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Spillovers among sovereign debt markets: identification by absolute magnitude restrictions

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Abstract

This paper studies spillovers among US and European sovereign yields. We provide a new method based on absolute magnitude restrictions of the impact matrix to identify the countries that were the main sources of spillovers. Despite the large size of shocks from euro area stressed countries, connectedness among sovereign yields declined between 2008 and 2012 due to financial fragmentation, particularly between countries with more divergent business and fiscal cycles. We show that none of the sovereign yields are insulated from foreign shocks and that shocks to the Greek bond market in 2010 explained 20-30% of the variance of sovereign yields in stressed countries, while in 2011-2012 Italy (not Spain) was the source of systemic risk.

Keywords: Spillovers, Contagion, Connectedness, Fragmentation, Sovereign Risk, SVAR identification.

JEL Classification: C3, G2
Non-Technical Summary

When the turmoil in the Greek sovereign bond markets started in 2009, only few commentators anticipated that it would spread up to the point of threatening the existence of the euro area. However, the sovereign debt crisis progressively affected most of euro area countries and even spread to the rest of the world, via fire sales vis-à-vis risky assets or flight to safety vis-à-vis safe assets. The euro area sovereign debt crisis is only the most recent of many instances in which policy makers were confronted with the possibility that spillovers on the cost of financing of governments, households and enterprises could endanger countries with otherwise relatively sound economic fundamentals.

Typically during financial crisis, the larger are the shocks, the larger the correlations among asset prices. However, appropriate economic policy measures require the identification of the source of the shocks at early stages. In this paper, we provide a method that identifies asset price shocks, singling out the countries whose assets were the main source of spillovers during the great recession and the euro area sovereign debt crisis. We show the volatility of countries’ shocks and the dynamic pattern of the propagation mechanism across countries. Moreover, we determine the economic factors that contributed to the change in the propagation during the euro area sovereign debt crisis.

The literature has studied issues of spillovers using a variety of econometric approaches. One of the key messages of this literature is that the identification of shocks is the main challenge in estimating the spillovers.

We propose a new method that allows us to pin down the sources of spillovers and we study the degree of connectedness among US, UK and euro area sovereign debt markets. We use the yields on 10-year benchmark sovereign bonds over the daily period January 2005 - August 2015, controlling for conventional monetary policy rates of the Federal Reserve (FED), Bank of England (BoE) and ECB, and global shocks such oil prices and macro news of the G10 countries.

We find that sovereign debt markets are all at times highly interconnected and none of the 12 sovereign yields we consider, including the US Treasury, are insulated from shocks from other markets.

We also find that total connectedness among sovereign markets declined steadily from end-2008 to end-2012, as the result of increased financial fragmentation (defined as a fall in cross-market linkages), which was not offset by the increased size of shocks. A cross-sectional analysis suggests that the increased market fragmentation between 2008 and 2012 was relatively higher between countries with more divergent business and fiscal
cycle developments. The degree of financial exposure and geographical distance did not play a role.

In addition, our results show that, in the first phase of the euro area sovereign debt crisis in 2010, between 12% and 35% of the variations of Italian, Spanish, Irish and Portuguese bond yields were due to shocks originating in Greece. Instead, in 2011 and 2012, when the sovereign yields in Italy and Spain reached 6-7%, about 10-20% of the variations of the Spanish yields were due to shocks originating in Italy, while less than 10% of the variations of the Italian yields were due to shocks originating in Spain. Furthermore, the spillovers from Italy to Belgium, Germany, the Netherlands, the UK and the US were much larger than those from Spain. Hence, our analysis finds that shocks originating in Greece and Italy contributed to developments in sovereign spreads during the hikes of the sovereign debt crisis, and suggests that shocks in Spain had a smaller impact.
1 Introduction

When the turmoil in the Greek sovereign bond markets started in 2009, only few commentators anticipated that it would spread up to the point of threatening the existence of the euro area. However, the sovereign debt crisis progressively affected most of euro area countries and even spread to the rest of the world, via fire sales vis-à-vis risky assets or flight to safety vis-à-vis safe assets. The euro area sovereign debt crisis is only the most recent of many instances in which policy makers were confronted with the possibility that spillovers on the cost of financing of governments, households and enterprises could endanger countries with otherwise relatively sound economic fundamentals.

Typically during financial crisis, the larger are the shocks, the larger the correlations among asset prices. However, appropriate economic policy measures require the identification of the source of the shocks at early stages. In this paper, we provide a method that identifies asset price shocks, singling out the countries whose assets were the main source of spillovers during the great recession and the euro area sovereign debt crisis. We show the volatility of countries’ shocks and the dynamic pattern of the propagation mechanism across countries. Moreover, we determine the economic factors that contributed to the change in the propagation during the euro area sovereign debt crisis.

The literature has used a variety of econometric approaches to study spillovers (for a survey, see Forbes (2012) and Diebold and Yilmaz (2015)). One of the key messages of this literature is that the identification of the source of the financial shocks among asset prices is the main issue, because of their extraordinary degree of contemporaneous co-movement.

The strong contemporaneous co-movement among asset prices makes zero restrictions on the contemporaneous reaction of other financial prices implausible. It is also neither straightforward nor advisable to impose sign restrictions: first, it is not clear ex-ante whether the spillovers are more likely to generate a positive correlation (for example, because of fire sales) or a negative correlation (for example, because of flight-to-safety) and second, it is practically impossible to generate mutually exclusive sign restrictions that would properly identify a set of asset price specific shocks. This difficulty arises by the fact that asset price shocks are less consensual in theory. Due to these difficulties Diebold and Yilmaz (2014) suggest using generalized impulse-response functions (GIRFs) that...
focus on correlations among asset price shocks and do not provide a causal interpretation of shocks.

One useful approach that provides a causal interpretation of shocks is the identification by heteroskedasticity (Sentana and Fiorentini (2001); Rigobon (2003)). As the variances of asset specific shocks change in different regimes, one can use additional moments in the data to identify the system and, thereby, extract the structural parameters. However, the identification by heteroskedasticity requires that the contemporaneous relationship between the variables do not change between the different volatility regimes. The volatility regimes are assumed to be known. Moreover, the statistical identification of the shocks is often associated with zero and sign restrictions in order to obtain a sensible economic identification (Ehrmann et al. (2011) and Ehrmann and Fratzscher (2015)).

The new identification method suggested in this paper relies on the assumption that, on average over the sample period, the magnitude of the instantaneous direct effect of the shock is larger in absolute value than the magnitude of the instantaneous spillover. This identification scheme fits the anecdotal evidence on the impact of specific crisis events on long-term sovereign yields and is in line with the evidence found in event studies. For example, Table 1 reports the reaction of long-term bond yields after country specific events. The evidence suggests that, in all the events under analysis, the instantaneous response of the interest rate on the market where the event occurred was larger in absolute term than the response of bond yields with the same maturity on all other sovereign markets.

The identification by magnitude restrictions bears similarity to those used in Kilian and Murphy (2012) and De Graeve and Karas (2014), who use sign restrictions together with elasticity bounds on the impulse responses to identify shocks. The similarity with our method is related to the fact that we also use bounds on the impact matrix to identify shocks. However, we do not impose sign restrictions because, as mentioned above, we want to remain agnostic about the sign of the response functions. Our method is particularly suited to study spillovers in the sovereign debt markets during the financial crisis, when sovereign yields were influenced by opposite forces, such as flight-to-safety and flight-to-liquidity on the one hand and fire sales on the other hand; and, as a consequence, the...
sign cannot be restricted a-priori.

Table 1: The impact of selected crisis events on 10-year benchmark sovereign bond yields

<table>
<thead>
<tr>
<th></th>
<th>AT</th>
<th>BE</th>
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<th>ES</th>
<th>FI</th>
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<th>GR</th>
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<td>-9</td>
<td>-9</td>
<td>-6</td>
<td>-7</td>
<td>-11</td>
<td>-1</td>
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<td>-5</td>
<td>-9</td>
<td>-10</td>
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</tr>
<tr>
<td>UK</td>
<td>-13</td>
<td>-9</td>
<td>-9</td>
<td>-14</td>
<td>-7</td>
<td>-9</td>
<td>-1</td>
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<tr>
<td>GR</td>
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<td>6</td>
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<td>2</td>
<td>3</td>
<td>0</td>
<td>22</td>
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<td>2</td>
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<td>-4</td>
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<td>-2</td>
<td>-1</td>
<td>1</td>
<td>11</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: Thomson Reuters.

Note: (1) On 29 September 2008, Finance Minister of Germany Peer Steinbrueck announced that a 35 billion credit line would be extended to Hypo Real Estate from the government and a consortium of German banks. (2) On 25 November 2008, the US Federal Reserve (FED) announced the launch of the large-scale asset purchase (LSAP) programme. (3) On 5 March 2009, Bank of England (BoE) announced the launch of the asset purchase facility (APF) programme. (4) On 5 March 2010, the Greek government revealed a revised budget deficit of 12.7% of GDP for 2009. (5) On 24 November 2011, the rating agency Fitch lowered the rating of the Portugal’s sovereign debt by one notch to the non-investment grade category, from triple-B-plus to double-B-plus, and maintained a negative outlook. (6) On 22 May 2013, the US FED announced that it would begin tapering back the LSAP programme.

We use the yields on 10-year benchmark sovereign bonds over the daily period January 2005 - August 2015. Specifically, we run a standard vector autoregression with exogenous variables (VARX), where the endogenous variables include 10 euro area sovereign yields, the US Treasury and the UK gilt yields, the conventional monetary policy rates of the Federal Reserve (FED), Bank of England (BoE) and European Central Bank (ECB) and as exogenous variables oil prices and macro news of the G10 countries. In order to study the dynamic effects particularly during the euro area sovereign debt crisis, when the shifts in volatility became extreme, the VARX is estimated using rolling correlations as in Diebold and Yilmaz (2013).

Our results show that sovereign debt markets are highly interconnected, as a large fraction of the variance of sovereign yields can be explained by foreign sovereign yield shocks, and that none of the 12 sovereign yields we consider, including the US Treasury, are insulated from shocks from other markets. In the first phase of the euro area sovereign debt crisis in 2010, between 12% and 35% of the variations of Italian, Spanish, Irish and Portuguese bond yields were due to shocks originating in Greece. Instead, in 2011 and 2012, when the sovereign yields in Italy and Spain reached 6-7%, about 10-20% of the variations of the Spanish yields were due to shocks originating in Italy and the spillovers from Italy to Belgium, Germany, the Netherlands, the UK and the US were much larger than those from Spain. Hence, shocks originating in Greece and Italy contributed to developments in sovereign spreads during the hikes of the sovereign debt crisis.

Interdependence among sovereign yields is studied using the variance decomposition
of shocks as in Diebold and Yilmaz (2014). We find that total connectedness among sovereign markets declined steadily from end-2008 to end-2012. Forbes and Rigobon (2002) and Rigobon (2003) show that the change in conditional correlations and therefore connectedness among asset prices can be the sole results of an increase in market volatility. Thus, to understand the drop in total connectedness, we separate the effects of volatility of asset price shocks from the change in the propagation mechanism. Specifically, we combine the measurement of connectedness developed by Diebold and Yilmaz (2014) with the concept of contagion developed by Forbes and Rigobon (2002), which is related to the structure of the economy measured by the cross-market linkages, which regulate the propagation mechanism of shocks. We show that the decline in connectedness was the result of a weaker propagation mechanism, not offset by the increased size of shocks. The weaker transmission of the shocks, namely the intensification of financial fragmentation, characterized both stressed and non-stressed countries. Market fragmentation improved substantially in 2013 settling in 2014 and 2015 to levels slightly below those recorded before the financial crisis started with Lehman Brothers' collapse. These results are in line with the findings by Caporin et al. (2013), who use quantile regression to estimate the parameters that govern the transmission, and by Ehrmann and Fratzscher (2013), who identify the shocks through heteroskedasticity.

We also perform a cross-sectional analysis to identify the economic factors behind the increase in fragmentation between 2008 and 2012. To address this question we regress the change in bilateral links among countries between 2008 and 2012 against a large number of regressors including geographical distance, differences in countries’ economic growth and fiscal space, and various measures of bilateral exposures associated to commodity trade, portfolio assets and bank claims. The results suggest that the increase in market fragmentation was relatively higher between countries with more divergent business and fiscal cycle developments. The degree of trade and financial exposures and geographical distance are statistically insignificant when controlling for economic distance between countries.

The remaining sections of the paper are structured as follows. Section 2 describes the methodology. Section 3 presents the data and the empirical model. Section 4 describes the results and shows the performance of our method relative to GIRFs. Section 5 concludes.
2 Econometric methodology

2.1 SVAR setup

A structural vector autoregression model (SVAR) can be written as:

\[ A_0 y_t = A_1 y_{t-1} + A_2 y_{t-2} + \ldots + A_K y_{t-K} + B \varepsilon_t, \]  

(2.1)

where \( y_t \) is the \( N \times 1 \) vector of endogenous variables, \( K \) is a finite number of lags, and the structural shocks \( \varepsilon_t \) are assumed to be white noise, \( \mathcal{N}(0, I_N) \). \( A_0 \) describes the contemporaneous relations between the variables, while matrices \( A_k, k \in [1, 2, \ldots, K] \), describe the dynamic relationships. The diagonal matrix \( B \) contains the standard errors of the structural shocks.

The system (2.1) implies a following structural moving average representation,

\[ y_t = B(L) \varepsilon_t, \]  

where \( B(L) \) is a polynomial in the lag operator. The system in (2.1) cannot be estimated directly, but needs to be estimated in its reduced form:

\[ y_t = A^*_{11} y_{t-1} + A^*_{21} y_{t-2} + \ldots + A^*_{P1} y_{t-P} + \varepsilon_t, \]  

(2.2)

where \( \varepsilon_t = A^*_{01} B \varepsilon_t \) and \( A^*_{k1} = A^{-1}_0 A_k \).

The moving average representation of (2.2) is \( y_t = C(L) u_t \). Therefore, the reduced form response function, \( C(L) \), is related to the structural impulse response function by \( B(L) = A_0 C(L) \). In other words, to identify the structural shocks and obtain the structural impulse responses, \( A_0 \) ought to be identified.

Given \( S = A_0^{-1} B \), \( A_0 \) is such that \( \Sigma_u = SS^t \), where \( \Sigma_u \) is the variance-covariance matrix of the reduced form errors. The decomposition \( \Sigma_u = SS^t \) is not unique. For any \( H \) such that \( HH^t = I \), the matrix \( SH \) also satisfies this condition. In this case, \( SH(SH)^t = SS^t = \Sigma_u \). Therefore, starting from any arbitrary \( \tilde{S} \), such that \( \Sigma_u = \tilde{S} \tilde{S}^t \) (i.e. a Cholesky decomposition of \( \Sigma_u \)), alternative decompositions can be found by post-multiplying by any \( H \). The entire set of permissible impact matrices is infinite and the impact matrix cannot be identified uniquely from data.

2.2 Identification: The magnitude restriction method

Prior assumptions are required to achieve identification. In this paper, rather than imposing a set of \( k(k-1)/2 \) restrictions that guarantees unique identification, we obtain the distribution of impulse response functions by retaining only those models that sat-
isfy prior constraints using the QR decomposition of Rubio-Ramirez et al. (2010), which works with the uniform Haar prior. Specifically, we impose restrictions on the size of contemporaneous spillovers at impact.

Let \( \hat{\psi}_{i,j,0} \) be the instantaneous response of variable \( i \) to shock \( j \) and \( \hat{\psi}_{j,j,0} \) the instantaneous response of variable \( j \) to the structural shock \( j \). We identify the orthogonal structural shock in market \( j \), \( \epsilon_j \), by assuming that \( |\hat{\psi}_{i,j,0}| < |\hat{\psi}_{j,j,0}| \forall i \neq j \). Intuitively, this assumption implies that the contemporaneous spillovers to other markets from shock \( \epsilon_j \) are smaller than the direct effect of shock \( \epsilon_j \) on market \( j \) at impact.

For a given \( H \) we obtain an estimate of \( A_0 \), denoted \( \hat{A}_0 \), and the impact response matrix \( \hat{A}_0^{-1}B \). With the diagonal elements of \( \hat{A}_0 \) normalized to 1, the off-diagonal elements can be written as:

\[
\hat{a}_{i,j,0} = \frac{\hat{\psi}_{i,j,0}}{\hat{\psi}_{j,j,0}}.
\]

Hence, the identifying restriction is \( |\hat{a}_{i,j,0}| < 1 \forall i \neq j \). For each \( H \), we keep the corresponding estimate of the impulse response functions (IRFs) only if the resulting \( \hat{A}_0 \) satisfies the restrictions on the size of spillovers.

Given the large number of shocks to be identified, a numerical algorithm is employed to facilitate the search of models that are consistent with priors and data (see Appendix A).

The sources of the sovereign yield shocks (i.e. country-specific supply developments or fiscal policy or unconventional monetary policy) remain unknown with our methodology. However, regardless of the economic interpretation, the identified shocks provide useful information about the source of the risk and how it transmits across assets.

2.3 An illustrative example

The idea behind the magnitude restriction method can be appreciated with a simple example. Consider a system with two variables, and let:

\[
A_0^{-1} = \begin{bmatrix} 1 & b \\ c & 1 \end{bmatrix}, \quad B = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}
\]

so that

\[
\alpha_{i,j} = \frac{\sigma_i}{\sigma_j}, \quad \text{where } \sigma_i \text{ and } \sigma_j \text{ denote the standard deviations of shocks of asset } i \text{ and } j, \text{ respectively.}
\]

Another plausible assumption is to normalize the instantaneous impact by the standard deviation of shocks: \( \alpha_{i,j} = \sigma_i/\sigma_j \). In this paper, we assume that \( \alpha_{i,j} = 1 \).
\[ A_0^{-1}B = \begin{bmatrix} \sigma_1 & b\sigma_2 \\ \alpha_1 & \sigma_2 \end{bmatrix} \quad \Sigma_u = \begin{bmatrix} \sigma_1^2 + b^2\sigma_2^2 & \sigma_1^2 + b\sigma_2^2 \\ \alpha_1^2 + b\alpha_2^2 & \alpha_1^2 + b^2\alpha_2^2 \end{bmatrix} \]

where \( A_0^{-1} \) is the impact matrix.

The identification problem arises because the variance-covariance matrix is symmetric and the system is characterized by three equations and four parameters: \( b, c, \sigma_1 \) and \( \sigma_2 \).

Define the estimated variance-covariance matrix as \( \hat{\Sigma}_u = \begin{bmatrix} \hat{\Sigma}_{11} & \hat{\Sigma}_{12} \\ \hat{\Sigma}_{21} & \hat{\Sigma}_{22} \end{bmatrix} \), then the three equations can be written as follows:

\[
\begin{align*}
\sigma_1^2 + b^2\sigma_2^2 &= \hat{\Sigma}_{11} \\
\alpha_1^2 + b\alpha_2^2 &= \hat{\Sigma}_{21} \\
c^2\sigma_1^2 + \sigma_2^2 &= \hat{\Sigma}_{22}.
\end{align*}
\]

A typical unique solution of this system is a zero restriction (i.e. \( b = 0 \)). Alternatively, one can impose prior information such as sign restrictions (i.e. \( b > 0 \) and \( c > 0 \)), or magnitude restrictions (i.e. \(|b|, |c| < 1\)). The latter methods only give a set of models that are consistent with prior information and data.

In order to explain the magnitude restriction method graphically, let us assume that the standard errors of the shocks are equal to one, \( \sigma_1 = 1 \) and \( \sigma_2 = 1 \). This allows us to discard two equations of the system, which reduces to one equation, \( c + b = \hat{\Sigma}_{21} \), and two unknowns, \( b \) and \( c \). Given the estimated covariance \( \hat{\Sigma}_{21} \), we can plot all solutions that identify \( b \) and \( c \).

Figure 1 shows three cases under the hypothesis that \( \hat{\Sigma}_{21} = 1 \) to the left panel, \( \hat{\Sigma}_{21} = 1.5 \) in the middle panel and \( \hat{\Sigma}_{21} = 0.5 \) to the right panel. The black lines provide all possible solutions to this equation. The number of solutions is infinite, but still bounded by the combinations of \( b \) and \( c \) lying on the black line.

Figure 1: Identification with magnitude restrictions: Solutions to \( c + b = \hat{\Sigma}_{21} \)
The magnitude restrictions \(|b|, |c| < 1\) that we impose can be represented by the shaded gray area in these graphs. The bold black line shows the intersection of the magnitude restrictions with all possible combinations of parameters consistent with data. This identified set is relatively small compared to the set of solutions when restrictions are not imposed. In this illustrative example with magnitude restrictions, we get \(b, c \in [0, 1]\) when \(\Sigma_{21} = 1\) (left panel), \(b, c \in [0.5, 1]\) when \(\Sigma_{21} = 1.5\) (middle panel) and \(b, c \in [-0.5, 1]\) when \(\Sigma_{21} = 0.5\) (right panel). Moreover, the higher the estimated covariance in absolute value, the smaller the set of accepted models (middle panel).

2.4 Measuring connectedness

Following Diebold and Yilmaz (2014), we measure financial connectedness by means of the forecast error variance decomposition (FEVD). Once shocks are identified and the appropriate \(H\) matrix is selected, one can compute the \(h\)-step ahead forecast error:

\[
y_{t+h} - E_t y_{t+h} = \sum_{\tau=0}^{h-1} C_r \tilde{S} H_{t+h-r}. \quad (2.4)
\]

Denoting by \(\psi_{i,j,h} (i, j \in [1, 2, \ldots, N])\) the \((i,j)\)th element of the orthogonalized impulse response coefficient matrix \(C(L)\tilde{S}H\) at horizon \(h\), the \(h\)-step ahead forecast error variance of variable \(i\) is:

\[
\varsigma^2_i(h) = \sum_{\tau=0}^{h-1} (\psi^2_{i,1-r} + \psi^2_{i,2-r} + \ldots + \psi^2_{i,N-r}), \quad (2.5)
\]

where \((\psi^2_{i,0} + \psi^2_{i,1} + \ldots + \psi^2_{i,h-1})\) provides the contribution of shock \(j\) to the \(h\)-step forecast error variance of variable \(i\). Hence, the percentage contribution of shock \(j\) to the \(h\)-step forecast error variance of variable \(i\) is:

\[
\omega^2_{i,j}(h) = \frac{\psi^2_{i,j,0} + \psi^2_{i,j,1} + \ldots + \psi^2_{i,j,h-1}}{\varsigma^2_i(h)}. \quad (2.6)
\]

The innovative idea of Diebold and Yilmaz (2014) is that you can treat FEVD as an adjacency matrix that defines a directed weighted network. The connectedness table describes how the adjacency matrix captures connectedness (see Table 2). The upper-left \(N \times N\) block contains the FEVD at horizon \(h\). The off-diagonal

\[\text{For more details see Diebold and Yilmaz (2014).}\]
In other words, forecast-error variance explained by shock $j$ connectedness from market $j$, $C_{i,j}$ of the off-diagonal elements. Total directional connectedness from market $j$, $C_{j}^{H}$ relations are symmetric, $C_{i,j} = C_{j,i}$. The diagonal elements define a market’s own connectedness, $C_{i,i}^{H} = \omega_{i}^{2}(h)$. The aggregate connectedness statistics are obtained by taking row and column sums of the off-diagonal elements. Total directional connectedness from market $j$ to other countries is defined as:

$$C_{i,j}^{H} = \frac{1}{N-1} \sum_{h=1}^{N} \omega_{i,j}^{2}(h)$$

(2.7)

In other words, $C_{i,j}^{H}$ is the sum of the $j$-th column elements of the FEVD except its own share, $\omega_{i,j}^{2}(h)$. This connectedness ‘to others’ provides the average share of the $h$-step forecast-error variance explained by shock $j$ and, therefore, it summarizes the importance of asset $j$ shocks in inducing fluctuations in all other markets.

Similarly, by taking sums over rows, a total directional connectedness to market $i$ elements describe pairwise directional connectedness measures from market $j$ to market $i$, $C_{i,j}^{W} = \omega_{i,j}^{2}(h)$. The network is weighted, $\omega_{i,j}^{2}(h) \in [0,1]$, and directed, $C_{i,j}^{W} \neq C_{j,i}^{W}$. The diagonal elements define a market’s own connectedness, $C_{i}^{W} = \omega_{i}^{2}(h)$.

Table 2: Connectedness Table

<table>
<thead>
<tr>
<th>$i_1$</th>
<th>$i_2$</th>
<th>$\cdots$</th>
<th>$i_N$</th>
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<tr>
<td>$y_1$</td>
<td>$\omega_{1,1}^{2}(h)$</td>
<td>$\omega_{1,2}^{2}(h)$</td>
<td>$\cdots$</td>
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<tr>
<td>$y_2$</td>
<td>$\omega_{2,1}^{2}(h)$</td>
<td>$\omega_{2,2}^{2}(h)$</td>
<td>$\cdots$</td>
<td>$\omega_{2,N}^{2}(h)$</td>
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<tr>
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<td>$\omega_{N,2}^{2}(h)$</td>
<td>$\cdots$</td>
<td>$\omega_{N,N}^{2}(h)$</td>
</tr>
</tbody>
</table>

| To Others | $\frac{1}{N-1} \sum_{h=1}^{N} \omega_{i,1}^{2}(h)$ | $\frac{1}{N-1} \sum_{i=1}^{N} \omega_{i,2}^{2}(h)$ | $\cdots$ | $\frac{1}{N-1} \sum_{i=1}^{N} \omega_{i,N}^{2}(h)$ | $\frac{1}{N-1} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \omega_{i,j}^{2}(h)$ |

Note: This table shows the connectedness across markets. The off-diagonal elements define the pairwise directional connectedness from market $j$ to market $i$, $C_{i,j}^{W} = \omega_{i,j}^{2}(h)$. The network is weighted, $\omega_{i,j}^{2}(h) \in [0,1]$, and directed, $C_{i,j}^{W} \neq C_{j,i}^{W}$. The diagonal elements define a market’s own connectedness, $C_{i,i}^{W} = \omega_{i}^{2}(h)$.

In an unweighted network $\omega_{i,j}$ is either 1 or 0, so that the adjacency matrix only specifies whether a relation exists or not, but does not specify the strength of the relation. In an undirected network relations are symmetric, $C_{i,j}^{W} = C_{j,i}^{W}$.

7Compared to Diebold and Yilmaz (2014), we prefer to scale ‘to others’ connectedness by $N-1$, because this statistic is bounded in the interval $[0,1]$ and it is easier to interpret. We adopt the same scaling for ‘from others’ connectedness.
from all shocks can be constructed:

\[
C_{i,i}^H = \frac{1}{N-1} \sum_{j=1}^{N} \omega_{i,j}^2(h) \tag{2.8}
\]

where \(C_{i,i}^H\) gives the average share of the \(h\)-step forecast-error variance of market \(i\) coming from shocks originating from all other markets.

Finally, total connectedness \((C^H)\) and total connectedness among a subset of assets \(S \subseteq \{1,\ldots,N\}\) \((C_{i,j|S}^H)\) can be computed as follows:

\[
C^H = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_{i,j}^2(h) \tag{2.9}
\]

\[
C_{i,j|S}^H = \frac{1}{N_{S}} \sum_{i \in S} \sum_{j \in S, i \neq j} \omega_{i,j}^2(h). \tag{2.10}
\]

### 3 Data and specification of the SVAR

We apply our method to study the transmission of shocks in the sovereign debt market of the US, the UK and ten euro area countries (Austria, Belgium, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal, Spain). Monetary policy rates in the three economies are included in order to control for conventional monetary policy shocks that shape the yield curves. Moreover, we control for other global factors such as the oil price and the macro news of the G10 economies.

We use 10-year benchmark sovereign yields and the 3-month Overnight Indexed Swap (OIS) rates provided by Thomson Reuters (see Figure 2). It is useful to point out the high degree of comovement between sovereign yields of Italian and Spanish assets, of US and UK assets and of assets issued by Austria, France, Germany and the Netherlands. This is the reason why a SVAR approach is needed to identify from which country the shock is originated.

The OIS rate uses an overnight rate index, such as the EONIA for euro-denominated products, the SONIA for sterling-denominated products or the Federal Funds Rate for US dollar-denominated products. Therefore, at short maturities, they are good indicators of the conventional monetary policy stance. The macro news is provided by Citibank and the Brent price in US dollar is provided by Thomson Reuters. The data is daily, running
from 3 January 2005 to 24 August 2015. The monetary policy rates, oil prices and the macro news absorb common effects on bond markets.

Specifically, the SVAR is characterised by the following three assumptions (i) monetary policy rates do not react contemporaneously to sovereign yield shocks, while conventional monetary policy shocks contemporaneously affect sovereign yields, (ii) oil price and macro news are exogenous and (iii) a shock to asset \( j \) is such that the instantaneous response of the sovereign yield \( j \) is larger in absolute value than the instantaneous response of sovereign yield \( i \).

The methodology remains agnostic about the underlying idiosyncratic fundamental shocks to sovereign yields, which could be associated to country-specific supply developments, fiscal or unconventional monetary policies. The novel of this framework is that we can identify the asset under stress and how it transmits across all other countries.

The predictive horizon \( H \) is important because connectedness measures are time dependent. Following Diebold and Yilmaz (2014) we focus on a medium-run horizon of \( H = 12 \) days, although we also consider results on a short-run horizon of \( H = 2 \) days. The latter results are not reported, but they are available upon request.

To study the dynamics of spillovers, contagion and connectedness, we follow Diebold and Yilmaz (2014) and estimate the VAR using a rolling window of 200 days. The VAR is estimated in levels with a constant and its lag length is selected using the Bayesian Information Criterion (BIC).

4 Results

This section provides the empirical results. Section 4.1 shows the rolling-sample (dynamic) analysis in the spirit of Diebold and Yilmaz (2014). Section 4.2 shows a counterfactual exercise to disentangle the importance of contagion versus the role of shocks in explaining connectedness in the spirit of Forbes and Rigobon (2002). Section 4.3 provides an explanation for the decline in contagion or the intensification of financial fragmentation during the financial crisis. Section 4.5 looks at impact of shocks to specific assets in more detail. Finally, Section 4.6 discusses how the results would differ using GIRF analysis.
4.1 IRFs, shocks and connectedness

The key inputs for the analysis are the IRFs. Given that we consider 15 markets and 46
200-day rolling windows, presenting and discussing 9660 IRFs is impractical. Here, we
present a sub-set of IRFs between the sovereign yields of the key countries (Greece, Italy,
Germany, UK and US).

First, we show how the impact matrix has changed over time (see Figure 3). Shocks
from sovereign yields in Greece (Italy) had a contemporaneous impact on Italy (Greece)
and Germany before the sovereign debt crisis started, while the impact in the UK and the
US has always been statistically insignificant. On average, the impact has been declining
over time, as we enter in the sovereign debt crisis. Interestingly, during the hikes of the
sovereign debt crisis in 2012, a positive adverse shock from Italy reduced German yields
at impact, owing to flight-to-safety as it will be further discussed. Conversely, shocks
to German yields also affected the British guilt and the US Treasury instantaneously.
Similarly, shocks from the UK and the US have affected each other jurisdiction and
Germany, but not Italy and Greece at impact.

[Insert Figures 3 here]

Second, we show IRFs over the two specific sovereign crisis periods: the October 2009-
July 2010 period, associated to the Greek crisis, and the May 2011-April 2012 period,
associated to the euro area break-up risk.

As shown in Figure 4, our results suggest that Greece played a key role in 2010,
driving up the sovereign yields in stressed countries (i.e. fire sales) and down those of
non-stressed countries, including the United States due to flight to safety and liquidity
considerations. Instead, Italy became a source of volatility in the second phase of the

[Insert Figures 4, here]

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area sovereign debt crisis was exacerbated, and so spillovers within the euro area became more important. These results further justify the relevance of the time-varying analysis of such shocks.

A complementary approach to appreciate the results is assessing the estimated standard deviation of the shocks to sovereign yields for each 200-day rolling-sample windows (see Figure 4). The shocks to yields on sovereign bonds issued by Germany, Austria, the Netherlands, Belgium, France, the UK and the US were small and relatively stable over the sample. Conversely, the shocks originating in Greece, Portugal, Ireland, Italy and Spain changed over time, and were at times very large. Shocks to US and UK sovereign yields stabilised with the launch of the quantitative easing programmes in 2009 and they declined steadily after the beginning of the sovereign debt crisis in the euro area.

Overall, the dynamics of the shocks obtained with our method are in line with anecdotal evidence. For example, at their peak, the size of the shock to sovereign yields is largest in Greece, followed by Portugal, Ireland, Italy and Spain. In addition, the dynamics of the shocks matches the conventional wisdom: there are large shocks in Greece and Ireland in 2010 resulting in the European troika programme\(^9\) and in 2012 due to the credit event on Greek sovereign debt; similarly, there are large shocks in Italy and Spain in 2011 and 2012 due to fears of euro area break-up; finally, bigger shocks in all sovereign assets are estimated since the crisis in inter-bank markets in August 2007.

When comparing our results with those obtained with GIRFs, the shocks estimated with the latter method are always far larger. These differences are then reflected in all the relevant measures of connectedness.

The estimated IRFs are then used to construct total connectedness measured using expression (2.9). The overall degree of connectedness among short-term money market rates and long-term sovereign yields is reported in Figure 6 and is constructed using all shocks to sovereign yields and monetary policy rates. Before Lehman Brothers’ bankruptcy, connectedness fluctuated between 80% and 85%. With the collapse of the financial sector after September 2008, connectedness declined sharply and steadily

\(^9\)The European troika addressing the financial crisis is formed by the EU Commission, the International Monetary Fund (IMF) and the European Central Bank.
reaching the lowest point in the summer of 2012. After the launch of the Outright Monetary Transactions (OMTs) by the ECB in September 2012, it rose again settling in 2015 at 75%. The results obtained using GIRF are very similar (see Figure 7).

However, the results obtained using GIRF and the magnitude restriction method differ substantially if we estimate connectedness excluding conventional monetary policy shocks. Figure 8 measures the degree of connectedness among long-term sovereign yields and is constructed excluding conventional monetary policy shocks using expression (2.10). Over the period January 2005 to September 2009, total connectedness among sovereign yields was very volatile and ranged between 60% and 78% according to the magnitude restriction method, while it fluctuated around 85% when using GIRF. The gap is the result of standard monetary policy shocks that are orthogonal to sovereign yield shocks when identified with the magnitude restriction method. In fact, the two methods reconcile at the end of 2009, when the zero-lower bound was reached and non-standard measures were introduced. Total connectedness declined steadily from 77% in October 2009 to 56% in the first half of 2012. The negative trend in connectedness was particularly notable from mid-2010 until the summer of 2012, when the fears of the break-up of the euro area exacerbated the financial crisis. It then increased steadily, returning to pre-crisis levels in 2014.

In order to understand the geographical forces behind the change in total connectedness, we split the sample in two sub-groups - the stressed countries (Greece, Ireland, Italy, Portugal and Spain) and the non-stressed countries (Austria, Belgium, France, Germany, the Netherlands, UK, US) - and study connectedness within each group and among groups. The results suggest that during the sovereign debt crisis bond markets were highly fragmented, due to a steady decline in connectedness from non-stressed to stressed countries. These results differ from those obtained with GIRFs by Claeys and Vaˇ s´ıˇ cek (2014), who find that connectedness among sovereign yields increased during the sovereign debt crisis. The discrepancy is due to the use of common factors from sovereign yield spreads in a Factor Augmented VAR. In this model, the contemporaneous spillovers between countries that were high before the crisis is absorbed by the principal component and connectedness is estimated to be much lower before the sovereign debt crisis. Instead qualitatively similar results are obtained by Ehrmann and Fratzscher (2015), who finds a substantial fragmentation among euro area government bond markets from 2010 onwards by exploiting the heteroskedasticity of changes in bond yields to identify the model.
stressed countries (see Figure 9), but also due to the sharp fall in connectedness in 2011 among the non-stressed countries and among the stressed countries. It is worth emphasising the large differences between the magnitude restriction method and GIRFs. The additional different results obtained with the two methods are discussed in Section 4.6.

4.2 What drives the change in connectedness?

Connectedness, spillovers and contagion are sometimes used interchangeably in the literature. In this paper, we have defined them as follows:

- **Connectedness**:
  \[ \Omega(h) = f(\Phi(A_1, ..., A_N), A_0^{-1}, B) \]
  As in Diebold and Yilmaz (2014), connectedness is constructed using the FEVD, which is a non-linear function of the cross-market linkage parameters defining the propagation mechanism, \( \Phi(A_1, ..., A_N) \) and \( A_0^{-1} \), and the size of the shocks, \( B \).

- **Spillovers**:
  \[ \Psi(h) = \Phi(A_1, ..., A_N)A_0^{-1}B \]
  Spillovers are measured using the IRFs, which are a non-linear function of \( \Phi(A_1, ..., A_N) \), \( A_0^{-1} \) and \( B \). While connectedness is always positive, spillovers can be positive or negative.

- **Contagion**:
  \[ \Upsilon(h) = \Phi(A_1, ..., A_N)A_0^{-1} \]
  As in Forbes and Rigobon (2002), contagion is related to the structure of the economy and is captured by the parameters defining the dynamic relationship among variables, \( \Phi(A_1, ..., A_N) \), and the matrix of contemporaneous effects, \( A_0^{-1} \).

Define with \( w \) a specific sample window: \( w \in [1, 2, ..., W] \), where \( W \) is the total number of rolling windows (in our case \( W = 51 \)), and with \( P = 1 \) the first pre-crisis rolling window covering the period 3 January 2005 - 7 October 2005. Then, to disentangle connectedness due to shocks from that due to contagion, we construct the following four measures of connectedness:

- connectedness when both contagion and shocks vary over time: \( \Omega(h)^w = f(\Upsilon^w, B^w) \);
The contribution of contagion and shocks to the overall connectedness is computed subtracting $\Omega(h)^P$ from $\Omega(h)^w,B^P$ and $\Omega(h)^w,T^P$, respectively. As described in the bottom panel, connectedness due to shocks increased by 25 percentage points at its peak in the summer of 2012. Despite this, total connectedness dropped due to a fall in contagion by 40 percentage points. Clearly, financial fragmentation characterized the euro area government debt market between 2009 and 2012 with the intensification of the sovereign debt crisis.

The counter-factual analysis is carried out in more detail investigating the role of stressed countries (see Figure 11) versus the role of non-stressed countries (see Figure 12). While the shocks originated in the stressed countries contributed positively to connectedness among all sovereign yields, the decline in cross-market linkages was a common fact. Both stressed and non-stressed countries contributed positively to the intensification of financial fragmentation, which suggest that fragmentation was a widespread phenomenon.

4.3 What is the mechanism explaining the decline in contagion?

Total connectedness among sovereign yields declined sharply between 2008 and 2012 due to a fall in the propagation mechanism of the shocks. Do economic fundamentals matter?
To address this question we regress the change in bilateral connectedness from asset \(i\) to asset \(j\) between 2008 and 2012 against a large number of regressors, which can be divided in two main groups: distance and exposure.

As for distance, we control for geographical distance and economic distance. Geographical distance is proxied by the simple distance between the most populated cities. We also consider the simple distance between capitals or the population-weighted distance, whether countries share a common official language or whether countries are contiguous. All these measures are provided by CEPII (Centre d’Etudes Prospectives et d’Informations Internationales). As for the economic distance, we consider (1) the squared difference between country \(i\) and country \(j\) of GDP growth cumulated over the period 2008-2012 and (2) the squared difference between country \(i\) and country \(j\) of the change in government debt-to-GDP ratio over the period 2008-2012. Countries’ GDP are provided by Thomson Reuters, while government debt-to-GDP ratios are provided by Eurostat.

As for exposure, we look at the role of exports, portfolio assets (equity as well as bond securities) and bank claims. Specifically, we consider (1) the change in exports’ share of country \(j\) vis-à-vis country \(i\) over the period 2008-2012 or its exports’ share in 2008; (2) the change in portfolio assets’ (equity securities’ [bond securities’] share of country \(j\) vis-à-vis country \(i\) over the period 2008-2012 or its portfolio assets’ (equity securities’ [bond securities’] share in 2008; (3) the change in bank claims’ share of country \(j\) vis-à-vis country \(i\) over the period 2008-2012 or its bank claims’ share in 2008. Country exports and portfolio assets are provided by the IMF, while bank claims are provided by the Bank for International Settlement (BIS).

The cross-section takes two alternative specifications:

\[
\Delta \omega_{ij}^2(h) = \beta_1 + \beta_2(\Delta g_i - \Delta g_j)^2 + \beta_3(\Delta f_i - \Delta f_j)^2 + \beta_4 d_{i,j} + \beta_5 x_{i,j} + \xi_{ij},
\]

\[
\Delta \omega_{ij}^2(h) = \beta_1 + \beta_2(\Delta g_i - \Delta g_j)^2 + \beta_3(\Delta f_i - \Delta f_j)^2 + \beta_4 d_{i,j} + \beta_5 x_{i,j} + \xi_{ij},
\]

where \(\Delta\) is the difference operator over the period 2008-2012, \(\omega_{ij}^2(h)\) denotes connectedness from asset \(i\) to asset \(j\), \(g_i\) is the log of real GDP in country \(i\), \(f_i\) is the debt-to-GDP ratio in country \(i\), \(d_{i,j}\) is geographical distance between country \(i\) and country \(j\), \(x_{i,j}\) measures the exposure of country \(j\) versus country \(i\) and \(\beta\) control for country \(i\)’s time-invariant unobserved heterogeneity.
Table 3: Factors affecting the change in contagion

<table>
<thead>
<tr>
<th>GDP dist</th>
<th>Debt Dist</th>
<th>Geo. Dist</th>
<th>Exposure</th>
<th>$R^2$</th>
<th>Obs.</th>
<th>Exposure$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Benchmark regression</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assets total</td>
<td>OLS</td>
<td>-0.14***</td>
<td>-6.69**</td>
<td>-0.01</td>
<td>0.64</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>HAC</td>
<td>(3.90)</td>
<td>(2.07)</td>
<td>(0.05)</td>
<td>(0.62)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>W. OLS</td>
<td>-0.10***</td>
<td>-6.69***</td>
<td>-0.01</td>
<td>0.64</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.68)</td>
<td>(3.43)</td>
<td>(0.06)</td>
<td>(0.58)</td>
<td></td>
</tr>
<tr>
<td>Assets equity</td>
<td>OLS</td>
<td>-0.17***</td>
<td>-6.51**</td>
<td>-0.03</td>
<td>-1.3</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>HAC</td>
<td>(4.65)</td>
<td>(2.01)</td>
<td>(0.16)</td>
<td>(1.20)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>W. OLS</td>
<td>-0.11***</td>
<td>-4.97*</td>
<td>0.06</td>
<td>-0.55</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.79)</td>
<td>(1.90)</td>
<td>(0.34)</td>
<td>(0.54)</td>
<td></td>
</tr>
<tr>
<td>Exports</td>
<td>OLS</td>
<td>-0.15***</td>
<td>-5.62*</td>
<td>0.04</td>
<td>0.52</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>HAC</td>
<td>(4.15)</td>
<td>(2.77)</td>
<td>(0.39)</td>
<td>(1.29)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>W. OLS</td>
<td>-0.11***</td>
<td>-3.99</td>
<td>0.12</td>
<td>-1.23</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.06)</td>
<td>(1.59)</td>
<td>(0.71)</td>
<td>(0.70)</td>
<td></td>
</tr>
<tr>
<td>Bank claims</td>
<td>OLS</td>
<td>-0.15***</td>
<td>-5.62***</td>
<td>0.04</td>
<td>0.52</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>HAC</td>
<td>(4.00)</td>
<td>(2.91)</td>
<td>(0.27)</td>
<td>(0.91)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>W. OLS</td>
<td>-0.11***</td>
<td>-4.03</td>
<td>0.11</td>
<td>0.25</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.14)</td>
<td>(1.60)</td>
<td>(0.70)</td>
<td>(0.87)</td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Exposure in 2008 levels

| Assets total | OLS | -0.15*** | -6.55** | -0.01 | 0.19 | 0.2 | 121 | 0.32 |
|  | HAC | (4.32) | (2.01) | (0.05) | (0.32) | | (0.52) | |
|  | W. OLS | -0.15*** | -6.50*** | -0.01 | 0.19 | 0.2 | 121 | 0.32 |
|  |  | (4.13) | (3.34) | (0.04) | (0.49) | | (0.75) | |
| Bank claims | OLS | -0.15*** | -5.62** | 0.06 | 0.34 | 0.18 | 132 | 1.00** |
|  | HAC | (3.87) | (2.98) | (0.36) | (0.87) | | (2.06) | |
|  | W. OLS | -0.11*** | -4.07 | 0.12 | 0.25 | 0.25 | 132 | 0.61 |
|  |  | (3.96) | (1.64) | (0.76) | (0.53) | | (1.24) | |

Note: Panel A (exc. the last column) provides the results of the following benchmark specification: $\Delta \omega_{i,j}^2(h) = \beta_1 + \beta_2(\Delta g_i - \Delta g_j)^2 + \beta_3(\Delta f_i - \Delta f_j)^2 + \beta_4d_{i,j} + \beta_5x_{i,j} + \eta_{i,j}$, where $\Delta$ is the difference operator over the period 2008-2012, $\omega_{i,j}^2(h)$ denotes connectedness due to cross-market linkages from asset $i$ to asset $j$, $g_i$ is the log of real GDP in country $i$, $f_i$ is the debt-to-GDP ratio in country $i$, $d_{i,j}$ is geographical distance between country $i$ and country $j$, $x_{i,j}$ measures the exposure of country $j$ versus country $i$ and $\beta_i$ control for time-invariant unobserved heterogeneity from country $i$. Exposure$^a$ provides the results when the model includes only exposure and fixed effects. Panel B provides the results when exposure is considered at the 2008 level: $\Delta \omega_{i,j}^2(h) = \beta_1 + \beta_2(\Delta g_i - \Delta g_j)^2 + \beta_3(\Delta f_i - \Delta f_j)^2 + \beta_4d_{i,j} + \beta_5x_{i,j} + \xi_{i,j}$. HAC are the White heteroscedasticity-consistent estimator. W. OLS are the weighted least squares estimator, where the weights are constructed as the inverse of the error variance of the estimated changes in connectedness obtained through bootstrap. Sample period: 3 January 2005 - 24 August 2015.
## Table 4: Factors affecting the change in connectedness

<table>
<thead>
<tr>
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<tbody>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>-0.17***</td>
<td>-2.64</td>
<td>0.13</td>
<td>0.69</td>
<td>0.28</td>
<td>121</td>
</tr>
<tr>
<td>0.01</td>
<td>(0.98)</td>
<td>(0.75)</td>
<td>(0.80)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HAC</td>
<td>-0.17***</td>
<td>-2.64</td>
<td>0.13</td>
<td>0.69</td>
<td>0.28</td>
<td>121</td>
</tr>
<tr>
<td>0.01</td>
<td>(0.82)</td>
<td>(0.82)</td>
<td>(0.85)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>W. OLS</td>
<td>-0.14***</td>
<td>-3.09</td>
<td>0.11</td>
<td>0.68</td>
<td>0.29</td>
<td>121</td>
</tr>
<tr>
<td>0.02</td>
<td>(1.14)</td>
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**Panel B: Exposure in 2008 levels**

| OLS       | -0.17***   | -2.57      | 0.12      | 0.01  | 0.27 | 121       | 0.05    |
| 0.01      | (0.95)     | (0.70)     | (0.03)    |       |      |           | (0.10)  |
| HAC       | -0.17***   | -2.57      | 0.12      | 0.01  | 0.27 | 121       | 0.05    |
| 0.01      | (0.95)     | (0.70)     | (0.03)    |       |      |           | (0.10)  |
| W. OLS    | -0.15***   | -2.99      | 0.1       | 0.06  | 0.28 | 121       | 0.27    |
| 0.02      | (1.19)     | (0.51)     | (0.12)    |       |      |           | (0.51)  |
| OLS       | -0.17***   | -2.77      | 0.15      | 0.28  | 0.26 | 132       | 0.97*** |
| 0.01      | (0.69)     | (0.93)     | (0.57)    |       |      |           | (1.84)  |
| Bank claims |
| W. OLS    | -0.17***   | -2.77      | 0.15      | 0.28  | 0.26 | 132       | 0.97*** |
| 0.01      | (0.69)     | (0.93)     | (0.57)    |       |      |           | (1.84)  |

Note: Panel A (exc. the last column) provides the results of the following benchmark specification: 
$$
\Delta \omega_{i,j}^{(h)} = \beta_i + \beta_1 (\Delta g_i - \Delta g_j)^2 + \beta_2 (\Delta f_i - \Delta f_j)^2 + \beta_3 d_{i,j} + \beta_4 \Delta x_{i,j} + \xi_{i,j},
$$
where $\Delta$ is the difference operator over the period 2008-2012, $\omega_{i,j}^{(h)}$ denotes connectedness from asset $i$ to asset $j$, $g_i$ is the log of real GDP in country $i$, $f_i$ is the debt-to-GDP ratio in country $i$, $d_{i,j}$ is geographical distance between country $i$ and country $j$, $x_{i,j}$ measures the exposure of country $j$ versus country $i$ and $\beta_i$ control for time-invariant unobserved heterogeneity from country $i$. Exposure° provides the results when the model includes only exposure and fixed effects. Panel B provides the results when exposure is considered at the 2008 level: 
$$
\Delta \omega_{i,j}^{(h)} = \beta_i + \beta_1 (\Delta g_i - \Delta g_j)^2 + \beta_2 (\Delta f_i - \Delta f_j)^2 + \beta_3 d_{i,j} + \Delta x_{i,j} + \xi_{i,j},
$$
HAC are the White’s heteroscedasticity-consistent estimator. W. OLS are the weighted least squares estimator, where the weights are constructed as the inverse of the error variance of the estimated changes in connectedness obtained through bootstrap. Sample period: 3 January 2005 - 24 August 2015.

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The change in connectedness is measured as the average bilateral connectedness in 2012 minus the average bilateral connectedness in 2008. Given that the use of generated dependent variables in the estimation can induce heteroskedasticity, next to the standard OLS estimates, we also consider two additional estimation techniques that produce consistent estimates in the presence of heteroskedasticity: the Whites heteroscedasticity-consistent estimator (HAC) and the weighted least squares estimator (W. OLS), where the weights are constructed as the inverse of the error variance of the estimated changes in connectedness obtained through bootstrap.

The results of the cross-section suggest that both commodity trade and financial exposure cannot explain the intensification of financial fragmentation (see Table 3) and the change in financial connectedness (see Table 4). The decline in cross-market linkages or the increase in financial fragmentation over the period 2008-2012 was relatively higher between countries more economically distant with more divergent business cycle and fiscal cycle developments.

At first instance, it could be argued that contagion is positively correlated with the exposure to the debt market (see last column of Tables 3-4, where exposure is the only regressor together with fixed effects). However, both the change in exposure and geographical distance are not statistically significant when controlling for economic distance. The same results are obtained when using the level of exposure in 2008 (see Panel B) and the alternative measures of geographical distance. This suggests that the country divergence in business and fiscal cycles during the crisis period played the key role in explaining financial fragmentation.

4.4 Directional connectedness from specific sovereign yields

As described in subsection 2.4, one can provide additional detailed evidence and assess the importance of specific shocks in inducing fluctuations in all other markets by making use of the FEVD and of directional connectedness using expression (2.7).

Figure 13 shows the estimated directional connectedness among countries’ sovereign yields due to shocks to sovereign yields stemming from specific sovereigns. There are specific developments in some periods, with larger effects that are in line with conventional wisdom. For example, connectedness from Greece doubled from 5% to 11% in the spring of 2010. Similarly, after the Deauville agreement on Private Sector Involvement on 18

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4 Given that we look at cross-border exposure, bilateral connectedness is rescaled to take into account the different degree of own connectedness in 2008 versus 2012.
October 2010, when it was agreed that private investors would share the burden of future defaults with the taxpayer, the results reveal Ireland to be the key source of spillovers, as connectedness from this market rose from 4% on average to 10%. Similarly, when the risk of euro area break-up unfolded in 2011 and 2012, connectedness from Italy rose from 3% in spring 2011 to 7% in the first half of 2012. Connectedness from Italy further increased to 8% in the autumn of 2012 after Mario Draghi’s “whatever it takes” speech on 26 July 2012 and the announcement in September 2012 of the Eurosystem’s OMTs in secondary sovereign bond markets. Increased connectedness after such a speech was a desired outcome as sovereign yields started a steady decline. On the other hand, connectedness from Spain has been rather stable, rising only marginally since the beginning of the euro area sovereign debt crisis. This suggests that Italy, and not Spain, was a key source of systemic risk in the sovereign debt markets in the euro area, an assessment corroborated by the impact of shocks from specific markets reported in the next subsection.

Finally, it is useful to note that there was an increase in connectedness from the UK in March 2009 and from the US in March 2009 and May 2013. On 5th and 18th of March, respectively, Bank of England and the Federal Open Market Committee (FOMC) announced they would purchase 75 billion GBP of sovereign and corporate bonds and 300 billion USD of long-term Treasury Bills, which was subsequently expanded. The average connectedness from the UK rose from 3% in the spring of 2009 to 5% at the end of 2009 and from the US rose from 3% in the spring of 2009 to 7% at the end of 2010. Since the FMOC announcement released on 22 May 2013, which financial markets perceived as the beginning of the end of accommodative monetary policy in the US, connectedness from the US rose marginally from 3% in May 2013 to 5% at the beginning of 2014.

4.5 Impact of shocks from and to specific assets

Finally, we can show the pairwise directional connectedness, which measure the effect of a specific shock on a specific market, represented by each element of Table 2. Given that we consider 15 markets, presenting such plots for each of the 210 pairwise directional measures is not feasible. Therefore, we present some relevant case studies, looking at shocks originating in the US, Greece, Italy and Spain.

Figure 14 shows connectedness from US sovereign yields to the sovereign yields of other countries. As discussed in the previous section, there is an increase in connectedness from the US after March 2009 and May 2013. The sovereign yields most affected are those of
the UK and Germany, followed by Austria, the Netherlands, Belgium and France. The stressed countries were mildly influenced.

A key case study is Greece. Figure 15 shows connectedness from the Greek sovereign yields to the sovereign yields of all other markets. Before the sovereign debt crisis, sovereign yield shocks were relatively small and a Greek shock affected its own sovereign yield marginally. Thereafter, the shocks become large and explain 40% of the variance of the Greek sovereign yields in 2010 and about 70% in 2013. The sovereign yields most affected by the developments in Greece are, in order of magnitude, Portugal, Ireland, Italy and Spain. Again, this is in line with conventional wisdom. Shocks stemming from Greek sovereign yields in 2010 explain 35% of the variance of Portuguese yields, 30% of the variance of Irish yields, 15% of the variance of Italian yields and 12% of the variance of Spanish yields. Interestingly, the German Bund was also affected: with the intensification of the crisis, international investors became more risk averse, and demanded more liquid and relatively safer assets. In 2011 and 2012, the shocks from Greece were subdued except in March 2012 when Greece declared a credit event. In 2013, sovereign yields in Greece declined sharply. Again, the countries positively affected by these developments were the stressed countries: Portugal (30%), Italy (15%), Ireland (13%) and Spain (12%). There was a period in the first half of 2013, with the peak in March 2013, when the Greek shocks also influenced non-stressed economies, such as the UK (18%) and the US (13%). During that period, the Greek sovereign yields declined from 23% in September 2012 to 10% in March 2013. This large shock contributed to the decline in yields in the stress countries as well as in France, Belgium and Austria and to an increase in yields, by a few basis points, in Germany, the Netherlands, the UK and the US. This could be due to a portfolio reallocation away from safer assets as international investors’ risk aversion receded.

Other interesting case studies are Italy and Spain. In 2011 and 2012, when the fear of a euro area break-up contributed to a sharp dynamics in sovereign yields, the shocks from Italy, shown in Figure 16 spilled over to the sovereign yields in other countries, contributing to a rise in connectedness vis-à-vis Spain (10%) and, again due to a portfolio reallocation towards safer assets, vis-à-vis Germany, the Netherlands, Austria, the UK and the US (10%). In 2012, the “whatever-it-takes” speech reversed the dynamics of Italian sovereign yields. They started to decline quickly, contributing to the decline in Spanish sovereign yields, but with the opposite effect on the UK and the US Treasuries. Conversely, the shocks stemming from Spanish sovereign yields affected its own sovereign

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12 All figures refer to peak effects.
market and, only to a limited extent, the developments in Italian sovereign yields (less than 10%), as shown in Figure 17. All other sovereigns did not record a significant change in the contribution of Spanish sovereign yield shocks.

4.6 What does traditional identification tell us about spillovers?

When using GIRFs, the resulting shocks are not orthogonal, and might therefore be incorrect. We show in Appendix B that the GIRF method can yield incorrect results particularly if shocks are relatively large.

For example, during the October 2009-July 2010 period, the effect of shocks stemming from Italy on Greek sovereign yields turn out to be positive using GIRFs and negative using our method (see Figure 13). The impact of GIRFs is sometime even larger than unity; during the October 2009-July 2010 period, an Italian shock amounting to 100 basis points identified using GIRFs implies an average increase at impact by 400 basis points in Greek sovereign yields, by 210 basis points in Portuguese sovereign yields and by 150 basis points in Irish sovereign yields. These results are counter-intuitive, given that Greece is believed to be the source of the crisis during this period.

The shocks estimated using GIRFs always overestimate the size of the shocks obtained with the magnitude restriction method. Moreover, during the periods preceding both the Lehman bankruptcy in September 2008 and the euro area sovereign debt crisis in October 2009, the estimated shocks on the sovereign yields exhibit trends that were steeper than those suggested by the magnitude restriction method (see Figure 10). These differences are then reflected in all the relevant measures of connectedness.

As already discussed, the results obtained using the magnitude restriction method and GIRFs differ substantially if total connectedness is generated by shocks to long-term sovereign yields only, before the beginning of the sovereign debt crisis in October 2009, due to the sizeable influence of monetary policy rates on sovereign yields, which we disentangled from sovereign yield shocks (see Figure 11).

The results obtained using the magnitude restriction method and GIRFs are generally very different when looking at directional connectedness (see Figure 9 and Figures 13-17). This suggests that shock identification is more important when detailed relationships are investigated. For example, connectedness estimated using GIRFs from Greece 15 and Ireland declined in 2009 and 2010, in contrast with conventional wisdom.
5 Conclusion

Spillovers, contagion and financial connectedness are an important subject in the macrofinance literature, which is investigated using a variety of methods. A key issue in macrofinance is the identification of structural shocks, because asset prices move simultaneously. Forbes and Rigobon (2002) suggest to split contagion from shocks by introducing an adjustment for heteroskedasticity, because the increase in conditional correlations among asset prices can be the results of an increase in market volatility. But the volatility regimes are assumed to be known. Diebold and Yilmaz (2014) suggest to study financial connectedness using the generalized forecast error variance decomposition, but the shocks resulting from the generalised impulse response functions (GIRFs) are not orthogonal.

In this paper, we combine the two strands of the literature, but we estimate spillovers, contagion and connectedness with orthogonal structural shocks. We propose the absolute magnitude restriction method, which identifies structural shocks in asset prices based on restrictions on the relative size of the contemporaneous impact of the shocks in different markets. This new method imposes bounds on the impact matrix, but it remains agnostic about the sign of the responses.

We apply the method to the US and European sovereign yield markets and find that US and European sovereign debt markets are highly inter-connected. Total cross-border connectedness among sovereign yields declined steadily between October 2008 and December 2012, due to financial fragmentation. Moreover, we find clear evidence that financial fragmentation was a common phenomenon among stressed and non-stressed countries, although contagion declined by a larger extent from non-stressed countries. A cross-sectional analysis that studies the factors behind the changes in bilateral connectedness between 2008 and 2012 suggests that financial market fragmentation increased further between countries with more divergent business and fiscal cycle developments, while financial exposure, trade exposure and geographical distance did not play a role.

Connectedness among sovereign yields improved in 2013 and in 2014 reached the level recorded before the financial crisis, suggesting that financial fragmentation in the sovereign debt market was no longer a key policy issue already in 2014.

One of the key advantages of the magnitude restriction method is that it can pin down the source of shocks. For example, in May 2013, when financial markets perceived a potential change in the US monetary policy, the influence of US sovereign yields on the developments in sovereign yields in other countries rose. Moreover, we find that in the first half of 2010 shocks stemming from Greek sovereign yields could explain 30% of the
variance of Portuguese yields, 25% of the variance of Irish yields, 15% of the variance of Italian yields and 10% of the variance of Spanish yields. Similarly, focusing on the 2011-2012 period, when financial markets were pricing in the risk of a break-up of the euro area, our method suggests that Italy, and not Spain, was a key source of systemic risk in the sovereign debt markets.

Finally, we find that shocks and spillovers estimated using GIRFs relative to the magnitude restriction method are often overestimated in size and financial connectedness among countries present different trends, which are more accelerated in crisis periods.

After the increase in the US monetary policy rate in December 2016, the FED has clearly stated that its plan is to raise rates very gradually, while the ECB continues its enhanced quantitative easing and negative interest rate policy; potentially generating large asset price spillovers across the Atlantic. The same shock identification scheme by absolute magnitude restrictions can be employed to study the monetary spillovers across economic areas, which we leave for future research.

References


Figure 2: Sovereign yields, monetary policy rates, oil price and macro news

Source: Thomson Reuters.

Note: This figure shows the benchmark sovereign yields at ten-year maturity for Ireland, Portugal, Greece, Italy, Spain, Belgium, France, Austria, the Netherlands, Germany, the US and the UK; the 3-month OIS rate for the US, the UK and the euro area; the Brent crude oil price per barrel in US dollar and the macro news of the G10 economies.
Figure 3: The dynamics of the impact matrix

Note: This figure shows the developments of the impact matrix on selected countries’ sovereign yields due to shocks to sovereign yields in Greece, Italy, Germany, UK and US. The blue line and the shaded area provide respectively the median estimate and the 68% error bands obtained using the magnitude restriction method. The dotted red line is the median estimate obtained using GIRFs. The vertical bars denote 9 August 2007 (the interbank credit crisis), 25 November 2008 (US LSAP), 5 March 2009 (BoE APF), 5 March 2010 (the Greek revised budget deficit), 18 October 2010 (the Deauville agreement upon Private Sector Involvement), 24 November 2011 (Fitch downgrade of Portugal’s sovereign debt), 26 July 2012 (Draghi’s speech), 22 May 2013 (FED tapering announcement), 22 January 2015 (ECB PSPP). Sample period: 3 January 2005 - 24 August 2015.
Figure 4: Impulse response functions during the Greek crisis

Note: This figure shows the IRFs on selected countries’ sovereign yields due to shocks to sovereign yields in Greece, Italy, Germany, UK and US estimated over the sample period 19 October 2009 - 23 July 2010. The blue line and the shaded area provide respectively the median estimate and the 68% error bands obtained using the magnitude restriction method. The dotted red line is the median estimate obtained using GIRFs.
Figure 5: Impulse response functions during the euro break-up risk crisis

Note: This figure shows the IRFs on selected countries’ sovereign yields due to shocks to sovereign yields in Greece, Italy, Germany, UK and US estimated over the sample period 2 May 2011 - 3 February 2012. The blue line and the shaded area provide respectively the median estimate and the 68% error bands obtained using the magnitude restriction method. The dotted red line is the median estimate obtained using GIRFs.
Figure 6: Standard deviation of the shocks to sovereign yields

Note: This figure shows the standard deviation of the shocks to sovereign yields in percentage points. The blue line and the shaded area provide respectively the median estimate and the 68% error bands obtained using the magnitude restriction method. The dotted red line is the median estimate obtained using GIRFs. The vertical bars denote 9 August 2007 (the interbank credit crisis), 25 November 2008 (US LSAP), 5 March 2009 (BoE APF), 5 March 2010 (the Greek revised budget deficit), 18 October 2010 (the Deauville agreement upon Private Sector Involvement), 24 November 2011 (Fitch downgrade of Portugal’s sovereign debt), 26 July 2012 (Draghi’s speech), 22 May 2013 (FED tapering announcement), 22 January 2015 (ECB PSPP). Sample period: 3 January 2005 - 24 August 2015.
Figure 7: Total connectedness among sovereign yields and monetary policy rates

Note: This figure shows total connectedness among sovereign yields and monetary policy rates ranging between 0 (no connectedness) and 1 (full connectedness). The blue line and the shaded area provide respectively the median estimate and the 68% error bands obtained using the magnitude restriction method. The dotted red line is the median estimate obtained using GIRFs. The vertical bars denote 9 August 2007 (the interbank credit crisis), 25 November 2008 (US LSAP), 5 March 2009 (BoE APF), 5 March 2010 (the Greek revised budget deficit), 18 October 2010 (the Deauville agreement upon Private Sector Involvement), 24 November 2011 (Fitch downgrade of Portugal’s sovereign debt), 26 July 2012 (Draghi’s speech), 22 May 2013 (FED tapering announcement), 22 January 2015 (ECB PSPP). Sample period: 3 January 2005 - 24 August 2015.
Figure 8: Total connectedness among sovereign yields excluding monetary policy shocks

Note. This figure shows total connectedness among sovereign yields excluding monetary policy shocks ranging between 0 (no connectedness) and 1 (full connectedness). The blue line and the shaded area provide respectively the median estimate and the 68% error bands obtained using the magnitude restriction method. The dotted red line is the median estimate obtained using GIRFs. The vertical bars denote 9 August 2007 (the interbank credit crisis), 25 November 2008 (US LSAP), 5 March 2009 (BoE APF), 5 March 2010 (the Greek revised budget deficit), 18 October 2010 (the Deauville agreement upon Private Sector Involvement), 24 November 2011 (Fitch downgrade of Portugal’s sovereign debt), 26 July 2012 (Draghi’s speech), 22 May 2013 (FED tapering announcement), 22 January 2015 (ECB PSPP). Sample period: 3 January 2005 - 24 August 2015.
Figure 9: Directional connectedness among sovereign yields of stressed and non-stressed countries

Note: This figure shows directional connectedness among sovereign yields of stressed and non-stressed countries due to shocks to sovereign yields ranging between 0 (no connectedness) and 1 (full connectedness). The non-stressed country group is composed of Austria, Belgium, France, Germany, Netherlands, UK and US. The stressed country group is composed of Greece, Ireland, Italy, Portugal and Spain. The blue line and the shaded area provide respectively the median estimate and the 68% error bands obtained using the magnitude restriction method. The dotted red line is the median estimate obtained using GIRFs. The vertical bars denote 9 August 2007 (the interbank credit crisis), 26 November 2008 (US LSAP), 5 March 2009 (BoE APF), 5 March 2010 (the Greek revised budget deficit), 18 October 2010 (the Deauville agreement upon Private Sector Involvement), 24 November 2011 (Fitch downgrade of Portugal’s sovereign debt), 26 July 2012 (Draghi’s speech), 22 May 2013 (FED tapering announcement), 22 January 2015 (ECB PSPP). Sample period: 3 January 2005 - 24 August 2015.
Figure 10: Decomposition of total connectedness among sovereign yields: contagion versus shocks

Note: This figure shows the decomposition of total connectedness (grand average) among sovereign yields between contagion and shocks. The grand average is constructed from the pairwise connectedness measures of all twelve sovereign yields excluding the monetary policy rates. The black line and the shaded area provide respectively the median estimate and the 68% error bands of total connectedness (grand average) obtained using the magnitude restriction method. The red line and the shaded area provide respectively the median estimate and the 68% error bands of total connectedness (grand average) when the distribution of sovereign yield shocks is fixed to the pre-crisis level. The green line and the shaded area provide respectively the median estimate and the 68% error bands of total connectedness (grand average) when cross-market linkages are fixed to the pre-crisis level. The vertical bars denote 9 August 2007 (the interbank credit crisis), 25 November 2008 (US LSAP), 5 March 2009 (BoE APF), 5 March 2010 (the Greek revised budget deficit), 18 October 2010 (the Dorville agreement upon Private Sector Involvement), 24 November 2011 (Fitch downgrade of Portugal’s sovereign debt), 26 July 2012 (Draghi’s speech), 22 May 2013 (FED tapering announcement), 22 January 2015 (ECB PSPP). Sample period: 3 January 2005 - 24 August 2015.
Figure 11: Contagion versus shocks - the role of stressed countries

Note: This figure shows the decomposition of total connectedness (grand average) among sovereign yields between contagion and shocks, where the distribution of shocks and cross-market linkages are fixed for Greece, Ireland, Italy, Portugal and Spain. The grand average (upper panel) and the change in connectedness (bottom panel) are constructed from the pairwise connectedness measures of all twelve sovereign yields excluding the monetary policy rates. The black line and the shaded area provide respectively the median estimate and the 68% error bands of total connectedness (grand average) obtained using the magnitude restriction method. The red line and the shaded area provide respectively the median estimate and the 68% error bands of total connectedness (grand average) when the distribution of sovereign yield shocks is fixed to pre-crisis level. The green line and the shaded area provide respectively the median estimate and the 68% error bands of total connectedness (grand average) when cross-market linkages are fixed to pre-crisis level. The vertical bars denote 9 August 2007 (the interbank credit crisis), 25 November 2008 (US LSAP), 5 March 2009 (BoE APF), 5 March 2010 (the Greek revised budget deficit), 18 October 2010 (the Deauville agreement upon Private Sector Involvement), 24 November 2011 (Fitch downgrade of Portugal’s sovereign debt), 26 July 2012 (Draghi’s speech), 22 May 2013 (FED tapering announcement), 22 January 2015 (ECB PSPP). Sample period: 3 January 2005 - 24 August 2015.
Figure 12: Contagion versus shocks - the role of non-stressed countries

Note: This figure shows the decomposition of total connectedness (grand average) among sovereign yields between contagion and shocks, where the distribution of shocks and cross-market linkages are fixed for Austria, Belgium, France, Germany, Netherlands, UK and US. The grand average (upper panel) and the change in connectedness (bottom panel) are constructed from the pairwise connectedness measures of all twelve sovereign yields excluding the monetary policy rates. The black line and the shaded area provide respectively the median estimate and the 68% error bands of total connectedness (grand average) obtained using the magnitude restriction method. The red line and the shaded area provide respectively the median estimate and the 68% error bands of total connectedness (grand average) when the distribution of sovereign yield shocks is fixed to pre-crisis level. The green line and the shaded area provide respectively the median estimate and the 68% error bands of total connectedness (grand average) when cross-market linkages are fixed to pre-crisis level. The vertical bars denote 9 August 2007 (the interbank credit crisis), 25 November 2008 (US LSAP), 5 March 2009 (BoE APF), 5 March 2010 (the Greek revised budget deficit), 18 October 2010 (the Dearville agreement upon Private Sector Involvement), 24 November 2011 (Fitch downgrade of Portugal’s sovereign debt), 26 July 2012 (Draghi’s speech), 22 May 2013 (FED tapering announcement), 22 January 2015 (ECB PSPP). Sample period: 3 January 2005 - 24 August 2015.
Figure 13: Directional connectedness among countries’ sovereign yields excluding monetary policy shocks

Note: This figure shows connectedness to countries’ sovereign yields due to shocks to sovereign yields ranging between 0 (no connectedness) and 1 (full connectedness). The blue line and the shaded area provide respectively the median estimate and the 68% error bands obtained using the magnitude restriction method. The dotted red line is the median estimate obtained using GIRFs. The vertical bars denote 9 August 2007 (the interbank credit crisis), 25 November 2008 (US LSAP), 5 March 2009 (BoE APP), 5 March 2010 (the Greek revised budget deficit), 14 October 2010 (the Deauville agreement upon Private Sector Involvement), 24 November 2011 (Fitch downgrade of Portugal’s sovereign debt), 26 July 2012 (Draghi’s speech), 22 May 2013 (FED tapering announcement), 22 January 2015 (ECB PSPP). Sample period: 3 January 2005 - 24 August 2015.
Figure 14: Pairwise directional connectedness to countries’ sovereign yields due to shocks to US sovereign yields

Note: This figure shows connectedness to individual countries’ sovereign yields due to shocks to US sovereign yields ranging between 0 (no connectedness) and 1 (full connectedness). The blue line and the shaded area provide respectively the median estimate and the 68% error bands obtained using the magnitude restriction method. The dotted red line is the median estimate obtained using GIRFs. The vertical bars denote 9 August 2007 (the interbank credit crisis), 25 November 2008 (US LSAP), 5 March 2009 (BoE APP), 5 March 2010 (the Greek revised budget deficit), 18 October 2010 (the Duisenberg agreement upon Private Sector Involvement), 24 November 2011 (Fitch downgrade of Portugal’s sovereign debt), 26 July 2012 (Draghi’s speech), 22 May 2013 (FED tapering announcement), 22 January 2015 (ECB PSPP). Sample period: 3 January 2005 - 24 August 2015.
Figure 15: Pairwise directional connectedness to countries’ sovereign yields due to shocks to Greek sovereign yields

Note: This figure shows connectedness to individual countries’ sovereign yields due to shocks to Greek sovereign yields ranging between 0 (no connectedness) and 1 (full connectedness). The blue line and the shaded area provide respectively the median estimate and the 68% error bands obtained using the magnitude restriction method. The dotted red line is the median estimate obtained using GIRFs. The vertical bars denote 9 August 2007 (the interbank credit crisis), 25 November 2008 (US LSAP), 5 March 2009 (BoE APF), 5 March 2010 (the Greek revised budget deficit), 18 October 2010 (the Deauville agreement upon Private Sector Involvement), 24 November 2011 (Fitch downgrade of Portugal’s sovereign debt), 26 July 2012 (Draghi’s speech), 22 May 2013 (FED tapering announcement), 22 January 2015 (ECB PSPP). Sample period: 3 January 2005 - 24 August 2015.
Figure 16: Pairwise directional connectedness to countries’ sovereign yields due to shocks to Italian sovereign yields

Note: This figure shows connectedness to individual countries’ sovereign yields due to shocks to Italian sovereign yields ranging between 0 (no connectedness) and 1 (full connectedness). The blue line and the shaded area provide respectively the median estimate and the 68% error bands obtained using the magnitude restriction method. The dotted red line is the median estimate obtained using GIRFs. The vertical bars denote 9 August 2007 (the interbank credit crisis), 25 November 2008 (US LSAP), 5 March 2009 (BoF APF), 5 March 2010 (the Greek revised budget deficit), 18 October 2010 (the Deauville agreement upon Private Sector Involvement), 24 November 2011 (Fitch downgrade of Portugal’s sovereign debt), 26 July 2012 (Draghi’s speech), 22 May 2013 (FED tapering announcement), 22 January 2015 (ECB PSPP). Sample period: 3 January 2005 - 24 August 2015.
Figure 17: Pairwise directional connectedness to countries’ sovereign yields due to shocks to Spanish sovereign yields

Note: This figure shows pairwise connectedness to individual countries’ sovereign yields due to shocks to Spanish sovereign yields ranging between 0 (no connectedness) and 1 (full connectedness). The blue line and the shaded area provide respectively the median estimate and the 68% error bands obtained using the magnitude restriction method. The dotted red line is the median estimate obtained using GIRFs. The vertical bars denote 9 August 2007 (the interbank credit crisis), 25 November 2008 (US LSAP), 5 March 2009 (BoE APP), 5 March 2010 (the Greek revised budget deficit), 18 October 2010 (the Dourville agreement upon Private Sector Involvement), 24 November 2011 (Fitch downgrade of Portugal’s sovereign debt), 26 July 2012 (Draghi’s speech), 22 May 2013 (FED tapering announcement), 22 January 2015 (ECB PSPP). Sample period: 3 January 2005 - 24 August 2015.
A Estimation algorithm

The estimation procedure consists of three steps. The reduced form VAR model is estimated in the first step. The structural shocks are identified in the second step. The third step is introduced to account for estimation uncertainty.

While randomly drawing orthonormal matrices using the QR decomposition is feasible when a small number of variables are used, numerical optimization becomes necessary when the number of variables is large because the probability of obtaining a successful draw is decreasing with the size of the system. Therefore, we introduce a numerical algorithm for computational convenience. Formally, the steps of the algorithm are the following:

1. **Estimate reduced-form VAR:** Given a chosen number of lags, $K$, a $VAR(K)$ is estimated by Ordinary Least Squares (OLS) to obtain an estimate of autoregressive coefficients $A(L)$ and of the variance-covariance of reduced form errors, $\Sigma_u$.

2. **Identification restrictions:** The reduced form IRF, $C(L)$, is related to the structural IRF via $B(L) = A_0 C(L)$ and reduced form errors, $u_t$, are related to structural shocks as $u_t = A_0^{-1} B \varepsilon_t$. The impact matrix, $S = A_0^{-1} B$, must satisfy: $\Sigma_u = SS'$. We then distinguish the approach used in small systems with few variables, and larger systems with many variables.

   **Small systems (used in Appendix B):**
   
   (a) The initial estimate of $\hat{S}$ is obtained by a Cholesky decomposition of the variance-covariance matrix of reduced form errors, $\hat{\Sigma} = \text{chol}(\hat{\Sigma}_u)$, giving an initial estimate of the IRF is $\hat{B}(L) = \hat{C}(L)\hat{S}$.

   (b) A $q \times q$ matrix $P$ is drawn from standard normal distribution, $N(0, 1)$ and the QR decomposition of $P$ is derived. Note that $P = QR$ and $QQ' = I$.

   (c) The initial estimate of the IRF is post-multiplied by $Q$, to obtain a candidate IRF $\hat{B}^*(L) = \hat{C}(L)\hat{S}Q$.

   (d) The steps 2b-2c of the small systems are repeated until the candidate IRFs, $\hat{B}^*(L)$, satisfy the identifying restrictions.

   **Large systems:**

   (a) The problem is initialized as in the small system case with $\hat{S} = \text{chol}(\hat{\Sigma}_u)$. 

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(b) A random matrix $A_0^{-1}$ is drawn satisfying the identifying restrictions. In particular, in the baseline estimation, we construct a matrix with 1s on the diagonal and with random numbers drawn from $[0,1)$ on the off-diagonal elements.

(c) Given $A_0^{-1}$, the matrix $Q^*$ is defined through the following minimization problem:

$$Q^* = \arg \min_Q (A_0^{-1} - A_0^*)^2$$

subject to

$$Q^* Q^{*\prime} = I$$

$$\hat{S} = \hat{S} Q^*$$

$$\hat{A}_0^{-1}(i,j) = \hat{S}_{ij}/\hat{S}_{jj} \forall i,j$$

$$c(A_0^{-1}) \geq 0$$

where $c(.) \geq 0$ represents the identifying restrictions. In other words, we look for an orthonormal matrix, $Q^*$, that implies a decomposition, $\hat{\Sigma}_u = (\hat{S} Q^*)(\hat{S} Q^*)'$, such that the resulting matrix of impact coefficients, $\hat{A}_0^{-1}$, is close to $A_0^{-1}$, and satisfies the identifying restrictions.

(d) In case the minimization does not converge to a feasible solution, steps 2b and 2c of the large systems are repeated. Once the minimization converges, the candidate IRF is calculated as $\hat{B}^*(L) = \hat{C}(L)\hat{S} Q^*$.

3. **Estimation uncertainty:** to account for estimation uncertainty, we repeat 1000 times steps 1-2, each time with a new artificially constructed data sample, $Y^*$. To construct data samples, we use re-sampling of errors. The new data sample is constructed recursively as $y^*_t = \hat{A}_1^* y^*_{t-1} + \ldots + \hat{A}_N^* y^*_{t-N} + \hat{u}^*_t$, starting from initial values $[y_0^*, \ldots, y_{N-1}^*]$. $\hat{A}_n^*$ are the estimated reduced form autoregressive coefficients and $\hat{u}^*_t$ are drawn randomly, with replacement, from the estimated reduced form errors, $\hat{u}_t$.

The point estimates and confidence bands are given by the median and relevant percentiles of the distribution of the retained IRFs.

**B Comparison with other identification methods**

[Diebold and Yilmaz (2014)] measures financial connectedness using a generalized variance

\[ \text{Note that this minimization could be carried out without the matrix } A_0^{-1}, \text{ for example using a constant objective function. However, the role of } A_0^{-1} \text{ in our algorithm is to ensure that we search the full space of permissible matrices, satisfying the identifying restrictions in a more complete way.} \]
decomposition. We can show that the results may differ if the size of the shocks differ across assets. In these cases, shock identification and orthogonality play a key role.

We can use the simple example from subsection 2.3 to compare the identification with magnitude restrictions with more standard identification methodologies, such as zero restrictions (or Cholesky decomposition) and GIRFs.

The estimated impact matrix obtained via Cholesky decomposition of the variance-covariance matrix when the variables are ordered as \([y_1 \ y_2]\) is:

\[
\hat{A}^{-1}_0 B = \begin{bmatrix}
\sqrt{\sigma^2_1 + b^2 \sigma^2_2} \\
\frac{c \sigma_1 + b \sigma_2}{\sqrt{c^2 \sigma^2_1 + \sigma^2_2}} \\
\sqrt{c^2 \sigma^2_1 + \sigma^2_2 - \frac{c \sigma_1 + b \sigma_2}{\sigma_1 + \sigma_2}} \\
\end{bmatrix}
\]

\(\hat{A}^{-1}_0 B\) coincides with the true \(A^{-1}_0 B\) only when \(b = 0\), namely when there is no instantaneous spillover from market 2 to market 1. If the variables are ordered as \([y_2 \ y_1]\) it can be similarly shown that the estimated and true impact matrix will coincide when \(c = 0\).

The GIRFs for the first period are:

\[
\hat{A}^{-1}_0 B = \begin{bmatrix}
\sigma_1 \sqrt{1 + c^2 x^2} \\
\frac{c \sigma_1 + b \sigma_2}{\sqrt{c^2 \sigma^2_1 + \sigma^2_2}} \\
\sigma_1 \sqrt{\frac{c^2 x^2 + x^2}{c^2 + 1}} \\
\end{bmatrix}
\]

In this case, \(\hat{A}^{-1}_0 B\) coincides with the true \(A^{-1}_0 B\) only when \(b = 0\) and \(c = 0\), namely only when there are no instantaneous spillovers between markets. Since in general the off-diagonal elements of \(\hat{A}^{-1}_0 B\) are non-zero, shocks identified with GIRFs are not orthogonal.

How big are the errors when measuring spillovers via GIRFs? To simplify, assume that \(\sigma_1 = x \sigma_2\), where \(x > 0\). Then the GIRFs are:

\[
\hat{A}^{-1}_0 B = \begin{bmatrix}
\sigma_1 \sqrt{1 + x^2} \\
\frac{c \sigma_1 + b \sigma_2}{\sqrt{c^2 \sigma^2_1 + \sigma^2_2}} \\
\sigma_1 \sqrt{\frac{x^2 + x^2}{c^2 + 1}} \\
\end{bmatrix}
\]

The true instantaneous impact of \(y_2\) on \(y_1\) is \(b\). The estimated instantaneous impact is \(\hat{b} = b(1 + x^2) \sqrt{b^2 + x^2}^{-1}\). Figure B.1 shows the relationship between \(b\) and \(\hat{b}\). The curved area represents the instantaneous impact obtained by using GIRFs, the transparent grey areas represents the bounds implied by the magnitude restriction method, while the red area represents the true \(b\). For presentational reasons we assume that \(b = c\).
Notice that $\hat{b} = 0$ when $b = 0$. However, when the standard deviations of the shocks differ substantially, for example when the standard deviation of the shock from $y_2$ is ten times that from $y_1$ (i.e. $x = 0.1$), the estimated instantaneous impact using GIRFs can reach 4, while the true instantaneous impact is close to zero. On the other hand, the magnitude restriction method is designed such that the estimated instantaneous impact cannot exceed one in absolute value. Yet, $\hat{b}$ is inside the transparent grey areas for certain parameters. This implies that the results obtained with the two methodologies in these specific cases may not differ considerably.

Figure B.2 shows the root square error of the estimated instantaneous impact when using GIRFs (yellow area) and when using magnitude restriction method (blue area). Again, the difference is most pronounced when the standard deviations of shocks differ substantially. A more general Monte Carlo simulation with four assets and for $b, c \in (-1, 1)$ reported below confirms the above results.
Figure B.2: The root squared error of the estimated spillovers: GIRFs versus the magnitude restriction method

Note: This figure shows the root squared errors of the estimated versus the true spillover using GIRFs (yellow area) and the magnitude restriction method (blue area).

For example, we find that the spillover from Italian to Greek sovereign yields is more than one when using GIRFs. The numerical example would suggest that this may be due to the standard deviation of shocks to Greek sovereign yields being considerably larger.

Let us extend the analysis to four markets and compares the results obtained with the magnitude restriction and GIRFs using Monte Carlo simulation. We use multiple models to simulate a time-varying model with the following SVAR:

\[
\begin{align*}
A^m_0 y^m_t &= A^m_1 y^m_{t-1} + B^m \varepsilon^m_t, \\
y^m_t &= (A^m_0)^{-1} A^m_1 y^m_{t-1} + (A^m_0)^{-1} B^m \varepsilon^m_t, 
\end{align*}
\]

(B.2)

where \( m = 1, 2, ..., M \) denotes the model.

We model a time-varying data generating process by assuming that the impact matrices, \( A^m_0 \), and the standard errors of the shocks, \( B^m \), are not constant. In the first exercise, the non-diagonal entries of \( (A^m_0)^{-1} \) are randomly selected from the interval \([-0.99, 0.99]\),

A time-varying model is simulated because a rolling window estimation is carried out in the empirical section.
The diagonal entries of matrix $B^m$, which give the standard errors of structural shocks, are also selected randomly from a uniform distribution $U(0.05, 0.45)$. We simulate the data across models assuming that $A^m_0 = A_1 \forall m$. The auto-regressive parameters in $A^m_0$ and the variance-covariance matrix, $\Sigma^m$, are estimated using OLS and the small sample distribution of parameters is obtained using bootstrap methods.

The performance of the identification schemes can be compared by using the root mean squared error (RMSE) of different statistics. The RMSE of the estimates of the IRFs is:

$$RMSE(irf) = \sqrt{\frac{1}{N^2 H M} \sum_{i=1}^N \sum_{j=1}^N \sum_{h=1}^H \sum_{m=1}^M \left( \hat{\Psi}_{i,j,h,m} - \Psi_{i,j,h,m} \right)^2}$$  \hspace{1cm} (B.3)

where $\Psi_{i,j,h,m}$ is the true IRF of variable $i$ to the shock $j$ at horizon $h$ for the model $m$, and $\hat{\Psi}_{i,j,h,m}$ is its median estimated counterpart. The RMSE of estimated ‘to others’ connectedness is:

$$RMSE(to others) = \sqrt{\frac{1}{N^2 H M} \sum_{j=1}^N \sum_{h=1}^H \sum_{m=1}^M \left( \hat{C}_{h,m}^{\bullet \rightarrow j} - C_{h,m}^{\bullet \rightarrow j} \right)^2}$$  \hspace{1cm} (B.4)

where $C_{h,m}^{\bullet \rightarrow j}$ is ‘to others’ connectedness for shock $j$ at horizon $h$ obtained from the model $m$, and $\hat{C}_{h,m}^{\bullet \rightarrow j}$ is its median estimated counterpart. Finally, the MSE of estimated grand average connectedness is:

$$RMSE(grand average) = \sqrt{\frac{1}{H M} \sum_{h=1}^H \sum_{m=1}^M \left( \hat{C}_{h,m} - C_{h,m} \right)^2}$$  \hspace{1cm} (B.5)

where $C_{h,m}$ is total connectedness at horizon $h$ obtained from the model $m$, and $\hat{C}_{h,m}$ is its median estimated counterpart.

Table B.1 reports the RMSEs obtained with the magnitude restriction method.
and zero restrictions with random ordering, the latter two employed by Diebold and Yilmaz (2014). In the first 10 periods, on average, the RMSEs based on IRF estimates are almost twice smaller when using magnitude restriction method relative to GIRFs and are also smaller than those obtained with zero restrictions.

Table B.1: The RMSE of the estimates for different identification methodologies

<table>
<thead>
<tr>
<th>horizon</th>
<th>Magnitude res.</th>
<th>GIRF</th>
<th>Zero res. (random order)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>1-10</td>
<td>1</td>
</tr>
<tr>
<td>IRFs</td>
<td>0.06</td>
<td>0.05</td>
<td>0.11</td>
</tr>
<tr>
<td>TO connectedness</td>
<td>0.23</td>
<td>0.12</td>
<td>0.42</td>
</tr>
<tr>
<td>Grand Average</td>
<td>0.07</td>
<td>0.09</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Figure B.3 shows directional connectedness ‘to others’ from all four markets for the first 10 models. The new suggested methodology performs reasonably well, as the true directional connectedness ‘to others’ is mostly included in the two-standard error bands. The same cannot be said for the estimates obtained using GIRFs, shown in Figure B.4: the true directional connectedness ‘to others’ is mostly outside the two-standard error bands. The RMSE of the estimated directional connectedness ‘to others’ is around 50 percent smaller when using the magnitude restriction method as compared to GIRFs (see Table B.1).

Estimates of total connectedness differ little across methodologies. As depicted in Figures B.5 and B.6, estimates obtained using magnitude restriction method are still closer to the true total connectedness, but the gain in precision in terms of RMSE is now less than 20 percent.

It is also interesting to explore how magnitude restriction method would perform when estimates obtained via GIRFs and the true model coincide. Theoretically, they coincide only when there are no contemporaneous spillovers between markets. To see how the new method performs when there are no spillovers, we repeat the described exercise now setting the non-diagonal entries of \((A_m^\circ)^{-1}\) to zero keeping the diagonal entries equal to one. Table B.2 reports the RMSE of the magnitude restriction method, GIRFs and zero restrictions with random ordering. Clearly, estimates obtained with GIRFs and zero restrictions are more precise since the identification restrictions exactly coincide with the true model, and the errors are only due to small sample estimation errors. Nonetheless,

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18 We show only the first 10 models to simplify the presentation, the figures for other models can be provided on request.
magnitude restriction method perform reasonably well. The RMSE of the IRF estimates, 0.01, and of the estimated directional connectedness ‘to others’, 0.06, are smaller than in the general case.

Table B.2: The RMSE of the estimates for different identification methodologies - model with no spillovers

<table>
<thead>
<tr>
<th></th>
<th>Magnitude res.</th>
<th>GIRF</th>
<th>Zero res. (random order)</th>
</tr>
</thead>
<tbody>
<tr>
<td>horizon</td>
<td>1-10</td>
<td>1-10</td>
<td>1-10</td>
</tr>
<tr>
<td>IRFs</td>
<td>0.02 0.01</td>
<td>0.00</td>
<td>0.00 0.02 0.02</td>
</tr>
<tr>
<td>TO connectedness</td>
<td>0.07 0.06</td>
<td>0.00</td>
<td>0.01 0.00 0.04</td>
</tr>
<tr>
<td>Grand Average</td>
<td>0.21 0.20</td>
<td>0.01</td>
<td>0.02 0.01 0.02</td>
</tr>
</tbody>
</table>

To summarize, if the focus is only on total connectedness, the different identification approaches produce similar results. However, whenever we are interested in less aggregate statistics, such as bilateral relations between countries or the importance of a specific shock, using the magnitude restriction method produces more precise estimates.
Figure B.3: Monte Carlo estimation: Directional connectedness ‘to’ others using the magnitude restriction method

This figure shows the estimated directional connectedness ‘to’ others for simulated markets obtained using the magnitude restriction method. The red line and the grey bands provide respectively the median estimate and the 95% error bands. The black line with circles is the true directional connectedness ‘to’ others.

Figure B.4: Monte Carlo estimation: Directional connectedness ‘to’ others using generalized IRFs

This figure shows the estimated directional connectedness ‘to’ others for simulated markets obtained using GIRFs. The red line and the grey bands provide respectively the median estimate and the 95% error bands. The black line with circles is the true directional connectedness ‘to’ others.
Figure B.5: Monte Carlo estimation: Total connectedness using the magnitude restriction method

Note: This figure shows the estimated total connectedness 'to' others for simulated markets obtained using the magnitude restriction method. The red line and the grey bands provide respectively the median estimate and the 95% error bands. The black line with circles is the true total connectedness 'to' others.

Figure B.6: Monte Carlo estimation: Total connectedness by using generalized IRFs

Note: This figure shows the estimated total connectedness 'to' others for simulated markets obtained using GIRFs. The red line and the grey bands provide respectively the median estimate and the 95% error bands. The black line with circles is the true total connectedness 'to' others.
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