

Public Liquidity and Bank Lending: Treasuries, Quantitative Easing, and Central Bank Digital Currency

Roberto Robatto*

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Abstract

What is the optimal supply of public liquidity by the Treasury and the central bank, and what are the impacts on the banking system and the economy? To answer these questions, I first show that, in the U.S. data, a higher debt-to-GDP ratio (i) reduces the share of credit to firms that is intermediated by banks, (ii) reduces GDP, but (iii) has no statistically significant effects on total investments. I then present a simple theoretical model that rationalizes these facts and derives three main implications. First, the optimal supply of public liquidity balances the direct benefits of having a larger stock of such assets with the negative effects on the share of credit intermediated by banks. This first result holds for many definitions of public liquidity: Treasuries, central bank reserves created by quantitative easing (QE), and central bank digital currency. Second, the same outcomes can be achieved with QE and central bank digital currency, provided that the interest paid on reserves created with QE is higher than the interest on digital currency. Third, the optimal size of QE is non-monotonic in the stock of Treasury debt, and there are multiple combinations of Treasury and central bank policies that maximize welfare.

1 Introduction

What is the optimal size of public liquidity? This question is important because liquid assets, including the “public” ones supplied by the government, affect the functioning of the financial system and of the whole economy. Traditionally, public liquidity has taken the form of Treasury

*University of Wisconsin-Madison, Wisconsin School of Business, United States. E-mail: robatto@wisc.edu. This is a large revision of a previous work circulated under the title "Public Liquidity and Bank Lending: The Perils of Large Interventions." I thank Todd Keister and seminar participants at Rutgers for helpful conversations and suggestions.

securities, which can be held either directly or through money market funds. However, in recent years, central banks have become key players in the determination of the supply of public liquidity by purchasing securities — mostly Treasury securities — and issuing interest-bearing reserves. In addition, several central banks are now discussing the possibility of offering a so-called central bank digital currency, which is another form of public liquidity.¹

Because public liquidity can take various forms — Treasuries, central bank reserves, and central bank digital currency — this paper provides a simple unifying theoretical framework that highlights their similarities, differences, and complementarities. In particular, some results will hold under different definitions of public liquidity, whereas others highlight how the Treasury and the central bank can achieve similar objectives with different tools or policy stances. The focus of the paper is on the medium- and long-run “average” supply of public liquidity, and I abstract from the analysis of how public liquidity should temporarily deviate from the average target because of acute financial crises.

To discipline the theoretical analysis, I first provide some evidence about the effects of public liquidity on the financial system and the economy. The literature has shown that an increase in public liquidity reduces the amount of private liquidity, that is, liquid assets such as deposits that are issued by banks and financial intermediaries (see, e.g., [Greenwood, Hanson, and Stein 2015](#); [Krishnamurthy and Vissing-Jorgensen 2015](#); [Li 2019a](#)). However, less is known about the impact on the provision of credit to the economy and, ultimately, investments. Yet, this is a crucial issue because if public liquidity policies affect lending by banks and firms’ investments, they are likely to generate sizable effects on the output, employment, and the financial system.

I show that an exogenous, permanent increase in the supply of Treasury securities (i.e., in the debt-to-GDP ratio) reduces the fraction of credit to firms that is intermediated by banks. That is, an increase in public liquidity triggers a shift in credit provision from bank lending to direct financing such as corporate bonds and commercial paper. I also study the effects on aggregate investments and GDP. Both quantities are negatively affected by an increase in the debt-to-GDP ratio, but only the effect on GDP is statistically significant. Taken together, these results suggest

¹Central bank digital currency is a highly debated topic among central bankers. China has already tested a central bank digital currency, and a large number of central banks around the world are studying its implementation, including the European Central Bank, Bank of England, and the Federal Reserve. In a recent statement, the chairman of the Federal Reserve, Jerome Powell, said that “we have an obligation to stay on the forefront of policy and technological innovation [...] as regard to payments, [...] CBDC (i.e., central bank digital currency)” and “[central bank digital currency] is one of those issues [...] that is more important to get it right than it is to be first.”

that a permanent increase in the debt-to-GDP ratio affects negatively GDP and that the channel that operates through the composition of the credit supply plays a more important role than the one that operates through the total level of investments. I derive these results using a structural VAR with long-run identification restrictions, although the results are robust to using a more standard identification based on the ordering of the variables (i.e., Cholesky). The long-run restrictions allow me to look at permanent increases in the debt-to-GDP ratio, such as those driven by time-varying attitudes of policymakers over the long-run average target of the debt-to-GDP ratio, as opposed to temporary increases that are related to business cycle fluctuations and thus likely not exogenous with respect to credit market conditions. This identification scheme is also consistent with the model, which abstracts from short-run fluctuations and thus can be interpreted as focusing on the medium- and long-run supply of public liquidity.

I then turn to the theoretical analysis. I provide a simple theoretical model that rationalizes the above facts using standard elements employed in widely-used macro-finance frameworks such as [Brunnermeier and Sannikov \(2014\)](#), [Gertler and Kiyotaki \(2010\)](#), and [He and Krishnamurthy \(2013\)](#). The model provides insights into the impact of public liquidity on the economy and guides the policy analysis.

In the model, agents outside the financial sector need liquid assets that are supplied by the government (i.e., public liquidity) and banks (i.e., private liquidity). Credit can be intermediated by banks or supplied directly from savers to firms. Two key assumptions are in common with the macro-finance literature cited above: banks have better technology — motivated by their ability to screen and monitor firms and other borrowers and thus identify better investment opportunities ([Diamond, 1984](#); [Freixas and Rochet, 2008](#)) — but are subject to moral hazard.² On the one hand, the technological advantage alone implies that it would be optimal to have banks making all the investments in the economy. On the other hand, the moral hazard alone implies that banks extract rents, and thus financial intermediation would shut down if bank debt had no liquidity value. When all the features of the model are combined, the equilibrium displays an outcome that resembles what we observe in practice. That is, savers directly supply a fraction of the credit in the economy and purchase some bank debt, so banks supply the remaining fraction of credit. In the model,

²While these two assumptions have been commonly employed to study macro-finance issues, their implications for the analysis of public liquidity have been overlooked. An exception is [Li \(2019b\)](#), in which banks demand public liquidity for insurance purposes, and in which the policy analysis focuses on liquidity injections during crises as opposed to the medium- and long-term supply of public liquidity studied here.

public liquidity is safer than bank debt because banks might fail. In particular, public and private liquidity are substitutes in normal times but private liquidity loses its value in times of financial distress. This approach is similar to Nagel (2016) and also consistent with the evidence discussed by Gorton (2017), who highlights how various securities issued by financial intermediaries and that are normally perceived as safe might lose their value and liquidity during crises.

An increase in public liquidity generates two effects with opposite implications. First, because public liquidity is safer than private liquidity, the higher liquidity services increase welfare. Second, the increase in the supply of public liquidity reduces the liquidity needs that must be met using banks' debt. Agents respond by holding less of banks' debt — to economize on the cost that originates from the moral hazard friction — and by investing more directly. In other words, an increase in the supply of public liquidity triggers a process of disintermediation that shifts investments from the financial sector to the non-financial sector, as documented in the empirical analysis. This second effect is detrimental to output and welfare — similar again to the data — because the disintermediation increases the fraction of direct investments, which are less productive. The optimal supply of public liquidity balances these two effects, and a too-large supply is not optimal because it triggers excessive disintermediation. These results are a common theme under the three definitions of public liquidity that I consider (i.e., Treasury securities, central bank reserves issued under quantitative easing, and central bank digital currency), although the strength of the forces and the details of the implementation differ among the three.

The baseline model includes only public liquidity in the form of Treasury supply. I then introduce a central bank that implements quantitative easing (QE) and possibly issues a central bank digital currency.

QE in the model is represented by the purchase of Treasury debt by the central bank, which is financed by issuing reserves. I focus on Treasury securities purchases because, in practice, they represent the bulk of the assets purchased by central banks. For instance, as of October 2020, about 80% of the assets purchased by the ECB fall under the Public Sector Purchase Program. To study QE in the model, I assume that Treasury debt provides less liquidity than bank deposits. This is in line with the fact that non-financial agents, in practice, hold a large amount of deposits despite the lower return they provide in comparison to holding Treasury securities — either directly or through money market funds.³ The overall effect of QE is to “transform” partially liquid Treasury debt into

³Holding a large amount of deposits in comparison to Treasuries is true not only for households but also for firms.

fully liquid debt issued by intermediaries, in two steps. First, the central bank transforms Treasury debt into reserves, which must be held by banks. Second, banks invest in reserves, which allows them to issue more deposits and, because reserves are risk-free, to reduce the overall risk of their liabilities. In a sense, quantitative easing enhances the liquidity services provided by Treasury securities. In practice, QE has other consequences too, as documented by the literature, but the model abstracts from them to highlight the new effect studied here.

Central bank digital currency is modeled along the lines of [Bordo and Levin \(2017\)](#), that is, by allowing non-financial agents such as households to deposit at the central bank (i.e., to invest in central bank reserves).⁴ The main result here is that any allocation that is achieved by quantitative easing can also be achieved when reserves are offered to the public through central bank digital currency and vice versa. There is an important difference among the two, however, which is crucial for implementation. To achieve any given allocation, the interest rate paid on reserves depends on whether reserves can be held by banks only, as is the case under QE, or by the public, as with digital currency. In particular, the central bank must pay a higher interest rate to banks to compensate them for the rents associated with moral hazard.

Finally, I perform some (preliminary) analysis of the interaction between the Treasury and the central bank. Using numerical examples, I consider the case in which the central bank chooses its policy optimally for any given supply of Treasury debt. Under this assumption, the optimal size of QE is non-monotonic in the amount of Treasury debt. If the supply of Treasury debt is low, the central bank buys a large amount of it in order to enhance its liquidity value as described above. However, if the supply of Treasuries is too high and is already creating a too-large disintermediation effect, QE would exacerbate the disintermediation. Thus, it is optimal for the central bank to supply no excess reserves (i.e., no reserves above the minimum required to have a well-functioning interbank market). For an intermediate range of Treasury debt, the central bank can achieve the maximum welfare using the appropriate size of the QE policy. In particular, welfare in this region is flat because, if the Treasury supply is slightly higher, for example, the central bank can reduce

Using data from the Flow of Funds, which capture the universe of firms in the US and not just public companies, firms held about \$3.5 trillion in cash-like assets at the end of 2018, and about three-quarters of these were in bank deposits. That is, only about \$0.9 trillion was invested in instruments other than deposits, such as Treasuries, munis, commercial paper, and reverse repos.

⁴[Bordo and Levin \(2017\)](#) suggest this can be done using account held directly at the central bank or using designated accounts at commercial banks which would hold the corresponding amount in segregated reserve accounts at the central bank.

the size of QE a bit to keep the “effective” supply of public liquidity unchanged and the share of bank credit unaltered.

1.1 Additional comparison with the literature

Several other papers study the effects of public liquidity and its interaction with the financial sector, but most of these studies focus only on one definition of public liquidity. [Holmström and Tirole \(1998\)](#) show that public liquidity is needed when the economy is subject to aggregate risk, but there are no negative effects of large public liquidity on financial intermediation. In [Greenwood, Hanson, and Stein \(2015\)](#), public liquidity crowds out the financial sector too, but this effect is positive in their model because of an externality generated by private intermediaries and modeled along the lines of [Stein \(2012\)](#). In [Li \(2017\)](#), public liquidity reduces liquidity premia and creates an incentive for intermediaries to take on more risk, which amplifies credit cycles and lengthens the duration of crises. [Bolton and Huang \(2017\)](#) study public liquidity in a framework in which domestic and foreign money provides liquidity. [Liu, Schmid, and Yaron \(2019\)](#) highlight another possible risk associated with a large supply of Treasury debt, namely, its negative effects on corporate financing. Finally, a recent theoretical literature has emerged to study the effects of central bank digital currency; [Keister and Sanches \(2020\)](#) and [Williamson \(2019\)](#) study the effects on payments and banks’ investments but abstract from the substitution with direct financing that is highlighted here, and [Fernández-Villaverde et al. \(2020\)](#) study the effects on maturity transformation and bank runs.

A separate literature has been focusing on the analysis of QE, either empirically or in the context of macro-finance models. Among the second set of paper, [Eggertsson and Woodford \(2003\)](#) show that asset purchases by the central bank do not have any impact in a model with a representative agent and no financial constraints, along the lines of the Ricardian equivalence and other Modigliani-Miller type results. However, QE produces effects in models with limited participation or in which financial intermediaries face constraints, such as [Gertler and Kiyotaki \(2010\)](#) and [Gertler and Karadi \(2011\)](#). [Gertler and Karadi \(2013\)](#) and [Vayanos and Vila \(2009\)](#) include purchases of Treasury securities, that have an impact because of frictions that affect the term structure of interest rates.

This paper is also related to a literature that studies financial policies in models in which credit

can be intermediated not only by banks but also by various forms of shadow banks. In particular, [Begenau and Landvoigt \(2016\)](#) and [Dempsey \(2020\)](#) show that higher capital requirements shift resources from regulated commercial banks to shadow banks, similar to an increase of public liquidity here. Tighter regulation is good because the substitution toward the shadow banking system reduces the negative impacts on welfare. A similar channel likely implies that the optimal supply of public liquidity is higher than the one that would be suggested by models that do not account for the non-bank supply of credit. However, I leave this issue for future research.

The model I use builds on [Benigno and Robatto \(2019\)](#), but there are important differences in comparison to that paper. A too-large supply of public liquidity is not feasible in [Benigno and Robatto \(2019\)](#) because of a limit on fiscal capacity, similar, for example, to [Greenwood, Hanson, and Stein \(2015\)](#). Absent such a limit, however, the optimal policy is similar to that of [Holmström and Tirole \(1998\)](#): an injection of public liquidity that drives private liquidity to zero and eliminates financial intermediation entirely. Indeed, that paper does not include the two key elements of the model that affect the policy trade-offs here, namely, intermediaries' moral hazard and technological advantage, which are instead borrowed from the macro-finance literature.

2 Government debt and credit intermediation: Empirical evidence

This section provides some empirical evidence that motivates the theoretical model. In particular, it shows that an increase in the ratio of government debt to GDP causes a reduction in the share of credit intermediated by banks.

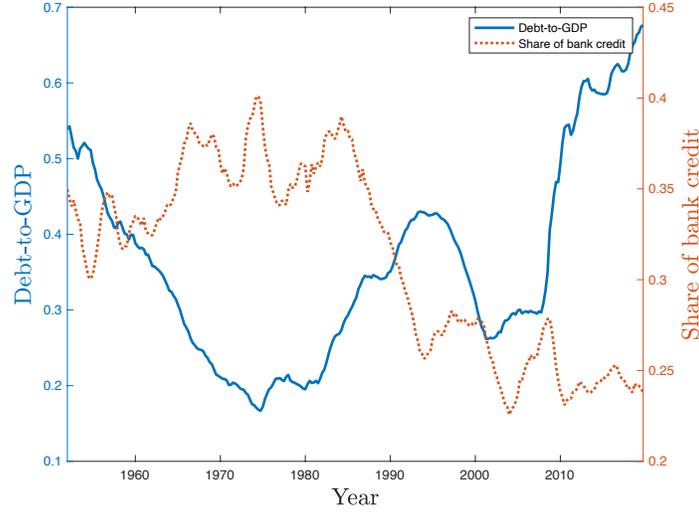
Using data from the Flow of Funds, I construct a quarterly time series for the share of credit to firms that is intermediated by banks. I define such a share as

$$(\text{share})_t = \frac{(\text{bank loans})_t}{(\text{bank loans})_t + (\text{corporate bonds})_t + (\text{commercial paper})_t + (\text{other loans})_t},$$

where $(\text{bank loans})_t$ denotes loans from depository institutions, and $(\text{other loans})_t$ includes all other loans extended to firms.⁵ [Figure 1](#) plots the evolution of the share of bank credit from

⁵[Dempsey \(2020\)](#) considers the same definition of $(\text{share})_t$ in his analysis of capital requirements with banks and shadow banks.

Figure 1: Share of credit intermediated by banks and debt-to-GDP ratio



Debt-to-GDP ratio (solid line, left axis), and share of bank credit (dotted line, right axis), for 1952-2019. Debt is defined as privately held gross federal debt and excludes debt held in government accounts or by the Federal Reserve. The share of bank credit is the ratio of bank loans (i.e., loans extended to firms by depository institutions) to the sum of bank loans, corporate bonds, commercial paper, and other loans.

1952 to 2019 (dotted line), together with the debt-to-GDP ratio (solid line). Debt is defined as privately-held gross federal debt, and excludes debt held in government accounts or held by the Federal Reserve. The correlation between the two series is -0.65.

To understand if there is a causal link between an increase in the debt-to-GDP ratio and a reduction in the share of bank credit, I need to identify an exogenous increase in debt to GDP. I do so using a VAR with long-run restrictions, and then I show that the results are robust to using a more standard identification approach based on the ordering of the variables in the VAR (i.e., the so-called Cholesky identification).

Let $Y_t = [\Delta \log (debt/GDP)_t, \Delta \log (share)_t]'$ and consider the reduced-form VAR

$$Y_t = A_1 Y_{t-1} + \dots + A_p Y_{t-p} + \begin{bmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \end{bmatrix}, \quad (1)$$

where p is the number of lags and $\varepsilon_t^1, \varepsilon_t^2$ are the reduced-form shocks. I first estimate the coefficients A_1, \dots, A_p , and then construct the impulse response to an exogenous shock identified as

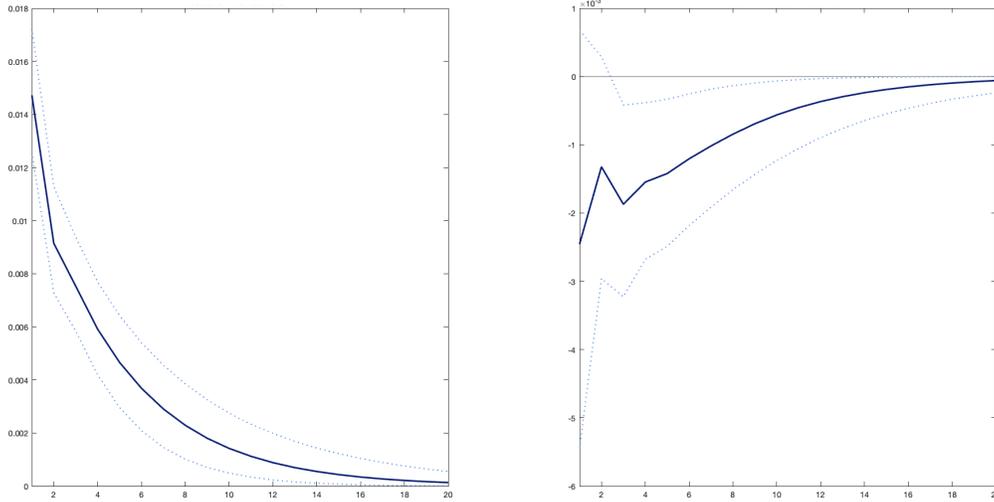
follows. I use the long-run restriction approach which identify two shocks: a shock that has transitory effects on the *level* of debt to GDP, and another shock, orthogonal to the first one, that has permanent effects on the *level* of debt to GDP.⁶ If the government has a target for the debt-to-GDP ratio that should be reached in the long run (i.e., in steady state), the second shock corresponds to an exogenous increase in this target. This approach is consistent with numerous models that include such a target; see, for instance, the general framework of Uhlig (2010) and the analysis of government debt and corporate financing of Liu, Schmid, and Yaron (2019). An increase in the target debt to GDP could be due, for instance, to changes in the attitude of policymakers over the “right” long-run average for the debt-to-GDP ratio.

The long-run identification scheme and the structure of the VAR that I use have the advantage of making sure that I focus on changes in debt to GDP that are not temporary and likely not driven by business cycle fluctuations. Indeed, business cycle fluctuations alone (i.e., without changes in the long-run target for the debt to GDP) give rise to temporary changes in government debt, but such changes are captured by the shock that has transitory effects on the level of debt to GDP. Instead, I focus on the other shock, that not only has permanent effects on debt to GDP but also is *orthogonal* to the temporary shock. Thus, as long as fluctuations in debt to GDP driven by the business cycle are accounted for by the temporary shock, my results are not driven by changes in the debt-to-GDP ratio that originate from short-run movements in the economy. In this regard, my identification is consistent with the model presented later, which abstracts from short-run fluctuations and, thus, focuses on the medium- or long-run average supply of public liquidity.

Figure 2 shows the impulse response using a VAR with $p = 2$ for the period 1952-2019. The left panel shows a temporary increase in $\Delta \log (debt/GDP)_t$, that is, a temporary increase in the growth rate of debt to GDP, which translates into a permanent increase in the level of debt to GDP. This is the permanent shock to debt to GDP that is identified using the long-run restriction. The right panel shows that this shock reduces the growth rate of the share of credit intermediated by banks, and the effect is statistically significant starting two quarters after the shock and for several periods after that. The dotted line in Figure 2 denote the 90% confidence bands, but the result is also significant when computing the 95% confidence bands; see Appendix A. In addition, I

⁶Long-run restrictions have been proposed by Blanchard and Quah (1989) and have been applied to both macro and finance questions. In the macro literature, for example, they have been used to study the effects of technology shocks on hours worked (Gali, 1999; Fisher, 2006). In finance, for example, they have been employed by Hansen, Heaton, and Li (2008) to study the pricing of risk exposure in the long run.

Figure 2: Impulse response to a shock that produces permanent effects on the debt-to-GDP ratio



Left panel: impulse response to $\Delta \log(\text{debt}/\text{GDP})_t$; right panel: impulse response to $\Delta \log(\text{share})_t$. The horizontal axis is quarters, and the shock takes place in period one. The VAR in Equation (1) is estimated using quarterly data from 1952 to 2008 and using $p = 2$. The solid line is the median impulse-response and the dotted lines denote the 90% confidence bands.

compute the cumulative effect of the shock on the share of bank credit in the long run.⁷ A 1% increase in the long-run average level of debt to GDP triggers a change in $(\text{share})_t$ of -0.259%, and the effect is statistically significant, with the 90% confidence interval given by [-0.692%, -0.046%].⁸ The result is also statistically significant if one uses the 95% confidence interval, as shown in the Appendix A.

Table 1 shows that the 90% confidence interval for the long-run effect is very robust to using various alternative specifications, and Appendix A presents the impulse responses, which are also very similar to those in Figure 2. The alternative specifications alter the baseline one by changing the number of lags (i.e., four vs. two), the frequency of the data (i.e., yearly vs. quarterly), and even the identification scheme (i.e., Cholesky vs. long-run restrictions). In particular, the alternative identification scheme (i.e., Cholesky) is based on the more standard ap-

⁷The cumulative effect is defined as the sum of the impulse response function of $\Delta \log(\text{share})_t$ over time — in principle, up to $t = \infty$ but in practice up to a finite but large horizon — which can be interpreted as the “integral” of the impulse-response function.

⁸To compute the long-run effect, I draw a large number of impulse responses that account for the uncertainty in the estimated reduced-form VAR and then compute the cumulative effect for each impulse response. I use the 50th percentile of these cumulative effects as the estimated effect and the 5th and 95th percentiles to compute the confidence interval.

Table 1: Debt-to-GDP ratio and credit intermediation by banks, robustness

Identification	Frequency	Lags	Long-run effect	90% confidence interval
Long-run restrictions (baseline)	Quarterly	2	-0.259%	[-0.692%, -0.046%]
Long-run restrictions	Quarterly	4	-0.198%	[-0.571%, -0.018%]
Long-run restrictions	Yearly	1	-0.239%	[-0.844%, -0.026%]
Cholesky ($share_t$ first)	Quarterly	2	-0.171%	[-0.501%, -0.013%]
Cholesky (debt to GDP first)	Quarterly	2	-0.208%	[-0.567%, -0.034%]

Effect of a 1% increase in the long-run average level of debt to GDP on the share of credit intermediated by banks. The first column shows the identification scheme, the second column shows the frequency of the data used, the third column shows the number of lags in the VAR, the fourth column shows the long-run effect on $(share)_t$, and the last column shows the 90% confidence interval.

proach of ordering of the variables of the VAR so that the variable ordered first does not respond, on impact, to a shock to the second variable. To perform the Cholesky identification, I define $Y_t = [\Delta \log (share)_t, \Delta \log (debt/GDP)_t]'$ so that $(share)_t$ is ordered first and, thus, does not respond on impact to a shock to debt to GDP. This is motivated by the idea that frictions in credit markets might prevent firms from quickly switching between bank and non-bank financing. Nonetheless, I also run the analysis by ordering debt to GDP first, and [Table 1](#) and [Appendix A](#) shows that the results are essentially unchanged.⁹ [Appendix A](#) provides more discussion of these robustness checks and some possible explanations of why, in this analysis, the results with Cholesky and long-run restrictions are so similar to each other. In any case, I argue that the fact that the results are quite alike despite the two identification schemes are very different provides

⁹An alternative method employed in the VAR literature is to use a narrative approach to identify exogenous changes in the variable of interest to be used as instruments. Because the variable of interest here is the ratio of government debt to GDP, which is related to government finances, a possible approach is to use news about increases in military spending related to wars, which are likely exogenous. [Ramey and Shapiro \(1998\)](#) identify three “war shocks” that led to large increases in military spending, namely, the Korean war, the Vietnam war, and the Carter-Reagan buildup, and [Ramey \(2011\)](#) adds a fourth shock, that is, 9/11. However, when added to the VAR following the approach in [Ramey \(2011\)](#), these shocks do not produce a permanent increase in the debt-to-GDP ratio and thus are not good instrument for my analysis. This is actually consistent with [Ramey \(2011\)](#)’s analysis in which, using a richer news time series, she finds that these war shocks produce an increase in the average marginal income tax. That is, the increase in military spending seems to be compensated by higher taxes, consistent with a not statistically significant response of debt to GDP.

a strong support for the main claim about the effect of government debt on the share of credit intermediated by banks.

Finally, I repeat the baseline analysis by replacing the share of bank credit in the VAR with investments to GDP and then with real GDP. That is, I compute the effects of a permanent increase in the debt-to-GDP ratio on investments-to-GDP ratio and real GDP using shocks identified with long-run restrictions. For investments, the impulse response function shows a drop on impact, which is however compensated by a later increase; the impulse responses are provided in [Appendix A](#). The long-run effect of a 1% increase in debt to GDP is negative (-0.136%) but not statistically significant (90% confidence interval: [-0.537%, 0.071%]). For GDP, the effect is negative both on impact and in the long run, and statistically significant. The long-run effect of a 1% increase in debt to GDP is -0.08% and the 90% confidence interval is [-0.237%, -0.003%].

Taken together, these results show that an increase in the long-run average target for the debt-to-GDP ratio produces a negative effect on GDP that acts mostly through the composition of investments between banks and non-banks, rather than the overall level of investments. It is possible that there are some effects on total investments too, but if that is the case, they are likely quantitatively less important and thus not picked up by the above analysis. The rest of the paper presents a model that rationalizes these findings using a framework that can be used to understand the effects of public liquidity supply and guide the policy analysis.

3 Model

This section presents a simple theoretical model that rationalizes the facts presented above, provides insights into the impact of public liquidity on the economy, and guides the policy analysis. In the model, financial intermediaries are characterized by two features that have been extensively used in the literature: they have a technological advantage over non-financial agents but are subject to moral hazard. A similar structure is common in several macro-finance frameworks such as [Brunnermeier and Sannikov \(2014\)](#), [Gertler and Kiyotaki \(2010\)](#), and [He and Krishnamurthy \(2013\)](#).¹⁰

¹⁰Some elements of the model are conceptually similar to the public liquidity analysis in [Holmström and Tirole \(1998\)](#), but the structure of the model builds on the simpler framework of [Benigno and Robatto \(2019\)](#). This also facilitates the policy analysis because [Benigno and Robatto \(2019\)](#) employs a representative agent, whereas [Holmström and Tirole \(1998\)](#) have two sets of agents. [Holmström and Tirole \(1998\)](#) have to take a stand on the objective function

3.1 Environment

There are two time periods ($t = 0, 1$), and time $t = 1$ is divided into two subperiods. There are three sets of players: households, financial intermediaries, and the government. At time $t = 1$, an aggregate shock realizes and affects the productivity of the investments made at $t = 0$. In particular, there are two possible states at $t = 1$: h (i.e., high) and l (i.e., low). State h realizes with probability $1 - \pi$ and state l with probability π . Depending on the conditions in the financial sector, a liquidity crisis can occur in state l .

The demand for liquid assets comes from households, which represent the non-financial sector, similar to several other related papers (e.g., Nagel, 2016; Stein, 2012). The model could be reformulated by assuming that liquidity is used by entrepreneurs, as in the classic model of Holmström and Tirole (1998). However, the results would be unchanged but would require a richer structure, clouding the logic of the results.

At time $t = 0$, households have some endowment that can be invested in productive projects, either directly by households themselves or indirectly by financial intermediaries. Financial intermediaries (i) have no resources and finance their investments by issuing defaultable debt (i.e., a state-contingent security); (ii) have access to better investment opportunities than households; and (iii) are plagued by a moral hazard problem. Absent moral hazard, welfare would be maximized if all investments were made by intermediaries. However, the moral hazard friction implies that intermediaries must be compensated with rents to make sure they do not misbehave. To earn sufficient rents, intermediaries pay a low return on the debt they issue, but households buy such debt anyway because of the liquidity it provides (i.e., intermediaries' debt represents the "private liquidity" in the economy).

A unit of investment made at $t = 0$ produces a payoff that depends on the realization of the state at $t = 1$ (i.e., h or l) and on whether the project is run by an intermediary or a household. If the project is run by an intermediary, the productivity at $t = 1$ is $A_h > 0$ in the high state and $A_l \geq 0$ in the low state. I impose the normalizations $(1 - \pi) A_h + \pi A_l = 1$ (i.e., the average productivity is normalized to one) and $A_l = 0$ (i.e., the payoff is zero in the low state).¹¹ If a project is run by a household, the productivity is only a fraction $1 - \phi$ of that of financial intermediaries. That is,

of the government and the Pareto weights assigned to the various agents, and note that altering the objective function can alter the optimal policy.

¹¹The assumption $A_l = 0$ implies $A_h = 1/(1 - \pi)$.

the productivity is $A_h(1 - \phi)$ in the high state and $A_l(1 - \phi) = 0$ in the low state, where the zero productivity in the low state follows from the normalization $A_l = 0$.

In the first subperiod of $t = 1$, households are subject to a liquidity constraint; that is, they must finance their consumption expenditure using intermediaries' debt (i.e., private liquidity) or government debt (i.e., public liquidity).¹² While government debt is risk-free and thus provides liquidity in all states, intermediaries' debt is fully defaulted on in the low state because intermediaries' projects do not produce any output in that contingency. As a result, in the low state, liquidity is effectively provided only by public debt.¹³

3.2 Households

Households enjoy utility from consumption at $t = 1$. Their utility is given by

$$(1 - \pi) [\log C_h + X_h] + \pi [\log C_l + X_l], \quad (2)$$

where C_h and C_l denote consumption in the first subperiod of $t = 1$, and X_h and X_l denote consumption in the second subperiod at $t = 1$.

Households have endowments at $t = 0$ and $t = 1$. At $t = 0$, they are endowed with an amount \bar{Y} of goods and an amount \bar{B} of government debt, which is interpreted as a Treasury security. At $t = 1$, they are endowed with goods \bar{Y}_h and \bar{Y}_l in the high and low state, respectively.¹⁴ The time-0 endowment, \bar{Y} , will only be invested in productive projects, either directly by households or by intermediaries.¹⁵

¹²The liquidity constraint is based on the implicit assumption that households cannot purchase goods at $t = 1$ using unsecured credit (i.e., they lack the commitment to repay unsecured debt). In addition, this constraint is equivalent to a formulation in which households' purchases are paid using collateralized debt, where the collateral is either government debt or intermediaries' debt. In this case, the assumption that households' direct investments cannot be used as collateral can be microfounded by extending the model to add a moral-hazard problem on the side of households. In particular, one can assume that a household can extract, at $t = 1$, private non-pledgeable benefits γ per unit of output. Assuming that $\gamma = 1 - \phi$ (i.e., the private benefits are equal to the output produced by the investments), one obtains that households' direct investments are subject to a 100% haircut and, thus, cannot be used as collateral. The analysis can be extended to the case in which $0 < \gamma < 1 - \phi$, in which households' direct investments would be partially collateralizable.

¹³One could extend the model by adding a positive payoff to intermediaries' projects in the low states, so that intermediaries could still provide some liquidity in the low state. However, this would complicate the exposition without altering the logic of the results.

¹⁴The results do not depend on the state-contingent nature of the time-1 endowment, and thus one can assume that $\bar{Y}_h = \bar{Y}_l$.

¹⁵All endowments of goods (i.e., \bar{Y} , \bar{Y}_h , and \bar{Y}_l) are assumed to be sufficiently large so that none of the results are

At $t = 0$, households choose investments K , holdings of intermediaries debt D , and holdings of government bonds B subject to the budget constraint

$$K + Q^D D + Q^B B \leq \bar{Y} + Q^B \bar{B}, \quad (3)$$

where Q^D is the price at which intermediaries issue their debt, and Q^B is the price of government debt. Similar to government debt, intermediaries' debt is modeled as a zero-coupon security with unitary face value, although subject to default as described next.¹⁶

In the first subperiod of $t = 1$, households choose consumption C_h and C_l . Consumption expenditures are subject to a liquidity constraint of the form

$$C_h \leq B + D, \quad (4)$$

$$C_l \leq B, \quad (5)$$

in the high and low state, respectively. That is, households can purchase C_h in the high state using government debt B and intermediaries' debt D as means of payment, whereas they can use only government debt in the low state. This is because intermediaries' investments have zero productivity in the low state, and thus intermediaries' debt is fully defaulted on in that state.

In the second subperiod of $t = 1$, consumption X_h and X_l is residually determined by all the resources that are left to each household:

$$X_h \leq \bar{Y}_h + \Pi_h + (B + D - C_h) + A_h(1 - \phi)K - T_h \quad (6)$$

$$X_l \leq \bar{Y}_l + \Pi_l + (B - C_l) - T_l. \quad (7)$$

In the high state, a household's resources are represented by its endowment \bar{Y}_h , the profits Π_h received from intermediaries, the liquidity $B + D - C_h$ that was not used in the first subperiod, and the output produced by the investments K , net of lump-sum taxes T_h that must be paid to the government. In the low state, the resources available are determined similarly, with the differences being that government bonds B are the only liquid assets, and investments K produce no output.

affected by the lack of resources.

¹⁶Formally, D is a state-contingent security with payoff equal to one in state h and zero in state l because D is fully defaulted on in the low state, as explained below.

Households maximize utility (2) subject to (3)-(7). Let μ_h and μ_l be the Lagrange multipliers of the liquidity constraints (4) and (5).

The first-order condition for the choice of government debt at $t = 0$ is

$$1 + (1 - \pi) \mu_h + \pi \mu_l = Q^B (1 - \pi) A_h (1 - \phi). \quad (8)$$

Equation (8) equates the marginal benefits of holding an additional unit of government debt B to its marginal cost. The benefits are represented by the payoff, which is one, plus the value of relaxing the liquidity constraint in the high state, μ_h , and in the low state, μ_l . The marginal cost is the payoff that can be obtained by investing in projects directly. Government debt costs Q^B , and if these resources were instead invested in projects, they would produce $A_h (1 - \phi)$ in the high state (i.e., with probability $1 - \pi$) and zero in the low state (i.e., with probability π).

The first-order condition for the choice of intermediaries' debt at $t = 0$ is

$$(1 - \pi) + (1 - \pi) \mu_h = Q^D (1 - \pi) A_h (1 - \phi). \quad (9)$$

This expression is similar to that in Equation (8), but the marginal benefits differ because intermediaries' debt D is repaid and provides liquidity only when it is not defaulted on (i.e., in the high state, that is, with probability $1 - \pi$). To clarify the household's choice of intermediaries' debt D , consider the spread $1 + S^D$ defined as

$$1 + S^D \equiv \frac{(1 - \pi) A_h (1 - \phi)}{(1 - \pi) \frac{1}{Q^D}}. \quad (10)$$

The right-hand side is the ratio of two gross returns: (i) the return that households earn by investing directly in projects, $(1 - \pi) A_h (1 - \phi)$, and (ii) the return earned on intermediaries' debt D , $(1 - \pi) / Q^D$. When the two returns are the same, the spread S^D is thus zero; when the return earned on intermediaries' debt is lower, the spread is positive. Using the definition of the spread S^D in (10), the first-order condition (9) can be expressed as

$$\mu_h = S^D.$$

When $S^D > 0$, intermediaries' debt pays a lower return in comparison to investing directly in

investments K . However, households want to purchase D anyway because it relaxes the liquidity constraint and thus provides a liquidity value. In particular, households purchase D up to the point at which their liquidity value — which is given by the Lagrange multiplier μ_h of the liquidity constraint in the high state — equals the spread S^D .

At $t = 1$, consumption choices are determined subject to the liquidity constraints (4) and (5). The optimality conditions are

$$\frac{1}{C_h} = 1 + \mu_h, \quad \frac{1}{C_l} = 1 + \mu_l, \quad (11)$$

where μ_h and μ_l are the Lagrange multipliers of the liquidity constraints (4) and (5), respectively, as described above. When the liquidity constraints (4) and (5) are not binding, their Lagrange multipliers are $\mu_h = \mu_l = 0$, and the marginal utilities of C_h and C_l are equalized to those of the second subperiod.

3.3 Financial intermediaries

Financial intermediaries operate in a fashion similar to [Gertler and Kiyotaki \(2010, 2015\)](#). That is, a fraction of households become bankers at $t = 0$ and operates financial intermediaries in the households' interest. Because households have linear utility at $t = 2$, bankers simply maximize the expected profits. Any realized profit is then returned to households at $t = 2$.

Intermediaries are plagued by a moral hazard problem. That is, they can extract private benefits from managing their projects at the expense of the resources that can be produced and are pledgeable to depositors. If intermediaries misbehave, they can extract, in the high state, an amount θA_h per unit invested, with $\theta < 1$ (recall that A_h is the output produced in the high state by a unit of investment made by an intermediary).¹⁷ The resources obtained by misbehaving are returned to the shareholders, and nothing is left for depositors.¹⁸ As a result, to ensure that intermediaries do not misbehave, they need to be compensated in the high state with a rent θA_h per unit of capital.

At time $t = 0$, an intermediary issues zero-coupon debt D at price Q^D to finance investments

¹⁷Because the productivity is zero in the low state anyway, intermediaries cannot extract any benefits by misbehaving in that state.

¹⁸This approach is an extreme version of the moral hazard problem introduced by [Holmström and Tirole \(1998\)](#).

K^I . Thus, the budget constraint is

$$K^I \leq Q^D D. \quad (12)$$

At $t = 1$, in the high state, the intermediary earns the output A_h per unit of the project and must reimburse its debt D . Thus, profits are given by

$$\Pi_h = A_h K^I - D. \quad (13)$$

Profits in the low state are zero, $\Pi_l = 0$, because the productivity of the investments is $A_l = 0$ in that state and the intermediary fully defaults on its debt.

The moral hazard friction described above implies that the intermediary's profits must be at least as large as the private benefits that the intermediary can extract by misbehaving:

$$\Pi_h \geq \theta A_h K^I. \quad (14)$$

Using $A_h = 1/(1 - \pi)$ (see [Section 3.1](#)), equations (12), (13), and (14) imply that intermediaries are willing to supply a positive amount of securities, $D > 0$, as long as

$$Q^D \geq \frac{1 - \pi}{1 - \theta}, \quad (15)$$

whereas they supply $D = 0$ if the price Q^D does not satisfy (15).

Without the moral hazard friction (i.e., if $\theta = 0$), condition (15) would simplify to $Q^D \geq 1 - \pi$. This is because issuing an additional unit of the debt security at price $Q^D = 1 - \pi$ allows intermediaries to invest an additional $1 - \pi$ units of resources, which produces $(1 - \pi) A_h = 1$ unit of output in the high state at $t = 1$. This unit of output suffices to repay the debt security but leaves the intermediary with no profits. If instead there is moral hazard (i.e., $\theta > 0$), the intermediary issues a security at a price $Q^D = \frac{1 - \pi}{1 - \theta} > 1 - \pi$. As a result, the intermediary produces more than one unit of output in the high state for each unit of debt issued and earns positive profits that provide an incentive to exert effort.

3.4 Government

The government collects lump-sum taxes T_h and T_l from households in the high and low state, respectively, to repay the initial debt \bar{B} . This section abstracts from policy interventions, which are later analyzed in [Section 3.9](#).

Because the government must repay the debt \bar{B} in both states, taxes T_h and T_l are the same:

$$T_h = T_l = \bar{B}. \quad (16)$$

In this paper, I focus on lump-sum non-distortionary taxation. Abstracting from distortionary taxation allows me to highlight a new channel that prevents the optimality of a large injection of public liquidity.¹⁹

3.5 Economy-wide accounting

To clarify the model, it is useful to clarify how resources are used. At $t = 0$, the endowment \bar{Y} is invested by households (i.e., K) and intermediaries (i.e., K^I):

$$K + K^I \leq \bar{Y}.$$

At $t = 1$, consumption in the two subperiods equals the households' endowments and the output produced by the investments K and K^I made at $t = 1$. That is,

$$C_h + X_h \leq \bar{Y}_h + A_h K^I + A_h (1 - \phi) (\bar{Y} - K^I), \quad (17)$$

$$C_l + X_l \leq \bar{Y}_l, \quad (18)$$

in the high and low state, respectively.

¹⁹Other papers such as [Greenwood, Hanson, and Stein \(2015\)](#) introduce distortionary taxation, which also generates a cost of a too-large supply of public liquidity. Thus, the distortionary taxation channel is complementary to the one derived in this paper.

3.6 Equilibrium definition

The equilibrium definition is standard. Given a level of Treasury debt \bar{B} , an equilibrium is a collection of prices of government debt and intermediaries' debt (i.e., Q^B and Q^D), intermediaries' debt and households' holding of government debt (i.e., D and B), time-zero investments by intermediaries and households (i.e., K^I and K), intermediaries' profits in the high and low state (i.e., Π_h and Π_l), and time-one consumptions in the high and low state (i.e., C_h, X_h, C_l , and X_l) such that households maximize their utility (2) subject to (3)-(7), intermediaries maximize profits (13) subject to their budget constraint (12) and the moral hazard constraint (14), the government budget constraint (16) holds, and the time-zero bond market clears, $B = \bar{B}$.

3.7 Moral hazard and technological advantage

The amount of resources that are intermediated by intermediaries depends on the strength of the moral hazard friction (i.e., θ) and of the technological advantage of intermediaries (i.e., ϕ). This section imposes a parameter restriction to obtain an equilibrium in which a fraction of the investments is made by intermediaries and the remaining fraction is made by non-financial agents (i.e., households), as is the case in practice.

I assume that the private benefits that can be extracted by financial intermediaries are large in comparison to the technological advantage that such intermediaries have with respect to households. More precisely,

$$\theta > \phi, \tag{19}$$

where θ measures the degree of the moral hazard problem and ϕ measures the gap between the technologies available to intermediaries and households, as described in the previous sections. Under (19), the moral hazard friction is severe enough that households prefer to manage some of the time-zero investments directly, even if their technology is worse than that of the financial intermediaries. If, instead, the technological advantage of intermediaries were greater than the moral hazard friction (i.e., $\theta \leq \phi$), it would be optimal for households to turn all their time-zero endowment \bar{Y} over to intermediaries because the technological advantage of the financial sector would more than offset the rent extracted to prevent the moral hazard problem. The observation that, in practice, only a fraction of the credit is intermediated by banks motivates the inequality in

(19).

To simplify the exposition and derivation, the rest of the paper restricts attention to a special case in which the moral hazard parameter θ is only slightly greater than the technological gap ϕ . Formally, I will consider the limiting case in which, even if (19) holds, the difference between θ and ϕ is arbitrary small, that is, $\theta \rightarrow \phi$. Appendix B shows that, absent the restriction $\theta \rightarrow \phi$, I need to account for the possibility that financial intermediaries do not operate if the moral hazard problem is very severe (i.e., if $\theta - \phi$ is sufficiently large).²⁰ In that case, the rents that intermediaries need to extract are so large that households decide do purchase zero intermediaries' debt. Because that case is not empirically relevant, the full analyses is deferred to the appendix.

3.8 Equilibrium

I now present the equilibrium under the parameter restrictions on moral hazard and technology described in Section 3.7. That is, the moral hazard parameter θ is greater than the technological advantage of intermediaries ϕ , but the two parameters are close to each other, that is, $\theta \rightarrow \phi$. Appendix B presents the equilibrium in the full model that does not include the restriction $\theta \rightarrow \phi$.

In general, given arbitrary values of θ and ϕ with $\theta > \phi$, households' choice of intermediaries' debt D trades off the benefits associated with the liquidity of this security with the costs associated with the rents extracted by intermediaries. The latter takes the form of a lower return on D — namely, the spread S^D defined in (10) between the return on D and the return that households earn by investing directly in projects. The implication of focusing on the case $\theta \rightarrow \phi$ is that intermediaries' rents associated with the moral hazard are (almost) exactly offset by the technological advantage of the financial sector. As a result, the spread on intermediaries' debt becomes arbitrarily close to zero.²¹

With a close-to-zero spread, the liquid security D pays essentially the same return as direct

²⁰More precisely, the appendix shows that the $\theta - \phi$ should be sufficiently large in comparison to the supply of Treasury debt \bar{B} .

²¹From a technical point of view, I focus on the limiting economy in which $\theta \rightarrow \phi$ and $\theta > \phi$, rather than the case $\theta = \phi$, because of a discontinuity in the limit. As long as $\theta > \phi$, the moral hazard premium is slightly more severe than the intermediaries' technological advantage. Thus, intermediaries must earn rents to avoid misbehavior, and the spread S^D must be positive — albeit very small. If, instead, $\theta = \phi$, the spread S^D is exactly zero because the intermediaries' rents are fully covered by the intermediaries' technological advantage. In this case, the amount of resources intermediated by the financial sector is not uniquely defined because households are indifferent between investing directly in projects or buying intermediaries' debt.

investments (which are illiquid). As a result, households find it optimal to hold the amount of D that is required to finance a level of consumption C_h that is arbitrarily close to the first-best level $C_h = 1$.

In the low state, liquidity is not sufficient to sustain the first-best level of consumption because D is defaulted on. Thus, consumption C_l is financed only by government debt: $C_l = \bar{B}$. If the government does not flood the economy with public liquidity (i.e., if $\bar{B} < 1$), the liquidity constraint (5) is binding, and, thus, the allocation of consumption is inefficient in comparison to the first best.

Even though the spread is (close to) zero in equilibrium, intermediaries earn positive profits: $\Pi_h > 0$. This is because the moral hazard parameter θ and the technological parameter ϕ are strictly positive, even if the gap between the two is arbitrarily small. As a result, intermediaries earn positive profits thanks to the better technology to which they have access. In turn, these profits provide the incentive to intermediaries to behave properly and not extract private benefits.

The next proposition summarizes the equilibrium under moderate moral hazard and focuses on the case in which the government does not flood the economy with public liquidity, that is, $\bar{B} < 1$. The policy analysis of Section 3.9 does not include this restriction and considers all possible levels of public liquidity. The proofs are provided in the Appendix.

Proposition 1. *Assume $\bar{B} < 1$. In the limiting economy in which $\theta \rightarrow \phi$ but $\theta > \phi$, the equilibrium is given by:*

- *Investments by intermediaries, K^I , and households, K , at $t = 0$:*

$$K^I = \frac{(1 - \bar{B})(1 - \pi)}{1 - \phi}, \quad K = \bar{Y} - \frac{(1 - \bar{B})(1 - \pi)}{1 - \phi};$$

- *Intermediaries' debt $D = 1 - \bar{B}$ and profits in the high state $\Pi_h = \phi \frac{1 - \bar{B}}{1 - \phi}$;*
- *Price of government debt, Q^B , and of intermediaries' debt, Q^D :*

$$Q^B = \frac{1}{1 - \phi} \left[(1 - \pi) + \pi \frac{1}{\bar{B}} \right], \quad Q^D = \frac{1 - \pi}{1 - \phi},$$

- *Spread: $S^D = 0$;*

- *Households' consumption in the first subperiod of $t = 1$:*

$$C_h = 1, \quad C_l = \bar{B} < 1;$$

- *Households' consumption in the second subperiod of $t = 1$:*

$$X_h = \bar{Y}_h + \bar{Y} \frac{1 - \phi}{1 - \pi} - 1 + \frac{\phi(1 - \bar{B})}{1 - \phi}, \quad X_l = \bar{Y}_l - \bar{B}.$$

3.9 Policy analysis: optimal supply of Treasury securities

This section analyzes the optimal supply of public liquidity by the government. On the one hand, increasing the supply of public liquidity at $t = 0$ allows households to have more liquidity in the low state. Absent any other effect, the optimal policy would thus be to flood the economy with as much liquidity as needed to satiate the demand from households, as in [Holmström and Tirole \(1998\)](#). On the other hand, however, the same increase in public liquidity crowds out financial intermediation. This second effect is detrimental for welfare because financial intermediaries have access to better investment opportunities than households do. The optimal policy balances these two forces.

In the context of the model, I assume that the government chooses the initial amount of government debt \bar{B} that households are endowed with. Alternatively, one can keep \bar{B} as exogenous and assume that the government issues new debt at $t = 0$, but the results would be unchanged. In practice, this policy experiment could be interpreted as the choice of a long-run average level of public debt, whereas possible deviations from the target in booms and recessions are not studied here and are left for future research. Because, in the model, the government implements its policy by choosing households' endowment of bonds \bar{B} at $t = 0$, the equilibrium for any given \bar{B} is just given by [Proposition 1](#). The only difference is that \bar{B} must now be interpreted as a policy variable rather than as an exogenous endowment. The next proposition characterizes the optimal policy.

Proposition 2. *Consider the limiting economy in which $\theta \rightarrow \phi$ but $\theta > \phi$. When the government chooses the initial level \bar{B} of government debt held by households, the optimal policy is*

$$(\bar{B})^* = \frac{\pi(1 - \phi)}{\pi(1 - \phi) + (1 - \pi)\phi} < 1. \quad (20)$$

It is useful to contrast [Proposition 2](#) with that of a model in which intermediaries have no technological advantage with respect to households (i.e., $\phi = 0$). The results in this case follow as a corollary of [Propositions 1 and 2](#).

Corollary 3. *Assume $\phi = 0$ and consider the limiting economy in which $\theta > \phi$ but $\theta \rightarrow \phi$. When the government chooses the initial level \bar{B} of government debt held by households, the optimal policy is $(\bar{B})^* = 1$. Under the optimal policy, intermediaries do not operate: $D = 0$ and $K^I = 0$.*

The case $\phi = 0$ produces a result similar to that of [Holmström and Tirole \(1998\)](#). Effectively, the government floods the economy with liquidity. Financial intermediation is shut down, but this outcome does not affect welfare because intermediaries are no better than households at making investments under the assumption $\phi = 0$. However, the case $\phi = 0$ implies that an increase in \bar{B} does not reduce output, and this is at odds with the evidence discussed in [Section 2](#), which shows a reduction in GDP triggered by an increase in Treasury debt. In contrast, the case $\phi > 0$ produces a reduction in output at $t = 1$ when \bar{B} goes up.²²

4 Public liquidity as central bank reserves

The baseline model rationalizes the fact that a higher supply of Treasury securities crowds out the private liquidity issued by financial intermediaries (i.e., their debt D) and the resources intermediated by the financial sector. However, a key component of public liquidity after the 2008 financial crisis is central bank reserves. (By “reserves,” I refer to interest-bearing reserves held above the minimum reserve requirement, if any.) This section shows that the same disintermediation arises if the central bank supplies public liquidity in the form of reserves.

To study central bank reserves and quantitative easing, I introduce a few additional elements into the model. First, there is a central bank that issues reserves that can be held only by banks. Because reserves are risk-free, this will allow banks to issue not only a risky security D but also a risk-free one, denoted by S . The central bank issues reserves to finance the purchase of government securities, similar to the quantitative easing programs implemented by many central banks, including the Federal Reserve, the Bank of Japan, and the European Central Bank. Second, in the model, government bonds provide less liquidity to households than deposits. This assumption is

²²Formally, $\partial \mathbb{E} [A (K^I + (1 - \phi) K)] / \partial \bar{B} |_{\phi=0} = 0$ and $\partial \mathbb{E} [A (K^I + (1 - \phi) K)] / \partial \bar{B} |_{\phi>0} < 0$.

in line with the liquidity properties of bank deposits and government securities in practice, even if the latter are held in the form of money market mutual fund shares. Indeed, in practice, the private non-financial sector holds most of its liquidity in bank deposits, despite this instrument typically having a lower return than money market funds and direct holdings of Treasuries. This fact suggests that bank deposits have a liquidity advantage that compensates for their lower return. This is true not just for households but also for firms, if one looks at all firms as opposed to just public firms. For instance, at the end of 2018, US firms held about \$3.5 trillion in cash-like assets, and about three-quarters of these assets were in some form of bank deposits. Only about \$0.9 trillion were invested in non-bank deposits: \$0.15 trillion in government securities (i.e., Treasuries and munis), \$0.6 trillion in money market funds, \$0.15 in commercial paper, and \$0.03 in reverse repos.²³

Sections 4.1-4.5 focus on the case in which central bank reserves can be held only by intermediaries and thus study quantitative easing. Section 4.6 studies central bank digital currency in the form of reserves that can be held by households.

4.1 Households

Households allocate the endowment of goods \bar{Y} and bonds \bar{B} between investments K , holdings of risky intermediaries' debt D and safe intermediaries debt S , and holdings of government bonds B . The budget constraint is now

$$K + Q^D D + Q^S S + Q^B B \leq \bar{Y} + Q^B \bar{B}, \quad (21)$$

where Q^D , Q^S , and Q^B are the price of risky intermediaries' debt, safe intermediaries' debt, and government bonds.

The liquidity constraints are now given by

$$C_h \leq (1 - \chi) B + S + D \quad (22)$$

$$C_l \leq (1 - \chi) B + S, \quad (23)$$

where $\chi \in [0, 1]$ captures the fact that Treasury securities provide fewer liquidity services than

²³Data are from the Flow of Funds and refer to the entire non-financial business sector.

bank deposits, as discussed above.

In the second subperiod of $t = 1$, consumption X_h and X_l are given by

$$X_h \leq \bar{Y}_h + \Pi_h + (B + S + D - C_h) + A_h(1 - \phi)K - T_h \quad (24)$$

$$X_l \leq \bar{Y}_l + \Pi_l + (B + S - C_l) - T_l. \quad (25)$$

These equations imply that the fraction χ of government bonds that do not provide liquidity in the first subperiod can be used to finance consumption of X_h and X_l .

The first-order conditions for B , D , and S are

$$1 + (1 - \pi)(1 - \chi)\mu_h + \pi(1 - \chi)\mu_l = Q^B(1 - \pi)A_h(1 - \phi), \quad (26)$$

$$1 + (1 - \pi)\mu_h + \pi\mu_l = Q^S(1 - \pi)A_h(1 - \phi), \quad (27)$$

$$(1 - \pi) + (1 - \pi)\mu_h = Q^D(1 - \pi)A_h(1 - \phi), \quad (28)$$

respectively, where μ_h and μ_l are the Lagrange multipliers of the liquidity constraints.

4.2 Financial intermediaries

Financial intermediaries can now issue not only risky assets but also risk-free assets S . This is possible because intermediaries can now invest in central bank reserves, which are safe and, thus, can back the issuance of the safe securities S .²⁴ In addition, one could relax the assumption that $A_l = 0$ and obtain some backing for risk-free assets by imposing $A_l > 0$, even if the central bank does not issue any reserves. However, for simplicity, I maintain the assumption that $A_l = 0$.

Intermediaries can, in principle, invest in government bonds. However, I conjecture that they decide to hold no bonds and later verify that this is the case as long as the haircut χ is sufficiently small. Thus, the budget constraint is

$$K^I + Q^R R^I \leq Q^D D + Q^S S, \quad (29)$$

²⁴Banks could, in principle, issue securities with a richer risk profile, when they hold some risky investments and some risk-free reserves. The approach I use, in which a bank tranches its payoff to issue risk-free debt S and risky debt D that is fully defaulted on in the low state, is without loss of generality if partially-defaulted securities provide liquidity at $t = 1$, and optimal if partially defaulted securities are not liquid.

where R^I are central bank reserves, which are modeled as zero-coupon securities, and Q^R is the price of such reserves.

Profits in the high and low states are

$$\Pi_h = A_h K^I + R^I - D - S, \quad (30)$$

$$\Pi_l = R^I - S, \quad (31)$$

respectively.

The moral hazard friction is unchanged and implies

$$\Pi_h \geq \theta A_h K^I, \quad (32)$$

$$\Pi_l \geq \theta R^I. \quad (33)$$

4.3 Government: central bank and Treasury

The central bank can purchase an amount B^{CB} of government debt and issue reserves R . To keep the notation in line with the rest of the model, I model reserves as zero-coupon securities with price Q^R at $t = 0$ and a unitary payoff at $t = 1$. The budget constraint of the central bank is

$$Q^B B^{CB} \leq Q^R R. \quad (34)$$

The Treasury operates the same way as in the baseline model, that is, by collecting lump-sum taxes T_h and T_l to repay the initial debt \bar{B} ; see [Equation \(16\)](#).

4.4 Equilibrium

The equilibrium concept is the same as described in [Section 3.6](#), with a few differences. First, the amount B^{CB} of securities purchased by the central bank is announced at the beginning of $t = 0$, before the time-zero market opens. Because some government securities are now purchased by the central bank, the respective market-clearing condition is now

$$\bar{B} = B + B^{CB}, \quad (35)$$

and recall that B and B^{CB} are the amounts of Treasury debt held by households and by the central bank, respectively. Second, the equilibrium definition includes the price and quantities of intermediaries' risk-free debt, Q^S and S , and of central bank reserves, Q^R and R , together with the market clearing condition for reserves $R^I = R$.

Under no central bank intervention, the equilibrium is very similar to that of the baseline model, with the difference driven by the lower liquidity of government bonds for households. Similar to **Proposition 1**, I maintain the focus on the case in which the moral hazard parameter θ and the technological advantage ϕ satisfy $\theta > \phi$ and $\theta \rightarrow \phi$, and the Treasury does not flood the economy with public liquidity, which is now formalized as $\bar{B} < 1/(1 - \chi)$.

Proposition 4. *Assume $\bar{B} < 1/(1 - \chi)$ and $B^{CB} = 0$. In the limiting economy in which $\theta \rightarrow \phi$ but $\theta > \phi$, the equilibrium is given by:*

- *Investments by intermediaries, K^I , and households, K , at $t = 0$:*

$$K^I = \frac{[1 - \bar{B}(1 - \chi)](1 - \pi)}{1 - \phi}, \quad K = \bar{Y} - \frac{[1 - \bar{B}(1 - \chi)](1 - \pi)}{1 - \phi};$$

- *Intermediaries' debt $D = 1 - \bar{B}(1 - \chi)$ and profits in the high state $\Pi_h = \phi \frac{1 - \bar{B}(1 - \chi)}{1 - \phi}$;*
- *Price of government debt, Q^B , and of intermediaries' debt, Q^D :*

$$Q^B = \frac{1}{1 - \phi} \left[(1 - \pi) + \frac{\pi(1 + \bar{B}\chi)}{\bar{B}} \right], \quad Q^D = \frac{1 - \pi}{1 - \phi},$$

and price and quantities of risk-free intermediaries' debt $Q^S = \frac{\pi + \bar{B}(1 - \pi)(1 - \chi)}{\bar{B}(1 - \phi)(1 - \chi)}$ and $S = 0$;

- *Central bank reserves $R = 0$ and price $Q^R = 1 - \pi \left(1 - \frac{1}{\bar{B}(1 - \chi)}\right)$;*
- *Spread: $S^D = 0$;*
- *Households' consumption in the first subperiod of $t = 1$:*

$$C_h = 1, \quad C_l = \bar{B}(1 - \chi) < 1;$$

- *Households' consumption in the second subperiod of $t = 1$:*

$$X_h = \bar{Y}_h + \bar{Y} \frac{1 - \phi}{1 - \pi} - 1 + \frac{\phi[1 - \bar{B}(1 - \chi)]}{1 - \phi}, \quad X_l = \bar{Y}_l - \bar{B}(1 - \chi).$$

4.5 Central bank reserves and optimal quantitative easing

While it is possible to solve for the equilibrium in closed form for a positive size of the quantitative easing policy B^{CB} , the equations are rather complicated and do not offer much intuition. Thus, I focus on showing that the optimal size of the quantitative easing policy is not too large and then clarify this result.

Proposition 5. (*Optimal quantitative easing*) *If the supply of government debt \bar{B} is sufficiently close to one, the optimal size of the quantitative easing policy B^{CB} is $(B^{CB})^* < \bar{B}$.*

The quantitative easing policy works as follows. Quantitative easing triggers a “liquidity transformation” of partially liquid government securities into fully liquid bank deposits S . More precisely, the central bank “transforms” partially-liquid B^{CB} into reserves R , which are fully illiquid for households and can be held only by banks. Then, banks “transform” reserves R into fully liquid risk-free deposits S . This process increases the effective liquidity that is available to households in equilibrium. However, as in the baseline model, this higher liquidity implies that households reduce their (risky) deposits D at banks, thereby reducing the resources that banks have available for investments. The optimal policy balances the benefits of the higher liquidity with the costs of the disintermediation, so that the optimal size of quantitative easing is not too large to avoid excessive disintermediation.

4.6 Central bank digital currency

Over the last few years, several central banks have discussed the possibility of central bank digital currency (CBDC), in the sense of letting not only banks but also the public deposit at the central bank. For instance, [Bordo and Levin \(2017\)](#) suggest that this could be implemented “via accounts held directly at the central bank” or “via specially designated accounts at supervised commercial banks, which would hold the corresponding amount of funds in segregated reserve accounts at the central bank.”

Introducing central bank digital currency in the model yields two main results. First, a central bank that issues central bank digital currency can replicate any outcome that is obtained with quantitative easing (QE), in terms of both investments, consumption, and welfare. Hence, central bank digital currency in this model does not offer any advantage with respect to quantitative easing.

However, the second result shows that there are crucial differences with QE when it comes to the implementation. In particular, the interest rate on reserves held by banks under QE must be higher than the interest rate on digital currency. This is because the higher interest on reserves held by banks compensates them for the rents that they extract to offset the moral hazard friction. Because of the different interest rates, the amount of reserves offered to banks will also differ from the amount of digital currency. However, the former effect driven by interest rates is exactly offset by the second one due to quantities, so that the overall dollar value of Treasury securities that are purchased by the central bank is the same under the two policies.

The model with central bank digital currency is very similar to that with the central bank that implements quantitative easing, with a few differences. Households can now invest in reserves R (i.e., the digital currency), which are sold by the central bank at price Q^R , and use them to purchase consumption in the first subperiod of $t = 1$. Thus, the budget constraint at $t = 0$ is

$$K + Q^D D + Q^R R + Q^B B \leq \bar{Y} + Q^B \bar{B},$$

and the liquidity constraints at $t = 1$ are

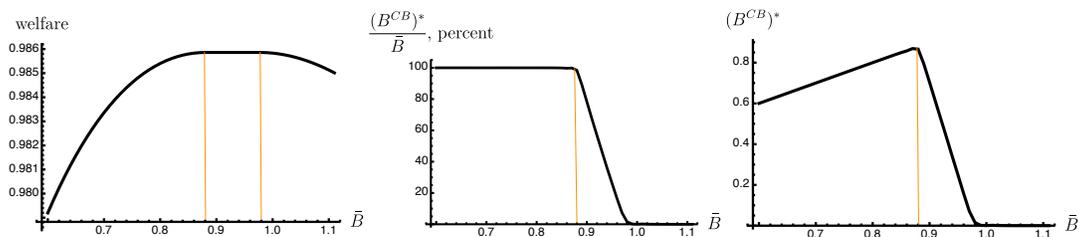
$$C_h \leq (1 - \chi) B + R + D,$$

$$C_l \leq (1 - \chi) B + R.$$

Because banks do not invest in reserves, they only issue risky debt D , similar to the version of the model with no central bank. Finally, the budget constraint of the central bank is the same as in the model with quantitative easing. The main results are summarized by the next proposition.

Proposition 6. *(Central bank digital currency) Assume $\theta > \phi$ and $\theta \rightarrow \phi$. Any allocation that can be achieved with quantitative easing can also be achieved with central bank digital currency, and vice versa. However, the price and amount of reserves are different under the two policies. In particular, $R^{qe} = (1 - \theta) R^{dc}$ (where R^{qe} and R^{dc} are the reserves issued under quantitative easing and digital currency), and $(Q^R)^{qe} < (Q^R)^{dc}$.*

Figure 3: Joint Treasury and central bank policy, numerical example



All figures are produced under the assumption that, for any level of \bar{B} , the central bank chooses the size of the quantitative easing policy (i.e., B^{CB}) to maximize welfare. Left panel: welfare as a function of Treasury debt \bar{B} . Middle panel: percentage of Treasury debt purchased by the central bank under the optimal policy. Right panel: level of Treasury debt purchased by the central bank under the optimal policy. Parameter values: $\phi = 0.015$; $\bar{Y} = 1$; $\pi = 0.1$; $\chi = 0.1$; $\bar{Y}_h = 1$; $\bar{Y}_l = 1$; $\theta \rightarrow \phi$.

5 Joint Treasury and central bank policies

This section uses the model to provide some (preliminary) insights into the optimal joint policy of the Treasury and central bank. In particular, I consider a central bank that chooses the optimal size of the quantitative easing policy (i.e., the optimal B^{CB}) and study how this choice varies with the level of Treasury debt \bar{B} . Because of the equivalence between quantitative easing and central bank digital currency shown in [Section 4.6](#), the results derived here are identical if the central bank issues digital currency as opposed to reserves created with quantitative easing.

Using numerical examples, I provide two main results. First, there is a range of Treasury debt levels that maximizes welfare (as opposed to a unique value), under the assumption that the central bank chooses its policy optimally. This result is represented in the left panel of [Figure 3](#), which plots welfare as a function of the level of Treasury debt \bar{B} . Second, the size of the central bank intervention is non-monotonic in the level of Treasury debt \bar{B} , as depicted in the right panel of [Figure 3](#). For low levels of \bar{B} , the liquidity provided by \bar{B} is too low, and it is optimal for the central bank to implement a very large quantitative easing policy and purchase all the Treasury securities that are not held by banks. In particular, in this model in which Treasury securities held by banks are essentially normalized to zero, the optimal quantitative easing policy when \bar{B} is low is to purchase the entire stock of Treasury debt, as shown in the middle panel of [Figure 3](#). As public debt grows (in the graphs, for $\bar{B} > 0.88$), the central bank purchases less and less

Treasury debt up until \bar{B} hits 0.98, at which point the central bank does not purchase anything. This is because, with $\bar{B} \in [0.88, 0.98]$, the Treasury and the central bank can jointly achieve the optimal level of “effective” public liquidity with a mix of a direct supply of Treasury securities and central bank reserves. In this range, the amount of central bank reserves is negatively related to the amount of Treasury debt. Finally, if $\bar{B} > 0.98$, the Treasury is already providing too much liquidity, which generates too much crowding out of banks’ investments. Hence, any additional intervention by the central bank that transforms and enhances the liquidity of Treasury debt — as discussed in [Section 4.5](#) — would further reduce the disintermediation created by the large supply of Treasury securities. As a result, it is optimal for the central bank to provide — in the model — no reserves. In practice, this would mean that the optimal policy when Treasury debt is high is to limit the amount of central bank reserves to the minimum amount that is necessary to have a well-functioning interbank market, system of payments, and repo market, which the model abstracts from. This also suggests the usefulness of any other central bank tool that helps to avoid distress in the interbank and repo markets while avoiding the need for large reserves, if Treasury debt is high. An example is provided by the standing fixed-rate repo facility, such as the one suggested by Joseph Gagnon and Brian Sack and discussed also by [Copeland, Duffie, and Yang \(2020\)](#).²⁵ This facility would essentially work like a discount window facility and would allow banks to borrow reserves as needed on days in which payments and interbank transactions are high (such as those in September 2019 in which the repo rate spiked), while keeping reserves low at other times.

6 Conclusions

I have presented an analysis of public liquidity that encompasses Treasury debt, central bank reserves and quantitative easing, and central bank digital currency. The empirical evidence suggests that an increase in the long-run average supply of Treasury debt to GDP reduces the fraction of credit to firms that is intermediated by banks, reduces GDP, but does not produce a statistically significant effect on investments. A simple theoretical model rationalizes these results in a framework that includes two assumptions commonly used in mainstream macro-finance models: (i) banks

²⁵Joseph E. Gagnon and Brian Sack, “Recent Market Turmoil Shows that the Fed Needs a More Resilient Monetary Policy Framework,” *Realtime Economic Issue Watch* (blog), PIIE—Peterson Institute for International Economics, September 26, 2019. Available at <https://www.piie.com/blogs/realtime-economic-issues-watch/recent-market-turmoil-shows-fed-needs-more-resilient-monetary>.

have access to better investment opportunities than those that can be financed directly by savers, and (ii) banks are subject to moral hazard, so that they extract some rents from their activity and finance only a fraction of the investments in the economy. The optimal supply of public liquidity trades off the beneficial, direct effect of public liquidity with the negative, indirect effect highlighted by the empirical analysis (i.e., more public liquidity reduces the size of the financial sector and the fraction of credit intermediated by banks). This trade-off holds under different definitions of public liquidity: Treasury debt, central bank quantitative easing, and central bank digital currency. A second result shows that any outcome that can be achieved with quantitative easing can also be obtained if the central bank issues digital currency, provided that the central bank pays a higher interest rate on reserves held by banks than on digital currency held by non-banks. Finally, when looking at the interaction between the Treasury and the central bank, the optimal size of the central bank intervention is non-monotonic in the stock of Treasury debt, and the optimal joint policy implies that there is an effective optimal level of public liquidity that can be achieved by different combinations of Treasury debt and central bank reserves supplied with quantitative easing or digital currency.

The theoretical analysis of this paper uses a simple qualitative model in order to highlight the similarities and complementarities between various forms of public liquidity. A possible step for future research is to use a richer model to quantify the forces presented here, although that approach likely requires analyzing Treasuries, quantitative easing, and central bank digital currency in separate frameworks.

References

- Begenau, Juliane and Tim Landvoigt. 2016. “Financial Regulation in a Quantitative Model of the Modern Banking System.” *Available at SSRN 2748206* .
- Benigno, Pierpaolo and Roberto Robatto. 2019. “Private money creation, liquidity crises, and government interventions.” *Journal of Monetary Economics* 106:42–58.
- Blanchard, Olivier and Danny Quah. 1989. “The Dynamic Effects of Aggregate Demand and Supply Disturbances.” *The American Economic Review* 79 (4):655–673.
- Bolton, Patrick and Haizhou Huang. 2017. “The capital structure of nations.” *Review of Finance* 22 (1):45–82.

- Bordo, Michael D and Andrew T Levin. 2017. “Central bank digital currency and the future of monetary policy.” .
- Brunnermeier, Markus K and Yuliy Sannikov. 2014. “A Macroeconomic Model with a Financial Sector.” *The American Economic Review* 104 (2):379–421.
- Copeland, Adam, Darrell Duffie, and Yilin (David) Yang. 2020. “Reserves Were Not So Ample After All: Evidence from the U.S. Treasury Repo Market.” .
- Dempsey, Kyle. 2020. “Macroprudential capital requirements with non-bank finance.” .
- Diamond, Douglas W. 1984. “Financial intermediation and delegated monitoring.” *The review of economic studies* 51 (3):393–414.
- Eggertsson, Gauti B. and Michael Woodford. 2003. “Zero bound on interest rates and optimal monetary policy.” *Brookings Papers on Economic Activity* 2003 (1):139–233.
- Fernández-Villaverde, Jesús, Daniel Sanches, Linda Schilling, and Harald Uhlig. 2020. “Central Bank Digital Currency: Central Banking For All?” .
- Fisher, Jonas DM. 2006. “The dynamic effects of neutral and investment-specific technology shocks.” *Journal of political Economy* 114 (3):413–451.
- Freixas, Xavier and J Rochet. 2008. “Microeconomics of Banking.”
- Gali, Jordi. 1999. “Technology, employment, and the business cycle: do technology shocks explain aggregate fluctuations?” *American economic review* 89 (1):249–271.
- Gertler, Mark and Peter Karadi. 2011. “A model of unconventional monetary policy.” *Journal of monetary Economics* 58 (1):17–34.
- . 2013. “QE 1 vs. 2 vs. 3...: A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool.” *International Journal of Central Banking* 9 (1):5–53.
- Gertler, Mark and Nobuhiro Kiyotaki. 2010. “Financial intermediation and credit policy in business cycle analysis.” *Handbook of monetary economics* 3 (3):547–599.
- . 2015. “Banking, Liquidity and Bank Runs in an Infinite Horizon Economy.” *American Economic Review* 105 (7):2011–43.
- Gorton, Gary. 2017. “The history and economics of safe assets.” *Annual Review of Economics* 9:547–586.
- Greenwood, Robin, Samuel G Hanson, and Jeremy C Stein. 2015. “A comparative-advantage approach to government debt maturity.” *The Journal of Finance* 70 (4):1683–1722.
- Hansen, Lars Peter, John C Heaton, and Nan Li. 2008. “Consumption strikes back? Measuring long-run risk.” *Journal of Political economy* 116 (2):260–302.
- He, Zhiguo and Arvind Krishnamurthy. 2013. “Intermediary asset pricing.” *American Economic Review* 103 (2):732–70.

- Holmström, Bengt and Jean Tirole. 1998. "Private and public supply of liquidity." *Journal of political Economy* 106 (1):1–40.
- Keister, Todd and Daniel R Sanches. 2020. "Should central banks issue digital currency?" .
- Krishnamurthy, Arvind and Annette Vissing-Jorgensen. 2015. "The impact of Treasury supply on financial sector lending and stability." *Journal of Financial Economics* 118 (3):571–600.
- Li, Wenhao. 2019a. "Liquidity Premium and the Substitution Between Bank Deposits and Treasuries." .
- . 2019b. "Public liquidity and financial crises." .
- Li, Ye. 2017. "Procyclical finance: The money view." .
- Liu, Yang, Lukas Schmid, and Amir Yaron. 2019. "The risks of safe assets." .
- Nagel, Stefan. 2016. "The liquidity premium of near-money assets." *The Quarterly Journal of Economics* :1927–1971.
- Ramey, Valerie A. 2011. "Identifying government spending shocks: It's all in the timing." *The Quarterly Journal of Economics* 126 (1):1–50.
- Ramey, Valerie A and Matthew D Shapiro. 1998. "Costly capital reallocation and the effects of government spending." In *Carnegie-Rochester conference series on public policy*, vol. 48. Elsevier, 145–194.
- Stein, Jeremy C. 2012. "Monetary policy as financial stability regulation." *The Quarterly Journal of Economics* 127 (1):57–95.
- Uhlig, Harald. 2010. "Some fiscal calculus." *American Economic Review* 100 (2):30–34.
- Vayanos, Dimitri and Jean-Luc Vila. 2009. "A preferred-habitat model of the term structure of interest rates." Tech. rep., National Bureau of Economic Research.
- Williamson, Stephen. 2019. "Central bank digital currency: Welfare and policy implications." .

Appendix

A Impulse-responses to a permanent shock to debt to GDP: robustness analyses

This appendix contains a few additional details about the effects of a permanent shock to the debt-to-GDP ratio analyzed in [Section 2](#). First, I show that the baseline specification delivers statistically significant results even when using 95% confidence intervals. Second, I present and discuss the impulse responses to a permanent shock to the debt-to-GDP ratio using the alternative specifications and identification assumptions described in [Table 1](#) to test the robustness of the baseline result. Third, I show the impulse responses to the investment-to-GDP ratio and to GDP.

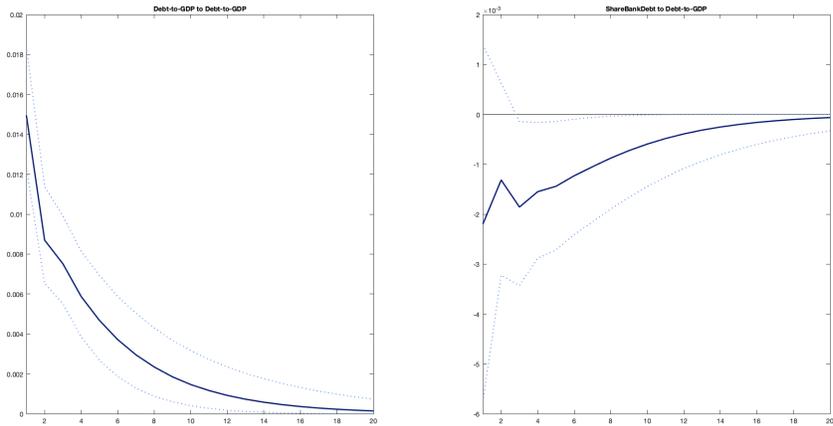
[Figure 4](#) shows the impulse responses of the baseline specification using 95% confidence bands. The results are essentially the same as in [Figure 2](#), which uses 90% confidence bands. The 95% confidence interval for the long-run effect is also similar and given by $[-0.811\%, -0.020\%]$, showing that the result is also significant at this stricter confidence level.

[Figure 5](#) shows the impulse responses obtained using long run restrictions and a VAR with quarterly data and four lags (top panel) and annual data and one lag (bottom panel). Overall, the results are qualitatively similar to the baseline presented in [Figure 2](#). The top panel shows that $\Delta \log (share)_t$ does not display any statistically significant movement on impact but it becomes negative after a few quarters. The impulse-response functions are not quite smooth, despite all the data is seasonally adjusted and a simple regression of both $\Delta \log (debt/GDP)_t$ and $\Delta \log (share)_t$ on quarterly seasonal dummies shows no statistically significant results. As a robustness, the bottom panel of [Figure 2](#) shows the impulse-response function obtained using data at yearly frequency. The pattern there is very similar but the impulse responses are much smoother, with no statistically significant effect of $\Delta \log (share)_t$ on impact (i.e., in period one in the plot) and a negative and significant effect after one year (i.e., in period two in the plot). Because the identification scheme is based on long-run restrictions and I am mostly interested in the long-run effects displayed in [Table 1](#), there is no downside in using data at yearly frequency if that allows for a more precise estimation of the coefficients of the VAR.

[Figure 6](#) shows the impulse responses obtained using a Cholesky identification. The top panel

Figure 4: Impulse responses of baseline specification, 95% confidence intervals

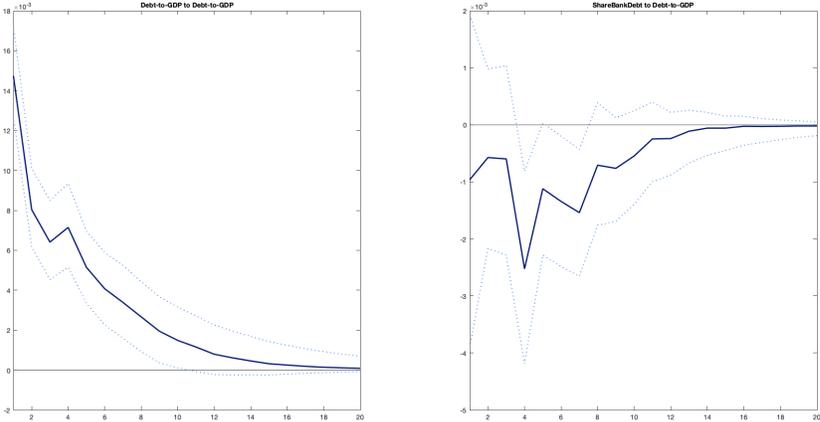
Long-run restrictions, quarterly data, VAR with 4 lags



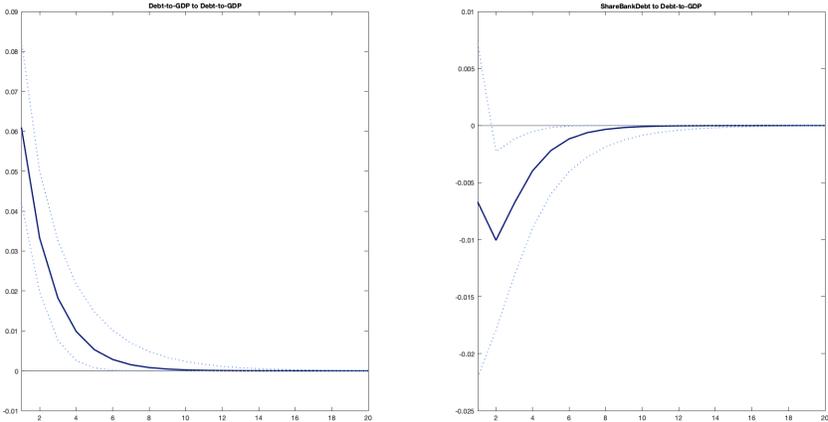
Impulse response to a shock that produces permanent effects on the debt-to-GDP ratio identified and estimated using the baseline approach (i.e., long-run restrictions, data at quarterly frequency, and a VAR with $p = 2$ lags). Horizontal axis: time periods (note that the shock takes place in period one). The solid line is the median impulse-response and the dotted lines denote the 95% confidence bands.

Figure 5: Impulse response to a shock that produces permanent effects on the debt-to-GDP ratio

Long-run restrictions, quarterly data, VAR with four lags



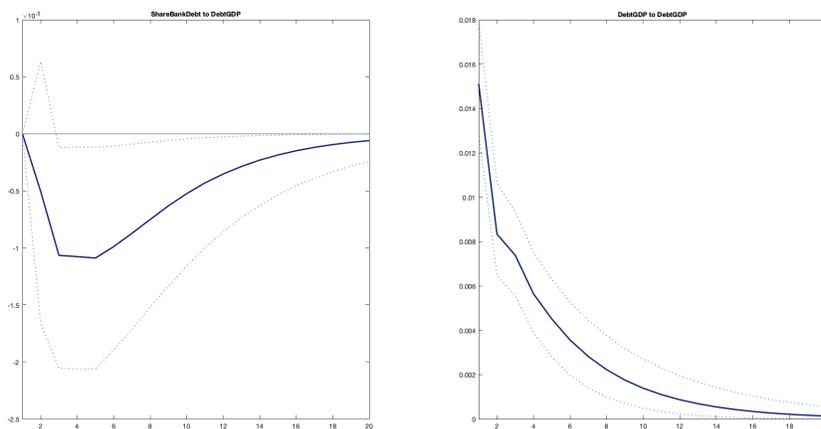
Long-run restrictions, yearly data, VAR with one lag



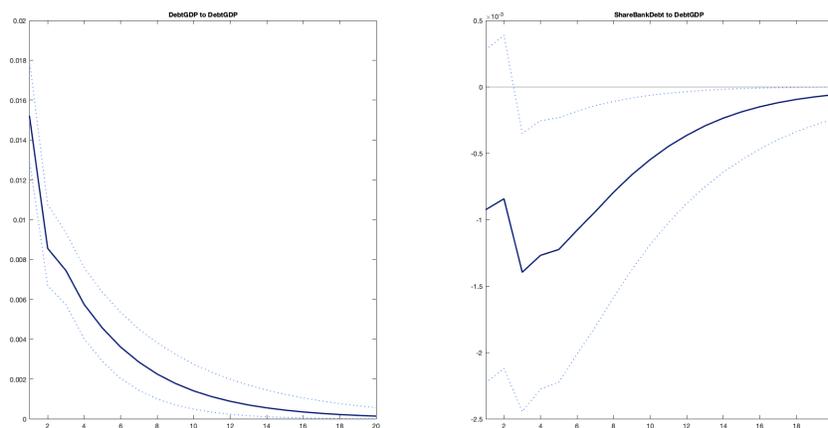
Horizontal axis: time periods (note that the shock takes place in period 1). The solid line is the median impulse-response, and the dotted lines denote the 90% confidence bands.

Figure 6: Impulse response to a shock that produces permanent effects on the debt-to-GDP ratio

Cholesky identification with $\Delta \log(\text{share})_t$ ordered first, quarterly data, VAR with two lags



Cholesky identification with $\Delta \log(\text{debt}/GDP)_t$ ordered first, quarterly data, VAR with two lags



Horizontal axis: time periods (note that the shock takes place in period 1). The solid line is the median impulse-response, and the dotted lines denote the 90% confidence bands.

shows the results when the first entry of the VAR is $\Delta \log (share)_t$, and the bottom panel shows the results when the first entry is $\Delta \log (debt/GDP)_t$. Overall, these impulse responses look very similar to those obtained using the long-run identification restrictions. I argue that, given the results obtained with long-run restrictions, it is not surprising that the Cholesky identification with $\Delta \log (share)_t$ ordered first delivers similar results. When $\Delta \log (share)_t$ is ordered first, the Cholesky identification restricts the response of $\Delta \log (share)_t$ to be zero on impact. This is consistent with the result obtained using the long-run restrictions, in which $\Delta \log (share)_t$ does not display any statistically significant effect on impact even though no restrictions are imposed on the behavior of $\Delta \log (share)_t$ right after the shock. That is, long-run restrictions and Cholesky with $\Delta \log (share)_t$ ordered first produce the same overall result — both in the short run and in the long run — with two drastically different identifications. In addition, when $\Delta \log (debt/GDP)_t$ is ordered first, the bottom panel of [Figure 6](#) shows that $\Delta \log (share)_t$ does not display any significant response in the first couple of quarters, even if the Cholesky identification does not impose any restriction on $\Delta \log (share)_t$ on impact, which is again similar to the results obtained using the other specifications.

Finally, [Figure 7](#) shows the impulse responses in the VARs in which $(share)_t$ is replaced with $(investments/GDP)_t$ and then with $(real\ GDP)_t$. Both investments and GDP drop on impact, but only the effect on GDP is significant in the long run as discussed in [Section 2](#).

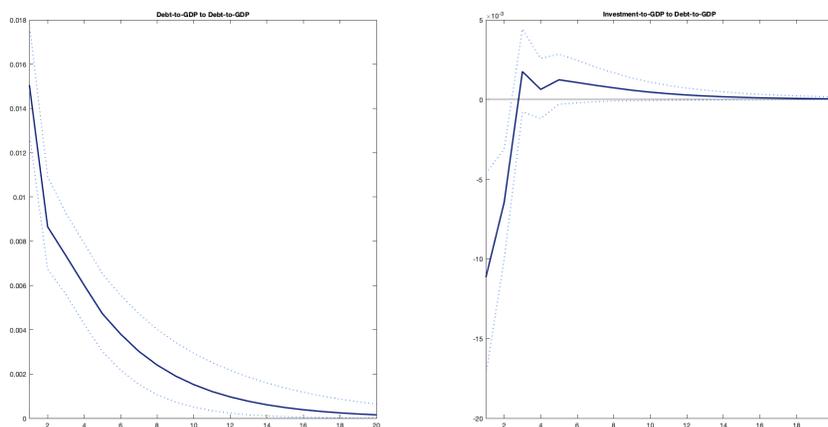
B Equilibrium with more severe moral hazard

This appendix analyzes the equilibrium in the general case with an arbitrary severe moral hazard problem; that is, [Equation \(19\)](#) holds and there are no further restrictions on θ and ϕ . The main trade-off described in [Section 3.8](#) still holds. That is, households trade off the liquidity of D (i.e., its ability to relax the constraint [\(4\)](#)) with the opportunity cost of holding D , namely, the spread S^D defined in [\(10\)](#), except now S^D is possibly large.

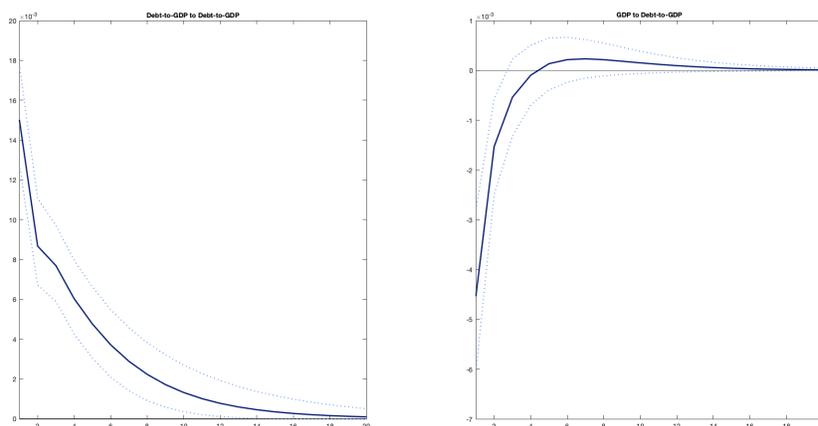
When no restriction is imposed on the moral hazard parameter θ other than [\(19\)](#), the spread S^D is positive in equilibrium and possibly large. As a result, households find it optimal to make choices so that their high-state consumption, C_h , is below the first-best level $C_h = 1$. The positive spread S^D on D is equivalent to a higher price Q^D at which intermediaries issue D , in comparison to the baseline version of the model. This higher price generates additional profits, in comparison

Figure 7: Impulse response to a shock that produces permanent effects on the debt-to-GDP ratio

VAR variables: $\Delta \log (debt/GDP)_t, \Delta \log (investments/GDP)_t$



VAR variables: $\Delta \log (debt/GDP)_t, \Delta \log (real\ GDP)_t$



Horizontal axis: time periods (note that the shock takes place in period 1). The solid line is the median impulse-response, and the dotted lines denote the 90% confidence bands. The VAR is estimated using quarterly data, two lags, and the identification is based on long-run restrictions.

to those earned by intermediaries of the economy of [Section 3.8](#). Such higher profits are necessary to provide incentives to intermediaries to behave well because the moral hazard problem is in general more severe here.

While the above paragraphs implicitly assume that intermediaries are active in equilibrium, the equilibrium is featured by a lack of financial intermediation for some levels of \bar{B} sufficiently close to one — and even if $\bar{B} < 1$. While this extreme case is not central to the analysis, it is helpful to discuss it to clarify the mechanics of the model. When \bar{B} is sufficiently close to one, there is enough liquidity in circulation so that households choose to finance a large amount of consumption C_h using only public debt, and hold $D = 0$. In this case, the value associated with relaxing the liquidity constraint in the high state, (4), is small, and so is its Lagrange multiplier μ_h . As a result, households' benefits from holding intermediaries' debt D are lower than the cost, represented by the spread defined in (10), confirming that households do prefer to choose $D = 0$. Alternatively, one could think that households would be willing to buy D only if the spread is small enough. However, with a small spread, intermediaries' profits are not high enough to discipline their behavior. Financial intermediation thus shuts down, as any attempt to intermediate resources would result in intermediaries' misbehavior and thus a default on their debt D even in the high state. The equilibrium with no financial intermediation arises even for values of government debt $\bar{B} > (1 - \theta) / (1 - \phi)$, where $(1 - \theta) / (1 - \phi) < 1$ because of (19). In addition, if $(1 - \theta) / (1 - \phi) < \bar{B} < 1$, not only will the financial sector shut down (i.e., $D = K^I = 0$), but also there will not be enough liquidity to satiate households' demand, and thus $C_h, C_l < 1$. If instead $\bar{B} = 1$, there is plenty of liquidity to finance $C_h = C_l = 1$, as discussed in [Section 3.9](#). The following proposition states the equilibrium, which is derived similarly to that of [Proposition 1](#); see the proof of [Proposition 1](#) in [Appendix C](#).

Proposition 7. *The equilibrium is:*

- *Investments by intermediaries at $t = 0$,*

$$K^I = \begin{cases} \frac{(1-\pi)[1-\theta-\bar{B}(1-\phi)]}{(1-\theta)(1-\phi)} & \text{if } \bar{B} < \frac{1-\theta}{1-\phi} \\ 0 & \text{if } \bar{B} \geq \frac{1-\theta}{1-\phi}, \end{cases}$$

and households, $K = \bar{Y} - K^I$;

- *Intermediaries' debt :*

$$D = \begin{cases} \frac{1-\theta-\bar{B}(1-\phi)}{1-\phi} & \text{if } \bar{B} < \frac{1-\theta}{1-\phi} \\ 0 & \text{if } \bar{B} \geq \frac{1-\theta}{1-\phi}; \end{cases}$$

- *Price of government debt, Q^B , and of intermediaries' debt, Q^D :*

$$Q^B = \begin{cases} (1-\pi) \frac{1}{1-\theta} + \pi \frac{1}{\bar{B}(1-\phi)} & \text{if } \bar{B} < \frac{1-\theta}{1-\phi} \\ \frac{1}{\bar{B}(1-\phi)} & \text{if } \bar{B} \geq \frac{1-\theta}{1-\phi}, \end{cases}$$

$$Q^D = \frac{1-\pi}{1-\phi};$$

- *Spread = $(\theta - \phi) / (1 - \theta)$;*
- *Consumption at $t = 1$, first subperiod:*

$$C_h = \begin{cases} \frac{1-\theta}{1-\phi} & \text{if } \bar{B} < \frac{1-\theta}{1-\phi} \\ \min \{1, \bar{B}\} & \text{if } \bar{B} \geq \frac{1-\theta}{1-\phi}, \end{cases}$$

and $C_l = \min \{1, \bar{B}\}$;

- *Consumption at $t = 2$, second subperiod:*

$$X_h = \begin{cases} \bar{Y}_h + \bar{Y} \frac{1-\phi}{1-\pi} - \frac{1-\theta-\phi}{1-\phi} - \bar{B} \frac{\phi}{1-\theta} & \text{if } \bar{B} < \frac{1-\theta}{1-\phi} \\ \bar{Y}_h + \bar{Y} \frac{1-\phi}{1-\pi} - \bar{B} & \text{if } \bar{B} \geq \frac{1-\theta}{1-\phi}, \end{cases}$$

$$X_l = \bar{Y}_l - \bar{B};$$

- *Intermediaries' profits in the high state:*

$$\Pi_h = \begin{cases} \theta \frac{1+\theta-\bar{B}(1-\phi)}{(1-\theta)(1-\phi)} & \bar{B} < \frac{1-\theta}{1-\phi} \\ 0 & \text{if } \bar{B} \geq \frac{1-\theta}{1-\phi}. \end{cases}$$

C Proofs

Proof of Proposition 1. The result follows directly from the definition of the equilibrium in [Section 3.6](#). That is, given \bar{B} , I solve for $Q^B, Q^D, D, B, K^I, K, \Pi_h, C_h, C_l, X_h, X_l$ using households' first-order conditions (8) and (9), with the Lagrange multipliers given by (11), the households' budget constraint at $t = 0$, (3), the households' liquidity constraints (4) and (5), and the time-1 resource constraints (17) and (18); the intermediaries' budget constraint (12), their profits (13), and the moral hazard constraint (14); and the market-clearing condition for government bonds $B = \bar{B}$. The spread can then be computed using (10). The results are then evaluated at $\theta \rightarrow \phi$. \square

Proof of Proposition 2. The optimal policy is computed by maximizing the utility function of households, (2), evaluated at the equilibrium of [Proposition 1](#), with respect to \bar{B} . That is, after plugging in the values of C_h, C_l, X_h, X_l from [Proposition 1](#) into (2), I take the first-order condition with respect to \bar{B} and solve for \bar{B} , which yields (20). \square

Proof of Proposition 4. Given Treasury debt \bar{B} and central bank purchases $B^{CB} = 0$, I solve for 17 variables: $Q^B, Q^D, Q^S, Q^R, D, S, B, R^I, R, K^I, K, \Pi_h, \Pi_l, C_h, C_l, X_h, X_l$. I use the following equations: the households' first-order conditions (26), (27), and (28), with the Lagrange multipliers given by (11), the households' budget constraint at $t = 0$, (21), the households' liquidity constraints (22) and (23), and the time-1 resource constraints (24) and (25); the intermediaries' budget constraint (29), their profits (30) and (31), and their moral-hazard constraints (32) and (33); the budget constraint of the central bank, (34); and the market-clearing condition for government bonds, (35), and the one for reserves, $R = R^I$. This is list of 16 equations; to obtain an additional one, I consider a financial intermediary that issues only risk-free debt S and invests in reserves R ; for this intermediary, the moral hazard constraint implies $(1 - \theta) Q^S = Q^R$. Thus, I solve these 17 equations in 17 variables and then evaluate them at $\theta \rightarrow \phi$. \square

Proof of Proposition 5. I first solve for the equilibrium (i.e., I solve for $Q^B, Q^D, Q^S, Q^R, D, S, B, R^I, R, K^I, K, \Pi_h, \Pi_l, C_h, C_l, X_h, X_l$) for a given \bar{B} and an arbitrary amount of central bank purchases $0 \leq B^{CB} \leq \bar{B}$, using the equations listed in the proof of [Proposition 4](#). I then evaluate

the utility function of households, (2), at the equilibrium values just computed, and differentiate it with respect to the policy variable B^{CB} . The result is evaluated at $B^{CB} \rightarrow \bar{B}$ and $\bar{B} \rightarrow 1$, yielding $-\frac{(1-\pi)\phi\chi}{(1-\phi)(1-\pi\chi)} < 0$. Thus, by continuity, welfare is decreasing in B^{CB} when B^{CB} is sufficiently close to \bar{B} , and \bar{B} is sufficiently close to one, implying the optimality of $(B^{CB})^* < 1$. \square

Proof of Proposition 6. Take an equilibrium obtained by the central bank with quantitative easing (QE), in which reserves are R^{qe} and their price is $(Q^R)^{qe}$. From the intermediaries' moral hazard constraint in the low state, (33), and using the definition of profits in (31), we have $(1 - \theta) R^I = S$ and thus, using the market clearing condition $R^I = R^{qe}$, $(1 - \theta) R^{qe} = S$. In addition, as shown in the proof of Proposition 4, we have $(1 - \theta) Q^S = (Q^R)^{qe}$. Now define $R^{dc} = S$ and $(Q^R)^{dc} = Q^S$. Then $(Q^R)^{dc}$ together with the prices Q^D and Q^B from the equilibrium with QE sustain the households' choices $D, B, K, C_h, C_l, X_h, X_l$ from the equilibrium with quantitative, together with R^{dc} . Intermediaries' choices of D and K^I from the equilibrium with QE is still optimal given prices. The budget constraint of the central bank holds with $Q^B B^{CB} = (Q^R)^{dc} R^{dc}$ because $(Q^R)^{dc} R^{dc} = (Q^R)^{qe} R$, using $(Q^R)^{dc} = Q^S$, $(1 - \theta) Q^S = (Q^R)^{qe}$, and $(1 - \theta) R^I = S$. Finally, the market clearing for Treasury debt is unchanged. Thus, if the original collection of prices and quantities under QE is an equilibrium, the same collection of prices and quantities with $(Q^R)^{dc} = Q^S$ and $R^{dc} = S$ is an equilibrium in the model with central bank digital currency. \square