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Fiorella De Fiore, Oreste Tristani

(Un)conventional policy and the effective lower bound

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Abstract

We study the optimal combination of conventional (interest rates) and unconventional (credit easing) monetary policy in a model where agency costs generate a spread between deposit and lending rates. We show that unconventional measures can be a powerful substitute for interest rate policy in the face of certain financial shocks. Such measures help shield the real economy from the deterioration in financial conditions and warrant smaller reductions in interest rates. They therefore lower the likelihood of hitting the lower bound constraint. The alternative option to cut interest rates more deeply and avoid deploying unconventional measures is sub-optimal, as it would induce unnecessarily large changes in savers' intertemporal consumption patterns.

Keyworks: optimal monetary policy, unconventional policies, zero-lower bound, asymmetric information

JEL codes: E44, E52, E61

Non-Technical Abstract

After the financial crisis of 2008-09, central banks have aggressively cut monetary policy rates, in many cases all the way to their lower bound. At the same time, many central banks have implemented so-called "non-standard" or "unconventional" monetary policy measures. However, standard and non-standard measures have been combined in different ways. Non-standard measures were introduced in late 2008 and then implemented for a number of years, both in the US and in the euro area. As far as standard monetary policy is concerned, the Federal Reserve cut its interest rates to near zero almost at the same time. The European Central Bank, on the contrary, did not immediately cut its main policy interest rate to zero. The rate on the main refinancing operations (MRO) was reduced sharply at the end of 2008 but it bottomed at 1% in May 2009 and did not fall below that threshold until mid-2012.

Which considerations should determine the mix of standard and non-standard policy responses? This paper presents a theoretical analysis of this issue. It specifically seeks to provide answers to the following questions. Do non-standard measures reduce the likelihood that interest rates reach the effective lower bound (ELB)? Should interest rate policy be used at all, once unconventional measures have been deployed? Looking forward to the return to normal conditions, the so-called “exit”, how long should non-standard policies be optimally kept in place?

We attempt to provide an answer to these questions in the context of a simple dynamic, general equilibrium model that features both sticky prices and financial frictions in the form of asymmetric information between borrowers (firms) and lenders (banks). Following a well-established literature, we assume that banks must pay a monitoring cost to audit borrowers that do not repay their loans. The monitoring cost captures all bankruptcy costs, including legal expenses and any losses associated with asset liquidation. We assume that monitoring costs are subject to stochastic shocks over time. We specifically have in mind time variations in losses associated with asset liquidation, which may increase markedly during crises for example due to fire sales. We interpret adverse financial shocks as increases in monitoring costs, which lead to higher bank lending rates and a reduction in lending.

In this model environment, we define non-standard measures in a stylised fashion. Rather than letting the central bank purchase assets from commercial banks, we assume that it can directly lend to firms, also subject to asymmetric information and a monitoring cost. The key difference is that the monitoring cost of the central bank is higher on average–hence the central

bank is normally less efficient–but it does not increase with adverse financial shocks. Given these assumptions, central bank intermediation is not desirable under normal circumstances. Following an adverse financial shocks, however, commercial banks' monitoring efficiency deteriorates. If the shock is sufficiently severe, private bank lending rates may increase up to the point where central bank intermediation becomes competitive.

Our main result is that, in the face of adverse shocks of a financial nature–shocks which reduce banks' monitoring efficiency–non-standard measures can be powerful substitutes of standard interest rate policy. Non-standard measures can mitigate the repercussions of the financial shock on the real economy and on inflation. They therefore reduce the need for interest rate cuts.

The alternative option to cut interest rates and avoid deploying non-standard measures would be suboptimal. Reductions in policy rates would at the same time dampen the increase in financing costs for borrowers and reduce asset returns for lenders. By contrast, non-standard measures can shield borrowers from unwarranted increases in lending rates without affecting lenders. Interest rate cuts remain however optimal in reaction to other demand-type shocks which do not directly arise from the financial sector.

We illustrate our results in a "financial crisis" scenario in which the economy is hit by a highly persistent financial shock. In this scenario, interest rate cuts and non-standard measures must be used in combination, because the crisis creates downward pressure on prices. In the normalization phase, interest rates are increased once deflationary pressures abate, but non-standard measures remain in place for a prolonged period of time due to the persistent effects of the crisis.

1 Introduction

In response to the financial and economic crisis of 2008-09, central banks have aggressively cut monetary policy rates, in many cases all the way to the effective lower bound (henceforth ELB), namely the rate below which it becomes profitable for financial institutions to exchange central bank reserves for cash.[1] At the same time, all central banks have implemented so-called "non-standard" or "unconventional" monetary policy measures.

However, standard and non-standard measures have been combined in different ways by different central banks (for a cross-country comparison see e.g. Lenza, Pill and Reichlin, 2010). Taking the expansion of the central banks' balance sheets as an indicator, non-standard measures were implemented in late 2008, after the failure of Lehman Brothers, both in the US and in the euro area. As far as standard monetary policy is concerned, the Federal Reserve cut its interest rates to near zero almost at the same time: the Federal funds rate reached 1% at the end of October and the 0.00-0.25% range in December. The European Central Bank, on the contrary, did not immediately cut its main policy interest rate to zero. The rate on the main refinancing operations (MRO) was reduced sharply at the end of 2008 but it bottomed at 1% in May 2009 and did not fall below that threshold until mid 2012.[2]

Some guidance on the sequencing of standard and non-standard measures can be obtained from the ELB literature which predates the financial crisis (see e.g. Reifschneider and Williams, 2000, Eggertsson and Woodford, 2003, Adam and Billi, 2006, and Nakov, 2008). The tenet of that literature is that standard interest rate policy is the best monetary policy tool in response to shocks leading to a fall in the natural rate of interest. Any other type of policy response should only be considered as a substitute for standard interest rate policy, once the latter is no longer available because the ELB constraint is binding. Indeed any non-standard measures involving the central bank balance sheet are ineffective in the standard new-Keynesian model.

A number of recent papers have reconsidered this issue and demonstrated that certain non-standard measures can be an effective tool in the presence of distortions which prevent the efficient allocation of financial resources – see e.g. Gertler and Karadi (2010), Gertler and Kiyotaki (2010), Del Negro et al. (2010), Eggertsson and Krugman (2010), Cúrdia and Woodford (2011) and Correia et al. (2016). Such measures have been described as "credit policy", i.e. measures aimed at offsetting impairments to the process of credit creation. As

[1] The recent experience of many developed countries has shown tht the lower bound for nominal interest rates is not zero, as previously assumed, but negative due to cash storage costs. This is why we refer to the "effective" lower bound (rather than the more traditional "zero" lower bound). In our theoretical model, however, cash storage costs are ignored, thus the effective lower bound is equal to zero.

[2] The interest rates were further reduced after the intensification of the sovereign debt crisis and during the following economic crisis. The rate on the depost facility reached zero in July 2012, before entering negative territory. The MRO rate was cut to zero in March 2016.

such, standard and non-standard measures can be complementary to each other. Their optimal mix and sequencing are no longer straightforward to determine.

Our paper analyses the optimal combination of standard and non-standard policies and seeks answers to the following questions. If non-standard measures can be targeted to the prevailing source of financial inefficiency, do they reduce the likelihood that interest rates reach the ELB? Should interest rate policy be used at all, once unconventional measures have been deployed? Looking forward to the return to normal conditions, the so-called "exit", how long should non-standard policies be optimally kept in place?

We attempt to answer these questions within a simple dynamic, general equilibrium model that features both sticky prices and financial frictions in the form of asymmetric information between borrowers and lenders. As in Bernanke, Gertler and Gilchrist (1999), we also assume that lenders must pay a monitoring cost to audit borrowers that do not repay their loans. The monitoring cost captures all bankruptcy costs, including legal expenses and any losses associated with asset liquidation. We however deviate from Bernanke, Gertler and Gilchrist (1999) in assuming that lending can potentially be provided by both commercial banks and the central bank. Private lending is normally more efficient, because commercial banks have a superior loan monitoring technology. As a result, only commercial banks will provide credit to the economy under normal conditions. In a crisis, however, commercial banks monitoring costs rise due to the increase in losses associated with asset liquidation. If the crisis is sufficiently severe, the central bank becomes a competitive lender and can replace commercial banks in providing loans to firms.[3]

Our main result is that under certain circumstances – notably in reaction to a specific type of financial shocks which reduce banks' monitoring efficiency – credit policy may be a strictly more efficient tool than policy interest rates.

Credit policy is desirable because it contributes to insulate the real economy from the increase in credit spreads caused by deterioration in banks' efficiency. The more effective the insulation, the lower the need to lower policy rates. In an illustrative example, we show that it can be optimal for the central bank not to cut rates to zero, and to implement non-standard measures instead. By contrast, reductions in policy rates without deploying non-standard

[3] Our modelling of unconventional policy mirrors the early phase of purchase of private sector securities in the US (the so-called QE1, which was implemented from December 2008 to March 2010) and the recent Corporate Bonds Purchase Programme in the euro area (started in June 2016).

measures would be inefficient. While they can (away from the ELB) insulate borrowers from the increase in financing costs induced by the financial shock, lower policy rates also induce savers to change their intertemporal consumption patterns. Given the financial nature of the shock, such changes are inefficient.

In general, the exact timing of implementation of standard and non-standard measures depends on the size of the monitoring advantage of commercial banks over the central bank – an object which is difficult to calibrate. Non-standard measures are more likely to be deployed in response to large financial shocks, while there is no scope for using them in reaction to demand, or technology shocks.

To develop an intuition for what optimal policy ought to do in reaction to financial shocks that increase credit spreads, we derive in closed form the target rule which would implement the Ramsey allocation under the timeless perspective in our simple model. We focus for simplicity on the case when the ELB is ignored. Compared to the model with frictionless financial markets, the target rule implies a stronger mean reversion of the price level. In response to a shock which increases the price level on impact, the price level falls over time and eventually returns to a value lower than its initial level – and viceversa.

In our simple model, financial shocks affect firms' marginal costs and have a cost-push component. As a result, while typically lowering interest rates on impact to cushion the adverse effects on the real economy, optimal policy also requires stabilizing inflation through a commitment to increase rates relatively quickly thereafter – notably increasing them long before non-standard measures are reabsorbed.

We test the robustness of this conclusion in a richer model with capital, where an increase in credit spreads directly affects aggregate demand, notably by depressing investment. In this case, interest rates are optimally increased much more slowly than in the simple model. However it remains true that non-standard measures tend to remain in place long after the policy interest rate has returned to its long run level.

Finally, we revisit the prescription of the simple new Keynesian literature that the likelihood of being at the ELB and the severity of the ensuing recession can be reduced by an appropriate policy commitment. More specifically the central bank should promise to keep interest rates low in the future for a longer period than optimal in the absence of the ELB. Such a promise, if credible, generates high inflation expectations, reduces the current real interest rate and stimulates the economy. When non-standard measures are ruled out, or unwarranted because adverse shocks are not of a financial nature, these prescriptions remain valid in our model. When non-standard measures are effectively deployed, however, keeping the policy rate low for an extended period of time may no longer be necessary.

This paper contributes to a recent literature that explains the lack of deflationary pressure during the Great Recession. In our model, because firms need to borrow in advance of production and debt is nominal, increases in the policy rate exert upward pressure on marginal costs and inflation. This effect has been named in the literature the "cost" channel (Ravenna and Walsh, 2010). Increases in credit spreads also rise the costs of finance and exert similar inflationary pressure through the so called "credit" channel (De Fiore and Tristani, 2012). Recent empirical evidence is consistent with these channels being active during the financial crisis. For instance, Gilchrist et al. (2015) analyses good-level transaction prices and firms' income and balance sheet data during the U.S. Great Recession. It finds that firms with limited internal liquidity and high leverage significantly increased their prices in 2008, a period characterized by disruptions in credit markets and a sharp contraction in output. Abbate et al. (2016) uses a VAR analysis where financial shocks are identified through sign restrictions to assess the extent to which these shocks account for the "missing disinflation" during the U.S. Great Recession. It finds that adverse financial shocks helped preventing deflation during the crisis and that the response of inflation can mainly be attributed to the cost channel.

Our work relates to Curdia and Woodford (2015), which analyses the optimal conduct of monetary policy in a New Keynesian (NK) model where credit spreads arise from a different type of financial friction. Curdia and Woodford (2015) derives an analytical target rule for the special case when credit spreads do not depend on the volume of loans. In this case, a main finding is that the target rule is identical to the one that arises in the standard NK model with frictionless financial markets. We also derive the optimal target rule but find that the presence of the credit channels does modify it from the benchmark NK case, creating different implications for the price level. The main reason for this difference is that in our model credit spreads depend on the corporate sector's leverage and on the resources needed to conduct banks' monitoring activity. In the numerical analysis of a more general case where credit spreads are allowed to depend on the volume of loans, Curdia and Woodford (2015) also finds that the standard NK target rule does not perfectly approximate the optimal policy, in line with our results.

The paper also relates to Harrison (2017), which studies the optimal combination of interest rate policy and quantitative easing in a New Keynesian model with portfolio adjustment costs. Under those conditions non-standard measures are implemented only when the interest rate hits the zero-lower bound.

The paper is structured as follows. In section 2, we describe the model and characterize the equilibrium. We also derive a system of log-linear equilibrium conditions, which we later use to develop an intuition for our numerical results. In section 3, we present the welfare analysis. We derive a second-order approximation to the welfare function and the first order conditions of the Ramsey allocation. This allows us to derive in closed form the target rule which, absent the ELB constraint, would implement the Ramsey allocation. In section 4, we outline the procedure we use to solve the model under the ELB constraint and we present our numerical results. In this section, we also show that our main results extend to a model where credit spreads directly affect aggregate demand by depressing investment. Section 5 offers some concluding remarks.

2 The model

The economy is inhabited by a representative infinitely-lived household, wholesale firms owned by risk-neutral entrepreneurs, monopolistically competitive retail firms owned by the households, zero-profit financial intermediaries, a government and a central bank. We describe in turn the problem faced by each class of agents.

2.1 Households

At the beginning of period t, households receive interest payments from the nominal financial assets acquired at time $t-1$. The households, holding an amount W_t of nominal wealth, choose to allocate it among existing nominal assets, namely money M_t , a portfolio of nominal state-contingent bonds A_{t+1}, and one-period deposits denominated in units of currency, D_t.

In the second part of the period, the goods market opens. Households' money balances are increased by the nominal amount of their revenues and decreased by the value of their expenses. Taxes are also paid or transfers received. The amount of nominal balances brought into period $t+1$, $\widetilde{M}_t$, is given by

$$\widetilde{M}_t \equiv M_t + P_t w_t h_t + Z_t - P_t c_t + T_t, \tag{1}$$

where h_t is hours worked, w_t is the real wage, Z_t are nominal profits transferred from retail producers to households, and T_t are lump-sum nominal transfers from the government. c_t denote a CES aggregator of a continuum $\eta \in (0,1)$ of differentiated consumption goods produced by retail firms, $c_t = \left[\int_0^1 c_t(\eta)^{\frac{\varepsilon-1}{\varepsilon}} d\eta\right]^{\frac{\varepsilon}{\varepsilon-1}}$, with $\varepsilon > 1$. P_t is the price of the CES aggregator.

Nominal wealth at the beginning of period $t+1$ is given by

$$W_{t+1} = A_{t+1} + R_t^d D_t + R_t^m \widetilde{M}_t, \tag{2}$$

where each of the state-contingent bonds in the portfolio A_{t+1} pays one unit of currency in a particular state in period $t+1$, and R_t^d is the gross interest paid on deposits at the end of period t. As in Woodford (2003), we allow end-of-period private money holdings to be remunerated at the rate R_t^m. In particular, we assume that monetary policy sets R_t^m at a level that is proportional (and possibly equal) to the risk-free rate paid on central banks' reserves, R_t^d.

The household's problem is to maximize preferences, defined as

$$E_o\left\{\sum_0^\infty \beta^t \left[u(c_t) + \kappa(m_t) - v(h_t)\right]\right\},$$

where $u_c > 0$, $u_{cc} < 0$, $\kappa_m \geq 0, \kappa_{mm} < 0$, $v_h > 0$, $v_{hh} > 0$, and $m_t \equiv M_t/P_t$ denotes real balances, subject to the budget constraints

$$M_t + D_t + E_t\left[Q_{t,t+1}A_{t+1}\right] \leq W_t, \tag{3}$$

together with (1) and (2).

Define $\Lambda_{m,t} \equiv \frac{R_t^d - R_t^m}{R_t^d}$. Under our assumption of proportionality between the remuneration of cash and the risk-free rate, $\Lambda_{m,t} = \Lambda_m$ for all t. The households' optimality conditions are then given by

$$R_t = R_t^d = E_t\left[Q_{t,t+1}\right]^{-1}$$

$$-\frac{v_h(h_t)}{u_c(c_t)} = w_t, \tag{4}$$

$$u_c(c_t) = \beta R_t E_t\left\{\frac{u_c(c_{t+1})}{\pi_{t+1}}\right\}, \tag{5}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$. The money demand is given residually by

$$\kappa_m(m_t) = \frac{\Lambda_m}{1-\Lambda_m} u_c(c_t).$$

The optimal allocation of expenditure between the different types of goods is given by $c_t(\eta) = \left(\frac{P_t(\eta)}{P_t}\right)^{-\varepsilon} c_t$, where $P_t(\eta)$ is the price of good η.

2.2 Wholesale firms

Wholesale firms, indexed by i, are competitive and owned by infinitely lived entrepreneurs. Each firm i produces the amount $y_{i,t}$ of an homogeneous good, using a linear technology

$$y_{i,t} = \omega_{i,t} l_{i,t}. \tag{6}$$

Here $\omega_{i,t}$ is an iid productivity shock with distribution function Φ and density function ϕ, which is observed at no cost only by firms.

At the beginning of the period, each firm receives an exogenous endowment τ, which can be used as internal funds. Since these funds are not sufficient to finance the firm's desired level of production, firms need to raise external finance. Before observing $\omega_{i,t}$, firms sign a contract with a financial intermediary to raise a nominal amount $P_t (x_{i,t} - \tau)$, where

$$x_{i,t} \geq w_t l_{i,t}. \tag{7}$$

Each firm i's demand for labor is derived by maximizing firm's expected profits, subject to the financing constraint (7).

Let $\overline{P}_t$ be the price of the wholesale homogenous good, $\frac{\overline{P}_t}{P_t} = \chi_t^{-1}$ the relative price of wholesale goods to the aggregate price of retail goods, and $(q_t - 1)$ the Lagrange multiplier on the financing constraint. Optimality requires that

$$q_t = \frac{1}{w_t \chi_t} \tag{8}$$

$$x_{i,t} = w_t l_{i,t} \tag{9}$$

implying that

$$\mathcal{E}(y_t) = \chi_t q_t x_t, \tag{10}$$

where $\mathcal{E}[\cdot]$ is the expectation operator at the time of the factor hiring decision.

Equation (10) states that wholesale firms must sell at a mark-up $\chi_t q_t$ over firms' production costs to cover for the presence of credit frictions and for the monopolistic distortion in the retail sector. Notice that all firms are ex-ante identical. Hence, we drop below the subscripts i.

The assumption that firms receive an endowment from the government at the beginning of the period is made for simplicity, in order to facilitate the analytical characterization of the optimal monetary policy and the computation of the numerical non-linear solution of the model. The absence of accumulation of firms' net worth implies that the persistence of the endogenous variables merely reflects the persistence of the exogenous shocks. Nonetheless, financial frictions provide an important transmission channel in our economy, through the credit constraint faced by firms and the endogenous spread charged by financial intermediaries. As documented in De Fiore and Tristani (2012), up to a linear approximation, the model with and without capital accumulation delivers qualitatively similar responses to both real and financial shocks. The characterization of optimal monetary policy is also broadly similar in these two cases.

2.3 The financial contract

In writing the financial contract we need to be explicit about what constitutes unconventional policy in our model. We will focus on an interpretation of non-standard measures in which the central bank replaces the private banking sector and does direct intermediation to firms.

Direct lending is closest to the Federal Reserve facilities set up for direct acquisition of high quality private securities (see also Gertler and Kiyotaki, 2010) and to the Corporate Bonds Asset Purchase Programme implemented by the Eurosystem since June 2016. As in both the Fed and the ECB cases, in our model the central bank lending program is financed though an increase in interest bearing banks' reserves. As a result, non-standard measures lead to a large increase in the central bank's balance sheet.

Direct lending in our model is entirely demand determined: central bank intermediation is chosen endogenously when it can be performed at a lower cost (spread) than private bank intermediation.

Finally, we design credit policy in such a way that the central bank takes on no credit risk. Together with the assumption that reserves are remunerated, this implies that the expansion of the central bank's balance sheet has no inflationary consequences, nor any implications for government finances.

The financial contract is structured as follows. External finance takes the form of either bank loans or direct lending from the central bank. Firms face the idiosyncratic productivity shock $\omega_{i,t}$, whose realization is observed at no costs only by the entrepreneur. If the realization of the idiosyncratic shock $\omega_{i,t}$ is sufficiently low, the value of firm production is not sufficient to repay the loans and the firm defaults.

The financial intermediaries (banks or the central bank) can monitor ex-post the realization of $\omega_{i,t}$, but a fraction of firm's output is consumed in the monitoring activity. These monitoring costs are associated with legal fees and asset liquidation in case of bankruptcy. We assume that commercial banks are on average more efficient monitors than the central bank, i.e. $\mu^c > \mu^b$, where μ^c and μ^b denote the steady state fraction of the firm output lost in monitoring by the central bank and by commercial banks, respectively.

Commercial banks collect deposits D_t from households. Deposits are the only funds available to finance loans in the economy. Each representative commercial bank uses a fraction γ_t of deposits to finance loans to firms, and deposits the remaining fraction, $1 - \gamma_t$, as reserves at the central bank. These reserves are remunerated at the market rate R_t^d and used in turn by the central bank to finance firms. The fraction γ_t of deposits lent by each commercial bank is then combined with a fraction γ_t of the firms' internal funds to finance the production of $\gamma_t q_t \chi_t x_{i,t}$ units of wholesale goods.

The contract stipulates a loan amount $x_{i,t} - \tau$. The firm needs to pay back a unit gross interest rate $R_{i,t}^b$ on bank loans and $R_{i,t}^c$ on loans from the central bank, which can differ across firms. The total repayment is $P_t\left(R_{i,t}^b\gamma_t + R_{i,t}^c\left(1-\gamma_t\right)\right)\left(x_{i,t}-\tau\right)$. The firm is able to meet those payments when $\omega_{i,t} \geq \overline{\omega}_{i,t}$, where $\overline{\omega}_{i,t}$ is the minimum productivity level such that the firm can pay the agreed return and is implicitely defined by

$$\overline{P}_t\overline{\omega}_{i,t}l_{i,t} = P_t\left(R_{i,t}^b\gamma_t + R_{i,t}^c\left(1-\gamma_t\right)\right)\left(x_{i,t}-\tau\right) \tag{11}$$

When $\omega_{i,t} < \overline{\omega}_{i,t}$, the firm goes bankrupt, and hands out all the production $\overline{P}_t A_t \omega_{i,t} l_{i,t}$, in units of currency. In this case, a constant fraction μ_t^j of the firm's output is destroyed in monitoring. The bank obtains $\gamma_t\left(1-\mu_t^b\right)\overline{P}_t A_t\omega_{i,t}l_{i,t}$ and the central bank $\left(1-\gamma_t\right)\left(1-\mu_t^c\right)\overline{P}_t A_t\omega_{i,t}l_{i,t}$.

Using the equilibrium relationshis $\frac{\overline{P}_t}{P_t} = \chi_t^{-1}$, $q_t = \frac{1}{w_t\chi_t}$ and $x_{i,t} = w_t l_{i,t}$ we can then rewrite $\overline{\omega}_{i,t}$ as

$$\overline{\omega}_{i,t} = \frac{\left(R_{i,t}^b\gamma_t + R_{i,t}^c\left(1-\gamma_t\right)\right)}{q_t}\left(1-\frac{\tau}{x_{i,t}}\right) \tag{12}$$

Define

$$f\left(\overline{\omega}\right) \equiv \int_{\overline{\omega}}^{\infty}\omega\Phi\left(d\omega\right) - \overline{\omega}\left[1-\Phi\left(\overline{\omega}\right)\right], \tag{13}$$

$$g^b\left(\overline{\omega};\mu^b\right) \equiv \int_0^{\overline{\omega}}\left(1-\mu^b\right)\omega\Phi\left(d\omega\right) + R^b\frac{1}{q}\left(1-\frac{\tau}{x}\right)\left[1-\Phi\left(\overline{\omega}\right)\right] \tag{14}$$

and

$$g^c\left(\overline{\omega};\mu^c\right) \equiv \int_0^{\overline{\omega}}\left(1-\mu^c\right)\omega\Phi\left(d\omega\right) + R^c\frac{1}{q}\left(1-\frac{\tau}{x}\right)\left[1-\Phi\left(\overline{\omega}\right)\right] \tag{15}$$

as the expected unit shares of output accruing respectively to the firm, the commercial bank and the central bank, after stipulating a financial contract that sets a lending rate R^b or R^c. Notice that

$$f\left(\overline{\omega}_{i,t}\right)+\gamma_t g^b\left(\overline{\omega}_{i,t};\mu_t^b\right)+\left(1-\gamma_t\right)g^c\left(\overline{\omega}_{i,t};\mu_t^c\right)=1-\left[\gamma_t\mu_t^b+\left(1-\gamma_t\right)\mu_t^c\right]G\left(\overline{\omega}_{i,t}\right) \tag{16}$$

where $G\left(\overline{\omega}_{i,t}\right)=\int_0^{\overline{\omega}_{i,t}}\omega\Phi\left(d\omega\right)$. On average, $\left[\gamma_t\mu_t^b+\left(1-\gamma_t\right)\mu_t^c\right]G\left(\overline{\omega}_{i,t}\right)$ of output is lost in monitoring.

The budget constraint for the commercial bank is

$$\left(1-\gamma_t\right)R_t^d P_t\left(x_{i,t}-\tau\right)+\gamma_t\overline{P}_t q_t\chi_t g^b\left(\overline{\omega}_{i,t};\mu_t^b\right)x_{i,t}\geq R_t^d P_t\left(x_{i,t}-\tau\right).$$

The first term on the left-hand side is the amount of reserves held at the central bank, gross of their remuneration, in units of currency. The second term is the gross nominal return to banks from extending credit of $\gamma_t P_t\left(x_{i,t}-\tau\right)$ units of money to firms. The righ-hand side is the cost of funds for the bank.

The central bank uses all its funds (reserves) to satisfy the demand for credit by firms. Its budget constraint is

$$\overline{P}_t q_t\chi_t g^c\left(\overline{\omega}_{i,t};\mu_t^c\right)x_{i,t}\geq R_t^d P_t\left(x_{i,t}-\tau\right).$$

The constraint says that the return to the central bank from lending $P_t\left(x_{i,t}-\tau\right)$ units of money to firms must be sufficient to cover for the costs of funds (the remuneration of reserves).

Each firm stipulates a contract that sets a fixed repayment on each unit of debt of $\overline{P}_t\chi_t q_t\overline{\omega}_{i,t}$. The contract also sets the fraction of deposits the commercial bank needs to devote to loans and the fraction to be held as reserves at the central bank. The informational structure corresponds to a standard costly state verification (CSV) problem (see e.g. Gale and Hellwig (1985)). The problem is

$$\max_{\overline{\omega}_{i,t},x_{i,t},\gamma_t} f(\overline{\omega}_{i,t})q_t x_{i,t}$$

subject to

$$\begin{aligned}
q_t g^b\left(\overline{\omega}_{i,t};\mu_t^b\right)x_{i,t} &\geq R_t^d\left(x_{i,t}-\tau\right) && (17)\\
q_t g^c\left(\overline{\omega}_{i,t};\mu_t^c\right)x_{i,t} &\geq R_t^d\left(x_{i,t}-\tau\right) && (18)\\
f\left(\overline{\omega}_{i,t}\right)+\gamma_t g^b\left(\overline{\omega}_{i,t};\mu_t^b\right)+\left(1-\gamma_t\right)g^c\left(\overline{\omega}_{i,t};\mu_t^c\right) &\leq 1-\left[\gamma_t\mu_t^b+\left(1-\gamma_t\right)\mu_t^c\right] && (19)\\
q_t x_t f(\overline{\omega}_{i,t}) &\geq \tau && (20)\\
0 &\leq \gamma_t\leq 1. && (21)
\end{aligned}$$

The optimal contract is the set $\{x_{i,t}, \overline{\omega}_{i,t}, \gamma_t\}$ that maximizes the entrepreneur's expected profits, subject to the profits of the private bank and those of the central bank being sufficient to cover their respective repayment on deposits, (17) and (18), the feasibility condition, (19), the entrepreneur being willing to sign the contract, (20), and the share $\gamma_{i,t}$ being between zero and one. Define $\lambda_{1i,t}$ and $\lambda_{2i,t}$ as the lagrangean multipliers associated to $\gamma_{i,t} \geq 0$ and to $\gamma_{i,t} \leq 1$, respectively.

The optimality conditions include (16) and

$$f(\overline{\omega}_{i,t})q_t = \frac{R_t^d}{1 - \frac{\left[\gamma_t\mu_t^b + (1-\gamma_t)\mu_t^c\right]\overline{\omega}_{i,t}\phi(\overline{\omega}_{i,t})}{\left[1-\Phi(\overline{\omega}_{i,t})\right]}} \frac{\tau}{x_{i,t}} \tag{22}$$

$$x_{i,t} = 1 + \frac{1 - f\left(\overline{\omega}_{i,t}\right) - \left(\gamma_t\mu_t^b + (1-\gamma_t)\,\mu_t^c\right) G\left(\overline{\omega}_{i,t}\right)}{f(\overline{\omega}_{i,t})\left[1 - \frac{\left(\gamma_t\mu_t^b + (1-\gamma_t)\mu_t^c\right)\overline{\omega}_{i,t}\phi(\overline{\omega}_{i,t})}{1-\Phi(\overline{\omega}_{i,t})}\right]}\tau \tag{23}$$

$$\eta_{1,t} - \eta_{2,t} = \frac{\left(R_{i,t}^c - R_{i,t}^b\right)\left(x_{i,t} - \tau\right)\frac{1}{\tau}\left[1 - \Phi\left(\overline{\omega}_{i,t}\right)\right]}{1 - \frac{\phi(\overline{\omega}_{i,t})\overline{\omega}_{i,t}\left[\gamma_t\mu_t^b + (1-\gamma_t)\mu_t^c\right]}{\left[1-\Phi(\overline{\omega}_{i,t})\right]}} \tag{24}$$

$$\eta_{1,t}\gamma_t = 0 \tag{25}$$

$$\eta_{2,t}\left(1 - \gamma_t\right) = 0 \tag{26}$$

together with $\lambda_{1i,t} \geq 0$ and $\lambda_{2i,t} \geq 0$. A formal derivation of the solution to the optimal contract can be found in appendix A.

Notice that equation (23) expresses the size of production of each firm, $x_{i,t}$, as a function of the threshold $\overline{\omega}_{i,t}$ and of aggregate variables only. Plugging this expression into equation (22), it becomes clear that threshold $\overline{\omega}_{i,t}$ is itself a function of aggregate variables only. Hence, $\overline{\omega}_{i,t} = \overline{\omega}_t$, and $x_{i,t} = x_t$, for all i.

The solution to the optimal contract is such that γ_t takes the value of either zero or one. If $R_{i,t}^c > R_{i,t}^b$, then $\eta_{1,t} > \eta_{2,t} \geq 0$. Now, if both $\eta_{1,t}$ and $\eta_{2,t}$ are strictly positive, at least one of conditions (25)-(26) cannot be verified. It follows that the only solution to the optimal contract is $\gamma_t = 0$ and $\eta_{2,t} = 0$. If $R_t^c < R_t^b$, then $0 \leq \eta_{1,t} < \eta_{2,t}$, implying that $\gamma_t = 1$ and $\eta_{1,t} = 0$. Only when the interest rates on commercial bank loans and on central bank loans are the same, $R_{i,t}^c = R_{i,t}^b$, any value of γ_t satisfies the optimality conditions of the contract. It is irrelevant which share of intermediation is conducted by commercial banks or by the central bank.

Given the solution to the CSV problem, the gross interest rate on loans extended to firms by the commercial bank, $R_{i,t}^b$, and the one extended to firms by the central bank, $R_{i,t}^c$, can be backed up from the condition for debt repayment, (11). They are implicitely given by

$$\overline{P}_t \overline{\omega}_t \chi_t q_t x_t = R^b_{i,t} P_t (x_t - \tau), \tag{27}$$

when $\gamma_t = 1$, and by

$$\overline{P}_t \overline{\omega}_t \chi_t q_t x_t = R^c_{i,t} P_t (x_t - \tau), \tag{28}$$

when $\gamma_t = 0$. From conditions (27) and (28), it follows that both commercial banks and the central bank charge a single loan rate to all firms, $R^b_{i,t} = R^b_t$ and $R^c_{i,t} = R^c_t$, for all i.

From (17) and (18), which hold as equality in equilibrium, we know that $g^c (\overline{\omega}_{i,t}; \mu^c_t) = g^b \left(\overline{\omega}_{i,t}; \mu^b_t\right)$. It follows from (14) and (15) that

$$\frac{1}{q_t}\left(1 - \frac{\tau}{x_t}\right) \left[1 - \Phi(\overline{\omega}_t)\right] \left(R^c_t - R^b_t\right) = \int_0^{\overline{\omega}_t} \left(\mu^c_t - \mu^b_t\right) \omega_t \Phi(d\omega).$$

Therefore, $R^c_t > R^b_t$ implies that $\mu^c_t > \mu^b_t$.

The optimal financial contract thus has to satisfy the conditions (22) and (23), together with

$$\gamma_t = \begin{cases} 1 \text{ if } \mu^c_t \geq \mu^b_t \\ 0 \text{ if } \mu^c_t < \mu^b_t \end{cases}. \tag{29}$$

Define now the spread between loan rates and the risk-free rate as $\Lambda^j_t = \frac{R^j_t}{R_t}$, for $j = b, c$. We can now use expressions (17)-(18) and (27)-(28) to relate those spreads to the thresholds for the idiosyncratic productivity shocks, $\overline{\omega}^j_t$,

$$\Lambda^j_t = \frac{\overline{\omega}^j_t}{g(\overline{\omega}^j_t; \mu^j_t)}. \tag{30}$$

2.4 Entrepreneurs

Entrepreneurs die with probability γ_t. They have linear preferences over the same CES basket of differentiated consumption goods as households, with rate of time preference β^e. This latter is sufficiently high so that the return on internal funds is always larger than the rate of time preference, $\frac{1}{\beta^e} - 1$, and entrepreneurs postpone consumption until the time of death.

As in De Fiore, Teles and Tristani (2011), we assume that the government imposes a tax ν on entrepreneurial consumption. It follows that

$$(1+\nu) \int_0^1 P_t(\eta) e_t(\eta) d\eta = \overline{P}_t (\omega_t - \overline{\omega}_t) \chi_t q_t x_t,$$

where $e_t(\eta)$ is the firm's consumption of good η. Notice that $\int_0^1 P_t(\eta)\, e_t(\eta) = P_t e_t$,where e_t is the demand of the final consumption good. We can then write

$$(1+\nu)\, e_t = f(\overline{\omega}_t) q_t x_t.$$

We consider the case where ν becomes arbitrarily large.[4] Consumption of the bankers approaches zero, $e_t \to 0$, and the consumption tax revenue,

$$T_t^e = \left(\frac{\nu}{1+\nu}\right) f(\overline{\omega}_t) q_t x_t, \tag{31}$$

approaches the total funds of the bankers that die.

2.5 Government

Revenues from taxes on entrepreneurial consumption are used by the government to finance the transfer τ. Funds below (in excess of) τ are supplemented through (rebated to) households lump-sum taxes (transfers), T_t^h. The budget constraint of the government is

$$T_t^e = \tau - T_t^h. \tag{32}$$

2.6 Retail firms

As in Bernanke, Gertler and Gilchrist (1999), monopolistic competition occurs at the retail level. A continuum of monopolistically competitive retailers buy wholesale output from entrepreneurs in a competitive market and then differentiate it at no cost. Because of product differentiation, each retailer has some market power. Profits, Z_t, are distributed to the households, who own firms in the retail sector.

Output sold by retailer η, $Y_t(\eta)$, is used for households' and entrepreneurs' consumption. Hence, $Y_t(\eta) = c_t(\eta) + e_t(\eta)$. The final good Y_t is a CES composite of individual retail goods $Y_t = \left[\int_0^1 Y_t(\eta)^{\frac{\varepsilon-1}{\varepsilon}} d\eta\right]^{\frac{\varepsilon}{\varepsilon-1}}$, with $\varepsilon > 1$.

We assume that each retailer can change its price with probability $1-\theta$, following Calvo (1983). Let $P_t^*(\eta)$ denote the price for good η set by retailers that can change the price at

[4] The reason for this assumption is that, with $e_t > 0$, it would be optimal for policy to generate a redistribution of resources between households and entrepreneurs. This would enable to exploit the risk-neutrality of the latter to smooth out consumption of the former. Since risk neutrality of entrepreneurs is a simplifying assumption needed to derive debt as an optimal contract, we eliminate this type of incentives for monetary policy by completely taxing away entrepreneurial consumption. Allowing entrepreneurs to consume would also require arbitrary choices on the weight of entrepreneurs to be given in the social welfare function.

time t, and $Y_t^*(\eta)$ the demand faced given this price. Then each retailer chooses its price to maximize expected discounted profits. The optimality conditions are given by

$$1 = \theta\pi_t^{\varepsilon-1} + (1-\theta)\left(\frac{\varepsilon}{\varepsilon-1}\frac{\overline{\Theta}_{1,t}}{\overline{\Theta}_{2,t}}\right)^{1-\varepsilon} \tag{33}$$

$$\overline{\Theta}_{1,t} = \frac{1}{\chi_t}Y_t + \theta E_t\left[\pi_{t+1}^{\varepsilon}\overline{Q}_{t,t+1}\overline{\Theta}_{1,t+1}\right] \tag{34}$$

$$\overline{\Theta}_{2,t} = Y_t + \theta E_t\left[\pi_{t+1}^{\varepsilon-1}\overline{Q}_{t,t+1}\overline{\Theta}_{2,t+1}\right], \tag{35}$$

where $\overline{Q}_{t,t+k} = \beta^k\left[\frac{u_c(c_{t+k})}{u_c(c_t)}\right]$.

Recall that the aggregate retail price level is given by $P_t = \left[\int_0^1 P_t(\eta)^{1-\varepsilon} d\eta\right]^{\frac{1}{1-\varepsilon}}$. Define the relative price of differentiated good η as $p_t(\eta) \equiv \frac{P_t(\eta)}{P_t}$ and divide both sides by P_t to express everything in terms of relative prices, $1 = \int_0^1 (p_t(\eta))^{1-\varepsilon} d\eta$. Now define the relative price dispersion term as

$$s_t \equiv \int_0^1 (p_t(\eta))^{-\varepsilon} d\eta.$$

This equation can be written in recursive terms as

$$s_t = (1-\theta)\left(\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right)^{-\frac{\varepsilon}{1-\varepsilon}} + \theta\pi_t^{\varepsilon}s_t. \tag{36}$$

2.7 Monetary policy

We characterize "standard" monetary policy as one where the central bank uses the nominal interest rate to implement the desired allocation, subject to a non-negativity constraint on the nominal interest rate

$$R_t \geq 0. \tag{37}$$

We define as "non-standard" monetary policy the ability of the central bank to affect allocations by intermediating credit directly. Commercial banks deposit part of their funds (households' deposits) at the central bank as reserves. These latter are remunerated at the risk-free rate R_t^d and used by the central bank to provide direct credit to firms.

The central bank also remunerates households' money holdings at a rate R_t^m that is proportional to the risk-free rate.

2.8 Market clearing

Market clearing conditions for money, bonds, labor, loans, wholesale goods and retail goods are given, respectively, by

$$M_t = M_t^s, \tag{38}$$

$$A_t = 0, \tag{39}$$

$$h_t = l_t, \tag{40}$$

$$D_t = P_t\left(x_t - \tau\right), \tag{41}$$

$$y_t = \int_0^1 Y_t\left(\eta\right) d\eta, \tag{42}$$

$$Y_t\left(\eta\right) = c_t\left(\eta\right) + e_t\left(\eta\right), \text{ for all } \eta. \tag{43}$$

2.9 Log-linearization

We log-linearize the system of equilibrium conditions. This helps us to characterize the main channels of transmission of shocks in our economy. It also enable us to show analytically how the presence of financial frictions affects the optimal monetary policy.

We log-linearize the equilibrium conditions around a steady state where $\gamma_t = p_t\left(\eta\right) = s_t = 1$, assuming the functional form for utility $u\left(c_t\right) - v\left(h_t\right) = \frac{c_t^{1-\sigma}}{1-\sigma} - \psi\frac{h_t^{1+\varphi}}{1+\varphi}$.

We define the efficient equilibrium as one where all financial frictions, as well as nominal price stickiness, are absent. Denote variables in such equilibrium with the e superscript, and a variable with a tilde hat as the log deviation of the variable from its steady state level. Because financial shocks are absent in the efficient equilibrium, $\widehat{Y}_t^e = \widehat{r}_t^e = 0$, where $\widehat{r}_t^e$ is the efficient real interest rate.

The system of log-linearized equilibrium conditions can be simplified to

$$\left(\alpha_3 - \alpha_1\right)\widehat{\Lambda}_t^j = \left(1+\sigma+\varphi\right)x_t + \left(\alpha_2+\alpha_4\right)\widehat{\mu}_t^j \tag{44}$$

$$x_t = E_t x_{t+1} - \sigma^{-1}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right) \tag{45}$$

$$\widehat{\pi}_t = \lambda\left[\left(\sigma+\varphi\right)x_t + \widehat{R}_t + \alpha_1\widehat{\Lambda}_t^j + \alpha_2\widehat{\mu}_t^j\right] + \beta E_t\widehat{\pi}_{t+1} \tag{46}$$

where

$$j = \begin{cases} b \text{ if } \widehat{\mu}_t^c \geq \widehat{\mu}_t^b \\ c \text{ if } \widehat{\mu}_t^c < \widehat{\mu}_t^b \end{cases}, \tag{47}$$

and where $x_t = \widehat{Y}_t - \widehat{Y}_t^e$ denotes the output gap. The coefficients α_1, α_2, α_3 and α_4 are defined in appendix B, and $\lambda \equiv (1-\theta)(1-\beta\theta)/\theta$. Notice that α_1 and α_3 can be signed and are always positive. Under our calibration, the coefficients α_2, α_4 and α_5 also take positive values.

To understand condition (47), notice that $j = b$ if $\gamma_t = 1$, or if $\overline{\omega}_t^c \geq \overline{\omega}_t^b$, while $j = c$ if $\gamma_t = 0$, or if $\overline{\omega}_t^c < \overline{\omega}_t^b$. From equation (30), it can be shown that $\frac{\partial \Lambda^j}{\partial \overline{\omega}^j}$ can be negative either for values of $\overline{\omega}_t^j$ close to zero, or for values falling in the right tail of the distribution of ω. Under parameterizations that delivers reasonable default rates, $\overline{\omega}^j$ always lie in the left tail of the distribution, so that $\frac{\partial \Lambda^j}{\partial \overline{\omega}^j} > 0$. At the same time, we know from equation (44) that $\Lambda_t^c \geq \Lambda_t^b$ if $\widehat{\mu}_t^c \geq \widehat{\mu}_t^b$, and $\Lambda_t^c < \Lambda_t^b$ if $\widehat{\mu}_t^c < \widehat{\mu}_t^b$.

Equation (44) shows that the spread between the loan rate and the policy rate, $\widehat{\Lambda}_t^j$, increases with the output gap, x_t. A larger demand for retail goods (and thus for wholesale goods to be used as production inputs) tightens the credit constraint of firms, since they need to finance a higher level of debt given the same amount of internal funds. The increased default risk generates a larger spread. The spread is also positively related to the shock to monitoring costs, $\widehat{\mu}_t^j$. The reason is that intermediaries need to set a higher repayment threshold to cover for increased monitoring costs, which results in larger credit spreads.

Equation (45) is a standard forward-looking IS-curve describing the determinants of the gap between actual output and its efficient level.

Equation (46) represents an extended Phillips curve. The first determinant of inflation in this equation is the output gap. Ceteris paribus, a higher demand for retail goods, and correspondingly for intermediate goods, implies that wholesale firms need to pay a higher real wage to induce workers to supply the required labor services. The second determinant is the nominal interest rate, whose increase also pushes up marginal costs due to the presence of the cost channel. The third term is the credit spread, $\widehat{\Lambda}_t^j$. A higher spread implies a higher cost of external finance for wholesale firms and therefore exerts independent pressure on inflation.

As in De Fiore and Tristani (2012), the credit spread and the nominal interest rate act as endogenous "cost-push" terms in our model. While raising marginal costs and inflation, an increase in either term also exerts downward pressure on economic activity. A higher nominal interest rate determines an output contraction through the ensuing increase in the real interest rate, which induces households to postpone their consumption to the future. An increase in the credit spread contracts activity through the increase in the financial markup q_t and the consequent fall in the real wage.

The shock to monitoring costs acts as an exogenous "cost-push" factor in the New-Phillips curve, as it creates inflationary pressures independently from those exerted by the output gap.

In our model, a positive shock to monitoring costs raises the cost of external finance and depresses economic activity. At the same time, it increases the spread that banks charge over the risk-free rate, and thus firms' marginal costs, which are passed through to higher prices for final consumption goods. In equilibrium, inflation rises in spite of the fall in the output gap. As a result, this shock does not lead the economy to hit the ELB under a simple Taylor-type of monetary policy rule. The central bank would react to such a shock by raising the policy instrument.

3 Welfare analysis

We characterize analytically the solution to the Ramsey problem under committment.

The welfare criterion in our analysis is the utility of the economy's representative household

$$W_{t_0} = E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^t U_t \right\},$$

where temporary utility is given by $U_t = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{h_t^{1+\phi}}{1+\phi}$.

We can provide an analytic approximate characterisation of optimal policy using the log-linear model conditions. Specifically, under the functional form for household's utility defined above, appendix C shows that the present discounted value of social welfare can be approximated to second order by

$$W_{t_0} \simeq c^{1-\sigma} \left[\varkappa - \frac{1}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} L_t \right] + t.i.p., \tag{48}$$

where $t.i.p.$ denotes terms independent of policy,

$$L_t \equiv \kappa_\pi \pi_t^2 + (\sigma + \varphi)\, x_t, \tag{49}$$

$\kappa_\pi = \frac{\varepsilon\theta}{(1-\theta)(1-\beta\theta)}$ and $\varkappa = \left(\frac{1}{1-\sigma} - \frac{1}{1+\phi} \right)$.

Define $\widetilde{\sigma} \equiv \sigma + \varphi$, $\widetilde{\lambda} \equiv \lambda \alpha_1 \alpha_5$ and $\widetilde{\alpha} \equiv \lambda \left[\widetilde{\sigma} + \alpha_1 \alpha_5 \left(1 + \widetilde{\sigma} \right) \right]$. The planner maximizes (49) subject to the linearized equilibrium condition (45), the New-Phillips curve rewritten as

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \widetilde{\alpha} x_t + \lambda \widehat{R}_t + \left[\widetilde{\lambda} \left(\alpha_2 + \alpha_4 \right) + \lambda \alpha_2 \right] \left[\gamma_t \widehat{\mu}_t^b + \left(1 - \gamma_t \right) \widehat{\mu}_t^c \right],$$

the ELB constraint

$$\widehat{R}_t \geq \ln \beta,$$

and the restriction

$$\gamma_t = \Psi\left(\widehat{\mu}_t^c - \widehat{\mu}_t^b\right). \tag{50}$$

Notice that the social planner does not choose γ_t. Equation (50) is a restriction to the Ramsey problem which ensures that the optimal allocation satisfies the optimality conditions of the CSV problem.

The first-order conditions of the Ramsey problem can be written as

$$\begin{aligned} \psi_t &= \frac{(\sigma+\varphi)\, x_t - \beta^{-1}\psi_t - \lambda^{-1}\widetilde{\alpha}\phi_{t-1}}{\widetilde{\alpha}\lambda^{-1}\sigma^{-1} - 1} \\ \phi_t &= -\varepsilon\widehat{\pi}_t + \phi_{t-1} + \sigma^{-1}\frac{\beta+\lambda}{\beta}\psi_{t-1} + \frac{\widetilde{\alpha}\lambda^{-1}\phi_{t-1} + \beta^{-1}\psi_{t-1} - (\sigma+\varphi)\, x_t}{\widetilde{\alpha}\lambda^{-1} - \sigma} \\ 0 &= \left(\widehat{R}_t - \ln\beta\right)\phi_t \end{aligned}$$

where ψ_t and ϕ_t are the lagrangean multipliers on the Euler equation and the ELB constraint, respectively (the New-Phillips curve multiplier, ν_t, has been substituted out).

3.1 Target rule without ELB and non-standard measures

We provide some intuition on what monetary policy ought to do in our model by abstracting from the ELB constraint and from the possibility that the central bank intervenes with non-standard policy measures. The aim is to disentangle the consequences of the nominal denomination of debt (the cost channel) and the costly state verification environment (the existence of endogenous credit spreads) for the optimal monetary policy.

Under the assumption that the ELB constraint can be ignored, and when $\gamma_t = 1$, the optimality conditions of the Ramsey problem can be rewritten in terms of the following target rule

$$\Delta x_t = -\varepsilon\left[1 + \frac{\alpha_1}{\alpha_3 - \alpha_1}\left(1 + \frac{1}{\sigma+\varphi}\right)\right]\widehat{\pi}_t + \frac{\sigma}{\sigma+\varphi}\varepsilon\left(\widehat{\pi}_t - \frac{\widehat{\pi}_{t-1}}{\beta}\right) + \frac{\lambda}{\beta}x_{t-1} \tag{51}$$

Equation (51) nests the target rule which implements optimal policy in the New Keynesian model, given by $\Delta x_t = -\varepsilon\widehat{\pi}_t$ (see eg Woodford, 2003). In that model, the target rule can be interpreted as the simple prescription to keep contracting the output gap as long as inflation is positive (and viceversa for negative inflation).

The introduction of the cost channel in the model is responsible for the last two terms in equation (51). In fact, when monitoring costs are zero, $\alpha_1 = 0$. To realize the implications of the cost channel for optimal policy, consider the prescription of the target rule in the first period after a shock has hit the economy. Because in steady state $x = \widehat{\pi} = 0$, in the first period

$\Delta x_t = -\varepsilon\left(1 - \frac{\sigma}{\sigma+\varphi}\right)\widehat{\pi}_t$. In response to a certain increase in inflation, the last two terms suggest that the initial contraction in the output gap should be smaller than in the model with frictionless financial markets. Intuitively, these terms take into account the cost-push inflationary effects of the increase in the nominal interest rate, which have to be implemented to induce a contraction of the output gap.

Finally, the existence of asymmetric information and credit spreads calls for a more aggressive policy response to current inflation – the coefficient is higher than in the frictionless case by the positive amount $\alpha_1/(\alpha_3 - \alpha_1)(1 + 1/(\sigma + \varphi))$. This is necessary to contain any additional inflationary pressures coming from credit spreads.

Equation (51) can also be written differently to highlight its implications on the price level. We then have

$$p_t = p_{t-1} - \frac{1}{\widetilde{\varepsilon}}\left[\beta\frac{\varphi+\sigma}{\sigma}\Delta x_t + \lambda\frac{\varphi+\sigma}{\sigma}(\varepsilon\sigma - 1)x_{t-1} + \lambda\widehat{R}_{t-1} + \lambda\alpha_1\widehat{\Lambda}_{t-1} + \beta\mathrm{E}_{t-1}\widehat{\pi}_t\right] \quad (52)$$

where $\widetilde{\varepsilon}$ is a positive reaction coefficient given by $\widetilde{\varepsilon} \equiv \varepsilon\beta\sigma^{-1}\left[\varphi + \frac{\alpha_1}{\alpha_3-\alpha_1}(1+\sigma+\varphi)\right]$.

Note that the NK model would require $p_t = p_{t-1} - (1/\varepsilon)\Delta x_t$. Should a positive output gap, and thus deflation, ever occur, the central bank should generate an inflationary period until output gap is stabilised and $\Delta x_t = 0$. The price level should return to its initial point.

In the case of our model, a return to the original price level is not sufficient. Note that all terms inside the square brackets on the right-hand side of equation (52) are positive. Should a positive output gap ever occur, the central bank should not only generate inflation until the output gap is stabilized and $\Delta x_t = 0$. Some additional upward pressure on prices may be warranted as long as lagged policy rates and credit spreads remain above their steady state levels. As a result, the price level will eventually return to a higher point than where it started. The price level will nevertheless remain, as in the NK model, trend stationary.

4 Numerical results

For our numerical analysis, we first derive analytically the first order conditions of the nonlinear Ramsey problem of an optimal planner. We then solve the resulting model using nonlinear, deterministic simulation methods. Given initial conditions for pre-determined variables and terminal conditions for non-predetermined variables, the path of all endogenous variables can be found as the solution of a large system of nonlinear equations at all simulation dates.[5]

A more complete solution to the system would include stochastic terms, e.g. using the collocation method as suggested by Judd (1998) or Miranda and Fackler (2002). A stochastic solution would in principle allow for precautionary policy motives, e.g. the possibility to target a slightly positive inflation rate in order to reduce the likelihood of hitting the ELB. Such precautionary effects, however, have been found to be negligible in the new Keynesian literature. Since, as we illustrate below, non-standard measures reduce the likelihood of reaching the ELB, precautionary effects are likely to be small also in our model. There should therefore be no loss of accuracy in our deterministic nonlinear solution, which is much simpler to compute and feasible also for relatively large models.

To simplify the solution procedure, we smooth out the two kinks in γ_t through a simple approximation. Specifically, we replace equation (29) with

$$\gamma_t = \Psi\left(\mu_t^b - \mu^c\right) \tag{53}$$

where we also assumed that μ^c is not affected by financial shocks that increase μ_t^b. The idea is that fluctuations in μ_t^b are mainly related to changes in asset liquidation costs, which can increase markedly during financial crises, for example due to fire sales. By contrast, central banks are not subject to liquidity constraints and can always perform an orderly asset liquidation. Hence μ^c remains constant over time.

The functional form for the function $\Psi(x)$ is $\Psi(x) = \frac{1}{2}\frac{e^{(\kappa x)}-e^{(-\kappa x)}}{e^{(\kappa x)}+e^{(-\kappa x)}}+\frac{1}{2}$ where κ is a parameter which can be tuned to improve the accuracy of the approximation at the points of discontinuity.

Parameter values are in line with the literature. More specifically, we set the elasticity of intratemporal substitution $\varepsilon = 11$ and the Calvo parameter $\theta = .66$. The discount factor is set as $\beta = 0.995$, to mimic the low interest rates environment which prevailed over the years before the financial crisis. For the utility parameters, we use standard values: $\sigma = 1.0$, $\phi = 0.0$. The contract parameters τ and σ_ω are set consistently with the parametrization used in De Fiore and Tristani (2012), which matches US data on the average annual spread between lending and deposit rates (approximately 2%) and on the quarterly bankruptcy rate (around 1%). These values imply that $\alpha_1 = 4.7$ and $(\alpha_3 - \alpha_1)^{-1} = 0.008$. Consistently with actual financial

[5] In practice we use Newton methods as implemented in the Dynare command "simul".

developments over the past 5 years, we assume very persistent monitoring cost shocks: they have a serial correlation coefficient equal to 0.95.

A new coefficient which we need to calibrate is μ^c, the monitoring cost of central bank lending activities. To gauge a value for this parameter, we draw from the euro area experience during the financial crisis, when the ECB intervened to offset impairments in the interbank market. In the first phase of the crisis, in 2008-09, lack of trust emerged between banks concerning each other's ability to repay interbank loans. Possibly as a consequence of higher interbank market rates, many banks chose to borrow directly from the ECB at the rate on the main refinancing operations, rather than from other banks at the prevailing interbank overnight rate (the EONIA). The spread between MRO and overnight rates, which was essentially zero on average in pre-crisis times, hovered between 50 and 70 basis points in 2009.

We interpret this hike in the MRO-overnight spread as due to an increase in the inefficiency of commercial banks in monitoring their loans to other banks, or equivalently as an increase in their monitoring costs. Only when the spread reached 50 basis points, the ECB became a competitive financial intermediary, i.e. it became more efficient than commercial banks at monitoring credit worthiness. We therefore interpret this spread level as a measure of the ECB's lower monitoring efficiency under normal circumstances. We set μ^c so as to imply a steady state credit spread between ECB loans and banks' loans of 50 basis points.

Finally, in line with a large fraction of the standard new-Keynesian literature, we study economic dynamics around an efficient steady state. The steady state is efficient thanks to a constant subsidy which removes the steady state wedge between marginal rate of substitution and marginal rate of transformation.

Figures 1 and 2 display the impulse responses to a μ_t^b shock under optimal policy in the simple model. As already discussed above, this shock generates an immediate increase in the loan-deposit rate spread, which pushes up firms' marginal costs and thus generates inflationary pressure. The increase in marginal costs also generates a persistent increase in the mark-up q_t and downward pressure on wages, inducing a reduction in both labour supply and the demand for consumption goods. Hence, the spread moves anti-cyclically.

Figures 1 focuses on the case in which the central bank responds solely with the standard policy instrument. The shock is such that spreads increase by approximately 100 basis points. Optimal policy requires a cut in interest rates, in spite of the inflationary pressure created by the increase in spreads. The main reason for this policy response is that the financial shock is inefficient, hence the fall in households' consumption is undesirable. The expansion in the monetary policy stance helps smooth the adjustment of households' consumption after the shock, at the cost of producing a short inflationary episode. As already apparent from the target rule, at the end of the adjustment period the price level reverts back to the original level and then crosses it to eventually end up *below* the starting value. The promise of a future fall in the price level keeps expectations of future inflation down. It ensures that only a short inflationary episode follows an inflationary shock, in spite of the impact fall in the policy rate when the shock hits. If we ignore the ELB constraint, the nominal rate falls to almost -3%, before returning relatively quickly towards the steady state to react to the increase in inflation. Once we impose the ELB constraint, the nominal rate is zero for two periods and then rises faster than in the absence of the ELB. Figures 1 thus illustrates a well-known property of the optimal interest rate policy: policy rates remain "low for longer" in the presence of the ELB – they increase later and faster than they would in the absence of that constraint.

Figure 2 displays impulse responses to a shock of the same size as in figure 1 when non-standard policy is also available (solid line), and compares it to the case when only standard policy can be used (dotted line). In the case where non-standard policy is implemented, the shock does not require the policy rate to reach the zero lower bound. The smaller response of the policy rate is nonetheless able to substantially reduce the volatility of both the output gap and inflation. Also, a lower fall in the future price level is necessary to limit the initially inflationary consequences of the shock.

Non-standard measures – i.e. central bank intermediation – are deployed as soon as the credit spread on banks' loans increases above 50 basis points. Given the persistence of the shock, this is the case for the entire period considered. Non-standard measures are therefore implemented irrespectively of the level of the policy rate. They can optimally be deployed when the interest rate has not yet reached the zero bound. Once the shock hits, the central bank starts providing loans to the economy at the same time as it lowers interest rates. This direct intermediation activity continues long after interest rates have essentially returned to their steady state. Standard monetary policy is again tightened quickly few quarters after the shock hits, because inflation is stabilised in spite of the fact that financial market conditions remain impaired.

The optimal combination of standard and non-standard measures delivers a superior outcome to the case in which non-standard measures are unavailable and the policy rate is kept "lower for longer": inflation is better stabilised and the output gap is smaller.

4.1 Robustness: a model with capital

In our model, a shock to monitoring costs acts as a purely "cost-push" factor, as it creates inflationary pressures independently from those exerted by the output gap. The increase in credit spreads induced by the shock exerts upward pressure on inflation by raising firms' marginal costs, without directly affecting aggregate demand. The contraction in real activity occurs because the increase in credit spreads raises the financial markup q_t, and consequently lowers the real wage and labor supply.

The pure cost-push nature of the financial shock in our simple model could have an impact on our conclusions concerning the optimal mix of standard and non-standard measures. In particular, policy interest rates are optimally increased quickly after a very persistent financial shock, because the ensuing increase in spreads puts upward pressure on marginal costs. We therefore check robustness of this conclusion to a model similar to the one described in section 2, the key difference being the presence of competitive firms operating an investment sector. These firms are endowed with a technology which transforms final consumption goods into capital goods, and experience an idiosyncratic productivity shock. As firms need to raise external finance to buy consumption goods, they need to stipulate a contract with financial intermediaries. The financial friction takes the form of asymmetric information on the idiosyncratic productivity shock and of monitoring costs, as in our simple benchmark model. We provide the details of that framework in appendix D.

Figures 3 presents the impulse responses to a μ_t^b shock under the optimal policy in the model with capital, when the interest rate is constrained by the ZLB and non-standard measures are implemented. The shock is calibrated to generate the same increase in market spreads as in figure 2. The figure shows the spreads offered by both banks (solid line) and the central bank (dashed line). Notice that inflation, the inverse of leverage, z_t, investment and output are plotted in percentage deviations from the steady state.

In this model, the exogenous increase in monitoring costs depresses investment through a sharp and persistent rise in the price of capital, which contributes to drive output down. The interest rate is reduced to limit the fall in output but less than in the model without capital, and it is later increased much more gradually. The reason is that the upward pressure on inflation exerted through the cost channel by a relatively higher interest rate on impact is compensated by the strong deflationary pressures induced by the severe reduction in investment and output. As a consequence, the overall reaction of inflation is similar to the one arising in the model without capital.

Our robustness analysis shows that, when credit spreads have a direct effect on some components of the aggregate demand (in this case, investment), the optimal reaction of the interest rate is milder on impact and its convergence to the steady state takes longer.

The main conclusions obtained in the benchmark model without capital, however, are unaffected. In reaction to financial shocks that negatively affect banks' monitoring efficiency, it is optimal to use unconventional policies. Once these measures have been deployed, it is sub-optimal to lower policy rates further. Moreover, under the type of shocks we consider, it is desirable to increase interest rates before unconventional measures are discontinued.

5 Conclusion

We have presented a microfounded model with credit market imperfections and nominal price rigidities, which we use to analyse the response of monetary policy to financial shocks in the presence of the ELB. The model can also sheed light on the role of non-standard policy measures both at the ELB and away from it.

We find that adverse financial shocks (notably a shock that increases banks' monitoring costs) can lead the economy to the ELB under optimal policy. Non-standard meaures can be effective in these situations. When adverse financial shocks impair the efficiency of private banks in intermediating finance, the ability of the central bank to provide direct credit to the economy mitigates the negative consequences of the shock on inflation and real activity. Cutting policy rates to zero may be unnecessary after non-standard measures have been implemented.

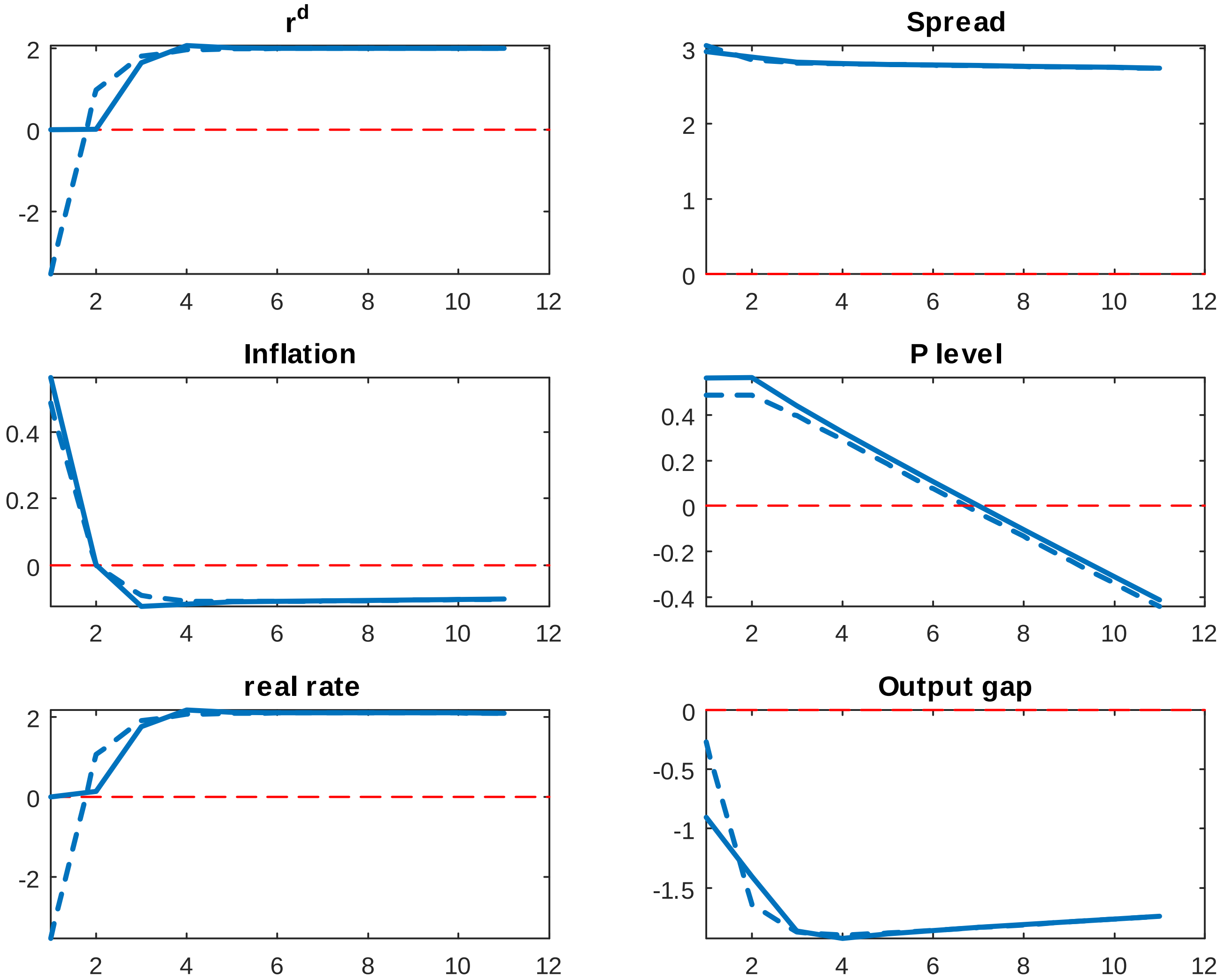

Figure 1: Response to a shock to μ under the optimal monetary policy: with ZLB (solid blue line) and without ZLB (dotted blue line). All variables are in levels.

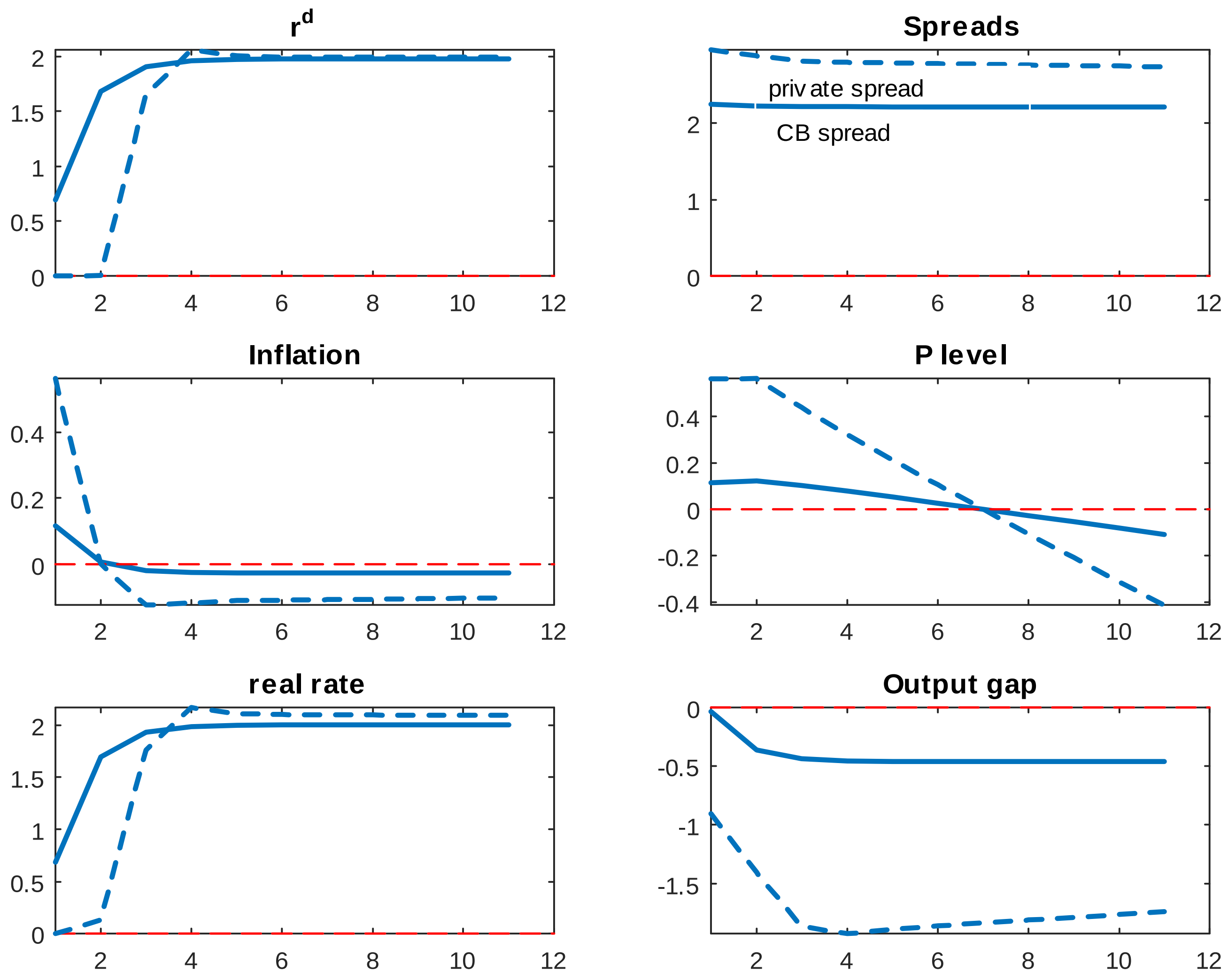

Figure 2: Response to a shock to μ under the optimal monetary policy: with ZLB and non-standard measures (solid blue line) and with ZLB only (dotted blue line). All variables are in levels.

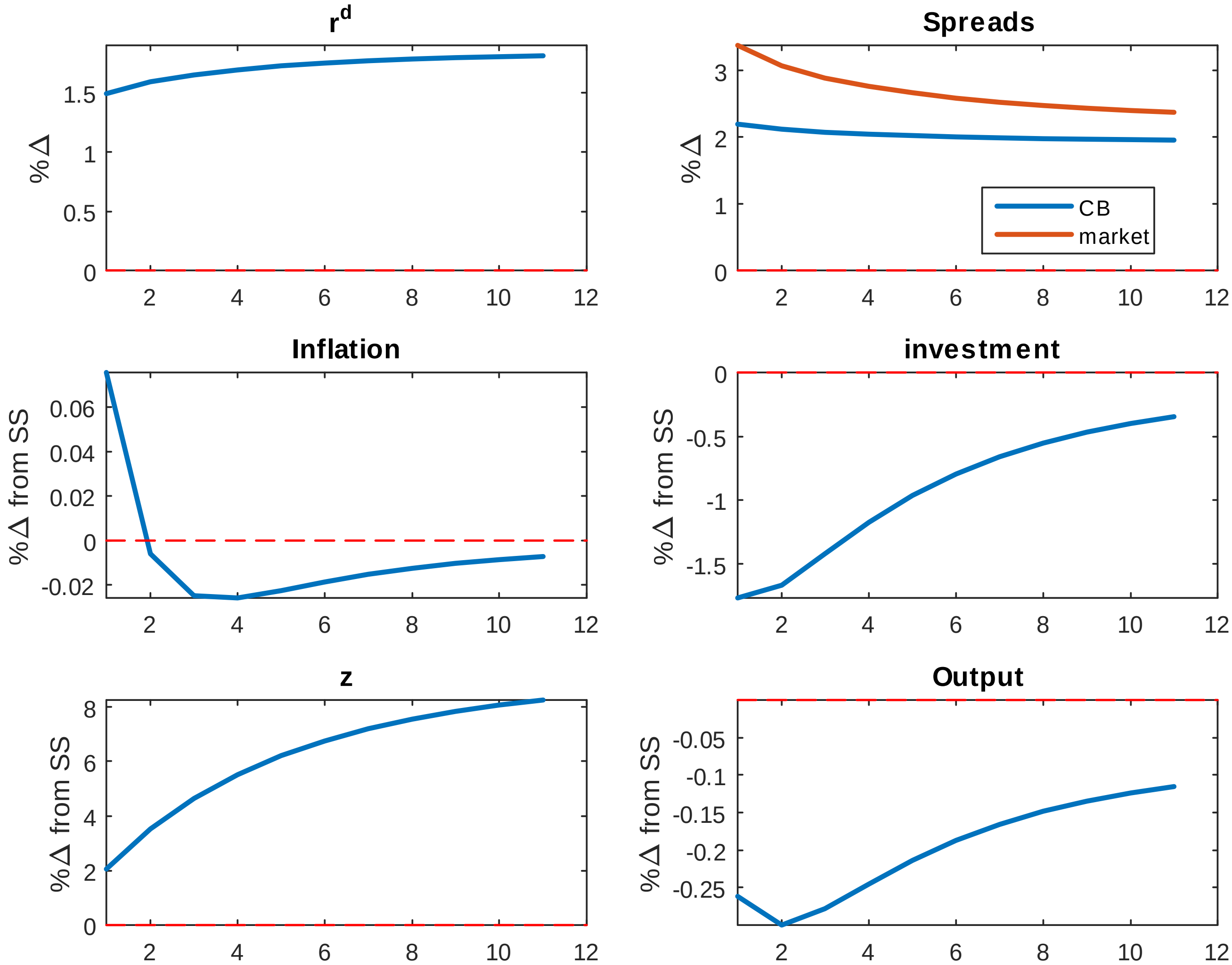

Figure 3: Impulse responses to an AR(1) μ shock with non-standard measures in the model with capital. The nominal interest rate and the spreads are plotted in levels. All other variables are in percentage deviations from the steady state.

6 Appendix

A. The financial contract

The optimal financial contract solves a standard costly state verification problem (see e.g. Gale and Hellwig (1985)). The problem is

$$\max_{\overline{\omega}_{i,t},x_{i,t},\gamma_t} f(\overline{\omega}_{i,t})q_t x_{i,t}$$

subject to

$$q_t g^b\left(\overline{\omega}_{i,t};\mu_t^b\right)x_{i,t} \geq R_t^d\left(x_{i,t}-\tau\right) \quad (54)$$

$$q_t g^c\left(\overline{\omega}_{i,t};\mu_t^c\right)x_{i,t} \geq R_t^d\left(x_{i,t}-\tau\right) \quad (55)$$

$$q_t x_t f(\overline{\omega}_{i,t}) \geq \tau \quad (56)$$

$$f\left(\overline{\omega}_{i,t}\right)+\gamma_t g^b\left(\overline{\omega}_{i,t};\mu_t^b\right)+\left(1-\gamma_t\right)g^c\left(\overline{\omega}_{i,t};\mu_t^c\right) \leq 1-\left[\gamma_t\mu_t^b+\left(1-\gamma_t\right)\mu_t^c\right]G\left(\overline{\omega}_{i,t}\right) \quad (57)$$

$$0 \leq \gamma_t \leq 1. \quad (58)$$

The functions $f(\overline{\omega}_{i,t})$, $g^b\left(\overline{\omega}_{i,t};\mu_t^b\right)$ and $g^c\left(\overline{\omega}_{i,t};\mu_t^c\right)$ are defined in equations (13)-(15) and denote the expected unit shares of output accruing respectively to the firm, the commercial bank and the central bank, after stipulating a financial contract that sets a lending rate R^b or R^c, respectively.

Denote with $\lambda_{1,t}^b$, $\lambda_{1,t}^c$ and $\lambda_{2,t}$ the lagrangean multipliers associated to constraints (13), (14) and (15), respectively, and $\eta_{1i,t}$ and $\eta_{2i,t}$ those associated to $\gamma_{i,t} \geq 0$ and to $\gamma_{i,t} \leq 1$. Condition (57) is used with equality to replace the $g^b\left(\overline{\omega}_{i,t};\mu_t^b\right)$ and $g^c\left(\overline{\omega}_{i,t};\mu_t^b\right)$ out in the first-order conditions of the problem.

Define also $z_{i,t} \equiv \frac{\tau}{x_{i,t}}$ as the share on internal funds over the size of production. We conjecture (and later verify) that $\lambda_{2,t}=0$. The first-order conditions of the problem are

$$f(\cdot)\frac{q_t}{z_{i,t}^2} = f_z(\cdot)\frac{q_t}{z_{i,t}}+\lambda_{1,t}^b\left(g_z^b\left(\cdot\right)q_t+R_t^d\right)+\lambda_{1,t}^c\left(g_z^c\left(\cdot\right)q_t+R_t^d\right) \quad (59)$$

$$0 = f_{R^b}(\cdot)\frac{q_t}{z_{i,t}}+\lambda_{1,t}^b g_{R^b}^b\left(\cdot\right)q_t+\lambda_{1,t}^c g_{R^b}^c\left(\cdot\right)q_t \quad (60)$$

$$0 = f_{R^c}(\cdot)\frac{q_t}{z_{i,t}}+\lambda_{1,t}^b g_{R^c}^b\left(\cdot\right)q_t+\lambda_{1,t}^c g_{R^c}^c\left(\cdot\right)q_t \quad (61)$$

$$g^b\left(\cdot\right)q_t \geq R_t^d\left(1-z_{i,t}\right) \quad (62)$$

$$g^c\left(\cdot\right)q_t \geq R_t^d\left(1-z_{i,t}\right) \quad (63)$$

$$\eta_{1,t}-\eta_{2,t} = f_\gamma(\cdot)\frac{q_t}{z_{i,t}}+\lambda_{1,t}^b g_\gamma^b\left(\cdot\right)q_t+\lambda_{1,t}^c g_\gamma^c\left(\cdot\right)q_t \quad (64)$$

Use the definitions (13), (14) and (15) to obtain partial derivatives of the functions $f(\overline{\omega}_{i,t})$, $g^b\left(\overline{\omega}_{i,t};\mu_t^b\right)$ and $g^c\left(\overline{\omega}_{i,t};\mu_t^b\right)$ and to rewrite equation (59) as

$$f(\overline{\omega}_{i,t})\frac{q_t}{z_{i,t}} = -\left[R_t^b\gamma_t + R_t^c\left(1-\gamma_t\right)\right] f_{\overline{\omega}}\left(\overline{\omega}_{i,t}\right) \tag{65}$$
$$+\lambda_{1,t}^b z_{i,t}\left(\begin{array}{c} R_t^d - R_t^b\left[1-\Phi\left(\overline{\omega}_{i,t}\right)\right] \\ -\left[R_t^b\gamma_t + R_t^c\left(1-\gamma_t\right)\right]\phi\left(\overline{\omega}_{i,t}\right)\left[\left(1-\mu_t^b\right)\overline{\omega}_{i,t} + R^b\frac{1-z}{q}\right]\end{array}\right)$$
$$+\lambda_{1,t}^c z_{i,t}\left(\begin{array}{c} R_t^d + R_t^c\left[\overline{\omega}_{i,t}\phi\left(\overline{\omega}_{i,t}\right) - 1 + \Phi\left(\overline{\omega}_{i,t}\right)\right] \\ -\left(R_t^b\gamma_t + R_t^c\left(1-\gamma_t\right)\right)\left(1-\mu_t^c\right)\overline{\omega}_{i,t}\phi\left(\overline{\omega}_{i,t}\right)\end{array}\right),$$

equation (60) as

$$0 = \gamma_t\frac{\left(1-z_{i,t}\right)}{q_t}f_{\overline{\omega}}\left(\overline{\omega}_{i,t}\right)\frac{q_t}{z_{i,t}} \tag{66}$$
$$+\lambda_{1,t}^b\left(1-z_{i,t}\right)\left[\gamma_t\left(1-\mu_t^b\right)\overline{\omega}_{i,t}\phi\left(\overline{\omega}_{i,t}\right) + 1 - \Phi\left(\overline{\omega}_{i,t}\right) - \gamma R_t^b\phi\left(\overline{\omega}_{i,t}\right)\frac{1-z_{i,t}}{q_t}\right]$$
$$+\lambda_{1,t}^c\left(1-z_{i,t}\right)\left[\gamma_t\left(1-\mu_t^c\right)\overline{\omega}_{i,t}\phi\left(\overline{\omega}_{i,t}\right) - \gamma R_t^c\phi\left(\overline{\omega}_{i,t}\right)\frac{1-z_{i,t}}{q_t}\right],$$

equation (61) as

$$0 = \left(1-\gamma_t\right)\frac{1-z_{i,t}}{q_t}f_{\overline{\omega}}\left(\overline{\omega}_{i,t}\right)\frac{q_t}{z_{i,t}} \tag{67}$$
$$+\lambda_{1,t}^b\left(1-\gamma_t\right)\left(1-z_{i,t}\right)\phi\left(\overline{\omega}_{i,t}\right)\left[\left(1-\mu_t^b\right)\overline{\omega}_{i,t} - R_t^b\frac{1-z_{i,t}}{q_t}\right]$$
$$+\lambda_{1,t}^c\left(1-z_{i,t}\right)\left[\begin{array}{c}\left(1-\mu_t^c\right)\left(1-\gamma_t\right)\overline{\omega}_{i,t}\phi\left(\overline{\omega}_{i,t}\right) \\ +1-\Phi\left(\overline{\omega}_{i,t}\right) - R_t^c\phi\left(\overline{\omega}_{i,t}\right)\left(1-\gamma_t\right)\frac{1-z_{i,t}}{q_t}\end{array}\right],$$

and equation (64) as

$$\eta_{1,t} - \eta_{2,t} = \left(R_t^b - R_t^c\right)\frac{1-z_{i,t}}{z_{i,t}}f_{\overline{\omega}_{i,t}}\left(\overline{\omega}_{i,t}\right) \tag{68}$$
$$+\lambda_{1,t}^b\left(R_t^b - R_t^c\right)\left(1-z_{i,t}\right)\phi\left(\overline{\omega}_{i,t}\right)\left[(1-\mu^b)\overline{\omega}_{i,t} - R^b\frac{1-z_{i,t}}{q_t}\right]$$
$$+\lambda_{1,t}^c\left(R_t^b - R_t^c\right)\left(1-z_{i,t}\right)\phi\left(\overline{\omega}_{i,t}\right)\left[\left(1-\mu^c\right)\overline{\omega}_{i,t} - R_t^c\frac{1-z_{i,t}}{q_t}\right].$$

Rearranging equations (66) and (67), we get that

$$\left(1-\gamma_t\right)\lambda_{1,t}^b = \gamma_t\lambda_{1,t}^c. \tag{69}$$

From equations (66) and (69), it follows that

$$\frac{1}{\gamma_t}\lambda_{1,t}^b z_{i,t} = \frac{1}{1-\frac{\left(\gamma_t\mu_t^b + \left(1-\gamma_t\right)\mu_t^c\right)\overline{\omega}_{i,t}\phi\left(\overline{\omega}_{i,t}\right)}{1-\Phi\left(\overline{\omega}_{i,t}\right)}}.$$

This latter condition, together with (69), ensures that constraints (54) and (55) are binding at the optimum.

We can therefore simplify equation (65) to get

$$f(\overline{\omega}_{i,t})\frac{q_t}{z_{i,t}} = \frac{R_t^d}{1-\frac{\left[\gamma_t\mu_t^b+(1-\gamma_t)\mu_t^c\right]\overline{\omega}_{i,t}\phi(\overline{\omega}_{i,t})}{[1-\Phi(\overline{\omega}_{i,t})]}},$$

and equation (68) to get

$$\eta_{1,t}-\eta_{2,t} = -\left(\frac{1-z_{i,t}}{z_{i,t}}\right)\frac{\left(R_t^b-R_t^c\right)}{1-\Phi\left(\overline{\omega}_{i,t}\right)-\left[\gamma_t\mu_t^b+\left(1-\gamma_t\right)\mu^c\right]\overline{\omega}_{i,t}\phi\left(\overline{\omega}_{i,t}\right)}.$$

Now use the definition $z_{i,t}\equiv\frac{\tau}{x_{i,t}}$ and condition (57) to rewrite the optimality conditions as

$$f(\overline{\omega}_{i,t})q_t = \frac{R_t^d}{1-\frac{\left[\gamma_t\mu_t^b+(1-\gamma_t)\mu_t^c\right]\overline{\omega}_{i,t}\phi(\overline{\omega}_{i,t})}{[1-\Phi(\overline{\omega}_{i,t})]}}\frac{\tau}{x_{i,t}} \quad (70)$$

$$g^b\left(\overline{\omega}_{i,t};\mu_t^b\right)q_t = R_t^d\left(1-\frac{\tau}{x_{i,t}}\right) \quad (71)$$

$$\eta_{1,t}-\eta_{2,t} = \frac{\left(R_{i,t}^c-R_{i,t}^b\right)\left(\frac{x_{i,t}}{\tau}-1\right)\left[1-\Phi\left(\overline{\omega}_{i,t}\right)\right]}{1-\frac{\phi(\overline{\omega}_{i,t})\overline{\omega}_{i,t}\left[\gamma_t\mu_t^b+(1-\gamma_t)\mu_t^c\right]}{[1-\Phi(\overline{\omega}_{i,t})]}} \quad (72)$$

$$\eta_{1,t}\gamma_t = 0 \quad (73)$$

$$\eta_{2,t}\left(1-\gamma_t\right) = 0 \quad (74)$$

together with $\eta_{1,t}\geq 0$ and $\eta_{2,t}\geq 0$. Notice that, from condition (70), $f(\overline{\omega}_{i,t})q_t x_{i,t} > R_t^d\tau$, which verifies our conjecture that $\lambda_{2,t}=0$.

Substituting the expression for q_t obtained from equation (70) in (71), and using condition (57) and (69), we can rewrite equation (71) as

$$\frac{x_{i,t}}{\tau} = 1+\frac{1-f\left(\overline{\omega}_{i,t}\right)-\left(\gamma_t\mu_t^b+\left(1-\gamma_t\right)\mu_t^c\right)G\left(\overline{\omega}_{i,t}\right)}{f(\overline{\omega}_{i,t})\left[1-\frac{\left(\gamma_t\mu_t^b+(1-\gamma_t)\mu_t^c\right)\overline{\omega}_{i,t}\phi(\overline{\omega}_{i,t})}{1-\Phi(\overline{\omega}_{i,t})}\right]}. \quad (75)$$

The optimality conditions can therefore be written as the equations (22)-(26) in the main text.

B. Coefficients

The coefficients of the system of log-linearized equilibrium conditions are given by

$$
\begin{aligned}
\alpha_1 &= -\frac{q}{R}\frac{\mu\frac{f\overline{\omega}}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}}-\frac{\phi^2}{f_{\overline{\omega}}}\right)}{(1-g_{\overline{\omega}}\Lambda)} \\
\alpha_2 &= \mu\frac{q}{R}\left[\Phi+\frac{f\overline{\omega}}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}}-\frac{\phi^2}{f_{\overline{\omega}}}\right)\frac{\frac{\mu\Phi}{g}}{(1-g_{\overline{\omega}}\Lambda)}-\frac{f\phi}{f_{\overline{\omega}}}\right] \\
\alpha_3 &= -\left(\frac{\mu\frac{f}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}}-\frac{\phi^2}{f_{\overline{\omega}}}\right)+(f_{\overline{\omega}}+\mu\phi)}{f+\frac{\mu f\phi}{f_{\overline{\omega}}}}\right)\frac{\overline{\omega}}{(1-g_{\overline{\omega}}\Lambda)} \\
\alpha_4 &= \frac{\mu\Phi}{g}\alpha_3+\frac{\mu\frac{f\phi}{f_{\overline{\omega}}}}{f+\frac{\mu f\phi}{f_{\overline{\omega}}}} \\
\alpha_5 &= (\alpha_3-\alpha_1)^{-1}
\end{aligned}
$$

C. Welfare approximation

Welfare is

$$W_{t_0}=E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^t U_t\right\},$$

where households' temporary utility is given by $U_t=u(c_t;\xi_t)-v(h_t)$. This latter can then be approximated as

$$
\begin{aligned}
U_t \simeq\; & U+u_c c\left(\widehat{c}_t+\frac{1}{2}\left(1+\frac{u_{cc}c}{u_c}\right)\widehat{c}_t^2\right)-v_h h\left(\widehat{h}_t+\frac{1}{2}\left(1+\frac{v_{hh}h}{v_h}\right)\widehat{h}_t^2\right)+u_{c\xi}c\widehat{c}_t\widehat{\xi}_t \\
& +u_\xi\left(\widehat{\xi}_t+\frac{1}{2}\left(1+\frac{u_{\xi\xi}}{u_\xi}\right)\widehat{\xi}_t^2\right)
\end{aligned}
$$

where hats denote log-deviations from the deterministic steady state and c and h denote steady state levels.

Under the functional form $U_t=\xi_t\frac{c_t^{1-\sigma}}{1-\sigma}-\frac{h_t^{1+\phi}}{1+\phi}$, and assuming that in steady state $\xi=1$, households' temporary utility can be rewritten as

$$
\begin{aligned}
U_t \simeq\; & \frac{c_t^{1-\sigma}}{1-\sigma}-\frac{h_t^{1+\phi}}{1+\phi}+c^{1-\sigma}\widehat{c}_t-\psi h^{1+\phi}\widehat{h}_t+\frac{1}{2}c^{1-\sigma}(1-\sigma)\widehat{c}_t^2-\frac{1}{2}\psi h^{1+\phi}(1+\phi)\widehat{h}_t^2 \\
& +c^{1-\sigma}\widehat{c}_t\widehat{\xi}_t+\frac{c^{1-\sigma}}{1-\sigma}\left(\widehat{\xi}_t+\frac{1}{2}\widehat{\xi}_t^2\right).
\end{aligned}
$$

We can now express hours and households' consumption as $h_t = \frac{s_t y_t}{A_t}$ so that $\widehat{h}_t = \widehat{s}_t + \widehat{y}_t - \widehat{a}_t$. Using this expression together with $c_t = y_t$, we can write utility as

$$\begin{aligned}\frac{U_t}{c^{1-\sigma}} \simeq\ & \frac{1}{1-\sigma} - \frac{\psi}{1+\phi}\frac{h_t^{1+\phi}}{c_t^{1-\sigma}} + \left(1 - \frac{\psi h^{1+\phi}}{c^{1-\sigma}}\right)\widehat{y}_t - \psi\frac{h^{1+\phi}}{c^{1-\sigma}}\widehat{s}_t - \frac{1}{2}\left(\frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi) - (1-\sigma)\right)\widehat{y}_t^2 \\ & + \frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi)\widehat{y}_t\widehat{a}_t + \widehat{\xi}_t\widehat{y}_t - \frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi)\widehat{s}_t\widehat{y}_t + \frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi)\widehat{s}_t\widehat{a}_t - \frac{1}{2}\frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi)\widehat{s}_t^2 \\ & + \frac{1}{1-\sigma}\left(\widehat{\xi}_t + \frac{1}{2}\widehat{\xi}_t^2\right) + \frac{\psi h^{1+\phi}}{c^{1-\sigma}}\widehat{a}_t - \frac{1}{2}\frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\psi)\widehat{a}_t^2\end{aligned}$$

or, given that s_t is of second order, as

$$\begin{aligned}\frac{U_t}{c^{1-\sigma}} \simeq \frac{1}{1-\sigma} - \frac{\psi}{1+\phi}\frac{h_t^{1+\phi}}{c_t^{1-\sigma}} \quad & + \left(1 - \frac{\psi h^{1+\phi}}{c^{1-\sigma}}\right)\widehat{y}_t - \psi\frac{h^{1+\phi}}{c^{1-\sigma}}\widehat{s}_t \\ -\frac{1}{2}\left(\frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi) - (1-\sigma)\right)\widehat{y}_t^2 \quad & + \frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi)\widehat{y}_t\widehat{a}_t + \widehat{\xi}_t\widehat{y}_t + t.i.p.s\end{aligned}$$

Assume a subsidy such that $\frac{\psi h^{1+\phi}}{c^{1-\sigma}} = 1$. Then

$$\frac{U_t}{c^{1-\sigma}} \simeq \frac{1}{1-\sigma} - \frac{1}{1+\phi} - \widehat{s}_t - \frac{1}{2}(\varphi+\sigma)\widehat{y}_t^2 + \left[(1+\varphi)\widehat{a}_t + \widehat{\xi}_t\right]\widehat{y}_t + t.i.p.s.$$

Now recall that $\widehat{y}_t^e = \frac{1}{(\sigma+\varphi)}\left[(1+\varphi)a_t + \widehat{\xi}\right]$. Then

$$\frac{U_t}{c^{1-\sigma}} \simeq \frac{1}{1-\sigma} - \frac{1}{1+\phi} - \widehat{s}_t - \frac{1}{2}(\sigma+\varphi)\widehat{y}_t^2 + (\sigma+\varphi)\widehat{y}_t^e\widehat{y}_t + t.i.p.s$$

This can be rewritten as

$$\frac{U_t}{c^{1-\sigma}} - \left(\frac{1}{1-\sigma} - \frac{1}{1+\phi}\right) \simeq -\frac{1}{2}\frac{\varepsilon\theta}{(1-\theta)(1-\beta\theta)}\widehat{\pi}_t^2 - \frac{1}{2}(\sigma+\varphi)x_t^2 + t.i.p.s.$$

D. A model with capital

We analyse in this section the robustness of our results to a richer model. We use a version of the model described in section 2. The key difference is the presence of competitive firms operating an investment sector. We describe here the various sectors, highlighting the differences relative to the model of section 2. The problems of the households and of the final production sector are unchanged, as well as those of the government and the central bank.

D.1 The capital producing sector

There is a continuum of competitive capital producing firms who produce at time $t-1$ new capital to be used in production at t, k_t. In order to produce new capital, they need to acquire old capital k_{t-1}, whose price in terms of consumption good is $\overline{q}_{t-1}$. The firms have the following production function

$$k_t = (1-\delta)\, k_{t-1} + \left[1 - \varphi\left(\frac{I_{t-1}}{I_{t-2}}\right)\right] I_{t-1}$$

where δ denotes the depreciation rate, I_{t-1} is investment in the composite final good in $t-1$, where $I_t = \left[\int_0^1 I_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$, and $\varphi(\cdot)$ is an adjustment cost function,which is increasing and convex. The price of the new capital in terms of consumption good is denoted with q_t. Firms' profits at time t are given by

$$\Pi_t^k = P_t\left(q_t k_{t+1} - I_t - \overline{q}_t k_t\right).$$

The first order conditions of the profit maximization problem are

$$\begin{aligned} \overline{q}_t &= (1-\delta)\, q_t \\ q_t\left[1 - \varphi\left(\frac{I_t}{I_{t-1}}\right) - \varphi'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] &= 1 \end{aligned}$$

D.2 The entrepreneurs

There is a continuum of risk-neutral, infinitely lived entrepreneurs, denoted with i, who die with probability ξ_t. At the end of period $t-1$, each entrepreneur buys new capital $k_{i,t}$ from capital producers at price q_t and transforms it into capital services $\overline{k}_{i,t}$ using a linear technology

$$\overline{k}_{i,t} = \omega_i k_{i,t},$$

where the random variable ω is i.i.d. across time and across entrepreneurs, with distribution Φ, density ϕ and mean unity. The shock ω is drawn after the capital $k_{i,t}$ is bought and is private information. Its realization can be observed by the financial intermediary at the cost of $\mu_t k_{i,t}$ units of capital.

During period t, entrepreneurs rent the capital to intermediate goods producing firms, earning a real return ρ_t, and then sell the undepreciated capital to capital producers at the end of the goods market, at price $\overline{q}_t$. Hence, firm i gross real return on capital at time t is

$$R_t^k = \frac{\rho_t + q_t(1-\delta)}{q_t}.$$

The firm's expected revenue is given by

$$P_t\left[\rho_t + q_t(1-\delta)\right]\omega_i k_{i,t} = P_t q_t R_t^k \omega_i k_{i,t}.$$

In order to dispose of the capital necessary for production at time t, the firm needs to raise external finance. At at the end of period $t-1$, in the financial market, firm i disposes of a nominal amount of internal funds $Z_{i,t-1}$. It therefore needs to raise additional funds in the amount $X_{i,t-1} - Z_{i,t-1}$, for total funds at hand $X_{i,t-1}$. Each firm is thus restricted to buy a stock of capital such that

$$P_t q_t k_{i,t} \leq X_{i,t}. \tag{76}$$

D.3 The financial contract

The derivation of the loan contract is unaffected, once we replace the exogenous internal funds τ_t in the benchmark model without capita with the accumulated internal funds $N_{i,t}$, and the markup q_t with the return on capital R_t^k. The first-order conditions are then given by equations (22)-(26).

D.4 Entrepreneurial net worth

At the beginning of t, firms' profits net of debt repayment are allocated to either entreprenurial consumption (gross of consumtpion taxes) or to the accumulation of firms' nominal funds,

$$R_t^k f\left(\overline{\omega}_{i,t}\right) X_{i,t} = Z_{i,t} + P_t \left(1+\tau\right) e_{i,t}.$$

Here $e_{i,t} = \left[\int_0^1 e_{i,t}\left(\eta\right)^{\frac{\varepsilon-1}{\varepsilon}} d\eta\right]^{\frac{\varepsilon}{\varepsilon-1}}$, with $\varepsilon > 1$.

Since the entrepreneurs postpone consumption to the time of death, aggregate consumption is given by

$$\left(1+\tau\right) P_t e_t = \xi_t R_t^k f\left(\overline{\omega}_t\right) X_t$$

and the aggregate accumulation of internal funds by

$$Z_t = \left(1-\xi_t\right) \frac{R_t^k}{z_{t-1}} f\left(\overline{\omega}_t\right) Z_t. \tag{77}$$

We consider the limiting case where consumption of entrepreneurs is fully taxed. As the tax rate is made arbitrarily large, the consumption of the entrepreneurs approaches zero, $e_t \to 0$, and the consumption tax revenue, $T_t^e = \frac{\xi_t R_t^k f(\overline{\omega}_t) X_t}{1+\tau}$, approaches the total funds of the entrepreneurs that die.

D.5 The intermediate goods sector

Firms in the intermediate goods sector are monopolistically competitive. They produce intermediate good j, with $j \in (0,1)$, using the technology

$$y_t\left(j\right) = A_t l_t\left(j\right)^{\alpha} \overline{k}_t\left(j\right)^{1-\alpha}, \tag{78}$$

where $\alpha \in (0,1)$ denotes the labor share, $l_t(j)$ and $\overline{k}_t(j)$ denote the amount of labor and capital services rented on the market by firm j, while A_t is an aggregate exogenous productivity shock.

Because of product differentiation, each intermediate good firm has some market power. We assume that each retailer can change its price with probability $1-\theta$, following Calvo (1983). The optimality conditions for price setting are given by (33)-(36) in the main text.

D.6 Market clearing

Market clearing conditions for money, bonds, labor, loans, wholesale goods and retail goods are given, respectively, by

$$M_t = M_t^s, \tag{79}$$

$$B_{t,t+1} = B_{t,t+1}^s, \tag{80}$$

$$h_t = l_t, \tag{81}$$

$$D_t = X_t - Z_t, \tag{82}$$

$$y_t = \int_0^1 y_t(j)\,dj, \tag{83}$$

$$y_t(j) = A_t l_t(j)_t^{\alpha} \overline{k}(j)^{1-\alpha}, \tag{84}$$

$$y_t(j) = c_t(j) + e_t(j) + I_t(j), \text{ for all } j, \tag{85}$$

where $\int Z_{i,t}di = Z_t$ and $\int X_{i,t}di = X_t$.

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Fiorella De Fiore (corresponding author)
European Central Bank, Frankfurt am Main, Germany; CEPR, London, United Kingdom; email: fiorella.de_fiore@ecb.europa.eu

Oreste Tristani
European Central Bank, Frankfurt am Main, Germany; email: oreste.tristani@ecb.europa.eu

Postal address 60640 Frankfurt am Main, Germany
Telephone +49 69 1344 0
Website www.ecb.europa.eu

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