

# Default Risk Mitigation in Derivatives Markets And Its Effectiveness

## DRAFT (WORK IN PROGRESS)

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### Abstract

Derivatives have become an integral part of the major financial institutions' business and the global derivatives market has grown into the largest market in the world by far. As for any other contract, derivatives are subject to default risk. The set of mechanisms employed by traders to mitigate default risk, such as netting and margining, vary across market types. While exchanges have not experienced any notable credit events in the recent past, over-the-counter markets suffered several, almost systemic events. It seems that the sets of default risk mitigation mechanisms employed by exchanges are more effective at mitigating default risk than those employed by over-the-counter markets.

The broader impacts of these mitigation mechanisms are not yet fully understood, though. In this paper we analyze the effect of different default risk mitigation mechanisms on wealth, market liquidity, and default rates.

We develop a model to investigate the effects of default risk mitigation mechanisms on market, credit, and liquidity risk. Our model captures some of the main characteristics of derivatives markets. The dynamic and non-linear nature of our problem, of liquidity and default in particular, render a formal modelling approach unpromising. We therefore use simulations to evaluate our model.

We find that there exist situations where default risk mitigation mechanisms reduce market liquidity, increase default rates as well as default severity, and the variance of agents' wealth. Such situations include periods of market stress. This means that default risk mitigation mechanisms might have a negative effect on wealth at times when market participants expect them to be most valuable.

*JEL classifications:* G19, G21

*Key words:* Derivative securities, over-the-counter markets, default risk, systemic risk, central counterparty

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## 1 Introduction

Time and again, derivatives markets have been afflicted by severe credit events. Recent examples include the meltdown of LTCM, the collapse of Enron, and the illiquidity of Metallgesellschaft. All these institutions held large derivatives positions, and their defaults had major adverse effects on the respective markets.<sup>1</sup> Since the early days of (modern) derivatives markets, traders have tried to manage default risk inherent in derivatives contracts through contractual innovations. These innovations include definitions of default events, enforcement procedures as well as provisions for margin requirements and netting. Such innovations can be viewed as mechanisms to mitigate default risk. Today, the contracts underlying derivatives transactions vary widely with regards to the mitigation mechanisms they employ. The implications of the various contractual specifications are ambiguous. We address the following research questions: (1) What are the differences in the mechanisms for default risk mitigation observed in derivatives markets?; and (2) How do these mechanisms affect the wealth of market participants, market liquidity, and default risk? A related historical issue, namely the evolution of derivatives contracts in terms of the mechanisms for default risk mitigation they employ, is investigated in Gibson & Murawski (2005).

Default risk, the risk of non-performance of a counterparty, is inherent in derivatives contracts as much as in any other contract. However, default risk in a derivatives contract appears considerably more complex and less predictable than the default risk in, say, a simple loan. Uncertainty about the payoff pertains not only its size but also its sign. This implies that each coun-

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<sup>1</sup> Cf. Greenspan (1998) regarding LTCM, Partnoy (2002) regarding Enron, and Stulz (1996) regarding Metallgesellschaft. Cf. also Steinherr (2000).

terparty to a derivatives contract is potentially both, a creditor and a debtor. Thus, derivatives contracts are *two-way* credit instruments. Moreover, credit exposures are time-varying and depend on the prices of the underlying assets. Therefore, liabilities in relation to derivatives contracts are correlated with the underlying price changes being hedged. For example, when the price of the underlying of a call option rises, the call option gains in value. However, it becomes riskier from a credit perspective since, in case of default, the holder loses more money than initially. This is sometimes called *wrong-way risk*.<sup>2</sup> Default risk in derivatives markets should not be neglected. One reason is the fact that derivatives markets frequently experienced, often severe, credit events. A second reason is the fact that, at the aggregate level, the global derivatives market is by far the largest market in the world. In December 2005, the notional amount outstanding of derivatives contracts was around \$270 trillion, compared to about \$12 trillion of debt or to the World's GDP of about \$36 trillion, as shown by the Bank for International Settlements (2005). A third reason to watch default risk is the fact that large counterparties in derivatives markets are highly levered. A realized loss in the notional position of fifty basis point at the largest counterparties would wipe out their respective risk-based capital.<sup>3</sup> Furthermore, derivatives markets are highly concentrated. At the end of 2005, about 96% of all derivatives contracts held by U.S. insured commercial banks were held by the five largest institutions in this group, as reported in OCC (2005).

The mechanisms to mitigate default risk include minimum capital requirements, margin requirements, netting, and central counterparties. Today, various combinations of these mechanisms can be found in derivatives markets.

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<sup>2</sup> Cf. e.g. Duffie & Singleton (2003).

<sup>3</sup> Cf. Walen (2004) and Section 4.

Whereas many over-the-counter (OTC) markets have virtually no minimum capital requirements and low margin requirements, exchange-traded derivatives are highly regulated, have very high margin requirements, and are usually intermediated by a central counterparty.

Indeed, losses due to counterparty defaults have been minimal in case of exchange-traded derivatives cleared by a central counterparty, as pointed out by Moody's Investor Service (1998). This is not true for OTC derivatives.<sup>4</sup> One might conjecture that the contractual innovations employed in exchange-traded derivatives markets have helped to contain default risk.

The single most important mechanism to mitigate default risk is probably collateral. It is attached to single contracts or to the net position of a counterparty as margin.<sup>5</sup> Margin increases the lower bound of the delivery rate in case a default occurs. The costs entailed in margin, however, render its benefits ambiguous. First, the collateral posted as margin might be used more profitably. Secondly, if a trader does not hold sufficient collateral, margin requirements might constrain the number of contracts traded and thus prevent her from implementing her optimal position. Thirdly, a reduction in the number of contracts traded reduces market liquidity and, again, might prevent a market participant from implementing her optimal position. Thus, margin requirements might preclude risk sharing among traders. As a result, while they do reduce credit exposure, their effects on the wealth of traders are ambiguous. In certain situations, margin requirements might even increase default risk in a market.

Various studies, discussed in Section 2 and including Brunnermeier & Petersen

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<sup>4</sup> See Footnote 1 and the references therein.

<sup>5</sup> In the following, we will refer to the assets used as security deposit as *collateral*. We will refer to the collateral attached to a contract or to a position of several contracts as *margin*.

(2005), Cuoco & Liu (2000), and Johannes & Sundaresan (2006), have analyzed the implications of margin requirements on agents' investment decisions and on asset values. These studies take the perspective of a single agent, with the exception of Brunnermeier & Petersen (2005). What is less well understood is the effect of margin requirements on “aggregate phenomena” such as market liquidity and default risk.

The ambiguity surrounding the net benefit of margin requirements is just one example of a larger issue. There is a tension between the management of the various categories of risk such as market, default or liquidity risk. In many situations the reduction of one of those risks increases another one. Therefore, the analysis of one such category of risks in isolation can be delusive.

More specifically, there has been a trend in recent years to increase margin requirements in relation to OTC derivatives. The aim of increasing margin rates is to reduce default risk or, put differently, to isolate price risk from default risk. From a more “holistic” perspective, at the level of wealth, such a measure might backfire and decrease wealth.

Another tension exists between public versus private costs and benefits in relation to the financial system. While we address the tension between the various categories of risk, we ignore the tension between public and private benefits. In the following, we provide a theoretical framework for the analysis of the mechanisms for default risk mitigation found in derivatives markets. More precisely, we develop a model that allows us to investigate the effects of default risk mitigation on aggregate wealth, market liquidity, and default rates. We analyze the effects on various categories of risk in a common framework rather than one at a time.

Our aim is to address our research questions with a model that captures the main characteristics of derivatives contracts and trading, in particular, the

interactions between market, default and liquidity risk. Some of these characteristics render a formal mathematical model unpromising or even infeasible. These include the dynamic nature of credit risk in derivatives contracts, the non-linearity of contract values and trading strategies, path-dependence of contract values and wealth, and the heterogeneity as well as the discreteness of the number of traders. We therefore use simulations for our investigation. Simulation allows for greater flexibility in model building, enables us to include non-linear phenomena in a satisfactory way, and can be truly dynamic. We develop a simulation framework that permits us to model derivatives trading and default risk mitigation in a process model. We explicitly model the trading positions and wealth of a number of banks taking into account netting, margining and a central counterparty. This allows us to endogenize default. According to one of our major findings, in a setting where banks are under severe stress due to adverse market conditions, the effects of default risk mitigation on wealth are ambiguous. While they do reduce loss-given-default in many cases, they impair banks' ability to hedge and thus have negative consequences for their wealth. These consequences are indirect as the mitigation mechanisms reduce market liquidity and hedge ratios while increasing default rates. The largest adverse effect is due to variation margin.

Our results reinforce two points. First, the legal framework of derivatives trading, often considered a minor detail, may have a major impact on both, the value of a single contract as well as on the wealth of a bank. Secondly, disentangling risks, such as market, liquidity and default risk, and considering these risks in isolation may be grossly misleading.

In addition, our results show that, in certain situations, default risk mitigation mechanisms, or specific combinations, might have a negative effect on systemic risk. Given the size of OTC derivatives markets, most of the contracts being

held by a small number of large banks<sup>6</sup>, we believe that our results warrant a closer examination of default risk mitigation mechanisms with regards to their impact on the stability of the financial system.

We proceed as follows. In Section 2 we briefly review the literature relevant to our study. In Section 3, we describe the different mechanisms employed in derivatives markets to mitigate default risk. Section 4 presents an overview of the current state of derivatives markets. We present our model in Section 5 and the simulation in Section 6. In Section 7, we discuss our results. Finally, Section 8 concludes.

## 2 Literature Review

The issue at the core of our study is the fact that derivatives contracts are essentially credit instruments as counterparties might possibly default on their contractual obligations. Obviously, the risk of non-performance might have a significant impact on the value of a derivatives contract. The question arising is how to protect against non-performance. If non-performance was solely due to exogenous uncertainty it could be addressed through appropriate regulatory structures including default penalties and collateral.

Dubey & Shubik (1979) and Shubik & Zhao (1991) show, among others, that in a setting where agents cannot insure all the uncertainty they face, it is economically efficient to have finite default penalties, that is, to allow for default. The mitigation of counterparty default risk in relation to derivatives markets has concerned market participants ever since the inception of derivatives markets, as Swan (2000) documents. The first means to mitigate this risk,

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<sup>6</sup> Cf. Section 4.



other than counterparty appraisal, was collateral. Over time, more sophisticated mechanisms evolved. A historical account of this evolution is provided by Loman (1931) and Moser (1994*a*). As we have pointed out, this concern lead to the development of clearinghouses. The evolution of specific clearing mechanisms up to the creation of central counterparties is described in Moser (1994*a*) and Moser (1998*b*).

Baer, France & Moser (1995) describe the operations of clearinghouses. Moser (1994*b*) discusses the private and public benefits of clearing in OTC markets. In terms of private benefits, clearing reduces the costs of making contractual payments, of collateralizing payment obligations, and of monitoring the financial well-being of counterparties. Public benefits include the centralization of information and the mutualization of risks. Centralized information gathering makes it possible for multilateral systems to identify system-wide problems that may escape notice in bilateral arrangements.

Duffee (1996) discusses the measurement of credit risks in derivatives contracts. He points out that standard measurement approaches to credit risk often fail in case of derivatives. One reason is their ignorance of the correlations among exposures on derivative instruments and the probabilities of default.

The incorporation of default risk into the valuation of derivatives contracts was first considered by Hull & White (1995) and has since been analyzed for a rather broad class of settings by Collin-Dufresne & Hugonnier (2002) and others.

Margining, or collateral, obviously affects the cash flows in relation to a contract. Taking the perspective of a single agent, Cox, Ingersoll & Ross (1981) show that variation margins of futures contracts result in stochastic dividends. Variation margin is also discussed at length by Duffie (1989). Other authors

have analyzed the impact of margining on wealth more broadly. Margining reduces loss-given-default of an agent but at the same time constrains her investment opportunities. Cuoco & Liu (2000) analyze the impact of margin requirements on optimal portfolio choice and hedging costs. In an empirical study of swaps markets, Johannes & Sundaresan (2006) show that market prices reflect the cost of collateral. In other words, traders do indeed recognize the opportunity cost of posting collateral. Constraints on the investment opportunity set are typically exacerbated when trading with many different counterparties. An agent might have a neutral (balanced) position with regards to market risk but when the offsetting contracts are with different counterparties, the agent might have a positive credit exposure, and he might have to post collateral. Obviously, this might entail significant inefficiencies.

Another default risk mitigation mechanism, (close-out) netting, is analyzed by Bergman, Bliss, Johnson & Kaufman (2003). They show that the protection offered by netting benefits major derivatives dealers and markets. However, its implications for smaller market participants and markets are ambiguous.

Default risk mitigation mechanisms do not only affect the nature of assets and liabilities between market participants, but also the *structure* of relationships emerging from trading. Whereas in case of bilateral contracts, counterparties hold positions with each other, forming a complex network of assets and liabilities, in case of cleared contracts counterparties only have a position with the central counterparty. This means that there is a systemic component to mitigation mechanisms. Recent work by Shin (2005) suggests that the structure of assets and liabilities between market participants affects the value of contracts and thus the wealth of agents. However, no work has yet demonstrated what the precise relation between structure and wealth is.

Systemic risk due to potential defaults of market participants has of course

been widely discussed in the literature. An excellent overview is given in De Bandt & Hartmann (2000) with an exhaustive list of references. Most of the literature on systemic risk focuses on bank run models and funding illiquidity. With the exception of Schinasi, Craig, Drees & Kramer (2000) who give an overview of derivatives markets and address stability as well as regulatory issues related to them, there is hardly any work that analyzes to what extent derivatives might trigger a systemic event.

The stability of a clearinghouse, and its systemic effects, might be weakened by moral hazard. Knott & Mills (2002) point out that the clearinghouse's customers—as a consequence of multilateral netting—may be encouraged to take on more risk thereby increasing the default risk of the clearinghouse.

In many jurisdictions, derivatives clearinghouses are subject to special laws.<sup>7</sup> Most importantly, they are typically exempt from standard bankruptcy law to allow for netting, close-out, and termination—privileges that in the past were usually not available to most other creditors. There is an ongoing debate what the effects of such provisions are on systemic risk in a financial system, as reviewed by Bliss & Kaufmann (2004).

Although clearinghouses might default, they do so rather rarely. Recent clearinghouse failures have occurred in Paris (1973), Kuala Lumpur (1983) and Hong Kong (1987), as discussed in Knott & Mills (2002). Nevertheless, clearinghouses are typically considered of very high credit quality as a report by Moody's Investor Service (1998) shows.

Systemic risk associated to clearing and settlement systems during the 1987 crash is analyzed by Brimmer (1989) and Bernanke (1990). The latter investigation points out that several clearinghouses, without government intervention, would have become illiquid. This finding underlines the fact that systemic

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<sup>7</sup> Today, the same often applies to OTC derivatives contracts.

risk in relation to clearinghouse should not be ignored.

Moser (1998*b*) discusses self-regulation of derivatives markets in general and the role of loss-sharing arrangements in particular. Kroszner (1999) argues that recent innovations in the legal system, well-functioning rating agencies, as well as the development of risk models might allow market participants to reach the same level of efficiency in OTC markets that were previously only possible with a central counterparty. Recent events, however, like the recapitalization of LTCM and the bankruptcy of Enron and their respective consequences, cast some doubt on this view, in particular, in the context of systemic risk.

Finally, the effect of competition between exchanges and their clearinghouses, based on fees and margin requirements, is discussed by Santos & Scheinkman (2001). They argue that competition does not necessarily lead to a “race to the bottom”, and under many circumstances leads to a “race to the top”.

### **3 Cost and Benefits of Mitigation Mechanisms**

Mechanisms employed in derivatives markets to mitigate default risk are described in Appendix A. In this section, we briefly discuss their costs and benefits. From now on, we will call contracts executed with a central counterparty *cleared contracts* and all other contracts *bilateral contracts*.

Costs and benefits of netting and margining should not be investigated at the level of a single contract but at the level of a single agent’s wealth or even at the level of aggregate wealth in a given market. For example, while margining might increase the value of a single contract, it might reduce market liquidity in this contract and thus reduce agents’ ability to hedge their wealth.

Netting as well as margining reduce current and potential future exposure and thus the realized loss conditional upon default.<sup>8</sup> Moreover, they reduce economic capital and regulatory capital requirements. A fully margined bilateral position has a zero credit weighting in the Basel framework. Furthermore, any position with a central counterparty has a zero credit weighting as well. In addition, frequent margining might prevent an agent from taking too much leverage by constraining her trading. This, in turn, might reduce the agent's probability to default. Margining might also expand the list of potential counterparties by levelling the playing field, thereby increasing competition.

By altering the structure of credit relationships in a market, a central counterparty reduces the risk of default contagion and thus the risk of a systemic event. This is discussed in more detail by Bernanke (1990) and Knott & Mills (2002). We point out, though, that some authors have voiced concerns about central counterparties. Moser (1998*a*) and Pirrong (1997) argue that central counterparties might lead to moral hazard and adverse selection. According to some authors including Schinasi et al. (2000), this is one reason why market participants do not adopt central counterparties for OTC trading at present. While margining has many uncontested benefits, it entails, often significant, costs. Assets posted as collateral might be put to more profitable use elsewhere. This is particularly true for cash. Furthermore, margining might prevent an agent from entering into a contract if she does not have sufficient collateral to cover the initial margin requirement. This in turn might affect market liquidity, that is, the availability of contracts for trading. The effects of margining on the wealth of an agent, lower exposure on the one hand versus potentially

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<sup>8</sup> The effect on potential future exposure is only true if potential future exposure is independent of netting and margining. In this section we assume that this is the case unless noted otherwise. The same applies to economic capital and regulatory capital requirements.

lower market liquidity and higher probabilities of default of counterparties on the other, are not straightforward and will be investigated in Section 7.

Both netting and margining change the timing as well as the size of cash flows of a derivatives contract. For example, variation margin generates payments by counterparties between contract initiation and maturity. The changes of cash flows obviously change the value of a contract. The direction of the change depends on the relation between default risk and the opportunity costs of collateral.

The use of a central counterparty typically requires an agent to enter into a loss-sharing agreement with the other members of the central counterparty. Furthermore, clearing by a central counterparty is subject to fees, although those are often priced at cost.

A central counterparty also has costs and benefits in terms of information. It reduces the costs related to the assessment of counterparties' credit qualities. Furthermore, by centralizing information on traders' information on positions, it typically has a better view of a trader's overall risk exposure than other market participants.

It is probably clear by now that the net benefits of default risk mitigation mechanisms depend on the economic setting. The various mechanisms have different effects on market, default as well as (market and funding) liquidity risk. Their net benefit might vary from trader to trader and over time.

We do point out at this stage that some market practitioners seem to assess the costs and benefits of collateral differently than we do. The International Swaps and Derivatives Association (ISDA), whose documentation provides the contractual framework for most OTC derivatives trading, writes in its guidelines for collateral usage (International Swaps and Derivatives Association, Inc. 2005*a*, p. 12, emphasis added):

“The mechanism by which collateral provides benefit is through improvement of the recovery rate. Collateral *does not* make it more or less likely that a counterparty will default and *does not* change the value of a defaulted transaction. Where collateral acts post-default it is to increase to offset the amount of recovery made to offset the loss.”

We will show in the remainder of this study that such a view might be misleading. Another example of differing views is provided in Johannes & Sundaresan (2006). The authors report that many institutions seem to assume that delivering collateral is costless. This, however, implies that credit risk can be eliminated at no cost—probably an unrealistic assumption.

## 4 Current Structure of Derivatives Markets

Let us now briefly describe those aspects of the current structure of derivatives markets that seem relevant to our analysis. We will distinguish between OTC markets and exchange markets. The former are typically decentralized, informal, lightly supervised and regulated, and market-discipline driven. OTC markets are very similar to interbank or interdealer markets, an informal network of bilateral relationships. Exchanges, on the other hand, are centralized, formal, regulated and rule-driven.

Derivatives volumes have soared, in particular during the last two decades. Their growth has not been slowed down by incidents like the collapse of LTCM and others. Table B.1 shows the development of notional amounts outstanding of derivatives contracts for the period from 1994 to 2004 for the U.S. derivatives markets.

Whereas up to around 1980, most derivatives contracts were traded on ex-

changes, this has changed substantially since then. In June 1998, only about 12.8 percent of all derivatives contracts were traded on exchanges (in terms of notional). Since then, this figure has been declining steadily, to 7.6 percent in December 2004, as reported by the Bank for International Settlements (2005). As we pointed out at the very beginning, the major disasters in derivatives markets in the last few years originated in OTC markets. The two best known incidences of the recent past are probably the defaults of LTCM and Enron. While LTCM held large positions in the interest rate and equity markets, Enron was one of the largest counterparties in the energy markets.

Today, there seems to be agreement among practitioners that the collapse of LTCM would probably have led, according to many observers, to a systemic event without government invention. As Greenspan (1998) recalled later that year, during the crisis, officials of the Federal Reserve Bank of New York expected that

“the act of unwinding LTCM’s portfolio in a forced liquidation would not only have a significant distorting impact on market prices but also in the process could produce large losses, or worse, for a number of creditors and counterparties, and for other market participants who were not directly involved with LTCM.”

Interestingly, the Federal Reserve, according to its chairman, facilitated rescue efforts of LTCM

“not to protect LTCM’s investors, creditors, or managers from loss, but to avoid the distortions to market processes caused by a fire-sale liquidation and the consequent spreading of those distortions through contagion.”



The collapse of Enron shattered the market for electricity and natural gas contracts in the U.S. and Europe, resulted in losses for many market participants, in some cases to default, and subsequently crippled the respective markets for several years. Richard Green (2002), at the time chairman of Acquila, one of the largest energy traders, testified that

“[...] the entire energy sector has experienced a state of upheaval since [...] the Enron bankruptcy. The troubling effects of these events have expanded to affect all energy traders, even those who had nothing to do with [...] Enron’s inappropriate practices. [...] Consequently, a substantial portion of the trading industry has reduced their trading activities or withdrawn altogether.”

At this stage, we point out that innovations in the legal systems such as ISDA master agreements have probably influenced the for central counterparty clearing. Before these innovations, collateralization, as well as netting, were problematic from a legal point of view, especially for cross-border transactions, and could be implemented effectively by central counterparties only. These legal uncertainties have largely been removed by the development of the ISDA framework.<sup>9</sup>

A well known fact that has been of concern for market participants and regulators alike is the high level of concentration in derivatives markets. As shown in Table B.2, the five largest U.S. insured commercial banks held 96.2% of total notional outstanding held by all banks in this group.

There is also concentration of contracts among end-users. At Fannie Mae and

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<sup>9</sup> ISDA developed a legal framework for over-the-counter trading of derivatives. Among its main benefits is the increased effectiveness, partly as a result of the removal of legal uncertainty, of (close-out) netting and of the attachment of collateral to derivatives trades in the various jurisdictions. Cf. Allen & Overy (2002)

Freddie Mac, the two large mortgage companies in the U.S., five counterparties accounted for almost 60% of total notional outstanding at Fannie Mae and 58% at Freddie Mac in 2001. The credit exposure of counterparties to Fannie Mae and Freddie Mac was \$7 billion and \$2.6 billion, respectively, at the time. Together, they represented 7.6% of the end-user market for \$-denominated interest rate derivatives, as reported in Falcon (2003).

In Table B.3, we show the ratio of credit exposure to risk-based capital for the seven largest U.S. derivatives dealers. In Table B.4 we show the ratio of total notional amount outstanding to risk-based capital at the seven largest dealers. These figures show how tightly run some of these organizations are. This probably reflects the fierce competition in these markets.

As mentioned above, OTC markets have far outgrown exchange markets in the last ten to twenty years despite several large defaults. It seems that market participants have reacted to these losses by increasing collateral requirements. While collateralization or margining has always been an important feature of OTC markets, as pointed out by Litzenberger (1992), it has gained in significance in the last few years. Whereas in the period from 2000 to 2004, notional amounts outstanding grew by a factor of approximately 2, reported collateral usage in the OTC markets grew by a factor of more than 5 in the same period. Reported and estimated collateral usage in the U.S. OTC derivatives markets are shown in Table B.5.

Johannes & Sundaresan (2006) report that nearly all swap transactions at major counterparties are collateralized. 56% of all OTC derivatives volume traded and 55% of credit exposure in relation to OTC derivatives are covered by collateral, as reported in International Swaps and Derivatives Association, Inc. (2005*b*). While a wide variety of assets is posted as collateral, by far the most popular form of accepted collateral is cash at 73%, according to Inter-

national Swaps and Derivatives Association, Inc. (2005*b*). Johannes & Sundaresan (2006) point out that it may also be the cheapest form of collateral in many cases because of the large haircuts required for risky securities and the valuation issues involved. A last fact concerns rehypothecation. International Swaps and Derivatives Association, Inc. (2005*b*) reports that more than three quarters of large and medium counterparties re-use collateral.

From the data just presented we draw several conclusions. First, derivatives markets are highly concentrated. Second, the large players have significant leverage ratios, with credit exposures often many times larger than their capital base. Third, collateral usage in OTC markets has grown significantly indicating that collateral levels in OTC markets have been increasing. Fourth, by far the most popular form of collateral is cash. And fifth, an increasing fraction of reusable collateral is rehypothecated.

## 5 Description of the Model

In this section, we describe a dynamic model where banks facing some exogenous random endowment trade a derivatives contract with each other to hedge the price (market) risk of their endowment. Upon delivery on the derivatives contract, banks may default. Step by step, we introduce various mechanisms to mitigate default risk and investigate their effects on wealth, default rates, loss-given-default, and market liquidity.

Our aim is to capture a number of characteristic features of derivatives markets. First, there is a high level of concentration among market participants. Secondly, market participants have significant credit exposures in relation to derivatives contracts. Thirdly, they can only pledge cash as collateral. And

lastly, default risk exposures to a central counterparty have a zero capital requirement.

The results of the model are presented and discussed in the subsequent section.

### 5.1 *Economic Setting*

We consider an economy with a real sector and a financial sector comprising  $N$  banks indexed by  $i = 1, \dots, N$ . Time is measured in discrete intervals  $t = 0, \dots, T$ . Banks are exposed to a stochastic short-term interest rate. The interest rate process,  $r$ , in the continuous-time limit, is given by

$$d \ln r_t = \left( u_t + \frac{1}{\sigma_t} \frac{\partial \sigma}{\partial t} \ln r_t dt \right) + \sigma_{r,t} dW_t, \quad (1)$$

where  $u_t$  and  $\sigma_{r,t}$  are the (time-dependent) drift and diffusion coefficients, respectively, and  $W$  represents a standard Brownian motion. Equation (1) resembles the model of Black, Derman & Toy (1990).

At time  $t = 0$ , banks are endowed with a certain amount of money  $m$ . The amount varies across banks, as described in Section 7. The wealth of bank  $i$  is denoted by  $W^i$ . At the beginning of every period  $t$ , agents receive a demand for a bond  $D$  with maturity  $t + T_D$ ,  $T_D \in \mathbb{N}$ , from the real sector, that is, their clients.

Client demand is uniformly distributed within  $[-lW_t^i, lW_t^i]$  with  $l \in \mathbb{R}^+$ . Note that client demand might either be positive (lending) or negative (borrowing). Clients might default on their obligations. We assume that the hazard rate is the same for all clients.<sup>10</sup> We denote the clients' hazard rate by  $h^c$ . For the time being, we also assume that all banks have the same rating level, which

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<sup>10</sup> Cf. Section 6.

we denote by  $h$ .<sup>11</sup>

Both, interest rate and hazard rate process can easily be calibrated to actual term structures as described in Black et al. (1990) and Skinner & Diaz (2001).

The interest rate at time  $t$  is given by

$$r_t = r_{t-1} \exp \left( u_t \Delta T + \sigma_{r,t} \sqrt{\Delta T} \right), \quad (2)$$

where  $\Delta T$  is the time interval. The hazard rate at time  $t$ , conditional upon no prior default, is given by

$$h_t = h_{t-1} \exp \left( v_t \Delta T + \rho_{h,r} \frac{\sigma_h}{\sigma_r} r_t \Delta T \right), \quad (3)$$

where  $v_t$  and  $\sigma_h$  denote the drift and diffusion coefficient, respectively, of the hazard rate, and  $\rho_{r,t}$  denotes the correlation between interest rate and hazard rate.<sup>12</sup>

The bond traded with clients is either a fixed-rate or a floating-rate bond. For half of the banks ( $i \bmod 2 = 1$ ), all lending is in floating-rate bonds whereas all borrowing is in fixed-rate bonds, and vice versa for the other half ( $i \bmod 2 = 0$ ). The value at time  $t$  of a defaultable bond maturing at time  $t + 1$  is given by

$$D_t(t + 1) = \mathbf{E}_t^{\mathbb{Q}} \left[ e^{-r_t \Delta T} (h_t^c \omega_{t+1} + (1 - h_t^c) D_{t+1}(t + 1)) \right], \quad (4)$$

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<sup>11</sup> This assumption can be interpreted such that all banks (or their derivatives trading units) have a similar credit rating which, we believe, fairly reflects the current situation in derivatives markets.

<sup>12</sup> The hazard rate process in Equation (3) extends the Black-Derman-Toy model in a straightforward manner. We choose it for reasons of consistency with our interest rate model.

where  $\omega_{t+1}$  is the recovery amount and  $D_{t+1}(t+1)$  is the promised payout at maturity  $t+1$ . In other words, the bond promises to pay  $D_{t+1}(t+1)$  at maturity  $t+1$ , but the promise may be broken at hazard rate  $h_t^c$ . If default occurs at time  $t$ , an amount  $\omega_{t+1}$  is paid at  $t+1$ , conditional upon no prior default.  $\mathbb{Q}$  denotes the risk-neutral probability measure. We assume that a risk-neutral probability measure exists and that all banks choose the same measure  $\mathbb{Q}$  when valuing the bond.<sup>13</sup> Under the risk-neutral probability measure  $\mathbb{Q}$ , these cash flows can be discounted at the risk-free interest rate, as shown by Duffie & Singleton (1999). If the bond has a time to maturity of more than one period and default occurs prior to maturity, we will assume that  $\omega$  is invested at the risk-free rate  $r$  until the bond's maturity.

We assume that all bonds have a notional amount of one unit of money. A bank lending an amount  $x$  to a client will thus enter into a position of  $q = \lfloor x/D \rfloor$  units of the bond, where  $\lfloor x \rfloor$  denotes the integer part of  $x$ . A bank entering, at time  $t$ , into a  $q$  units of the bond contract with maturity  $t + T_D$  will exchange the notional principal of  $-qD_t(t + T_D)$  at initiation and of  $qD_{t+T_D}(t + T_D)$  at expiry, and will make or receive interest payments in all periods  $t \in \{t + 1, \dots, t + T_D\}$ .<sup>14</sup>

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<sup>13</sup> We cannot assume that  $\mathbb{Q}$  is unique as this would imply market completeness. However, we will later assume that banks cannot hedge default risk, implying an incomplete market.

<sup>14</sup> Note that  $D_t(t + T_D)$  reflects the default risk of the clients. Therefore, the payments at contract initiation include a compensation for the default risk in the bond.

Client demand exposes the banks to both, interest rate risk and default risk. Default risk cannot be hedged, rendering the market incomplete.<sup>15</sup> Interest rate risk can be hedged by trading in a swap contract. By entering into a swap contract, a bank agrees to pay the agreed swap rate and to receive the current interest rate (long position), or vice versa (short position). More, precisely, by executing at time  $t$  a swap contract maturing at time  $t = T_S$ ,  $T_S \in \mathbb{Z}$ , counterparties agree to exchange interest payments at an agreed swap rate,  $s(t+T_S)$ , against floating interest rates. We assume that the swap contract has the same notional principal as the bond, that principals are not exchanged, and that the swap has the same time to maturity as the bond, that is,  $T_S = T_D$ . The swap contract is subject to default risk.

In order to trade in the swap, a bank submits an order to the market. An order has the form  $(q^i, s^i)$ , where  $q^i$  is the number of contracts the agent wants to trade and  $s^i$  is the swap rate. The swap rate  $s^i$  determines the payoff of the contract, which can be both, positive and negative. When a bank submits an order, it sets the swap rate  $s^i$  such that the contract has a value of zero.

Orders are submitted sequentially but in random order, that is, the order in which banks submit orders in the different periods (and in the different simulation runs) changes. A trade takes place whenever two orders match. Our trading mechanism thus resembles a discriminatory continuous double auction. Such a trading mechanism is used in most of today's security markets. It determines the structure of contractual relationships between agents.

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<sup>15</sup> One might argue that this assumption is unrealistic due to the existence of credit derivatives. However, supply of such derivatives might be insufficient to cover demand. More importantly, credit derivatives are also subject to default risk. We therefore believe our assumption to be reasonable.

Every swap position might be subject to a margin requirement of  $\phi \in \mathbb{R}^+$  units of money per contract traded. This (initial) margin has to be posted at the date when a bank initiates a position.

Contracts might also be subject to variation margin. In this case, the change in the value of a bank's position between two dates is settled in money. Bank  $i$  holding  $q_{t-1}^i$  swap contracts at date  $t-1$  has a variation margin requirement of  $q_{t-1}^i (S_t(\bar{s}_t) - S_{t-1}(\bar{s}_{t-1})) + q_t^i (S_t(\bar{s}_t) - S_t(s_t^i))$  at date  $t$ , where  $S(s)$  denotes the value of the contract given swap rate  $s$ , and  $\bar{s}$  denotes settlement prices. In other words, the value of the position of the previous period is set to zero (first term) and so is the value of the contracts traded in the current period (second term). The second term is necessary since contracts might be traded at a price different from the current period's settlement price and therefore already have to be “marked to market” in the period in which they are traded. This means that variation margin eliminates current exposure of a position. By default, contracts are traded bilaterally, that is, directly between banks (what we previously called bilateral contracts).

The value of a swap contract is zero at initiation and zero at expiry.<sup>16</sup> A swap contract results in periodical cash flows comprising the “coupon” payment (differential between current interest rate and swap rate) and variation margin (differential between previous period's and current period's contract value). Obviously, margining changes the cash flow pattern.

To summarize, banks are exposed to a risk factor  $r$  through client demand for a bond. Clients default on their obligations with hazard rate  $h^c$ . They hedge their interest rate exposure by trading in a swap contract. The market is characterized by a time horizon  $T$ , the number of banks  $N$ , the interest rate

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<sup>16</sup> The value of the swap obviously depends on the hazard rate of the counterparty. As we assume a single hazard rate across banks at the moment we drop the reference to the counterparty, i.e.,  $S^i(\cdot) = S(\cdot)$  for all  $i = 1, \dots, N$ .



process  $r$ , the client's hazard rate process  $h^c$ , the margin requirements  $\phi$ , and the risk limit  $l$ . Banks are characterized by their initial endowment in money  $m$ .

We point out at this stage that all these parameters can be calibrated to quantities observed in actual markets. We will do so in our simulations in order to resemble actual markets as closely as possible.

### 5.3 Banks' Optimization Problem

An time  $t = 0$ , a bank holds  $m$  units of money. Over time, it builds up a portfolio of long and short positions in the bond. At time  $t$ , the money holding of a bank is given by

$$m_t = r_{t-1}m_{t-1} - q_t^c(t+T_D)D_t(t+T_D) + q_t^c(t)D_t(t) + \sum_{\tau=t}^{t+T_D} \psi(q_t^c(\tau), D_t(\tau)), \quad (5)$$

where  $q^c$  reflects the aggregate position with the client sector and  $\psi(\cdot)$  denotes the interest payment of a particular position.  $\psi$  is given by

$$\psi(q_t^c(\tau), D_t(\tau)) = \begin{cases} q_t^c(\tau)^+ r_t - q_t^c(\tau)^- s_{t+T_D-\tau}(\tau) & \text{if } i \bmod 2 = 1, \\ q_t^c(\tau)^+ s_{t+T_D-\tau}(\tau) - q_t^c(\tau)^- r_t & \text{if } i \bmod 2 = 0, \end{cases} \quad (6)$$

where  $x^+$  denotes  $\max\{0, x\}$  and  $x^- := \max\{0, -x\}$ . The expression above reflects the fact that half of the banks lend at fixed rates and borrow at floating rates, and vice versa for the other half. As indicated by Equation (6), fixed rate bonds entered into at time  $t$  have a coupon payment of  $s_t(t+T_D)$ , that is, the coupon is set equal to the swap rate in the respective period.<sup>17</sup>

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<sup>17</sup> This assumption is made for simplicity.

Equation (5) reflects the cash flows resulting from bond contracts maturing at  $t$  and contracts entered into at  $t$  (with maturity  $t + T_D$ ) as well as interest payments. The wealth of a bank at  $t$  is given by

$$W_t = m_t + \sum_{\tau=t}^{t+T_D} q_t^c(\tau) D_t(\tau). \quad (7)$$

A bank's wealth at time  $t$  thus comprises its money holdings and the current value of the bond holdings not yet matured.

We assume that a bank will always try to eliminate its exposure to interest rate risk. Whenever a bank enters into a fixed-rate bond, it tries to enter into a swap position of the same quantity.<sup>18</sup> In other words, the optimal quantity in the swap contract is given by

$$q^* = \begin{cases} (q^c)^- & \text{if } i \bmod 2 = 1, \\ (q^c)^+ & \text{if } i \bmod 2 = 0. \end{cases} \quad (8)$$

Note that banks' trading in the swap contract will not only be affected by client demand but but also by client as well as counterparty default. The latter renders trading demand dynamic.

We now explain the implications of the various mitigation mechanisms in relation to the swap contract on a bank's cash flow and its wealth. We discuss four cases: (1) no margining, (2) initial margin only, (3) initial and variation margin, as well as (4) initial and variation margin with a central counterparty. These four cases are supposed to resemble the combinations of mitigation mechanisms found in actual markets.

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<sup>18</sup> There exist situations where complete hedging is optimal for banks. Bauer & Ryser (2004) show that this the case, e.g., in the presence of high asset volatility.

In line with some empirical facts described in Section 4, we assume that contracts across a single counterparty can be netted; in case of a central counterparty they can be netted across counterparties, of course. Furthermore, we assume that only money can be posted as collateral.

*No margining:* In case of no margining, a swap contract results in interest (“coupon”) payments only, that is,

$$m_t = r_{t-1}m_{t-1} - q_t^c(t + T_D)D_t(t + T_D) + q_t^c(t)D_t(t) + \sum_{\tau=t}^{t+T_D} \psi(q_t^c(\tau), D_t(\tau)) + \sum_{\tau=t}^{t+T_S} q_t(\tau) [r_t - s_{t+T_S-\tau}(\tau)]. \quad (9)$$

The last term in the equation reflects the interest payments in relation to the swap contracts.<sup>19</sup> The bank’s wealth is given by

$$W_t = m_t + \sum_{s=t}^{t+T_D} q_t^c(s)D_t(s) + \sum_{j=1}^{N-1} \sum_{\tau=t}^{t+T_S} q_t^j(\tau)S_t(\tau), \quad (10)$$

where  $q^j$  denotes a position in the swap contract with bank  $j$ . The last term in the equation above reflects the net value of the swap position.

*Initial margin:* When initial margin has to be posted by banks, a swap contract may result in a cash flow at time of its initiation. We will assume that margin requirements can be offset across maturities. From now on, we will denote the total (initial) margin requirement of the bank at time  $t$  by  $\Phi_t$ . The margin requirement can be expressed as

$$\Phi_t = \sum_{j=1}^{N-1} \left| \sum_{\tau=t}^{t+T_S} q_t^j(\tau)\phi \right|. \quad (11)$$

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<sup>19</sup> Note that for those banks where  $i \bmod 2 = 1$ ,  $q$  is negative, and vice versa for all other banks.

$\Phi$  is always positive. The expression above reflects the fact that margin requirements in our model are symmetric (both counterparties to a contract have the same margin requirement per unit of contract). A bank entering into  $q$  units of a contract with a another bank has to deliver  $|q| \phi$  units of money as margin and will receive  $|q| \phi$  units of money from the other bank (assuming that the two banks had no contracts with each other before).

The money holding of the bank at time  $t$  is now given by

$$\begin{aligned}
m_t = & r_{t-1}m_{t-1} - q_t^c(t + T_D)D_t(t + T_D) + q_t^c(t)D_t(t) \\
& + \sum_{\tau=t}^{t+T_D} \psi(q_t^c(\tau), D_t(\tau)) + \sum_{\tau=t}^{t+T_S} q_t(\tau) [r_t - s_{t+T_S-\tau}(\tau)] - (\Phi_t - \Phi_{t-1}).
\end{aligned} \tag{12}$$

The last term reflects the changes to the money holding due to changes in the margin requirement. A bank's wealth can be expressed as

$$W_t = m_t + \sum_{s=t}^{t+T_D} q_t^c(s)D_t(s) + \sum_{j=1}^{N-1} \sum_{\tau=t}^{t+T_S} q_t^j(\tau)S_t(\tau) + \Phi_t. \tag{13}$$

The last term in the equation above denotes the collateral delivered by the bank.

*Initial and variation margin:* In case variation margin is charged, the changes in the swap contracts' values are settled periodically. Obviously, this significantly affects the cash flow of a bank. Its money holding at time  $t$  is now given

by

$$\begin{aligned}
m_t = & r_{t-1}m_{t-1} - q_t^c(t + T_D)D_t(t + T_D) + q_t^c(t)D_t(t) \\
& + \sum_{\tau=t}^{t+T_D} \psi(q_t^c(\tau), D_t(\tau)) \\
& + \sum_{\tau=t}^{t+T_S} q_t(\tau) [r_t - s_t(\tau)] \\
& + \sum_{j=1}^{N-1} \sum_{\tau=t}^{t+T_S} q_t^j(\tau) (S_t(\tau) - S_{t-1}(\tau)) \\
& - (\Phi_t - \Phi_{t-1}).
\end{aligned} \tag{14}$$

The expression above reflects the assumption that variation margin is based on the expected value of the swap. The bank's wealth is given by Equation (13).

*Initial margin, variation margin, and central counterparty:* In the presence of a central counterparty, the margin requirement changes. A bank only has one position in the swap contract, namely with the central counterparty.  $\Phi$  is now given by

$$\Phi_t = \left| \sum_{\tau=t}^{t+T_S} q_t(\tau) \phi \right|, \tag{15}$$

and the money holding changes accordingly

$$\begin{aligned}
m_t = & r_{t-1}m_{t-1} - q_t^c(t + T_D)D_t(t + T_D) + q_t^c(t)D_t(t) \\
& + \sum_{\tau=t}^{t+T_D} \psi(q_t^c(\tau), D_t(\tau)) \\
& + \sum_{\tau=t}^{t+T_S} q_t(\tau) [r_t - s_t(\tau)] \\
& + \sum_{\tau=t}^{t+T_S} q_t(\tau) (S_t(\tau) - S_{t-1}(\tau)) \\
& - (\Phi_t - \Phi_{t-1}).
\end{aligned} \tag{16}$$

A bank's wealth is now given by

$$W_t = m_t + \sum_{s=t}^{t+T_D} q_t^c(s) D_t(s) + \sum_{\tau=t}^{t+T_S} q_t(\tau) S_t(\tau) + \Phi_t. \quad (17)$$

In the presence of margining, trading in the swap contract is constrained by the bank's holding of money as well as by the solvency constraint. Due to these constraints, the bank may not be able to implement its optimal position in the swap contract. In such a case, it will not be able to fully hedge its exposure to interest rate risk.

Banks default when their wealth falls below zero (asset-based insolvency). In case of a default, a bank is liquidated, that is, money and receivables are distributed proportionally to creditors.

In our model, banks have common knowledge about the parameters of both interest rate and hazard rate process, that is, about  $u$ ,  $\sigma_r$ ,  $v$ ,  $\rho_{h,r}$  and  $\sigma_h$ . At any given time, banks therefore trade at the same swap rate.<sup>20</sup>

## 6 Simulation

We now turn to the analysis of the model described in the previous section. We will briefly discuss the methodology we use, describe the calibration of the model as well as the different scenarios we analyze, and explain a sample run of the simulation.

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<sup>20</sup> This implies that banks do not charge liquidity premia. We will come back to this issue further below.

As Shubik (1999) suggests, we analyze trading in a process model, that is, a model that explicitly takes the mechanisms “carrying” the trading process into account. In our case, the mechanisms governing the trading process are the trading mechanisms as well as the different default risk mitigation techniques. This model, analyzing market liquidity and default risk, is inherently dynamic and non-linear.

We do point out that certain aspects of default risk mitigation mechanisms, in particular, netting and margining, have each been analyzed with other, analytical modelling approaches. However, to the best of our knowledge, nobody has analyzed these mechanisms in a common framework. This, however, is necessary to study the interaction between the different phenomena, such as market liquidity and default risk.

As mentioned already, we use simulation to assess the implications of our model. This allows us to analyze aggregate phenomena like market liquidity and its effects on aggregate wealth with a high degree of flexibility at the level of their microeconomic foundations. Some of the scenarios we investigate would be inaccessible by analytical modelling approaches. These benefits, however, come at the expense of a lower degree of analytical tractability. We think, though, that in our case the benefits outweigh the costs.

A point of critique often brought forward against simulation is the number of free parameters and a resulting high degree of flexibility in “fitting the model to the facts”. We address this issue in two ways. First, we set up the model such that all parameters can be related to quantities observed in actual markets. We then calibrate the model such that its parameters are in line with the quantities observed.

Table 1  
Description of model parameters

The following table lists the market parameters for the simulation.

Type	Parameter	Description
Market	$T$	Time horizon
	$N$	Number of banks
	$r, u, \sigma_r$	Dynamics of short-term interest rate
	$T_D$	Time to maturity of bond
	$T_S$	Time to maturity of swap
	$\phi$	Margin requirement per swap contract
Real sector	$h, v, \rho_{h,r}, \sigma_h$	Hazard rate dynamics
Banks	$m$	Initial amount of money

## 6.2 Model Calibration

We now turn to the calibration of the model parameters. As we mentioned in the Introduction, the most interesting question to us is whether there exist situations where default risk mitigation mechanisms deteriorate the market outcome in terms of wealth, default rates, and losses given default. We investigate cases where the banking system is under severe and sustained stress. We will therefore choose an environment where banks experience highly fluctuating interest rates as well as high default rates. The set of model parameters is displayed in Table 1.

*Time horizon:* We fix the time horizon of the model,  $T$ , to 100 periods. With this choice we keep computational time at a reasonable level. We think of one period in the model as representing one month in calendar time.

*Term structure:* We calibrate our term structure to prices of U.S. treasury securities in the period from January 1996 to April 2004. The level of interest rates fluctuates widely. Figure 1 displays the short-term interest rate during



Table 2  
Parameter values

The table below shows values of the model parameters used in the simulation.

Type	Parameter	Value
Market	$T$	100
	$N$	25
	$r, u, \sigma_r$	Empirical term structure (see text)
	$T_D$	48
	$T_S$	48
	$\phi$	95%, 97%, 99% VaR of swap contract
Real sector	$h, v, \rho_{h,r}, \sigma_h$	See text
Banks	$m$	Empirical distribution (see text)

this period. In addition, the shape of the yield curve changes as well. The yield curve at various times during the time period we cover is shown in Figure 2.

*Instrument maturities:* We fix the maturities of the two instruments, the bond and the swap, to 48 periods, that is,  $T_D = T_S = 48$ .<sup>21</sup>

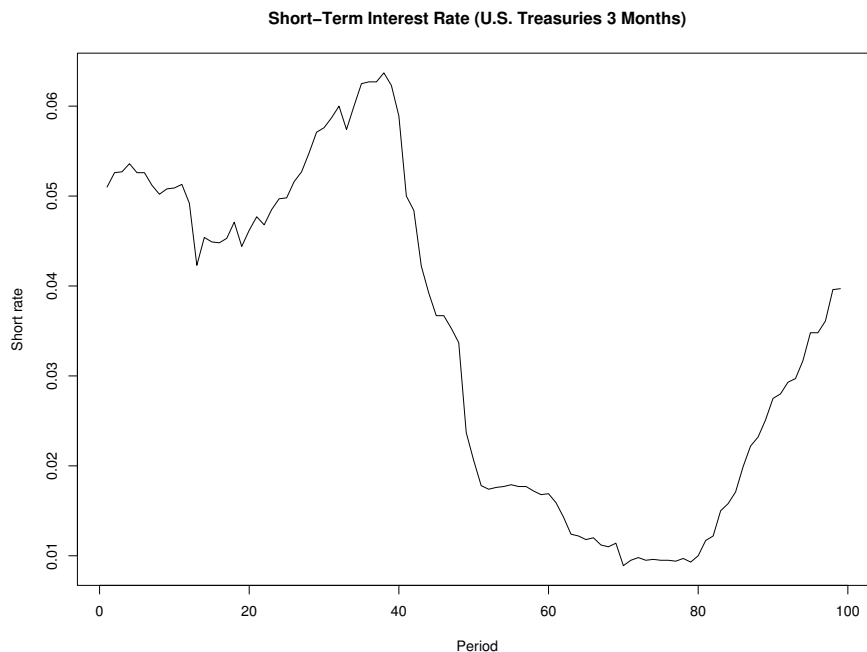
*Margin:* We will consider three levels of initial margin,  $\phi$ . More precisely, we will consider scenarios where  $\phi$  is set equal to 95%, 97%, and 99% of the 1-month value-at-risk of a swap contract. As described in Knott & Mills (2002), for example, clearinghouses typically set initial margins within this range. The time horizon is usually shorter, though. As we consider the somewhat ideal case of full cross-margining (contracts of different maturities are offset against each other in the margin calculation), initial margin would typically be higher. We consider our choice of the 1-month value-at-risk a reasonable approximation.

*Number of agents:* We fix the number of agents,  $N$ , to 25. As described in Sec-

<sup>21</sup> We believe that this is a reasonable approximation of the average maturity of the instruments on a bank's balance sheet. For derivatives, cf. OCC (2005).

Fig. 1. 3-months U.S. Treasury rate

The figure below shows the 3-months U.S. Treasury rate from January 1996 to April 2004 at monthly intervals.



tion 4, the seven largest U.S. insured commercial banks currently hold about 96 percent of total notional outstanding held by all members of this group of banks, and the largest 25 banks hold about 99 percent. We therefore consider the choice of 25 banks in our model as reasonable.

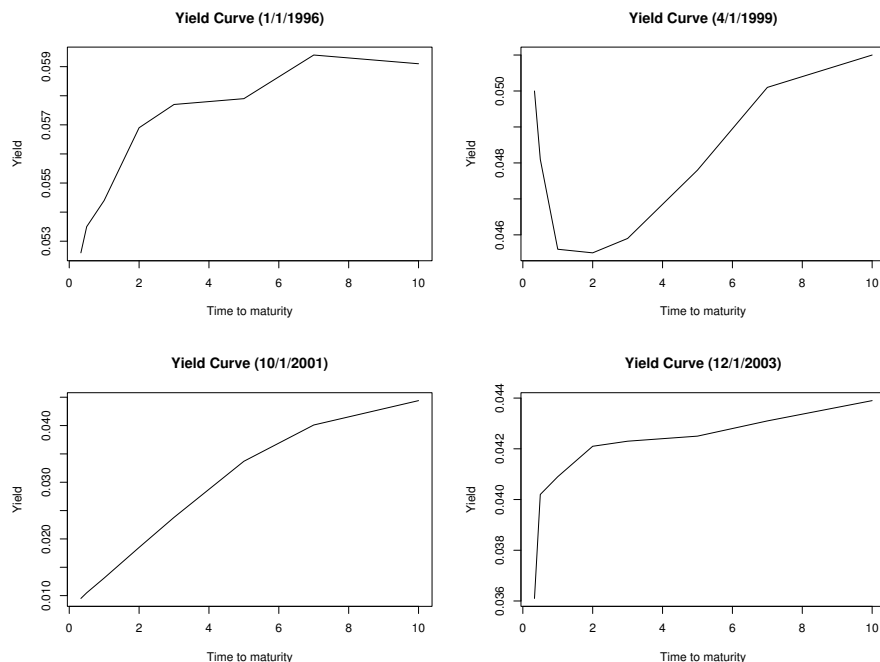
*Hazard rate of clients:* As we mentioned at the beginning of this section, we would like to investigate a scenario where banks are under severe stress in terms of both, interest rate risk and default risk. We therefore set  $h^c$  such that a significant amount of banks could not sustain losses without hedging. We set  $v \equiv 0$ ,  $\sigma_h \equiv 0.01$ , and  $\rho_{r,h} \equiv -0.5$ .<sup>22</sup>

*Initial amount of money:* We will endow banks with initial amounts of money (equity) such that their distribution reflects the empirical distribution of eq-

<sup>22</sup> For the choice of  $\sigma_h$  cf. the analysis by Skinner & Diaz (2001), and for  $\rho_{r,h}$  cf. Duffee (1998).

Fig. 2. Yield curve of U.S. Treasuries

The figure below shows the yield curve of U.S. Treasuries with maturities from 3 months to 10 years.



uity of the largest derivatives dealers (OCC 2005). The empirical cumulative distribution function of initial wealth is shown in Figure 3.

### 6.3 Sets of Default Risk Mitigation Mechanisms

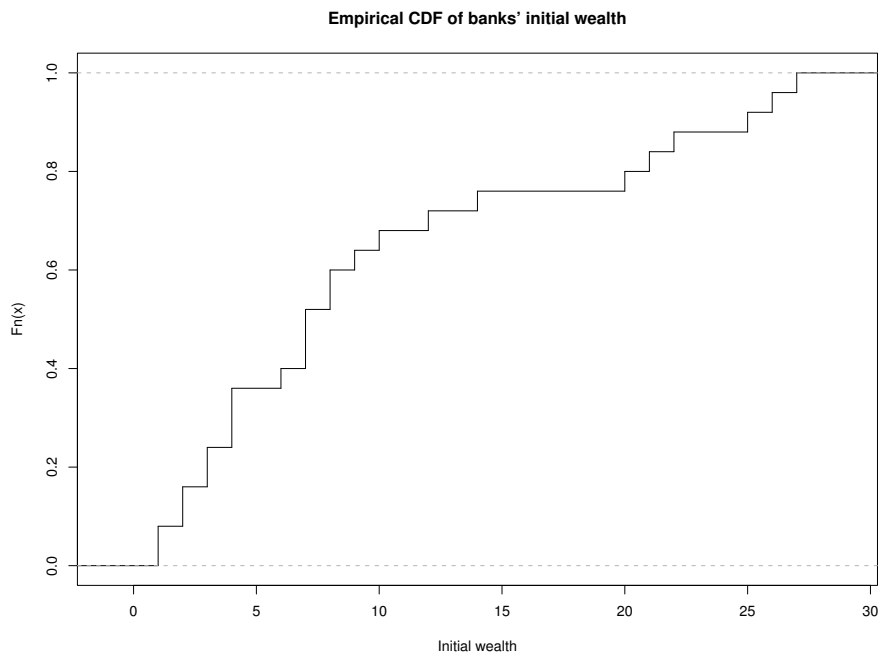
We will consider three sets of mitigation mechanisms. The first set, “IM”, considers the case where counterparties charge initial margin. The second set, “IM & VM”, also takes variation margin into account. Finally, the third set, “CCP”, includes the previous set as well as a central counterparty. These combinations of default risk mitigation mechanisms are the ones most often observed in practice.<sup>23</sup>

Money holdings and wealths of the banks given the three different sets of

<sup>23</sup> Cf. Appendix A.

Fig. 3. Empirical cumulative distribution function of bank's initial wealth

The figure below shows the empirical cumulative distribution function of banks' initial wealth. Initial wealth is normalized.



mitigation mechanisms are computed according to the expressions described in Section 5.

#### 6.4 A Sample Run

We now briefly describe a sample run of the simulation. At the beginning of a run, banks receive their endowment in money,  $m$ . At the beginning of every period, banks receive random client demand and enter into a position,  $q^c$ , in the bond with the client at its expected value.<sup>24</sup> The size of client demand is constrained by a bank's wealth as well as a solvency constraint, as described above. Subsequently, banks submit an order for the swap contract to the market. The size of the order is set such that the trade in the swap equals

<sup>24</sup> Cf. Footnote 14.

the number of fixed-rate bonds in the portfolio, as described in Section 5. The size of the order might be constrained by the margin requirement,  $\phi$ , as well as by the solvency constraint. After trading, the change of the interest rate,  $r$ , is revealed and positions are settled. Settlement includes mature bond positions, mature swaps positions, coupon payments, as well as initial and variation margin in relation to the swap positions.<sup>25</sup>

As described in Section 5, a bank might be solvent but not have sufficient money to make the payments due in a given period. In this case, banks are provided with liquidity in the form of a short position in a one-period bond at the current interest rate.<sup>26</sup> The provision of liquidity is constrained by a bank's wealth.

If a bank becomes insolvent, it defaults and its positions are liquidated at current market prices, together with the margins this bank held and delivered.

## 7 Discussion of Results

We now turn to the evaluation of our model. Each parameter configuration discussed below was simulated 100 times. Before we turn to the investigation of risk mitigation mechanisms, we briefly describe the “base case”, that is, the case where banks do not trade in the swap at all.

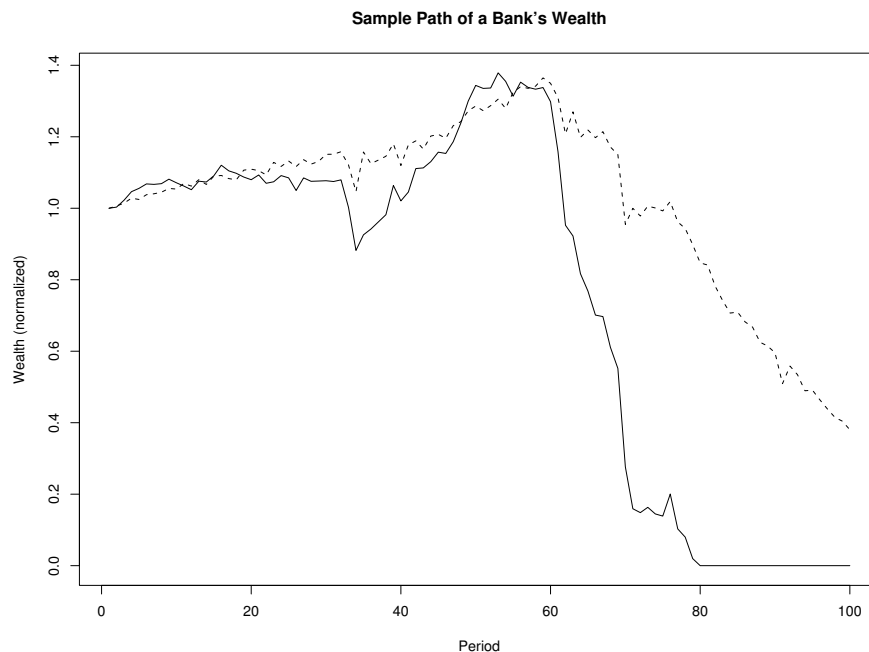
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<sup>25</sup> To compute the payments vector, we use an implementation of the algorithm described in Eisenberg & Noe (2001).

<sup>26</sup> We assume that liquidity is supplied from outside, e.g., by a central bank. As we use actual interest rates, they will include any liquidity premium observed during a difficult market environment.

Fig. 4. Sample path of a bank's wealth

The figure below shows the sample paths of a bank's wealth (floating-fixed portfolio) without hedging (solid line) and with hedging (dashed line).



### 7.1 Base Case

As we described in detail in Section 6, banks build up a portfolio of bond positions. Half of the banks have long positions in floating-rate bonds and short positions in the fixed-rate bond (*floating-fixed portfolio*), and vice versa for the other half (*fixed-floating portfolio*). Given the interest rate environment in the simulation, banks with a floating-fixed portfolio will experience a net loss, while banks with a fixed-floating portfolio will generate a significant profit. Absent any hedging activities, several banks with a floating-fixed portfolio will default at some time during the simulation. Hedging their interest-rate exposure can prevent those banks from becoming insolvent. Figure 4 shows the sample paths of wealth of a bank with a floating-fixed portfolio without and with hedging. The benefit of hedging is rather obvious.

In Table 3 we show simulation results for the base case (BC) as well as the case of hedging with no default risk mitigation mechanisms (0). While we include terminal wealth for the sake of completeness, the more meaningful measure as concerns wealth implications is the standard deviation of wealth. As banks try to hedge all of their exposure, standard deviation is an appropriate measure to analyze the effectiveness of hedging.

Hedging reduces the standard deviation of wealth, our main measure of the effectiveness of hedging, by 19.3 percent. Terminal wealth increases by 5.2 percent while the default rate decreases by 36.4 percent. In addition, the relation between the hedge ratio and the standard deviation of wealth over time is statistically significant at the one-percent level.

## *7.2 Overall Effects of Mitigation Mechanisms*

Before analyzing the effects of the various sets of mitigation mechanisms in more detail, we give an overview of the simulation results for a generic parameter configuration. In Table 3, we show simulation results for the base case (no hedging), the case of hedging without mitigation mechanisms, as well as cases of trading with the three sets of risk mitigation mechanisms described above. Initial margin is set at the 95% value-at-risk. In Table B.6, we show the results of regression analyses of the default risk mitigation mechanisms on the various measures.

The introduction of initial margin does not considerably reduce the benefits of hedging. The increase of the standard deviation of wealth and of the default rate are not statistically significant. However, initial margin increases the av-

Table 3

Overview of simulation results for a generic parameter configuration

The table below shows the effects of the three sets of default risk mitigation mechanisms we investigate. 0 denotes the case where banks trade in the swap contract but do not employ any risk mitigation mechanisms. Initial margin is set at the 95% value-at-risk level.  $\sigma_W$ : standard deviation of  $W$ ;  $W_T$ : terminal wealth;  $d$ : default rate;  $LGD$ : loss given default *per default*;  $XD$ : total demand - total supply;  $V$ : volume traded;  $\theta$ : ratio of swap contracts to fixed-rate bonds. All values are averages across banks. Standard deviations are normalized.

	BC	0	IM	IM & VM	CCP
$\sigma_W$	0.492	0.397	0.401	0.498	0.498
$W_T$	166.0	174.7	169.8	162.2	161.4
$d$	0.176	0.112	0.140	0.240	0.240
$LGD$	n/a	0.227	9.94	5.74	5.62
$XD$	n/a	-26.9	-26.4	-28.1	-27.6
$V$	n/a	19.0	18.6	15.4	15.4
$\theta$	0	0.812	0.814	0.802	0.802

erage loss given default.<sup>27</sup> It also reduces trading volume, albeit slightly. The latter two effects are statistically significant.

The introduction of variation margin considerably deteriorates the benefits of trading in the swap contract. In terms of standard deviation of wealth, it removes all the benefits of trading. It increases the standard deviation of wealth by 24 percent and reduces trading volume by 16.7 percent. While it decreases loss given default, compared to case IM, by 42.3 percent, it increases the default rate by 71 percent.

A central counterparty, in our set-up, cannot effectively reduce the negative effects of variation margin. It does, however, reduce loss given default by 2.1 percent.

<sup>27</sup> In the following, when we refer to loss given default we will mean the average loss per default in relation to swap contracts. It does not include losses in relation to bonds traded with clients.



Table 4

Effects of increased initial margin on the market outcome

The table below shows the effects of increases of initial margin on the market outcome. 0 denotes the case where banks trade in the swap contract but do not employ any risk mitigation mechanisms. Initial margin is set at the 95% (IM95), 97% (IM97), and 99% (IM99) value-at-risk level. Case IM95 is the same as IM before.  $\sigma_W$ : standard deviation of  $W$ ;  $W_T$ : terminal wealth;  $d$ : default rate;  $LGD$ : loss given default *per default*;  $XD$ : total demand - total supply;  $V$ : volume traded;  $\theta$ : ratio of swap contracts to fixed-rate bonds. All values are averages across banks. Standard deviations are normalized.

	BC	0	IM95	IM97	IM99
$\sigma_W$	0.492	0.397	0.401	0.403	0.406
$W_T$	166.0	174.7	169.8	167.6	164.9
$d$	0.176	0.112	0.140	0.156	0.172
$LGD$	n/a	0.227	9.94	13.7	18.5
$XD$	n/a	-26.9	-26.4	-26.9	-25.6
$V$	n/a	19.0	18.6	18.4	18.1
$\theta$	n/a	0.812	0.814	0.813	0.815

We now turn to a more detailed analysis of the default risk mitigation mechanisms.

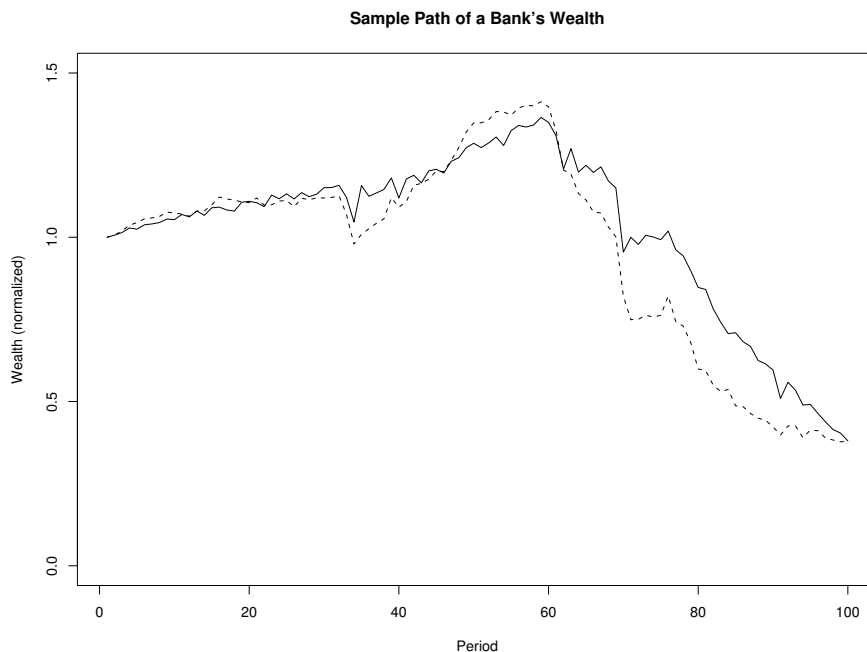
### 7.3 Initial Margin

We consider various levels of initial margin and analyze the effects of an increased initial margin requirement on the standard deviation of wealth, default risk, and market liquidity. As before, we compare trading with initial margin to the cases of no trading (BC) and to trading without any risk mitigation mechanisms (0).

As shown in Table 4, the effects of initial margin persist when its level is increased within the range commonly observed in financial markets. An increase in the level of initial margin reduces trading volume and increases both, de-

Fig. 5. Effect of variation margin on wealth

The figure below shows a sample path of a bank's wealth (floating-fixed portfolio) in the case of initial margin only (solid line) and initial as well as variation margin (dashed line).



fault rates and default severity. Its effects on the standard deviation of wealth are negligible, though, and not statistically significant.

#### 7.4 Variation Margin

Marking-to-market of positions and settlement of the differences in cash has ambiguous effects on a bank's wealth. While it reduces credit exposure, variation margin entails opportunity costs in terms of foregone interest and potentially lower capacity to trade bonds and swaps. Thus, it might exacerbate market movements in either direction.

Figure 5 shows sample paths of a bank's wealth in case of initial margin only as compared to initial and variation margin. The wealth trajectory clearly re-

Table 5  
Effects of variation margin on the market outcome

The table below shows the effects of variation margin, given certain levels of initial margin, on the market outcome. 0 denotes the case where banks trade in the swap contract but do not employ any risk mitigation mechanisms. Initial margin is set at the 95% (IM95), 97% (IM97), and 99% (IM99) value-at-risk level.  $\sigma_W$ : standard deviation of  $W$ ;  $W_T$ : terminal wealth;  $d$ : default rate;  $LGD$ : loss given default *per default*;  $XD$ : total demand - total supply;  $V$ : volume traded;  $\theta$ : ratio of swap contracts to fixed-rate bonds. All values are averages across banks. Standard deviations are normalized.

	BC	0	IM95 & VM	IM97 & VM	IM99 & VM
$\sigma_W$	0.492	0.397	0.498	0.499	0.499
$W_T$	166.0	174.7	162.2	160.5	159.1
$d$	0.176	0.112	0.240	0.260	0.272
$LGD$	n/a	0.227	5.74	9.99	16.0
$XD$	n/a	-26.9	-28.1	-28.2	-27.4
$V$	n/a	19.0	15.5	15.3	15.0
$\theta$	n	0.812	0.802	0.799	0.798

flects the effects of variation margin on wealth, namely, that variation margin exacerbates changes in wealth due to market movements.

Table 5 shows the negative effects of variation margin on the market outcome. It deteriorates the market outcome in terms of all our measures and eliminates the benefits of trading.

### 7.5 Central Counterparty

Finally, we investigate the effects of a central counterparty. The simulation results for the cases IM & VM & CCP are displayed in Table 6.

Table 6  
Effects of a central counterparty on the market outcome

The table below shows the effects of a central counterparty, given certain levels of initial margin and variation margin, on the market outcome. 0 denotes the case where banks trade in the swap contract but do not employ any risk mitigation mechanisms. Initial margin is set at the 95% (IM95), 97% (IM97), and 99% (IM99) value-at-risk level.  $\sigma_W$ : standard deviation of  $W$ ;  $W_T$ : terminal wealth;  $d$ : default rate;  $LGD$ : loss given default *per default*;  $XD$ : total demand - total supply;  $V$ : volume traded;  $\theta$ : ratio of swap contracts to fixed-rate bonds. All values are averages across banks. Standard deviations are normalized.

	BC	0	IM95 & VM & CCP	IM97 & VM & CCP	IM99 & VM & CCP
$\sigma_W$	0.492	0.397	0.498	0.499	0.499
$W_T$	166.0	174.7	161.4	160.4	158.2
$d$	0.176	0.112	0.240	0.252	0.272
$LGD$	n/a	0.227	5.62	9.66	15.9
$XD$	n/a	-26.9	-27.6	-27.4	-27.3
$V$	n/a	19.0	15.4	15.3	15.0
$\theta$	0	0.812	0.802	0.799	0.802

One would expect a central counterparty to lift at least some of the inefficiencies introduced by initial and variation margin. In our setting, most of the inefficiencies arise from variation margin. The size of variation margin is not effected by the introduction a central counterparty. However, a central counterparty does reduce default rates and losses given default, albeit not considerably. A central counterparty slightly increases the hedge ratio, an effect of multilateral netting. This effect is small since the market for the swap in our simulations is highly concentrated due to the distribution of bank size.

## 7.6 *Model Limitations*

While we do believe that our model captures certain key characteristics of derivatives markets, in particular, the interaction between market, credit, and liquidity risk, it has certain limitations. We now discuss what we consider the most important ones. First, our assumption that banks try to completely hedge their exposure to market risk may be unrealistic. If this is so, the negative impact of default risk mitigation mechanisms might be less pronounced. Secondly, we consider a derivatives market consisting of banks (hedgers) only. In this market, demand and supply are usually not balanced due to the size distribution of banks. In actual markets, any excess demand or supply might be balanced by third parties including speculators. In such a case, the impact of default risk mitigation mechanisms on market liquidity, and on default, might be less noticeable than in our model. On the other hand, close to one hundred percent of notional is held by the largest banks acting in derivatives markets; therefore, our model might reflect actual markets rather well. Fourthly, in our model a shortfall in market liquidity is reflected in traded quantities only and not in prices. As we analyze wealth effects, we believe that this does not af-

fect our results, though. Fifthly, we ignore information effects of margining and of a central counterparty. Margining might provide more timely information about the financial strength of a counterparty. A central counterparties pools information about positions of market participants and is often in a superior position to manage risk than single counterparties. The benefits of such information effects from the perspective of a single market participants are probably ambiguous, however. Finally, and most importantly, in evaluating default risk mitigation mechanisms we ignore externalities of derivatives markets and the banking system. Taking information effects and externalities (public or social costs) into account would probably change any cost-benefit analysis of such mechanisms. Such analysis, however, is beyond the scope of this investigation.

## 8 Conclusion

The aim of our study was to analyze direct and indirect effects of various sets of default risk mitigation mechanism on wealth. We conduct our analysis in a setting where banks are under severe stress, that is, a setting where default risk mitigation mechanisms, so one might think, are needed most.

We find that, in such a setting, the effects of default risk mitigation on wealth are negative. They increase default rates. The introduction of initial margin also increases default severity (losses given default) while variation margin and a central counterparty reduce it. Furthermore, default risk mitigation mechanisms impair banks' ability to hedge.

Many of their adverse consequences are indirect. They reduce market liquidity and hedge ratios and, subsequently, increase default rates as well as, in

case of initial margin, losses given default. The largest adverse effect is due to variation margin.

Margins are justified on the basis that they reduce losses given default and help to significantly reduce credit risk. They look particularly appealing in the presence of high deadweight costs of default. However, as the results of our simulations indicate, default risk mitigation mechanisms might fail to deliver on expectations. Our results show that certain default risk mitigation mechanisms do reduce losses given default, while others may increase them.

Variation margin, in particular, might exacerbate market movements. This is especially inefficient when these movements are only transient fluctuations without permanent changes in market fundamentals. In practice, though, such a differentiation is often impossible.

While variation margin (marking to market) does seem appealing from various perspectives, its consequences should be considered carefully, especially during a crisis. At present there seems to be a trend towards stricter capital, that is, margin, requirements for OTC derivatives. In combination with hard solvency constraints, such regulation might indeed help to fuel a crisis instead of preventing one.

We believe that our findings warrant a closer examination of some the issues raised, in particular, the interaction of market, credit, and liquidity risk, the rôle of information and its distribution, as well as the trade-off between private and public costs and benefits in relation to derivatives markets.

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## A Description of Default Risk Mitigation Mechanisms

In the following, we describe the most important default risk mitigation mechanisms. We focus on those mitigation mechanisms that differ across contracts traded in today's derivatives markets. We analyze netting, margining, rehypothecation and central counterparties. We ignore certain other mitigation mechanisms such as minimum capital requirements, definitions of default events and enforcement procedures either because they seem to be rather similar across contracts traded or because they seem to be less relevant.

A derivatives contract with its default risk mitigation mechanisms is necessarily embedded in the legal code of the respective jurisdiction. Therefore, many of the contractual innovations discussed below were preceded by changes in corporate law. This is particularly true for certain forms of netting and margining. As an example, amendments to the U.S. Bankruptcy Code in the 1970s and 1980s assign a special status to collateral in derivative transactions and were key to the success of collateral, as described by Johannes & Sundaresan (2006). Today, the important mitigation mechanisms including the ones discussed below are supported by the legal codes of the most important jurisdictions, as described in Allen & Overy (2002).

### A.1 *The Nature of Default Risk in Derivatives Contracts*

Like any other contract, a derivatives contract is a *promise to perform*. At delivery date, a counterparty to the contract might choose not to perform on its obligations, that is, she might choose to default. This choice might indeed be optimal to one side of the contract. The right to exercise such a choice can be viewed as a nonperformance option. In granting such options, market participants recognize that the cost of absolute performance assurances can exceed the value of trading benefits and might act as barriers to trade. In other words, default can be welfare-improving, as Dubey & Shubik (1979) and others showed.

The credit exposure in relation to derivatives contracts is highly complex. It depends on the prices of the underlying assets and thus varies over time. Often, credit exposure changes rapidly and in large amounts. Furthermore, credit risk in derivatives interacts not only with market risk but also with liquidity, operational, and other types of risks. These interactions and the resulting conceptual and measurement issues are not yet well understood.

The aim of default risk mitigation mechanisms is to reduce credit exposures by reducing loss amounts when nonperformance occurs (loss given default), and to reduce the probabilities of nonperformance states (probabilities of default). The off-balance sheet character of derivatives contracts makes it difficult indeed for counterparties to evaluate the financial health of an institution and

its contingent liabilities. Data on individual exposures and their fluctuations is largely proprietary. As a result, information to assess the creditworthiness of a counterparty (probability of default) is often insufficient. In such cases, the reliance on collateral appears to be a rather valuable risk-mitigation mechanism.

To illustrate the mitigation mechanisms we will make use of the following simple example: Suppose trader A holds a long position of ten units of a forward contract with trader B, and a short position of ten units of the same contract with trader C. A thus has a “net zero” position, that is, when the price of the underlying changes, a gain in one position is exactly offset by a loss in the other position. We assume that the probability of default of each of the three traders is the same and strictly positive.<sup>28</sup>

## A.2 Netting

Netting allows a trader to offset obligations with a counterparty.<sup>29</sup> Without netting, a defaulting counterparty might “cherry pick” profitable contracts and disclaim unprofitable ones. Netting allows a trader to cancel offsetting transactions and “net” their values thus reducing credit exposure. The most common form of netting employed in derivatives markets is *close-out netting*, allowing for the netting of contractual obligations in the event of a counterparty default. In the remainder, whenever we speak of netting we will mean close-out netting.

Suppose that trader A in the example above has the initial long position of 10 units with trader B, as well as additional, offsetting short position of 10 units with the same trader. A is now “net zero” with B. Let us assume further that the long position is now seasoned and has a value of 10. Accordingly, the short position is worth -10 to A. Assume now that B defaults. Without netting, the creditors of B might cherry-pick the short position (with positive value to B). A would thus have to fully deliver on the short position and might receive nothing from the long position. With a netting provision in place, A would cancel both transactions and “net” their respective values, reducing her obligation to 0.

Netting of obligations with a single counterparty is called *bilateral netting*. A further reduction of credit exposure might be achieved by *multilateral netting*,

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<sup>28</sup> Later on, we will gradually add various default risk mitigation mechanisms to the forward contract. A forward contract subject to *both* initial and variation margin *and* cleared by a central counterparty is commonly called a *futures* contract. We will stick with the term *forward* although at some stage the contract will have metamorphosed into a futures contract.

<sup>29</sup> Of the mitigation mechanisms discussed here, margining was the first to be employed by traders. Netting and central counterparties were introduced subsequently. Cf. Moser (1994a).

that is, netting across counterparties. Let us return to the initial set-up of our example where trader A holds a long position of ten units with trader B and an offsetting short position of ten units with trader C. If B defaults, A's "net zero" position is turned into a short position. With multilateral netting in place, A would be able to cancel both transactions and retain its initial net zero position. Multilateral netting is only possible with a central counterparty, though, as described below.

By reducing the contractual obligations resulting from a position of several transactions, netting reduces the loss-given-default of a position. As netting affects the credit risk borne by a trader, it also affects her solvency, and thus her probability of default.

### A.3 Margining

A further mechanism to reduce default risk is margining. Its purpose is to establish a lower bound for the delivery rate in case of a counterparty default. Margin is supposed to cover not just current exposure but also potential future exposure and replacement costs of contracts. A growing number of derivatives contracts are subject to margin requirements. In the following, we will distinguish two types of margin: *initial margin* and *variation margin*.

Initial margin is charged at contract initiation. Its size is usually related to the potential future exposure of the contract and is often based on the contract's value-at-risk. Initial margin is increasingly supplemented by variation margin. In this case, the contracts are regularly marked to market and the differences in value are settled in cash. This difference is called variation margin. It sets the current exposure of a contract to zero. Variation margin is usually settled daily or even more frequently.

If netting of contracts is provided for, margin requirements are based on the net position with a single counterparty (in case of bilateral netting) or across counterparties (in case of a central counterparty).

Initial margin may be covered by a variety of collaterals. As the value of collateral (except cash) fluctuates, collateral is typically subject to a haircut. Variation margin has to be covered by cash. Both, initial and variation margin might thus lead to (funding) liquidity issues.<sup>30</sup>

Let us return to our example. Trader A would have to post collateral to both, trader B and trader C although she holds a balanced position. At the same time, she will receive collateral from both counterparties. To avoid such sit-

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<sup>30</sup> Variation margin is one of the two features by which futures contracts are differentiated from a forward contracts. The other difference is the type of trading mechanism and clearing. Whereas futures contracts are traded on exchanges and cleared by a central counterparty, forward contracts are traded over the counter and are usually not cleared.

uations, counterparties today provide each other with the right the re-use collateral, called *rehypothecation*. In this case, the collateral received from one counterparty can be used to cover a margin requirement of another counterparty. The amount of collateral required to support a portfolio of positions is thus decreased, often dramatically. In our example, trader A would be allowed to re-use the collateral received from B to cover the margin requirement of C and vice versa. Thus, A would not need any collateral of its own for her position.

We point out at this stage that minimum capital requirements like those in the Basel accord can also be viewed as a form of margining.

#### A.4 Central Counterparties

Default risk can be further reduced by the use of a central counterparty. A central counterparty, as part of a derivatives clearinghouse, intermediates contracts, that is, it becomes the legal counterparty to every contract in a given market.<sup>31</sup> A central counterparty enables offsetting of obligations. It occurs when the aggregated claims against any counterparty are netted against the aggregate of the counterparty's claims against all other counterparties—what we called multilateral netting above. The current liabilities of the central counterparty and its members are the net of these obligations.

A central counterparty typically has a balanced market position (except when one or more members are in default) but has current credit exposure. Credit risk arises because a change in the price of the underlying asset could cause one counterparty to owe a considerable amount on its position, particularly if the contract is highly leveraged.

If one of its members defaults, the clearinghouse usually has the right to liquidate the member's position as well as the member's margin. The member's margin might not be sufficient to cover the central counterparty's losses in case of default. To protect itself from this risk, a central counterparty typically requires its members to enter into a loss-sharing agreement in the form of a *default fund*. Additional losses are often covered by insurance.

A central counterparty is usually one of several services offered by a derivatives clearinghouse. Due to broad diversification, high margin requirements, and other credit enhancements, central counterparties are typically of very high credit quality. Additional services offered by derivatives clearinghouses include contract management, collateral management, and payment processing. These services often lower administrative and processing costs of contracts. In addi-

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<sup>31</sup> Traditionally, central counterparties were only employed in relation to exchange-traded contracts. More recently, however, central counterparties are also available for certain OTC contracts. The most prominent example is SwapClear operated by LCHClearNet.

tion, clearinghouses, by way of their central counterparty, provide anonymity. Most trading mechanisms, including exchanges and brokers, nowadays provide anonymity at the trading stage (pre-trade anonymity). However, only central counterparties also provide anonymity at the settlement stage (post-trade anonymity).

The benefits offered by clearinghouses do not come for free but are subject to fees. Many derivatives clearinghouses are run as non-profit organizations or are subject to competition so that it can be assumed that the fees are priced near their costs.

There is a subtle issue in relation to central counterparties that is often ignored. It is related to the question whether the benefits of multilateral netting can be achieved by bilateral netting and rehypothecation. To answer this question, let us reconsider our simple example. We assume that the traders have charged each other with initial and variation margin. This means that the positions have no current credit exposure as it has been offset by variation margin. Let us now assume that trader B defaults. In this case, A is left with an open position with C. In other words, A is committed to honoring her obligations with C. This position carries potential future exposure and replacement cost risk. A might try to replace the defaulted contract with B by trading in the market. However, such a contract might not be available, for example, during a market crisis.<sup>32</sup> A will have to cover future losses arising from the contract with C by the initial margin received from B and, if this is not sufficient, her own resources. In case of multilateral netting, such a situation would not arise since a default of the central counterparty would cancel the entire (net) position. Only if the margins received by A covered the entire potential future exposure would the resulting default risk be the same (zero) in both cases. However, margin rarely covers the entire potential future exposure as this would be too costly.<sup>33</sup>

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<sup>32</sup> Such a situation appears somewhat artificial. However, it might very well occur during a market crisis (it did occur, e.g., after the terrorist attacks on September 11, 2001). Market participants might refrain from trading for several reasons, one being the disruption of the trading infrastructure. Clearing in particular and risk mitigation mechanisms in general are about extreme events and should thus be evaluated in such settings.

<sup>33</sup> The potential future exposure of many derivatives contracts is, theoretically, infinite.





Table B.1  
Notional amounts outstanding of derivatives contracts

This table shows the development of notional amounts outstanding of derivatives contracts by product category. Figures reported in billions of \$. Source: OCC (2005).

	94Q4	95Q4	96Q4	97Q4	98Q4	99Q4	00Q4	01Q4	02Q4	03Q4	04Q4	05Q3
Futures & forwards	8,109	7,399	8,041	9,550	10,918	9,390	9,877	9,313	11,374	11,393	11,373	11,927
Swaps	4,823	5,945	7,601	9,705	14,345	17,779	21,949	25,645	32,613	44,803	56,411	62,127
Options	2,841	3,516	4,393	5,754	7,592	7,361	8,292	10,032	11,452	14,605	17,750	19,636
Credit derivatives				55	144	287	426	395	635	1,001	2,347	5,094
Total	15,774	16,861	20,035	25,064	32,999	34,817	40,543	45,386	56,074	71,082	87,880	98,783

Table B.2  
Concentration of derivatives contracts

This table shows concentration in derivatives markets. In 2005, the five largest banks held 96.2% of total notional outstanding. Figures only include U.S. insured commercial banks as. Current international data is not available. The underlying dataset comprises 805 banks. Source: OCC (2005).

	\$	%	\$	%	\$	%
	Top-5	Total derivs	Rest 800	Total derivs	All 805	Total derivs
Futures & forwards	10,501	10.6	1,426	1.4	11,927	12.1
Swaps	60,827	61.6	1,300	1.3	62,127	62.9
Options	18,589	18.8	1,047	1.1	19,636	19.9
Credit derivatives	5,065	5.1	29	0.0	5,094	5.2
Total	94,981	96.2	3,802	3.8	98,783	100.0

Table B.3

Ratio of credit exposure to risk-based capital

This table shows the ratio of credit exposure to risk-based capital for the seven largest U.S. derivatives dealers. Current international data is not available. Figures reported in %.

Data for 99Q3 reflects the merger between Bank of America and NationsBank. Prior quarters are not merger-adjusted and may not be comparable. 01Q4 data reflects the merger between Chase Manhattan and Morgan Guaranty. Prior quarters represent Chase Manhattan's data. 02Q2 reflects the merger between First Union and Wachovia. Prior quarters represent First Union's data. 04Q4 reflect the merger between JPMC and Bank One. Source: OCC (2005).

	96Q4	97Q4	98Q4	99Q4	00Q4	01Q4	02Q4	03Q4	04Q4	05Q3
JP Morgan Chase	265.8	329.5	380.3	416.0	442.5	589.2	654.4	844.6	592.7	657.7
Morgan Guaranty	507.7	806.4	820.3	873.3	817.4	873.7				
Bank of America	112.0	92.2	90.3	119.8	114.5	141.7	204.9	221.7	232.9	174.4
NationsBank	120.1	68.2	80.8							
Citibank	162.1	204.9	202.5	176.3	190.6	167.4	201.1	267.1	305.3	345.6
Wachovia	30.3	16.3	17.5	20.5	55.5	83.9	102.5	80.6	77.6	76.8
HSBC Bank USA				32.2	44.7	72.4	127.2	288.5	301.6	461.9

Table B.4

## Notional to risk-based capital

This table shows the ratio of notional to risk-based capital, also called the “margin”. Notional is reported in millions of \$, margin is reported in basis points. A margin of 1 basis point implies that a realized loss in the notional position of 1 basis point is equivalent to the institution’s risk-based capital. Source: Walen (2004) based on FDIC data.

	Notional	Margin
JP Morgan Chase	39,622,611	11.341
HSBC Bank USA	1,572,083	54.802
Citibank	13,701,733	40.982
Bank of America	14,891,391	37.164
Wachovia Bank	2,604,746	120.292
Bank of New York	608,233	135.382
Bank One	1,151,411	197.740

Table B.5

## Volume of collateral used in OTC derivatives markets

This table shows the development of collateral usage in OTC derivatives markets over the last six years. Numbers are in millions of \$. Source: International Swaps and Derivatives Association, Inc. (2005*b*)

	2000	2001	2002	2003	2004	2005
Reported	138	145	289	491	707	854
Estimated	200	250	437	719	1,017	1,209

Table B.6

Factor analysis of the impact of default risk mitigation mechanisms

This table shows the results of two-way anova analyses of the impact of default risk mitigation mechanisms on various measures. We introduce three factors, *IM*, *VM*, *CCP*, representing the three different default risk mitigation mechanisms, and a blocking variable, *EXPOT*, discriminating the exposure type of the bank (fixed-floating v. floating-fixed).  $\sigma_W$ : standard deviation of wealth,  $W$ ;  $d$ : default rate; *LGD*: loss given default *per default*;  $V$ : volume traded.

Bold face indicates  $p$ -values below 0.01. Standard errors are shown in brackets.

	$\sigma_W$	$d$	<i>LGD</i>	$V$
<i>Intercept</i>	<b>0.419</b>	<b>0.112</b>	<b>0.227</b>	<b>18.965</b>
	(0.010)	(0.024)	(0.764)	(0.206)
<i>EXPOT</i>	<b>0.045</b>			
	(0.010)			
<i>IM</i>	0.004	0.003	<b>9.717</b>	<b>-0.399</b>
	(0.013)	(0.034)	(1.080)	(0.291)
<i>VM</i>	<b>0.097</b>	<b>0.100</b>	<b>-4.209</b>	<b>-3.104</b>
	(0.013)	(0.034)	(1.080)	(0.291)
<i>CCP</i>	0.000	0.000	<b>-0.111</b>	-0.028
	(0.013)	(0.034)	(1.080)	(0.291)
Residual SS	0.191	0.191	210.0	651.6
$R^2$	0.607	0.395	0.694	0.879
Adj. $R^2$	0.591	0.345	0.668	0.869

SS: sum of squares