Volatility, Valuation Ratios, and Bubbles: An Empirical Measure of Market Sentiment

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Two views of the equity premium

Based on valuation ratios (Campbell and Thompson, *RFS*, 2008) and on index option prices (Martin, *QJE*, 2017)
Very roughly, think of $D/P$ as revealing $\mathbb{E} R - \mathbb{E} G$, and interest rates and option prices as revealing $\mathbb{E} R$; then the gap between the two reveals $\mathbb{E} G$.

Specifically, today:

1. Relate dividend yields to expected returns and dividend growth using a twist on the Campbell–Shiller methodology
2. Introduce a bound on expected returns based on interest rates and option prices
3. Derive a bound on expected dividend growth by playing off (1) against (2)
Campbell–Shiller decomposition (1)

Notation: log dividend yield $dp_t = \log(D_t/P_t)$; log return $r_{t+1}$; log dividend growth $g_{t+1}$

- Campbell and Shiller (1988) famously showed that, up to a linearization,

$$dp_t = \frac{k}{1 - \rho} + \sum_{i=0}^{\infty} \rho^i \mathbb{E}_t \left[ r_{t+1+i} - g_{t+1+i} \right] \quad \text{where} \quad \rho \approx 0.97$$

- These are expected log returns, not expected returns
- Low expected log returns may be consistent with high expected returns if returns are volatile, right-skewed, or fat-tailed
- All three plausibly true in late 1990s, so the distinction between log returns and simple returns matters
Campbell–Shiller decomposition (2)

\[ d\rho_t = \frac{k}{1 - \rho} + \sum_{i=0}^{\infty} \rho^i (\rho_{t+1+i} - \rho_{t+1+i}) - \frac{\rho(1 - \rho)}{2} \sum_{i=0}^{\infty} \rho^i (dp_{t+1+i} - \bar{dp})^2 \]

- In the late ’90s \( dp_t \) was 2.2 sd below its mean (using CRSP data 1947–2017)
- Ignoring the second order term is equivalent to understating \( \mathbb{E}_t \rho_{t+1+i} - \rho_{t+1+i} \) by 14.5 pp for one year, 3.1 pp for five years, or 1.0 pp for 20 years
  - In long sample, 1871–2015, numbers are even bigger: 25.3 pp for one year, 5.5 pp for five years, 1.8 pp for 20 years, or 1.0 pp for ever
- Thus the CS decomposition may “cry bubble” too soon
An alternative approach (1)

- Campbell and Shiller loglinearize

\[ r_{t+1} - g_{t+1} = dp_t + \log \left( 1 + e^{-dp_{t+1}} \right) \]

- We start, instead, from

\[ r_{t+1} - g_{t+1} = y_t + \log \left( 1 - e^{-y_t} \right) - \log \left( 1 - e^{-y_{t+1}} \right) \]

where

\[ y_t = \log \left( 1 + \frac{D_t}{P_t} \right) \]

- \( y_t \), unlike \( dp_t \), is in natural units: if \( D_t / P_t = 2\% \) then \( y_t = 1.98\% \) whereas \( dp_t = -3.91 \)
Result

We have the loglinearization

\[ y_t = (1 - \rho) \sum_{i=0}^{\infty} \rho^i (r_{t+1+i} - g_{t+1+i}) \]

where \( \rho = e^{-\bar{y}} \approx 0.97 \).

On average, this relationship holds exactly—no linearization needed:

\[ \bar{y} = \bar{r} - \bar{g} \]
An alternative approach (3)

- We have already seen that the Campbell–Shiller approximation may lead one to conclude too quickly that the market is bubbly, as

\[ dp_t < -\frac{k}{1 - \rho} + \sum_{i=0}^{\infty} \rho^i \mathbb{E}_t (r_{t+1+i} - g_{t+1+i}) \]

- Our variant is a conservative diagnostic for bubbles. If \( y_t \) is far from its mean then

\[ y_t \geq (1 - \rho) \sum_{i=0}^{\infty} \rho^i \mathbb{E}_t (r_{t+1+i} - g_{t+1+i}) \]

  - *Far from its mean*: \( \mathbb{E}_t [(y_{t+i} - \bar{y})^2] \leq (y_t - \bar{y})^2 \) for all \( i \geq 0 \)
  - In AR(1) case, “far” means “one standard deviation”
Information in valuation ratios (1)

- If $y_t$ follows an AR(1) with autocorrelation $\phi_y$,

\[ E_t (r_{t+1} - g_{t+1}) = \text{constant} + \frac{1 - \rho \phi_y}{1 - \rho} y_t \]

- In the unit root case $\phi_y = 1$, we have $y_t = E_t (r_{t+1} - g_{t+1})$

- So we use $y_t$ to forecast $r_{t+1} - g_{t+1}$

- We estimate the regression freely, but results are almost identical if we estimate $\rho$ and $\phi_y$ from time series, then use the formula above

- AR(1) is not critical: key is that we have a forecast of $E_t y_{t+1}$. Will show AR($k$) later
### Information in valuation ratios (2)

<table>
<thead>
<tr>
<th>RHS$_t$</th>
<th>LHS$_{t+1}$</th>
<th>$\hat{a}_0$</th>
<th>s.e.</th>
<th>$\hat{a}_1$</th>
<th>s.e.</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>$r_{t+1} - g_{t+1}$</td>
<td>$-0.067$</td>
<td>[0.049]</td>
<td>$3.415$</td>
<td>[1.317]</td>
<td>7.73%</td>
</tr>
<tr>
<td></td>
<td>$r_{t+1}$</td>
<td>$-0.018$</td>
<td>[0.050]</td>
<td>$3.713$</td>
<td>[1.215]</td>
<td>10.51%</td>
</tr>
<tr>
<td></td>
<td>$-g_{t+1}$</td>
<td>$-0.049$</td>
<td>[0.028]</td>
<td>$-0.298$</td>
<td>[0.812]</td>
<td>0.32%</td>
</tr>
<tr>
<td>$dp_t$</td>
<td>$r_{t+1} - g_{t+1}$</td>
<td>$0.417$</td>
<td>[0.146]</td>
<td>$0.107$</td>
<td>[0.042]</td>
<td>7.58%</td>
</tr>
<tr>
<td></td>
<td>$r_{t+1}$</td>
<td>$0.500$</td>
<td>[0.138]</td>
<td>$0.114$</td>
<td>[0.041]</td>
<td>9.92%</td>
</tr>
<tr>
<td></td>
<td>$-g_{t+1}$</td>
<td>$-0.083$</td>
<td>[0.085]</td>
<td>$-0.007$</td>
<td>[0.024]</td>
<td>0.19%</td>
</tr>
</tbody>
</table>

**Table:** S&P 500, annual data, 1947–2017, dividends reinvested monthly at CRSP 30-day T-bill rate. Hansen–Hodrick standard errors.

- Relative importance of $r$ and $g$ is sample specific: $g$ more important in long sample.
- But coefficient estimates for $r - g$ are stable.
Information in options (1)

- We start from an identity

$$\mathbb{E}_t r_{t+1} = \frac{1}{R_{f,t+1}} \mathbb{E}^*_t (R_{t+1} r_{t+1}) - \text{cov}_t (M_{t+1} R_{t+1}, r_{t+1})$$

- $M_{t+1}$ is an SDF. Risk-neutral $\mathbb{E}^*_t$ satisfies

$$\frac{1}{R_{f,t+1}} \mathbb{E}^*_t (X_{t+1}) = \mathbb{E}_t (M_{t+1} X_{t+1})$$

- We assume that $\text{cov}_t (M_{t+1} R_{t+1}, r_{t+1}) \leq 0$
  
  - Similar to the negative correlation condition of Martin (2017)
  - Loosely, requires that investors are sufficiently risk-averse wrt $R_{t+1}$

- We then have

$$\mathbb{E}_t r_{t+1} \geq \frac{1}{R_{f,t+1}} \mathbb{E}^*_t (R_{t+1} r_{t+1})$$
Information in options (2)

\[ \mathbb{E}_t r_{t+1} \geq \frac{1}{R_{f,t+1}} \mathbb{E}_t^* (R_{t+1}r_{t+1}) \]

- Doesn’t require that the market is complete
- Doesn’t require any distributional assumptions (eg lognormality)
- Allows for the presence of constrained and/or irrational investors
- Holds with equality for a log investor who chooses to hold the market
- This investor’s perspective works well empirically for forecasting
  - the market as a whole (Martin, QJE, 2017)
  - individual stocks (Martin and Wagner, JF, 2019)
  - currencies (Kremens and Martin, AER, 2019)
Using the result of Breeden and Litzenberger (1978), we show

$$\frac{1}{R_{f,t+1}} \mathbb{E}_t^* (R_{t+1} r_{t+1}) = r_{f,t+1} + \frac{1}{P_t} \left\{ \int_0^{F_t} \frac{\text{put}_t(K)}{K} dK + \int_{F_t}^{\infty} \frac{\text{call}_t(K)}{K} dK \right\}$$

This gives the lower bound $\mathbb{E}_t r_{t+1} - r_{f,t+1} \geq LVIX_t$

Bootstrapped $p$-value for the mean of $r_{t+1} - r_{f,t+1} - LVIX_t$ being negative is 0.097
A sentiment index

- Putting the pieces together,

\[
\mathbb{E}_t g_{t+1} = \mathbb{E}_t (r_{t+1} - r_{f,t+1}) + r_{f,t+1} - \mathbb{E}_t (r_{t+1} - g_{t+1}) \\
\geq \underbrace{\text{LVIX}_t + r_{f,t+1} - \mathbb{E}_t (r_{t+1} - g_{t+1})}_{B_t}
\]

- We replace \(\mathbb{E}_t (r_{t+1} - g_{t+1})\) by the forecast based on \(y_t\):

\[
B_t = \text{LVIX}_t + r_{f,t+1} - (\hat{a}_0 + \hat{a}_1 y_t)
\]

with \(\hat{a}_0\) and \(\hat{a}_1\) calculated on a rolling basis so \(B_t\) is observed at \(t\)

- The bound \(\mathbb{E}_t g_{t+1} \geq B_t\) relies on two key assumptions:
  - the modified NCC
  - a stable statistical relationship between valuation ratios and \(r - g\)
The sentiment index

![Graph showing the sentiment index from 2000 to 2015 with blue line representing $B_t$ and red line representing $B_{dp,t}$.]

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The three components of the sentiment index, $B_t$
Allowing $y_t$ to follow an AR($k$)
Sentiment index vs. detrended volume (1)
Figure: Correlation between $B_{t+k}$ and detrended volume at time $t$. 

**Sentiment index vs. detrended volume (2)**
Sentiment index vs. crash probability index (1)

Figure: $B_t$ and crash probability (Martin, 2017)
Sentiment index vs. crash probability index (2)

\[ \text{corr}(B_{t+k}, P_t) \]

**Figure:** Correlation between \( B_{t+k} \) and crash probability at time \( t \).
Conclusion

- Volatility and valuation ratios have long been linked to bubbles
- We use some theory to make the link quantitative
- We have tried to make choices in a conservative way to avoid “crying bubble” prematurely, and/or overfitting
- Signature of a bubble: valuation ratios, volatility, and interest rates are simultaneously high