MONETARY POLICY IN INCOMPLETE MARKET MODELS: THEORY AND EVIDENCE

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- Workhorse model in public economics: Bewley-Imrohoroglu-Huggett-Aiyagari incomplete markets model.
 - Matches joint distribution of earnings, consumption and wealth
 - Generates realistic distribution of MPCs
 - Can generate realistic consumption responses to transitory income and transfers.

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 - Matches joint distribution of earnings, consumption and wealth
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- Workhorse model in monetary economics: Representative-Agent New-Keynesian model.
 - Nominal rigidities allow output to be demand determined.
 - Meaningful role for monetary policy.
 - Can match the data.

- Research frontier: Combine
 - Representative-Agent New-Keynesian model.
 - Aiyagari model.
 - $\hookrightarrow AiyaGalí$
 - Allows for demand determined output and
 - Consumption responses in line with the data

- Research frontier: Combine
 - Representative-Agent New-Keynesian model.
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 - Allows for demand determined output and
 - Consumption responses in line with the data
- Our Objective: Estimate the new model.
 - ► Incomplete Markets (No Ricardian Equivalence): Shocks → Government budget → Fiscal Policy → Prices, ...
 - Consequence I: Need to estimate the response of fiscal policy.
 - Consequence II:
 - Re-estimate other parameters (Price-rigidity, ...)

METHODOLOGY: IRF-MATCHING

- Methodology: Impulse Response Function Matching.
 - 1. Use identified technology shocks. Need to take into account:
 - Monetary policy response (FFR).
 - Fiscal policy response (Govt. Spending, Revenue, Transfers, Debt).
 - 2. Use identified monetary policy shocks. Need to take into account:
 - Fiscal policy response (Govt. Spending, Revenue, Transfers, Debt).
- Model Impulse Response Functions.
 - 1. Compute non-linear IRF to MIT shock
 - Following Boppart, Krusell & Mitman (2018) can treat as numerical derivative in sequence space to provide a linearized solution to the model with aggregate risk.
 - 2. Pick parameters to minimize distance between model & data IRF.

NEUTRAL TECHNOLOGY SHOCKS

- ▶ Bocola et al (2016).
- Identified innovations to labor-augmenting technology. Details
- Series extends back to 1947.

MONETARY POLICY SHOCKS

- ▶ Romer-Romer (2004) extended by Wieland-Yang (2017).
- These are residuals from a regression of the target federal funds rate on lagged values and the Federal Reserve's information set based on Greenbook forecasts.
- Series extends back to 1969.
- Results are qualitatively similar when we use monetary policy shocks measured with high frequency identification, but those series are much shorter.

CONSTRUCTION OF FISCAL VARIABLES

- Measure government Spending, Revenue, and Transfers in the data.
- Source: NIPA
- Coverage: Federal, State and Local government.
- Ensure that variables are defined consistently with their meaning in the model and that the budget constraint holds.

Variable Construction Details

ESTIMATING IRFS

- ▶ Outcome variable *X*.
- Identified shock ξ .
- Estimated IRF:

$$100 * (\log(X_{t+k}) - \log(X_{t-1})) = \beta_k \log(\xi_t) + \varepsilon_t$$

WHAT HAPPENS AFTER A TECHNOLOGY SHOCK?



WHAT HAPPENS AFTER A TECHNOLOGY SHOCK?



WHAT HAPPENS AFTER MONETARY POLICY SHOCK?



WHAT HAPPENS AFTER MONETARY POLICY SHOCK?



MODEL: HOUSEHOLDS

- Continuum of ex-ante identical households
- Preferences over consumption and leisure
- Stochastic (uninsured) labor productivity
- Can save in one-period uncontingent assets
- No borrowing
- HH budget constraint:

$$Pc + a' = (1 + r^{a})a + P(1 - \tau)whs + T$$

where *P*: price level, *c*: consumption, *a*: nominal savings, r^a : return on savings, τ : tax, *w*: real wage, *h*: hours, *s*: productivity, *T*: transfers

Model, Detailed Exposition

MODEL: PRODUCTION AND PRICES

► Hours and Wages:

- Recruiting firms aggregate differentiated HH labor services
- Sell to intermediate goods produces
- Union sets nominal wages as if subject to Rotemberg (1982) adjustment costs

MODEL: PRODUCTION AND PRICES

► Hours and Wages:

- Recruiting firms aggregate differentiated HH labor services
- Sell to intermediate goods produces
- Union sets nominal wages as if subject to Rotemberg (1982) adjustment costs
- Output and Prices:
 - Final good produces aggregate continuum of intermediates
 - Intermediate production Cobb-Douglas in capital and labor
 - Intermediate firms set prices as if subject to Rotemberg (1982) adjustment costs

MODEL: GOVERNMENT

Government taxes labor income and provides nominal transfers:

 $\tilde{T}(wsh) = -T + \tau Pwsh$

- Government fully taxes firm profits $P_t d_t$
- Government taxes capital income at rate τ_k
- ▶ Government issues nominal bonds *B^g*
- Exogenous unvalued expenditures G_t
- Government budget constraint given by:

$$B_{t+1}^g = (1+i_t)B_t^g + G_t - P_t d_t - \tau_k (r_t^k - \delta)K_t - \int \tilde{T}_t(w_t s_t h_t) d\Omega$$

MONETARY POLICY IN COMPLETE MARKETS

The complete markets economy arises as a special case when there is no idiosyncratic risk:

$$\begin{split} Y_{t}^{CM} &= Z_{t}H_{t}^{CM} &= C_{t}^{CM} + g_{t} + F + \frac{\theta}{2} \left(\pi_{t}^{CM} - \overline{\Pi}\right)^{2} Y_{t}^{CM} \\ w_{t}^{CM} (1 - \tau_{t}) (C_{t}^{CM})^{-\sigma} &= D(H_{t}^{CM})^{\phi} \\ u_{c} (C_{t}^{CM}) &= (C_{t}^{CM})^{-\sigma} &= \beta \frac{1 + i_{t+1}}{1 + \pi_{t+1}^{CM}} u_{c} (C_{t+1}^{CM}) = \beta (1 + r_{t+1}^{CM}) (C_{t+1}^{CM})^{-\sigma} \\ (1 - \varepsilon) + \frac{\varepsilon}{1 - \alpha} \frac{w_{t}^{CM}}{Z_{t}} &= \theta \left(\pi_{t}^{CM} - \overline{\Pi}\right) \pi_{t}^{CM} - \frac{1}{1 + r_{t}^{CM}} \theta \left(\pi_{t+1}^{CM} - \overline{\Pi}\right) \pi_{t+1} \frac{Y_{t}}{Y_{t}} \end{split}$$

Note that output is linear in hours, Y = ZH, and that the function describing the disutility of labor is g(h)

Complete Markets:

Steady state in CM: C_{ss}^{CM} , H_{ss}^{CM} , Y_{ss}^{CM} , w_{ss}^{CM}

Monetary Policy shock:

$$i_0=i^*,i_1,i_2,\ldots,i_t,\ldots,i^*$$

~ . .

Consumption/Hours/Output/Wages Responses:

Consumption:
$$\gamma_t^C = \frac{C_t^{CM}}{C_{ss}^{CM}}$$

Hours/Output: $\gamma_t^H = \gamma_t^Y = \frac{H_t^{CM}}{H_{ss}^{CM}} = \frac{Y_t^{CM}}{Y_{ss}^{CM}}$
Wages: $\gamma_t^w = \frac{w_t^{CM}}{w_{ss}^{CM}}$,

Incomplete Markets:

- Steady state in IM: C_{ss}^{IM} , H_{ss}^{IM} , Y_{ss}^{IM} , w_{ss}^{IM}
- Take scaled CM Monetary Policy shock:

$$\begin{aligned} 1 + i_0^{IM} &= 1 + i_{ss}^{IM}, 1 + i_1^{IM} = (1 + i_{ss}^{IM}) \frac{1 + i_1}{1 + i^*}, \\ 1 + i_2^{IM} &= (1 + i_{ss}^{IM}) \frac{1 + i_2}{1 + i^*}, \dots, 1 + i_t^{IM} = (1 + i_{ss}^{IM}) \frac{1 + i_t}{1 + i^*}, \dots \end{aligned}$$

Households receive real transfers in addition to labor earnings:

$$\begin{split} \Gamma_{t}^{IM} &= d_{t}^{IM} + \tau w_{t}^{IM} H_{t}^{IM} + \frac{B_{t+1}^{IM} - B_{t}^{IM}(1+i_{t}^{IM})}{P_{t}^{IM}} - g_{t}^{IM}, \\ \Gamma^{IM,ss} &= d^{IM,ss} + \tau w^{IM,ss} H_{ss}^{IM,ss} + \frac{B^{IM,ss} - B^{IM,ss}(1+i^{IM,ss})}{P^{IM,ss}} - g^{IM,ss} \end{split}$$

Each household *i* receives a share $\lambda_{i,t}$ of the transfer at time *t*, such that $\int \lambda_{i,t} di = 1$. We denote $\gamma_t^{\Gamma} = \Gamma_t^{IM} / \Gamma_{ss}^{IM}$.

HOUSEHOLD PROBLEM

Solve the following dynamic program in response to the monetary policy shock:

$$V_{t}(a_{i,t}, s_{i,t}) = \max_{\substack{c_{i,t}^{IM}, a_{i,t+1} \ge 0}} u(c_{i,t}^{IM}, h_{i,t}) + \beta \mathbb{E}_{s_{t+1}} V_{t+1}(a_{i,t+1}, s_{i,t+1})$$

subj. to $c_{i,t}^{IM} + a_{i,t+1} = \frac{(1 + i_{t}^{IM})}{(1 + \pi_{t}^{IM})} a_{i,t} + (1 - \tau) \gamma_{t}^{w} \gamma_{t}^{H} w_{ss}^{IM} h_{i,ss}^{IM} s_{i,t}$
 $+ \lambda_{i,t} \gamma_{t}^{\Gamma} \Gamma_{ss}^{IM} + \Delta_{i,t}$

Note: $\Delta_{i,t}$ does not depend any subsequent choices.

• Define an individual specific time *t* transfer $\Delta_{i,t}$:

$$\begin{split} & \Delta_{i,t} \\ = & (\gamma_t^C - 1)c_{i,t}^{IM,ss} - (\gamma_t^H \gamma_t^w - 1)w_{ss}^{IM}(1 - \tau_{ss})s_{it}h_{i,t}^{IM,ss} \\ & - & \lambda_{i,t}(\gamma_t^\Gamma - 1)\Gamma^{IM,ss} + a_{it}(\frac{1 + i^{IM,ss}}{P^{IM,ss}} - \frac{P_{t-1}^{IM}}{P^{IM,ss}}\frac{1 + i_t^{IM}}{P_t^{IM}}) \end{split}$$

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▶ The total transfer received by a household is then given by:

$$\begin{aligned} \Delta_{i,t}^{Total} &= \Delta_{i,t} + \lambda_{i,t} (\gamma_t^{\Gamma} - 1) \Gamma^{IM,ss} \\ &= (\gamma_t^{C} - 1) c_{i,t}^{IM,ss} - (\gamma_t^{H} \gamma_t^{w} - 1) w_{ss}^{IM} (1 - \tau_{ss}) s_{it} h_{i,t}^{IM,ss} \\ &+ a_{it} (\frac{1 + i^{IM,ss}}{P^{IM,ss}} - \frac{P_{t-1}^{IM}}{P^{IM,ss}} \frac{1 + i_t^{IM}}{P_t^{IM}}) \end{aligned}$$

• By comparison define the rep. agent counterpart of Δ is:

$$\overline{\Delta}_{t} = (\gamma_{t}^{C} - 1)C_{ss}^{IM} - (\gamma_{t}^{H}\gamma_{t}^{W} - 1)w_{ss}^{IM}(1 - \tau_{ss})H_{ss}^{IM} - (\gamma_{t}^{\Gamma} - 1)\Gamma_{ss}^{IM} + A(\frac{1 + i_{ss}^{IM}}{P_{ss}} - \frac{P_{t-1}^{IM}}{P_{ss}}\frac{1 + i_{t}^{IM}}{P_{t}^{IM}}),$$

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so that the difference makes the various redistributions clear:

$$\begin{split} \tilde{\Delta}_{i,t} &= \Delta_{i,t} - \overline{\Delta}_t \\ &= \underbrace{(\gamma_t^C - 1)(c_{i,t}^{IM,ss} - C_{ss}^{IM})}_{\text{Redistributes toward high } c \text{ if } \gamma_t^C > 1 \\ &- (\gamma_t^H \gamma_t^w - 1) w_{ss}^{IM} (1 - \tau_{ss})(s_{it}h_{i,t} - H_{ss}^{IM}) \\ &- (\lambda_{i,t} - 1)(\gamma_t^\Gamma - 1) \Gamma_{ss}^{IM} \\ &+ (a - A)(\frac{1 + i_{ss}^{IM}}{P_{ss}} - \frac{P_{t-1}^{IM}}{P_{ss}} \frac{1 + i_t^{IM}}{P_t^{IM}}) \end{split}$$

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Redistribution through dividends and transfers

$$+ (a-A)(\frac{1+i_{ss}^{lM}}{P_{ss}}-\frac{P_{t-1}^{lM}}{P_{ss}}\frac{1+i_{t}^{lM}}{P_{t}^{lM}})$$

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Redistribution across asset holders

EQUIVALENCE BETWEEN IM AND CM

THEOREM

Consider the CM economy $\{C_t^{CM}, H_t^{CM}, w_t^{CM}, \pi_t^{CM}, 1+i_t\}$. The IM economy with transfers $\Delta_{i,t}$ as above and taking $1+i_t = (1+i_{ss}^{IM})\frac{1+i_t}{1+i^*}$ has the same aggregate consumption, hours, wages and inflation rates as the complete markets case. Furthermore, individual consumption, hours, and savings satisfy

$$\begin{aligned} c_{i,t}^{IM} &= \gamma_t^C c_{i,t}^{IM,ss} \\ h_{i,t}^{IM} &= \gamma_t^H h_{i,t}^{IM,ss} \\ a_{i,t+1}^{IM} &= \frac{P_t}{P_{ss}} a_{i,t+1}^{IM,ss}, \end{aligned}$$

for a price sequence P_t . Real bonds are unchanged, $B_t = \frac{P_t}{P_{ss}}B_{ss}$ and transfers are adjusted to balance the government period-budget constraint.

EQUIVALENCE (SPECIAL CASE)

- Consumption = Output, $\gamma_t^C = \gamma_t^Y$ (No fixed costs, no adjustments costs (as-if), $G = 0, \tau = 0$)
- Profits distributed proportional to wages: $w_t sh_t + \lambda(s)\Gamma_t = Z_t sh_t$.

RESULT

Then the IM economy with transfers:

$$\Delta_{i,t} = (\gamma_t^Y - 1)(c_{i,t}^{IM,ss} - Z_t s_{it} h_{i,t}^{IM,ss})$$

has the same aggregate consumption, hours, wages and inflation rates as the complete markets case. Furthermore, individual consumption, hours, and savings satisfy

$$c_{i,t}^{IM} = \gamma_t^C c_{i,t}^{IM,ss} \tag{1}$$

$$h_{i,t}^{IM} = \gamma_t^H h_{i,t}^{IM,ss} \tag{2}$$

$$a_{i,t+1}^{IM} = \frac{P_t}{P_{ss}} a_{i,t+1}^{IM,ss},$$
 (3)

OTHER POLICIES

Redistribute Profits Lump-Sum

Redistributes towards low-productivity hhs

• $\Delta C^{IM} > \Delta C^{CM}$ if profits go up.

► Tighten Monetary policy → profits go up.
→ IM-consumption responses muted

► Loosen Monetary policy → profits go down.
→ IM-consumption increase smaller

• Effect of undone wealth redistribution.

► Prices increase → Distributes towards low asset hhs

• Prices decrease \rightarrow Distributes towards high asset hhs

CALIBRATION OVERVIEW

- Household side follows Krueger, Mitman and Perri (2016)
- Frisch elasticity of 1
- Markup of 10%
- ▶ G 17% of SS Output
- Transfers 12% of SS Output
- Debt / GDP 0.63
- Profits 0% of SS Output
- Tax τ 32%
- Steady state nominal interest rate 4%, inflation 2.7%
- ► To be estimated:
 - Slope of NK Price Philips Curve
 - Slope of NK Wage Philips Curve
 - Elasticity of investment to q

RESULTS, TECHNOLOGY SHOCK



- Slope of NK Price Philips Curve : 0.0055.
- Slope of NK Wage Philips Curve: 0.0055.
- Elasticity of investment to q: 0.35.

TECHNOLOGY SHOCK + NO POLICY RESPONSE



TECHNOLOGY SHOCK + ONLY MP RESPONSE


TECHNOLOGY SHOCK + ONLY FP RESPONSE



COMPARISON TO COMPLETE MARKETS

As-if complete markets (using IM model r_t):



RESULTS, MONETARY POLICY SHOCK

Monetary policy shock: .25pp nominal interest rate increase (pers. .8)



CONCLUSIONS

A simple AiyaGalí model generates impulse responses that are similar to those in the data.

Next step is to improve the estimation

- The effects of market incompleteness can be analyzed theoretically.
- Fiscal and monetary policies interact and should be studied jointly.

Thanks!

Additional Slides

Price Level Determinacy in Incomplete Market Models

PRICE LEVEL INDETERMINACY

- Sargent and Wallace (1975): Interest rate target determines only expected inflation.
- Price level is left indeterminate.
- Next: Price level determinacy in a large class of incomplete market models.
- ► Government budget constraint is in nominal terms. Satisfied for all prices ⇒ Not FTPL.

STEADY STATE PRICE LEVEL

HUGGETT ECONOMY: ASSET MARKET



STEADY STATE PRICE LEVEL

INDETERMINACY



STEADY STATE PRICE LEVEL



- i · nominal interest rate
- B. nominal bonds G: nominal government spending
- r : real interest rate
- T: nominal tax revenue
- π : inflation rate

PRECAUTIONARY SAVINGS

- Failure of the permanent income hypothesis (Campbell and Deaton (1989), Attanasio and Davis (1996), Blundell, Pistaferri and Preston (2008), Attanasio and Pavoni (2011)):
 - Precautionary Savings: A permanent income gain does increase household consumption less than one-for-one.
 - A permanent decrease in government spending by one dollar and a simultaneous permanent tax rebate of the same amount to private households lowers real total aggregate demand - the sum of private and government demand.

PRECAUTIONARY SAVINGS AND STEADY STATE PRICES

Steady State (fixed real interest rate):

- Higher steady state price level lowers real government consumption (given monetary and nominal fiscal policy).
- Lowers the real tax burden for the private sector by the same amount.
- Private sector demand does not substitute one-for-one for the drop in government consumption (Precautionary savings up).
- Aggregate demand-price curve is downward sloping.
- Steady state price level equates aggregate real demand and real supply.

STEADY STATE PRICE LEVEL: FULLY PRICE-INDEXED BONDS *B*^{real}



- i : nominal interest rate
- B: nominal bonds
- r : real interest rate
- π : inflation rate
- G: nominal government spending
- T: nominal tax revenue

STEADY STATE PRICE LEVEL: Aggregate (Goods) Demand



- i · nominal interest rate
 - B: nominal bonds
 - G: nominal government spending
- r : real interest rate π : inflation rate
- T: nominal tax revenue

STEADY STATE PRICE LEVEL: COMPLETE MARKETS



i : nominal interest rate

B: nominal bonds

r : real interest rate

 π : inflation rate

G: nominal government spending

T: nominal tax revenue

Monetary and Fiscal Policy, Technology, Liquidity

STEADY STATE PRICE LEVEL: ASSET AND GOODS MARKET



Steady State Price Level: Expansionary Fiscal Policy $\Delta G > 0$



Steady State Price Level: Tighter Monetary Policy $\Delta i > 0$



Steady State Price Level: Higher Liquidity Demand $\Delta\sigma>0$



STEADY STATE PRICE LEVEL: PRODUCTIVITY INCREASE $\Delta Y > 0$



Model, Details

MODEL: HOUSEHOLDS

Continuum of ex-ante identical households with preferences:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) - g(h_t) \right\}$$

where:

$$\begin{array}{lll} u(c) & = & \log(c) \\ g(h) & = & \psi \frac{h^{1+1/\varphi}}{1+1/\varphi} \end{array}$$

and $\beta \in (0,1)$ is the discount factor.

- Households' labor productivity $\{s_t\}_{t=0}^{\infty}$ is stochastic
- ► $s_t \in \mathscr{S} = \{s^1, \dots, s^N\}$ with transition probability characterized by $p(s_{t+1}|s_t)$

MODEL: RECRUITING FIRMS

A representative, competitive recruting firm aggregates a continuum of differentiated households labor services indexed by $j \in [0, 1]$ and nominal wages per efficiency unit W_{it} :

$$H_t = \left(\int_0^1 s_{jt} (h_{jt})^{\frac{\varepsilon_W - 1}{\varepsilon_W}} dj\right)^{\frac{\varepsilon_W}{\varepsilon_W - 1}}$$

Given a level of aggregate labor demand H, demand for the labor services of household j is given by:

$$h_{jt} = h(W_{jt}; W_t, H_t) = \left(rac{W_{jt}}{W_t}
ight)^{-arepsilon_w} H_t.$$

where W_t is the (equilibrium) nominal wage,

$$W_t = \left(\int_0^1 s_{jt} W_{jt}^{1-\varepsilon_w} dj\right)^{\frac{1}{1-\varepsilon_w}}$$

MODEL: WAGE SETTING

- A union sets a nominal wage $W_{jt} = \hat{W}_t$ for an effective unit of labor to maximize profits.
- Quadratic wage adjustment as in Rotemberg (1982):

$$s_{jt}\frac{\theta_w}{2}\left(\frac{\hat{W}_t}{\hat{W}_{t-1}}-1\right)^2 H_t.$$

Union's wage setting problem is to maximize

$$V_{t}^{w}(\hat{W}_{t-1})$$

$$\equiv \max_{\hat{W}_{t}} \int \left(\frac{s_{jt}(1-\tau_{t})\hat{W}_{t}}{P_{t}}h(\hat{W}_{t};W_{t},H_{t}) - \frac{g(h(\hat{W}_{t};W_{t},H_{t}))}{u'(C_{t})} \right) dj$$

$$- \int s_{jt}\frac{\theta_{w}}{2} \left(\frac{\hat{W}_{t}}{\hat{W}_{t-1}} - 1 \right)^{2} H_{t}dj + \frac{1}{1+r_{t}}V_{t+1}^{w}(\hat{W}_{t})$$

Symmetry: $h_{jt} = H_t$ and $\hat{W}_t = W_t$. Real wage $w_t = \frac{W_t}{P_t}$. $C_t = \text{aggregate consumption.}$

MODEL: WORKER HOUSEHOLDS

Can write their problem recursively:

$$V(a,s;\Omega) = \max_{c \ge 0, h \ge 0, a' \ge 0} u(c,h) + \beta \sum_{s \in \mathscr{S}} p(s'|s) V(a',s';\Omega')$$

subject to

$$Pc + a' = (1 + i)a + P(1 - \tau)whs + T$$

 $\Omega' = \Gamma(\Omega)$

Ω(a,s) ∈ M is the distribution on the space X = A × S.
 Γ equilibrium object determines evolution of Ω.

MODEL: FINAL GOODS PRODUCTION

A final good producer aggregates a continuum of intermediate goods indexed by $j \in [0, 1]$ and with prices p_j :

$$Y_t = \left(\int_0^1 y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Given a level of aggregate demand Y, cost minimization for the final goods producer implies that the demand for the intermediate good j is given by

$$y_{jt} = y(P_{jt}; P_t, Y_t) = \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon} Y_t,$$

where P_t is the (equilibrium) price of the final good and can be expressed as

$$P_t = \left(\int_0^1 P_{jt}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$$

MODEL: INTERMEDIATE GOODS PRODUCTION

Production technology is linear in labor:

$$y_{jt}=Z_t n_{jt},$$

where Z_t is aggregate productivity.

Marginal costs given by

$$mc_{jt}=\frac{w_t}{Z_t}.$$

Price adjustment costs a la Rotemberg (1982):

$$\frac{\theta}{2} \left(\frac{P_{jt}}{P_{jt-1}} - 1 \right)^2 Y_t.$$

Fixed cost of production:

 $Z_t \Phi$.

MODEL: GOVERNMENT

Government taxes labor income and provides nominal transfers:

$$\tilde{T}(wsh) = -T + \tau Pwsh.$$

- Government fully taxes firm profits $P_t d_t$
- Government issues nominal bonds B^g
- Exogenous unvalued expenditures G_t
- Government budget constraint given by:

$$B_{t+1}^g = (1+i_t)B_t^g + G_t - P_t d_t - \int \tilde{T}_t(w_t s_t h_t) d\Omega.$$

EQUILIBRIUM

Definition: A monetary competitive equilibrium is a sequence of prices P_t , tax rates τ_t , nominal transfers T_t , nominal government spending G_t , bonds B_t^g , a value functions v_t , policy functions a_t and c_t , h_t , H_t , pricing functions r_t and w_t , and law of motion Γ , such that:

- 1. v_t satisfies the Bellman equation with corresponding policy functions a_t, c_t, h_T given price sequences r_t, w_t .
- 2. Prices are set optimally by firms.
- 3. Wages are set optimally by middlemen.
- 4. For all $\Omega \in \mathscr{M}$: Markets clear
- 5. Aggregate law of motion Γ generated by a' and p.

Focus on steady state equilibria where all real variables are constant, and constant rate of inflation.

Neutral Technology Shocks

TECHNOLOGY SHOCKS

- Need to compare impulse responses to *the same* shocks in the data and in the model.
- Labor-augmenting, or Harrod-neutral shocks are typically used among major stochastic disturbances in the model. Need to identify them in the data.
 - Arbitrary CRS aggregate production function:

$$Y = F(K_1, ..., K_k, Z_t L_1, ..., Z_t L_n, t).$$

- Solow residual $\frac{\dot{Z}}{Z} + \frac{\partial F/\partial t}{F}$ does not isolate neutral shocks.
- Neither do SVARs. E.g., identification with long run restrictions pick up all shocks that have a long run effect on output per worker.
- Methodology to identify neutral shocks proposed in Bocola, Hagedorn and Manovskii (20xx).

BHM IDENTIFICATION STRATEGY

Identification Theorem: [Reformulation of Uzawa (1961)] A permanent Harrod-neutral technology shock is the only shock with the following (balanced-growth) properties for some time T. An innovation which increases the level of the shock by x percent at time 0 implies for all $t \ge T$

- \blacktriangleright \uparrow in agg. output Y by x percent,
- \uparrow in investment I_j by x percent,
- \blacktriangleright \uparrow in capital K_j by x percent,
- \uparrow in agg. consumption C by x percent,
- ▶ No effect on labor inputs L_m,
- No effect on the marginal product of capital F_{K_i} ,
- \uparrow in the marginal product of labor F_{L_m} by x percent.

BHM IMPLEMENTATION STRATEGY

Observe time series $\mathbf{D}_{\mathbf{t}}$ of growth rates of *n* macroecon variables. Wlg:

$$\mathbf{D}_{\mathbf{t}} = \Delta Z_t \mathbf{1}_{\mathbf{n}} + \tilde{S}_t,$$

where ΔZ_t is the growth rate of the neutral technology (in logs), and \tilde{S}_t is a vector of states. E.g.,

$$\Delta \log(Y_t) = \Delta Z_t + \Delta \log \left[F\left(\frac{K_{1,t}}{Z_t}, \dots, \frac{K_{J,t}}{Z_t}, L_{1,t}, \dots, L_{M,t}; \theta_t \right) \right].$$

 $F(\cdot)$ is unknown and unrestricted. \tilde{S}_t is unobserved.

Strategy:

- 1. Assume a time series model for the behavior of $[\Delta Z_t, \tilde{S}_t]$, indexed by the vector of parameters Λ .
- 2. Estimate the parameters' vector Λ given identifying restrictions.
- 3. Conditional on the estimation of Λ and given a time series for \mathbf{D}_t , estimate the realization of ΔZ_t using smoothing techniques.



Fiscal Variables Construction

CONSTRUCTION OF FISCAL VARIABLES, DETAILS

Source: NIPA Table 3.1, line numbers in brackets

Spending = Consumption expenditures [21] + Gross government investment [39] + Net purhases on nonproduced assets [41] - Consumption of fixed capital [42]

Revenue = Total receipts [34]

- Subsidies [30]
- Current transfer receipts from the rest of the world [18]
- + Current surplus of government enterprises [19]

Transfers = Current transfer payments [22]

+ Capital transfer payments [40]

- Current transfer receipts from the rest of the world [18]

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