Low pass-through and high spillovers in NOEM: what does help and what does not

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Abstract

This paper jointly analyses two major challenges of the canonical NOEM model: i) combining a relatively important exchange rate pass-through at the border with low pass-through at the consumer level, and ii) generating significant endogenous international business cycle synchronization. We show that introducing input trade for price-maker firms rehabilitate the model regarding the pass-through disconnect, while adding a distribution sector lacks flexibility to do so. Moreover, these two extensions of the canonical model mitigate the expenditure switching effect, with implications in terms of international synchronization.

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1 Introduction

New open economy macroeconomic models are well known to face two major challenges. First, they have a clear difficulty to combine a relatively important exchange rate pass-through at the border with low pass-through at the consumer level. Second, they struggle to replicate endogenously the high empirically observed international synchronization of business cycles. The literature has tackled the first or the second of these puzzles, but as far as we know, there has been no clear attempt to link both issues. The low pass-through problem has been mainly addressed by augmenting the standard model with a domestic distribution sector according to the intuition of Burstein, Neves and Rebelo (2003) and Corsetti and Dedola (2005). The most promising avenue to enhance the endogenous cross-border spillovers of country specific shocks via the trade channel is the formal introduction of international trade in intermediate inputs following the lines of Huang and Liu (2007), Burstein, Kurtz and Tesar (2008) or Johnson (2014). In the present contribution, we bring a particular attention to the respective structural implications of distribution sector and foreign intermediate input in an otherwise standard open New Keynesian model before illustrating them in a general equilibrium framework examining both real and nominal effects. In this regard, our work complements the careful empirical partial equilibrium analysis of Campa and Goldberg (2010) which focused on the nominal side and it offers an alternative way to verify their claim that "the dominant channel for CPI sensitivity is through the cost arising from imported input use in goods production", what we will refer to as "input trade" in this paper.

In the standard NOEM, the main culprit for the too close connection between the pass-through to import and consumption prices is the simplifying assumption that all the imported goods end-up in the consumption price index at their border price. This amounts to consider that domestic end-users combine final home and foreign goods in a proportion dictated by the ratio of imports to domestic demand. While relatively acceptable for economies in which this ratio remains low, like the United States or Japan, such an assumption loses any credibility when attempting to model the numerous very open economies characterized by imports equal or larger than domestic demand. Input trade is shown to circumvent the issue by incorporating a share of imported goods into the consumption basket indirectly, via the final home goods produced from the combination of own and imported value added. In a world of flexible prices and multilateral trade,\textsuperscript{1} the input trade assumption would yield exactly the same outcome as

\textsuperscript{1}i.e. where trade partners of a particular country are not necessarily the same on the import and the export sides, implying among others different exchange rates.
the standard NOEM regarding the consumption price index. Switching to a pure bilateral trade model, input trade neutralizes the structural pass-through to import prices at the prorata of the requirement of home made goods for the production of foreign goods, following the intuition developed in Georgiadis, Gräb and Khalil (2018). In addition, if one considers that price stickiness might be different for foreign exporters and for domestic firms when selling on the home market, the channel through which import prices reach the consumption price index is strongly modified. Foreign goods that enter consumption with few transformations by the domestic firms are valued in the consumption basket at the (relatively flexible) border price, while those that are integrated as factors in the production of the domestic output to satisfy the domestic demand are valued at the (relatively sticky) domestic producers’ price.\(^2\)

On these grounds, we show that input trade is very efficient in reproducing the disconnect between short-run exchange rate pass-through at the border and at the consumer price level, the former being estimated about ten times larger than the latter in the literature (see for example Campa and Goldberg, 2010, Burstein and Gopinath, 2014, Table 4, or Ortega and Osbat, 2019). On the contrary, the distribution channel lacks flexibility to do so. As highlighted by Corsetti et al. (2008), the distribution channel modifies the perceived mark-up of the foreign exporters on the home market, partly substituting out the influence of their own marginal cost and exchange rate by the price of the home distribution services. This increases the weight of the home produced value added in the CPI, but reduces concurrently the pass-through to import price. In this sense, the distribution sector mechanism suffers the same criticism as this addressed by Corsetti et al. (2008) to strengthening import price nominal rigidity: any decrease in the pass-through to the consumption price comes through a reduced import price pass-through. Instead of combining home prices with foreign marginal costs, input trade operates exactly the other way round, by combining import prices with domestic marginal costs. The final outcome on consumer price might a priori be relatively similar, but the implied dynamics of the import price are definitely different: the pass-through at the border is left relatively unchanged compared to the canonical model while the relatively rigid domestic producers’ prices delay the influence of exchange rate and import prices on the consumer price index.

International input trade offers a nice rationale for modelling very open economies: in a globalized world, the production process does not care much about borders, and geographically limited economies are more likely to observe an important international trade, linked to both

\(^2\)See our discussion in Section 4.1 for the intuition why domestic producer prices would be stickier than import price at the border.
transit and intermediate inputs. This intuition is confirmed by OECD data which display a strong positive relationship between trade openness and import content of exports in cross section (see left subplot of Figure 2 below). This is not only true in levels, but also in growth rates: in a cross-sectional average of OECD countries, the rise in import content of exports the last two decades follows the trade openness increase nearly one by one (see right subplot of Figure 2 below). In this respect, input trade offers a potential explanation for a stable consumption price pass-through despite the observed surge of imports to domestic demand ratios in most industrialized economies.

Beside their different implications regarding the pass-through to the price index of the final good bundle, both the foreign intermediate inputs in production and distribution services assumptions alter the relative price of home versus foreign goods and the elasticity of imports to this relative price. According to Huang and Liu (2007), and in the spirit of Obstfeld and Rogoff (1995), the expenditure switching effect, driven by this elasticity, is the very reason why the terms of trade externality hinders the business cycles synchronizing role of the aggregate demand externality in the aftermath of, e.g. country-specific monetary policy shocks. From this perspective one may expect that both mechanisms might elicit cross-border co-movements.

Consistent with this intuition, we complement the literature in two directions. First, we underline that business cycle synchronization brought about by the two extensions of the canonical NOEM is shock-specific: they improve international spillovers for country specific shocks for which the country policy rate reacts countercyclically but business cycle synchronization is at best unchanged for others, as e.g. demand shocks. Second, we compare the relative efficiency of the two extensions in improving international spillovers. For input trade, we show analytically why the elasticity of substitution between foreign and domestic goods at the intermediate production level plays a critical role. As in Burnstein et al. (2008), the larger the complementarity, the stronger the effect of trade input. Third, for all the shocks considered (monetary policy, productivity, risk premium and preference), input trade helps concurrently to solve the pass-through disconnect puzzle and to improve the cross-border correlation of CPI inflations.

Section 2 below describes succinctly the households’ preferences, the production structure and the determinants of the demand for imports. The extensions of the model to a distribution sector and to input trade have strong implications for the consumption home bias, computed as the share of domestically produced goods in the consumption basket, foreign value added included. This is underlined in Section 3 that motivates the careful introduction of input trade by (i) the unrealistic implications of the standard model augmented or not with a distribution
sector and (ii) by the 85% correlation between trade openness and import content of exports computed for OECD economies. Section 4 focuses on the nominal side of the economy and establishes formally the structural exchange rate pass-through from the border to the consumption basket. It derives the exact roles played by domestic distribution services and input trade as well as the slope of the various Phillips curves, respectively. In Section 5, we compute the elasticity of imports to the relative price of foreign/domestic goods, which drives the expenditure switching effect and affects the real variables of the economy. The consequences of the structural analysis conducted in Sections 3 to 5 are tested in Section 6 within a dynamic general equilibrium exercise allowing to link the interactions between the nominal and real reactions. The general equilibrium effects bring an interesting contradiction to the partial equilibrium outcome of Campa and Goldberg (2010) that more substitutability between home and foreign intermediate inputs reduces the price effect of exchange rate shocks. In a first stage, the attention is focused on exchange rate shocks, and in a second stage it is extended to others country specific shocks, with a particular attention to the implications of the studied mechanisms in terms of international real and nominal spillovers. This part complements the structural analysis of Sections 3 to 5 and show that input trade systematically dominates the distribution channel in generating endogenously cross-border CPIs correlation while the reverse hold true concerning real GDPs comovements. Section 7 concludes.

2 Households preferences and production structure

Households allocate consumption ($C_t$) and investment ($I_t$) between homogeneous final goods produced domestically and imported from abroad according to the usual CES preference:

$$
\zeta_t = \left[ \phi_H \frac{1}{\lambda} \zeta_{h,t} \frac{1}{\lambda} + (1 - \phi_H) \frac{1}{\lambda} \zeta_{f,t} \frac{1}{\lambda} \right] \frac{1}{\lambda}, \quad \zeta \in \{ C, I \}
$$

with $\phi_H$, the steady-state share of domestically produced final goods, also called home bias, and $\lambda$, the Armington elasticity of substitution between domestic and imported distributed goods. "D" stands for "distributed", meaning that, in the spirit of Burstein et al. (2003), traded goods require logistic services in order to reach the final users. For simplicity and as in Burstein et al. (2003), let’s assume a Leontief distribution technology to produce retail goods, where $\delta$ domestic
goods are required to bring one unit of homogenous good to retail stores,\(^3\) such that
\[
\zeta_{j,t}^D = \min \left\{ (1 + \delta) Y_{j,t}; \frac{1 + \delta}{\delta} Y_{h,t}^d \right\} \quad \text{with } j \in \{h, f\},
\]
where \(Y_{h,t}\) (resp. \(Y_{f,t}\)) represent the home produced homogenous goods and the "d" superscripts stands for the homogenous goods used for distribution services. The corresponding price index\(^4\) is
\[
P_t = \left[ \phi_H P_{h,t}^{D,1-\lambda} + (1 - \phi_H) P_{f,t}^{D,1-\lambda} \right]^{\frac{1}{1-\lambda}} \tag{2}
\]
where \(P_{j,t}^D = \frac{1}{1+\delta} P_{j,t} + \frac{\delta}{1+\delta} P_{h,t}, \quad \text{with } j \in \{f, h\}. \tag{3}
\]

Home homogenous goods \((Y_{h,t})\) for domestic purposes and for exports are produced by firms acting on a perfectly competitive market. These firms buy intermediate inputs on a market in monopolistic competition and combine them using a Dixit-Stiglitz aggregator. Domestic intermediate inputs, indexed by \(i\) on the unit circle, are obtained from domestic value added \((Y_t)\) compounded with foreign homogenous final goods \((Y_{f,t})\) through a CES technology. The domestic value added is produced with a Cobb-Douglas technology from the services of capital and labour rented from domestic households. On these grounds, all the domestic intermediate firms share the same marginal cost
\[
MC_{h,t}(i) = \left[ (1 - \phi_m) MC_{y,t}^{1-\lambda_m} + \phi_m P_{f,t}^{1-\lambda_m} \right]^{\frac{1}{1-\lambda_m}}, \quad \text{for } i \in [0, 1] \tag{4}
\]
with \(MC_{y,t} = \frac{w_t}{\alpha^\alpha (1 - \alpha)^{1-\alpha} r_k^{\alpha \alpha} e_{h,t}^{\alpha}}\)
\[\text{where } r_k^t \text{ is the rental rate of capital, } w_t, \text{ the price of labor, } \alpha, \text{ the Cobb-Douglas capital share and } e_{h,t}^t, \text{ the AR}(1) \text{ total factor productivity exogenous process. The weight of foreign inputs in the CES technology is represented by } \phi_m, \text{ while } \lambda_m \text{ is the elasticity of substitution between home and foreign inputs. The foreign production structure is exactly symmetric.}
\]

Beside the role of foreign goods for direct final use and production in the home economy, let us extend the purposes of imports to transit goods \((TG_t)\), i.e. goods that enter the economy and are re-exported without a substantial domestic value added.\(^5\) Therefore, the demand for

\(^3\)We simplify somehow the structure of Corsetti and Dedola (2005) who distinguish traded and non-traded goods. This refinement is not necessary for the point we intend to make here.
\(^4\)For simplicity we assume that investment and consumption bundles share the same home bias and trade elasticity. This implies that their price is alike and in the rest of the paper we will mostly refer to it as the consumption price index.
\(^5\)Such transit goods can be important for very open and small economies such as Belgium (Duprez, 2014) or The Netherlands, as they often relate to port effects (e.g. Antwerp, Rotterdam).
foreign goods can then be written as

$$M_t = \frac{C_{f,t} + I_{f,t}}{1 + \delta} + Y_{f,t} + TG_t,$$

(5)

where $$\zeta_{f,t} = (1 - \phi_H) \left( \frac{P_{f,t}}{P_t} \right)^{-\lambda} \zeta_t$$, with $$\zeta \in \{C, I\}$$

(6)

and $$Y_{f,t} = \phi_m \left( \frac{P_{f,t}}{MC_{h,t}} \right)^{-\lambda_m} Y_{h,t}.$$  

(7)

Note that the imported share of retail foreign goods within the home private domestic demand is decreasing in the size of the distribution services.

The standard NOEM set-up considers implicitly a steady-state import content of exports ($$\phi_x^m$$) reduced to zero, against the empirical evidence. In our generalized set-up, the import content of exports ($$\phi_x^m$$) may be viewed as composed of transit goods ($$tg/\bar{x}$$) and of the foreign goods contained in exported domestic production, such that

$$\phi_x^m = tg/\bar{x} + \left(1 - \frac{tg}{\bar{x}}\right) m_{f,t}.$$

(8)

As long as the import content of exports is not accounted for, the larger the degree of trade openness that characterizes an economy, the more unrealistic the consequences of the model in terms of homes bias and exchange rate pass-through to consumption prices. This is underlined in the next section.

3 Home bias and weights in the consumption price index

Equations (2), (4), (6) and (7) highlight the key role of the home bias parameter, $$\phi_H$$, and of the weighting parameter for foreign inputs in production, $$\phi_m$$, for both the real and nominal features of the modelled economy. On the real side, together with their respective associated substitution elasticities $$\lambda$$ and $$\lambda_m$$, they translate the differential between domestic and foreign prices into demands for real imports. On the nominal side, they contribute to determine the weights of border import prices in the consumption price index and in the producers’ price index. This section focuses on the latter aspect and analyses how trade openness, distribution margins and import content of exports, either under the form of transit goods or foreign intermediate inputs, affect the share of the import price $$P_{f,t}$$ in the end-user price $$P_t$$. 

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3.1 Home bias, trade openness, distribution, transit and input trade

Substituting equations (6) and (7) into (5) at steady-state highlights the direct relationship between the home bias parameter, $\phi_H$, and the import-to-absorption ratio, $\bar{m}/(\bar{c} + \bar{i})$.

Lemma 1: Assuming (i) balanced trade at steady-state and (ii) that only the production of goods and services for private demand and exports requires foreign inputs, trade openness is computed as

$$\frac{\bar{m}}{\bar{c} + \bar{i}} \left( \frac{1 - \phi_H^m}{1 - \rho_m} \right) = \left( \frac{1 - \phi_H}{1 + \delta} + \frac{\rho_m}{1 - \rho_m} \right)$$

with $\rho_m = \phi_m \left( \frac{P_f}{MC_h} \right)^{-\lambda_m} = \frac{\bar{y}_f}{\bar{y}_h}$. (10)

Proof. From equations (5), (6), (7) and (8), the real imports can be expressed as

$$M_t = \frac{1 - \phi_H}{1 + \delta} \left( \frac{P_{f,t}}{P_{t}} \right)^{-\lambda} (C_t + I_t) + \phi_m \left( \frac{P_{f,t}}{MC_{h,t}} \right)^{-\lambda_m} Y_{h,t} + \frac{\bar{y}g}{\bar{x}} X_t.$$ (11)

Under assumption (ii), $^6$ the demand for the domestically produced homogeneous goods (private demand, distribution services and exports) is given by

$$Y_{h,t} = \phi_H + \delta \left( \frac{P_{f,t}}{P_{t}} \right)^{-\lambda} (C_t + I_t) + \left( 1 - \frac{\bar{y}g}{\bar{x}} \right) X_t.$$ (12)

At steady-state, assuming balanced trade and substituting for $\bar{y}_h$ into $\bar{m}$, one obtains

$$\bar{m} \left( 1 - \frac{\bar{y}g}{\bar{m}} \right) = (\bar{c} + \bar{i}) \left( \frac{1 - \phi_H}{1 + \delta} + \frac{\phi_m \left( \frac{P_f}{MC_h} \right)^{-\lambda_m}}{1 - \phi_m \left( \frac{P_f}{MC_h} \right)^{-\lambda_m}} \right).$$

Equation (8) allows to substitute out the transit good ratio. □

The term on the left-hand side of equation (9) indicates that one considers only the imports that actually enter the economy to be transformed in a way or another. The terms on the right-hand side represent what imports will be used for: either they directly feed the private domestic demand or they are used for production purposes. Finally, $\rho_m$ is the steady-state share of the value of foreign inputs into the value of the domestic homogeneous good. Expression (9) helps to assess how assumptions regarding parameters $\delta, \phi_x^m$ and $\rho_m$ affect the structural home bias.

$^6$This amounts to say that government consumption is produced from domestic value added only, i.e. that for this particular type of good $\phi_H$ is equal to one and $\rho_m$ is zero.
Proposition 1 For \( \frac{\bar{m}}{\bar{c}+\bar{i}} \) and \( \delta \in \mathbb{R}_+ \), \( \phi^m_x \in [0,1] \), \( \rho_m \in [0, \phi^m_x] \), the home bias \( \phi_H \) can be expressed as

\[
\phi_H = 1 - \frac{1+\delta}{1-\rho_m} \left[ \frac{\bar{m}}{\bar{c}+\bar{i}} (1-\phi^m_x) - \rho_m \right],
\]

with \( \phi_H \in [0,1] \Leftrightarrow \frac{1+\delta}{\delta} (1-\phi^m_x) \frac{\bar{m}}{\bar{c}+\bar{i}} - 1 \leq \rho_m \leq (1-\phi^m_x) \frac{\bar{m}}{\bar{c}+\bar{i}} \).

It is (i) linearly decreasing in the trade openness measured by \( \frac{\bar{m}}{\bar{c}+\bar{i}} \), (ii) decreasing in the size of the distribution sector \( \delta \), (iii) linearly increasing in the import content of exports \( \phi^m_x \), and (iv) increasing and convex in the intermediate foreign goods \( \rho_m \) required for the domestic production process.

Let us illustrate Proposition 1 by setting particular values to the consumption and investment shares in GDP. We take arbitrarily the values for the euro area, which, according to the Fagan et al. (2005) database, average respectively to 0.56 and 0.20 for the period 1995 – 2014. Note that in OECD countries, these ratios sum up to values comprised in a relatively narrow range between 0.70 and 0.85, while the import-to-GDP ratio evolves from 0.12 (Japan) up to 1.40 (Luxemburg). Trade openness is a much more likely source of variance in country-specific home bias values than absorption. On these grounds, let us consider that the chosen values for \( \frac{\bar{c}+\bar{i}}{\bar{y}} \) is a valid approximation for any OECD country.

Figure 1 graphs several lines representing the evolution of the home bias \( \phi_H \) as a function of the trade openness \( \frac{\bar{m}}{\bar{c}+\bar{i}} \) and letting it vary for different values of the parameters mentioned in Proposition 1 according to equation (13). All the lines drawn in Figure 1 are straight and downward sloping, as stated in Proposition 1-item (i). Whatever the assumptions regarding \( \delta \), \( \phi^m_x \) or \( \rho_m \), a larger exposition to international trade means more foreign goods in final consumption and hence a lower value for the home bias. For the calibration of the canonical NOEM, i.e. \( \delta = \phi^m_x = \rho_m = 0 \), equation (13) implies that the home bias is exactly equal to one minus the import content of domestic absorption, \( \frac{\bar{m}}{\bar{c}+\bar{i}} \). This is represented by the black solid line in Figure 1 which emphasizes that such a calibration prevents to model very open economies, like Ireland for example, as it leads to unrealistically small or even negative home bias. This shortcoming gets even worse as the size of the distribution sector increases, as indicated by the downward rotation from the black solid line to the pink line, illustrating Proposition 1-item (ii). An increase in \( \delta \) lowers the required home bias because more domestic goods are used in the distribution process of foreign goods: would the home bias be constant, the share of imports in GDP should decrease. On the contrary, an increase in the import content of exports \( \phi^m_x \) operates
Figure 1: Home bias $\phi_H$ as a function of trade openness $\frac{m}{y}$

an upward rotation from the black solid line to the blue one, as stated in Proposition 1-item (iii). At given $\rho_m$ (equal to zero in the case of the blue line), a higher import content of exports $\phi^m_x$ (set to 0.2) implies more transit goods, which is equivalent to reducing the penetration of foreign goods within the economy, such that a lower share of foreign goods directly dedicated to final use is needed to target a given import-to-GDP ratio. If the import content of exports is fixed (still set at 0.2 for the red solid line), an increase in $\rho_m$ necessarily ends up in a reduced share of transit goods. As expressed in Proposition 1-item (iv), the home bias increases with the required foreign intermediate inputs in a convex way. Noteworthy, the home bias is the steady-state share of domestically produced goods in the consumption basket, but as soon as $\rho_m$ is strictly positive, this share must be understood as "import content of production included".

So far, the solid black, pink, blue and red lines of Figure 1 have helped to illustrate Proposition 1. Let us go one step further, and consider the particular case of the euro area, characterized by an import-to-GDP ratio of 0.2.\textsuperscript{7} The OECD evaluates the import content of exports for the euro area around 0.28. However, one may suspect many double counting due to the lack of distinction between intra- and extra- euro area trade. More accurate evaluations by van der Helm

\textsuperscript{7}As computed by van der Helm and Hoekstra (2009) who correct for the intra-zone trade.
and Hoekstra (2009) and by Amador et al. (2015) sets the import content of extra area exports around 0.20. Figure 1 displays that, for $\bar{m}/\bar{y} = 0.2$, increasing $\phi^m_x$ from 0 to 0.2 pushes the home bias from 0.74 to 0.78 as long as $\rho_m = 0$. For $\rho_m = 0.12$, it raises up to 0.9 which is still on the low side as it would imply, according to equation (8), that transit goods make 9 percent of euro area exports, a most probably exaggerated value. Therefore, taking seriously empirical estimates of the import content of exports into account allows to calibrate much higher home bias with all the consequences it might have regarding, first, exchange rate volatility and the exchange rate disconnect, as pointed by Wang (2010), and second, the pass-through to consumption prices, as emphasized in the next sections of the present contribution.

Noteworthy, import content of exports and trade openness are strongly correlated among OECD economies, as displayed on the left part of Figure 2. This empirical evidence offers a good rationale to break the strict inverse relationship between home bias and import-to-GDP, such that the home bias of wide open economies has no reason anymore to be much smaller than this of less open economies. Let us illustrate this point with the examples of Sweden and Ireland. For the 1995 – 2014 period, the import-to-GDP ratios of these two economies average to 0.38 and 0.75 respectively, while their import content of exports is measured by the OECD at 0.29 and 0.42 respectively. The dashed red line (for Sweden) drawn on Figure 1 displays that, for an economy twice as open as the euro area, given a larger import content of exports, it is still possible to reach a home bias equal to 0.9. In the case of Ireland, the measured import content of exports is too small with respect to the trade openness to allow to obtain a home bias larger then 0.74. This is however much more satisfactory than the zero implied by the canonical model.

This reasoning is not only valid when comparing different economies, but also if one considers the evolution of an economy through time. As displayed on the right panel of Figure 2, the import-to-GDP ratio has increased in most OECD countries between the years 1995 and 2014. For the extreme case of Japan, it has nearly tripled. Under the traditional view that $\phi^m_x = \rho_m = 0$, this should have come with a strong decrease of the home bias and all the consequences in terms of increased exchange rate pass-through to consumption price. However, OECD numbers show that this global increase in trade openness goes along nearly one to one with an increase in import content of exports. If one gives some credit to these numbers, it might be the case that the increase in trade openness led actually to an increase of the home bias in some countries. The upward trend in both import-to-GDP ratios and import content of exports shown on the right panel of Figure 2 supports strongly the pioneering studies of Feenstra (1998) and Yi (2003)
on the rise of foreign value added in domestic production. It is clear from equation (2) that this has direct implications for the respective weight of import and domestic production in the consumption price index.

### 3.2 Composition of the consumption price index

According to equation (2), the price of consumption may be viewed as a weighted average of the import price and the domestically produced goods price. Beside the reaction of each of these prices to changes in the exchange rate, the steady-state proportion of the two homogenous goods entering the composition of the final good is the key element of the pass-through to consumption price. Log-linearized around steady-state, the price of the final good that is either consumed or invested can be represented by

\[
\hat{p}_t = \phi_H \hat{p}_{h,t} + \frac{1 - \phi_H}{1 + \delta} (\hat{p}_{f,t} + \delta \hat{p}_{h,t}) ,
\]

where the second term is the retail price of the foreign good, i.e. after inclusion of domestic distribution services.
Proposition 2 The log-linearized consumption price may be rewritten as

\[ \hat{p}_t = \phi^{m,d}_c \cdot \hat{p}_{f,t} + \left( 1 - \phi^{m,d}_c \right) \cdot \hat{p}_{h,t} \] (15)

with \[ \phi^{m,d}_c = \frac{1 - \phi_H}{1 + \delta} \] (16).

Given the parameters set of Proposition 1, \( \phi^{m,d}_c \), the share of foreign goods that enter directly into the consumption basket at the import price value is

(i) increasing linearly in trade openness \( \hat{m}/(\bar{e} + \bar{i}) \),

(ii) invariant in the size of the distribution sector \( \delta \),

(iii) decreasing linearly in the import content of exports \( \phi^n_x \),

(iv) decreasing and concave in the intermediate foreign inputs \( \rho_m \), required for the domestic production process. Furthermore, the import content of consumption, \( \phi^m_c = \phi^{m,d}_c + \rho_m \left( 1 - \phi^{m,d}_c \right) \), is decreasing in the intermediate foreign inputs \( \rho_m \).

Items (i)-(iv) of Proposition 2 are direct corollaries of the items (i)-(iv) stated in Proposition 1. In particular, the invariance of the weight \( \phi^{m,d}_c \) with respect to the size of the distribution sector \( \delta \) is obtained by isolating \( \phi^{m,d}_c \) on one side of the equality in equation (13). The last part of item (iv) deserves some more explanation. Domestic production is a broader concept than domestic absorption such that, when replacing imports that directly enter the consumption/investment bundle \( \phi^{m,d}_c \) by imports used in production \( \rho_m \), a lower share of import in final good is required to match the import-to-GDP ratio. Proposition 2 stresses that, when targeting both the import-to-GDP ratio and the import content of exports, the distribution channel mechanism influences the consumption price exchange rate pass-through only via its direct role on the import price pass-through (cf. item (ii)) while foreign intermediate inputs modifies the respective weights of the import and domestic producers prices (cf. item (iv)). In order to catch the full implications of the distribution sector and import content of exports mechanisms on the exchange rate pass-through towards consumption price, it is necessary to develop the Phillips curves of the foreign exports and domestic producers. This is the object of the next section.

4 Exchange rate pass-through: a structural analysis

4.1 Firms price setting

Intermediate domestic firms act in a monopolistic competition environment and adapt their price to the targeted market. Following Calvo (1983), they reset optimally their price according to
the macroeconomic circumstances with a given probability, say $\xi_m$ when exporting and $\xi$ when selling on the domestic market. The corresponding New Keynesian Phillips curves first-order approximations around steady-state are respectively

$$\hat{\pi}_{h,t} = \beta E_t \hat{\pi}_{h,t+1} - \frac{(1 - \xi)(1 - \beta \xi)}{\xi} \hat{\mu}_{h,t}, \quad (17)$$

$$\hat{\pi}^*_{f,t} = \beta E_t \hat{\pi}^*_{f,t+1} - \frac{(1 - \xi_m)(1 - \beta \xi_m)}{\xi_m} \hat{\mu}^*_{f,t}, \quad (18)$$

with

$$\hat{\mu}_{h,t} = \bar{\pi}_{h,t} - \left[ \frac{\eta - 1 - \delta}{\eta - 1} \hat{mc}_{h,t} + \frac{\delta}{\eta - 1} \hat{h}_{h,t} \right], \quad (19)$$

$$\hat{\mu}^*_{f,t} = \bar{\pi}^*_{f,t} - \left[ \frac{\eta^* - 1 - \delta^*}{\eta^* - 1} [\hat{mc}_{h,t} - s_t] + \frac{\delta^*}{\eta^* - 1} \hat{h}_{f,t} \right]. \quad (20)$$

The symbol "**" identifies foreign economy variables. Parameter $\beta$ represents the psychological discount factor in the domestic economy. The coefficient $\eta$ is the steady-state value of the price elasticity of demand of the firms in monopolistic competition. The aggregate time-varying mark-ups $\hat{\mu}_{h,t}$ and $\hat{\mu}^*_{f,t}$ are determined by the differences between the aggregate price on the targeted market and the drivers of the optimal pricing strategy, all expressed in real terms, i.e. relative to the domestic end-user price $\bar{p}_h$. Under local currency pricing, the latter are the real marginal cost ($\hat{mc}_{h,t}$), expressed in foreign currency for exporting firms through the real bilateral exchange rate $s_t$, and the distribution services priced by local firms. Equation (3) indicates that the pricing decision of a firm affects only a share of the retail price on the targeted market, a share that decreases with the importance of distribution requirements. As pointed by Corsetti and Dedola (2005), this reduces the induced variation of market shares compared to what would be implied purely by the demand elasticity, and firms’ mark-ups increase accordingly. As such, the price of the foreign distribution services becomes a key element in the exporting firms’ pricing decision and dilutes somehow the influence of the exchange rate.

Interestingly, Corsetti et al. (2008) and Huang and Liu (2007) consider a pretty low nominal stickiness for firms both on the domestic and export markets. In this regard, they follow quite literally micro studies estimating that, in average, firms reset their price after 4.3 months (see e.g. Bils and Klenow, 2004). On the other hand, estimated nominal rigidity of the domestic New Keynesian Phillips curve is estimated much higher in macromodels like e.g. Christiano, Eichenbaum and Evans (2005) or Smets and Wouters (2007). The Huang and Liu (2001, 2007) contributions on production chains offer a nice intuition that helps reconcile the discrepancy between macro- and micro-based estimations. In the real world, firms are mostly trading with

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8For simplicity, we assume that all the firms, domestic or foreign, selling on the home (resp. foreign) market share the same market power.

9In this sense, $\bar{p}_{h,t}$ and $\bar{p}^*_{f,t}$ must be read as $\bar{p}_{h,t} = \bar{p}_h - \bar{n}$ and $\bar{p}^*_{f,t} = \bar{p}^*_{f,t} - \bar{n}$.
firms, along a production process made of several intermediate steps and the price of the final good is only set at the very last stage. The New Keynesian Phillips curve is built from the horizontal integration of intermediate firms acting in monopolistic competition and totally ignores the vertical integration dimension. As a consequence, the dynamics of the observed macro price series (e.g. the GDP deflator) can only be reproduced through an estimated large degree of price stickiness, which reflects the modelling shortcut. However, when intermediate firms export, be it to foreign firms or households, the cross-border price reflects only one stage of production, such that aggregate international price dynamics require much less nominal rigidity to be matched, more in line with micro studies. In the absence of more information about intermediate prices, the input-output structure and the average number of production steps, the empirical DSGE literature estimates an overall large domestic producers’ price rigidity that mimics the accumulation of small intermediate price rigidities. In the rest of the paper we will rest on this simplified representation instead of following Huang and Liu (2001, 2007) in a more careful representation of the production stages. In this logic, from then on we consider that firms reoptimize their price after 4.5 months when exporting ($\xi_m = 0.33$) while they do it only after 3 quarters on the domestic market ($\xi = 0.75$).

4.2 ERPT at the border

The import price inflation for the domestic economy is actually the foreign export price inflation in domestic currency obtained symmetrically from equations (18), (20) and (4) by switching systematically on/off the "*" symbolizing the foreign economy. We compute the exchange rate pass-through as the coefficient multiplying the contemporaneous exchange rate in the import price Phillips curve when the latter is rewritten in terms of price level instead of price inflation. This allows to obtain measures comparable with Corsetti et al. (2008) who model price stickiness with Rotemberg adjustment costs instead of the Calvo probability. The expressions obtained can be interpreted as a structural elasticity of import prices with respect to exchange rate. It is a ceteris paribus concept comparable to, e.g., the Campa and Goldberg (2005) reduced

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10 This modelling choice has the advantage to break the implicit link imposed by Huang and Liu (2007) between the number of production stages, i.e. overall nominal rigidity, and the proportion of foreign intermediate inputs in the production process. They consider that all the firms set their price à la Taylor (1980) for two quarters. At every intermediary stage, there is a requirement for some amount of intermediate foreign inputs. Therefore, increasing the numbers of production steps yields at the same time to more foreign value added in the final domestic production and accumulates price rigidities between the first production step and the final good used for consumption.
form pass-through estimates. In Proposition 3 below we operate a clear distinction between the distribution sector and input trade assumptions for the sake of clarity.\textsuperscript{11} The fully general case is developed in the technical appendix.

Proposition 3 For $\beta^*$, $\beta$, $\xi_m$ and $\xi_m \in [0,1]$, for $\eta$ and $\eta^*$ strictly larger than one, for $\delta$ and $\delta^* \in \mathbb{R}^+$, the structural exchange rate pass-through towards import price at the border is equal to

$$\text{ERP}^{TP}_{MP}(\rho_m=\rho_m^*=0) = \Psi_f \cdot \frac{\eta - 1 - \delta}{\eta - 1},$$

$$\text{ERP}^{TP}_{MP}(\delta=0) = \Psi_f \frac{1 - \rho_m^* \eta}{\eta - 1} \Psi_f^*,$$

with $\Psi_f = \frac{(1 - \xi_m^*(1 - \beta^* \xi_m^*))}{(1 - \xi_m^*) (1 - \beta^* \xi_m^*) + \xi_m^*(1 + \beta^*)}$

and $\Psi_f^* = \frac{(1 - \xi_m)(1 - \beta \xi_m)}{(1 - \xi_m)(1 - \beta \xi_m) + \xi_m(1 + \beta)}$.

Note that

(i) $\Psi_f$ (resp. $\Psi_f^*$) is decreasing and convex in $\xi_m^*$ (resp. $\xi_m$);

(ii) $\text{ERP}^{TP}_{MP}(\rho_m=0)$ is linearly decreasing in $\delta$. The larger $\eta$, the less steep the slope;

(iii) $\text{ERP}^{TP}_{MP}(\delta=0)$ is increasing (resp. decreasing) in $\rho_m$ (resp. $\rho_m^*$).

Proof. cf. technical appendix. \hfill \blacksquare

The pass-through towards import price at the border is limited in the short run by the proportion of firms that do not re-optimise their price. Intuitively, the higher $\xi_m$, the more rigid are prices, and their sensitivity to changes in the exchange rate is delayed.\textsuperscript{12} A larger distribution services requirement, $\delta$, makes exporters’ mark-up less sensitive to own costs and exchange rate, and reduces the pass-through of the exchange rate to the import price at the border.

\textsuperscript{11}Note that dealing with intermediate foreign inputs ($\rho_m > 0$) and the distribution sector ($\delta > 0$) at the same time makes the derivation of the pass-through a bit cumbersome. Indeed, the pass-through to domestic producers depends on the pass-through to border prices via $\rho_m$ in the marginal cost, and the reverse holds true via $\delta$ in the foreign exporters price mark-up. For this reason, the pass-through to import price requires the computation of the pass-through to domestic price and vice versa. This is made clear in the proof of Proposition 3 (in appendix) but we restrain from this complicity in the text.

\textsuperscript{12}This point is first introduced by Smets and Wouters (2002). Note that, switching from a CES aggregator to a Kimball (1995) one, the nominal rigidity could be combined with the curvature of the demand to flatten the slope of the import price Phillips curve. A higher curvature corresponds to a final demand whose elasticity increases with price, such that market shares loss of deviating from competitors becomes more important, and adjusting firms tend to absorb more exchange rate fluctuations in their mark-ups.
border. As highlighted by Corsetti and Dedola (2005), the lower the demand elasticity, the stronger the potential of the distribution margin to decrease $ERPT^{MP}$.

Interestingly, both the distribution channel and the input trade mechanisms allow to obtain a pass-through to import prices at the border that is incomplete under flexible prices, i.e. for $\Psi_f = \Psi_f^T = 1$. For the distribution services, the reason for the path-through incompleteness lies in the increased mark-up of the foreign exporting firms, as reported supra. For the input trade, the explanation comes from the marginal cost of the foreign exporting firms, that include a share $\rho^*_m \eta^*/(\eta^* - 1)$ of home produced goods. For the latter share, the exchange rate effect on the import price cancels out, as highlighted by Georgiadis, Gräb and Khalil (2018). This is the economic intuition behind items (iii) of Proposition 3 that establishes that the pass-through to import price decreases with the integration of home produced goods in the foreign production process. On the contrary, if the home economy uses more foreign intermediate inputs, the exchange rate is partially cancelled out back and forth, and pass-through increases.

4.3 ERPT towards domestic production price

In models that do not consider intermediate foreign inputs in the production process, the relative price of currencies does not affect the domestic price Phillips curve. However, given the internationalization of the production process briefly documented in Section 3, the share of foreign value-added contained into a domestic final good is certainly not negligible. In the production process with $\rho^*_m > 0$, the exchange rate affects the marginal cost of domestic producers via its role in the determination of import prices. The structural pass-through to domestic producers prices, $ERPT^{DP}$, is equal to the coefficient multiplying the exchange rate in equation (17) when the latter is rewritten in terms of price level rather than inflation.

**Proposition 4** For the same parameters set as in Proposition 3, and for $\xi \in [0, 1]$, the structural exchange rate pass-through towards domestic producers price is equal to

$$ERPT^{DP} = \Psi_h \cdot \frac{\rho_m \eta}{\eta - 1} \cdot ERPT^{MP}$$

with

$$\Psi_h = \frac{(1 - \xi)(1 - \beta \xi) \frac{n - 1 - \delta}{\eta - 1}}{(1 - \xi)(1 - \beta \xi) \frac{n - 1 - \delta}{\eta - 1} + \xi(1 + \beta)}.$$ 

Note that (i) $\Psi_h$ is decreasing and convex in $\xi$ and decreasing and concave in $\delta$. The convexity in $\xi$ decreases with $\eta$. (ii) $ERPT^{DP}$ increases linearly with $\rho_m$. 
Expression (23) makes clear that the pass-through of the exchange rate to the domestic producers’ price is limited twice: first via the combination of nominal and real rigidities that apply to the price dynamics of imported intermediate goods, specified supra, and second, via the combination of nominal and real rigidities that drive the price dynamics of domestically produced goods.

4.4 ERPT towards the consumption price index

All the results gathered at this stage allow to establish some conclusions regarding the transmission of the relative value of the domestic currency to the consumption deflator. They are formally stated in the following Corollaries.

Corollaries of Propositions 2, 3 and 4

- **C0**: equation (15) may be turned into

  \[
  ERPT^{CP} = \phi^m_d ERP^{MP} + \left(1 - \phi^m_d\right) ERP^{DP};
  \]

  \[
  \text{(24)}
  \]

- **C1**: The parameters affecting the slope of the import price Phillips curve, i.e. \(\xi_m^*, \eta, \) and \(\delta,\) make it possible to match any \( ERP^{MP}.\) International trade in intermediate inputs may also help, through \(\rho^*_m,\) though his potential is more limited in this respect and is reduced further by \(\rho_m\) (cf. Proposition 3);

- **C2**: for \(\rho_m = 0\) and \(\phi^m_x = 0,\) the relationship between \( ERP^{CP} \) and \( ERP^{MP} \) is strictly linear in \(\bar{m}/(\bar{c} + \bar{i}),\) which renders extremely unlikely to simultaneously match the two pass-throughs, notably for large trade openness. Neither the slope of the import price Phillips curve nor the distribution channel are able to break this linear relationship;

- **C3**: Allowing \(\phi^m_x > 0\) offers the required flexibility to circumvent C2 by decreasing \(\phi^m_c.\) However, as long as \(\rho_m = 0,\) it may induce an unrealistically large degree of import content of export under the form of transit goods;

- **C4**: Allowing \(\rho_m > 0\) rebalances equation (24) away from the import price pass-through towards the domestic production price pass-through, which is much smaller (cf. e.g. equation (23)). At given \(\phi^m_x > 0,\) a marginal increase of \(\rho_m\) is much more efficient to decrease \(\phi^m_c\) than a marginal increase in \(\phi^m_x\) (cf. Proposition 2-items (iii) and (iv)), reducing the need for large import content of exports in general and for transit goods in particular.
According to Corollaries C3 and C4, it is possible for a more open economy to face the same consumption price structural pass-through as this of a less open economy. The intuition is exactly similar to this developed supra for the home bias: the initial handicap of a large trade openness can be circumvented by more import content of exports, which in OECD economies are indeed observed to be strongly positively correlated with trade openness. Furthermore, the required import content of exports may be relatively reduced if it is mainly composed of intermediate foreign inputs rather than of transit goods.

4.5 A numerical illustration

In order to illustrate Corollaries C1-C4, let us give specific values to the most obvious ratios and coefficients to help assess numerically the implication of the reviewed pass-through attenuating mechanisms for the euro area. As in Figure 1 we pose $c_{x}/y = 0.56$ and $i_{x}/y = 0.20$. The import-to-GDP ratio for extra trade has been evaluated by van der Helm and Hoekstra (2009) at 0.20. The parameters appearing in the Phillips curve equations are calibrated at fairly standard values: the discount rate $\beta$ is set equal to 0.99 and the elasticity of substitution between intermediate goods on markets in monopolistic competition is set equal to 4.5.

Illustrating Corollaries C1-C4: flexible import prices  Figure 3 depicts how three of the mechanisms discussed so far affect the pass-through to consumption price and via which channel under perfectly flexible import prices ($\xi_{m} = 0$) for an economy calibrated for the euro area. The upper-left panel illustrates Corollary 2: as the size of the distribution sector increases, the importance of the exchange rate in the foreign exporters pricing decision (cf. equations (20) and (21)) declines, reducing the pass-through towards import price. The size of the distribution sector does not affect $\phi_{c}^{m,d}$, the weight of import price in the consumption price index, such that the decrease in the pass-through to import price is transmitted directly to the consumption price with a proportion factor equal to $\phi_{c}^{m,d}$, as emphasized by Corollary C2. The distribution margin, i.e. $\delta/(1 + \delta)$, has to increases up to 64% in order to reduce both $ERPT^{MP}$ and $ERPT^{CP}$ by fifty percent.

Corollary C3 is put into perspective on the upper-right panel of Figure 3. The larger the share of transit goods in the exports, the lower the steady-state share $\phi_{c}^{m,d}$ of imports entering the final good bundle at their border price, and a lower pass-through to consumption price can be obtained with a given and unchanged pass-through to import price. In order to halve the
initial $ERPT^{CP}$, it is necessary to increase the import content of exports up to fifty percent, which is unrealistically high for the euro area, especially under the form of transit goods.

In the third experiment, illustrated by the lower-left panel of Figure 3, we follow van der Helm and Hoekstra (2009) and Amador et al. (2015) and set the import content of extra area exports ($m_x$) at 20 percent while increasing the share of foreign input in domestic production, and symmetrically the share of home inputs in foreign production. According to equation (8), for $p_m$ set at zero, $\phi^m_x$ is only made up of transit goods and increasing the share of foreign intermediate inputs lowers the share of transit goods in $\phi^m_x$. As the foreign economy uses more home intermediate inputs, the import price pass-through decreases somehow and this movement is limited by the simultaneous rise in foreign intermediate inputs, as stated in Proposition 3-item (iii). With $p_m > 0$, the exchange rate affects the domestic production price, even though still far less than the import price. For a given trade openness, at given $\phi^m_x > 0$, intermediate foreign inputs for domestic production limit the direct influence of import price to consumption price by rebalancing the weights in final consumption from imported to home produced goods. Using this strategy, the exchange rate pass-through to the consumption price can be halved from the initial 0.26 to 0.13 with foreign inputs requirement making up 8.5 percent of production.

In order to better emphasize the importance of Corollary C4 and the potential role of the international integration of the production process to explain low pass-through to consumption
prices, let us consider an economy for which trade is two times more important than for the euro area, as Sweden for example (cf. Figure 1). According to Figure 1, for $\delta = \phi^m_x = \rho_m = 0$, the pass-through to consumption price would be equal to a huge 0.50. However, the OECD computes that Swedish import content of exports amounts to 0.29 (average 1995-2014). As displayed on the lower right panel of Figure 3, for $\rho_m = 0$, this corresponds still to an unrealistically high pass-through to consumption of 0.32. Rising foreign intermediate inputs up to the maximum, i.e. 29 percent of the domestic production, the pass-through to consumption price decreases to 0.17, slightly above the targeted value in the three previous experiments.\(^{13}\) The latter exercise confirms the intuition raised earlier that the nominal side of a quite open economy is not necessarily much more affected by exchange rate fluctuations than this of a more closed one. Instead, the effect of exchange rate fluctuations on domestic prices strongly depends on the extent to which the economy is integrated into global value chains, and whether that integration is limited to exports or extends to total domestic production. It is straightforward to carry on this illustration to relationship between import price nominal stickiness and pass-throughs.

**Illustrating Corollaries C1-C4: import price rigidities and pass-throughs** The slope of the foreign exporters New Keynesian Phillips curve is a key element to help target the empirically observed short run import price $ERP_T$. The latter has been estimated at 0.40 (France and Germany) by Burstein and Gopinath (2014) and at 0.35 (EA) by Özyurt (2016). For the sake of the exercise, let us assume $ERP^{MP}_T = 0.4$ is a valid number for the euro area. As illustrated on Figure 4 (left panel), for a given elasticity of substitution $\eta$, such a short run import price pass-through can be obtained by combining the Calvo probability, $\xi^*_m$, the size of the distribution sector, $\delta$, and the share of domestic goods in the foreign production inputs, $\rho^*_m$. However, as long as $\phi^m_x = \rho_m = 0$, for $ERP^{MP}_T = 0.4$, the corresponding short run pass-through to the consumption price index is strictly equal to 0.105, which is more than the double of the estimates reported by Burstein and Gopinath (2014) or Ortega et al. (2018) for the $ERP^{CP}_T$. Naturally, the reverse holds true: given $\xi^*_m = 0.33$, one may target an $ERP^{CP}_T$ equal to the empirical evidence, 0.04, by setting the distribution margin $\delta/(1 + \delta)$ at 0.67, but the corresponding $ERP^{MP}_T$ drops then to 0.15. This remains true for any combination of $\xi^*_m$, $\delta$ and $\rho^*_m$, illustrating the role and limitation of these parameters mentioned in Corollaries C1

\(^{13}\)In this simulation, we consider that the trade partner of the Swedish economy uses Swedish intermediate inputs in a proportion of 8.5 percent of its own product, such that the import price exchange rate pass-through evolves from 0.88 ($\rho_m = 0$) to 0.92 ($\rho_m = 0.29$)
and C2.

On Figure 4 (right panel), we adapt the calibration to take into account the import content of export (diamond) and the intermediate foreign inputs (blue). As for flexible prices, setting the import content of exports equal to 0.20 allows to reduce somehow the consumption price pass-through. However, if at the same time foreign intermediate inputs are increased from 0 to 7 percent of the production, the consumption price pass-through shrinks to 0.04. Note that the impact of allowing for an import content of production on the structural import price pass-through comes only from the demand of the foreign economy. For \( \rho_m = 0 \), the blue and red solid lines would be perfectly superposed, whatever the value of \( \rho_m^* \), while the effect on the structural consumption price pass-through would still continuously decrease with \( \rho_m \).

5 A structural analysis of the expenditure switching effect

So far, the structural analysis has been entirely focused on the nominal side of the economy. It is extended here towards the real implications of the mechanisms under scrutiny. We know from equations (6) and (7) and from Proposition 1 that altering the home bias parameter through the distribution parameter or input trade will also affect the demand for imports. Moreover, the latter is directly influenced by the respective sensitivities of the import and domestic prices.
to exchange rate fluctuations. In this sense, any mechanism able to mitigate the pass-through to the import price, producers’ price or consumer price will not only affect the nominal side of the economy, but has real consequences by altering the channels and the determinants of the demand for imports. This is made clear in the following proposition, illustrated by Figure 5 below.

**Proposition 5** Substituting for (12) into equation (11) and loglinearizing, the real imports equation of the home economy may be written as

\[
\hat{m}_t = (-\lambda) \Omega_1 (\hat{p}_{f,t} - \hat{p}_{h,t}) + (-\lambda_m) \Omega_2 (\hat{p}_{f,t} - \hat{m}_{ch,t}) \\
+ \Omega_3 \left( \frac{c + i}{c + i} \hat{c}_t + \frac{i}{c + i} \hat{i}_t \right) + \hat{x}_t \phi^m_x
\]

with

\[
\Omega_1 = \frac{\bar{c} + i (1 - \phi_H)}{\bar{m}} (1 + \delta)^2 [\phi_H (1 - \rho_m) - \rho_m \delta] ; \quad \Omega_2 = 1 - \frac{1 - \phi_H \bar{c} + i}{1 + \delta} \frac{\phi^m_x - \rho_m}{1 - \rho_m} ; \\
\Omega_3 = \frac{\bar{c} + i - \phi_H (1 - \rho_m) + \rho_m \delta}{\bar{m}}.
\]

According to this expression, a weaker slope of the import price Phillips curve obtained via parameters $\xi_m$, $\delta$ and $\rho_m^*$ limits structurally the expenditure switching effect linked to changes in the import price relative to the domestic producers’ price. This is also true for the import content of exports, $\phi^m_x$, and the foreign value added in production, $\rho_m$, as long as $\lambda_m$ remains contained with respect to $\lambda$. Finally, as long as $\phi^m_x = \rho_m = 0$, the distribution sector leaves the elasticity of real imports to absorption ($\Omega_3$) unchanged at 1, while this elasticity decreases in $\phi^m_x$ and $\rho_m$.

**Proof.** Concerning the relative prices, Propositions 3 establishes that parameters $\xi^*_m$, $\delta$ and $\rho^*_m$ driving the slope of the import price Phillips curve either delay or limit the transmission of relative currency fluctuations towards import price. Furthermore, both distribution services and input trade establish a link between border import price and domestic producers’ price that reduces the price gap: the first one operates via the import price mark-up (cf. equation (20)) and the second one via the domestic producers’ marginal cost (cf. equation (4)). Beside this, the coefficients of the relative prices in equation (25) also matter and one observes that

(i) $\Omega_1$ is decreasing and convex in $\delta$ and $\Omega_2$ is invariant in $\delta$;

(ii) $\Omega_1$ is decreasing and concave in $\phi^m_x$ and $\Omega_2$ is invariant in $\phi^m_x$;

(iii) $\Omega_1$ is decreasing and concave in $\rho_m$ and $\Omega_2$ is increasing and convex in $\rho_m$, such that, for $\lambda$ sufficiently large compared to $\lambda_m$, the first one dominates. ■
In the absence of distribution services and input trade, the elasticity of imports to the relative price of imports is simply equal to the Armington trade elasticity multiplied by the non-imported share of absorption.\textsuperscript{14} From equation (3), we see that the requirement of distribution services has the natural consequence of decreasing the volume of trade that is affected by the price competition between final home and foreign goods. From Proposition 2-items (iii)-(iv), we know that the same holds true for the import content of exports and production: higher $\phi_x^m$ and $\rho_m$ involve a decrease of $\phi_c^{m,d} = (1 - \phi_H)/(1 + \delta)$, i.e. the share of the border import price in the consumption price index, which is affected by the expenditure switching effect linked to variations in the relative price. However, once input trade is introduced, the demand of domestic producers for imports depends also on the relative price of imports with respect to the cost of the domestic production factors, via $\Omega_2$. This channel counterbalances the decrease of $\phi_c^{m,d} = (1 - \phi_H)/(1 + \delta)$ and reinforces the expenditure switching effect in proportion of the corresponding trade elasticity, $\lambda_m$. Finally, note that the elasticity between real imports and absorption remains unchanged and equal to one as long as the import content of exports is set to zero. Once the latter becomes positive, the motives behind imports are broadened to transit and production purposes and the sensitivity of imports to private aggregate demand decreases.

Proposition 5 is illustrated on Figure 5 which produces a numerical example for the calibration used so far, i.e. $\bar{m}/(\bar{c} + \bar{i}) = 0.26$, corresponding to the euro area. The left panel displays the strong effect of an increase of the distribution margin $\delta/(1 + \delta)$ from 0 to 67 percent on the coefficient of the relative price in the import demand, abstracting from the trade elasticity: coefficient $\Omega_1$ drops from 0.74 to 0.07. On the second panel, one observes that transit goods have a lower potential in mitigating the expenditure switching effect, but they affect the import/absorption relationship as synthesized by $\Omega_3$. Finally, the third panel illustrates the effect of foreign intermediate inputs in production on the expenditure switching effect. In the case of a Leontief production technology, the potential of this mechanism to reduce the expenditure switching effect is as important as this of the distribution sector. However, it is opposed by a strong force in opposite direction once departing from the pure complementarity assumption.

We conclude from this that, assuming perfect complementarity between home and foreign inputs in the domestic production process, input trade and distribution services have a rather similar potential to reduce the expenditure switching effect, but the sensitivity to absorption is reduced in the former mechanism compared to the latter.

\textsuperscript{14}One easily compute that $\Omega_1(\delta = \phi_x^m = \rho_m = 0) = \phi_H = 1 - \frac{\bar{m}}{\bar{c} + \bar{i}}$. 

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Up to now, the whole analysis has been conducted on the grounds of static equations and partial equilibrium. Beside the structural pass-through, the strength of the relationship between exchange rate and prices after a shock that modifies the relative price of currencies is also affected by general equilibrium effects. This part of the analysis requires a comprehensive macroeconomic set-up. For this exercise, we rely on the two-country model described in de Walque et al. (2017), which entails all the open economy characteristics discussed supra. As far as the domestic economies are concerned, the model is entirely based on Smets and Wouters (2007). In order to focus on the differences in dynamic responses due to the chosen assumption regarding (i) the slope of the Phillips curve, (ii) the size of the distribution sector and (iii) the import content of export and of production, the model is calibrated for the euro area and the partner economy is assumed to be fully symmetric, sharing all the parameters that are made explicit in the Calibration Appendix.\textsuperscript{15} Without loss of generality and for the benefit of the exercise, let us fix the Armington elasticity of substitution between foreign and domestic retail goods at 3, a value

\textsuperscript{15}The households CES equation (1) composing the end-users bundles from home and foreign goods and the equivalent CES function for intermediate firms are augmented with an adjustment cost as in Erceg, Guerrieri and Gust (2006) in order to smooth somehow the expenditure switching.

Figure 5: Coefficients of the import demand and expenditure switching effect (beside trade elasticities) for $\frac{m}{\delta+1} = 0.26$.

6 A general equilibrium two-country application

Up to now, the whole analysis has been conducted on the grounds of static equations and partial equilibrium. Beside the structural pass-through, the strength of the relationship between exchange rate and prices after a shock that modifies the relative price of currencies is also affected by general equilibrium effects. This part of the analysis requires a comprehensive macroeconomic set-up. For this exercise, we rely on the two-country model described in de Walque et al. (2017), which entails all the open economy characteristics discussed supra. As far as the domestic economies are concerned, the model is entirely based on Smets and Wouters (2007). In order to focus on the differences in dynamic responses due to the chosen assumption regarding (i) the slope of the Phillips curve, (ii) the size of the distribution sector and (iii) the import content of export and of production, the model is calibrated for the euro area and the partner economy is assumed to be fully symmetric, sharing all the parameters that are made explicit in the Calibration Appendix.\textsuperscript{15} Without loss of generality and for the benefit of the exercise, let us fix the Armington elasticity of substitution between foreign and domestic retail goods at 3, a value

\textsuperscript{15}The households CES equation (1) composing the end-users bundles from home and foreign goods and the equivalent CES function for intermediate firms are augmented with an adjustment cost as in Erceg, Guerrieri and Gust (2006) in order to smooth somehow the expenditure switching.

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close to the one estimated by de Walque et al. (2017) for the period 1970Q1-2014Q4.\footnote{Such a value is admittedly high compared to the trade elasticities usually found in the NOEM literature, that are more around unity. However, it is not the value of the elasticity per se which is of interest, but how its implications in terms of expenditure switching effect are modified by the different variants examined. A careful analysis of the interaction between the trade elasticities and the two mechanisms studied here - input trade and distribution - is one of the topics of an estimated sequel of the present paper.} The log-linearized uncovered interest rate parity condition is given by

\begin{equation}
\dot{S}_t = E_t \dot{S}_{t+1} + \hat{\dot{r}}^*_t - \hat{\dot{r}}_t - \rho_n \hat{\dot{f}} a_t + \epsilon_s^t,
\end{equation}

where $\dot{S}_t$ represents the nominal exchange rate in relative deviation from steady-state, $\hat{\dot{r}}^*_t$ is the absolute percentage variation of the foreign short-term nominal interest, and $\hat{\dot{r}}_t$ its home counterpart. $\hat{\dot{f}} a_t$ is the percentage deviation with respect to steady-state of the domestic holdings of net foreign assets, ensuring the solution stability. Finally, $\epsilon_s^t$ is an auto-regressive process of order one, capturing exogenous variations in international financial market conditions.\footnote{Note that in this two-country symmetric set-up, this term is the only potential source of asymmetry. We limit it by calibrating $\rho_n$ to $10^{-5}$, a fairly low value that still ensure stability and robustness of the simulations.} Beside $\epsilon_s^t$, any shock in the domestic or the foreign economy that affects one of the interest rates through the reaction of the monetary policy, will modify the bilateral exchange rate. The monetary policy is represented by a Taylor rule based on Smets and Wouters (2007):

\begin{equation}
\hat{\dot{r}}_t = \rho_r \hat{\dot{r}}_{t-1} + (1 - \rho_r) (\rho_x \hat{\pi}_{c,t} + \rho_{yy} \hat{y}^d_t) + \rho \Delta_{yy} \big( \hat{y}^d_t - \hat{y}^d_{t-1} \big) + \epsilon_t^r,
\end{equation}

where $\hat{y}^d_t$ represents the differential between real domestic value added and potential domestic value added measured as the GDP prevailing in a counterfactual economy with flexible prices and wages.

The strength of the relationship between prices and the exchange rate can then be assessed by

\begin{equation}
PERR^i_{l,t} = \frac{\sum_{j=0}^{t} \hat{\pi}_{i,j}^l}{\sum_{j=0}^{t} \Delta S^l_j}, \quad l \in \{f,h,c\}
\end{equation}

where $PERR$ is the acronym for Price to Exchange Rate Ratio, a measure introduced in the exchange rate pass-through literature by Shambaugh (2008). Index $l$ associated with inflation specifies for which specific price - import, $f$, domestic producers, $h$, or consumer, $c$ - the relationship is computed. Index $i$ on each variable stresses that the strength of this co-movement is actually shock dependent.
6.1 Dynamic responses to an unexpected depreciation

As the first objective of the present contribution is to deal with the exchange rate pass-through, it seems natural to start the exercise with the study of the macroeconomic dynamics after an unexpected depreciation of the home currency. The persistence of the UIP autoregressive process $\varepsilon_t$ is set equal to 0.80 and the size of the shock is chosen to generate an on impact depreciation by one percent for the benchmark NOEM with $\delta = \phi_2^m = \rho_m = 0$. The UIP shock has the distinctive feature to be common to both economies and the full symmetry assumption adopted supra implies that the reaction of the foreign economy exactly mirrors this of the home economy.

Let us observe and discuss the implications of departing from the benchmark model by playing with (i) the slope of the import price Phillips curve, (ii) the size of the distribution sector and (iii) input trade in order to target a structural pass-through to consumption price, $ERP^{CP}$, of 0.04. The corresponding parametrizations can readily be inferred from Figure 4 and the impulse response functions obtained for the four variants considered are displayed on Figure 6.

6.1.1 Varying the slope of the import price NKPC

In the benchmark simulation (full black line), the slope of the import price Phillips curve is determined solely by the Calvo probability of not re-optimizing, $\xi^*_m$, which is set to 0.33 such that $ERP^{MP} = 0.4$ and $ERP^{CP} = 0.105$ (cf. Figure 4). The admittedly high trade elasticity implies that the difference in import and producers’ prices in the home (resp. foreign) economy triggers a strong reallocation of the global demand away (resp. towards) from foreign (resp. home) goods. The surge in foreign demand for home produced goods more than compensates the negative effect of the imported inflation on the home private absorption and pushes domestic producers price upwards.

The full red line on Figure 6 displays the consequences of increasing the Calvo probability to 0.56, which corresponds to an average price duration of 7 months instead of the initial 4.5. This has the consequence of drawing the structural pass-through downwards such that $ERP^{MP} = 0.15$ and $ERP^{CP} = 0.04$. The nominal rigidity determines the hump-shaped profile of the import price reaction which is directly transmitted to the consumption price. As the slope of the import price Phillips curve flattens, the price differential is reduced, thereby limiting the expenditure switching effect and the general equilibrium forces that generate inflation in the domestic producers’ price. Given the weight of the latter ($1 - \phi_c^{m,d} = 0.74$) in the consumption price index, this efficiently supplements the delayed reaction of import price to
limit the transmission of exchange rate to consumption price.

As already stated in the related literature, the low pass-through to consumption price obtained via the nominal stickiness in the import price Phillips curve is reached at the cost of unrealistically low transmission of the exchange rate to import price in the short run. Said differently, the initial reduction of the $\text{PERR}_{c,t}$ ratio the first four quarters after the shock is the outcome of the modification of the profile of the $\text{PERR}_{f,t}$ ratio. Therefore, the lack of flexibility found in the case of the ERPT disconnect in Section 4 also reflects here in the $\text{PERRs}$. This is one major argument on which Corsetti at al. (2008) grounds to introduce a distribution sector à la Burstein et al. (2003) in NOEM, in the hope of circumventing this identified weakness. Let us observe in a dynamic framework whether their intuition is indeed verified.

6.1.2 Varying the need for domestic distribution services

Setting the foreign exporters’ Calvo probability back to its initial value of 0.33, a structural pass-through to consumption price of 0.04 coherent with empirical findings can be obtained by considering a distribution margin $\delta/(1 + \delta)$ equal to 0.67. As stated in Propositions 3 and 5, at given trade openness, the size of the distribution sector alters the import demand. First, in the foreign exporters’ mark-up, it partially substitutes the own marginal cost and exchange rate for the price of home distribution services, reducing the relative price gap between foreign and home goods. Second, it attenuates the import sensitivity to this relative price, $\Omega_1$. The latter is reduced by a factor ten compared to the benchmark (cf. Figure 5, left panel). Both elements contribute to limit sharply the expenditure switching effect. The impulse responses to a UIP shock (full blue lines) computed on Figure 6 for the net trade and real GDP displays how the mechanism wipes the growth prospect of a depreciation off.

For the given UIP shock, the real and nominal consequences of increasing $\delta$ reduce the endogenous reaction of the monetary policy, which pushes upwards the exchange rate. It also enhances the foreign marginal cost, mirror of the home one, but dampens the reaction of the domestic producers’ price, and thus of domestic distribution prices. As the distribution margin increases, this second element opposes and dominates the first one within the foreign exporting firms’ mark-up.\footnote{The differential between the nominal stickiness in the import price and domestic producers' price Phillips curve plays an important role here, through the increased persistence associated to the home real marginal cost compared to this associated with the foreign one.} Given the parametrization choice and according to equation (20), the domestic producers’ price represents now $\delta/(\eta - 1) = 57$ percent of the foreign exporters’ mark-up, and the
import price reaction is strongly tuned down with respect to the benchmark (black full line) or the higher import price Calvo (red line). This also implies that the dynamics of the consumption price are more influenced by the domestic producers’ price which accounts now for 80 percent of it, computed as $(1 - \phi_c^{m,d}) + \phi_c^{m,d} \Psi / \eta$. Finally, the producers’ price itself is strongly affected by the depressed foreign demand for home goods relative to the benchmark NOEM (black full line).

Figure 6: Impulse responses to a UIP shock

Domestic distribution services are more efficient than nominal stickiness to obtain a low transmission of exchange rate fluctuations to consumption price. However Figures 6 illustrates
that the general equilibrium mechanisms at work do not strongly modify the conclusion stated in Corollary C2 for a static environment: in both cases, the lower transmission of exchange rate towards consumption price is reached by reducing the (empirically observed large) transmission to import price. This is particularly well illustrated by the Shambaugh (2008) $\text{PERR}_{s,t}$ and $\text{PERR}_{c,t}$ indicators. Reducing the gap between import and consumption prices at the retail level either via a flattening of the import price Phillips curve only (full red line) or through distribution services (full blue line) pushes down the $\text{PERR}$ concept of consumption price pass-through but at the cost of an ab initio strong deformation of the exchange rate-import price relationship. As such, when estimating an open economy model, the Calvo parameter will help capture the import price dynamics, the assumption of a distribution sector may supplement the trade elasticity $\lambda$ in dealing with some features of the observed real series, but none of them offers a credible potential to reconcile the high exchange rate/import price connectedness with the low transmission of currency price to consumption price. This conclusion is not astonishing as the very essence of both mechanisms is to bring the import price dynamics closer to the producers’ price one, reducing the relative price gap. This is obviously not the case for the trade input extension (cf. Corollaries C3 and C4).

6.1.3 Varying the import content of exports and of production

Transit goods and intermediate foreign inputs operate directly by rebalancing the respective weights of import price at the border and domestic prices within the end-user price index as highlighted in Proposition 2. Integrating import content of exports under the form of transit goods only by setting $\phi^m_x = 0.2$ (euro area calibration) and $\rho_m = 0$ would be equivalent to reducing the actual trade openness of the economies. As for the Calvo and the distribution sector, Figure 6 (dashed black line) focuses on the case where $\text{ERPT}^{CP} = 0.04$, which is obtained for an import content of production, $\rho_m$, set equal to 10 percent. It considers perfect complementarity between home and foreign inputs in the production process ($\lambda_m = 0$ in equation (12)), such that the expenditure switching effect is fully driven by $\Omega_1$, which is reduced to 0.27 according to Figure 5.

The larger $\rho_m$, the stronger the domestic marginal cost reaction to the exchange rate, which is translated into higher domestic producers’ price. Concurrently, for the chosen calibration, the respective weights of import price and domestic price into the consumption price index (i.e. $\phi^{m,d}_c$ and $1 - \phi^{m,d}_c$) are reset to 0.09−0.91 instead of 0.26−0.74 for the previous exercises displayed on Figure 6. Indeed, a share $\rho_m/(1 - \rho_m)$ of imports enters the consumption price index indirectly,
via a flat domestic price Phillips curve ($\xi = 0.75$), instead than directly, through a much steeper import price Phillips curve ($\xi^* = 0.33$). Depreciation becomes less inflationary and the real private home (resp. foreign) demand reacts less negatively (resp. positively). For the Leontief production function studied here, the foreign firms’ demand for home produced intermediate inputs does not increase with the depreciation of the domestic currency. Furthermore, the share of foreign goods concerned by the home households’ decision regarding their final goods composition is nearly divided by three compared to the benchmark case (black line) such that the reaction of the real variables is strongly mitigated, as observed above for the distribution services. At an horizon of four years, the import price to exchange rate relationship $PERR_{f,t}$ is left relatively unchanged with respect to the benchmark, while the relative sensitivity of the consumption price, $PERR_{c,t}$, is strongly reduced, extending the validity of Corollaries C3 and C4 to the case of a general equilibrium analysis.

6.1.4 On the role of foreign/domestic inputs substitutability ($\lambda_m$)

Let us now depart from the Leontief technology driving the mix of home/foreign intermediate inputs for the domestic firms in monopolistic competition and allow them to react more flexibly to changes in the imports relative price. We fix the trade elasticity $\lambda_m = \lambda = 3$ such that firms adopt the same behavior as the households regarding relative prices. A priori, one could think that such a flexibility should help reduce further the transmission of exchange rates to domestic prices. In the words of Campa and Goldberg (2010): "Calibrated price effects of exchange rates and import prices are smaller when economies can more flexibly substitute away from imported components into domestic components when producers are confronted with an adverse cost shock". However, Campa and Goldberg (2010) are reasoning in partial equilibrium and do not take into account the consequences of substitutability ($\lambda_m > 0$) on the expenditure switching effect (cf. Proposition 5).

Figure 7 compares the benchmark NOEM economy ($\delta = \phi^m_x = \rho_m = 0$, full black line) with two variants with import content of exports ($\phi^m_x = 0.2$) and import content of production ($\rho_m = 0.1$): the previous Leontief case ($\lambda_m = 0$, dashed black line) and the CES case ($\lambda_m = \lambda = 3$, full blue line). We learned from Sections 4 and 5 that parameter $\lambda_m$ does not intervene in any of the structural equations driving the exchange rate pass-through, but that it indeed affects the expenditure switching effect as in equation (25). Figure 5 displays that for $\rho_m = 0.1$ and $\lambda_m = \lambda$, the surge in $\Omega_2$ more than compensates for the drop in $\Omega_1$, and we observe in Figure 7 that this results in a reaction of the real variables that is pretty close to this obtained
for the benchmark economy. The domestic currency depreciation brings about an increased foreign demand for home produced goods, both as intermediate inputs and for final use. This demand pushes upwards the domestic producers’ price and, via this channel, the consumption price. The general equilibrium consequences on producers’ price of the expenditure switching effect ruin the intuitions based on a partial equilibrium reasoning. However, compared with the benchmark economy, inputs trade still helps to efficiently solve the puzzle of a high pass-through at the border and a low one at the consumption level, whatever the considered degree of substitutability at the firm level.

Figure 7: on the role of the domestic/foreign inputs substitutability
6.1.5 An example of a small open economy

Beside the fact that the import content of exports, especially under the form of intermediate inputs, helps combine low consumption price pass-through with high import price pass-through, it also offers a convincing narrative to model small open economies contrarily to the distribution sector. There is a priori no reason for a very open economy to require more distribution services than a less open one. Even worse, the pink line on Figure 1 highlights that this potential mechanism is much more limited for very open economies than for relatively closed ones. On the contrary, according to the left panel of Figure 2, it makes sense to establish a direct relationship between the openness of an economy and the importance of transit goods and intermediate foreign inputs.

On these grounds, let us modify the symmetric two country set-up to represent the trade relationship between the euro area and, say, Sweden, a small economy with an independent monetary policy and a degree of trade openness twice as large as this of the euro area ($\bar{m}/\bar{y} = 0.38$). The euro area accounts for roughly 50% of the Swedish international trade while Sweden represents 3% of the euro area imports. According to these numbers, the standard NOEM with no import for exports nor production would consider that the home bias of Sweden is one half (cf. Figure 1); the corresponding impulse responses to a 1% depreciation of the Swedish krona with respect to the euro are represented by the full black lines on Figure 8. They are characterized by a high reaction of consumer prices, interest rate, net trade and private demand. Note that in this example, the negative effect of the depreciation on private demand more than compensate the positive net trade effect, and leads to a small decline in real GDP.

Taking into account the import content of exports, $\phi^m_x$, evaluated at 0.29 (1995-2014 average) by the OECD for Sweden, and considering that it is entirely made of imported intermediate inputs, one may compute from equation (13) a home bias, $\phi_H$, equal to 0.9 (cf. Figure 1).\(^{19}\) Completing this information with the numbers presented in the calibration Appendix gives an exchange rate pass-through at the border equal to 0.38, close to the one reported by Campa and Goldberg (2010) for Sweden, and an exchange rate pass-through to consumer price equal to 0.04, respecting roughly the factor ten between these two measures usually reported in the literature (see Ortega et al., 2019). The impulse responses to the same UIP shock as considered in the standard model are represented by the dashed (resp. dotted) black lines on Figure 8 for an Armington trade elasticity equal to zero (resp. equal to three) at the firm level. The profile

\(^{19}\) Such a value for the share of foreign intermediate inputs in production is roughly in line with the input-output results for Sweden reported by Campa and Goldberg (2010) measuring it at 0.26.
of the price responses is quite different from the benchmark, with producers’ price rising quicker on impact due to their imported inputs, and stabilizing after one year due to the reaction of the domestic real activity. As noted earlier, the degree of substitutability between foreign and domestic inputs matters a lot in this respect: the lower it is, the lower the expenditure switching effect and the less the depreciation is growth enhancing. The profile of the producers’ price is directly transmitted to the consumer price index, of which it constitutes 87%, reducing considerably the transmission of the exchange rate shock in the short run with respect to the traditional calibration (full black lines). In the medium to long run, the relationship between exchange rate and consumption price depends critically on the $\lambda$ and $\lambda_m$ trade elasticities. The goal of the present contribution is only to draw the attention to this point, the next step, left for future research, would be to try to extract this information from the data by estimating the full model.

Figure 8: An illustration for a SOE, calibrated on Sweden, interacting with the euro area

In the blue solid lines of Figure 8, we consider a case where the home bias is imposed to be 0.9 by the modeler, without any rigorous calibration of inputs trade. Note that this is equivalent to considering that the import content of exports, $\phi^m_x$, is equal to 0.74 instead of 0.29 and consists
only in transit goods. Such a modelling strategy allows to recover the empirical coherence of a structural pass-through to import price ten times larger than this to consumption price, but the corresponding dynamic reactions to an exchange rate shock are exaggeratedly muted by such an artificial isolation of the nominal side of the economy. This illustrates that input trade helps small open economies to reconcile their large trade openness with a limited transmission of foreign prices to the consumption price in the short run, delaying it to further horizons via the larger nominal stickiness observed at the aggregate domestic producing sector level.

6.2 On spillovers

The previous subsection established some important results. First, the conclusions of Corollaries C1 to C4 still hold in general equilibrium for a set of fairly standard parameters. Second, under this calibration the intuition behind Proposition 5 is confirmed, reasserting the important link between the exchange rate pass-through and the real economy via the expenditure switching effect. Backing on the pioneering logic of Obstfeld and Rogoff (1995), this settles a direct channel between pass-through and international co-movements, formalized in the following corollary.

**Corollary to Proposition 5** *In a two-country symmetric framework, any mechanism that reduces the expenditure switching effect mitigates the terms of trade externality. This enhances (resp. limits) the international synchronization of real business cycles in the aftermath of country specific shocks to which the country policy rate reacts countercyclically (resp. procyclically). The last sentence of Proposition 5 states that import contents of exports under the form of either transit goods or foreign intermediate inputs reduce structurally the aggregate demand externality, while the distribution sector does not. As a result, one may expect the last mechanism to be more efficient than the former in eliciting real synchronization, whatever the considered shock.*

Let us consider the pure exchange rate shock studied so far that affects both economies in an inverse way and pushes them on divergent paths. Inflation increases in the economy with a depreciated currency, depressing its domestic absorption. The larger the expenditure switching effect, the more this economy adjusts imports downwards and exports upwards, increasing the chance that depreciation is growth enhancing. As the reverse holds for the trade partner, country specific inflations and outputs diverge. For this particular shock, any mechanism able to reduce the expenditure switching effect attenuates the discrepancy as illustrated on Figures 6 and 7.
Following Obstfeld and Rogoff (1995), the reasoning may be expanded to assess ex-ante the international consequences of any country-specific shock, using the UIP condition. A shock that pushes the home demand upwards requires more home and foreign productions. This is the usual aggregate demand externality (cf. e.g. Huang and Liu, 2007) synthesized by parameter $\Omega_3$ in equation (25). If at the same time the monetary authority lowers its policy rate, either as an endogenous expected reaction or as the initial unexpected shock, the monetary easing sustains the initial increase in the home economic activity through the implied devaluation, which handicaps the foreign economy. This is the so-called terms of trade externality. Again, the larger the expenditure switching effect, the more the terms of trade externality foils the aggregate demand externality, reducing the capacity of the model to generate ample co-movements. On the contrary, for all the shocks to which the central bank reacts by pushing the policy rate in the same direction as this of output, the monetary policy tempers somehow the demand in the home economy. The larger the expenditure switching effect, the more the implied home currency devaluation benefits to the foreign economic activity.

According to this typology, one expects that all the mechanisms mentioned in Proposition 5 contribute to improve the capacity of a model to generate endogenous co-movements by reducing the terms of trade externality after country-specific monetary policy shocks, productivity shocks and mark-up shocks.20 On the contrary, they would hinder this possibility in the aftermath of demand shocks in general. However, note that for shocks emanating directly from the demand for consumption and investment, the aggregate demand externality plays an important role. A mechanism that downsizes the elasticity between import and domestic private demand would inevitably reduce further the potential for demand shocks to drive real business cycles coordination. All this may be quickly verified with the symmetric two-country model used so far in Section 6 and running simulations for different types of shocks. The results of these experiments are displayed in Tables 1 and 2 below, the first one focusing on monetary and technology shocks, and the second one on risk-premium shocks à la Smets and Wouters (2007) and preference shocks. The persistence of the exogenous shock processes are fixed at conventional values in the literature. The two tables report cross-country correlations for real GDP and consumption prices, and the shock-dependent correlations between exchange rate with import price inflation on the one hand and with consumption price inflation on the other hand. In a two-country

20Following this reasoning, the results obtained in Proposition 5 regarding the role of the demand elasticity of the CES domestic production function are fully coherent with the conclusion of Burnstein et al. (2008), i.e., that low substitutability increases the potential of input trade to elicit real business cycle synchronization.
symmetric set-up, net trades are perfectly negatively correlated and for this variable we write down the ratio of the standard deviation of net trade to this of real GDP as an indicator, though imperfect, of the expenditure switching effect.

Table 1. Simulated correlations for mon. pol. and prod. shocks

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<tr>
<th>$\xi_m$</th>
<th>$\delta$</th>
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Monetary policy shocks (i.i.d.)

| $\text{corr}(\hat{y}_t, \hat{y}_t^*)$ | 0.07 | 0.12 | 0.36 | 0.21 | 0.00 |
| $\text{std}(\hat{nx}_t)/\text{std}(\hat{y}_t)$ | 0.79 | 0.69 | 0.62 | 0.73 | 0.69 |
| $\text{corr}(\hat{\pi}_{c,t}, \hat{\pi}_{c,t}^*)$ | -0.45 | -0.29 | -0.40 | -0.18 | -0.22 |
| $\text{corr}(\hat{\pi}_{f,t}, \hat{\Delta \delta}_t)$ | 0.94 | 0.83 | 0.85 | 0.94 | 0.94 |
| $\text{corr}(\hat{\pi}_{c,t}, \hat{\Delta \delta}_t)$ | 0.75 | 0.58 | 0.62 | 0.50 | 0.50 |

Total factor productivity shocks (AR1: 0.9)

| $\text{corr}(\hat{y}_t, \hat{y}_t^*)$ | 0.13 | 0.14 | 0.47 | 0.37 | 0.00 |
| $\text{std}(\hat{nx}_t)/\text{std}(\hat{y}_t)$ | 2.16 | 2.09 | 0.68 | 0.64 | 1.90 |
| $\text{corr}(\hat{\pi}_{c,t}, \hat{\pi}_{c,t}^*)$ | 0.78 | 0.90 | 0.61 | 0.68 | 0.74 |
| $\text{corr}(\hat{\pi}_{f,t}, \hat{\Delta \delta}_t)$ | 0.72 | 0.61 | 0.81 | 0.78 | 0.67 |
| $\text{corr}(\hat{\pi}_{c,t}, \hat{\Delta \delta}_t)$ | 0.30 | 0.11 | 0.34 | -0.07 | -0.07 |

Table 1: Simulated correlations for mon. pol. and prod. shocks

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Monetary policy shocks (i.i.d.)

| $\text{corr}(\hat{y}_t, \hat{y}_t^*)$ | 0.07 | 0.12 | 0.36 | 0.21 | 0.00 |
| $\text{std}(\hat{nx}_t)/\text{std}(\hat{y}_t)$ | 0.79 | 0.69 | 0.62 | 0.73 | 0.69 |
| $\text{corr}(\hat{\pi}_{c,t}, \hat{\pi}_{c,t}^*)$ | -0.45 | -0.29 | -0.40 | -0.18 | -0.22 |
| $\text{corr}(\hat{\pi}_{f,t}, \hat{\Delta \delta}_t)$ | 0.94 | 0.83 | 0.85 | 0.94 | 0.94 |
| $\text{corr}(\hat{\pi}_{c,t}, \hat{\Delta \delta}_t)$ | 0.75 | 0.58 | 0.62 | 0.50 | 0.50 |

Total factor productivity shocks (AR1: 0.9)

| $\text{corr}(\hat{y}_t, \hat{y}_t^*)$ | 0.13 | 0.14 | 0.47 | 0.37 | 0.00 |
| $\text{std}(\hat{nx}_t)/\text{std}(\hat{y}_t)$ | 2.16 | 2.09 | 0.68 | 0.64 | 1.90 |
| $\text{corr}(\hat{\pi}_{c,t}, \hat{\pi}_{c,t}^*)$ | 0.78 | 0.90 | 0.61 | 0.68 | 0.74 |
| $\text{corr}(\hat{\pi}_{f,t}, \hat{\Delta \delta}_t)$ | 0.72 | 0.61 | 0.81 | 0.78 | 0.67 |
| $\text{corr}(\hat{\pi}_{c,t}, \hat{\Delta \delta}_t)$ | 0.30 | 0.11 | 0.34 | -0.07 | -0.07 |

Note: the model has been simulated for 6000 periods and the moments are computed discarding the first 500 ones.

As expected from the previous discussion, the baseline model with no-distribution-no-foreign-intermediate-inputs performs poorly in terms of cross-border GDP correlations if we consider that the economy is hit uniquely by uncorrelated country-specific monetary policy shocks. Consumption prices become negatively correlated: in the home economy surprised by a hike in the policy rate, prices are pushed downwards directly in reaction to the shock, and indirectly through the ensuing currency appreciation. In the foreign economy the effect of the depreciation dominates and the consumption price increases.

More nominal stickiness at the import price level reduces the connection of both import
and consumption prices with the exchange rate. The latter point implies an improvement in the cross-border correlation of consumption price inflations. The reduction of the expenditure switching effect observed from the relative volatility of net exports with respect to GDP helps marginally to increase the GDPs’ co-movement. The introduction of a (large) distribution sector does also reduce the conditional correlation between consumer price inflation and exchange rate variations. As for the UIP shock (cf. $PERR_s$ on Figure 6, full blue line), this outcome is obtained at the cost of a reduction of the link between import price and currency price. This mechanism does not help much to obtain more cross-border CPI co-movements, but it improves strongly the GDPs synchronization by reducing the expenditure switching effect. Finally, as long as perfect complementarity is assumed at the firm level, import content of exports and production are also efficient mechanisms to induce endogenous GDPs co-movements. This confirms the results obtained by Huang and Liu (2007). However, in their multi-stage production sector with firms setting prices according to fixed duration Taylor contract, these authors develop a set-up in which the importance of intermediate foreign inputs raises together with overall producers’ price stickiness. We show that their results for GDPs synchronization still hold at constant slope for the aggregate Phillips curve of the production sector. Furthermore, we extend their observations to the pass-through puzzle. Indeed, foreign value added in domestic production reduces the consumer price-exchange rate connection while leaving the import price strongly correlated with the relative price of currency. This helps to improve the cross-border correlation of the consumption price indices. The last column of the upper part of Table 1 is pretty interesting: it indicates that when the expenditure switching effect is revitalized via the substitutability between home and foreign intermediate inputs, the cross border GDP synchronization collapses, while all the properties observed in the previous column regarding the nominal side of the economy remain valid. Finally, as highlighted in the Corollary to Proposition 5, the reduction of the link between import and private aggregate demand induced by the alternatives demand for import (cf. parameter $\Omega_3$ of equation (25)) limits the aggregate demand externality with respect to the distribution sector mechanism, explaining the relative poorer performance in terms of real GDPs coordination.

A home technology shock depresses the producers price and the central bank reacts by decreasing its policy rate, depreciating the currency. The implied movements in import prices mitigates somehow the drop in home consumption price while it decreases foreign consumption prices, yielding a positive wealth effect. For the benchmark NOEM calibration (left column) such a shock yields some real GDP international co-movement and much more correlation in
CPI inflations. When the import price stickiness is increased from 0.33 to 0.56 the expenditure switching effect is only slightly affected and real synchronization is not much modified. On the nominal side, the increased persistence in import price emphasizes the above described import price effect and the CPI inflations are even more connected. The distribution sector operates exactly in the other direction: it strongly decreases the expenditure switching effect, tuning down the terms of trade externality and improving real co-movements. On the nominal side, as the home economy exports less, domestic producers’ prices decrease further, the central bank lowers even more its policy rate. Among the drivers of the import price, the increase in the marginal cost of the foreign producers more than compensates the reduction in the price of domestic distribution services and the import price-exchange rate correlation increases relatively to the benchmark NOEM. However, due to the relative drop in producers’ prices, this is not translated in the same proportion into an increased CPI-exchange rate correlation. In the foreign economy the wealth effect of the appreciation increases, but this demand is now more fulfilled by foreign producers whose prices increase. In the end, the cross-border CPI inflations correlation turns out to be somewhat lower. Input trade produces very similar outcomes on the real side: under perfect complementarity of home/foreign intermediate inputs the terms of trade externality is strongly reduced and real outputs synchronization is enhanced. As already displayed and discussed by Burstein, Kurtz and Tesar (2008) for technology shocks, and for the reasons made explicit in Proposition 5, the more firms have the possibility to switch between foreign and domestic inputs, the lower the cross-border real correlation. On the nominal side, the input trade story is pretty different from the distribution services one. The home currency depreciation rises the price of the foreign intermediate input, braking the drop of domestic producers’ marginal cost. At the same time, the share of import price in the CPI index drops from 0.26 to 0.10, closing down the CPI-exchange rate correlation independently of the degree of substitutability $\lambda_m$. Corollary to Proposition 5 is still verified for technology shocks that typically move economic activity and the policy interest rate in opposite directions and input trade is the only mechanism allowing to combine low exchange rate-CPI correlation while keeping a high exchange rate-import price interrelationship.

Table 2 displays the same co-movements statistics for a Smets and Wouters (2007) risk-premium shock, i.e. a shock that drives a wedge between the central bank risk-free rate and the interest rate actually faced by households. Such a shock has the distinctive feature of

\[21\text{Even though somewhat less than under the distribution sector mechanism, due to the above mentionned decrease in aggregate demand externality, via parameter $\Omega_3$ (equation (25)).}\]
moving investment and consumption together. The Corollary to Proposition 5 also applies to this shock to which the policy rate reacts procyclically. If the risk-premium narrows in the home economy, home domestic demand increases together with the demand for imports which is pushed even further by the home currency appreciation resulting from the central bank endogenous reaction. Confronted with increased import prices and home economy external demand, the foreign economy reduces its own domestic demand. The strong positive GDPs correlation is entirely driven by the respective net trades that slow down the home GDP and encourage the foreign one. Any mechanism that reduces the terms of trade externality tends to decrease this positive correlation and it is more particularly the case for the foreign intermediate inputs than for the distribution sector, as the former also reduces the import elasticity to absorption, $\Omega_3$ (cf. Proposition 5), and therefore the aggregate demand externality. Under input trade (last two columns on the right), the decrease in import price implied by the initial shock affects the producers’ marginal cost and calm down their price increase compared to the other models. This is translated in the CPI index with a larger weight as the home bias increases from 0.74 to 0.90, reducing the conditional correlation between consumption price and exchange rate and eliciting cross-border CPIs’ correlation.
Table 2. Simulated correlations for demand-like shocks

<table>
<thead>
<tr>
<th>Parameters</th>
<th>ρ = 0</th>
<th>ρ = 0</th>
<th>ρ = 2</th>
<th>ρ = 0.2 and ρ = 0.12</th>
</tr>
</thead>
<tbody>
<tr>
<td>ξ^*_m = 0.33</td>
<td>0.64</td>
<td>0.64</td>
<td>0.57</td>
<td>0.36</td>
</tr>
<tr>
<td>ξ^*_m = 0.56</td>
<td>2.94</td>
<td>2.81</td>
<td>2.40</td>
<td>1.54</td>
</tr>
<tr>
<td>ξ^*_m = 0.33</td>
<td>0.64</td>
<td>0.87</td>
<td>0.72</td>
<td>0.85</td>
</tr>
<tr>
<td>ξ^*_m = 0.33 and δ = 0</td>
<td>0.73</td>
<td>0.61</td>
<td>0.71</td>
<td>0.79</td>
</tr>
<tr>
<td>ρ_m = 0</td>
<td>0.40</td>
<td>0.21</td>
<td>0.34</td>
<td>0.09</td>
</tr>
<tr>
<td>ρ_m = 0</td>
<td>0.40</td>
<td>0.21</td>
<td>0.34</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Risk-premium shocks (AR1 0.2)

<table>
<thead>
<tr>
<th>ρ^* x</th>
<th>0 = ρ_m</th>
<th>ρ^* x = 0 = ρ_m</th>
<th>ρ^* x = 0 = ρ_m</th>
<th>λ_m = 0</th>
<th>λ_m = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr(ŷ_t, ỹ^*_t)</td>
<td>0.64</td>
<td>0.64</td>
<td>0.57</td>
<td>0.36</td>
<td>0.47</td>
</tr>
<tr>
<td>std(ň_xt)/std(ŷ_t)</td>
<td>2.94</td>
<td>2.81</td>
<td>2.40</td>
<td>1.54</td>
<td>2.09</td>
</tr>
<tr>
<td>corr(π_xt, ỹ^*_t)</td>
<td>0.64</td>
<td>0.87</td>
<td>0.72</td>
<td>0.85</td>
<td>0.90</td>
</tr>
<tr>
<td>corr(π_xt, Δšt)</td>
<td>0.73</td>
<td>0.61</td>
<td>0.71</td>
<td>0.79</td>
<td>0.75</td>
</tr>
<tr>
<td>corr(π_xt, Δš)</td>
<td>0.40</td>
<td>0.21</td>
<td>0.34</td>
<td>0.09</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Preference shocks (AR1 0.9)

<table>
<thead>
<tr>
<th>ρ^* x</th>
<th>0 = ρ_m</th>
<th>ρ^* x = 0 = ρ_m</th>
<th>ρ^* x = 0 = ρ_m</th>
<th>λ_m = 0</th>
<th>λ_m = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr(ŷ_t, ỹ^*_t)</td>
<td>0.57</td>
<td>0.56</td>
<td>0.59</td>
<td>0.47</td>
<td>0.48</td>
</tr>
<tr>
<td>std(ň_xt)/std(ŷ_t)</td>
<td>9.00</td>
<td>8.79</td>
<td>5.58</td>
<td>4.07</td>
<td>7.34</td>
</tr>
<tr>
<td>corr(π_xt, ỹ^*_t)</td>
<td>0.45</td>
<td>0.70</td>
<td>0.27</td>
<td>0.59</td>
<td>0.82</td>
</tr>
<tr>
<td>corr(π_xt, Δšt)</td>
<td>0.85</td>
<td>0.71</td>
<td>0.88</td>
<td>0.91</td>
<td>0.86</td>
</tr>
<tr>
<td>corr(π_xt, Δš)</td>
<td>0.51</td>
<td>0.34</td>
<td>0.55</td>
<td>0.31</td>
<td>0.19</td>
</tr>
</tbody>
</table>

ξ^*_m: Calvo probability of not reoptimizing import price, δ: proportion of distribution services per unit of final good, ρ^*_x: import contents of export, ρ_m: proportion of foreign inputs for the production of one intermediate good, λ and λ_m: Armington trade elasticity for households and firms respectively.

Note: the model has been simulated for 6000 periods and the moments are computed discarding the first 500 ones.

Interestingly, for consumption preference shocks, the consumption and investment react in opposite directions in the hit country. In the canonical NOEM, this has the consequence that aggregate demand externality is somehow reduced while the terms of trade externality plays fully for the directly imported part of consumption, while it only limits the reduction of imports for investment purposes. In this configuration, limiting the expenditure switching effect is much less detrimental for the real GDPs correlation whatever the assumption/column considered. Regarding the pass-through disconnect and the cross-country correlation of the CPIs, conclusions are left unchanged: international trade in intermediate inputs is definitely more efficient than the distribution sector assumption.

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^22The same hold true for investment relative price shocks (not shown here).
7 Conclusion

In the previous lines we have tried to better understand the respective macroeconomic implications of a distribution sector and of the inclusion of foreign inputs in production before price differentiation. The first mechanism has usually been advocated to be efficient in solving the exchange rate pass-through puzzle and the second one to significantly improve the international synchronization of business cycles. Our systematic comparison brings the surprising conclusion that the allocation of roles should actually be reversed. The distribution sector is very good at mitigating the connection between the exchange rate and the consumption price, but this occurs via an attenuation of the exchange rate/import price relationship, in opposition to empirical evidence. On the contrary, this mechanism is pretty efficient in generating cross-border real spillovers after country specific shocks. It improves strongly spillovers after shocks that move output and the relative price of currency in opposite directions and they are not much deteriorated, if at all, after demand-like shocks.

This reversal of the outcome with respect to the primary goal also holds for input trade, that is the inclusion of import price in the marginal cost of domestic producers with a market power and staggered prices. It systematically reduces the link between consumption price and exchange rate in the short run without affecting the strong relationship between import price and exchange rate. For all the shocks considered, the mechanism increases the cross-border correlation between consumption prices. Under perfect complementarity, it is also efficient in improving the synchronization of the real economic activity except after demand-like shocks. In our view, the input trade mechanism provides a strong economic rationale for potentially high home bias even for the very open economies, with all the benefits in terms of exchange rate disconnect it may imply.

The ultimate test would be to bring the model to the data and to check how a two-country estimated model endowed with the two channels discussed at length in this paper allows to indeed (i) improve the estimation of both the import and consumption price dynamics and (ii) increase the real and nominal synchronization of macro-variables. This is at the agenda of future research.
References


[34] Özyurt, S. (2016), *Has the exchange rate pass-through recently declined in the euro area?*, ECB WP n°1955.


Technical Appendix

Proof of Propositions 3 and 4

The domestic producers’ Phillips curves for the home and foreign markets are given by equations (17) to (18). The corresponding foreign producers’ Phillips curves are obtained by symmetry by systematically switching on/off the "**" indicator. The log-linearized marginal cost is equal to

\[ \tilde{m}_{ch,t} = \frac{(1 - \rho_m) \eta}{\eta - 1} \hat{A}_{h,t} + \frac{\rho_m \eta}{\eta - 1} \hat{p}_{f,t} \]

with \( \hat{A}_{h,t} = \alpha \hat{r}_{h,t}^k + (1 - \alpha) \hat{w}_{h,t} - \epsilon_{h,t}^o \)

Rewriting the inflations equations for the home domestic production and import in terms of price levels, we get

\[ \hat{p}_{h,t}^r + \hat{p}_{c,t} = \Gamma_{h,t} + \Psi_h \left[ \frac{(1 - \rho_m) \eta}{\eta - 1} \hat{A}_{h,t} + \frac{\rho_m \eta}{\eta - 1} \hat{p}_{f,t}^r \right] \]
\[ \hat{p}_{f,t}^r + \hat{p}_{c,t} = \Gamma_{f,t} + \Psi_f \left[ \frac{\eta - 1 - \delta}{\eta - 1} \left\{ \frac{(1 - \rho_m)^* \eta^*}{\eta^* - 1} \hat{A}_{h,t}^* + \frac{\rho_m^* \eta^*(-1)}{\eta^* - 1} \hat{p}_{f,t}^r \right\} + \hat{s}_t \right] + \frac{\delta \Psi_f}{\eta - 1} \hat{p}_{f,t}^r, \]

where

\[ \Gamma_{h,t} = \frac{\hat{p}_{h,t-1} + \beta \hat{p}_{h,t+1}}{1 + \beta + \frac{(1 - \xi)(1 - \beta \xi)}{\xi} \frac{\eta - 1 - \delta}{\eta - 1}} \]
\[ \Gamma_{f,t} = \frac{\hat{p}_{f,t-1} + \beta \hat{p}_{f,t+1}}{1 + \beta + \frac{(1 - \xi)(1 - \beta \xi)}{\xi} \frac{\eta - 1 - \delta}{\eta - 1}} \]
\[ \Psi_h = \frac{(1 - \xi)(1 - \beta \xi) \frac{\eta - 1 - \delta}{\eta - 1} + \xi(1 + \beta)}{(1 - \xi)(1 - \beta \xi) \frac{\eta - 1 - \delta}{\eta - 1} + \xi(1 + \beta)} \]
\[ \Psi_f = \frac{(1 - \xi^*_m)(1 - \beta \xi^*_m)}{(1 - \xi^*_m)(1 - \beta \xi^*_m) + \xi^*_m(1 + \beta^*_{m})} \]

Substituting for \( \hat{p}_{h,t}^r \) into \( \hat{p}_{f,t}^r \) (and for \( \hat{p}_{h,t}^* \) into \( \hat{p}_{f,t}^* \)) first and for \( \hat{p}_{f,t}^r \) into \( \hat{p}_{f,t}^r \) afterwards, one obtains

\[ \hat{p}_{f,t}^r = \left( \frac{\Psi_f^* - \Psi_f^{p*}}{\Psi_f^{p*}} \right) \hat{s}_t + \hat{p}_{f,t}^* \hat{A}_{h,t}^* + \frac{\Psi_f^*}{\Psi_f^{p*}} \left( \hat{p}_{f,t}^* \hat{A}_{h,t}^* + \hat{p}_{f,t}^* \right) + \frac{\Psi_f^*}{1 - \Psi_f^{p*}} \]

\[ \text{where} \]
\[ \Psi_f^* = \frac{\Psi_f}{\frac{1 - \delta^*}{\eta - 1} \Psi_h \frac{\rho_m \eta}{\eta - 1}} \eta^* - 1 - \delta^* (1 - \rho_m) \eta \]
\[ \tilde{p}_{f,t}^* = \frac{\Psi_f}{\frac{1 - \delta^*}{\eta - 1} \Psi_h \frac{\rho_m \eta}{\eta - 1}} \eta^* - 1 - \delta^* \rho_m \eta \]
\[ \tilde{p}_{f,t}^* = \frac{\Psi_f}{\frac{1 - \delta^*}{\eta - 1} \Psi_h \frac{\rho_m \eta}{\eta - 1}} \frac{\eta - 1 - \delta}{\eta - 1} \]
\[ \tilde{R}_{f,t}^* = \frac{\Gamma_{f,t} + \Psi_f \left[ \frac{(1 - \rho_m) \eta}{\eta - 1} \hat{A}_{h,t} - \hat{p}_{c,t} \right]}{1 - \frac{\delta}{\eta - 1} \Psi_h \frac{\rho_m \eta}{\eta - 1}} \frac{\delta}{\eta - 1} \]
The coefficient of the exchange rate is the structural ERPT:

\[
ERPT^{MP} = \frac{\Psi_f^s - \Psi_f^{ps} \Psi_f^{ss}}{1 - \Psi_f^{ps} \Psi_f^{pp}}
\]

and

\[
ERPT^{MP}_{\xi=\xi^*} = \frac{(\eta - 1)^2 - \delta \eta + \delta}{(\eta - 1)^2 - \delta \rho_m \eta},
\]

\[
ERPT^{MP}_{\rho_m=\rho^*_m} = \frac{\Psi_f \eta - 1 - \delta}{\eta - 1},
\]

\[
ERPT^{MP}_{\delta=\delta^*} = \frac{1 - \rho_m^* \eta}{\eta - 1} \Psi_f \Psi_f.
\]
## Calibration of the two-country model

Table A1: Calibration of the two-country symmetric model (on euro area)

<table>
<thead>
<tr>
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<th>Big ratios</th>
<th>Monetary policy</th>
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</thead>
<tbody>
<tr>
<td>$\frac{\epsilon}{\bar{y}}$</td>
<td>0.56</td>
<td>interest rate persistence ($\rho_r$) 0.9</td>
</tr>
<tr>
<td>$\frac{\bar{r}}{\bar{y}}$</td>
<td>0.20</td>
<td>reaction to inflation ($\rho_\pi$) 1.6</td>
</tr>
<tr>
<td>$\frac{\bar{m}}{\bar{y}}$</td>
<td>0.20</td>
<td>reaction to output gap ($\rho_{yy}$) 0.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>reaction to output gap variation ($\rho_{\Delta yy}$) 0.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Households</th>
<th>Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>external habit</td>
<td>0.7</td>
<td>Cobb-Douglas capital share ($\alpha$) 0.33</td>
</tr>
<tr>
<td>hh relative risk aversion</td>
<td>1.2</td>
<td>capital depreciation rate 0.025</td>
</tr>
<tr>
<td>inv. elast. of effort w.r.t. wage</td>
<td>2</td>
<td>inv. adjustment cost 4</td>
</tr>
<tr>
<td>Calvo prob. wage</td>
<td>0.75</td>
<td>demand price elasticity ($\eta$) 4.5</td>
</tr>
<tr>
<td>hh Armington trade elasticity ($\lambda$)</td>
<td>3</td>
<td>Calvo prob. dom. price ($\xi$) 0.75</td>
</tr>
<tr>
<td>foreign/dom. goods adjust. cost</td>
<td>4</td>
<td>Calvo prob. imp. price ($\xi^*_m$) 0.33</td>
</tr>
<tr>
<td></td>
<td>firms’ Armington trade elasticity ($\lambda_m$) 3 or 0</td>
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</tr>
<tr>
<td></td>
<td>foreign/dom. goods adjust. cost</td>
<td>4</td>
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