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Michal Andrle, Jan Brůha, Serhat Solmaz On the sources of business cycles: implications for DSGE models



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Abstract

What are the drivers of business cycle fluctuations? And how many are there? By documenting strong and predictable co-movement of real variables during the business cycle in a sample of advanced economies, we argue that most business cycle fluctuations are driven by one major factor. The positive co-movement of real output and inflation convincingly argues for a demand story. This feature—robust across time and space—provides a simple smell test for structural macroeconomic models. We propose a simple statistic that can compare data and models. Based on this statistic, we show that the recent vintage of structural economic models has difficulties replicating the stylized facts we document.

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DSGE models.

Nontechnical Summary

In this paper we investigate sources of economic fluctuations—their number, their nature, and their implications for economic modeling. Our empirical approach allows us to reach strong conclusions with relatively modest identification assumptions: we apply a dimension-reduction technique—dynamic principal component analysis—on data of advanced economies. We focus exclusively on business cycle frequencies, without any intention to explain long-run trends in the data or high-frequency fluctuations.

We consider real GDP, real consumption, real investment, real exports, real imports, the unemployment rate, and core inflation. We use median inflation as our preferred measure of inflation. Median inflation eliminates outliers and diminishes high-frequency variation without ex-ante eliminating particular components of the consumer basket. We present results for the U.S. and summary statistics for the cross section of advanced countries.

We carry out our analysis both in the time and in the frequency domain. In the time domain, we use statistical filters to isolate the part of the time series that corresponds to the frequencies of interest and then the common component is identified using the standard dynamic principal component filter. The results in the time domain can be transparently illustrated using intuitive graphs. In the frequency domain, we create the dynamic principal components using frequency domain filtering directly and ask how much the first or second dynamic principal components fit the spectral density. The analysis in the frequency domain is immune to criticism of pre-filtering of the time series by statistical filters. Both exercises carry the same message.

There are three main messages of our analysis. First, we document great regularities in business cycle co-movements of key macroeconomic variables across multiple economies. Our dynamic principal component analysis of the data identifies that there is one dominant source of real co-movements, typically explaining more than two thirds of cyclical fluctuations. We conclude that the business cycle dynamics of key macroeconomic data can be largely, although not completely, explained by a single source of variation.

Second, the analysis of both real variables and inflation confirms their tight co-movement and allows us to venture an interpretation of the dominant principal component as a 'demand factor.' Attention to the treatment of inflation dynamics is essential to our results on output-inflation co-movement. Low-frequency developments of inflation linked to changes in long-term inflation expectations are complemented with cyclical dynamics by and large due to demand shocks. Using the log of the price level or inflation itself would not acknowledge the often time-varying nature of long-run inflation expectations or inflation targets and might elicit concerns about stationarity of the series. Using the first difference of inflation in the analysis, however, leads in our view to a misspecification which prevents uncovering the Phillips curve relationship in the data, as the cyclical frequencies are suppressed and the high frequencies emphasized

Third, the results of our agnostic analysis carry implications for theoretical economic models regarding the number of shocks and the properties of a dominant structural shock. Independent of all structural interpretations of our finding, it is clear that any structural economic or econometric model of business cycles must be able to generate the principal component structure that we present. In this sense the principal-component space of the data is a very strong testable restriction. We argue that the recent vintage of structural economic models fails this test—these models cannot explain business cycle dynamics.

1. Introduction

What are the drivers of business cycle fluctuations, how many are there, and what are the implications for structural models? This paper attempts to shed more light on these perennial and profoundly difficult questions. It illustrates that most business cycle fluctuations in advanced and some emerging economies are driven by a single major source. We label it the 'demand shock,' due to its properties. We document that the strong and predictable co-movement of real variables during the business cycle is well explained by a single unobserved principal component. The positive co-movement of real output and inflation, reminiscent of the 'Phillips correlation,' convincingly argues for a demand story, not for the technology-driven fluctuations of the Real Business Cycle (RBC) theory. While both the demand and technology-shock-driven business cycle hypotheses may be consistent with one dominant source of co-movement of real variables, the strong co-movement of the dominant component with inflation is a decisive piece of evidence that argues for a demand-driven explanation.

The results of our analysis bear important consequences for structural macroeconomic models in terms of the nature and number of driving forces needed to reconcile the models with observed data. Our results suggest that the structure of empirical macro-models, notably of Dynamic Stochastic General Equilibrium (DSGE) models, should imply that only a few structural shocks drive the dynamics of the model at business cycle frequencies, with one shock being the dominant one. Of course, many other shocks contribute to the overall dynamics and specific episodes. Essentially, the stochastic singularity of many macro models, though currently often considered a vice, seems to be a virtue if handled appropriately. Further, the response of the economy to the dominant shock must result in very tight positive co-movement between most GDP components, employment, and inflation. In this paper, we document that most prominent DSGE models today are not compatible with our empirical findings on the number of factors and the nature of co-movement in the macroeconomic data.

Our findings thus constitute a powerful smell test for DSGE model misspecification and should prove useful in further model development.

The co-movement of macroeconomic variables is surprisingly strong and stable across countries and in time. As most practitioners and policymakers know, it simply does not happen that investment plummets while private consumption remains resilient or rallies during a recession. Again, it does not happen that the unemployment rate drops when output slumps. It just does not happen, except in many DSGE models. Yet, what may not be clear from the outset—given all the buzz about the Great Moderation, the Great Turbulence, stochastic volatility, and regime switches—is the surprising degree of business cycle fluctuation stability in time and across economies that we document in this paper. In short, we confirm the argument of Cochrane (1994) that business cycles are 'all alike' in many important ways. As Kindleberger and Aliber (2005) also suggest, despite each individual crisis or cycle being a product of a unique set of circumstances, the more things change, the more they stay the same.

Our empirical approach boils down to multi-country dynamic principal component analysis of data at business cycle frequencies. We focus exclusively on business cycle frequencies, with no intention to explain long-run trends in the data, or every high-frequency wiggle. We use non-parametric spectral analysis to estimate dynamic principal components or—with a slight abuse of terminology—factors present in the data. We demonstrate that the first dynamic principal component itself can

¹ Henceforth, we use the terms 'factor' and 'component' interchangeably unless stated otherwise.

explain up to 80% of the business cycle variation in real macroeconomic aggregates across a variety of countries. Despite the frequency domain nature of the analysis, we present most of our results in the time domain, using simple and intuitive charts. Our empirical strategy is most closely related to the index (factor) model by Sargent and Sims (1977) and the investigations of Burns and Mitchell (1946) on the nature of the 'reference cycle.'

There are three original contributions of this paper in our view. First, we document great regularities in post-war business cycle co-movements of key macroeconomic variables across multiple economies. Going beyond cross-correlations, our dynamic principal component analysis of the appropriately transformed data identifies that there is usually one dominant source of real co-movements, typically explaining more than two thirds of cyclical fluctuations. Second, the analysis of both real variables and inflation reveals their tight co-movement—often doubted in the literature—and allows us to venture structural identification of the dominant principal component as a 'demand factor.' The use of inflation—instead of the price level—and its deviations from the trend or long-term inflation expectations is a key ingredient for our results. Third, the results of our agnostic analysis carry important implications for theoretical economic models regarding the number of shocks and properties of a dominant structural shock in a way that, to our best knowledge, has not yet been demonstrated.

We anticipate three major possible objections to our analysis. First, one may dismiss the results as obvious, trivial, and known to everybody. Second, one could view our results as spurious, a result of creative statistical analysis, and continue to believe there is little co-movement of output and inflation, since the Phillips curve's slope is deemed flat or varying in time. And third, we expect a claim that our tools are too simple and we should use a parametric, estimated—and why not a DSGE—model. We do our best to dispel all these objections through the paper. For readers with the view that an argument in favor of demand-driven business cycles is redundant since it is all obvious, the heavy focus of the literature on various flavors of technology shocks and models incapable of producing positive co-movement of consumption and investment may come as a surprise. For instance, a prominent textbook by Gali (2008) does not feature a single model with an impulse-response function resembling aggregate demand fluctuations. For those who believe that demonstrating positive co-movement of output and inflation is impossible, we demonstrate what assumptions and data analysis are needed to avoid the impression that the Phillips curve is in flux. To convince the reader that our results are not just a statistical fluke or data mining, we provide sensitivity analysis of our computations. Finally, this paper does not employ an explicit estimated DSGE model or the like, since our goal is to use data and a minimal set of assumptions to obtain strong implications for testing and falsifying structural models themselves.

The structure of the paper is the following: in the next Section 2 we place our investigation into the context of economic research. In Section 3 we introduce and discuss the methods used in the paper. In Section 4 we describe the results for the U.S. and summarize evidence for the rest of the countries in our sample. In Section 5 we assess the implications of our results for macroeconomic modeling, and in Section 6 we conclude. Additional materials, such as non-core graphs and sensitivity and robustness checks, are included in the Appendix.

There is a related paper to this research: Andrle et al. (2016). The difference between the two papers is that in this one we put more emphasis on the implications of our findings for structural macroeconomic models. On the other hand, Andrle et al. (2016) is more exploratory and the reader is referred to it for a detailed description of more countries and for various sensitivity analyses.

2. Related Literature

Our paper is related both to the literature that seeks to identify the sources of business fluctuations and to the research on empirical testing of structural macroeconomic models. This section places our paper in the context of both streams of literature.

2.1 Related Literature on the Sources of Business Cycles

Despite the voluminous literature focused on the sources of business cycles, there seems to be no clear consensus. There is no clear consensus on the number of drivers of business cycles or on their nature, namely, whether they are more in line with the real business cycle (RBC) tradition or with aggregate demand and expectations shocks. Cochrane (1994) carried out an extensive exercise and concluded that there is not enough evidence that economic fluctuations are caused by popular candidates (technology, money, oil, credit). Similarly, Rebelo (2005) overviews a list of possible causes of business cycle fluctuations, centered on the RBC theory. Shapiro and Watson (1988) carry out a thorough empirical investigation, concluding that 'aggregate demand accounts for between 20 and 30 percent of the variation in output at business cycle horizons.' After the Great Recession events of 2008 onwards there has been a revival of theories linking cyclical fluctuations to financial or risk shocks and credit creation; see Christiano et al. (2014), for instance. Recently, excess optimism, changes in agents' confidence, and self-fulfilling prophecies have been proposed as sources of fluctuations; see Angeletos et al. (2014) as an example.

Empirical literature often points to a real-nominal 'dichotomy,' meaning that real shocks mostly drive real variables and nominal shocks nominal ones. This is quite a frequent finding of DSGE models—see for instance Smets and Wouters (2007b) or Justiniano et al. (2010), where inflation is dominated by cost and wage-push shocks. There are three important papers dealing with a question similar to the one we ask, and we thus build on their methodology—Sargent and Sims (1977), Giannone et al. (2005), and Kydland and Prescott (1990). In the two first papers the authors estimate dynamic factor models in the frequency domain and investigate co-movement among a subset of real and nominal variables. In both papers the authors reach the conclusion that there are two major sources of economic dynamics—a real shock driving real variables and a nominal shock, orthogonal to the real one, driving nominal variables. Sargent and Sims (1977, pp. 68) reach a conclusion that 'one index [factor] model ... delivers high coherences for all of the real variables except business formation ... and low coherences for the price indexes ... But adding the second index results in high multiple coherences for the two prices ... [such that] the coherences for the other real variables remain about as they were with one-index [model].' Giannone et al. (2005) conclude that U.S. macroeconomic dynamics are driven by two shocks. GDP and other real variables are driven by the real shock, the output deflator is driven mainly by the nominal shock, and the Phillips curve relation is weak, according to the paper. We are inspired by, and revisit some of the analysis in Kydland and Prescott (1990). The crucial difference is the treatment of nominal variables—the price level versus inflation. The authors focus on cyclical dynamics of the price level, not inflation, and use the fact that the price level is counter-cyclical as an argument in favor of the real business cycle theory. However, pro-cyclical inflation, lagging behind the activity cycle, is consistent with a counter-cyclical price level.

We believe that the results of our analysis have implications both for the number of shocks driving business cycles and for the real-nominal dichotomy. We present evidence for one dominant shock that drives business cycles, and this shock moves real variables and inflation in the same direction. Our results therefore do not confirm the real-nominal dichotomy and support the short-run Phillips correlation. Two of the reasons for our different results are the transformation of variables—we use

inflation instead of the price level, for instance—and the focus on business cycle frequencies. In an inflation-targeting economy, output should be related to the deviation of inflation from its target, or from long-term inflation expectations, if there is a 'Phillips correlation' in the economy. This would be so for demand shocks in New-Keynesian DSGE models. Our analysis acknowledges that low-frequency dynamics of inflation, due to an explicit or implicit inflation target, are a factor of their own, and inflation in relation to its trend or the inflation target is the variable of interest. Both inflation and interest rates are tied to the explicit inflation target and long-term inflation expectations.

2.2 Related Literature on Testing Structural Macroeconomic Models

Given the prominence of macroeconomic structural models in contemporary macroeconomics, it is not surprising that there is a huge amount of papers that aim at evaluating and testing structural macroeconomic models such as DSGE models. These papers differ in the features tested and also in the underlying metrics. The features examined can range from testing assumptions of trends, through forecast accuracy relative to simple statistical benchmarks, to selected features of the data. There are various metrics used, from very abstract ones such as Bayes factors through to very transparent ones that can be easily summarized and that directly reveal where a potential problem with a particular model is.

In this paper, we concentrate on a particular feature—co-movement along business cycles: we aim at evaluating whether structural models are capable of generating strong co-movements at cyclical frequencies. Although we do not question that correct specification of the trend component is important for practical forecasting, DSGE models are primarily models of business fluctuations, not models of long-run growth determinants, hence cyclical frequencies should be of main interest.

We are also close to papers that summarize their findings using transparent statistics. We would like to highlight the potential problems with the recent vintage of DSGE models, so we present our conclusions using pictures and simple summary numbers. We believe that this is more informative than providing abstract metrics such as likelihood ratio statistics or Bayes factors. Two papers are therefore especially close to our paper. Herbst and Schorfheide (2012) propose a method for evaluating DSGE models using co-movements of macroeconomic variables. Similarly to us, they consider the application of the statistics of interest both to actual data and to model-simulated data. The difference is that they concentrate on forecast densities and, more importantly, they are interested in growth rates. We instead are interested in the cyclical implications of structural models, so our statistics are directly targeted at the frequencies of interest.

Faust and Gupta (2012) is another paper close to ours. They use posterior predictive analysis to check the implied properties of estimated DSGE models. Similarly to us, they conclude that canonical DSGE models have problems with generating strong co-movements in real variables. Contrary to us, they investigate the implied correlations of growth rates; in our research we show that using proper transformation of variables is even more revealing about the co-movement.

3. Empirical Models and Methods

Our main tool is dynamic principal component analysis (DPCA), which can be applied both to data and to a model (either directly to the model's reduced form or indirectly to simulated data). DPCA is based on the seminal work of Brillinger (1981), which was popularized in empirical macroeconomics by Forni et al. (2000). DPCA is a dimensionality reduction technique which is essentially based on eigenvalue decomposition of the spectral density. At the same time, it can be reverted back to the time domain, which results in a two-sided filter that can be used to filter the common component. Somewhat more formally, DPCA aims at decomposition of observed time series $x_{i,t}$ using the following representation:

$$x_{i,t} = \chi_t + \xi_{i,t},$$

where $x_{i,t}$ is the observed series, χ_t is the low-dimensional common component, and $\xi_{i,t}$ is the idiosyncratic noise, which is uncorrelated with the common component χ_t , and only 'weak' correlation among elements of ξ_t is allowed. A set of K time series is fully explained by K principal components, with potentially a small number of principal components explaining most of the dynamics.

Frequency domain DPCA starts with the estimation of the multivariate spectral density $\Sigma^{\chi}(\omega)$ of the observed process x_t , from which the spectral density of the common component χ_t is obtained by selecting dominant eigenvalues. By selecting the dominant eigenvalues at each frequency, one estimates the spectral density $\Sigma^{\chi}(\omega)$ of the common component. This is essentially a frequency domain filter. This frequency domain filter can be reverted back to the time domain to obtain a two-sided filter that relates the observed series to the common component:

$$\chi_t = \sum_{l=-L}^{L} \Lambda_l x_{t+l},\tag{1}$$

where $\{\Lambda_l\}_{l=-L}^L$ are the weights of the time domain filter.² For L>0, the resulting time domain filter can easily account for the lead-lag relationship among the variables (such as is the case for unemployment and output cycles).

L > 0 implies that the filter is two-sided and that the common component cannot be inferred at the beginning and at the end of the sample. Nevertheless, the two-sided nature of the filter for the common component in the time domain is not a big issue for us since we are not interested in real-time forecasting but in ex-post analysis of the data. Therefore, we stick to the two-sided formulation as in Forni et al. (2000).

We present the results for DPCA in both the time and frequency domain. In the *time domain*, we isolate cycles using the band-pass filter (Fitzgerald-Christiano) and the high-pass Hodrick-Prescott filter and then we apply the time domain filter (1) to such isolated cycles. We chose L=2 as the filter can account for lead-lag relationships in the data. The lead-lag relationships of DPCA increase the fit of the model, ⁴ although for our data the gain in fit is not dramatic: if we applied static principal

 $[\]overline{{}^2}$ Ideally, one would choose $L = \infty$, which is obviously infeasible in practice. However, for a typical example, filter weights with a finite and 'small' L give a very accurate approximation of $\{\Lambda_l\}_{l=-\infty}^{\infty}$.

³ In fact, in our empirical analysis we use exactly the same approach in estimating the multivariate spectral matrix (the Bartlett non-parametric approach with the same setting of the smoothing window) as described by Forni et al. (2000).

⁴ This is obviously true in large samples. In small samples, it may happen that static principal component analysis fits the data better than DPCA because of imprecise estimation of the spectral density.

component analysis to our data, the results and implications would be qualitatively unchanged with a slightly lower fit. The main reason for using dynamic PCA is the time shift of unemployment with respect to output (Okun's law) and of inflation and interest rates if included in the computations.

To measure co-movement in the time domain, we use the statistics introduced by Stock and Watson (2002). In particular, let χ_{it}^k be the common component for the series x_{it} estimated using k first dynamic principal components based on the time domain filter (1). Our preferred statistic is the analogy of the \Re^2 statistic of linear regression:

$$\Re^{2}(k) = 1 - \frac{\sum_{t=1}^{T} (x_{it} - \chi_{it}^{k})^{2}}{\sum_{t=1}^{T} (x_{it} - \bar{x}_{i})^{2}},$$
(2)

where \bar{x}_i is the sample mean of x_{it} .

The frequency domain representation—as already mentioned—is centered on the multivariate spectral density $\Sigma^x(\omega)$. Let $\{\lambda_{(i)}(\omega)\}_{i=1}^n$ be ordered eigenvalues of $\Sigma(\omega)$ at frequency ω . Since $\Sigma(\omega)$ is positive semi-definite for each frequency ω , all eigenvalues are non-negative. Therefore, for a stationary time series, Y, we consider the following statistics:

$$\mathscr{S}_{Y}(\boldsymbol{\omega},k) \equiv \frac{\sum_{i=1}^{k} \lambda_{(i)}(\boldsymbol{\omega})}{\sum_{i=1}^{n} \lambda_{(i)}(\boldsymbol{\omega})},\tag{3}$$

which intuitively tells us the percentage of the variability explained by the k principal components at frequency ω .

The computation of the spectral density estimate using raw, unfiltered data is a subtle issue, since some of our macro variables are non-stationary. When working with non-stationary data, spectral estimates cannot be carried out without some modification. We use the non-parametric Bartlett approach on first log differences (when meaningful), which renders the problem stationary. This does not pose a problem for the measure (3), as it is invariant with respect to first differencing all series. Indeed, it can be shown that:

$$\mathscr{S}_{Y}(\omega, k) = \mathscr{S}_{\Lambda Y}(\omega, k), \tag{4}$$

for all ω , such that both sides are defined.⁵ It implies that for non-stationary I(1) time series, the statistics (3) can just be estimated for first differences of series and this holds for all $\omega \neq \pm 2\pi n$, where $n \in \mathbb{N}_+ \cup 0$. Moreover, some other statistics of interest, such as coherence, also remain unchanged if both series are pre-processed by the difference filter. Formally, if $\mathcal{C}_{x,y}(\omega)$ is the coherence between series x and y, then it holds that:

$$\mathscr{C}_{x,y}(\boldsymbol{\omega}) = \mathscr{C}_{\Lambda x,\Lambda y}(\boldsymbol{\omega}),$$

for all ω for which both expressions are defined.⁶

The analysis turns out to be robust with respect to whether our computations are carried out in the time or frequency domain. In the frequency domain, we create the dynamic principal component using frequency domain filtering and ask how much the first or second dynamic principal components

⁵ This statement can be easily generalized: the equivalence would hold if each of the series was pre-filtered by the same linear filter. This follows from the easily seen fact that such filtering would scale up all eigenvalues of the spectral density matrix by the same number.

⁶ See Koopman (1974, pp. 149).

fit the spectral density. This information can also be represented by revealing plots, though in the frequency domain. The analysis in the frequency domain is immune to criticism of pre-filtering of the time series by statistical filters. That said, it is useful to note that the often-heard opinion that the use of statistical filters, say the Hodrick-Prescott filter, always causes spurious cycles is misguided; see Pollock (2013), who proves that 'this idea is largely mistaken.'

For each country in our sample, we consider the following set of variables: real GDP, real consumption, real investment, real exports, real imports, the unemployment rate, and the short-term interest rate. The common component based on first principal component analysis is then projected onto the cyclical dynamics of inflation.

For our goals, it is crucial that we ask how the cyclical dynamics of real variables are related to the cyclical dynamics of inflation. We do this again in two ways. In the time domain, we compare the dynamics of the first dynamic component to the dynamics of the deviation of inflation from its trend (henceforth called the *inflation cycle*). We compare the dynamics of the inflation cycle with the output cycle. In the frequency domain, we compute and report the coherence between inflation and output as well as between inflation and the isolated first dynamic component. We employ median inflation as our preferred inflation. Median (or, more generally, trimmed-mean) inflation eliminates outliers and lowers high-frequency variation without ex-ante eliminating particular components of the consumer basket. However, with the exception of the U.S. and Australia, we have to construct our own median inflation measures with data available only from the early 1990s using the Haver Analytics database.

So, why don't we put inflation directly into the dynamic principal component model? The only reason is that the median inflation data for most countries span a much smaller sample size than the macroeconomic data on other variables, which would restrict our analysis too much. This is why we choose to compare the inflation dynamics with the common component estimated on real variables instead. Inflation, therefore, does not affect the estimates of the unobserved principal components. Nevertheless, we can do this for the U.S. and we present the result, which supports our conclusions.

Our results do not confirm the real-nominal dichotomy and support the Phillips correlation. Two of the reasons for our different results are the transformation of variables—we use inflation instead of the price level, for instance—and the focus on business cycle frequencies. In an inflation-targeting economy, output should be related to the deviation of inflation from its target, or from long-term inflation expectations, if there is a 'Phillips correlation' in the economy. This would be so for demand shocks in New-Keynesian DSGE models. Our analysis acknowledges that low-frequency dynamics of inflation, due to an explicit or implicit inflation target, are a factor of their own, and inflation in relation to its trend or the inflation target is the variable of interest. Both inflation and interest rates are tied to the explicit inflation target and long-term inflation expectations.

4. Main Empirical Results

In this section we document the strong co-movement among the cyclical components of the main macroeconomic variables and inflation. We show this for the United States and for the cross section of advanced countries. The United States is an obvious choice for its 'benchmark' status earned by the size of the economy and the length and quality of the statistical data. In the accompanying

⁷ Andrle et al. (2013) show this point using euro area data. Elimination of high-frequency variation using median inflation (i.e., the extreme case of trimmed means) has also been suggested by Meyer and Zaman (2013) in the forecasting context.

paper (Andrle et al., 2016), the interested reader may find evidence also for Germany and Japan. Moreover, the accompanying paper contains a set of robustness tests related to the sample size and the method for extracting the cycles in the time domain.

4.1 The United States

In the case of the U.S. economy, our empirical findings are the most robust ones. Figure 1 clearly demonstrates in the time domain that the first dynamic principle component can explain a large proportion of the variation of the business cycle in the U.S. Virtually every cyclical component of GDP, with the exception of real exports and short-term interest rates, is more than 80% explained using a single dynamic principal component. In the case of the short-term interest rate this is due to the fact that monetary policy is not easily described as following some sort of pro-active Taylor rule in the late 1980s to early 1990s. The case of exports is different, since U.S. exports are the imports of their trading partners and thus should be well approximated by the trade-weighted linear combination of the explained import components of partner regions, and in our analysis they are put mostly into the second dynamic principal component.⁸

In the frequency domain—without pre-filtering in the time domain—the results hold as well. Figure 4 shows the proportion of the spectral density explained by the first two dynamic principal components over the whole range of frequencies. Apparently, the fit of the spectral density using one principal component over the business cycle is particularly good for imports and investment. For exports, one needs the second principal component, which makes the fit of the spectral density of exports almost perfect over business cycles.

We present the results for both the Christiano-Fitzgerald and Hodrick-Prescott filters. The key difference is that the HP filter does not exclude high data frequencies and the filter cutoff between low and cyclical frequencies is not as sharp as for the Christiano-Fitzgerald band-pass filter. The results for the HP filter (in both the time and frequency domain) show that the results also hold for data pre-filtered by this popular filter in both the shorter and full sample; see Figure 2 and Figure A2.

These results are not affected much by extending the sample to before the 'Great Moderation' episode. We estimated the DPCA model for the data since 1955 and featuring two periods of what most economist agree is different volatility of macroeconomic aggregates in the U.S.—a period of volatile business cycles, followed after the mid-1980s by the Great Moderation, which was abruptly put to an end by the Great Recession starting in 2007. The relative explanatory power of the first principal component is changed a little, with an expected deterioration of the short-term interest rate fit before 1985—an era of a volatile policy rate, the Gold Exchange Standard, and two major oil price shocks. The variance of the first principal component changes, but the filter loadings (the coefficients of the model) are constant. This means that the relative variances among the cycles of real variables did not change significantly either during the Great Moderation or during the recent Great Recession. The sample starts in 1955Q2 and ends in 2012Q4 (see Figure A1 in the Appendix). This simple calculation has potential consequences for the specification of models with time-varying co-

⁸ To investigate this hypothesis we used data from the IMF's Global Projection Model database and computed the implied export gap using constant trade weights and imports of China, the Eurozone, Emerging Asia, Japan, Latin America, and Remaining Countries. Figure A11 presents the results and suggests that more formal and detailed investigation of co-movements and spillovers could explain the data in a more comprehensive way. A multi-country restricted factor model is left for further research.

efficients, namely, that the stochastic volatility can be relevant but the dynamics driving the relative co-movement of the variables may be kept constant.⁹

A thorough consideration of inflation dynamics is key to our analysis and an important piece of evidence in favor of demand shocks. It is the explicit use of inflation—instead of the price level—and considerations about the implicit and subsequently explicit inflation target of the Federal Reserve that allow us to demonstrate the close co-movement of output and the deviation of inflation from the target. Central banks today operate in a regime rather closer to inflation targeting than to price-level targeting. Clearly, low-frequency movements of inflation are driven by perceptions of the inflation target, as embodied in long-term inflation expectations, or long-term nominal bond yields. However, the cyclical component of inflation is obtained using the band-pass and HP filter for consistency with the other countries in our baseline calculations. Using a measure of ten-years-ahead long-term inflation expectations¹⁰ though would lead to similar removal of the 'trend' process from inflation; see Figure A10 in the Appendix. The high-frequency dynamics of core inflation are lower than in the case of headline CPI, since our measure is the Cleveland Fed's median inflation.

Viewed through the lens of our analysis, there is little evidence for the nominal-real dichotomy in the U.S.: inflation lags behind the output cycle in a relatively stable and predictable way. The strength of the output-inflation co-movement can be recognized from Figure 3, which depicts the cyclical component of core inflation and the normalized first dynamic principal component (essentially the output cycle). The figure also shows the estimated coherence along with 95% confidence intervals between median inflation and output (and between median inflation and the first estimated dynamic component). Figure A3 in the Appendix then shows the results for the full sample. Unlike in the case of real variables, monetary policy conduct following Chairman Paul Volcker led to lower variance of inflation around long-term inflation expectations, which we adjusted by normalizing the series to the Great Moderation mean variance. Yet, apart from the amplitude change, the co-movement between inflation and real variables is preserved.

Our results thus indicate strong and stable co-movement between key real macro variables and inflation in the course of business cycle. The first dynamic component has such dominant explanatory power that we do not venture any identification of other types of macroeconomic disturbances. The positive co-movement of the dominant component (and output) with the inflation cycle motivates the labeling of the component as a 'demand factor' or demand shock. We do not observe the demand shock directly and cannot link it to particular events.

Let's not forget that data transformations are important for seeing clear results. If growth rates were used instead of a band-pass filter, the DPCA fit would deteriorate, which can be seen from Figure 5. The logic is clear as soon as one looks at the graph of the transfer function of the difference operator, 1-L, which amplifies high frequencies relative to business cycle and low frequencies. Nevertheless,

⁹ To check the robustness to the methodology chosen, we also redid the calculations using standard static PCA instead of DPCA. The co-movement among real variables is clearly visible even for static PCA, which does not allow for lead-lag relationships among the variables; see Figures A6 and A7.

¹⁰ 10Y-ahead long-term inflation expectations are obtained from the Survey of Professional Forecasters (SPF) at the Philadelphia Fed. The FRB/US measure of the implicit inflation target—variable PTR in the FRB/US model—can also be used as a proxy for the unobserved inflation target (thanks to Bob Tetlow for providing the data), as it reaches the sample before SPF 10Y expectations; see Andrle (2012) for an empirical analysis and demonstration of the consistency of the New-Keynesian expectational Phillips curve with the observed data dynamics. What our analysis also says is that while the cyclical dynamics around long-term inflation expectations seem to be driven by the economic cycle, the dynamics of long-term inflation expectations are a different issue altogether.

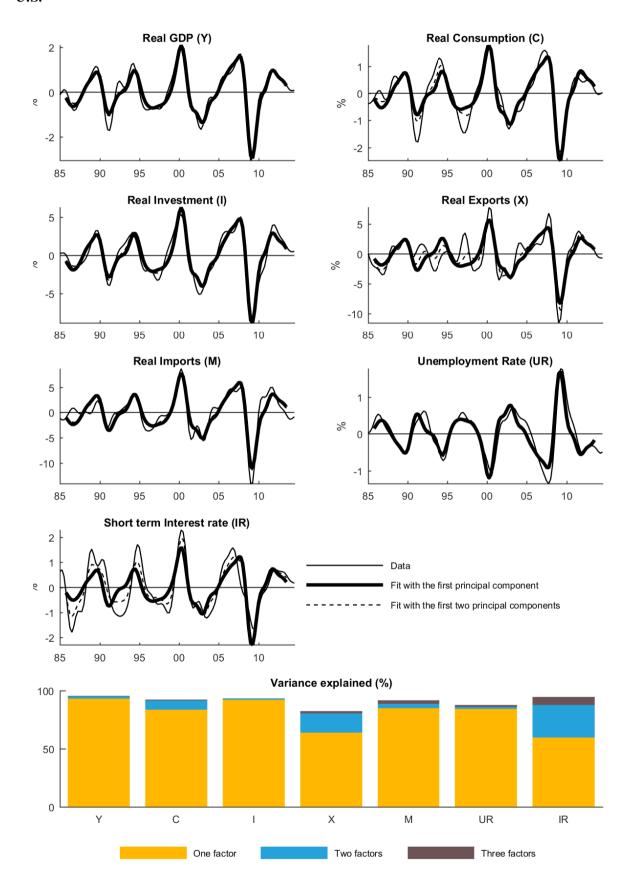
¹¹ The plot is phase-aligned, i.e., the inflation cycle is shifted by a mean lag.

¹² Computed using the wild bootstrap; see Wu (1986).

despite the deterioration of the fit, the co-movement among real variables is still there, although it is not as impressive as for the cyclical components of real variables. Figure A8 in the Appendix presents normalized growth rates of GDP components to highlight that strong co-movement is easily discernible. For inflation, we have already argued that economic theory has a strong say in terms of data transformation, specifically linking the deviation of inflation from its target (and thus long-term expectations) to output dynamics. Often, the contributions to the literature, namely, factor models, search for co-movement of the first difference of inflation with output growth or the output gap. Such attempts necessarily fail, like those attempts which ignore inflation targets. This is especially easy to understand in the case of inflation-targeting countries that underwent a disinflation process, such as Canada, the Czech Republic, and Poland.

Finally, the length of the U.S. data enables us to plug median mean inflation directly into the DPCA analysis. We did this, and the results are available in Figure A4, which shows the fit in the time domain for the HP cycles (the results for the band-pass cycles look similar). For output, consumption, investment, and unemployment, one principal component produces an excellent fit. The common component based on the first principal component for exports, the short-term interest rate, and median inflation explains about 50% of the volatility. The relatively low explanatory power of the first principal component is due to high volatility of these series during the 1960s and 1970s. Nevertheless, the filter loadings have the same sign. We conclude that this exercise confirms our finding that the relative variance of some variables may change, but the co-movement is stable.

Figure 1: Cyclical Components (Christiano-Fitzgerald Filter): Data and Fit with DPCA—the U.S.





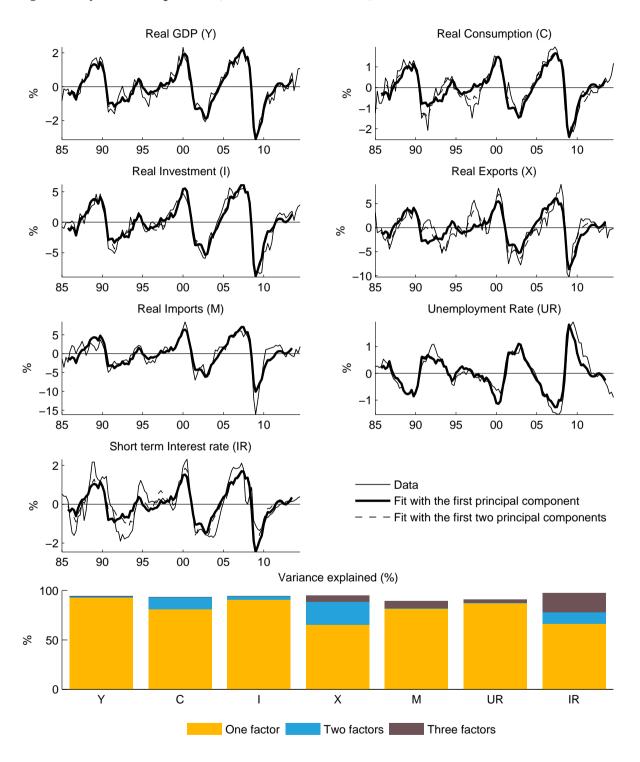
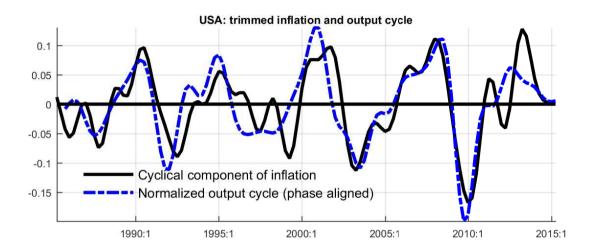
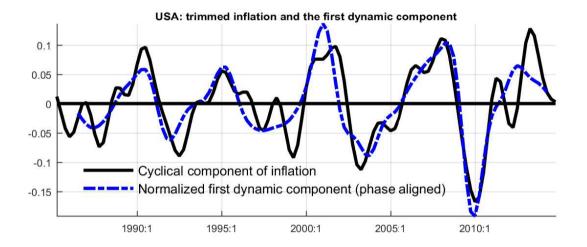


Figure 3: Inflation and the Real Economy—the U.S. (post 1985)





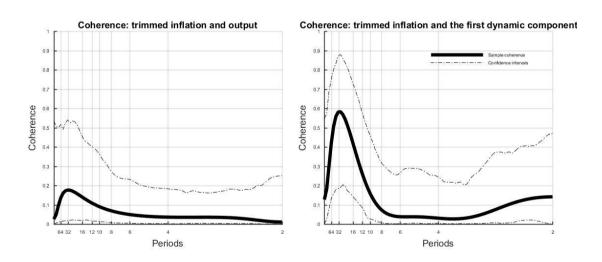


Figure 4: Proportion of the Spectral Density Explained by the First Two Dynamic Components—the U.S.

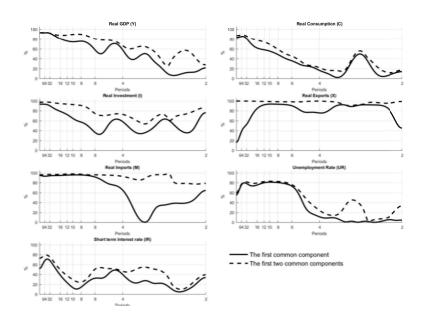
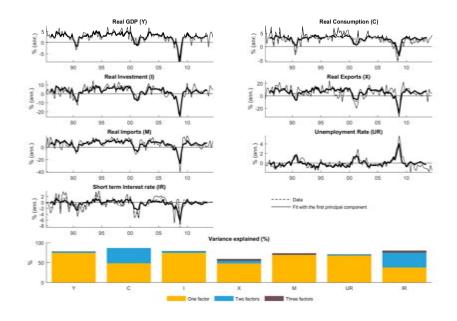


Figure 5: Growth Rates: Data and Fit with DPCA—the U.S.



4.2 Summary Statistics

In this subsection, we report summary statistics for all the countries in our dataset. We collected data for a set of advanced and several emerging market countries at quarterly frequency. The set consists of: Australia, Austria, Belgium, Canada, the Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey, the U.K., and the U.S. Our benchmark analysis starts from 1985 (or later according to data availability). The choice of this year is motivated by the change in the relative volatilities of inflation and real activity (the Great Moderation) in developed countries around the mid-1980s. Nevertheless, we carry out our exercise with a longer sample for countries where a larger sample is available. The co-movement among real variables remains stable even after the larger sample has been used. We also find a cyclical similarity of inflation and real activity when the change in their relative volatilities is taken into account.

First, we report the boxplots¹³ of how our model fits the co-movement in the cyclical parts of real variables. Figure 6 shows the fit for the sample post 1985 for the first three dynamic components (organized by rows) and for the two popular filters used (the Christiano-Fitzgerald band-pass filter and the HP filter—organized by columns). Apparently, for most countries, the first dynamic principal component already explains most of the dynamics in output, investment, imports, and unemployment.¹⁴ The first two dynamic principal components then explain a high proportion of the dynamics in all the variables. Figure A5 in the Appendix depicts the same exercise for all the data in our sample. Apparently, the fit is robust to the inclusion of the period before 1985 where available. The analysis reveals that for all the countries the dispersion of the percentage explained is larger for exports, the short-term real rate, and consumption, as indicated in the discussion above. The co-movement of inflation and real variables also seems quite strong for all countries in the sample. Figure 7 reports the summary results on the co-movement between the inflation cycle and cycles in real variables for all countries in the sample. It reports the coherence and cross-correlation of the inflation cycle with output and of inflation with the first dynamic principal component. Apparently, the results argue for relatively high co-movement between inflation and the real economy over the business cycle. 15

¹³ The boxplots are organized as follows: in each box, the central mark is the median, the edges of the box are the 25th and 75th percentiles, the whiskers extend to the most extreme data points that are not outliers, and the outliers are plotted individually. Observations are defined as outliers if they are larger than $Q_{75} + 1.5(Q_{75} - Q_{25})$ or smaller than $Q_{25} - 1.5(Q_{75} - Q_{25})$, where Q_{25} and Q_{75} are the 25th and 75th percentiles.

¹⁴ The exact figures on the fit using the first principal component (based both on two-sided DPCA and on static PCA) for all countries in our sample are given in Tables A1 and A2 in the Appendix.

¹⁵ Interestingly, for each country in our sample, there is a lag $k \in (0, ..., 4)$ for which the correlation between cyclical inflation and the cyclical component of output is positive and significantly different from zero at the 5% level.

Figure 6: Boxplot Summary Statistics

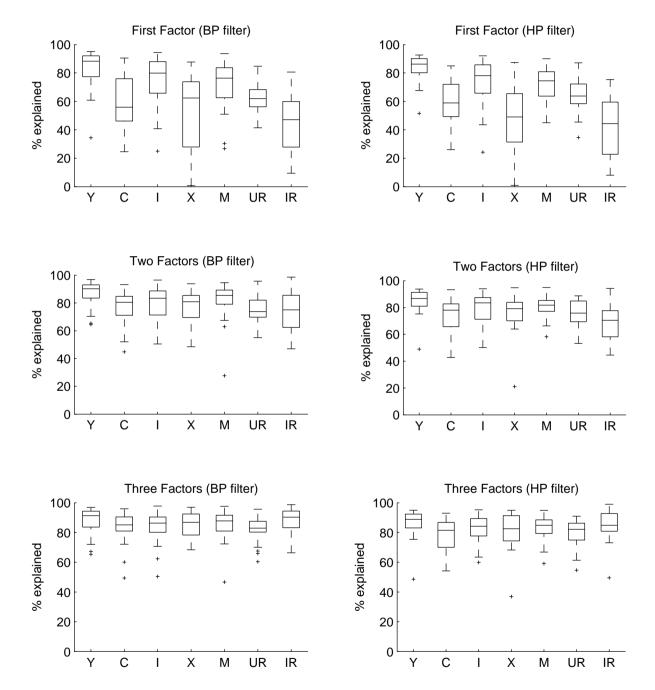
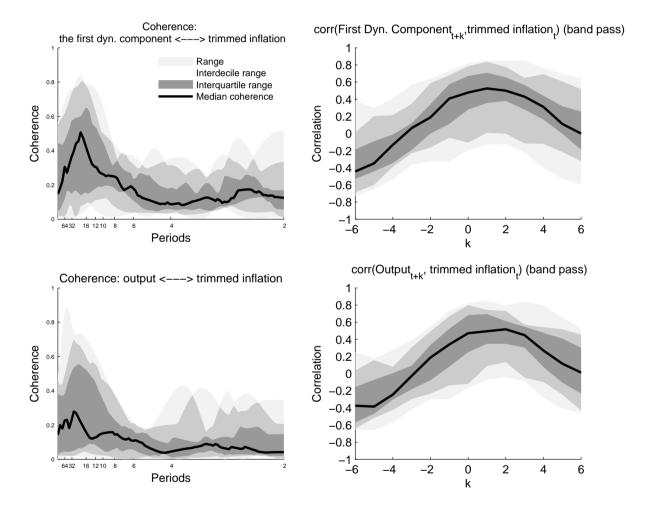


Figure 7: Summary Statistics: Coherence and Correlations between Inflation and Real Activity



5. Implications for Macroeconomic Models

5.1 Economics of Co-Movement

Our findings that the macroeconomic cycles of both real and nominal variables are well explained by a single dominant dynamic principal component bear important implications for structural economic modeling. More specifically, an economic model—for instance a DSGE model—that does not feature a structural shock that dominates the cyclical frequencies of consumption, investment, output, hours worked, and inflation is very likely to be misspecified. This refers to both (i) the variance contribution of the shock and (ii) the direction of co-movement of relevant variables. To the best of our knowledge, most DSGE models in the literature would struggle to explain the principal-component space of the data in a satisfactory manner. We do not propose a solution either, but we do offer a testing procedure.

When a DSGE model lacks a dominant structural factor with the above-mentioned properties, the misspecification will cause remaining structural shocks or measurement errors to be correlated. The requirements placed on models are rather strict. At business cycle frequency, not only should a single shock be dominant and result in positive co-movement of real variables with inflation, but also there is little leeway for the shape of the impulse-response function to such a shock. A misspecified structural shock inevitably leads to cross-correlation of other shocks and disturbances, a telltale sign of misspecification. Some prominent DSGE models in the literature fail to satisfy the restrictions on co-movement argued for in this paper and are therefore misspecified and easily falsified. Standard RBC-like TFP shocks are ruled out, mostly due to their implausible effects on inflation. The literature has thus proposed other types of shocks. For instance, the state-of-the-art model by Justiniano et al. (2010) proposes investment-specific shocks as an explanation of the business cycle. Their model explains a large proportion of output, investment, and hours worked, but fails to explain private consumption or inflation. The reason is that after an expansionary investment-specific shock, private consumption actually drops and increases only after about five quarters; see Figure A9.

The variance decompositions are an extremely useful tool and support our argument for model misspecification checks. As our discussion above indicates, the business cycle frequency dynamics of the main GDP components, unemployment, and inflation are dominated by one source of variation in the data across many countries. The modeling analogue of the analysis is variance decomposition of the model variables by the source of structural shocks. The paper by Justiniano et al. (2010) is very transparent in presenting the statistics with the identical definition of cyclical frequencies as in this paper. In their variance decomposition by, restated here for convenience, it is clear that the investment shock cannot explain more than 10% of the consumption dynamics and the preference shock fills the void. It can also be seen that inflation is driven by cost and wage-push shocks and not by investment-specific shocks, further undermining the investment shock as a plausible explanation of the business cycle. In Christiano et al. (2014), Table 5, one can see that the business cycle dynamics of consumption are essentially unaffected by the risk shock, amounting to 3% of the total, with almost half of the variance driven by the 'demand' shock, which, on the other hand, hardly affects any other variable. Although some 11% of the consumption dynamics are driven by the investment shock, the IRF has the wrong sign, which is not reflected in the variance decomposition.

¹⁶ We choose Justiniano et al. (2010) and Christiano et al. (2014) for illustration mainly due to their major influence on empirical modeling, quality of execution, and the exemplary scholarship and transparency of their research.

¹⁷ The line of research using financial sector disturbances and risk shocks also seems not to fit the bill. For instance, the prominent study by Christiano et al. (2014) fails to invoke robust co-movement of consumption and investment as robustly observed in the data. The consumption is explained by the risk shocks only at low frequencies, not at business-cycle frequencies.

Table 1: Variance Decomposition at Business Cycle Frequency, Justiniano et al. (2010)

Series / Shock	Policy	Neutral	Government	Investment	Price mark-up	Wage mark-up	Preference
Output	0.05	0.25	0.02	0.50	0.05	0.05	0.07
	[0.03, 0.08]	[0.19, 0.33]	[0.01, 0.02]	[0.42, 0.59]	[0.03, 0.07]	[0.03, 0.08]	[0.05, 0.10]
Consumption	0.02	0.26	0.02	0.09	0.01	0.07	0.52
	[0.01, 0.04]	[0.20, 0.32]	[0.02, 0.03]	[0.04, 0.16]	[0.00, 0.01]	[0.04, 0.12]	[0.42, 0.61]
Investment	0.03	0.06	0.00	0.83	0.04	0.01	0.02
	[0.02, 0.04]	[0.04, 0.10]	[0.00, 0.00]	[0.76, 0.89]	[0.02, 0.06]	[0.01, 0.02]	[0.01, 0.04]
Hours	0.07	0.10	0.02	0.59	0.06	0.07	0.08
	[0.04, 0.10]	[0.08, 0.13]	[0.02, 0.03]	[0.52, 0.66]	[0.04, 0.09]	[0.04, 0.11]	[0.06, 0.12]
Wages	0.00	0.40	0.00	0.04	0.31	0.23	0.00
	[0.00, 0.01]	[0.30, 0.52]	[0.00, 0.00]	[0.02, 0.07]	[0.23, 0.41]	[0.16, 0.32]	[0.00, 0.01]
Inflation	0.03	0.14	0.00	0.06	0.39	0.34	0.02
	[0.02, 0.06]	[0.09, 0.21]	[0.00, 0.00]	[0.02, 0.13]	[0.29, 0.50]	[0.26, 0.42]	[0.01, 0.04]
Interest Rates	0.17	0.09	0.01	0.47	0.05	0.04	0.16
	[0.13, 0.22]	[0.06, 0.12]	[0.00, 0.01]	[0.37, 0.56]	[0.03, 0.07]	[0.03, 0.07]	[0.11, 0.23]

Misspecification of a structural model can be recognized from the correlation patterns of shocks and shock decompositions. Once the model falls short of a dominant shock with robust co-movement of real and nominal variables at business cycle frequencies, the estimated shocks will be significantly correlated. Inspecting the auto-covariance function of the shocks is thus an easy and useful test for misspecification of the model and fits within the best tradition of econometrics, where 'residuals' matter and are inspected closely. The cross-correlation of shocks is visually apparent in the shock decomposition of the observed data, where one gets 'fish graphs,' with the contribution of one shock almost perfectly offsetting that of another. For instance, a positive investment shock often increases output and investment but depresses private consumption, while a 'consumption' preference shock expands consumption and lowers investment. To fit the observed profile of consumption and investment, the shocks must be negatively correlated and their individual contributions offsetting. Correlated estimated 'structural' shocks are problematic, since most of the model properties, namely, the impulse-responses, the variance decompositions, and the counterfactual simulations, assume that they are independent. All these properties are invalidated by misspecification manifested in correlated shocks and researchers should carefully inspect their models for cross-correlation of shocks.

5.2 Specification Testing

Based on our findings, we believe that any structural economic model must at least be able to generate the principal component structure of the data it is supposed to represent. Note that this requirement is void of any structural interpretation of the data-based factors or the theory of the model. It is not enough to explain real variables and fail to explain inflation, or to explain output and investment dynamics and miss consumption dynamics. In this sense the principal-component space of the data is a very strong restriction on every model and can be a useful device for testing and falsifying hypotheses, together with the tests using structural shocks. ¹⁸

Our analysis suggests two both intuitive and formal statistical tests to assess model performance and misspecification. Maximum-likelihood or other estimators will converge to some extreme and models can be compared in relative terms with each other using more or less sophisticated measures, posterior odds for instance. We can devise two additional tests—one *ex-ante*, based on the principal-component space of the model, and the other *ex-post*, based on the cross-correlation of the estimated shocks.

Before conducting a thorough analysis of shocks and measurement errors it is critical to check if the model can explain the business cycle dynamics with one dominant source of variance, or, more formally, if the model-induced principal-component space is close to the principal-component space of the data and if the impulse responses 'make sense' in light of robust stylized facts on co-movement that most practitioners are aware of. This check can help prevent structural macroeconomic modeling from becoming a "degenerative research program"—as Farmer (2012) puts it—that is characterized by adding additional 'structural' shocks to explain the dynamics of an expanding set of observable variables.

Cross-correlation test Ex-post, given a parameterized model, a finite-sample distribution of shock cross-correlation of shocks can be established for the hypothesis of correct specification. Given such null-hypothesis finite-sample distribution of cross-correlations, it is easy to test whether the estimated cross-correlations devivate from the correct specification in a statistically significant sense. When important cross-correlations is detected, the sources of the cross-correlation needs to be investigated.

Principle-component space test It is possible to construct a simple test statistic based on the principal-component space. Let $F(Y_T)$ be a suitable function of data Y_T of length T, such as (3) or (2), which can be compared with the implication of a model \mathscr{M} or with the distribution of the same statistic, applied to a simulated series of length T, Y_T^s , from model \mathscr{M} . While the share of the dominant eigenvalue of the spectral density can be computed exactly (i.e., without simulation) for a model having a linear state space form, the comparison of F(Y) with the empirical distribution of $F(Y_T)^s$ has the advantage of addressing finite sample issues in the sense that, for 'reasonable' statistics, the empirical distribution of $F(Y_T)^s$ will consistently estimate (as $S \to \infty$) the implied distribution of $F(Y_T)$ under the null that model \mathscr{M} is the true one.

Further, the mapping from structural shocks to principal components (factors) can easily be investigated. In the time domain, this follows trivially from the fact that the factors are a function of the data and its covariance structure. The principal factor is then the result of all the independent structural shocks, with appropriate weighting.

¹⁸ The requirement that models approximate well the dynamic-principal-component space of the data does not exclude models with stochastic singularity, on the contrary, as long as the sources of the dynamics capture the dominant eigenvalues closely enough.

We carried out the test for two prominent models: Justiniano et al. (2010) and Smets and Wouters (2007a). In both cases, we took the data used in the estimation of each model¹⁹ \mathscr{D} and applied the time domain statistics (2). Then, we simulated a large number (500) of the series from the model²⁰ of the same length as the original data and compared the empirical distribution of this statistic on the simulated series with those on the data.

Figure 8 shows the results for the Smets and Wouters model. We show the results in the time domain for three data transformations: the HP filter, the band-pass filter, and growth rates. Evidently, the model underestimates the co-movement for investment and hours worked (especially if cyclical frequencies are isolated), while overestimating the co-movement for the real wage. It would seem that the model gets the inflation-output co-movement right, but this impression vanishes when the signs of the factor loadings are inspected. Figure 9 displays the PCA factor loadings. Apparently, the mean of the loadings for inflation has the opposite sign to that in the data: in the data the loading for inflation has the same sign as the loading for consumption (and the rest of the real variables), while most simulations with the model would imply the opposite sign. Apparently, the shocks that dominate in the Smets and Wouters (2007a) model move inflation and output in the opposite direction. Given the nature of the monetary policy reaction function, the same holds for the interest rate.

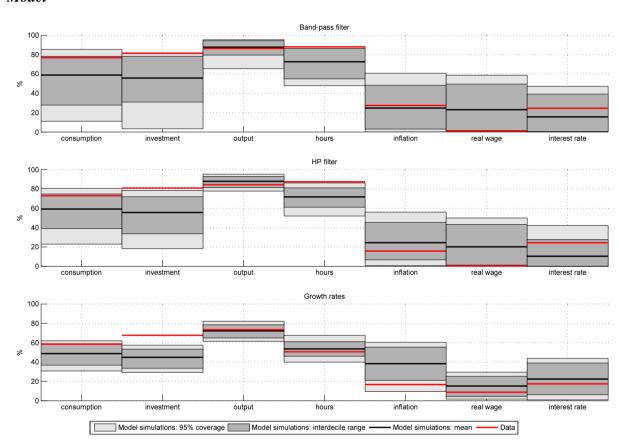
Figure 10 shows the results for the Justiniano et al. (2010) model, again for the three data transformations. Consistently with the previous section, the most pronounced results are obtained when growth rates are not used. The Justiniano et al. (2010) model is not able to generate the cyclical comovement of private consumption with the rest of the real macro variables, and also employment is not fully synchronized. This should not be surprising, since the dominant shock in the model—the investment-specific shock—produces a negative response of consumption to a boom in investment and output; see Figure A9.

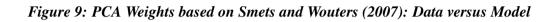
¹⁹ The data of Smets and Wouters (2007a) were downloaded from the article website of the American Economic Review. The data of Justiniano et al. (2010) come from Haver Analytics, following the appendix to the paper on the Journal of Monetary Economics website.

²⁰ For the simulation of the Smets and Wouters (2007a) model, we use the Dynare implementation by Volker et al. (2011). For the Justiniano et al. (2010) model, we use the Gensys code by Christopher Sims.

²¹ If χ_t is the first dominant factor (principal component), the fitted series can be written as $\hat{x}_{it} = \lambda_i \chi_t$ (in the static case). Since the factor and its loadings are not identified, we normalize them on the output weight and report $\lambda_i/\lambda_{\text{output}}$. A positive (negative) value of this ratio means that the variable is cyclical (counter-cyclical). For the case of the dynamic model, the fitted series are given as $\hat{x}_{it} = \sum_{k=-K}^{K} \lambda_{ik} \chi_{t-k}$. In that case, we report $\tilde{\lambda}_i \equiv \sum_k \lambda_{ik}$ as the factor loading.

Figure 8: R^2 fit of DPCA for Various Transformations: Data versus Smets and Wouters (2007) Model





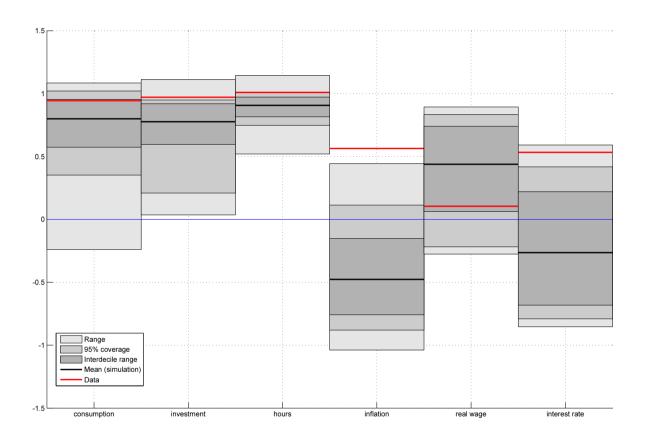
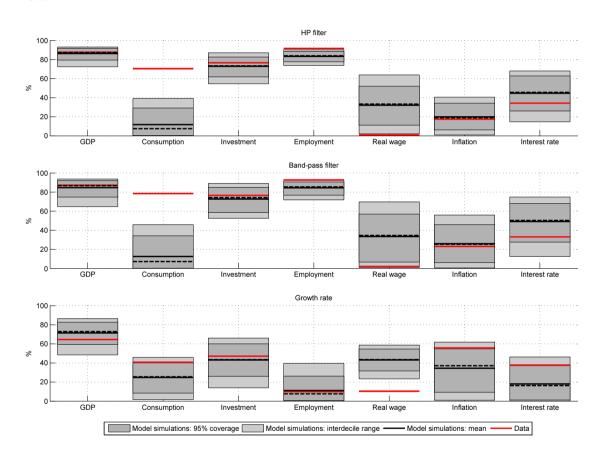


Figure 10: \mathbb{R}^2 fit of DPCA for Various Transformations: Data versus Justiniano et al. (2010) Model



6. Conclusions

In this paper we provide an empirical investigation of the sources of economic fluctuations, their number and their nature, and their implications for economic modeling and policy analysis. We reach a conclusion that the business cycle dynamics of key macroeconomic data can be largely explained by a single source of variation. Since this dominant unobserved principal component behind the business cycle explains the positive co-movement of the output cycle and inflation, we label the principal component as a 'demand factor.' We describe the properties of the demand factor and argue that structural economic models have great difficulties delivering structural shocks resembling our robustly estimated dominant principal component.

Our analytical approach allows us to reach strong conclusions with relatively modest identification assumptions. We employ straightforward dynamic principal component analysis to analyze key real and nominal macroeconomic data for OECD countries. The analysis decomposes the data into a set of orthogonal contributions of a number of components. The first dynamic principal component clearly dominates in terms of explained variance, so other components are not explicitly analyzed or identified. The effects of the dominant component also satisfy the sign restriction one would expect from intuitively understood demand, namely, positive co-movement of output and inflation, which renders the factor its label—a demand factor. We document that the set of stylized facts leading to a demand factor continue to hold for a set of OECD countries and in time. Further, our findings are invariant to the use of both time and frequency domain techniques which do not rely on time domain filtering.

The absence of the real-nominal dichotomy is an important result, highlighting the importance of variable definitions in the analysis as a shield against misspecification. We have illustrated that there is positive co-movement of output and inflation at business cycle frequencies, a key result allowing us to argue for a demand-like explanation of business cycles. Why do Phillips curve estimates or dynamic factor model analysis usually fail to find a stable relationship, claiming a real-nominal dichotomy, while our results do find one? The key is our focus on business cycle frequency and thus our data transformation. The cyclical dynamics of inflation are akin to the deviation of inflation from an inflation target or long-term inflation expectations and the theory predicts that this 'inflation gap' should positively co-move with the cyclical component of output. We do find this positive relationship. If we were to follow the literature and use the first difference of inflation (to render it stationary) with demeaned GDP growth, the task of finding a positive relationship would be much harder due to a transformation that amplifies high-frequency disturbances in the data and has weaker theoretical support.

The existence of a dominant 'demand' factor behind the business cycle dynamics of the data has strong implications for structural economic models. To sum up, we argue that the current vintage of DSGE models lacks a dominant demand shock that would explain the business cycle dynamics. This is no ado about nothing—most models fail to coherently explain up to 80% of key macroeconomic variables. Also, our analysis creates a shopping list for model builders in terms of the nature of the behavior a shock must exhibit in order to be considered a plausible source of business cycles. No model known to us can explain the positive co-movement of consumption, investment, and inflation to the degree and with the duration implied by the data and the estimated demand factor. Our analysis is tractable and powerful in testing and falsifying economic models and driving forces, while relying on a minimal set of assumptions.

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Appendix A: Additional Graphs and Tables

Figure A1: Cyclical Components: Data and Fit with DPCA (Christiano-Fitzgerald Filter)—the U.S. (Full Sample)

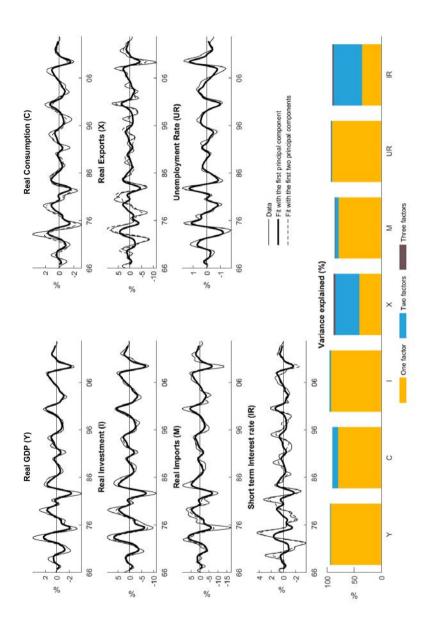


Figure A2: Cyclical Components (Hodrick-Prescott Filter): Data and Fit with DPCA—the U.S. (Full Sample)

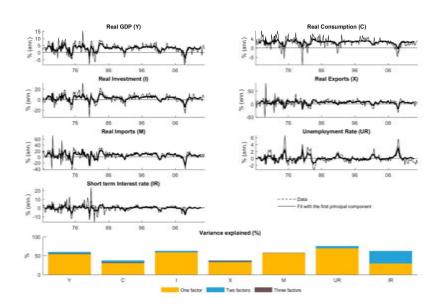
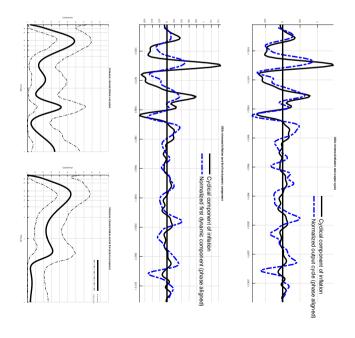
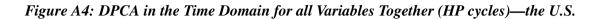


Figure A3: Inflation and the Real Economy—the U.S. (Full Sample)





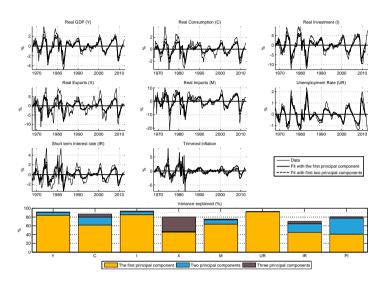


Figure A5: Boxplot Summary Statistics (Full Sample)

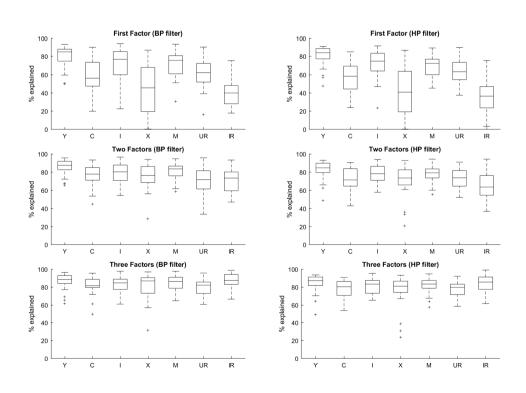


Figure A6: Cyclical Components: Data and Fit with Static PCA (Christiano-Fitzgerald Filter)—the U.S.

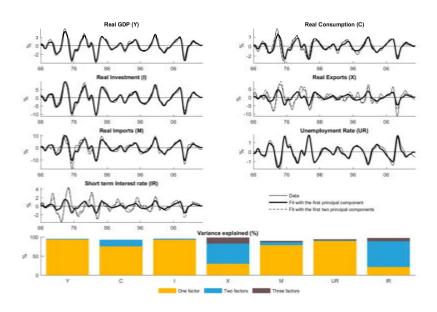


Figure A7: Cyclical Components (Hodrick-Prescott Filter): Data and Fit with Static PCA—the U.S.

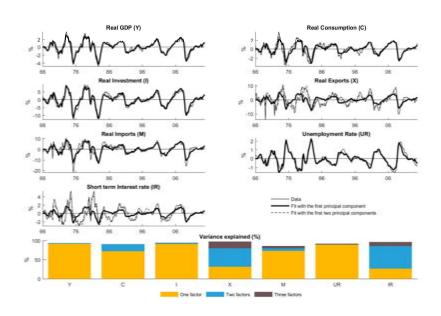
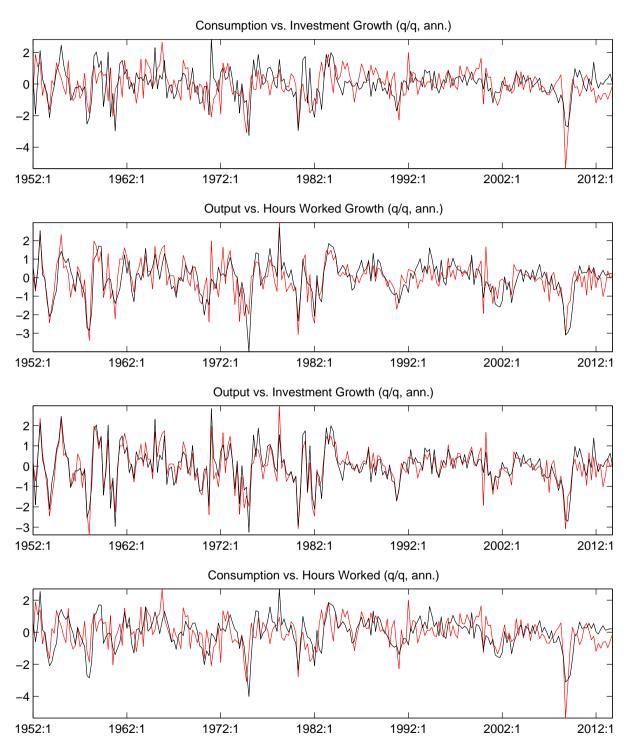
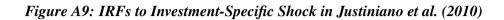


Figure A8: Growth Rates of Macroeconomic Data Consistent with Justiniano et al. (2010)



Note: Investment contains durable consumption; consumption consists of non-durable consumption only. The series are normalized to equal variance. Source: Haver Analytics



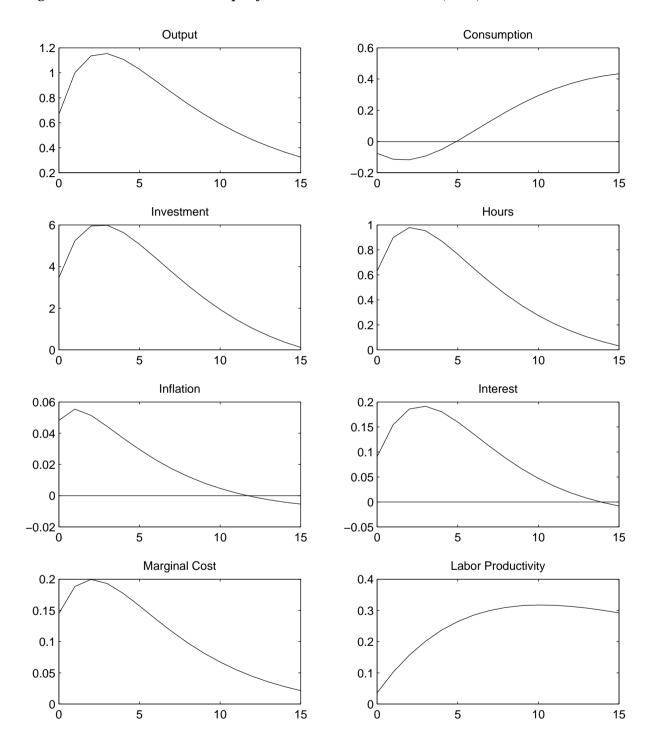


Figure A10: Inflation Components—Decomposition

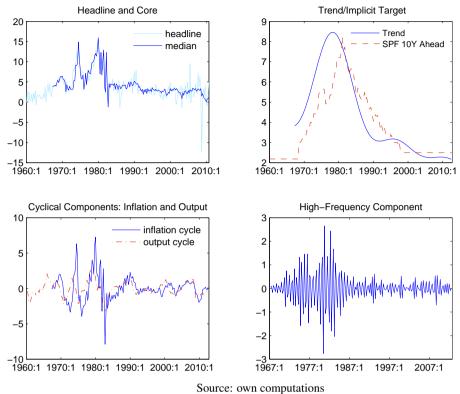
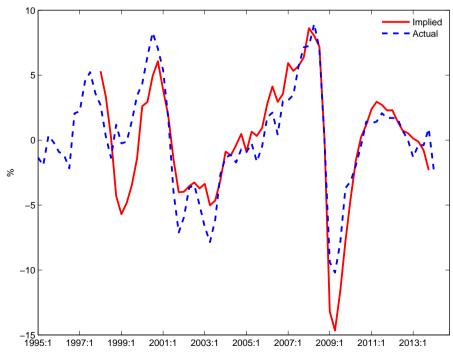


Figure A11: U.S. Export Cycle—Actual and Implied



Source: own computations based on IMF Global Projection Model database

Table A1: Fit of the First Dynamic Principal Component: Band-pass Cycle

Country / Variable	Y	С	I	X	M	UR	IR
Australia	60.8	66.4	70.2	29.8	81.1	83.2	61.3
Austria	90.3	35.4	65.8	84.9	81.8	41.4	73.0
Belgium	86.4	44.9	68.7	87.7	89.4	59.5	54.6
Canada	91.2	65.1	69.8	74.0	74.2	84.7	52.7
Czech Republic	88.4	26.5	87.9	45.4	64.6	65.8	24.2
Denmark	88.8	47.7	81.7	71.1	85.4	67.3	22.9
Finland	91.9	77.0	88.0	34.0	81.2	59.0	38.8
France	92.1	49.5	88.3	78.8	89.2	60.3	63.8
Germany	92.4	60.8	92.3	78.8	78.6	62.0	75.2
Greece	64.2	46.6	51.0	12.1	30.4	43.7	17.7
Hungary	72.8	50.7	40.8	27.9	59.1	67.7	35.9
Ireland	75.3	78.7	65.6	73.7	75.0	70.2	22.4
Italy	88.2	55.1	80.4	60.2	85.5	55.0	53.5
Japan	92.9	60.6	86.9	62.3	75.6	63.8	27.5
Korea	84.9	85.3	88.4	0.8	76.6	83.3	69.2
Luxembourg	78.1	25.4	24.9	87.1	85.7	59.8	59.2
Mexico	92.8	87.6	90.0	17.2	93.7	63.4	47.1
Netherlands	91.9	51.4	64.4	66.3	76.3	52.4	80.6
New Zealand	69.5	55.9	84.7	7.0	61.9	55.1	43.3
Norway	67.3	75.5	43.6	27.9	61.2	67.0	53.7
Poland	85.9	24.6	87.3	68.7	62.8	58.9	41.0
Portugal	34.4	43.5	45.5	4.5	26.9	48.2	9.5
Slovakia	82.6	58.8	79.5	26.6	59.5	56.6	39.6
Spain	81.6	78.9	77.3	70.1	80.2	77.0	25.9
Sweden	95.1	47.6	80.6	72.8	83.3	67.4	57.7
Switzerland	84.2	38.0	72.9	59.5	69.1	52.9	71.1
Turkey	93.0	90.5	94.4	87.5	50.9	81.6	28.0
UK	88.7	68.0	79.9	33.9	66.6	57.9	35.4
USA	92.9	83.4	91.9	63.7	84.7	84.1	59.5

Table A2: Fit of the First Static Principal Component: Band-pass Cycle

Country / Variable	Y	С	I	X	M	UR	IR
Australia	59.0	66.2	69.8	30.6	81.0	82.5	59.3
Austria	90.3	36.2	65.8	85.0	81.9	37.4	72.6
Belgium	86.0	45.5	68.3	87.5	89.1	60.5	53.2
Canada	90.9	65.5	69.1	73.7	74.6	84.0	51.7
Czech Republic	87.7	15.5	86.5	37.2	68.1	46.1	4.90
Denmark	84.2	45.4	83.9	68.4	88.1	72.1	18.6
Finland	90.8	75.1	87.8	35.6	82.1	57.9	32.8
France	91.3	46.1	88.0	78.1	89.1	57.2	59.6
Germany	92.0	58.0	91.5	78.5	78.1	62.0	73.6
Greece	91.3	65.9	71.3	0.10	26.4	72.4	2.10
Hungary	80.7	28.6	43.6	21.8	57.7	49.2	6.20
Ireland	74.9	75.7	64.8	74.3	75.5	70.4	17.8
Italy	87.1	54.3	79.8	59.8	86.1	52.9	48.4
Japan	92.7	55.0	85.9	61.3	73.6	61.6	27.0
Korea	97.5	90.9	92.4	2.10	83.8	85.2	0.20
Luxembourg	76.4	24.7	19.2	86.8	85.0	59.3	58.2
Mexico	92.5	86.9	89.4	15.8	93.5	58.6	45.9
Netherlands	91.7	50.8	62.1	64.3	75.0	50.2	79.1
New Zealand	69.4	55.3	85.0	7.60	62.2	52.7	33.7
Norway	70.5	71.3	36.8	0.00	57.5	61.8	46.2
Poland	84.6	17.7	87.6	67.9	56.6	61.3	46.7
Portugal	46.6	73.6	81.0	6.3	76.2	54.2	0.30
Slovakia	83.0	58.9	80.2	23.5	58.1	54.1	42.8
Spain	81.7	79.4	77.5	68.4	78.0	74.7	22.1
Sweden	94.2	42.2	79.6	71.9	84.5	65.1	48.1
Switzerland	82.4	37.0	72.4	55.7	62.9	53.1	69.4
Turkey	94.2	90.9	94.4	87.6	44.9	76.1	15.3
UK	87.3	66.7	78.2	34.9	67.6	54.1	24.2
USA	92.8	83.2	91.9	63.0	83.6	83.1	58.9

Appendix B: Data Sources and Specification

The specification of the data is described below. All computations were performed in Matlab by MathWorks. Data and codes for the paper are available upon request. Weighted median inflation for EU countries was computed using data from EUROSTAT as represented in the Haver Analytics database, Level 3. In the case of the United States the weighted median inflation is from the FRB of Cleveland. Core inflation for Australia was obtained from the Reserve Bank of Australia website and ten-years-ahead inflation expectations from the Survey of Professional Forecasters (SPF) by the FRB of Philadelphia. The 'PTR' variable (a proxy for the inflation target) of the FRB/US model was kindly provided by Bob Tetlow. Seasonal data adjustment was provided by the source authority, otherwise the default Bureau of Census X12/ARIMA algorithm was applied in its default setting.

Table B1: OECD Data

Countries	Variables Collected per Country	Data Source
Euro Area15	Private final consumption expenditure, value, GDP expenditure approach	
Australia, Austria,	Private final consumption expenditure, volume	
Belgium, Canada,	Gross domestic product, value, market prices	
Finland, France,	Gross domestic product, volume, market prices	
Germany, Ireland,	Gross fixed capital formation, total, value	
Italy, Japan,	Gross fixed capital formation, total, volume	
Korea, Luxembourg,	Imports of goods and services, value, National Accounts basis	OECD Economic Outlook
Mexico, Netherland,	Imports of goods and services, volume, National Accounts basis	No. 94
New Zealand, Norway,	Exports of goods and services, value, National Accounts basis	
Poland, Portugal,	Exports of goods and services, volume, National Accounts basis	
Spain, Sweden,	Core inflation index	
Switzerland, United Kingdom,	Unemployment rate	
United States of America	Short-term interest rate	

Table B2: National Source Data

Czech Rep: GDP: Final Consumption Expenditure: Households (SWDA, Mil.CZK)	Czech Statistical Office
Czech Rep: GDP: Final Consumption Exp: Households (SWDA, Mil.Chn.2005.CZK)	Czech Statistical Office
Czech Republic: Gross Domestic Product (SWDA, Mil.CZK)	Czech Statistical Office
Czech Republic: Gross Domestic Product (SWDA, Mil.Chn.2005.CZK)	Czech Statistical Office
Czech Republic: GDP: Gross Fixed Capital Formation (SWDA, Mil.CZK)	Czech Statistical Office
Czech Republic: GDP: Gross Fixed Capital Formation (SWDA, Mil.Chn.2005.CZK)	Czech Statistical Office
Czech Republic: GDP: Imports of Goods and Services (SA, Mil.CZK)	Czech Statistical Office
Czech Republic: GDP: Imports of Goods and Services (SA, Mil.Chn.2005.CZK)	Czech Statistical Office
Czech Republic: GDP: Exports of Goods and Services (SA, Mil.CZK)	Czech Statistical Office
Czech Republic: GDP: Exports of Goods and Services (SA, Mil.Chn.2005.CZK)	Czech Statistical Office
Czech Republic: Unemployment Rate, % of Labor Force (SA, %)	Czech Statistical Office
Czech Republic: PRIBOR: 3 Month (Avg, %)	Czech National Bank
Denmark: Private Consumption Expenditure (SA, Mil.Kroner)	Danmarks Statistik
Denmark: Private Consumption Expenditure (SA, Mil.Chn.2005.Kroner)	Danmarks Statistik
Denmark: Gross Domestic Product (SA, Mil.Kroner)	Danmarks Statistik
Denmark: Gross Domestic Product (SA, Mil.Chn.2005.Kroner)	Danmarks Statistik
Denmark: Gross Fixed Capital Formation (SA, Mil.Kroner)	Danmarks Statistik
Denmark: Gross Fixed Capital Formation (SA, Mil.Chn.2005.Kroner)	Danmarks Statistik
Denmark: GDP: Imports of Goods and Services (SA, Mil.Kroner)	Danmarks Statistik
Denmark: GDP: Imports of Goods and Services (SA, Mil.Chn.2005.Kroner)	Danmarks Statistik
Denmark: GDP: Exports of Goods and Services (SA, Mil.Kroner)	Danmarks Statistik
Denmark: GDP: Exports of Goods and Services (SA, Mil.Chn.2005.Kroner)	Danmarks Statistik
Denmark: Harmonized Unemployment Rate (SA, %)	Statistical Office of the European Communities
Denmark: Interbank Offered Rate: 3-months (AVG, %)	Danmarks Nationalbank
Greece: GDP: Private Consumption (NSA, Mil.Euros)	Hellenic Statistical Authority (ELSTAT)
Greece: GDP: Private Consumption (NSA, Mil.Chained.2005.Euros)	Hellenic Statistical Authority (ELSTAT)
Greece: Gross Domestic Product (NSA, Mil.Euros)	Hellenic Statistical Authority (ELSTAT)
Greece: Gross Domestic Product (NSA, Mil.Chained.2005.Euros)	
	Hellenic Statistical Authority (ELSTAT)
Greece: GDP: Gross Fixed Capital Formation (NSA, Mil.Euros)	Hellenic Statistical Authority (ELSTAT)
Greece: GDP: Gross Fixed Capital Formation (NSA, Mil.Chained.2005.Euros)	Hellenic Statistical Authority (ELSTAT)
Greece: GDP: Imports of Goods & Services (NSA, Mil.Euros)	Hellenic Statistical Authority (ELSTAT)
Greece: GDP: Imports of Goods & Services (NSA, Mil.Chained.2005.Euros)	Hellenic Statistical Authority (ELSTAT)
Greece: GDP: Exports of Goods & Services (NSA, Mil.Euros)	Hellenic Statistical Authority (ELSTAT)
Greece: GDP: Exports of Goods & Services (NSA, Mil.Chained.2005.Euros)	Hellenic Statistical Authority (ELSTAT)
Greece: Labor Force Survey: Unemployment Rate (SA, %)	Hellenic Statistical Authority (ELSTAT)
Hungary: Final Consumption Expenditure: Private (SA, Bil.Forints)	Central Statistical Office
Hungary: GDP: Final Consumption Expenditure: Private (SWDA,Bil.Ch.2005.Forints)	Central Statistical Office
Hungary: Gross Domestic Product (SA, Bil.Forints)	Central Statistical Office
Hungary: Gross Domestic Product (SWDA, Bil.Ch.2005.Forints)	Central Statistical Office
Hungary: Gross Fixed Capital Formation (SA, Bil.Forints)	Central Statistical Office
Hungary: GDP: Gross Fixed Capital Formation (SWDA,Bil.Ch.2005.Forints)	Central Statistical Office
Hungary: Imports of Goods & Services (SA, Bil.Forints)	Central Statistical Office
Hungary: GDP: Imports of Goods & Services (SWDA,Bil.Ch.2005.Forints)	Central Statistical Office
Hungary: Exports of Goods & Services (SA, Bil.Forints)	Central Statistical Office
Hungary: GDP: Exports of Goods & Services (SWDA,Bil.Ch.2005.Forints)	Central Statistical Office
Hungary: Unemployment Rate (SA, %)	Central Statistical Office
Hungary: Yield on 3-Month Government Debt Securities (EOP, % per annum)	National Bank of Hungary

Table B3: National Source Data

Slovakia: GDP: Final Consumption of Households (SA, Mil.EUR)	Statistical Office of the Slovak Republic
Slovakia: GDP: Final Consumption of Households (SA, Mil.Chn.2005.EUR)	Statistical Office of the Slovak Republic
Slovakia: Gross Domestic Product (SA, Mil.EUR)	Statistical Office of the Slovak Republic
Slovakia: Gross Domestic Product (SA, Mil.Chn.2005.EUR)	Statistical Office of the Slovak Republic
Slovakia: GDP: Gross Fixed Capital Formation (SA, Mil.EUR)	Statistical Office of the Slovak Republic
Slovakia: GDP: Gross Fixed Capital Formation (SA, Mil.Chn.2005.EUR)	Statistical Office of the Slovak Republic
Slovakia: GDP: Import of Goods and Services (SA, Mil.EUR)	Statistical Office of the Slovak Republic
Slovakia: GDP: Import of Goods and Services (SA, Mil.Chn.2005.EUR)	Statistical Office of the Slovak Republic
Slovakia: GDP: Export of Goods and Services (SA, Mil.EUR)	Statistical Office of the Slovak Republic
Slovakia: GDP: Export of Goods and Services (SA, Mil.Chn.2005.EUR)	Statistical Office of the Slovak Republic
Slovakia: Unemployment Rate [Registered] (SA, %)	Central Office of Labour, Social Affairs and Family
Slovakia: New Household Deposits: Redeemable at Notice: Up to 3 Months (%)	National Bank of Slovakia
Slovenia: GDP: Final Consumption: Households (SWDA, Mil.EUR)	Statistical Office of the Republic of Slovenia
Slovenia: GDP: Final Consumption: Households (SWDA, Mil.Chn.2000.EUR)	Statistical Office of the Republic of Slovenia
Slovenia: Gross Domestic Product (SWDA, Mil.EUR)	Statistical Office of the Republic of Slovenia
Slovenia: Gross Domestic Product (SWDA, Mil.Chn.2000.EUR)	Statistical Office of the Rep of Slovenia
Slovenia: GDP: Gross Fixed Capital Formation (SWDA, Mil.EUR)	Statistical Office of the Republic of Slovenia
Slovenia: GDP: Gross Fixed Capital Formation (SWDA, Mil.Chn.2000.EUR)	Statistical Office of the Republic of Slovenia
Slovenia: GDP: Imports of Goods and Services (SWDA, Mil.EUR)	Statistical Office of the Republic of Slovenia
Slovenia: GDP: Imports of Goods and Services (SWDA, Mil.Chn.2000.EUR)	Statistical Office of the Republic of Slovenia
Slovenia: GDP: Exports of Goods and Services (SWDA, Mil.EUR)	Statistical Office of the Republic of Slovenia
Slovenia: GDP: Exports of Goods and Services (SWDA, Mil.Chn.2000.EUR)	Statistical Office of the Republic of Slovenia
Slovenia: Unemployment Rate (%)	International Monetary Fund / IFS
Slovenia: Money Market Rate (% per annum)	International Monetary Fund / IFS
Turkey: Res/Nonresident HHs Final Consump Exp on Economic Territory(SA,Thous.TL)	Turkish Statistical Institute
Turkey: Res/Nonres HHs Final Consump Exp on Economic Territory (SA, Thous.98.TL)	Turkish Statistical Institute
Turkey: Gross Domestic Product (SA, Thous.TL)	Turkish Statistical Institute
Turkey: Gross Domestic Product (SA, Thous.98.TL)	Turkish Statistical Institute
Turkey: Gross Fixed Capital Formation (SA, Thous.TL)	Turkish Statistical Institute
Turkey: Gross Fixed Capital Formation (SA, Thous.98.TL)	Turkish Statistical Institute
Turkey: Exports of Goods & Services (SA, Thous.TL)	Turkish Statistical Institute
Turkey: Exports of Goods & Services (SA, Thous.98.TL)	Turkish Statistical Institute
Turkey: Imports of Goods & Services (SA, Thous.TL)	Turkish Statistical Institute
Turkey: Imports of Goods & Services (SA, Thous.98.TL)	Turkish Statistical Institute
Turkey: Unemployment Rate (SA, % of Labor Force)	Turkish Statistical Institute
Turkey: Weighted Average Interest Rates for TL Deposits: Up to 3 Months(% p.a.)	Central Bank of the Republic of Turkey

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