justified by the negative sign of the estimates of ρ_{1j}. These results hold for both re-scaling methods. They are also consistent with our earlier estimates of Table 3. This means that the threshold factor C_1 can efficiently summarize the sources of the OL relationship threshold effects examined, over time. This factor may be thus prove a useful policy tool in indicating possible shifts in the relationship between unemployment and economic activity and taking appropriate actions to offset adverse effects of such shifts.

Finally, regarding the issue of endogeneity of factor C_1, results demonstrate that ignoring this leads to different estimates of the parameters of the threshold model, especially the βs and γ. The estimates of γ and their standards errors imply that this issue is important, especially in regime "**2**", where λ_2 is clearly different than zero at 5% (or less). Further support for estimating dealing with the endogeneity of factor C_1 can be obtained by the values of AIC reported in the table. These show that the versions of the model controlling for the endogeneity of C_1 fit better the data, compared to those that do not.

Table 5: PC analysis and model estimates based on the threshold common-factor variable

Panel A: Estimates based on standardized variables $\tilde{z}_j$

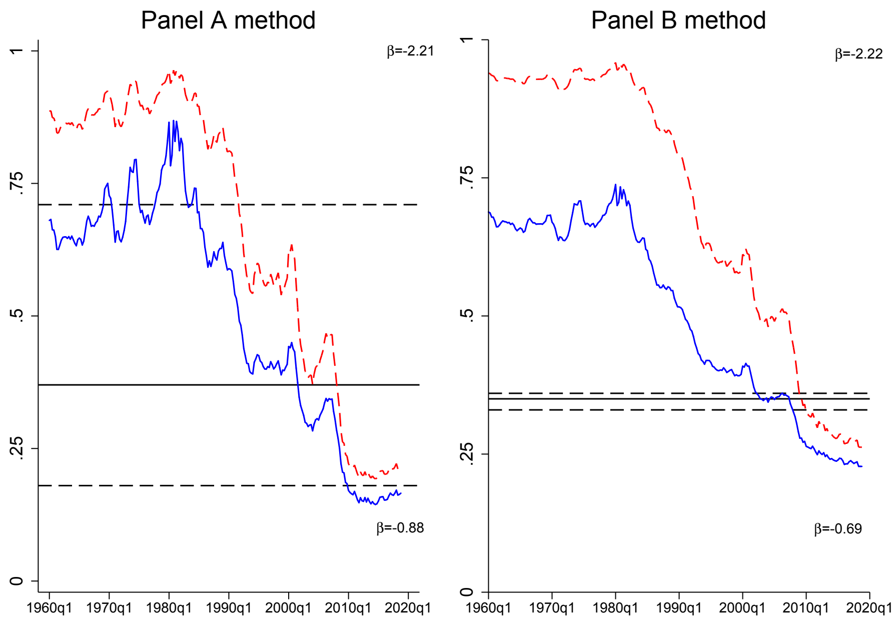
i, v	1	2	3	4	5	6	7	8	9	10	11
λ_i	7.1800	2.0200	0.9300	0.3200	0.2000	0.1300	0.0800	0.0600	0.0300	0.0060	0.0040
ρ_v^2	0.6500	0.8400	0.9300	0.9500	0.9700	0.9800	0.9900	0.9950	0.9990	0.9996	1.0000
j	$labsh$	u_n	pr_f	u_l	d	σ_y^2	i_s	$i_{s,wx}$	dp	i_l	$lmanu$
ρ_{1j}	0.31	0.35	-0.23	-0.28	-0.35	0.26	0.30	0.31	-0.28	0.28	0.32
		β_1	β_2	$\boldsymbol{\beta}$	$100\omega_1$	$100\omega_2$	γ	δ	$\mathcal{D}_2$	AIC	LM^{sup}
						Ignoring Endogeneity					
		-1.02	-1.09	-2.11			8.80	0.36	70.62	-867.32	[0.000]
		(0.15)	(0.21)				{5.10,9.10}	{0.35,0.37}			
						Controlling for endogeneity					
		-0.88	-1.33	-2.21	0.03	-0.7	6.90	0.37	70.60	-868.70	[0.000]
		(0.17)	(0.25)		(0.03)	(0.30)	{4.00,9.30}	{0.18,0.71}			

Panel B: Estimates based on scaled variables $\ddot{z}_j$

i, v	1	2	3	4	5	6	7	8	9	10	11
λ_i	0.5200	0.1300	0.0600	0.0200	0.0100	0.0080	0.0060	0.0030	0.0020	0.0005	0.0001
ρ_v^2	0.6800	0.8400	0.9200	0.9500	0.9700	0.9800	0.9900	0.9960	0.9990	0.9998	1.0000
j	$labsh$	u_n	pr_f	u_l	d	σ_y^2	i_s	$i_{s,wx}$	dp	i_l	$lmanu$
ρ_{1j}	0.27	0.38	-0.34	-0.22	-0.37	0.28	0.18	0.17	-0.34	0.19	0.42
		β_1	β_2	$\boldsymbol{\beta}$	$100\omega_1$	$100\omega_2$	γ	δ	$\mathcal{D}_2$	AIC	LM^{sup}
						Ignoring endogeneity					
		-0.17	-1.96	-2.13			8.10	0.25	90.03	-866.68	[0.000]
		(0.30)	(0.38)				{7.20,8.40}	{0.24,0.27}			
						Controlling for endogeneity					
		-0.69	-1.53	-2.22	0.04	-0.50	8.20	0.35	72.41	-868.10	[0.000]
		(0.18)	(0.38)		(0.03)	(0.20)	{6.25,8.90}	{0.33,0.36}			

Notes: The table presents results of the PCA of the variables z_j triggering significant threshold effects and estimates of model (7) using the first PC factor C_1, as the threshold variable. These estimates cover the case that the endogeneity of C_1 is ignored and is controlled based on our suggested copula method. Panel A presents results for the case that the PCA analysis is based on standardized transformation of z_j, i.e., $\tilde{z}_j = \frac{z_j - \mu_{z_j}}{\sigma_{z_j}}$, while Panel B for transformation $\ddot{z}_j$, thus scaling z_j in the interval [0,1]. Bootstrap standard errors and confidence intervals are, respectively, given in parentheses and braces, whilst the LM^{sup} probability values are given in brackets.

Figure 4: Transition Probability (Principal Components)



Notes: This figure graphically presents estimates of the factor C_1 against the transition probability of the model using C_1 as the threshold variable. This is done for the two methods scaling the data. The first is based on standardized variables $\tilde{z}_j = \frac{z_j - \mu_{z_j}}{\sigma_{z_j}}$, while the second is based on $\dddot{z}_j = \frac{z_j - z_j^{\min}}{z_j^{\max} - z_j^{\min}}$, scaling z_j in the interval $[0, 1]$. As before, the series in blue is the PC, and in red dashed we have the transition probabilities. See also notes to Figure 1.

8 Conclusions

Okun's law is a building block of many macro-econometric models and often considered an empirical regularity. We examined the possibility of asymmetries in that relationship. Specifically, we considered a nonlinear, smooth transition regression model, exploring a variety of candidate threshold variables. Our paper makes two main contributions. The first is methodological, the second is empirical.

In terms of **methodology**, our work has two distinct components:

(a) We allow for endogeneity in the threshold variable using copulas to capture the dependence between the disturbance term and the threshold variable in the context of a LSTR model. The copula approach admits normal and non-normal dependencies, thus adding an additional flexibility in modelling the endogeneity.

(b) We suggest a *new* testing procedure to test for smooth transition threshold effects against linearity under endogeneity of the threshold variable.

Both the copula based approach and the linearity testing procedure suggested are evaluated throughout a MC study.

Regarding **empirics**, which is our second main contribution, we establish that threshold effects can be detected in Okun's relationship. We find a combination of structural and policy-related variables accounts for changes in the Okun's law trade off in recent decades. This conclusion is bolstered by combing these threshold candidates into a single factor. We found regime-like behavior with Okun's coefficient rising (or flattening) over time: from around ≤ -2 over the 1960s-1980s, then a slow transition to a value around ≥ -1. Thus the unemployment gap is increasingly associated with a smaller output gap. Moreover, whilst the Great Recession accelerated that rise, interestingly, the bulk of the change occurred beforehand. This, in turn, corroborates our finding that both structural and non structural factors were at play. Of our candidates, special emphasis lies with manufacturing employment, female participation, the natural rate of unemployment and the private and public debt series.

The work done here, moreover, could be extended. An interesting controversy in the literature is whether OL holds outside the US and in particular for countries characterized by less flexible labor and product markets. More generally, though, our endogenous threshold LSTR model could prove useful for additional applications such as in studies of growth, trade and finance where threshold models have often been used to analyze asymmetries (albeit in a more restrictive form than that considered here) and may thus yield new or more robust insights.

References

Andrews D. (1993). Tests for Parameter Instability and Structural Change With Unknown Change Point. *Econometrica*, 61, 821–856

Andrews, D. and W. Ploberger (1994). Optimal Tests When a Nuisance Parameter is Present only under the Alternative, *Econometrica*, 62, 1383–1414.

Antonakis, J., Bendahan, S., Jacquart, P. and R. Lalive (2014). Causality and Endogeneity: Problems and Solutions, in *The Oxford Handbook of Leadership and Organizations*, edited by D. Day, Oxford University Press.

Auer, P. and Gervini, D. (2008). Choosing principal components: A new graphical method based on model selection, *Communication in Statistics - Simulation and Computation*, 37, 962-977.

Auerbach, A. and Y. Gorodnichenko, (2012). Measuring the Output Responses to Fiscal Policy, *American Economic Journal: Macroeconomics*, 4(2), 1–27.

Attfield, C. L. F and B. Silverstone (1997) Okun's Coefficient: A Comment, *Review of Economics and Statistics*, 79, 2, 326–329

Bernanke B, Boivin J, Eliasz P. (2005). Measuring monetary policy: a factor augmented autoregressive (FAVAR) approach. *Quarterly Journal of Economics*, 120: 387–422.

Bullard, J. (2014) The Rise and Fall of Labor Force Participation in the United States, *Federal Reserve Bank of St. Louis Review*, 96, 1, 1-12.

Chan, K. S. (1993). Consistency and Limiting Distribution of the Least Squares Estimator of a Threshold Autoregressive Model, *Annals of Statistics*, 21, 520–533.

Christopoulos, D., McAdam, P. and E. Tzavalis (2019). Dealing with Endogeneity In Threshold Models using Copulas: Revisiting the Foreign Trade Multiplier, *Journal of Business & Economic Statistics*, forthcoming.

Courtney, H. G. (1991). The Beveridge curve and Okun's law: a re-examination of fundamental relationships in the United States. MIT: PhD thesis.

Cuaresma, J. C (2003). Okun's Law Revisited, *Oxford Bulletin of Economics and Statistics*, 65(4), 439–451.

Daly, M. C., Fernald, J. G., Jordà, Ò. and F. Nechio (2018). Shocks and Adjustments: The View through Okun's Macroscope, mimeo, Federal Reserve Bank of San Francisco (version June 3).

Davidson, R. and J. G. MacKinnon (2007). Improving the reliability of bootstrap tests with the fast double bootstrap, *Computational Statistics & Data Analysis*, 51(7), 3259–3281.

Escribano, Á. and Jordà, Ò (1999). Improved Testing and Specification of Smooth Transition Regression Models, in *Nonlinear Time Series Analysis of Economic and Financial Data*, edited by P. Rothman, Springer, pp. 289–319.

Fernald, J. G. (2018). A Quarterly, Utilization-Adjusted Series on Total Factor Productivity. FRBSF Working Paper 2012-19 (current update).

Fernald, J. G., Hall, R. E., Stock, J. H. and M. W. Watson (2017). The Disappointing Recovery of Output after 2009, *Brookings Papers on Economic Activity*, 48(1), 1–81.

Fernald, J. G. and C. Wang (2016) Why Has the Cyclicality of Productivity Changed? What Does It Mean?, *Annual Review of Economics*, 8(1), 465-496.

Franses, P. H. and D. van Dijk (2000). *Non-linear Time Series Models in Empirical Finance*, Cambridge University Press.

Freeman, D. G. (2001). Panel Tests of Okun's law for Ten Industrial Countries, *Economic Inquiry*, 39 (4), 511–523.

Giannone, D. , M. Lenza and G. E. Primiceri (2018) Economic Predictions with Big Data: The Illusion of Sparsity, Federal Reserve Bank of New York Staff Reports 847.

Gilchrist, S. and E. Zakrajšek (2012). Credit Spreads and Business Cycle Fluctuations, *American Economic Review*, 102 (4), 1692–1720.

Gordon, R. G. (1984). Unemployment and Potential Output in the 1980s, *Brookings Papers on Economic Activity*, 15 (2), 537–64.

Granger, C. W., and T. Teräsvirta (1993). *Modelling non-linear economic relationships*, Oxford University Press.

Hansen, B. E. (1996). Inference when a Nuisance Parameter is not Identified under the Null Hypothesis, *Econometrica*, 64, 413–430.

Hansen B. E. (1997). Approximate Asymptotic p Values for Structural-Change Tests. *Journal of Business & Economic Statistics*, 15, 60–67.

Hansen, B. E. (1999). Threshold Effects in Non-Dynamic Panels: Estimation, Testing and Inference, *Journal of Econometrics*, 93, 345–368.

Harris, R., and B. Silverstone (2001). Testing for asymmetry in Okun's law: cross-country comparison, *Economics Bulletin*, 5, 1–13

Huang, H.-C. and S-C. Lin (2008). Smooth-time-varying Okun's coefficients, *Economic Modelling*, 25(2), 363–375.

IMF (2010). Unemployment dynamics during recessions and recoveries: Okun's Law and beyond. WEO chapter, April.

Joe, H. (2014). *Dependence Modeling with Copulas*, Chapman & Hall.

KılıÇ, R. (2011). Testing for Cointegration and Nonlinear Adjustment in a Smooth Transition Error Correction Model, *Journal of Time Series Analysis*, 32, 647–660.

KılıÇ, R. (2016). Tests for Linearity in STAR Models: SupWald and LM-type Tests, *Journal of Time Series Analysis*, 37, 660–674.

Klump, R., McAdam, P. and A. Willman (2007). Factor Substitution and Factor-Augmenting Technical Progress in the United States: A Normalized Supply-Side System Approach, *Review of Economics and Statistics*, 89(1), 183-192.

Knotek, E. S. (2007). How useful is Okun's law?, *Economic Review*, Q IV, 73–103.

Kourtellos, A., Stengos, T. and C. M. Tan (2013). The Effect of Public Debt on Growth in Multiple Regimes, *Journal of Macroeconomics*, 38, 35–43.

Kourtellos, A., Stengos, T. and C. M. Tan (2016). Structural Threshold Regression, *Econometric Theory*, 32, 4, 827–860.

Lee, J. (2000). The robustness of Okun's law: Evidence from OECD countries, *Journal of Macroeconomics*, 22(2), 331–356.

Leybourne, S., Newbold, P. and D. Vougas (1998). Unit Roots and Smooth Transitions, *Journal of Time Series Analysis*, 19, 83–97.

Luukonen, R., Saikkonen P. and T. Teräsvirta (1988). Testing Linearity against Smooth Transition Autoregressive Models, *Biometrika*, 75, 491–499.

Mankiw, N. G. (2015) *Macroeconomics*, Worth Publishers.

Mayes D. and M. Virén (2002) Asymmetry and the problem of aggregation in the Euro area. *Empirica*, 29:47–73.

Marcellino, M., Stock, J. H. and M. W. Watson (2006). A comparison of direct and iterated multistep AR methods for forecasting macroeconomic time series, *Journal of Econometrics*, 135, 499–526

Neely, C. J. (2010). Okun's law: output and unemployment, *Economic Synopses*, Federal Reserve Bank of St. Louis.

Okun, A. M. (1962). Potential GNP: Its Measurement and Significance, *Proceedings of the Business and Economics Statistics Section*, 98–104, Alexandria, VA: American Statistical Association.

Peres, P. R., D. J. Jackson and Somers, K.M. (2004). How many principal components? Stopping rules for determining the number of non-trivial axes revisited, *Computation Statistics & Data Analysis*, 49, 974-997.

Park, S. and S. Gupta (2012). Handling Endogenous Regressors by Joint Estimation Using Copulas, *Marketing Science*, 31, 4, 567–586.

Patton, A. J. (2006). Modelling Asymmetric Exchange Rate Dependence, *International Economic Review*, 47, 527–556.

Perman, R. and C. Tavéra (2007). Testing for convergence of the Okun's law coefficient in Europe. *Empirica*, 34(1), 45–61.

Perman, R., S. Gaetan, and C. Tavéra (2015). Okun's law – A Meta-analysis, *Manchester School*, 83, 1,101-126.

Prachowny, M. F. J. (1993). Okun's law: Theoretical Foundations and Revised Estimates, *Review of Economics and Statistics*, 75 (2), 331–336.

Reinhart, C. M., V. R. Reinhart and K. S. Rogoff (2012). Public debt overhangs: Advanced-economy episodes since 1800, *Journal of Economic Perspectives*, 26, 69–86.

Rothman, P., (1998). Forecasting Asymmetric Unemployment Rates, *Review of Economics and Statistics*, 80(1), 164–168.

Silvapulle, P., Moosa, I. and M. Silvapulle (2004). Asymmetry in Okun's law, *Canadian Journal of Economics*, 37 (2), 353–374.

Silverman, B. W. (1986). *Density Estimation for Statistics and Data Analysis*, Chapman & Hall, London.

Teräsvirta T. (1994). Specification, estimation, and evaluation of smooth transition autoregressive models, *Journal of the American Statistical Association*, 89: 208–218.

Timm, N. H. (2002). *Applied Multivariate Analysis*, Springer.

van Dijk, D., Teräsvirta, T. and P. H. Franses (2002). Smooth Transition Autoregressive Models – A Survey of Recent Developments, *Econometric Reviews*, 21(1), 1–47.

Virén, M. (2001). The Okun curve is non-linear, *Economics Letters*, 70(2), 253–257.

Woodford, M. (2012) Methods of policy accommodation at the interest-rate lower bound, in *The Changing Policy Landscape*, Federal Reserve Bank of Kansas City, Jackson Hole Wyoming: 185–288.

Wu, J. C. and F. D. Xia (2016) Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound, *Journal of Money, Credit, and Banking*, 48(2–3), 253–291.

APPENDICES

A Monte Carlo

In this section, we present the data generating processes (DGP) and results of the Monte Carlo (MC) exercise touched upon in the main text. Our first exercise examines the performance of the estimation method suggested to estimate the location parameter δ accurately and the ability of our method to successfully control for the endogeneity of threshold variable z_{it} in the LSTR model. The second exercise (in Appendix B) evaluates the power performance of test statistic LR^{sup}.

The DGP that we consider to estimate δ is as follows:[1]

$$y_t = (\beta_{11}+\beta_{12}x_{2t}+\beta_{13}x_{3t})(1-\boldsymbol{g}(z_t;\gamma,\delta))+(\beta_{21}+\beta_{22}x_{2t}+\beta_{23}x_{3t})\boldsymbol{g}(z_t;\gamma,\delta)+\varepsilon_t, \quad \text{(A.1)}$$

where $x_{2t} \sim IID\mathcal{N}(0,1)$ and $x_{3t} \sim IID\mathcal{N}(0,1)$ are exogenous, i.e., $\mathbb{E}(\varepsilon_t|x_{2t}) = \mathbb{E}(\varepsilon_t|x_{3t}) = 0$, and the values of the slope coefficients across the two regimes of the model are given as follows:

$$\text{Regime 1:} \quad \beta_{11} = \ \ 0.5, \qquad \beta_{12} = 2 \qquad , \ \beta_{13} = 1,$$

$$\text{Regime 2:} \quad \beta_{21} = -0.7, \quad \beta_{22} = \{1.5, 3.0\} \ , \ \beta_{23} = 0.$$

These values imply quite small differences in the slope coefficients of model (A.1) across its two regimes, which are difficult to detect. They are chosen to highlight the ability of our method to lead to accurate estimates of δ even for very small in size shifts of the slope coefficients of the model.

For the regression error term ε_t and threshold variable z_t of the above mentioned DGP, we consider the normal and the Student-t distribution, with four degrees of freedom. Using data from the Student's t distribution will show if our method, based on the transformations of the quantile function of the standard normal distribution , i.e., Φ^{-1}, can be proved robust to a misspecification of the true distributions of $\varepsilon_{h,t}$ and z_t, like the Student's t often assumed in econometrics. For reasons of space, we present results for the cases that the distribution of ε_t, and correlation structure between $\varepsilon_{h,t}$ and z_t do not change across the two regimes of the model. For z_t,

[1] Both the DGP and the values of its parameters considered in our simulation analysis are close to those considered in the simulation studies of, for instance, Lundbergh, Teräsvirta and van Dijk (2003).

we assume $z_t \sim IID(z_\mu, 1)$, with $z_\mu = 3.0$, while, for the correlation coefficient between $\varepsilon_{h,t}$ and z_t, we consider the following set of values: $\rho_{\varepsilon z} = \{0.0, 0.55, 0.75\}$, for $h = \{1, 2\}$. The structure of the threshold variable is given as follows:

$$z_t = \delta + c_{\varepsilon z} v_t + \zeta_t \tag{A.2}$$

where $\varepsilon_t \sim t_{DF=4}$ and $\zeta_t \sim t_{DF=4}$, for all t. The value of coefficient $c_{\varepsilon z}$ is chosen to control for the degree of correlation between ε_{it} and z_{it}

The threshold value δ is set to the 25% percentile (1^{st} quantile) of the distribution of z_t. We also examined threshold values at the 75% percentile (3^{rd} quantile) of this distribution, but the MC results do not change qualitatively. The values of the speed of adjustment considered are set to $\gamma = \{1.5, 3.5\}$. These values reflect the cases that the transition between the two regimes is low and high, respectively. Following other studies, we treat the above values of γ as known in our analysis, reflecting that our interest is focused on the estimation bias of δ. Since we are focused on the performance of our method to control for the endogeneity of z_t on the estimates of δ, we do not report results for the remaining parameters of the model, i.e., ω_1, ω_2 for reasons of space.

We consider sample sizes of $T = \{50,\ 250\}$ observations and carry out $1,000$ iterations. For all iterations, we calculate the bias and the root mean square error of the estimator of δ. In Table A.1, we present average values of the above metrics, over all iterations, denoted as BIAS and RMSE, respectively. The table presents different sets of results. Panel A presents results ignoring the problem of the threshold variable endogeneity in the estimation. Panel B controls for this problem, based on our method by including in the rhs of (A.1) the bias correction terms $z^*_{h,t}, h = \{1, 2\}$.

Panel C presents results for the case that Kourtellos's et al. (2016) approach for threshold models, appropriately modified for the LSTR model, is employed to control for the endogeneity problem of z_t. This approach adjusts equation (A.1) by estimating the expectation terms $\mathbb{E}(\varepsilon_t | z_t \leq \delta)$ and $\mathbb{E}(\varepsilon_t | z_t > \delta)$, capturing the bias of z_t across the two regimes of the model, based on the inverse Mills ratio terms assuming that both ε_t and z_t are normally distributed. To calculate these ratios, we assume that $z_t \sim IID\mathcal{N}(z_\mu, 1)$ has the following single factor presentation:

$$z_t = z_\mu + 0.95\zeta_t + e_{zt}, \tag{A.3}$$

where $\zeta_t \sim IID\mathcal{N}(0, 0.5)$ and $e_{zt} \sim IID\mathcal{N}(0, 0.5)$ and $z_\mu = 3.0$. Finally, Panel D presents results for the case that our method is applied to the case that $\varepsilon_{h,t}$ and z_t are jointly Student-t distributed.

Results lead to several interesting conclusions. First, they indicate that ignoring the endogeneity of threshold variable z_t causes serious biases in the estimates of threshold parameter δ. Specifically, it tends to overestimate the true value of δ. Moreover, as expected, the bias of δ is substantial in magnitude when the correlation between z_t and ε_t is high, i.e., 0.75. The magnitude of the bias also depends on the different values of γ considered. The bias is bigger, the smaller value of γ considered, i.e. $\gamma = 1.5$, than $\gamma = 3.5$, and it remains even if T increases.

A second conclusion is that our method can successfully control for the endogeneity problem of the threshold variable. The method can substantially reduce the estimation bias of δ. This is true even if the sample size is small (i.e., $T = 50$). As expected, the bias of δ reduces as the sample increases (i.e., $T = 250$). The bias also reduces, when γ increases (i.e., the LSTR model approaches the TAR, where the shifts of the model are faster across the two regimes). Similar conclusions to the above also hold for the RMSE. These results are also robust to the different distribution of $\varepsilon_{h,t}$ and z_t considered, namely the Student's t. This is clearer for the case of the larger size of $T = 250$. Note that, for the case that both $\varepsilon_{h,t}$ and z_t are normally distributed, the performance of our method successfully compares to that of KST. Actually, our method seems to have better small sample (i.e., $T = 50$) performance relative to KST.

Finally, the results indicate that the estimation of the augmented regression model, which controls for the endogeneity of the threshold variable, leads to unbiased estimates of δ even if there is no threshold variable endogeneity (i.e., $\rho_{\varepsilon z} = 0$). This result is more clear cut for the cases that T is large.

References

Lundbergh, S., Teräsvirta, T. and D. Van Dijk (2003), Time–Varying Smooth transition Autoregressive Models, *Journal of Business & Economic Statistics*, 21, 104-121.

TABLE A.1: Monte Carlo results of the BIAS and RMSE of the estimator of δ

γ	T	$\rho_{\varepsilon z} = 0.0$		$\rho_{\varepsilon z} = 0.55$		$\rho_{\varepsilon z} = 0.75$		$\rho_{\varepsilon z} = 0.0$		$\rho_{\varepsilon z} = 0.55$		$\rho_{\varepsilon z} = 0.75$	
		(i): $b_{22} = 1.5$						(ii) $b_{22} = 3.0$					
		Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
Panel A: Ignoring the endogeneity of z_t (z_t and ε_t are normally distributed)													
1.5	50	−0.613	1.732	−0.975	2.986	−1.002	3.238	−0.811	2.420	−0.990	3.257	−0.991	3.464
1.5	250	−0.213	0.417	−0.827	1.697	−1.043	2.113	−0.153	0.324	−0.529	0.982	−0.716	1.250
3.5	50	−0.398	0.961	−0.816	2.120	−0.929	2.533	−0.331	1.197	−0.647	1.651	−0.694	1.879
3.5	250	−0.020	0.057	−0.439	0.899	−0.782	1.528	−0.012	0.049	−0.174	0.343	−0.358	−0.592
Panel B: Controlling for the endogeneity of z_t (z_t and ε_t are normally distributed)													
1.5	50	−0.775	1.611	−0.729	1.829	−0.567	1.799	−0.732	1.522	−0.650	1.588	−0.485	1.561
1.5	250	−0.456	0.662	−0.343	0.614	−0.293	0.647	−0.381	0.546	−0.245	0.425	−0.165	0.421
3.5	50	−0.619	1.331	−0.503	1.287	−0.409	1.289	−0.522	1.098	−0.424	1.007	−0.266	0.884
3.5	250	−0.149	0.204	−0.135	0.204	−0.114	0.167	−0.073	0.111	−0.065	0.091	−0.072	0.092
Panel C: Controlling for the endogeneity of z_t based on Mill's ratios (z_t and ε_t are normally distributed)													
1.5	50	−0.891	2.363	−0.935	2.433	−0.928	2.423	−0.818	2.130	−0.863	3.202	−0.867	2.271
1.5	250	−0.398	0.811	−0.425	0.863	−0.428	0.873	−0.314	0.635	−0.315	0.661	−0.312	0.653
3.5	50	−0.691	1.559	−0.678	1.514	−0.667	1.500	−0.530	1.218	−0.572	1.295	−0.585	1.317
3.5	250	−0.091	0.189	−0.126	0.231	−0.129	0.234	−0.034	0.088	−0.057	0.125	−0.062	0.031
Panel D: Controlling for the endogeneity of z_t (z_t and ε_t are Student-t distributed)													
1.5	50	−0.920	2.516	−0.993	2.615	−0.901	2.378	−0.878	2.425	−0.902	2.434	−0.846	2.214
1.5	250	−0.240	1.104	−0.516	1.381	−0.683	1.638	−0.133	0.847	−0.334	1.010	−0.518	1.295
3.5	50	−0.678	2.012	−0.696	1.954	−0.630	1.790	−0.599	1.838	−0.619	1.737	−0.499	1.393
3.5	250	−0.004	0.401	−0.231	0.508	−0.337	0.656	−0.035	0.218	−0.108	0.245	−0.150	0.271

Notes: The table presents average values of the $Bias$ and $RMSE$ of the estimates of γ and δ estimates, over 1000 iterations, when both z_t and ε_t are normally and then Student-t distributed, with four degrees of freedom. This is done under different values of slope parameter: $b_{22} = \{1.5, 3\}$ and the speed-of-transition coefficient: $\gamma = \{1.5, 3.5\}$. For the correlation coefficient between ε_t and z_t, we consider the following set of values: $\rho_{\varepsilon z} = \{0.00, 0.55, 0.75\}$, for both regimes of the model.

B Power of the Test Statistic $LR^{\sup}$

We present values of the power performance of statistic $LR^{\sup}$ for the LSTR model (A.1), employed in our previous MC exercise, where the threshold parameter value δ is set at its 3rd quantile. For reasons of space, we do not report results for the case that δ is at the 1st quantile and we consider only the case of $\beta_{22} = 1.5$. The remaining slope parameters are set as in the previous MC exercise. We also consider the same simulation scenarios with that exercise for $\rho_{\varepsilon z} = \{0.0, 0.55, 0.75\}$ and $T = \{50, 250\}$.

To calculate the power of the test statistic, we need first to obtain its distribution under the null hypothesis H_0: $\beta_1 = \beta_2 = \beta$ and obtain its critical value, corresponding to the 95^{th}-quantile of its simulated distribution. To this end, for each iteration we generate the error term $\varepsilon_t \sim IID(0,1)$ and threshold variable $z_t \sim IID(3,1)$ from a joint distribution of them, while x_{2t} and x_{3t} are generated as $x_{2t} \sim IID\ \mathcal{N}(0,1)$ and $x_{3t} \sim IID\mathcal{N}(0,1)$, respectively. Given these generated series, we then generate series y_t under null hypothesis based on model $y_t = x_t'\beta + \varepsilon_t$, where $\beta_1 + \beta_2 = \frac{1}{2}(\beta_1 + \beta_2)$; β_1 and β_2 are defined as in the previous exercise.

Based on the generated series y_t, next we estimate model (A.1) under both the null and alternative hypotheses and, then, we calculate the test statistic $LR^{\sup}$, over all possible values of γ and δ, based on $1,000$ iterations. In so doing, note that we trim out the top and bottom 10 percentiles of the distribution of δ, while for γ we rely on the set of values $Q_\gamma = \left[\frac{1}{10\sigma_z}, \frac{100}{\sigma_z}\right]$.[2] Also, under the alternative hypothesis, the model is adjusted by the bias correction terms $z^*_{h,t}, h = \{1,2\}$ to control for the endogeneity of the threshold variable.

Given the critical values of $LR^{\sup}$, at the 5% level, the power of the test, which represents rejection frequencies of the above null hypothesis, is calculated by generating data under the alternative hypothesis the alternative hypothesis H_a: $\beta_1 \neq \beta_2$, namely model (A.1). The remaining steps of the MC exercise are as above. For each iteration, the error term ε_t, the variables x_{2t} and x_{3t}, and the threshold variable z_t are generated as before, while in estimating δ we trim out the top and bottom 10 percentiles of its distribution, while for γ we rely on the set of values $Q_\gamma = \left[\frac{1}{10\sigma_z}, \frac{100}{\sigma_z}\right]$. To see if the power of the test depends on ignoring the issue of the endogeneity of the threshold variable z_t, we also present results for the case that model (A.1) is not adjusted by the terms $z^*_{h,t}, h = \{1,2\}$ to calculate statistic $LR^{\sup}$.

The results of the power of the test are reported in Table B.1. These indicate that

[2] We found that extension of Q_γ to include higher values of γ does not affect our simulation results.

the power of the test statistic LR^{sup} is high and close to unity for all the cases of γ and $\rho_{\varepsilon z}$ considered. They also indicate that ignoring the endogeneity of the threshold variable leads to a version of the test statistic which has less power. However, this is more clear for the case that $\gamma = 3.5$. Another interesting result is that the power performance does not depend on the adjustment of model (1) for possible endogeneity of threshold variable (see model (6)) when $\rho_{\varepsilon z} = 0$. This result was expected. Unless there is a serious degrees of freedom problem, it means that augmentation of this auxiliary regression by the transformed variables z^*_{ht} does not affect the power of statistic LR^{sup} and, hence, it can be safely implemented, in practice, for all possible values of the correlation coefficient $\rho_{\varepsilon z}$. The above results are robust to the different values of γ and T considered.

TABLE B.1: Power of test statistic LR^{sup} with $\beta_{22} = 1.5$

	No Endogeneity		Controlling For Endogeneity	
$c_{\epsilon z}$	$\gamma = 1.5$			
0.00	0.910	0.926	0.896	0.936
0.55	0.837	0.886	0.870	0.934
0.75	0.781	0.826	0.868	0.925
$c_{\epsilon z}$	$\gamma = 3.5$			
0.00	0.852	0.893	0.827	0.896
0.55	0.766	0.833	0.812	0.896
0.75	0.687	0.762	0.804	0.895
T	50	250	50	250

Notes: The table presents the power of test statistic

$$LR^{\text{sup}} \equiv \sup_{(\gamma,\delta)\in Q_\gamma \times Q_z} LR(\gamma, \delta)$$

under the alternative hypothesis H_a: $\beta^{(1)} \neq \beta^{(2)}$, at the 5% nominal size, for model (A.1). The critical values of the test statistic are simulated under the null H_0: $\beta^{(2)} = \beta^{(2)}$, for alternative N and T. We use the parameter values from Appendix A but concentrate on the $\beta_{22} = 1.5$ case.

Supplementary Material
For Internet Appendices

C Additional Material

C.1 Additional Figures

FIGURE C.1: Comparisons of FRED Labor Income Share Series

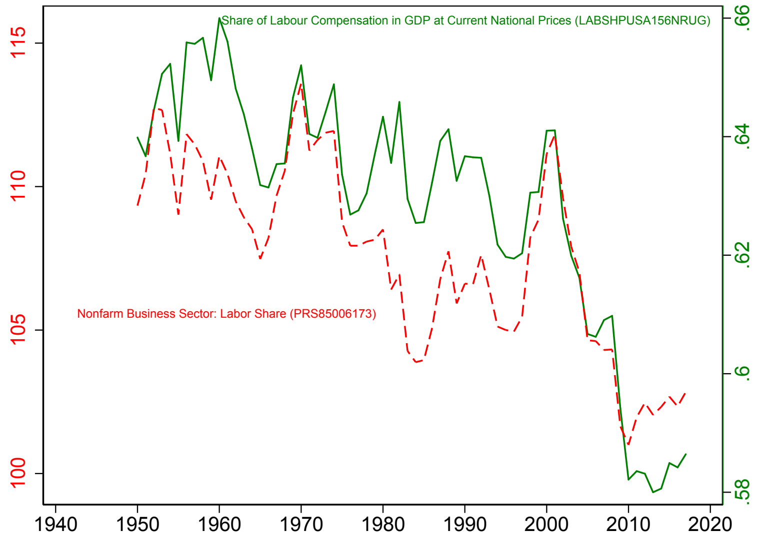
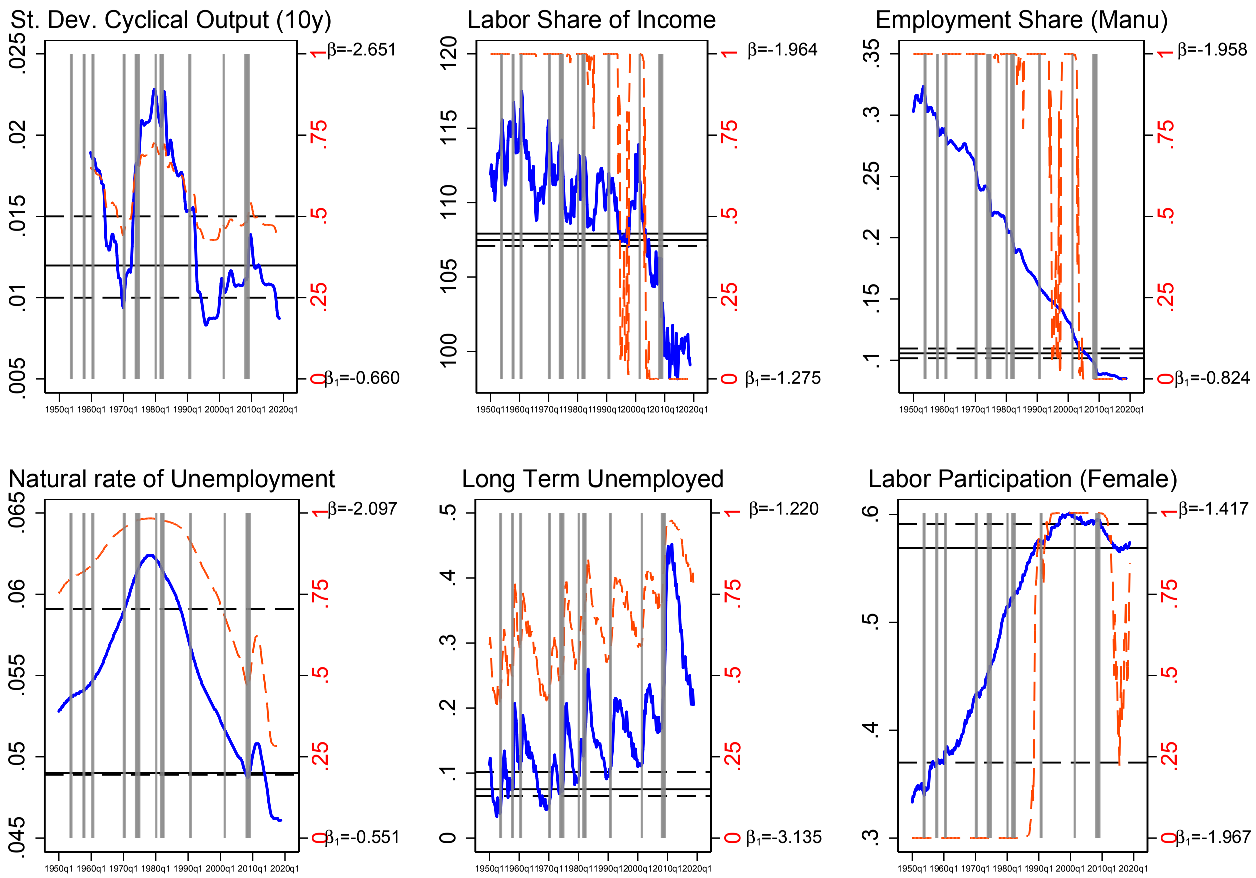
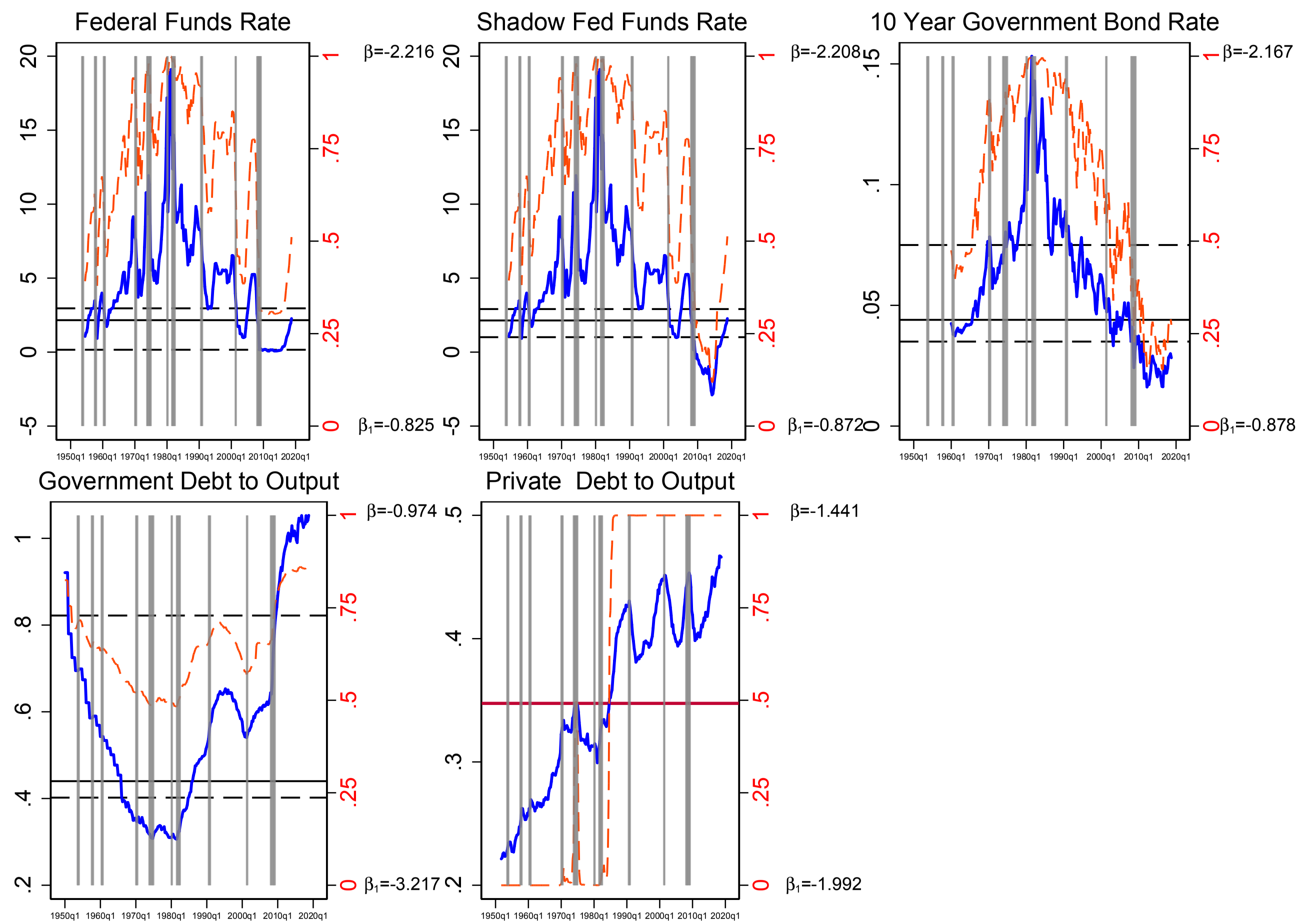


Figure C.2: Threshold Variables, Threshold and Transition Probabilities (Structural Variables), NBER dates



Notes: Blue solid lines represent the threshold variable, z, red dashed lines represent the $\{0,1\}$ transition probabilities and the solid black horizontal line represents the threshold, δ (with black dashed confidence intervals). The β values on the rhs axis are the Okun parameters in the respective regimes. As before, shaded gray areas represents NBER recession dates at quarterly frequency.

FIGURE C.3: Threshold Variables, Threshold and Transition Probabilities (Policy and Financial), NBER dates



Notes: See notes to Figure C.2.

C.2 Additional Tables

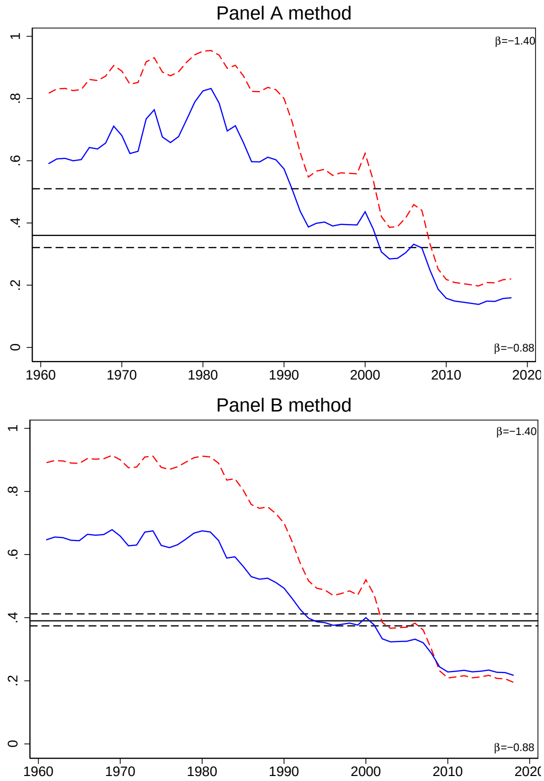
Table C.1: Summary Statistics of Thresholds

Variable	Obs	Mean	Median	Std. Dev.	Min	Max	δ
$labsh$	276	109.294	110.502	4.524	98.150	117.495	107.919
u_l	276	0.161	0.141	0.090	0.033	0.452	0.075
u_n	276	0.055	0.055	0.005	0.046	0.062	0.049
pr_f	276	0.501	0.537	0.092	0.333	0.601	0.569
d	276	0.581	0.572	0.212	0.306	1.053	0.440
dp	269	0.353	0.356	0.071	0.221	0.467	0.347
i_s	258	4.824	4.560	3.609	0.070	19.100	2.160
$i_{s,wx}$	258	4.679	4.560	3.826	-2.890	19.100	2.140
i_l	236	0.061	0.058	0.029	0.016	0.153	0.044
$\sigma_{y^c_{t(10)}}$	237	0.014	0.013	0.004	0.008	0.023	0.020

D Annual Data Analysis

In this Section of the Appendix, we provide estimates of the model with a single factor of the threshold variables based on annual frequency of the data.

Figure D.1: Transition Probability (Annual Frequency of the Data)



Notes: See also notes to Figure 4.

TABLE D.1: Estimates of model (7) with a single factor threshold based on annual data

Panel A: Estimates based on standardized variables $\tilde{z}_j$

β_1	β_2	$\boldsymbol{\beta}$	$100\omega_1$	$100\omega_2$	γ	δ	$\mathcal{D}_2$	AIC	LM^{sup}
				Ignoring Endogeneity					
-1.03	-1.12	-2.15			8.90	0.36	69.26	-220.77	
(0.30)	(0.45)				[5.100.9.44]	[0.351,0.371]			
				Controlling for Endogeneity					
-0.88	-1.40	-2.28	0.02	-0.50	6.40	0.36	70.50	-218.80	9.30
(0.36)	(0.52)		(0.03)	(0.25)	[4.11,8.45]	[0.321,0.510]			[0.000]

Panel B: Estimates based on scaled variables $\ddot{z}_j$

β_1	β_2	$\boldsymbol{\beta}$	$100\omega_1$	$100\omega_2$	γ	δ	$\mathcal{D}_2$	AIC	LM^{sup}
				Ignoring Endogeneity					
-1.16	-0.95	-2.11			17.70	0.37	70.17	-219.93	
(0.25)	(0.37)				[7.21,8.425]	[0.370,0.412]			
				Controlling for Endogeneity					
-0.88	-1.40	-2.28	-0.04	-1.70	8.20	0.39	66.67	-219.93	12.80
(0.36)	(0.52)		(0.04)	(0.87)	[7.210,8.425]	[0.374,0.412]			[0.000]

Notes: See notes to Table 5.

Dimitris Christopoulos
Athens University of Economics & Business, Athens, Greece; email: dk.christopoulos@gmail.com

Peter McAdam
European Central Bank, Frankfurt am Main, Germany; email: peter.mcadam@ecb.europa.eu

Elias Tzavalis
Athens University of Economics & Business, Athens, Greece; email: e.tzavalis@aueb.gr

Postal address 60640 Frankfurt am Main, Germany
Telephone +49 69 1344 0
Website www.ecb.europa.eu

PDF ISBN 978-92-899-3976-8 ISSN 1725-2806 doi:10.2866/120359 QB-AR-19-126-EN-N