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Asset pricing and the propagation of financial shocks

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Abstract

This study augments the neoclassical growth model with a mechanism that creates a novel transmission channel through which financial shocks propagate to the real economy. By affecting agents’ ability to finance consumption expenditures, financial frictions create a demand for safe assets that exposes the economy to asset quality shocks. My main finding is that this mechanism provides a potential explanation for the co-movement observed during the 2007-2009 financial crisis in the eurozone. My results also suggest that these shocks are a plausible source of aggregate risk that could explain business cycle fluctuations as well as standard asset pricing puzzles. Finally, introducing this transmission mechanism into the neoclassical growth model increases the welfare cost of business cycle fluctuations by several orders of magnitude.

- JEL: E32, E44, G10
- Keywords: Liquidity constraints, equity premium puzzle, Great Recession.
Non-technical Summary

A central feature of the financial crisis is that a large share of the so-called toxic financial products were considered very safe before the shock hit. Many of these products had a high credit rating and were widely used as pledgeable collateral in the years preceding the crisis. The second key feature of the subprime crisis is that the initial shock that triggered the recession was highly transitory. Recent evidence also shows that the financial shock caused a reduction in lending by European banks that were exposed to these products.

The propagation mechanism of traditional dynamic stochastic general equilibrium (DSGE) models is not sufficiently powerful to generate large and persistent recessions in response to small shocks that have little intrinsic persistence. Moreover, standard macroeconomic frameworks are unable to generate differences in risk premia across asset classes.

The first main contribution of the present paper is to address these issues by developing a dynamic general equilibrium model that matches the risk premia on safe and risky assets observed in the data. In such a model, a shock to the economy’s safe asset stock that is calibrated to generate cumulated losses in the banking sector of a plausible magnitude generates a deep and persistent recession, a fall in consumption as well as a stock market crash and a flight-to-safety effect.

The novel mechanism studied in this paper creates a tight link between the asset pricing implications of macroeconomic models and their ability to propagate and amplify the effects of macroeconomic shocks. The more persistent recession obtained in the version of the model that matches the equity premium is due to the combination of two factors. First, rebuilding the economy’s stock of collateral after an adverse financial shock is costly and takes time. Second, with habits, the low intertemporal elasticity of substitution in consumption that is needed to match asset pricing moments creates a strong consumption smoothing motive.

In such an environment, agents’ main priority after a financial shock is to rebuild the economy’s stock of safe assets, since collateral is needed to access the credit market and finance consumption expenditures. On impact, a larger fall in consumption can be avoided by reallocating resources to the financial sector. But, in a general equilibrium environment, this reallocation of resources comes at the cost of delaying the recovery by crowding out business investment in the real economy. The recession is therefore less severe on impact but lasts much longer.
1 Introduction

One main lesson from the 2007-2009 global crisis is that financial shocks are a more important source of aggregate risk than what is typically predicted by the generation of models developed during the Great Moderation period. The structural change brought about by the crisis has led to the emergence of a new class of macroeconomic models in which the introduction of financial frictions opens the door for studying the effects of financial shocks. While much progress has been achieved, most of this literature has however focused on macroeconomic implications and very few studies have attempted to simultaneously explain asset pricing puzzles in environments where financial shocks are a key source of aggregate risk.

This article addresses this question by augmenting a dynamic general equilibrium model with financial frictions that create a novel transmission mechanism through which financial shocks originating in the financial sector propagate to the real economy. Relative to some recent influential contributions to this literature (e.g., Jermann and Quadrini 2012; Kiyotaki and Moore 2012; Kurlat 2013; Christiano, Motto, and Rostagno 2014; Gorton and Ordonez 2014; Saki 2015; Bigio 2015; Boissay, Collard, and Smets 2015), the main departure is to investigate the role played by frictions affecting households’ ability to obtain financing in advance when credit is needed to finance consumption expenditures. The key distinguishing feature of this mechanism is that agents’ access to credit in turn depends on the value of pledgeable assets they have accumulated over time, the supply of which is endogenously determined by an asset-producing sector. The second main difference is that I investigate whether the shock transmission mechanism that I introduce can help resolve asset pricing anomalies by studying higher-order terms in the Taylor expansion.

I study these questions in a model in which the assets used as collateral are safe in the sense that investors require a low risk compensation to accept to hold them. Relative to the premium on riskier assets, which in my case is the equity premium, this low risk premium is not only due to the lower sensitivity of the safe asset price to business cycle fluctuations, but also to the fact that the asset can be used as collateral to obtain credit. This "specialness" of safe assets can be thought of as reflecting money-like convenience services in the pricing of assets whose function in the financial system is similar to that of a standard medium of exchange such as money (e.g., Greenwood and Vayanos 2010; Krishnamurty and Vissing-Jorgensen 2011, 2012; Venkateswaran and Wright 2013; Caballero and Fahri 2014). This specific characteristic is captured
by introducing different types of assets in a model in which the quantitative magnitude of the risk premiums observed across asset categories can be reproduced.

The asset-producing sector, which is meant to capture the role played by the shadow financial sector in the economy, produces assets that provide liquidity services to their holders. Relative to the commercial bank whose role is to provide credit, the difference is that the shadow financial sector is endowed with a technology that can be used to produce safe assets by combining funds collected from the non-financial sector and hours worked. Safe assets are then purchased by consumers who need a collateral to obtain credit from the commercial bank.

Financial shocks are exogenous shocks that affect the quality of the safe asset stock accumulated by the agent. There are several reasons to think that asset quality shocks were particularly relevant during the crisis period. There is by now a clear consensus that the mass downgrade of securities that were widely used as collateral in the pre-crisis years played a central role in triggering the global financial downturn. The unprecedented size and severity of the credit rating downgrades observed at the onset of the crisis (e.g., Benmelech and Dlugosz 2010), and the collapse in securitization issuance by the private sector led to a massive reduction in the quantity of assets that were considered safe (e.g., IMF 2012). In my environment, this sudden deterioration in asset quality takes the form of a financial shock that reduces the stock of assets that banks accept as collateral.

My first main finding is that this mechanism provides a potential explanation for the co-movement observed during the 2007-2009 financial crisis in the eurozone. In my economy, a small negative financial shock generates a large and persistent drop in output, consumption and investment as well as a large decline in stock prices. My second main result is that the model augmented with this novel propagation mechanism has the potential to resolve standard asset pricing anomalies, since the high equity premium as well as the low premium on the safe asset can be reproduced. This mechanism also provides an explanation for the leading indicator properties of equity prices observed in the data and endogenously generates positive autocorrelation in output growth. Overall, the main contribution of this article is to show that small exogenous variations in asset quality have the potential to generate asset pricing dynamics as well as business cycle predictions that are in line with developments observed during the financial crisis and at business cycle frequency.

Whereas the initial change in asset quality is exogenous, the mechanism through which the effects of small financial shocks are amplified and propagated to the rest of
the economy is endogenously determined. Relative to a standard neoclassical growth model, this stronger propagation mechanism is obtained by interacting financial frictions with a non-standard preference specification that increases the volatility of marginal utility (e.g., Jaccard 2014).

The introduction of habit formation generates a standard consumption-smoothing motive and the more persistent recession obtained with this mechanism arises from the need to rebuild the stock of collateral after a negative asset quality shock. If agents have a strong preference for consumption smoothing, their main priority is to rebuild their safe asset stock in order to regain access to the credit market as quickly as possible. In an economy in which a collateral is needed to obtain credit, the only way to achieve consumption smoothing after a negative asset quality shock is therefore to allocate resources to safe asset production. On impact, a larger fall in consumption can be avoided by quickly rebuilding the stock of collateral, but this reallocation of resources comes at the cost of crowding out investment in the other sectors of the economy. Restoring the stock of collateral is necessary in order to avoid a larger adjustment, but since this more stable consumption path is achieved by reducing the amount of resources allocated to the non-financial sector, this gain comes at the cost of a more persistent reduction in output. Interacting financial shocks with habits therefore creates a recession that is less painful on impact but that lasts much longer.

Financial frictions modify the propagation mechanism of an otherwise standard real business cycle model by introducing a time-varying wedge into the agent’s intratemporal consumption-leisure optimality condition. In my environment, this wedge is determined by the ratio between the Lagrange multiplier on the financing-in-advance constraint and the agent’s marginal utility, and therefore provides a measure of ease of access to credit. As illustrated by Figure 1, which plots the model-implied financing wedge, the large and countercyclical fluctuations observed in the data clearly suggest that this mechanism is not only relevant at business cycle frequency but also during the crisis period. Relative to the class of models in which this term is constant or exogenously specified, I also show that interacting financial frictions with this preference specification gives rise to endogenous fluctuations in this wedge that are countercyclical and almost as volatile as in the data.

As regards the response of asset prices to anticipated shocks, an interesting implication of this propagation mechanism is its ability to generate a boom in equity prices in response to positive news about the future. While the literature aimed at identifying the effects of news shocks finds that positive news about the future leads
to a stock market boom (e.g., Beaudry and Portier 2006; Barsky and Sims 2011), as discussed in Christiano et al. (2010), reproducing this effect in standard models can be challenging. In my economy, a positive news shock generates a gradual increase in equity prices that is followed by a bust. This increase in equity prices in response to positive news that will hit the economy in the near future is a by-product of the more realistic persistence in output growth obtained with this mechanism. While the risk-free rate increases in response to a positive news shock, the fact that agents expect a persistent increase in output after the realization of the shock raises the discounted value of future profits. Relative to a standard model, the more persistent increase in dividends that is expected once the positive news materializes enables this model to generate a small but significant increase in the value of the firm in response to positive news about the future.

In terms of welfare implications, my main finding is that this transmission mechanism generates a much larger welfare cost of business cycle fluctuations than what is generally obtained in standard versions of the neoclassical growth model. This result suggests that the welfare implications of financial shocks may be very different and the large cost that I obtain opens the question of whether policy measures aimed at containing the effects of such shocks have received sufficient attention.

**Relation to the literature**

This paper primarily builds on the literature that jointly studies asset pricing and business cycle facts in production economy models (e.g., Jermann 1998; Tallarini 2000; Boldrin, Christiano and Fisher 2001; Danthine and Donaldson 2002; Uhlig 2007; Campanale, Castro and Clementi 2010; Rudebusch and Swanson 2012; Gourio 2012; Jaccard 2014). Relative to these studies, the main difference is that I study the asset pricing implications of a novel propagation mechanism, which is able to resolve asset pricing anomalies in a model in which financial shocks are the main source of aggregate risk.

Relative to the literature that uses Epstein-Zin-Weil preferences (e.g., Weil 1989, 1990; Epstein and Zin 1989) to resolve asset pricing puzzles, an interesting difference is that my mechanism creates a tight link between the model’s asset pricing implications and the dynamics of macroeconomic variables. In particular, while the introduction of Epstein-Zin-Weil preferences leads to a separation of quantity and asset price determination (e.g., Tallarini 2000; Lucas 2003), my model’s endogenous propagation mechanism is considerably stronger in the version that generates realistic asset pricing predictions. Habit formation in the composite of consumption and leisure (e.g., Jaccard 2014) generates larger movements in marginal utility, which in this model amplify...
the fluctuations in the wedge created by the financing-in-advance constraint.

My approach is also inspired by the seminal contribution of Kiyotaki and Moore (1997) who showed how fluctuations in the price and quantity of collateral can affect the transmission of shocks in a model in which agents face collateral constraints. Relative to their mechanism or the type of frictions studied in Jermann and Quadrini (2012), I consider the case of an agent that needs to obtain a loan in order to finance consumption expenditures. In this respect, my mechanism shares some similarities with the literature on cash-in-advance constraints (e.g., Abel 1985; Svensson 1985; Lucas and Stokey 1987; Cooley and Hansen 1995; Hairault and Portier 1995; Cooley and Quadrini 1999; Alvarez, Atkeson and Kehoe 2002). At the same time, a key difference is that the amount of borrowing that can be obtained, which in turns affects agents’ consumption decisions, depends on the resell value of a pledgeable asset whose supply is endogenously determined by an asset producer.

My approach is also related to a recent strand of the literature in which capital quality shocks are a main source of business cycle fluctuations (e.g., Gourio 2012; Gertler and Karadi 2011). Relative to these studies, since my financial shock does not directly affect the production function or the law of motion of physical capital, the main difference is that these shocks originate in a different segment of the economy. The financial sector produces safe assets, but does not retain any residual risk on its balance sheet after having sold these assets to credit-constrained agents. In this sense, this sector is meant to capture the role played by the shadow financial sector in providing money-like securities to the real economy (e.g., Singh and Stella 2012; Gennaioli, Shleifer and Vishny 2013; Moreira and Savov 2014).

As in Kiyotaki and Moore (2012), I augment the neoclassical growth model with financial frictions and study the macroeconomic effects of shocks that propagate to the rest of the economy through financial frictions. Relative to their version of the liquidity shock hypothesis, my mechanism addresses a point recently raised by Shi (2015) and Biggio (2012) in the sense that in my model a negative financial shock generates a persistent recession as well as a large fall in equity prices. By contrast, liquidity shocks generate a negative co-movement between equity prices and output in the baseline version of the Kiyotaki-Moore model. As shown by Nezafat and Slavik (2015), introducing this mechanism into a neoclassical growth model nevertheless provides a solution to other important asset pricing puzzles.

This paper is also closely connected to the real business cycle literature (e.g., Kydland and Prescott 1982; Long and Plosser 1983; King, Plosser and Rebelo 1987; King
and Rebelo 1999) since my model reduces to the neoclassical growth model when the borrowing constraints are not binding. As initially pointed out by Cogley and Nason (1995), the propagation mechanism of the baseline real business cycle model is too weak and cannot generate the positive autocorrelation in output growth observed at business cycle frequency. In my case this issue is overcome by introducing financial frictions while the approach followed by Chang, Gomes and Schorfeide (2002) focuses on the labor market. My mechanism introduces an endogenous time-varying wedge into the agent’s intratemporal consumption-leisure optimality condition and is, in this respect, also related to the labor wedge literature recently summarized in Shimer (2009), and whose link with the real business cycle model’s endogenous propagation mechanism is discussed in Gourio and Rudanko (2014).

This paper is also related to a more recent work that studies the macroeconomic implications of safe asset shortages. The mechanism through which the financial sector was able to create safe assets at the cost of exposing the system to a panic is described in Caballero (2010). Caballero and Fahri (2015) develop a theory of the macroeconomic effects of safe asset shortages that is motivated by the secular downward trend in equilibrium real interest rates observed for more than two decades. Gourinchas and Jeanne (2012) put the spotlight on the role that countercyclical fluctuations in real interest rates could play in attenuating the effects of safe asset shortages.

Another recent strand of the literature studies the role played by information about the underlying collateral during the financial crisis. In Gorton and Ordonez (2014), information is costly to produce and the lack of information production during periods of economic booms makes the economy vulnerable to small shocks. While in this model the origin of a crisis remains exogenous, its duration crucially depends on the stock of information produced in the economy. Kurlat (2013) shows how asymmetric information about asset qualities affects the transmission of aggregate shocks. The core mechanism relies on an asymmetric information problem which prevents buyers from distinguishing assets that are useless lemons from high quality assets. Bigio (2015) studies the interaction between limited enforcement and asymmetric information and shows that increases in capital quality dispersion can reproduce some key regularities observed during the Great Recession. Finally, Gorton and Ordonez (2015) endogenize the creation of safe assets and show that whether a credit boom is sustainable very much depends on the characteristic of the technology shock. In a similar vein, Boissay, Collard, and Smets (2015) provide a mechanism that can explain why credit booms can sometimes end up in a banking crisis.
2 The environment

The economy is composed of four representative sectors: a household sector, a shadow financial sector, a bank, and a non-financial sector. Households are credit-constrained in the sense that they need to obtain a loan from banks in order to finance a fraction of their consumption expenditures. The amount of financing in advance that they receive from the banking sector in turn depends on the quantity of high-quality liquid instruments that they can pledge. The fact that households internalize this leverage constraint creates a demand for a liquid asset that is produced by the shadow financial sector. These liquid instruments, which I refer to as safe assets, are subject to asset quality shocks that modify the stock of collateral that households can use to obtain credit.

Households

The main liquidity friction arises from the fact that households need to obtain a loan from the banking sector in order to finance part of their consumption expenditures. The relationship between consumption $c$ and the quantity of credit that households obtain from the banking sector $l$ is given by the following financing-in-advance constraint:

$$c_t \leq \xi l_t,$$

where $\xi$ is a parameter that determines the tightness of the constraint.

Furthermore, I assume that the quantity of credit that bankers are willing to extend in turn depends on the quantity of high-quality liquid assets held by households. This collateral constraint, which provides an upper bound on the quantity of credit that can be obtained to finance consumption expenditures, is given as follows:

$$l_t \leq \varpi p_s t \gamma s_{t+1},$$

where the leverage parameter is denoted by $\varpi$. The quantity of credit that can be obtained depends on the resell value of the stock of safe assets available in period $t$, which I denote as $s$, and where $p_s$ is the market price of the safe asset. The specifications of preference and technology are compatible with balanced growth and $\gamma$ denotes the deterministic growth rate of the economy along the balanced growth path.

The sequential budget constraint faced by the representative household in period $t$ is given as follows:
\[ \pi_T e_t + w_{Ft} N_{Ft} + w_{St} N_{St} = c_t + r_L L_t + p_{St} x_t + p_{Ft} (\gamma e_{t+1} - e_t), \] (3)

On the revenue side, the representative agent firstly receives a dividend income \( \pi_T \) from owning firms in the other sectors of the economy, where \( e \) denotes the quantity of equity, which represents the risky asset, held by households. As far as the allocation of time is concerned, the representative agent divides his or her time endowment between leisure activities \( L \), hours worked in the final goods-producing sector \( N_F \), and hours worked in the shadow financial sector \( N_S \). Normalizing the total time endowment to 1, the allocation of time constraint takes the following form:

\[ N_{Ft} + N_{St} + L_t = 1, \] (4)

The wage rates paid by firms in the non-financial and shadow financial sectors are denoted by \( w_F \) and \( w_S \), respectively.

On the expenditure side of the budget constraint, \( r_L \) denotes the cost of obtaining credit from the banking sector. The stock of safe asset \( s \) that households retain on their balance sheet depends on the quantity purchased from the shadow financial sector and evolves over time according to the following law of motion:

\[ x_t = \gamma s_{t+1} - (1 - \tilde{\eta}_t)s_t, \] (5)

where \( x \) represents the quantity of new assets that is purchased from the shadow financial sector. The rate at which the stock of safe asset depreciates has both a deterministic and a random component:

\[ \tilde{\eta}_t = \eta + \log z_t, \]

where \( \eta \) is the constant depreciation rate.

The asset quality shock, which is denoted as \( z \), takes the form of a disturbance that affects the rate at which the stock of safe assets depreciates. The stochastic shock has a zero mean and follows an autoregressive process of order one:

\[ \log z_t = \rho \log z_{t-1} + \varepsilon_t, \]

where the random disturbance \( \varepsilon \) is normally distributed.

The representative household derives utility from consuming a market consumption good \( c \) and leisure \( L \). To maximize the model’s ability to explain asset pricing facts,
I assume that habits are formed over the mix of consumption and leisure, where the reference level or habit stock is denoted \( h \) (e.g., Jaccard 2014). Net utility is given by the difference between the composite good \( c(\psi + L^v) \) and the reference level, \( h \). The two labor supply parameters, \( v \) and \( \psi \), control the Frisch elasticity of labor supply and determine the steady-state time allocation.\(^1\) The modified discount factor and the curvature parameter are denoted\(^2\) \( \tilde{\beta} \) and \( \sigma \), respectively. The law of motion that governs the accumulation of the habit stock is given as follows:

\[
\gamma h_{t+1} = bh_t + (1-b)c_t(\psi + L^v_t), \tag{6}
\]

where \( 0 \leq b \leq 1 \) is a memory parameter that controls the rate at which the habit stock depreciates.

The representative agent optimally chooses consumption, the quantity of credit, the number of hours worked in the two sectors, the stock of safe assets, the stock of risky assets, and controls the evolution of his or her habit stock in order to maximize expected lifetime utility:

\[
\max_{c_t, h_t, N_{St}, N_{St+1}, c_{t+1}, h_{t+1}} E_0 \left\{ \sum_{t=0}^{\infty} \frac{\tilde{\beta}^t}{1-\sigma} [c_t(\psi + L^v_t) - h_t]^{1-\sigma} \right\},
\]

subject to constraints (1) to (6).

**The shadow financial sector**

The shadow financial sector is endowed with a production technology that enables firms in this sector to produce a high-quality liquid instrument by combining funds obtained from the non-financial sector with labor. Profits in the shadow financial sector depend on the quantity of new assets that is produced \( x \) and sold to households at price \( p_S \). Safe assets are produced and then distributed to the rest of the economy and do not remain on the shadow sector’s balance sheet once sold. The quantity of new liquid instruments produced in period \( t \) is given by the following production function:

\[
x_t = m_t^\phi N_{St}^{1-\phi}, \tag{7}
\]

where \( m \) denotes funds received from the non-financial sector. Each period, the shadow financial sector chooses the number of hours worked and the quantity of funds that

\(^1\)These parameters are restricted to ensure concavity in \( L \) and that both goods are always normal goods.

\(^2\)where \( \tilde{\beta} = \tilde{\beta}^1 \gamma^{1-\sigma} \)
maximizes profits,

$$\max_{m_t, N_{St}} \pi_{St} = p_{St}x_t - w_{St}N_{St} - r_Mm_t,$$

subject to constraint (7), and where $r_M$ represents the cost of borrowing funds from the non-financial sector.

**Firms in the non-financial sector**

Firms in the non-financial sector produce a final output good $y$ using physical capital $k$ and labor $N_F$ as productive inputs. In addition to the production decision, I assume that another main function of the non-financial sector is to determine its investment policy. Investment, which I denote by $i$, is financed through retained earnings and determines the stock of financial capital that firms have at their disposal. In parallel to the investment decision, the non-financial sector needs to decide how to optimally allocate its stock of financial capital between the different sectors of the economy. Financial capital, which is denoted by $a$, is a composite capital good that can be divided between the amount of physical capital used in the production function $k$, funds lent to the shadow financial sector $m$, and funds deposited in the banking sector $d$.

$$a_t = k_t + m_t + d_t, \quad (8)$$

The accumulation of financial capital is subject to an adjustment cost and evolves over time according to the following law of motion:

$$\gamma a_{t+1} \leq (1 - \delta)a_t + \left( \theta_1 \left( \frac{i_t}{a_t} \right)^{1-\epsilon} + \theta_2 \right) a_t \quad (9)$$

Following Baxter and Crucini (1993) and Jermann (1998), among others, I adopt a specification of adjustment costs that penalizes changes in investment that are large relative to the existing stock of financial capital. Since an increase in investment needs to be financed by reducing the dividend paid to shareholders, this friction could capture a cost that managers incur when they need to convince shareholders to accept a reduction in dividends to finance investment projects. One possible interpretation is that convincing shareholders to finance large investment projects is more costly than financing smaller ones, since large projects need to be financed through higher amounts of retained earnings.
Given the capital allocation decision faced by management, profits in the final good sector are given as follows:

\[ \pi_{Ft} = Ah_t^\alpha N_{Ft}^{1-\alpha} + r_M m_t + r_{Dt} d_t - w_{Ft} N_{Ft} - i_t, \]  

where \( r_D \) is the rate at which funds deposited in the banking sector are remunerated, and where the production function for the final output good is given by:

\[ y_t = Ah_t^\alpha N_{Ft}^{1-\alpha} \]  

Since the objective of this paper is to study the effects of asset quality shocks, the benchmark version of the model abstracts from technology shocks and total factor productivity, which is denoted by \( \Lambda \), is held constant. The response of the main variables to a technology shock will be discussed in section 6. Each period, managers in the final goods-producing sector choose the optimal number of hours worked, investment, the evolution of the stock of financial capital, and how capital is allocated between the different sectors of the economy to maximize the firm’s market value, which is equal to the present discounted value of all current and future expected cash flows:

\[ \max_{N_{Ft}, i_t, k_t, d_t, m_t, \alpha_{t+1}} E_0 \sum_{t=0}^{\infty} \frac{\beta^t}{\lambda_0} \pi_{Ft}, \]

where \( \frac{\beta^t}{\lambda_0} \) is the discount factor of the representative agent who is the owner of the firm, subject to constraints (8) to (11).

**The commercial bank**

The banking sector serves as an intermediary between the representative firm and the household sector. To keep the analysis tractable, I assume that the role of the banking sector is to collect funds from the representative firm that are then lent to the household sector. To keep the banking sector as simple as possible, banks do not hold any other type of assets or liabilities and their balance sheet is given as follows:

\[ l_t = d_t \]

Taking into account this constraint, the role of managers in the banking sector is to choose the profit-maximizing quantity of loans to extend to the household sector:

\[ \pi_{Bl} = r_{Lt} l_t - r_{Dt} d_t \]
Market equilibrium

A competitive equilibrium in the economy is a sequence of prices \( \lambda, \varphi, \mu, \chi, q, w_S, w_F, r_M, r_L, r_D, p_S, p_E \) where \( q \) is Tobin’s \( Q \), \( \lambda \) is marginal utility, \( \varphi \) is the Lagrange multiplier associated with equation (6), \( \mu \) and \( \chi \) are the Lagrange multipliers associated with the financing in advance and collateral constraints (1) and (2), respectively, and quantities \( c, d, l, i, x, k, m, N_F, N_S, a, s, h, c \) and \( y \) that satisfy efficiency conditions of households and firms in the different sectors of the economy as well as the economy-wide resource constraint:

\[
y_t = c_t + i_t,
\]

for all states, for \( t=1,\ldots,\infty \), and given initial values for the three endogenous state variables, \( a, h \) and \( s \).

The financing-in-advance wedge

Relative to the neoclassical growth model, the key difference is the introduction of financial frictions that create a wedge into the agent’s consumption-leisure optimality condition. With the introduction of habits, the first-order condition with respect to consumption and hours worked in the non-financial sector are given as follows:

\[
\left\{ \left[ c_t (\psi + L_t^\nu) - h_t \right]^{\sigma - \varphi} + \varphi_t (1 - b) \right\} (\psi + L_t^\nu) = \lambda_t + \mu_t,
\]

Relative to the frictionless case, the new term in equation (13) drives a wedge between the agent’s marginal utility of consumption, which in my case is given by the left-hand side of equation (13), and the marginal utility of wealth \( \lambda \). As in a standard cash-in-advance model, the marginal utility of consumption must exceed the marginal utility of wealth in a model in which consumption expenditures need to be financed in advance. The difference between the two, which is given by the Lagrange multiplier \( \mu \), determines the value of liquidity services (e.g., Walsh 2003). With this internal specification, the effect of habits cancels out and the only difference relative to the neoclassical growth model is the introduction of a wedge, captured by the term \( \mu/\lambda \), into the consumption-leisure optimality condition:

\[
\psi + L_t^\nu = \frac{c_t w_{Ft}^{\nu-1}}{w_{Ft}} \left( 1 + \frac{\mu_t}{\lambda_t} \right)
\]

This term shows how the tightness of the financing-in-advance constraint in (1) affects the agent’s optimal consumption and leisure choice. Using the household’s
optimality conditions, the financing-in-advance wedge can be further decomposed into a first term that captures the cost of credit and a second term that reflects the effect of collateral scarcity:

\[ \xi \frac{\mu}{\lambda_t} = r_{Lt} + \frac{\chi}{\lambda_t} \]  

(16)

First, the cost of obtaining credit from the commercial bank has a direct impact on the financing-in-advance wedge through the borrowing rate \( r_{Lt} \) that households have to pay to obtain credit. Second, the Lagrange multiplier \( \chi/\lambda \), which measures the tightness of the collateral constraint (2), shows how the collateral scarcity effect impacts the financing conditions faced by households. This decomposition illustrate the two channels through which the cost of credit and the availability of collateral affect the financing conditions faced by households, which in turns determine the ease at which consumption expenditures can be financed.

**Asset price dynamics**

The optimality condition characterizing the evolution of equity holdings can be used to derive a textbook asset pricing formula linking the price of equity, \( p_E \), to the expected discounted sum of future dividends:

\[ p_E = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (\pi_{Tt+1} + p_{Et+1}) \]  

(17)

Similarly, the households’ optimal demand for pledgeable assets yields the following asset pricing formula for the safe asset3 \( p_S \):

\[ p_S = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{1 - \frac{\omega}{\lambda_t}} (1 - \tilde{\eta}_{t+1}) p_{St+1} \]  

(18)

Relative to the risky asset, the first main difference is that the pricing condition of the safe asset is directly affected by the asset quality shock through the stochastic depreciation rate \( \tilde{\eta} \). Second, the safe asset provides liquidity services and the tightness of the collateral constraint is captured by the effect of the Lagrange multiplier \( \chi/\lambda \) on the pricing formula. Changes in the Lagrange multiplier therefore have a direct

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3 Equity returns and the return on the safe asset between periods \( t \) and \( t + 1 \) are respectively given by:

\[ r^e_{t,t+1} = \frac{\pi_{Tt+1} + p_{Et+1}}{p_E} \quad \text{and} \quad r^s_{t,t+1} = \frac{(1 - \tilde{\eta}_{t+1}) p_{St+1}}{1 - \frac{\omega}{\lambda_t}} \frac{p_S}{p_{St}} \]
effect of the pricing of safe assets contributing to the unique cyclical properties of this particular asset category.

Finally, the risk-free rate \( r^f \) can be defined as the rate of return that would be obtained from investing in a risk-free one-period asset that yields a certain payoff in \( t+1 \):

\[
\frac{1}{r^f_t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}
\]  

\( (19) \)

3 Parameter selection

In order to assess the role that asset quality shocks could have played during the crisis, the model is calibrated using pre-Great Recession data. I will then assess whether the model is able to capture the co-movement between financial and business cycle variables observed during the crisis once the model economy is hit by a shock of plausible magnitude. A first set of parameters is calibrated using moment conditions that can be derived from the structure of the model or by exploiting steady state conditions. A second set of parameter values, for which a priori knowledge is weak, is chosen to maximize the model’s ability to reproduce a series of empirical facts that characterize the eurozone economy.

Curvature parameter, technology parameters and population growth

The deterministic growth rate of the economy \( \gamma \) is calibrated using data on population growth, which is available on an annual basis since 1960. Between 1960 and 2008, the average rate of population growth for the country group that is now forming the eurozone is 0.5% per year, which implies a value for the quarterly growth rate of 1.0013.

The capital share parameter in the production function of the final output good \( \alpha \) is set to 1/3, which implies a labor share of 2/3. As documented in the literature, increasing the curvature parameter \( \sigma \) in a model with production helps to increase the equity risk premium, but comes at the cost of an increase in the risk-free rate that is at odd with the facts (e.g., Weil 1989). To minimize the role played by this parameter, I set \( \sigma \) to 1.4.

Frisch elasticity of labor supply and steady state time allocation

\footnote{Shimer (2009) argues that the microeconomic behaviour of the neoclassical growth model becomes unreasonable when \( \sigma \) is larger than 1.}
In Jaccard (2014), I show that the curvature parameter $\nu$ controls the Frisch elasticity of labor supply. In the real business cycle literature, a Frisch elasticity of at least 3 is usually needed to account for business cycle fluctuations (e.g., King and Rebelo 1999). Recent findings however suggest that the Frisch elasticity could be smaller than what is usually assumed in the macroeconomic literature (e.g., Hall 2009; Chetty et al. 2011). Given the lack of consensus regarding the Frisch elasticity of labor supply in Europe, I choose a value for $\nu$ that implies an elasticity of 1.5, that is half the value reported in Prescott (2004) using cross-country differences in aggregate hours.

The second labor supply parameter is chosen to ensure that in the steady state, agents spend on average about twenty percent of their time on work-related activities. This restriction pins down this parameter value, which can be calculated by combining the first-order conditions with consumption and hours worked in the final good sector:

$$\psi = \left(1 + \frac{\mu}{\chi} N_F \xi \nu L^{\nu-1} \right) \frac{1}{1 - \alpha} - L^\nu$$

**Financial frictions**

The transmission of asset-quality shock to the real economy crucially depends on the financing-in-advance and collateral constraint parameters $\xi$ and $\varpi$. These parameters are calibrated under the assumption that constraints (1) and (2) are always binding. The validity of this assumption will then be verified by means of model simulations by checking that the two Lagrange multipliers $\mu$ and $\chi$ are always strictly positive in large samples of simulated data. Using data on consumption and consumer credit, equation (1) can be used to derive a first moment condition to estimate the financing-in-advance parameter $\xi$:

$$\xi = \frac{E(c_t)}{E(l_t)}$$

Similarly, from equation (2), the leverage parameter $\varpi$ can be estimated using the following moment condition:

$$\varpi = \frac{E(l_t)}{\gamma E(p_{St}\delta_{t+1})}$$

Using aggregate data on final consumption expenditures and consumer loans, which are available on a quarterly basis since 1997, I find an average value for the financing-in-advance parameter of 2.96. The leverage parameter is estimated by using data on short-term liquid assets held by the household sector. Using the definition of short-term
assets reported in the integrated economic and financial accounts, short-term assets are composed of currency and deposits, short-term debt securities and money market fund shares. As an average over the period from 1999Q4 to 2008 Q3, this implies a value for the leverage parameter of 0.13.\(^5\)

**Additional long-run restrictions**

Data on financial investment by the household sector can be used to put some discipline on the parameters that determine the size of the shadow financial sector. As an average over the period from 1999 to 2008, net acquisition of short-term assets by the household sector represented eleven percent of gross domestic product:

\[ E(\frac{p_Sx}{y}) = 0.114 \]

Given the structure of my model, the steady state quantity of output produced by the shadow financial sector \(p_Sx\) corresponds to the amount that the representative agent invests in liquid asset. This information can be exploited to derive an indirect restriction that pins down the deterministic rate of asset depreciation \(\eta\).

Following the practice in the real business cycle literature, the depreciation rate of physical capital \(\delta\) can be pinned down by using data on the investment share of output. As an average over the period from 1995 to 2008, the investment to output ratio is given by:

\[ E(i/y) = 0.225 \]

Including this long-run restriction into the loss function will identify the physical capital depreciation parameter \(\delta\).

**Moment matching procedure**

Given the lack of exploitable information on the remaining model parameters, this second set is chosen to maximize the model’s ability to reproduce the volatility at quarterly frequency of the following macroeconomic aggregates: output growth \(\sigma_{\Delta y}\) consumption growth \(\sigma_{\Delta c}\) investment growth \(\sigma_{\Delta i}\) and hours worked in the final goods sector \(\sigma_{\Delta_{NF}}\). To maximize the model’s ability to simultaneously reproduce asset market facts, I also include the equity risk premium \(E(r_E - r_F)\) and the mean risk-free rate \(E(r_F)\) into the loss function. In addition to the two depreciation rates \(\delta\) and \(\eta\) which

\(^5\)The data used to compute these two moment conditions are described in the data appendix.
will be identified by the two restrictions discussed above, the six corresponding parameters are the habit parameter $b$, the subjective rate of time discount $\beta$, the capital adjustment cost parameter $\epsilon$, the capital share in the production function of safe assets $\phi$, the persistence of the shock process $\rho_Z$, and the shock standard deviation $\sigma_Z$. These parameters are chosen in order to minimize the following loss function $\ell$:

$$\ell = [\Psi - f(\Theta)]' \Omega [\Psi - f(\Theta)],$$

where $\Psi = [\sigma_{\Delta y}, \sigma_{\Delta c}, \sigma_{\Delta i}, \sigma_{\Delta_{\text{NP}}}, E(r_E - r_F), E(r_F), E(p_{x/y}), E(i/y)]'$ is the vector of moments to match, and where $\Theta = [\delta, \eta, b, \beta, \epsilon, \phi, \rho_Z, \sigma_Z]'$ denotes the vector of model parameters. The loss function is minimized for the following combination of parameter values:

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\eta$</th>
<th>$b$</th>
<th>$\beta$</th>
<th>$\epsilon$</th>
<th>$\phi$</th>
<th>$\rho_Z$</th>
<th>$\sigma_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.017</td>
<td>0.057</td>
<td>0.58</td>
<td>0.98</td>
<td>5.27</td>
<td>0.60</td>
<td>0.986</td>
<td>0.0024</td>
</tr>
</tbody>
</table>

### 4 Results

The outcome of this empirical procedure is reported in Tables 1 and 2 below, which compare the model implications with the data. The data used to estimate the empirical moments are described in the data appendix. As shown by the results reported in Table 2 and in the first six rows of Table 1, the model is able to match the moments that were targeted. While investment is slightly less volatile than in the data, it is still possible to reproduce the main business cycle regularities in a model that generates a high equity premium as well as a low mean risk-free rate (e.g., Mehra and Prescott 1985; Weil 1989). As shown by the results reported in Table 2, it is also possible to match the two key steady state ratios that determine the size of the shadow financial sector as well as the investment to output ratio. Relative to a standard real business cycle model (e.g., King, Plosser and Rebelo 1988; King and Rebelo 1999), the fact that the model is able to generate the high volatility of hours worked in the final-goods sector is a noteworthy improvement.

As regards model implications that were not explicitly targeted in the loss minimization procedure, as shown in Table 1, it is possible to generate fluctuations in equity prices, whose standard deviation is denoted by $\sigma_{\Delta p_E}$, that are almost as volatile as in the data. The model is also able to reproduce the sizeable difference between the equity and the safe asset risk premiums, as the predicted excess return between the safe asset
return and the risk-free rate, i.e., \( E(r^e - r^f) \) is close to zero, as suggested by the data. While the model is able to generate volatile fluctuations in hours worked, this success comes at the cost of underestimating the volatility of labor productivity \( \sigma_{\Delta w} \) whose model-generated volatility lies outside the 95% confidence interval. At the same time, the excessively low volatility that I obtain illustrates that the sluggish dynamics of real wages documented in many studies can also be reproduced in a model that abstracts from nominal rigidities.

<table>
<thead>
<tr>
<th>Table 1: Asset Pricing and Business Cycle Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{\Delta y} ) &amp; ([0.33, 0.48]) &amp; 0.39 &amp; 0.39</td>
</tr>
<tr>
<td>( \sigma_{\Delta c} ) &amp; ([0.24, 0.35]) &amp; 0.28 &amp; 0.28</td>
</tr>
<tr>
<td>( \sigma_{\Delta i} ) &amp; ([1.18, 1.74]) &amp; 1.41 &amp; 1.32</td>
</tr>
<tr>
<td>( \sigma_{\Delta N_F} ) &amp; ([0.33, 0.54]) &amp; 0.41 &amp; 0.39</td>
</tr>
<tr>
<td>( E(r^e - r^f) ) &amp; ([-4.23, 14.22]) &amp; 4.99 &amp; 4.99</td>
</tr>
<tr>
<td>( E(r^f - 1) ) &amp; ([1.03, 4.56]) &amp; 2.79 &amp; 2.79</td>
</tr>
<tr>
<td>( E(r^b - r^f) ) &amp; ([-1.97, 1.71]) &amp; -0.13 &amp; 0.12</td>
</tr>
<tr>
<td>( \sigma_{\Delta w} ) &amp; ([0.22, 0.36]) &amp; 0.27 &amp; 0.14</td>
</tr>
<tr>
<td>( \sigma_{\Delta p_E} ) &amp; ([7.1, 9.6]) &amp; 8.16 &amp; 7.49</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2: Steady State Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E(i/y) ) &amp; ([0.222, 0.229]) &amp; 0.225 &amp; 0.225</td>
</tr>
<tr>
<td>( E(p_S x/y) ) &amp; ([0.087, 0.141]) &amp; 0.114 &amp; 0.114</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3: Correlation with Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho(\Delta_c, \Delta_y) ) &amp; ([0.45, 0.78]) &amp; 0.64 &amp; 0.77</td>
</tr>
<tr>
<td>( \rho(\Delta_i, \Delta_y) ) &amp; ([0.34, 0.72]) &amp; 0.56 &amp; 0.83</td>
</tr>
<tr>
<td>( \rho(\Delta_{N_F}, \Delta_y) ) &amp; ([0.21, 0.72]) &amp; 0.51 &amp; 0.94</td>
</tr>
<tr>
<td>( \rho(\Delta_{p_E}, \Delta_y) ) &amp; ([0.14, 0.60]) &amp; 0.40 &amp; 0.79</td>
</tr>
</tbody>
</table>
Table 4: Lead-lag correlation equity prices-output

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(\Delta_y, \Delta_{pE}(+1))$</td>
<td>0.31</td>
<td>0.11</td>
<td>0.54</td>
<td>0.41</td>
</tr>
<tr>
<td>$\rho(\Delta_y, \Delta_{pE}(+2))$</td>
<td>0.10</td>
<td>0.09</td>
<td>0.46</td>
<td>0.36</td>
</tr>
<tr>
<td>$\rho(\Delta_y, \Delta_{pE}(+3))$</td>
<td>0.02</td>
<td>0.07</td>
<td>0.31</td>
<td>0.32</td>
</tr>
<tr>
<td>$\rho(\Delta_y, \Delta_{pE}(+4))$</td>
<td>0.01</td>
<td>0.06</td>
<td>0.21</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Note: In Table 1, the standard deviation of quarterly output growth is denoted by $\sigma \Delta y$, whereby in the model and in the data the growth rate is expressed as the quarter-over-quarter log difference, i.e., $\Delta y = (\ln y_t - \ln y_{t-1}) \cdot 100$. $E(r_e - r_f)$ is mean annualized excess returns of equity returns over the risk-free rate. $E(r^s - r_f)$ is the mean annualized excess returns of the safe asset return over the risk-free rate. In Table 4, $\rho(\Delta_y, \Delta_{pE}(+1))$ is the correlation between output growth at time $t$ and the growth rate of equity prices at time $t + 1$.

As far as co-movement is concerned, as shown by the results reported in Table 3, the model is able to predict the sign of all correlation coefficients. As in the data, consumption, investment, hours worked and equity prices are procyclical. While reproducing correlations in a model with only one shock is always a challenge, it is still possible to generate correlation coefficients that are lower than what is usually obtained in a standard real business cycle model. Moreover, as illustrated by results reported in Table 4, its ability to reproduce the leading indicator properties of equity prices observed in the eurozone is another main distinguishing feature of this mechanism. As in the data, equity prices in the model lead output, implying that for various horizons we have $\rho(\Delta_y, \Delta_{pE}(-k)) > \rho(\Delta_y, \Delta_{pE}(+k))$.

**Impulse response analysis**

The continuous blue line in Figure 3 shows the response of output, consumption, investment and hours worked in the non-financial sector to a negative financial shock. The response of equity prices, the safe asset price, the risk-free rate and the stock of safe asset are shown in Figure 4.

As documented by Cogley and Nason (1995) and Chang, Gomes, and Schorfheide (2002), while the baseline neoclassical growth model is able to reproduce the volatility and co-movement between the main business cycle variables, it is unable to propagate the effect of technology shocks and fails to explain the persistence of quarterly output growth observed at business cycle frequency. As can be seen from the top left panel of Figure 3, a main distinguishing feature of this mechanism is its ability to produce
the hump-shaped response of output that is needed to reproduce this stylized fact. In
the pre-crisis sample, the autocorrelation coefficient of quarterly output growth is 0.43
and reaches 0.65 if the entire sample period is taken into account. In the benchmark
model, combining these particular types of shocks with this endogenous propagation
mechanism generates an autocorrelation coefficient of 0.67.

To illustrate that this model implication depends not only on the propagation mech-
anism but also on the source of shocks, I report the impulse response of output to
a technology shock that has the same persistence and whose volatility is chosen to
produce a response of comparable magnitude. As illustrated in Figure 5, the main
difference is that the propagation mechanism is much weaker in the case of technology
shocks in the sense that the response of output remains predominantly driven by the
exogenous dynamics of the shock. Another way to illustrate this difference is to com-
pare the autocorrelation coefficients obtained under the two scenarios. While in the
model with financial shocks, the hump-shaped response that is obtained generates an
autocorrelation coefficient of 0.67, this coefficient falls to 0.02 when technology shocks
are the only source of fluctuations. Clearly, introducing financial shocks into the analy-
sis offers a potential resolution to this long-standing issue, which has always been one
of the main weaknesses of real business cycle models (e.g., King and Rebelo 1999).

As can be seen from the top left panel of Figure 4, a negative financial shock also
generates a large drop in equity prices. It is therefore possible to generate this co-
movement between output and equity prices in a model that is also able to replicate
the volatility of equity prices as well as the equity premium. Moreover, this mechanism
generates a response of equity prices that is less persistent than that of output, which
enables the model to reproduce the lead-lag pattern documented in Table 4.

It is difficult to find a precise empirical counterpart for the safe asset return. In
Table 1, the empirical value for the safe asset premium has been computed using data on
a collateralized bond index that includes asset-backed or mortgage-backed securities.6
If this particular index is used as a proxy for the safe asset returns, the empirical excess
return that I obtain is slightly negative, suggesting that this asset was considered an
insurance in the years that preceded the crisis. Data on euro-based collateralized bonds
being only available for a small sample, the empirical value that is reported in Table
1 should however be interpreted with care. The purpose of reporting this moment is
more to illustrate that the model is able to generate a significant difference between
the two asset categories that have been introduced. The asset used as collateral is a

6See the data appendix.
safe asset in the sense that the model-implied excess return is close to zero, while the excess returns required by investors to hold a risky asset such as equity is about 5%, as suggested by the data.

The different cyclical properties of equity and of the safe asset can be better understood by analyzing the response of these two asset prices to a negative financial shock, which is depicted in the two top panels of Figure 4. While equity prices fall sharply and recover very slowly, the safe asset price increases on impact and then falls before recovering gradually. As illustrated in the lower left panel of Figure 4, on impact the shock generates a decline in the risk-free rate, which through its effect on the valuation, generates an increase in the safe asset price. The slightly countercyclical dynamics of the safe asset price is therefore driven by this decline in the risk-free rate that dominates on impact.

The fall in the safe asset price that occurs several quarters after the shock can be explained by the decline in demand induced by the asset quality shock. Through the effect of \( \bar{\eta} \) on the valuation, the asset quality shock has a direct impact on the pricing equation by lowering the expected capital gain component (see equation 18). This S-shaped response, which is therefore the product of these two competing forces, makes this asset a safer investment than equity and this specific cyclical property explains the difference between the equity and the safe asset premium generated by the model and observed in the data.

**Risk-free rate dynamics**

One major weakness of production economy models with habits is their tendency to generate excessive risk-free rate variations. In Jermann (1998) and Boldrin, Christiano and Fisher (2001) for instance, the different mechanisms that are developed allow modified versions of the neoclassical growth model to match the equity premium as well as the mean risk-free rate. However, the high risk-free rate volatility that is obtained in these two studies, 11.5% and 24.6%, respectively, is of several orders of magnitude higher than the value observed in the data. In the paper that initially defined the equity premium puzzle, using data since 1889, Mehra and Prescott (1985) find an empirical value for the risk-free rate standard deviation of 5.7%. Similarly, Cechetti, Lam, and Mark (1990) who used data since 1892 find a risk-free rate standard deviation of 5.27%.

In my case, the risk-free rate volatility that I obtain in this model is 5.6%.7 In-

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7See Jaccard (2014) for a detailed discussion of the risk-free rate volatility implied by this preference specification. To obtain an accurate approximation of the risk-free rate standard deviation, it is necessary to use a third-order approximation to the policy function.
Introducing a law of motion for the habit stock that depreciates slowly (e.g., Campbell and Cochrane 1999; Constantinides 2000), as in equation (6), helps to generate a high equity premium while at the same time obtaining a risk-free rate that is more stable than what is usually found in the literature that uses habits to explain asset pricing puzzles (e.g., Cochrane 2005).

Using European data since 1988\(^8\) gives an empirical value for the risk-free rate standard deviation of 2.33%, that is a value more than twice lower than what is predicted by the model. Despite some important progress, this illustrates that explaining the low risk-free rate volatility observed in more recent samples remains a challenge for this class of models. To illustrate that the real effect of financial shocks does not depend on implausibly large fluctuations in the risk-free rate, Figure 6 compares the response of output obtained for the set of parameter values discussed above with an alternative calibration that allows the model to reproduce a lower risk-free rate standard deviation. Relative to the benchmark model, a risk-free rate volatility of less than 3% can for instance be obtained by decreasing the adjustment costs parameter. As shown by the impulse response of output to the same financial shock, the model is still able to generate a persistent decline in output in the version that generates more realistic risk-free rate fluctuations. This illustrates that the effects of financial shocks on output do not depend on the fact that the model generates risk-free rate variations that are higher than what is found using more recent data samples. Moreover, with lower adjustment costs, financial shocks would still generate the co-movement between output and equity prices observed during the crisis. With low adjustment costs, the model would however fail to explain the difference between the equity and the safe asset risk premiums observed in the data and the high volatility of equity prices could no longer be reproduced.

Finally, as shown by the lower left panel of Figure 4, while on impact a negative financial shock generates a decline in the risk-free rate, the persistent increase that occurs immediately after the shocks seems at variance with the flight-to-quality effect documented in the literature. This issue is further investigated in section 6.

Other main limitations

The model’s inability to reproduce the volatility of short-term asset holdings observed in the data is another main limitation of the analysis. Equations (1) and (2) imply that the volatility of the resell value of safe assets \(p_{ss}\) should be equal to the

---

\(^8\)As described in the data appendix, financial market facts are computed using French financial market data, which are available for a longer time period than eurozone-wide data.
volatility of consumption, while in the data short-term asset holdings are about three times more volatile than consumption. This well-known limitation has been documented in the cash-in-advance literature and one potential solution to this problem would be to introduce velocity shocks (e.g., Ireland 1996).

Second, since this is not the main focus of this paper, I have assumed the simplest possible form of balance sheet for financial intermediaries. Relative to what is observed in the data, this counterfactually implies that loans are always equal to the amounts deposited and a richer structure would be needed to reproduce the time-variation in the loan-to-deposit ratio observed in reality.

Finally, the characterization of the supply of safe assets remains very stylized and abstracts from many potentially important aspects. In the data, debt securities issuance by the non-financial sector or by the government is for instance strongly countercyclical, which provides empirical support for the idea that agents rebuild their safe asset stock during periods of recession. A richer model would however be needed to study the contribution of the different sectors of the economy to the overall stock of safe assets available in the economy.

5 The endogenous propagation mechanism

The model’s endogenous propagation mechanism can be better understood by illustrating how the financing-in-advance constraint affects the agent’s consumption-leisure optimality condition. Rearranging equation (14), I obtain the following expression for $\mu/\lambda$:

$$\frac{\mu_t}{\lambda_t} = (1 - \alpha) \frac{y_t}{N_{t+1}} \frac{(\psi + L^p_t)}{c_t L^p_t} - 1$$

This expression can be used to construct a model-implied wedge, which is shown in Figure 1. The unprecedented fall in output observed during the Great Recession was accompanied by a large increase in the financing-in-advance wedge, which confirms that this mechanism played a key role during the crisis period. As shown in Table 5, which focuses on the pre-crisis period, fluctuations in $\mu/\lambda$ are not only relevant during the crisis period but also at business cycle frequency, since this wedge is countercyclical and highly volatile in the pre-crisis sample as well.

Overall, these stylized facts confirm that augmenting standard models with mechanisms that are able to generate large and countercyclical fluctuations in this wedge
are needed to understand the crisis period and the eurozone business cycle in general. In my model, this is achieved by introducing a financing-in-advance constraint and the large increase in \( \mu/\lambda \) observed during the financial crisis period can be interpreted as a tightening of financing conditions faced by households. As illustrated by Figure 2, which shows the results of the eurozone bank lending survey, this model prediction is consistent with the sharp deterioration in financing conditions observed during the crisis period.

In terms of business cycle implications, as shown in Table 5\(^9\), this mechanism generates countercyclical fluctuations in the financing in advance wedge that are almost as volatile as in the data. As illustrated by the model’s ability to simultaneously reproduce the volatility of output, consumption and hours worked, relative to a baseline real business cycle model, these endogenous variations in \( \mu/\lambda \) help to bring the model’s main business cycle predictions into closer conformity with the data.

### Table 5: The financing in advance wedge

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>95% confidence interval</td>
<td>Estimated empirical moments</td>
</tr>
<tr>
<td>( \sigma \Delta_{\mu/\lambda} )</td>
<td>[1.66, 2.71]</td>
<td>2.06</td>
</tr>
<tr>
<td>( \rho(\Delta_y, \sigma \Delta_{\mu/\lambda}) )</td>
<td>[-0.42, 0.25]</td>
<td>-0.09</td>
</tr>
</tbody>
</table>

Note: \( \sigma \Delta_{\mu/\lambda} \) is the quarterly standard deviation of the wedge growth rate, while \( \rho(\Delta_y, \sigma \Delta_{\mu/\lambda}) \) denotes its correlation with output growth.

#### Interacting financial frictions with habit formation

To illustrate the role played by the preference specification, Figure 7 below compares the response of output, the financing wedge, consumption and the financial capital stock to a negative asset quality shock.

The red continuous line shows the response of these variables for the benchmark calibration discussed above. The blue dotted line shows the same impulse response but in the case in which habit formation is switched off by setting \( b = 1 \). As can be seen by comparing the dynamics of output and of the financing wedge shown in the two

---

\(^9\)In Table 5, the standard deviation of the financing in advance wedge \( \mu/\lambda \) is denoted \( \sigma \Delta_{\mu/\lambda} \), while \( \rho(\Delta_y, \sigma \Delta_{\mu/\lambda}) \) is the correlation between output growth and the growth rate of \( \mu/\lambda \).
upper panels of Figure 7, the introduction of habit formation strengthens the model’s endogenous propagation mechanism by generating an increase in $\mu/\lambda$ that is more gradual but also considerably more persistent. The introduction of habit formation affects the dynamics of the financing wedge through its effects on the elasticity of intertemporal substitution (EIS), since a lower EIS exacerbates the agent’s aversion to changes in consumption, whose dynamics are closely linked to the availability of credit and collateral.

Relative to the case without habits, and as illustrated by the bottom right and left panels of Figure 7, the economy as a whole is able to avoid a larger short-term fall in consumption by sharply reducing financial capital accumulation, which is a standard result in models with habit formation. The novel effect due to financing frictions is that consumption smoothing is achieved by rebuilding the safe asset stock as quickly as possible. Aggregate investment has to fall, which implies that the stock of financial capital declines, but at the same time more resources need to be allocated to safe asset production since rebuilding the stock of collateral is the only way to avoid a larger fall in consumption. In terms of the portfolio allocation decision in the non-financial sector, consumption smoothing implies that $\alpha$ has to decline, which is the standard effect, while $m$ needs to rise:

$$a_t = k_t + m_t + d_t$$

When consumption smoothing is the main priority, the only way to avoid a larger fall in consumption on impact is therefore to reduce the quantity of capital allocated to the production of the final output good $k$. The amplification mechanism arises from the fact that the fall in $k$ leads to a lower level of output, which in turn reduces income in the future. This decline in income further reduces financial capital accumulation which in turn lowers the quantity of capital that can be allocated to safe asset production. The resulting reduction in safe asset production generated by this mechanism further reduces the amount of liquidity in advance that can be obtained from the banking sector, which in turn contributes to the tightening of the liquidity-in-advance constraint.

The more gradual but persistent increase in the financing wedge obtained under habits illustrates this trade-off. A larger fall in consumption is avoided today but is obtained at the cost of a more persistent decline in safe asset production. Relative to the no habit case, the tightening of the financing in advance constraint is less pronounced

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10 Since firms in the non-financial sector are owned by the representative agents, the allocation decision is determined by the agents’ stochastic discount factor.
on impact, but becomes more severe several quarters after the shock, as the larger reduction in financial capital needed to stabilize consumption creates tighter financing conditions. On impact, a larger adjustment is avoided but this stability is obtained at the cost of a much deeper recession.

As shown by the blue dotted line, the adjustment is much quicker in the model without habits. If consumption smoothing is no longer a key priority, agents choose to absorb the effect of the shock by adjusting consumption and investment hardly reacts in this case. On impact, the tightening of the financing-in-advance constraint is more severe but as shown by the bottom right panel, the stock of financial capital need not adjust so dramatically in this case and the reduction in liquidity production is less pronounced. The fact that the safe asset shortage is less severe in this case implies that financing conditions improve much more rapidly in the model without habits, as can be seen by comparing the dynamics of $\mu/\lambda$ shown in the top right panel across the two cases.

**Simulating the effect of a large transitory shock**

The model with asset quality shocks that is calibrated using pre-Great Recession data is able to match some key characteristics of the eurozone business cycle and generates a large welfare cost of business cycle fluctuations. Given the pre-crisis values for the model’s main structural parameters, another test is to compare the model’s predictions with crisis data in order to evaluate its ability to generate recessions of the magnitude observed during the financial crisis period.

In the context of my model, a financial crisis can be interpreted as a shock that reduced the quantity of assets that lenders accept as collateral. The shock’s standard deviation is calibrated to simulate the effects of a shock, which on impact leads to a 1.5 percent unexpected reduction in the quantity of safe assets than can be used as collateral. The persistence parameter $\rho_Z$ is set to 0.7, which implies that the effects of the asset quality shock on the stochastic depreciation rate are fully dissipated after four years. More specifically, this implies that the stochastic component of the depreciation rate stands at 1.05% one quarter after the shock, at 0.5% one year after the shock, and is back to zero about four years after the shock. Over the 16-quarter period, the calibration of the shock implies a cumulated impact of the shock on the safe asset stock of about 5%, or 1.25% per year.11

The response of the total depreciation rate $\tilde{\eta}_t$ to this particular asset quality shock

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11 Abstracting from new purchases and from the deterministic component $\eta_t$ after 16 quarters, we have that:
\[ \tilde{n}_t = \eta + \log z_t, \]

is depicted by the blue line in Figure 9, where the stochastic component is denoted by \( \log z_t \). In Figure 9, the red dotted line shows the same experiment except that it is in the case of a shock that reduces the stock of collateral by 2.5% on impact and which implies a cumulated impact on the total safe asset stock over the 16 quarter period of about 8%, or 2% per year.

Figure 10 shows the corresponding response of output growth and compares it to the data.\(^{12}\) The blue continuous line shows the effect of the shock on year-on-year output growth under the first experiment. On impact, the negative financial shock reduces the stock of safe asset by 1.5% and leads to a decline in year-on-year output growth by about two percent. As discussed above, the hump-shaped response of output generated by these types of shocks implies that the maximum effect occurs with a delay. In the case of a shock that has a cumulated impact on the safe asset stock over the 16 quarter period of 1.25% per year, the maximum effect on output occurs after one year and leads to a decline in year-on-year output growth of about 3%. The red dotted line shows the effect of the shock on output under the second experiment. A shock that leads to a reduction in the stock of collateral accepted by banks of 2% per year over a 16-quarter period has a maximum impact on output growth that leads to a year-on-year decline of about 4.5% one year after the shock hit.

### 6 Additional implications

Another key stylized fact observed during the crisis period is that the price of high-quality assets that were not directly affected by the mass downgrades increased durably.

\[ s_{16} = (1 - \tilde{n}_{16})(1 - \tilde{n}_{15}) \ldots (1 - \tilde{n}_1)s_0 \]

where:

\[ \frac{s_{16} - s_0}{s_0} \simeq 5\% \]

The cumulated impact of the shock implies a decline in the safe asset stock after four years of about 5%, which corresponds to an annual rate of 1.25%.

\(^{12}\) where year-on-year output growth in the model and in the data is calculated as follows:

\[ g_{y,t-4} = (\log(y_t) - \log y_t(-4))100 \]
As discussed in section 4, while the initial decline in the risk-free rate shown in Figure 4 is consistent with this observation, the persistent increase that is obtained afterwards seems more difficult to reconcile with the facts reported in Gourinchas and Jeanne (2012); for instance, suggesting that the crisis triggered a persistent decline in the risk-free rate. In Gourio (2012) for example, the price of a short-term bond that is in zero net supply increases during recessions, which is consistent with the flight-to-quality effect documented in the literature.

As shown by a recent literature that studies how money-like convenience services affect the valuation of assets that are used as pledgeable collateral (e.g., Greenwood and Vayanos 2010; Krishnamurty and Vissing-Jorgensen 2011, 2012; Caballero and Fahri 2014), in the case that I am considering, it is possible to derive a measure of the flight-to-quality effect that differs from the textbook formula shown in equation (20).

\[ p_{Bt} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \]  

(20)

If an asset provides liquidity services, and as illustrated by the asset pricing equation of the safe asset shown in equation (18), the pricing equation depends on an additional term that captures the effects of the tightness of the collateral constraint on the valuation of a money-like asset. To illustrate the impact of this effect on the pricing equation of a risk-free bond, I adjust the textbook formula derived above to study how money-like convenience services affect the flight-to-quality effect in my model. The price of the corresponding one-period risk-free bond can be expressed as follows:

\[ \tilde{p}_{Bt} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{1 - \omega \frac{\chi_t}{\lambda_t}} \]  

(21)

where \( \chi_t \) is the Lagrange multiplier on the collateral constraint. This pricing equation could be justified by assuming the existence of a one-period risk-free bond in zero net supply that is not subject to asset quality shocks. In this case, the collateral constraint faced by the agent is given as follows:

\[ l_t \leq \gamma \omega (p_{St} s_{t+1} + \tilde{p}_{Bt} b_{t+1}) \]

Figure 11 shows the response of the price of a risk-free bond implied by the textbook formula shown in equation (20) and compares it with the price of a corresponding one-period bond whose value has been adjusted for collateral service, as in equation (21). Once this effect is taken into account, and as illustrated by the blue continuous line,
it is possible to generate a persistent increase in bond prices in response to a negative financial shock. Once the money-like feature of an asset is taken into account, the price of a risk-free bond that is not directly affected by a negative asset quality shock therefore increases, an implication that is consistent with the flight-to-quality effect documented in the literature.

**Are the financing-in-advance and collateral constraints always binding?**

The advantage of solving the model using perturbation methods (e.g., Adjemian et al. 2014) is that this solution method is time efficient. This is particularly important in my case since I need to simulate the model a very large number of times in order to find the minimum of a loss function that contains eight moments. The disadvantage of this solution method is that it requires differentiability and therefore that it cannot handle discontinuities such as occasionally binding constraints, for instance. Since occasionally binding constraints can be an issue in models with financial frictions, Figure 12 below plots the value of $\mu/\lambda$ and $\chi/\lambda$ that I obtain in a sample of 100,000 observations. As can be seen from Figure 12, negative values for one of these Lagrange multipliers were never observed, suggesting that the case of a negative Lagrange multiplier is very unlikely to happen under this calibration. This confirms that solving the model under the assumption that the financing-in-advance and collateral constraints are always binding is a valid strategy.

**Estimating financial and technology shocks using Bayesian techniques**

The deep structural parameters of the model have been estimated using pre-crisis data and under the assumption that financial shocks were the only source of business cycle fluctuations. Given values for these structural parameters, another question is whether financial shocks can still play a relevant role once these shocks are set to compete against technology shocks. To answer this question, I estimate the shock processes of financial and technology shocks with Bayesian methods using consumption and output growth as observable variables.\(^{13}\) All other parameters are kept at the pre-crisis values that I obtained using the procedure described in section 3.

As illustrated by Figure 13, it is reassuring to see that this procedure explains part of the financial crisis with a large increase in the rate at which the stock of safe asset depreciates, which corresponds to the case of a negative financial shock. The estimated

\(^{13}\)Since I have two observable variables and two shocks, and given the structure of the model, this procedure will produce shock processes that will allow the model to perfectly match the dynamics of consumption and output observed in the data.
shock persistence and standard deviation is 0.985 and 0.0046, respectively, and over the sample period 1995 Q1 to 2015 Q2, financial shocks still explain about half of the fluctuations in output growth once technology shocks are introduced into the analysis.

**Technology shocks**

The response of output, consumption, investment and hours worked in the non-financial sector to a negative total factor productivity shock are shown in Figure 15. The response of equity prices, the safe asset price, the risk-free rate and the safe asset stock are shown in Figure 16. While the co-movement between macroeconomic variables and equity prices observed during the financial crisis could be explained by a negative technology shock, the sharp increase in the risk-free rate obtained in this case is more difficult to reconcile with the data. The increase in the risk-free rate obtained in the case of technology shocks is so strong that even if the pricing equation of the corresponding one-period risk-free bond is adjusted for money-like convenience services, as performed above in the case of financial shocks, it would still not be possible to generate a flight-to-quality effect. Relative to the case of financial shocks, a by-product of this spike in the risk-free rate is the durable fall in the safe asset price obtained in the case of a negative technology shock.

7 Conclusion

Most of the literature initiated by the seminal contribution of Bernanke, Gertler, and Gilchrist (1999) has focused on the role played by the net worth of firms and households in amplifying business cycle fluctuations. In comparison to the vast literature on the financial accelerator, the quantitative implications of models in which constraints affecting consumption decisions are the main source of friction remain largely unexplored. This article addresses this question by studying the case of an economy in which liquid financial instruments have a role in facilitating access to credit. In such an environment, my first main finding is that small asset quality shocks have the potential to explain the co-movement between macroeconomic and financial market variables observed during the crisis period. With this endogenous propagation mechanism, small fluctuations in asset quality also have the potential to generate business cycle fluctuations of a magnitude that is empirically plausible, while at the same time providing a solution to standard asset pricing puzzles.
8 References


Figure 1: Model implied financing-in-advance (FIA) wedge (blue continuous line) and output (red dotted line). Q/Q growth rates 2000Q2-2014Q14.

Figure 2: Euro area bank lending survey: consumer credit and other lending. 2003Q1-2014Q14.
10 Appendix B

Figure 3: Impulse response of output, consumption, investment and hours worked in the non-financial sector to a negative financial shock. y axis: log-deviation from steady state. x axis: quarters after the shock.

Figure 4: Impulse response of equity prices, the safe asset price, the risk-free rate and the safe asset stock to a negative financial shock. y axis: log-deviation from steady state. x axis: quarters after the shock.
Figure 5: Impulse response of output to a technology and financial shock.

Figure 6: Impulse response of output to a financial shocks. Benchmark model vs. low adjustment case.
Figure 7: Impulse response of output, the financing in advance wedge, consumption and financial capital to a negative financial shock. The red continuous line shows the responses for the benchmark calibration. The blue dotted line shows the no habit case.
Figure 9: Calibration of a transitory asset quality shock that destroys 1.5% and 2.5% of the existing safe asset stock on impact.

Figure 10: Response of year-on-year output growth to a transitory asset quality shock that destroys on impact 1.5% and 2.5% of the safe asset stock.

Figure 11: Impulse response of the price of a risk-free bond (red dotted line) vs. bond price.
adjusted for collateral services (blue continuous line).

Figure 12: Simulated data for $\mu/\lambda$ and $\chi/\lambda$, 100'000 observations.

Figure 13: Estimated financial shock in the model with financial and technology shocks for the sample period 1995q1-2015q1
Figure 15: Impulse response of output, consumption, investment and hours worked in the non-financial sector to a negative technology shock. y axis: log deviation from steady state. x axis: quarters after the shock.

Figure 16: Impulse response of equity prices, the safe asset price, the risk-free rate and the safe asset stock to a negative technology shock. y axis: log deviation from steady state. x axis: quarters after the shock.
11 Appendix C

Data appendix to section 3

\[ \xi = \frac{E(c_t)}{E(l_t)}, \quad \varpi = \frac{E(l_t)}{\gamma E(p_{St, t+1})} \]

\[ E(p_{Sx}/y) = 0.114 \]

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<th>Description</th>
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<td>Stat. Office of the EC 1995q1-2008q3</td>
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<td>Final consumption expenditures</td>
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<td>Safe asset investment flow, (p_{Sx})</td>
<td>Investment in ST liquid asset by households</td>
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Data appendix to section 4

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$^{14}$Secured against specific assets or receivables (ABS), mortgages (MBS) or cash flows from a whole business segment (Whole Business Securitizations) in each case via a special purpose vehicle.
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