Extreme spectral risk measures: an application to futures clearinghouse margin requirements

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Outline

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- VaR and ES based margins
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Clearinghouses (CH) act as counterparty to trade
CH manage counterparty risk by setting margins
Literature has focused on statistical models for setting margins
Examples of models employed include Extreme value theory, gaussian, historical distribution, conditional distributions etc
Application of statistical models is to estimate VaR (probability of default/quantile or loss)
PAPER examines properties and estimates of potential candidate measures for setting margins
Statistical models determine default probability

Margin Requirements for a Short Position and a Distribution of Price Changes
Left Tail of Fat-tailed and Normal Distribution
Risk Measures

- **Value at Risk (VaR)** – quantile of loss distribution
  \[ \text{VaR}_\alpha = q_\alpha \]

- **Expected Shortfall (ES)** – average of losses beyond VaR
  \[ \text{ES}_\alpha = \frac{1}{1-\alpha} \int_{q_\alpha}^{1} q_p \, dp \]

- **Spectral Risk Measures (SRM)** – risk measure related to user’s risk aversion function
  \[ M_\phi = \int_{0}^{1} \phi(p) q_p \, dp \]
Histogram of profits and losses

95% VaR = 1.704
Risk measures and margin setting

- All risk measures have key parameter – confidence levels for VaR and ES, degree of risk aversion for SRM
- Confidence levels set in arbitrary fashion but degree of risk aversion can be obtained by user of risk measure
- CH would select CARA based on their risk appetite
- Risk measures react in similar way to key parameters eg. If CARA increases SRM increases
- VaR assumes user is risk lover whereas ES assumes user is risk neutral
- SRM assumes user is risk averse
Risk measures and margin setting

- VaR able to measure default probability associated with margin
- ES able to measure default probability associated with margin
- VaR not coherent whilst ES and SRMs are coherent
- VaR not subbadditive – investor would break up margin accounts to get reduction in margin requirement
- ES tells CH of loss that they should expect conditional that VaR is exceeded
Properties of SRMs

- SRMs are coherent
- SRMs are not based on confidence interval
- SRMs are based on risk aversion function
- User of SRMs decide on their risk aversion function
- Potential risk aversion function is exponential

\[ \phi_\gamma(p) = \frac{e^{-p/\gamma}}{\gamma(1 - e^{-1/\gamma})} \]

\[ M_\phi = \int_0^1 \phi(p) q(p) dp = \int_0^\gamma \frac{e^{-(1-p)/\gamma}}{\gamma(1-e^{-1/\gamma})} q(p) dp \]
Properties of SRMs

- **Non-negativity**: weights non-negative
- **Normalization**: weights sum to 1
- **Weakly increasing**: Weights attached to higher losses at least weights attached to lower losses
- For high weights associated with high losses, expect higher risk aversion associated with these higher weights
- Weights should rise faster as $p$ rises further
Figure: Exponential Risk-Aversion Functions
Extreme risk and margin setting

- Use Peaks over Threshold (GPD) approach
- Model realisations of random variable over high threshold
- Shape parameter indicates tail property with literature supporting fat-tailed property
- GPD parameters incorporated into VaR engine to give risk measures
Data and Preliminary Analysis

- Use heavily traded (e.g., S&P500, FTSE100, DAX, Hang Seng, Nikkei 225) futures between 1/1/91 – 31/12/03)
- QQ plots indicate fat-tails
- QQ plots show tail threshold values
- Tail index plots confirm tail threshold values
- GPD tail parameters reasonable in terms of literature
- Fit of Exceedences to GPDs good
Figure: Tail Index Plots as Functions of Numbers of Exceedances
Comparison of alternative risk based margins

- Use high confidence intervals to reflect CH concern with large losses and potential defaults
- ES larger than VaRs and ES similar to SRM:
  eg. SRMs with CARA of 100 similar to ES with 0.99 confidence level and VaR with 0.995 confidence level
- Risk measures increase for increasing confidence intervals/risk aversion
- All measures are reasonably symmetric margins for long and short positions
- ES more precise than VaR and reasonably similar to SRM
Figure : Generalised Pareto VaRs of Futures Positions at Extreme Confidence Levels
Figure: Generalised Pareto Expected Shortfalls of Futures Positions at Extreme Confidence Levels
Figure: Spectral-Exponential Risk Measures of Futures Positions
Conclusions and future work

CH impose margins to protect against extreme price movements.

Three risk measures outlined and estimated:
- SRMs and ES coherent
- SRMs attractive by including user’s risk aversion
- Extreme SRMs similar in magnitude and reasonably precise

Future work will compare actual margins set by A CH with 3 risk measure estimates.