Firm Heterogeneity and Credit Risk Diversification

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ECB-CGFS Conference, Frankfurt
November 9, 2005

* Any views expressed represent those of the authors only and not necessarily those of the Federal Reserve Bank of New York or the Federal Reserve System.

Obtaining credit loss distributions

- Credit loss distributions tend to be highly non-normal
  - Skewed and fat-tailed
  - Even if underlying stochastic process is Gaussian
  - Non-normality due to nonlinearity introduced via the default process
- Typical computational approach is through simulation for a variety of modeling approaches
  - Merton-style model
  - Actuarial model
- Closed form solutions, desired by industry & regulators, require strict homogeneity (in addition to distributional) assumptions
  - Basel 2 Accord
- What is the cost of imposing such homogeneity?

Credit risk modeling literature

- Most approaches use a single abstract, *un*observed systematic risk factor

- Contingent claim (options) approach (Merton 1974)
  - Model of firm and default process
  - KMV (Vasicek 1987, 2002)
  - CreditMetrics: Gupton, Finger and Bhatia (1997)

- Vasicek's (1987) formulation forms the basis of the New Basel Accord (Gordy 2003). It is, however, highly restrictive as it imposes a number of homogeneity assumptions

- A separate and growing literature on correlated default intensities
  - Schönbucher (1998), Duffie and Singleton (1999), Duffie and Garleanu (2001), Duffie, Saita and Wang (2005)

- Default contagion models
  - Davis and Lo (2001), Giesecke and Weber (2004)

Our contribution: conditional modeling and heterogeneity

- The loss distributions discussed in the literature typically do not explicitly allow for the effects of macroeconomic variables on losses. They are *unconditional* models.
  - Exception: Wilson (1997), Duffie, Saita and Wang (2005)

- In Pesaran, Schuermann, Treutler and Weiner (JMCB, forthcoming) we develop a credit risk model *conditional* on observable, global macroeconomic risk factors

- In this paper we de-couple credit risk from business cycle variables but allow for
  - Different probability of default by rating
  - Different systematic risk sensitivity across firms ("beta")
  - Different error variances across firms

Preview of paper

- Derive theoretical results on limiting loss distribution when risk factors are normal
  - Where possible, results generalized to non-normal distributions (e.g. Student-t)

- There is a rich and complex interaction between the underlying model parameters and the resulting loss distributions

- Theory says:
  - Neglecting parameter heterogeneity can lead to underestimation of expected losses (EL)
  - Once EL is controlled for, such neglect can lead to overestimation of unexpected loss (UL or VaR)

- Empirical study confirms theoretical results
  - Large, two-country (Japan, U.S.) portfolio
  - Credit rating information (firm-specific unconditional default risk) very important
  - Return specification important (conditional independence)

Generating portfolio loss distributions

- We are primarily interested in generating (conditional) portfolio loss distributions

$$L_{t+1} = \sum_{i=1}^{N} w_{i,t} L_{i,t+1}, \sum_{i=1}^{N} w_{i,t} = 1,$$

$$\sum_{i=1}^{n} w_{i,t}^{2} \rightarrow 0 \text{ as } N \rightarrow \infty \qquad \text{granularity condition}$$

$$L_{i,t+1} = \mathrm{I}\left(V_{i,t+1} < D_{i,t+1}\right) \times LGD_{i,t+1}$$

$LGD_{i,t+1}$ = 100%

- If interest is confined to simple portfolio-level analysis (passive portfolio management), not critical to elaborate asset/firm-level default process
  - However, required for active portfolio management, i.e. actively changing the portfolio weights

Firm returns

- Firm equity evolves as random walk with drift

$$\ln\left(E_{i,t+1}\right) = \ln\left(E_{i,t}\right) + \mu_i + \xi_{i,t+1} \quad \xi_{i,t+1} \sim iidN(0, \sigma_{\xi_i}^2)$$

- so that our basic firm return equation is

$$r_{i,t+1} = \mu_i + \boldsymbol{\gamma}_i^{\prime}\mathbf{f}_{t+1} + \sigma_i \varepsilon_{i,t+1}, \quad \sigma_{\xi_i}^2 = \boldsymbol{\gamma}_i^{\prime}\boldsymbol{\gamma}_i + \sigma_i^2$$

- Multi-factor formulation

$$\xi_{i,t+1} = \boldsymbol{\gamma}_i^{'}\mathbf{f}_{t+1} + \sigma_i \varepsilon_{i,t+1} \quad \varepsilon_{i,t+1} \sim iidN(0,1);\ \mathbf{f}_{t+1} \sim N(\mathbf{0}, \mathbf{I}_m)$$

- Note that the multi-factor nature of the process matters only when the factor loadings $\boldsymbol{\gamma}_i$ are heterogeneous across firms

Cross firm return correlations

- Given the basic firm return equation, we may derive pair-wise return correlation

$$r_{i,t+1} = \mu_i + \boldsymbol{\gamma}_i' \mathbf{f}_{t+1} + \sigma_i \varepsilon_{i,t+1}, \quad \sigma_{\xi_i}^2 = \boldsymbol{\gamma}_i' \boldsymbol{\gamma}_i + \sigma_i^2$$

$$\rho_{ij} = \frac{\boldsymbol{\delta}_i' \boldsymbol{\delta}_j}{\left(1 + \boldsymbol{\delta}_i' \boldsymbol{\delta}_i\right)^{1/2} \left(1 + \boldsymbol{\delta}_j' \boldsymbol{\delta}_j\right)^{1/2}}, \ \boldsymbol{\delta}_i = \frac{\boldsymbol{\gamma}_i}{\sigma_i}$$

Cross firm default correlations

- Firm default condition

$$z_{i,t+1} = I\left(r_{i,t+1} < \lambda_{i,t+1}\right) \Rightarrow E\left(z_{i,t+1}\right) = \pi_{i,t+1}$$

- From return to default correlation

$$\rho_{ij}^{*} = \frac{E(z_{i,t+1} z_{j,t+1}) - \pi_{i,t+1}\pi_{j,t+1}}{\sqrt{\pi_{i,t+1}(1-\pi_{i,t+1})}\sqrt{\pi_{j,t+1}(1-\pi_{j,t+1})}}$$

- Default correlation depends on return correlation and default probabilities

$$\pi_{i,t+1} = \pi_i = \Phi\left(\frac{\lambda_{i,t+1} - \mu_i}{\sqrt{\sigma_i^2 + \boldsymbol{\gamma}_i' \boldsymbol{\gamma}_i}}\right)$$

$$E\left(z_{i,t+1} z_{j,t+1}\right) = E_f\left[\Phi\left(a_{i,t+1} - \boldsymbol{\delta}_i' \mathbf{f}_{t+1}\right)\Phi\left(a_{j,t+1} - \boldsymbol{\delta}_j' \mathbf{f}_{t+1}\right)\right] \quad a_{i,t+1} = \frac{\lambda_{i,t+1} - \mu_i}{\sigma_i}$$

$$= \Phi_2\left[\Phi^{-1}\left(\pi_{i,t+1}\right), \Phi^{-1}\left(\pi_{j,t+1}\right), \rho_{ij}\right]$$

Portfolio loss distribution

- Assume for simplicity that loss-given-default = 100%. Then portfolio loss is

$$\ell_{N,t+1} = \sum_{i=1}^{N} w_{it} z_{i,t+1}.$$

- Problem of N assets (e.g. loans), each with weight w_i

- Granularity condition:

$$\sum_{i=1}^{N} w_{it} = 1, \ \sum_{i=1}^{N} w_{it}^2 = O(N^{-1}), \ w_{it} \geq 0.$$

Portfolio loss distribution (cont'd)

- Allow factors to have some serial dependence

$$\mathbf{f}_{t+1} = \mathbf{\Lambda f}_t + \boldsymbol{\eta}_{t+1},\ \boldsymbol{\eta}_{t+1} \mid \mathcal{I}_t \sim iidN(\mathbf{0}, \mathbf{\Omega}_{\eta\eta}),$$

$$Var(\mathbf{f}_{t+1} \mid \mathcal{I}_t) = \sum_{s=0}^{\infty} \mathbf{\Lambda}^s \mathbf{\Omega}_{\eta\eta} \mathbf{\Lambda}'^s = \mathbf{I}_m.$$

- Now ready to consider asymptotic portfolio properties

$$\ell_{N,t+1} \mid \mathcal{I}_t \quad \text{as} \quad N \to \infty$$

Portfolio loss in Vasicek model

- Vasicek (1987) was the first to propose a solution under full homogeneity

- Loans are tied together via a single, unobserved systematic risk factor (“economic index”) f and same correlation ρ

$$r_{i,t+1} = \gamma f_{t+1} + \sigma \varepsilon_{i,t+1}, \quad \begin{pmatrix} \varepsilon_{i,t+1} \\ f_{t+1} \end{pmatrix} \mid \mathcal{I}_t \sim iidN(\mathbf{0}, \mathbf{I}_2).$$

$$\rho_{ij} = \rho = \frac{\gamma^2}{\sigma^2 + \gamma^2}.$$

$$\pi = E(\ell_{N,t+1}) = \sum_{i=1}^{N} w_{it} E(z_{i,t+1}) = E(z_{i,t+1}) = \Pr(r_{i,t+1} \leq \lambda) = \Phi(\lambda).$$

Portfolio loss in Vasicek model (cont'd)

- Vasicek's model becomes quite simple

$$r_{i,t+1} = \sqrt{\rho}\, f_{t+1} + \sqrt{1-\rho}\, \varepsilon_{i,t+1},$$

- All firms have the same default probability, correlation and default threshold

$$\lambda = \Phi^{-1}\left(\pi\right)$$

- Default correlation is

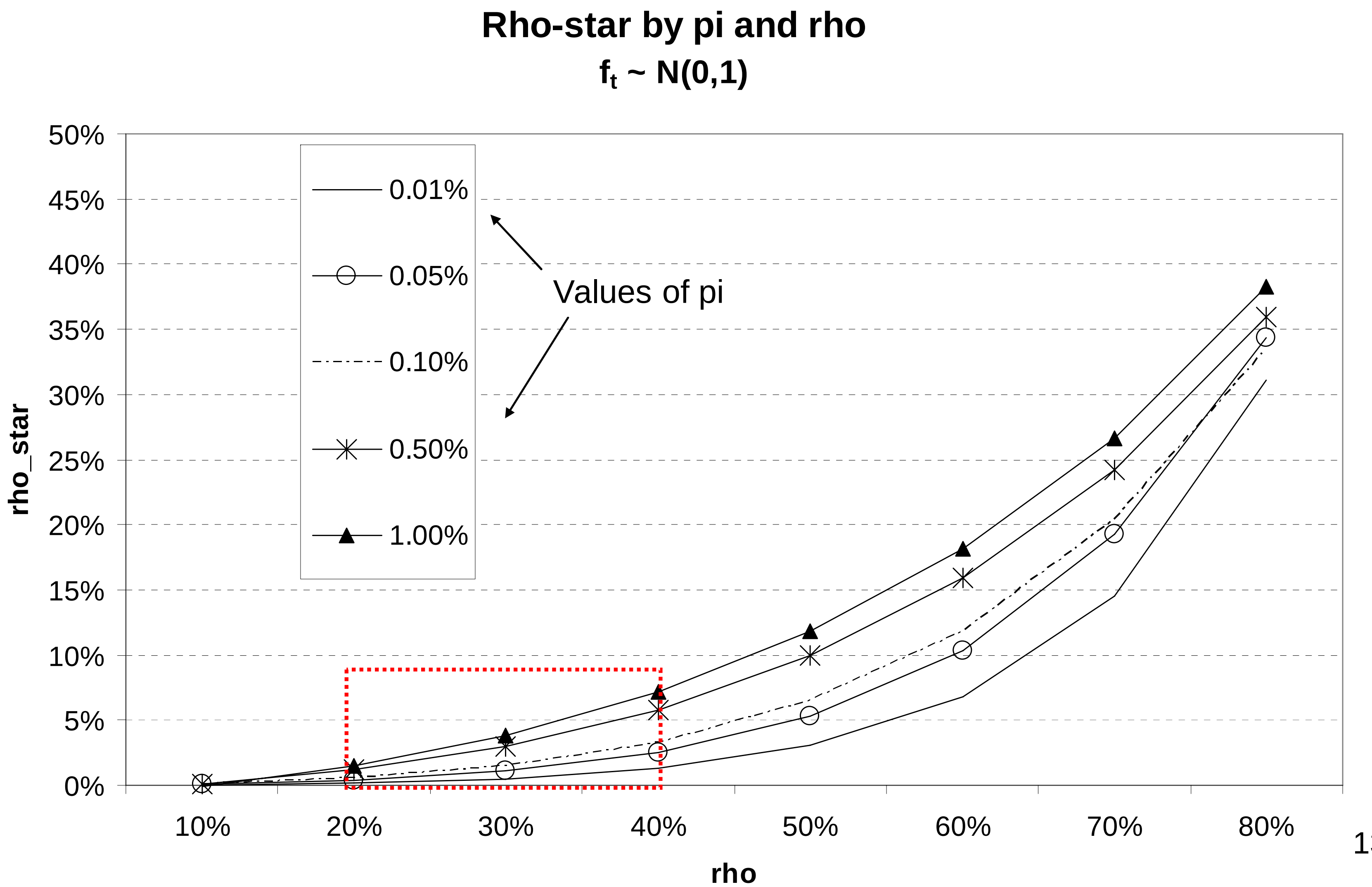
$$\rho_{ij}^{*} = \rho^{*}(\pi,\rho) = \frac{\Phi_2[\Phi^{-1}(\pi),\Phi^{-1}(\pi),\rho] - \pi^2}{\pi(1-\pi)}.$$

- Under Student-t distribution, default correlation is

$$\rho^{*}\left(\pi,\rho,v\right) = \frac{E_f\left\{\left[T_v\left(\frac{T_v^{-1}(\pi)}{\sqrt{1-\rho}} - \sqrt{\frac{\rho}{1-\rho}}\, f_{t+1}\right)\right]^2\right\} - \pi^2}{\pi(1-\pi)}$$

Default correlation

- Default correlation (ρ*) depends on return correlation (ρ) and probability of default (π)



Vasicek limit distribution

- As $N \to \infty$, the loss distribution converges to a distribution which depends on just π and ρ
  - –fraction of portfolio lost denoted by *x*

$$f_\ell(x \mid \mathcal{I}_t) = \sqrt{\frac{1-\rho}{\rho}} \left\{ \frac{\phi\left[\frac{\sqrt{1-\rho}\,\Phi^{-1}(x) - \Phi^{-1}(\pi)}{\sqrt{\rho}} \right]}{\phi[\Phi^{-1}(x)]} \right\}, \text{ for } 0 < x \leq 1,\ \rho \neq 0,$$

$$F_\ell(x \mid \mathcal{I}_t) = \Phi\left[\frac{\sqrt{1-\rho}\,\Phi^{-1}(x) - \Phi^{-1}(\pi)}{\sqrt{\rho}} \right].$$

- These two parameters π and ρ drive the shape of the loss distribution

- Portfolio loss variance: $\lim_{N\to\infty} Var(\ell_{N,t+1} \mid \mathcal{I}_t) = \pi(1-\pi)\rho^*.$

Introducing heterogeneity

- Allowing for firm heterogeneity is important
  - Firm values are subject to specific persistent effects
  - Firm values respond differently to changes in risk factors ("betas" differ across firms)
    - Note this is different from uncertainty in the parameter estimate
  - Default thresholds need not be the same across firms

- But it [heterogeneity] gives rise to an identification problem
  - Direct observations of firm-specific default probabilities are not possible
  - Classification of firms into types or homogeneous groups would be needed
  - In our work we argue in favor of grouping of firms by their credit rating: π_R

Introducing parameter heterogeneity

- Parameter heterogeneity can be introduced through the standard random coefficient model

$$\boldsymbol{\theta}_i = \boldsymbol{\theta} + \mathbf{v}_i\,, \quad \mathbf{v}_i \sim iid\left(\mathbf{0}, \Omega_{vv}\right)$$

where $\mathbf{v}_i$ is independent of $\mathbf{f}_{t+1}$ and ε_{t+1}

- Could be the case for middle market & small business lending where it would be very hard to get estimates of θ_i
  - Use estimates from elsewhere of θ and Ω_{vv}

- Could allow for more general types of heterogeneity
  - Firms grouped in sectors/countries where parameters have a common distribution within but not across types

  ➔ Hierarchical heterogeneity

Introducing simple heterogeneity

- EL for Vasicek fully homogeneous case

$$EL_{t+1} = \Pr\left(\delta f_{t+1} + \varepsilon_{i,t+1} < a\right) = \Phi\left(\frac{a}{\sqrt{1+\delta^2}}\right)$$

$$\delta = \frac{\gamma}{\sigma}, \quad a = \frac{\lambda - \mu}{\sigma}$$

- Heterogeneity is introduced through a_i

$$a < 0 \leftarrow a_i = a + v_i\,,\ v_i \sim iidN\left(0, \sigma_v^2\right)$$

Can be thought of as heterogeneity in default thresholds and/or expected returns

EL ↑ under parameter heterogeneity

- Now we can compute portfolio expected loss

$$EL_{t+1} = \Phi\left(\frac{a}{\sqrt{1+\delta^2+\sigma_v^2}}\right)$$

- Can be viewed as an example of Jensen's inequality

$$\Phi\left[\frac{E(a_i)}{\sqrt{1+\delta^2}}\right] < E\left(\Phi\left[\frac{a_i}{\sqrt{1+\delta^2}}\right]\right)$$

UL $\downarrow$ under parameter heterogeneity, for a given EL

- For the same given value of EL, examine the impact on UL

$$\rho_{ij} = \rho = \frac{\delta^2}{1 + \delta^2 + \sigma_v^2}$$

- Since UL is increasing in ρ
  – as degree of parameter heterogeneity $\uparrow$, i.e. $\sigma_v^2 \uparrow$, $\rho \downarrow$ and UL $\downarrow$

UL ↓ under parameter heterogeneity

- More generally, if the parameters for different groups come from different (parameter) distributions, there is further scope for portfolio loss variance reduction
  - For example, systematic differences across regions and/or sectors

- So active portfolio management by mixing across groups results in lower (asymptotic) portfolio variance than concentrating exposure in one group

Empirical application

- Two countries, U.S. and Japan, quarterly equity returns, about 600 U.S. and 220 Japanese firms

- 10-year rolling window estimates of return specifications and average default probabilities by credit grade
  - First window: 1988-1997
  - Last window: 1993-2002

- Then simulate loss distribution for the 11th year
  - Out-of-sample
  - 6 one-year periods: 1998-2003

- To be in a sample window, a firm needs
  - 40 consecutive quarters of data
  - A credit rating from Moody's or S&P at end of period

Merton default model in practice

- Approach in the literature has been to work with market and balance sheet data (e.g. KMV)
  - Compute default threshold using value of liabilities from balance sheet
  - Using book leverage and equity volatility, impute asset volatility

- We use credit ratings in addition to market (equity) returns
  - Derive default threshold from credit ratings (and thus incorporate private information available to rating agencies)
  - Changes in firm characteristics (e.g. leverage) are reflected in credit ratings

- We use arguably the two best information sources
  - Market: μ_i and σ_i
  - Rating agency: π_i

$$\left.\begin{array}{l}\text{Market: } \mu_i \text{ and } \sigma_i \\ \text{Rating agency: } \pi_i\end{array}\right\} \lambda_i \qquad \pi_R \Leftrightarrow \mathrm{DD}_R$$

Modeling conditional independence

- The basic factor set-up of firm returns assumes that, conditional on the systematic risk factors, firm returns are independent

- A measure of conditional independence could be the (average) pair-wise cross-sectional correlation of residuals (in-sample)

- Similarly, we can measure degree of unconditional dependence in the portfolio
  - (average) pair-wise cross-sectional correlation of returns (in-sample)

- Broadly, a model is preferred if it is “closer” to conditional independence

Simulated out-of-sample default correlations

2003

	$\hat{\pi}_R$	$\hat{\rho}$	$\hat{\rho}_R^*$
AAA/AA	0.10	9.20%	0.004%
A	0.58	9.20%	0.018%
BBB	10.59	9.20%	0.170%
BB	63.03	9.20%	0.615%
B	542.88	9.20%	2.437%
CCC	4,977.60	9.20%	5.865%

	AAA/AA	**A**	**BBB**	**BB**	**B**	**CCC**
AAA/AA	**0.004%**	-	-	-	-	-
A	0.009%	**0.018%**	-	-	-	-
BBB	0.026%	0.054%	**0.170%**	-	-	-
BB	0.048%	0.100%	0.320%	**0.615%**	-	-
B	0.086%	0.183%	0.607%	1.199%	**2.437%**	-
CCC	0.102%	0.224%	0.792%	1.636%	3.560%	**5.865%**

EL ↑ under parameter heterogeneity

- The impact on expected losses (EL) of allowing for fixed effects

	Simulated EL	
Year	**Vasicek**	**Fixed Effect**
1998	1.23%	1.72%
1999	1.60%	2.17%
2000	2.10%	2.67%
2001	2.28%	2.93%
2002	2.74%	3.28%
2003	3.26%	3.65%

UL ↓ under parameter heterogeneity

- The impact on unexpected losses (UL) of allowing for fixed effects, after equalizing EL

	Simulated UL	
Year	**Vasicek**	**Fixed Effect**
1998	1.47%	1.39%
1999	1.94%	1.84%
2000	2.22%	2.10%
2001	1.75%	1.68%
2002	1.98%	1.90%
2003	2.48%	2.40%

Return & default correlation, UL

- For 2003, $\hat{\pi} = 3.26\%$

Model Specifications	Parameter Restrictions	$\overline{\hat{\rho}_{i,j}}$	$\overline{\hat{\rho}^*_{i,j}}$	Simulated UL
Vasicek	$\hat{\pi}_i = \hat{\pi}\ ; \hat{\rho}_{i,j} = \hat{\rho}, \hat{\rho}^*_{i,j} = \hat{\rho}^*$	9.20%	1.80%	2.48%
Rating (σ^2)	$\hat{\pi}_i = \hat{\pi}_R\ i \in R;\ \hat{\rho};\ \hat{\rho}^*_{i,j} = \hat{\rho}^*_{R,R'}$	9.20%	0.26%	1.51%
Fixed Effects (σ_i^2)	$\hat{\pi}_i = \hat{\pi}_R\ i \in R$	9.06%	0.18%	1.16%
CAPM	$\hat{\pi}_i = \hat{\pi}_R\ i \in R$	10.09%	0.32%	1.27%

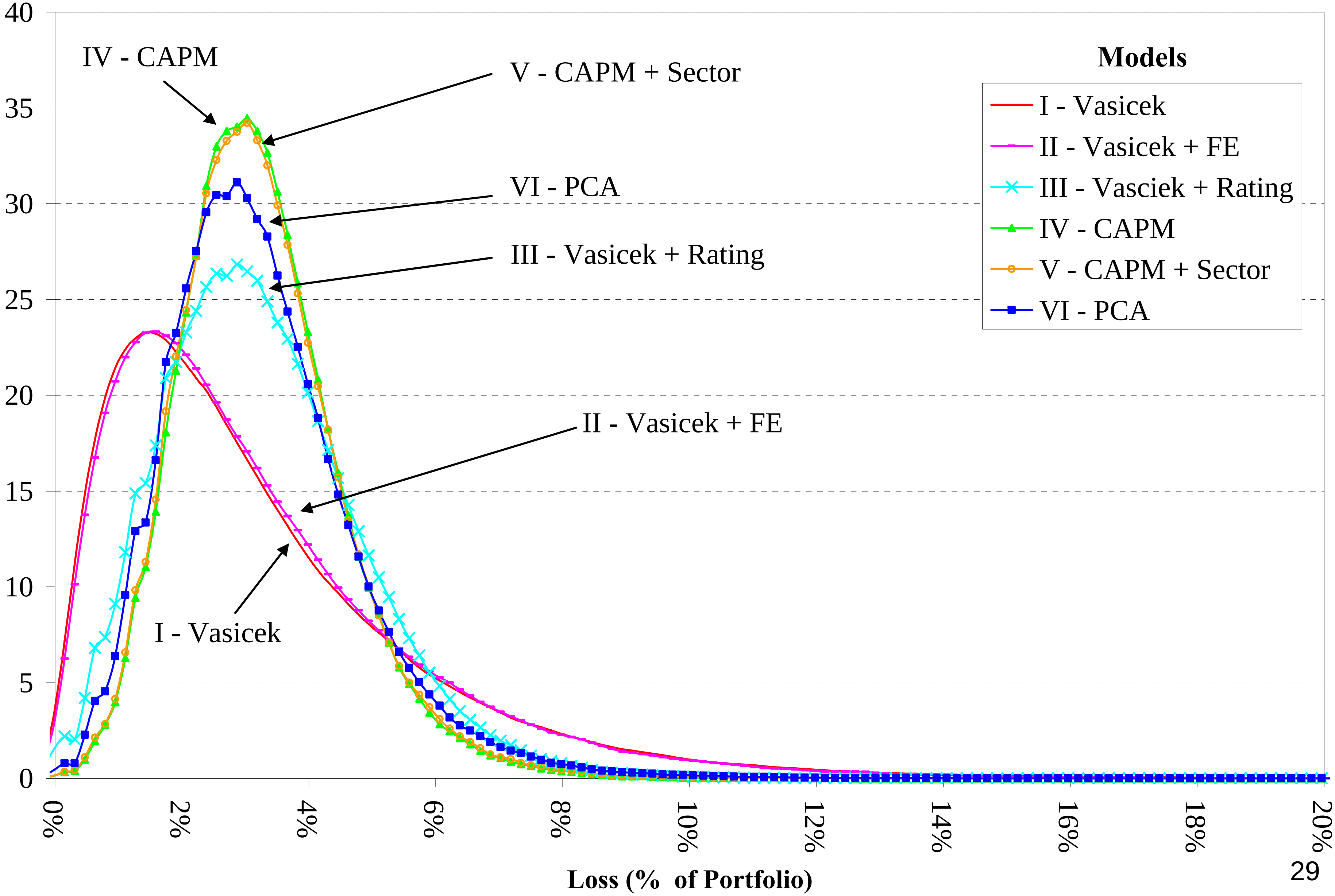
Details of loss distribution

- For 2003, $\hat{\pi} = 3.26\%$

2003:**US & Japan Pooled**

Model Specifications	**UL**	**Skew.**	**Kurt.**	**99.9% VaR**
I Vasicek	2.48%	1.8	8.0	17.47%
II Fixed Effect (σ^2)	2.40%	1.7	7.5	16.84%
III Rating (σ^2)	1.51%	0.6	3.7	9.46%
IV CAPM	1.27%	0.8	4.7	9.21%
V Sector CAPM	1.28%	0.8	4.7	9.20%
VI PCA	1.51%	1.1	5.9	11.15%

Loss distributions across models, 2003



Concluding remarks

- Firm typing along unconditional probability of default (PD) seems very important
  - Can be achieved using credit ratings
  - Within types, further differentiation using return parameter heterogeneity can matter
- Neglecting parameter heterogeneity can lead to underestimation of expected losses (EL)
- Once EL is controlled for, such neglect can lead to overestimation of unexpected loss (UL or VaR)
- Well-specified return regression allows one to comfortably impose conditional independence assumption required by credit models
  - In-sample easily measured using correlation of residuals
  - Measuring and evaluating out-of-sample conditional dependence requires further investigation

Thank You!

http://www.econ.cam.ac.uk/faculty/pesaran/