Monetary Policy and Bubbles in a New Keynesian Model with Overlapping Generations

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Motivation

- Asset price bubbles: ubiquitous in the policy debate...
  - key source of macro instability
  - monetary policy: cause and cure

...but absent in workhorse monetary models
  - no room for bubbles in the New Keynesian model
  - no discussion of possible role of monetary policy
Motivation

- Asset price bubbles: ubiquitous in the policy debate...
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- Present paper: modification of the basic NK model to allow for bubbles

- Key ingredients:
  1. overlapping generations of finitely-lived agents
  2. transitions to inactivity ("retirement")
Related Literature

- Monetary models with bubbles: Samuelson (1958),..., Asriyan et al. (2016) ⇒ flexible prices
- Monetary policy and bubbles in sticky price models:
  - Bernanke and Gertler (1999, 2001): ad-hoc bubble
  - Galí (2014): 2-period OLG, constant output
  - Present paper:
    - many-period lifetimes
    - variable employment and output
    - nests standard NK model as a limiting case
A New Keynesian Model with Overlapping Generations

- Individual survival rate: $\gamma$ (Blanchard (1985), Yaari (1965))
- Two types of individuals:
  - "Active": manage own firm, work for others.
  - "Retired": consume financial wealth
- Probability of remaining active: $\nu$ (Gertler (1999))
- Labor force (and measure of firms): $\alpha \equiv \frac{1-\gamma}{1-\nu\gamma} \in (0, 1]$
Consumers

- Complete markets (including annuity contracts)
- Consumer’s problem:

\[
\max E_0 \sum_{t=0}^{\infty} (\beta \gamma)^t \log C_{t|s}
\]

\[
\frac{1}{P_t} \int_{0}^{\alpha} P_t(i) C_{t|s}(i) di + E_t \{ \Lambda_{t,t+1} Z_{t+1|s} \} = A_{t|s} \left[ + W_t N_{t|s} \right]
\]

\[
A_{t|s} = Z_{t|s} / \gamma
\]
Firms

- Technology:
  \[ Y_t(i) = \Gamma^t N_t(i) \]
  where \( \Gamma \equiv 1 + g \geq 1 \).

- Price-setting à la Calvo
Labor Markets and Inflation

- Wage equation:
  \[ \mathcal{W}_t = \left( \frac{N_t}{\alpha} \right)^{\varphi} \]
  where \( \mathcal{W}_t \equiv \mathcal{W}_t / \Gamma^t \) and \( N_t \equiv \int_0^\alpha N_t(i) \, di \).

- Natural level of output: setting \( 1 / \mathcal{W}_t = \mathcal{M} \)
  \[ Y_t^n = \Gamma^t \mathcal{Y} \]

  with \( \mathcal{Y} \equiv \alpha \mathcal{M}^{-\frac{1}{\varphi}} \). Remark: invariant to bubble size.
Asset Markets

- Aggregate stock market

\[
Q_t^F = \sum_{k=0}^{\infty} (\nu \gamma)^k E_t \{ \Lambda_{t,t+k} D_{t+k} \}
\]

*Remark*: same discount rate as labor income.

- Aggregate bubble:

\[
Q_t^B = B_t + U_t
\]

where \( B_t \equiv \sum_{s=-\infty}^{t-1} Q_t^B \geq 0 \) and \( U_t \equiv Q_t^B \geq 0 \)

- Equilibrium condition:

\[
Q_t^B = E_t \{ \Lambda_{t,t+1} B_{t+1} \}
\]
Characterization of Equilibria

- Balanced Growth Paths
- Bubble-Driven Fluctuations around a Balanced Growth Path
Balanced Growth Paths

- Aggregate consumption function

\[
C = (1 - \beta \gamma) \left[ Q^B + \frac{\gamma}{1 - \frac{\Gamma \nu \gamma}{1+r}} \right]
\]

- In equilibrium \((C = \gamma)\):

\[
1 = (1 - \beta \gamma) \left[ q^B + \frac{1}{1 - \frac{\Gamma \nu \gamma}{1+r}} \right]
\]

where \(q^B \equiv Q^B / \gamma\).

- Bubbleless BGP \((q^B = 0)\)

\[
\frac{\Gamma \nu}{1 + r} = \beta
\]

Remark #1: \(r\) increasing in \(\nu\)

Remark #2: \(\nu < \beta \iff r < g\)
Balanced Growth Paths

- Bubbly BGP:

\[
q^B = \frac{\gamma (\beta - \frac{\Gamma \nu}{1+r})}{(1 - \beta \gamma)(1 - \frac{\Gamma \nu \gamma}{1+r})} > 0
\]

\[
u = \left(1 - \frac{1 + r}{\Gamma}\right) q^B \geq 0
\]

where

\[
\frac{1 + r}{\Gamma} \leq 1 \iff r \leq g
\]

\[
\frac{\Gamma \nu}{1 + r} < \beta \iff r > r_0
\]

- Existence condition:

\[
u < \beta
\]
Figure 1. Balanced Growth Paths

- Bubbly BGP without bubble creation
- Bubbly BGP with bubble creation
- Bubbleless BGP
Equilibrium Dynamics (I)

- Goods market clearing:
  \[ \hat{y}_t = \hat{c}_t \]

- Aggregate consumption function:
  \[ \hat{c}_t = (1 - \beta \gamma)(\hat{q}_t^B + \hat{x}_t) \]

where

\[ \hat{x}_t = \Lambda \Gamma v \gamma E_t\{\hat{x}_{t+1}\} + \hat{y}_t - \frac{\Lambda \Gamma v \gamma}{1 - \Lambda \Gamma v \gamma}(i_t - E_t\{\pi_{t+1}\}) \]

with \( \Lambda \equiv \frac{1}{1+r} \)

- Aggregate bubble dynamics:
  \[ \hat{q}_t^B = \Lambda \Gamma E_t\{\hat{q}_{t+1}^B\} - q^B(i_t - E_t\{\pi_{t+1}\}) \]
Equilibrium Dynamics (II)

- New Keynesian Phillips curve

\[ \pi_t = \Lambda \Gamma \nu \gamma E_t \{ \pi_{t+1} \} + \kappa \hat{y}_t \]

- Monetary Policy

\[ \hat{i}_t = \phi_\pi \pi_t + \phi_q \hat{q}_t^B \]

- Assumption: no fundamental shocks, focus on bubble-driven fluctuations
Bubble-Driven Fluctuations

- Implied dynamic IS equation:

\[ \hat{y}_t = \Phi E_t \{ \hat{y}_{t+1} \} - \Psi (i_t - E_t \{ \pi_{t+1} \}) + \Theta \hat{q}_t^B \]

- Equilibrium dynamics

\[ A_0 x_t = A_1 E_t \{ x_{t+1} \} \]

where \( x_t = [\hat{y}_t, \pi_t, \hat{q}_t^B] \)' and

\[
A_0 \equiv \begin{bmatrix}
1 & \Psi \phi_\pi & \Psi \phi_q - \Theta \\
-\kappa & 1 & 0 \\
0 & q^B \phi_\pi & 1 + q^B \phi_q
\end{bmatrix} ;
A_1 = \begin{bmatrix}
\Phi & \Psi & 0 \\
0 & \Lambda \Gamma \nu \gamma & 0 \\
0 & q^B & \Lambda \Gamma
\end{bmatrix}
\]

- Conditions for stationary, bubble-driven fluctuations
Figure 2. Determinacy and Indeterminacy Regions

For $r = 0.00335$, the determinacy region is shown on the left side of the figure. The blue area indicates determinacy, while the yellow areas represent indeterminacy in 1-dimensional and 2-dimensional cases.

For $r = 0.0035$, the determinacy region is shown on the right side of the figure. The blue area indicates determinacy, while the yellow areas represent indeterminacy in 1-dimensional and 2-dimensional cases.

For $r = 0.0037$, the determinacy region is shown on the bottom left side of the figure. The blue area indicates determinacy, while the yellow areas represent indeterminacy in 1-dimensional and 2-dimensional cases.

For $r = 0.004$, the determinacy region is shown on the bottom right side of the figure. The blue area indicates determinacy, while the yellow areas represent indeterminacy in 1-dimensional and 2-dimensional cases.
Figure 3. Simulated Bubble-Driven Fluctuations Around a Bubbly BGP
An Example with an Stochastic Bubble

- Assumed bubble process:

\[ q_t^B = \begin{cases} \frac{v}{\beta \delta} q_{t-1}^B + u_t & \text{with probability } \delta \\ u_t & \text{with probability } 1 - \delta \end{cases} \]

where \( \{u_t\} > 0 \) is white noise with mean \( \bar{u} \geq 0 \).

- Equilibrium output and inflation:

\[ \hat{y}_t = (1 - \beta \gamma) \Omega (\Theta - \phi_q) q_t^B \]

\[ \pi_t = \kappa \Omega (\Theta - \phi_q) q_t^B \]

where \( \Omega \equiv 1 / [(1 - \beta \gamma)(1 - v / \beta) + \kappa (\phi_{\pi} - v / \beta)] > 0. \)

- Simulated bubble driven fluctuations (\( \phi_{\pi} = 1.5, \phi_q = 0 \)) (*)
Figure 5. Simulated Bubble-Driven Fluctuations around the Bubbleless BGP
Assumption: inflation and output gap stabilization mandate

*Strategy #1*: "leaning against the bubble" to rule out bubble-driven fluctuations
Figure 4.
The Effectiveness of “Leaning against the Bubble” Policies
Figure 7. Bubble-Driven Fluctuations: Monetary Policy and Bubble Volatility
Bubbles and Monetary Policy Design

- Assumption: inflation and output gap stabilization mandate
- **Strategy #1**: Hard "lean against the bubble policy" to rule out bubble-driven fluctuations
- **Strategy #2**: Neutralize effects of bubble fluctuations on aggregate demand

\[ \hat{\pi}_t = \phi_\pi \pi_t + \left( \Theta / \Psi \right) \hat{q}_t^B \]
Figure 6. Bubble-Driven Fluctuations: Monetary Policy and Macro Volatility
Assumption: inflation and output gap stabilization mandate

*Strategy #1*: "Lean against the bubble policy" to rule out bubble-driven fluctuations

*Strategy #2*: Neutralize effects of bubble fluctuations on aggregate demand

\[ \hat{i}_t = \phi_{\pi} \pi_t + \left( \frac{\Theta}{\Psi} \right) \hat{q}_t^B \]

*Strategy #3*: Direct inflation targeting

\[ \hat{i}_t = \phi_{\pi} \pi_t \]

with \( \phi_{\pi} \) arbitrarily large
Concluding Remarks

- Bubbly equilibria may exist in the NK model once we depart from the infinitely-lived representative consumer assumption. Room for bubble-driven fluctuations.
- More likely in an environment of low natural interest rates.
- No obvious advantages of "leaning against the bubble" policies (relative to inflation targeting), plus some risks (e.g. may amplify bubble fluctuations)
- Need for instruments alternative to interest rate policy?
- Caveats/potential extensions

(i) Rational bubbles. But non-rational bubbles can be readily accommodated.
(ii) ZLB has been ignored. Potential interesting interaction with bubbles (e.g. by raising underlying natural rate, bubbles may lower the risk of hitting the ZLB).
(iii) No role for credit supply factors; may be needed to boost the size of bubble effects.