EVALUATING INTERCONNECTEDNESS IN THE FINANCIAL SYSTEM ON THE BASIS OF ACTUAL AND SIMULATED NETWORKS

Multiple levels of analysis are required to assess banks’ fragility in a complex banking system. On the one hand, network analysis using existing data for the euro area shows a banking structure which is well integrated across euro area countries, with some banks playing an important role at the euro area level while others have a more domestic focus. On the other hand, a dynamic network modelling approach can illustrate important aspects and fragilities of interbank activity in a simulated network in the absence of real micro data.

This special feature first describes a static approach to financial network analysis and then gives a specific illustration of a dynamic network in a stress-testing context. Both provide important insights for financial stability analysis. The static analysis of existing financial networks and the use of a simulated network for stress testing exploit information on the microstructure of banking activities to characterise the robustness of the banking sector to operating shocks. This is a unique application of conceptual and analytical techniques that have only recently been introduced in financial analysis.

INTRODUCTION

Network relationships in a financial context are exposures and liabilities recorded on or off balance sheet, or reflect financial activities in general. Mutual exposures of financial intermediaries are, on the one hand, beneficial as they generally allow for a more efficient allocation of financial assets and liabilities and are a sign of better diversified financial institutions. On the other hand, when large shocks affect the financial system, financial networks can accelerate the shock’s initial impact by propagating it throughout the system. The unit of analysis in macro-prudential analysis has traditionally been at the level of countries and/or sectors, providing information on sources of fragility for the financial sector as a whole. However, as the recent crisis revealed the intrinsic dependence of stability on institution-level relationships, macro-prudential analysis has begun to focus on information concerning individual institutions.

Accordingly, one can view a financial exposure or liability within a network as a relationship (or edge) of an institution (node) vis-à-vis another whereby the relationship portrays a potential channel of transmission between institutions. This simple – or static – representation of a network does not specify how transmission mechanisms transfer shocks throughout the network and, in particular, makes no assumptions as to the institution’s behaviour when confronted with a shock stemming from one of the relationships. A static network is therefore most valuable in its ability to summarise stylised facts of the network architecture as a whole, which can be very useful in macro-prudential analysis. Information derived from the static network includes the identification of central or systemic groups of institutions or nodes in the network. For example, one standard method of identifying the centrality of a node alone is the “between-ness” measure, i.e. the number of shortest paths from all nodes to all others that pass through that given node. Another method is eigenvector centrality, that is, a measure of the node’s influence in the network, assigning relative scores to all nodes on the assumption that connections to high-scoring nodes contribute more to the score of the node in question than equal connections to low-scoring nodes. These and related network centrality measures enable a simple identification of systemic nodes and the general structure of a network. Observed over time, these measures can reveal the evolution of important aspects of the network relevant to its systemic robustness, identifying, for example, certain network vulnerabilities or its ability to dampen or exacerbate shocks.
A second class of network models – those that are dynamic – imposes additional characteristics on the nodes. These characteristics allow the transmission of shocks across the system to be modelled. Specifically, studying dynamic network models is justified by the substantial volatility of some financial networks. A trade-off between the richer nature and robustness of the results of the model and the specificity brought by the behavioural assumptions at the nodes is relevant. This is especially important when the behaviour at the nodes significantly impacts their systemicity or the vulnerability of the system as a whole. Therefore, it is particularly important to work in as general a model setting as possible, i.e. taking into account various possible network structures and looking at exogenous shocks from different angles. A notable example of this second class of models is used in the context of stress testing, whereby the response modelled at the nodes will allow risk maps of contagion effects of exogenous shocks to be formulated.

STRUCTURAL VULNERABILITY AND HIGH INTERCONNECTION IN CROSS-HOLDINGS OF BANK SECURITIES

A network rests on the definition of who (the nodes) and what (the links). Banks’ interbank activity, at both the individual and the aggregate level, motivates the use of network analysis. At the country level, the Bank for International Settlements’ consolidated banking statistics provide information on foreign bank claims which are a prominent and the most studied form of bank interconnection. Microstructure studies, however, concentrate on proprietary supervisory information and have a narrower national or market-specific context, depending on a wide range of links: claims and obligations computed from balance sheets, return correlations, joint investment, or the same pool of depositors. In principle, various relationships between banks can be analysed, even simultaneously.

Structural issues relevant to financial surveillance

From a system perspective, the architecture of a network and its potential fragility support macro-prudential analysis in many ways. Different network structures can deal with shock propagation in different ways, and there are a number of measures to classify such network typologies. Notably for existing networks, Watts and Strogatz find that actual networks are generally highly clustered in groups adjacent to one another but not to other groups in the network. These are also known as small-world networks. The transmission of information within this structure is very quick and has been found to be important for spreading news, human disease and internet viruses. The forms of bank interaction, therefore, can illustrate how rapidly shocks can spread across classes of banks or across the banking sector, both nationally and internationally.

1 Simulations depend on the strategic interaction of financial agents. Hence, understanding the relationships is essential to predict future outcomes, i.e. to limit — or possibly prevent — negative effects before they affect the whole system, including feedback mechanisms and realistic behavioural responses. A key challenge is specifying a network with different agents; see K. Anand et al., “Epidemics of rules, rational negligence and market crashes”, European Journal of Finance, forthcoming; O. Castrén and I. K. Kavonius, “Balance sheet interlinkages and macro-financial risk analysis in the euro area”, ECB Working Paper Series, No 1124, 2010; and O. Castrén and M. Rancan, “Macro-Networks - an application to Euro Area Financial Accounts”, Padova University, 2012. All include different sectors in the same framework.


7 This is in contrast to regular networks, characterised by a large value for the average path length and a high degree of clustering, and random networks, which have a low average shortest path and a small degree of clustering.

8 Similar mechanisms could help explain rumours, fears and excessive euphoria spread across professional investors and financial markets.
In addition to the description of the structure of banks’ interlinkages as a unit in itself, *measures based at the bank level* (i.e. at the node level) give a wider perspective on system fragilities. In fact, banks’ centrality may be understood by reference to three different structural attributes within the network: a bank’s *degree*, *between-ness* and *closeness*. Network activity would best be analysed using a degree-based measure, whereas an analysis of a node’s control of network activity would benefit from a measure based upon between-ness. A measure based upon closeness would be the best solution when looking at independence or efficiency. These *centrality* measures evaluate importance based on the position of a node in relation to the others, and each covers a different aspect of the centrality/power. For example, a high degree of a node (the number of connections or edges the node has to other nodes) is associated with the node’s ability to accentuate the spread of a shock, making the network more *fragile*. By contrast, the network is considered to be more *robust* whenever nodes facilitate more risk-shifting and therefore act as shock absorbers.

The importance of financial actors is measured extensively with such metrics by identifying critical institutions, such as banks too connected to fail, illustrating the effects in the event of a loss or a shock, or identifying nodes of the financial network serving a particularly important role.10

**Application of structural surveillance to cross-holdings of bank securities**

One rich source of information illustrating the usefulness of network analysis in macro-prudential work is the network with banks as nodes and the cross-holding of securities as links, here referred to as the *securities network*. As observed from the collateral used for Eurosystem operations, in March 2012 this network had 1,530 bank groups (or nodes) and 13,121 group relationships formed by banks holding securities issued by each other amounting to a total of around €914 billion (see Chart C.1 which groups the banks according to the nationality of the issuer/user).

The present analysis relies on observations which are available on a weekly basis starting in October 2008, i.e. over 174 periods. The number of holding relationships or links of each bank with another bank is 17 on average (simple average) and 69 when using a value-weighted average (the value of securities representing the link). Thus, this network is characterised by low density; it is a very sparse network. Indeed the diameter – the greatest distance between any pair of nodes – comprises only seven nodes and the average path length is 2.51, indicating that typically banks are not “too distant” from each other in this type of relationship. This is a consequence of well-connected nodes being linked to less well-connected ones, as indeed a

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9 Centrality metrics are different and thus a node with many links will have a high value in terms of degree but may have a marginal position in the overall structure, while a node with a lower degree value but which is more central can matter more in the overall structure. See L. C. Freeman, “Centrality in Social Networks: I. Conceptual Clarification”, *Social Networks*, Vol. 1, 1979.

10 For example, in G. von Peter, “International banking centres: a network perspective”, *BIS Quarterly Review*, December 2007, centrality measures are used to identify Germany, France, the United Kingdom, Switzerland and the United States as international banking centres.
low level of assortativity indicates. While the concentration of banks in the network is also low (a low clustering coefficient), the larger weighted coefficient implies strong relationships between the nodes (see Table C.1).

Metrics computed at bank level may help to ascertain the type of the securities network and thus its vulnerability. Financial institutions having the largest number of connections play a hub role, and those whose securities are widely held by other counterparties are, of course, particularly relevant in measuring system fragility (see Chart C.2).

In addition, holding or issuing a bank security distinguishes banks as users or issuers. Banks are quite specialised: around 13% both issue and hold securities, while the remainder are only issuers (78%) or only users (9%), indicating the hierarchical and “intermediate” nature of financing based on securities-holding (see Chart C.3). The value of the securities held is the weight of the link.

Therefore, the direction and size of the interconnections give further nuance to the analysis. The direction provides information distinguishing between the cause and consequence of potential shocks. The typical degree notion of connectedness becomes the dual in- and out-degrees, clarifying the concepts of users and issuers of securities. Likewise, the number of securities held qualifies the strength of the relationship, with larger volumes representing more significant links, and thus enabling the linkages to be weighed (see Table C.2).

To illustrate how the analysis is enhanced by the use of both direction and size, it is useful to consider eigenvector and alpha centrality. Each node is given a starting random positive amount of influence. Each node then splits its influence evenly, dividing it among its outward neighbours and receiving from its inward neighbours in kind, continuing until every institution is giving out as

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11 Assortativity describes nodes’ preference to attach to others that are similar or different in some way and is often operationalised as a correlation between two nodes. There are several ways to capture such a correlation. The two most prominent measures are the assortativity coefficient and neighbour connectivity.

12 A number of factors affect the level of securities held by different institutions, such as the introduction of a limit on the use of unsecured bank bonds as collateral.
much as they are taking in and the system has reached a steady state. Clearly, institutions with larger links, being well connected, have greater influence. The amount of influence each has in this steady state is its eigenvector centrality.

Alpha centrality enhances this process by, first, allowing nodes to have external sources of influence that do not stem from the links themselves, and then trading off external influence against that associated with a connection. Alpha centrality therefore captures an innate centrality of a node that is independent of the number, direction and size of its relationships. A nil value of alpha denotes that only external influence matters, whereas very large values denote that only the innate characteristic of the bank matters. A bank has a positive alpha value for links from or to banks with high scores and a negative value for links from or to banks with low scores.\(^{13}\) The large dispersion of alpha centrality around nil supports the notion of much bank diversity, with banks both influencing and being influenced by each other and being connected both as users and issuers (without much innate influence on average).

In addition, banks with a high level of between-ness are more fragile in the event of the failure of other banks, while at the same time they are systemically more important as their difficulties have a bigger impact on the network than banks with a low level of between-ness. Closeness also detects systemic importance, quantifying a bank’s distance from or to all other banks. While intrinsically denoting fragility, high-closeness banks may also be protected by other big and “healthy” close institutions in the event of the failure of a peripheral bank.

Overall, the standard deviation of centrality measures is very high (see Table C.2). Accordingly, banks have very different positions in the network, with some being very central while others have a negligible role in the system. In particular, the distributions of between-ness and centrality show

\(^{13}\) In addition, Kleinberg’s centrality scores do not take into account the weights of links but identify hubs and authorities: important hubs send links to banks that have high authority scores, while a bank is a good authority if it is pointed to by many good hubs. In addition, it can be useful to identify banks linked to very important banks via the Bonacich power centrality (P. Bonacich, “Power and centrality: a family of measures”, *The American Journal of Sociology*, Vol. 92, 1987).
that many nodes are almost isolated, with between-ness and closeness scores being essentially negligible.

Importantly, the evolution over time of the distribution of bank-level values can help to illustrate changes in the network’s structure. This is the case for between-ness, for instance, whose average across banks can help to ascertain the (average) fragility of the banking system as a whole (see Chart C.4). This measure seems to be sensitive to developments associated with a high impact on confidence, with between-ness measuring a bank’s willingness to “become more connected” to other banks (i.e. increased confidence).

Overall, measures at the network and node levels confirm that the security network has a centralised structure, with some important banks connected with many other peripheral ones. Moreover, the analysis of the securities network shows that the structure is well integrated across countries, with some banks playing an important role at the European level and others at the domestic level. Single measures alone may not be sufficient to analyse the securities network, as multiple levels of analysis are required to assess banks’ network fragility in a complex banking system.

INTERCONNECTIONS GAUGED FROM A SIMULATED NETWORK OF BANK LOANS AND DEPOSITS

The introduction of the euro created a large and integrated euro area money market allowing euro area banks to lend to and fund themselves via other euro area banks across national borders. This facilitated financial transactions and trade between euro area countries. However, since the outbreak of the financial crisis in mid-2007, which inter alia led to severe disruptions in the interbank market, particular attention has been paid to the potential counterparty risks incurred by banks via their bilateral interbank exposures.

To model how shocks to one (or more) financial entities can have contagious effects throughout the financial system, a dynamic network modelling approach is warranted. However, since data on bank-to-bank bilateral exposures are not generally available, an alternative method is proposed which uses individual banks’ aggregate interbank exposures to simulate a wide range of possible interbank networks. Once the interbank interconnectedness structures have been simulated, a dynamic analysis of how and to what extent shocks to different entities propagate throughout the

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14 The three changes illustrated relatively arbitrarily show recent developments with a profound impact – both positive and negative – on the subsequent movement of this measure.
15 Interbank lending and borrowing constitute a significant part of EU banks’ balance sheets (up to 20%).
banking system can be conducted. Such an analysis is useful, for example, in a stress-testing context to gauge the impact on specific banks or the banking system as a whole of shocks to one or more banks.

The following paragraphs describe how the interbank structures are simulated before giving an illustrative analysis of an exogenous shock to one or more banks.

Random network model
A sample of 89 European, mostly euro area, banks is used. Notably, only interbank relations between the EU banks are considered, i.e. any cross-border linkages with non-EU banks are ignored. As data on the individual banks’ bilateral exposures are not readily available, they are derived from their total interbank placements and deposits. Individual bank data used to parameterise the model are taken from the Bureau van Dijk Bankscope database and banks’ financial reports.

An interbank network is randomly generated based on banks’ interbank placements and deposits and taking into account the geographical breakdown of banks’ activities. Once the distribution of interbank networks has been calibrated, the system can be shocked to assess how specific shocks are transmitted throughout the system and to gauge the implications for the overall resilience of the banking sector. The shock is typically a given bank’s default on all of its interbank payments. It then spreads across the banking system, transmitted by the interbank network of the simulated bilateral exposures.

The model consists of three main building blocks: the interbank probability map, the random interbank network generator and the equilibrium interbank payments.

Interbank probability map
Bank-by-bank bilateral interbank exposures are not readily available. Thus, in order to define the interbank linkages, a probability structure (a probability map) is needed. For this purpose, the European Banking Authority (EBA) disclosures on the geographical breakdown of individual banks’ activities (here measured by the geographical breakdown of exposures at default) were used. This provides a proxy for the likelihood with which banks lend to and/or borrow from each other given their presence on the same market and client relationship. The probabilities were defined at the country level, i.e. the exposures were aggregated within a country and the fraction of these exposures towards banks in a given country was calculated. These fractions were assumed to be probabilities that a bank in a given country makes an interbank placement to a bank in another (or the same) country. With this aim, banks were first grouped into two sub-categories within countries: banks with a domestic scope of activity and banks with international activities. The classification was based on a ratio calculated as the share of cross-border intra-EU exposures in total exposures.

Generating interbank networks
The interbank probability map enables various structures of the interbank network to be studied, even when only aggregated interbank loan/deposit data are available. The basic notion is to reconstruct, using a random generator, the linkages between banks from the reported interbank placements and deposits. An iterative procedure to establish a realisation of the network is...
applied, whereby a pair of banks is randomly drawn – all pairs have equal probability – and the pair is kept as an edge (link) in the interbank network with a probability given by the probability map. If the drawn link is kept as an interbank exposure, then the random number is generated (from the uniform distribution on \([0,1]\)) indicating the percentage of reported interbank liabilities (\(IL\)) of the first bank in the pair coming from the second bank in the pair. (The amount is appropriately truncated to account for the reported interbank assets (\(IA\)) of the second bank.) If not kept, then the next pair is drawn and accepted with a corresponding probability or not. Ultimately, the stock of interbank liabilities and assets is reduced by the volume of the assigned placement. The procedure is repeated until no more interbank liabilities are left to be assigned as placements from one bank to another. Analysing many different interbank structures instead of just one specific structure (either observed at the reporting date or – if not available – estimated, for example by means of entropic measures) generates the dynamic and unstable interbank structures that are confirmed by many studies.21 Chart C.5 illustrates one realisation from the whole distribution of network structures for the EU banking sector generated using the random

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network modelling approach. The width of the arrows indicates the size of exposures (logarithmic scale) and the colouring scale (from light to dark green) denotes the probability (inferred from the interbank probability map) that a given bank grants an interbank deposit to the other bank. Most of the connections are between banks from the same country, but the connectivity between the biggest domestic banking systems is also quite high (the German, Spanish and British banking systems in particular).

Equilibrium interbank payments

Once the set of interbank structures has been generated, the impact of shocks to the networks can be analysed by looking at their impact on the equilibrium interbank payment transactions. For this purpose, a clearing payments vector is defined, in line with the concept introduced by Eisenberg and Noe. The assessment of the size of the interbank contagion is based on the so-called interbank clearing payments vector defined by the vector solving the following equation (minimum and maximum are entrywise):

\[ p^* = \min \left\{ \max \left\{ C - IA + IL + \pi^T \pi^* 0 \right\} \right\} \]

where \( C \) is a vector of banks’ capital buffers and \( \pi^* \) is a transposed matrix of the relative interbank exposures with \( \pi_{ij} \) entry defined as bank \( i \) interbank deposits from bank \( j \) divided by the total interbank deposits of bank \( i \).

The expression \( C - IA + IL \) can be interpreted as banks’ own funding sources adjusted by the net interbank exposures; the ultimate interbank payments are derived as the equilibrium of flows in the interbank network. If \( p_{ij}^* \) is equal to \( IL_{ij} \), then bank \( i \) returns all its interbank obligations. The contagious default on the interbank deposits is detected if \( p_{ij}^* \) is less than \( IL_{ij} \); this means that bank \( i \) defaults on its interbank payments. The loss is then proportionately spread among its creditors using the matrix of the relative exposures.

The risk of “fire-sale” losses

If some part of its interbank funding were to evaporate, a bank may need to shed part of its securities portfolio \( S_i \) in order to meet its obligations; the less interbank assets it receives back, the higher the liquidation need. This may adversely affect the mark-to-market valuation of the banks’ securities portfolios and further depress their capacity to pay back their interbank creditors. Consequently, this mechanism may lead to a spiral effect of fire sales of securities. It is assumed that the extent of the devaluation of the securities portfolios is related to the share of the liquidated securities in the total volume of securities held by banks. All in all, there is an implicit assumption that banks do not use eligible securities as collateral to obtain central bank funding in such emergency circumstances; instead they sell outright part of their securities portfolio.

In order to quantify this fire-sale mechanism, an auxiliary measure of the conditional amount of securities sold by bank \( i \) given all other banks pay back \( (p) \) units of their interbank deposits is introduced. This is the sum of the securities (denoted \( SecSold(p) \)) that may be sold by banks covering

22 The proneness of the network to contagion may be directly assessed based on the network topology. The most recent concepts are presented in B. M. Tabak, M. Y. Takami, J. M. C. Rocha and D. O. Cajueiro, “Directed Clustering Coefficient as a Measure of Systemic Risk in Complex Banking Networks”, Working Papers Series, No 249, Central Bank of Brazil, 2011.


The securities sold depress the market value of the existing portfolio, which reduces banks’ loss-absorption capacity.

The framework is applied to study contagion triggered by a given bank’s default on its interbank obligations.

Contagion is a tail risk, highly non-linear; its impact is very limited in as many as 99% of simulated interbank structures.

The securities sold depress the market value of the existing portfolio, which reduces banks’ loss-absorption capacity.

The new equilibrium interbank payments can be computed with a new loss-absorption capacity which is equal to the initial capital level less the devaluation of the securities. It is assumed that the liquidation value of the portfolio of securities is related to the part of the portfolio that may be disposed of (and de facto to the interbank payments vector $p$) in the following way:

$$S_i(p) = \exp(-\alpha \cdot \text{SecSold}(p)/TS) \cdot S_i,$$

where $TS$ is the aggregate volume of securities in banks’ portfolios (a proxy for general securities market depth) and $\alpha$ is the sensitivity parameter. The sensitivity can be gauged by looking at estimates from studies of the impact of bond trading on prices. The higher the supply amount of liquidated securities, the lower their expected market value. To simplify, it is implicitly assumed that all the securities are marked to market, so liquidation of part of the securities portfolio affects the valuation of the whole portfolio, which may not be the case for held-to-maturity bonds.

Simulations

The simulation of contagious defaults on interbank debt can be performed following an event-driven concept, where an exogenous shock to the ability of a bank (or a group of banks) to satisfy its creditors affects other institutions’ solvency through the linkages in the network. In the following illustration, “bank triggers” of contagion are analysed only within internationally active banks. It is assumed that one of these banks defaults on its interbank deposits. Then 20,000 scenarios of the interbank network are generated and, for each such structure, the clearing payments vector of the interbank system is calculated. In order to illustrate the fire-sale mechanism, the interbank payments equilibrium is simulated with the securities’ value sensitivity parameter equal to 0.15. Consequently, for the sake of comparability the results are reported in the form of the distribution of the capital adequacy ratio reduction attributable to the interbank (contagion) losses.

As shown in Chart C.6, contagious bank defaults are a tail-risk phenomenon. In 99% of the scenarios of the randomly generated networks, the average reduction of banks’ capital adequacy ratios (CAR) does not exceed 0.2 percentage point which, in general, should not depress banks’ capital base (the CAR reduction exceeds 1 percentage point in only one out of one


26 Following studies by Mitchell et al., ibid., it is calibrated in such a way that a sell-off of 17.5% of banks’ securities portfolios leads to a 2.7% discount in the mark-to-market valuation of the portfolios.
thousand realisations). Inclusion of the fire-sale mechanism increases the potential contagious losses but the additional reduction of CAR is rather limited (the average decrease in 1% of the worst-case networks amounts only to 6 basis points). The results are quite homogenous across countries. One natural feature of the interbank losses is their apparent non-linearity since they start to have an adverse effect in the system once linkages of a certain size between certain banks are present in the network. Nevertheless, in most of the cases, the event-driven shock is contained by the diversified interbank connections.

Moreover, at least two important mechanisms may mitigate the risk (and size) of the interbank contagion. First, the Basel II rules on large exposures limit the size of exposure to counterparties and mitigate the risk of contagion. Second, banks actively manage the counterparty trading limits and in many cases they may still have enough time to reduce exposure to an institution perceived as having the potential to get into financial trouble.

CONCLUDING REMARKS

This special feature discusses the use of network analysis based on existing and simulated information in the context of financial stability. Given that only minimal information on financial institutions’ interlinkages is in the public domain, the two approaches are a practical means of gaining insight into the interconnection of financial firms. Since the objective of monitoring and assessing such interlinkages can vary depending on the policy question at hand, the approaches highlight broad technical aspects that are fundamental from a macro-prudential perspective for existing data, and present a novel way of testing more dynamic issues on the basis of simulations.

Both the more static analysis of existing financial networks – in this case the securities network – and the dynamic use of a simulated analysis for stress testing constitute new approaches to understanding the fragilities related to activities linking banks with one another. Both exploit information on the microstructure of banking activities to characterise the robustness of the banking sector as a whole to localised operating shocks.