

# Optimal Inflation Target in an Economy with Menu Costs and Zero Lower Bound

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Understanding Inflation: lessons from the past, lessons for  
the future?

ECB

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# Question

- ▶ Since 80s, countries follow policy of *inflation targeting* (IT)
  - Declare medium-term inflation target (2%)
  - Keep inflation as close as possible to this number

Question: What is the IT a Central Bank should have?

# Trade-Offs for IT: Cost and Benefit

- ▶ **Benefit of higher IT:** lower output volatility
  - Summer (91), Blanchard et al (10)
  - Increase average nominal interest rates
  - With ZLB, more room to reduce rates during recessions
- ▶ **Cost of higher IT:** lower aggregate productivity
  - Higher gap between new and old prices
  - Inefficient price dispersion of relative price
  - Misallocation of inputs of production

# What I do?

- ▶ Cost of raising inflation: price dispersion
  - Capture pricing behavior
  - Pricing model: menu cost with idiosyncratic shocks
  - Interaction  $\Rightarrow$  low cost of inflation
- ▶ Benefit of raising inflation: business cycle stabilization
  - Incorporate pricing model to New Keynesian model
    - Rich set of aggregate shocks
    - Taylor rule subject to a ZLB
  - Reproduce US business cycle
- ▶ Optimal inflation target of 3%

# Literature Review: Trade-off Quantification

- ▶ Walsh09, William09 and Billi11: IT higher than 2%
  - Log-linear approx. Calvo model around zero trend inflation
  - Arbitrary loss function
- ▶ CoGoWi13: IT around 1%
  - Use household welfare function with Calvo pricing
  - Robust to time and state dependent models (Taylor, Menu Cost)
  - Inconsistent with micro-pricing behavior (easy aggregation)
- ▶ This paper: 3% IT
  - Consistent with micro-pricing behavior (not easy aggregation)

# Roadmap

## 1. Model

## 2. Calibration

- ▶ Business cycle implication
- ▶ Micro-behavior implications

## 3. Optimal inflation target

- ▶ Cost of a higher IT
- ▶ Benefit of a higher IT
- ▶ Robustness

Model

# Environment

- ▶ Representative household
  - Consume  $C_t$ , supply labor  $L_t$  and save  $B_t$
- ▶ Continuum of monopolistic firms  $i \in [0, 1]$ 
  - Produce intermediate inputs  $y_{ti}$
- ▶ Competitive final good firm FGP more
  - Produces final output  $Y_t$  with CES aggregator
- ▶ Government Gov. more
  - Set nominal rate  $R_t$  with Taylor rule subject to ZLB
  - Finance stochastic expenditure  $\eta_{tg}$  with lump-sum transfers



# Representative Household

$$\max_{C_t, L_t, B_t} U_0 \quad s.t.$$

$$P_t C_t + B_t = W_t L_t + \int \Phi_{it} di + T_t + \eta_{t-1q} R_{t-1} B_{t-1}$$

$$U_t = u_t(C_t, L_t) + \beta \mathbb{E}_t [U_{t+1}^{1-\sigma_{ez}}]^{1-\frac{1}{\sigma_{ez}}}$$

- $\int \Phi_{it} di, T_t$  : firms' profit and lump-sum transfers
- $P_t, W_t$  : price of final good and labor
- $\eta_{tq}$  : risk premium shock
  - Main shock that trigger the ZLB
- $U_t, u_t$  : value function with risk-sensitive ( $\sigma_{ez}$ ) and period utility
  - Main cost of ZLB  $\Rightarrow$  business cycle fluctuations
  - Calibrate  $\sigma_{ez}$  to match risk premium

# Intermediate Monopolistic Firms

- ▶ Technology for output:  $y_{ti} = A_{ti} x_{ti}^\alpha (\eta_{tz} l_{ti})^{1-\alpha}$ 
  - $\eta_{tz}$ , : aggregate TFP shock
  - $l_{ti}, x_{ti}$  : labor and final good (material) input  
⇒ Flatter Phillips curve, higher cost inflation

- ▶  $A_{ti}$  : firms' idiosyncratic shocks
  - Main motive of price changes

$$\Delta \log(A_{ti}) = \begin{cases} \eta_{t+1i}^1 & \text{with prob. } p \\ \eta_{t+1i}^2 & \text{with prob. } 1 - p \end{cases} ; \eta_{ti}^k \sim i.i.d. N(0, \sigma_{ak})$$

- ▶ Stochastic menu cost of changing prices ( $\theta_{ti}$ ) in units of labor

$$\theta_{ti} \sim i.i.d. \begin{cases} 0 & \text{with prob. } hz \\ \theta & \text{with prob. } 1 - hz \end{cases}$$

# Intermediate Monopolistic Firms Problem

$$\max_{p_{ti}} \mathbb{E}_0 [Q_t \Phi_{ti}] \quad s.t.$$

$$\Phi_{ti}/P_t = Y_t \tilde{p}_{ti}^{-\gamma} (\tilde{p}_{ti} - \iota(1 - \tau)(w_t/\eta_{t,z})^{1-\alpha}) - I(p_{t-1i} \neq p_{ti})w_t\theta_{ti}$$

- $Q_t$  : nominal discount factor
- $\Phi_{ti}/P_t$  : firms' real profit
- $w_t, \iota((1 - \tau_L)w_t)^{1-\alpha}$  : real wage and marginal cost
- $\tilde{p}_{ti} = \frac{p_{ti}A_{ti}}{P_t}$  : firms' adjusted relative price
- $\tau$  : subsidy to marginal cost
  - Match demand elasticity and level of markups

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# Equilibrium Definition

**Equilibrium definition** An equilibrium is a set of stochastic processes for (i) consumption, labor supply, and bonds holding  $\{C, L, B\}_t$  for the representative consumer; (ii) pricing policy functions for firms  $\{p_{ti}\}_t$  and inputs demand  $\{n_{ti}, l_{ti}\}$  for the monopolistic firms; (iii) final output and inputs demand  $\{Y_t, \{y_{ti}\}_i\}_t$  for the final producer and (iv) nominal interest rate  $\{R\}_t$ :

1. Given prices,  $\{C, L, B\}_t$  solve the consumer's problem.
2. Given prices,  $\{Y_t, \{y_{ti}\}_i\}_t$  solve the final good producer problem.
3. Given the prices and demand schedule, the firm's policy  $p_{ti}, n_{ti}, l_{ti}$  is optimal.
4. Nominal interest rate satisfies the Taylor rule.
5. Markets clear at each date:

$$\int_0^1 (l_{ti} + I(p_{ti} \neq p_{t-1i})\theta_{ti}) di = L_t$$

$$Y_t - \int_0^1 x_{ti} di = C_t + \eta_{tg}$$

# Calibration

# Calibration: Preferences and Technology

$g$	$\beta$	$\sigma_{np}$	$\chi$	$\alpha$	$\tau$	$\sigma_{ez}$
0.0017	0.999	2	0.5	0.5	0.2	-5.3
2% growth	4% RR	GrHeHu88	IS 45%	17% MaUps	Cost BC	

► Model frequency: monthly

► Preferences and technology:

$$\begin{aligned} \circ u_t &= \frac{(C_t - \eta_{z,t} L_t^{1+\chi})^{1-\sigma_{np}}}{1-\sigma_{np}} ; U_t = u_t + \beta \mathbb{E}_t [U_{t+1}^{1-\sigma_{ez}}] \frac{1}{1-\sigma_{ez}} \\ \circ y_{ti} &= A_{ti} x_{ti}^\alpha (\eta_{tz} l_{ti})^{1-\alpha} ; \frac{\eta_{tz}}{\eta_{t-1z}} = (1+g)^{1-\rho_z} \left( \frac{\eta_{t-1z}}{\eta_{t-2z}} \right)^{\rho_z} \exp(\sigma_z \epsilon^z) \end{aligned}$$

► Cost of business cycle: Risk premium 4%

► Firms demand elasticity: 3

◦ Consistence with micro-estimates

# Calibration: Structural Shock and Taylor Rule

ZLB

$(\phi_r, \phi_\pi, \phi_x, \phi_{dy})$	$(\rho_r, \sigma_r 100)$	$(\rho_z, \sigma_z 100)$	$(\rho_g, \sigma_g 100)$	$(\rho_q, \sigma_q 100)$
(0.87, 2, 0.22, 0)	(0, 0.05)	(0.97, 0.012)	(0.95, 0.21)	(0.94, 0.125)

- ▶ Taylor rule: Del negro et. al. (2007)

$$R_t^* = \left( \frac{1 + \bar{\pi}}{\beta} \right)^{1 - \phi_r} (R_{t-1}^*)^{\phi_\pi} \left[ \left( \frac{1 + \pi_t}{1 + \bar{\pi}} \right)^{\phi_{\bar{\pi}}} \left( \frac{X_t}{X_{ss}} \right)^{\phi_{\bar{y}}} \right]^{1 - \phi_\pi} \left( \frac{X_t}{X_{t-1}} \right)^{\phi_{dy}} \eta_{rt}$$
$$R_t = \max\{1, R_t^*\}$$

- ▶ Exogenous shocks AR(1):  $\eta_{tx} = \eta_{ss,x}^{1-\rho_x} \eta_{t-1,x}^{\rho_x} e^{\epsilon_{tx}}$  with  $x \in \{r, g, q\}$ 
  - gover. and monetary: Del negro et. al. (2007)
  - risk premium innovations: international ZLB frequency of 14%
- ▶ Next: model fit with US business cycle
  - 1960:Q1 to 2015:Q4 (HP trend)



# Business Cycle Moments: Model and Data

	Standard Deviation			Correlation With Output		
	Model			Model		
	Data	Median	IC [2,98]	Data	Median	IC [2,98]
Output	1.46	1.35	[1.13,1.78]	1.00	1.00	[1.00,1.00]
Labor	1.31	1.24	[1.04,1.64]	0.87	0.98	[0.96,0.99]
Interest Rate	0.35	0.67	[0.56,0.82]	0.41	0.49	[0.36,0.62]
Real Wage	0.87	0.66	[0.56,0.86]	0.07	0.98	[0.96,0.99]
Inflation	0.27	0.32	[0.27,0.41]	0.18	0.95	[0.92,0.97]

- ▶ Model matches volatility of main aggregate variables
- ▶ Model matches correlation with output (except real wage)

# Estimation: Menu Cost and Idiosyncratic Shocks

Data UK CPI

$\theta$ : menu cost	$hz$ : prob. zero menu cost	$p$	$(\sigma_1^a, \sigma_2^a)$
0.128	0.058	0.63	(0.210, 0.024)

- ▶ SMM with
  - UK CPI price quotes (similar to US)
  - Average resources spend on price adjustment (0.4% revenue)
- ▶ Next: model fit with micro-data

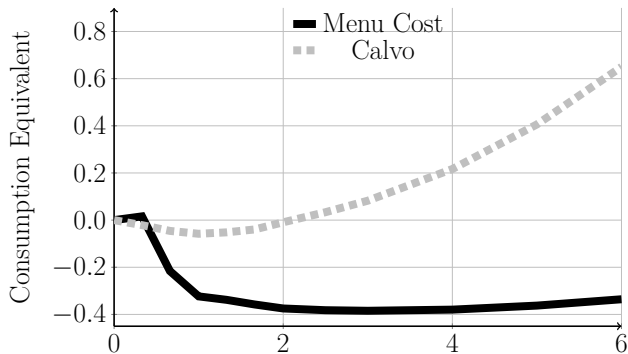
# Micro-Price Statistics: Model and Data

Moments Absolute Value of Price Change	Data	Model
Mean	0.124	0.133
Standard deviation	0.112	0.120
Skewness	1.324	1.325
5th percentile	0.008	0.006
90th percentile	0.288	0.300
Frequency of price change	0.105	0.105
Ratio free to total price adjustment	—	0.557

- ▶ Zero menu cost  $\Rightarrow$  Small price changes
- ▶ Fat tails in idiosyncratic shocks  $\Rightarrow$  Large price changes

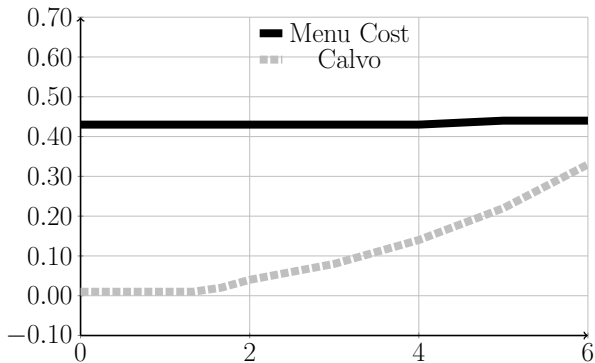
# Optimal Inflation Target

# Optimal Inflation Target: Consumption Equivalent w.r.t. Zero Inflation



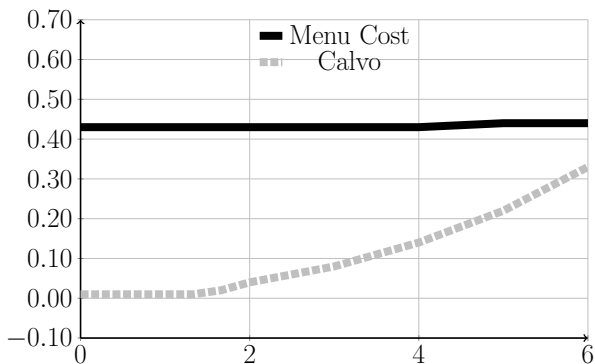
Optimal inflations: Calvo 1%, Menu Cost 3%

## Mean Price Dispersion (percentage)



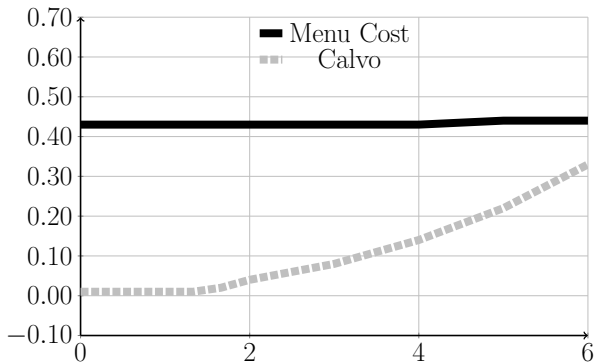
Calvo: small price dispersion in levels/large elasticity w.r.t. IT

## Mean Price Dispersion (percentage)



Menu cost: large price dispersion in levels (large idiosyncratic shocks)

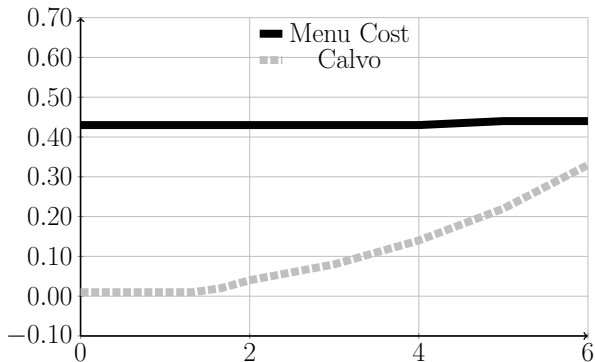
## Mean Price Dispersion (percentage)



⇒ small elasticity w.r.t. IT (small cost of inflation)



## Mean Price Dispersion (percentage)



Observation: in menu cost model one of every two price changes is due to “Calvo”

# Intuition of Low Cost of Inflation: $\tilde{p}_t = \frac{p_t A_t}{P_t}$

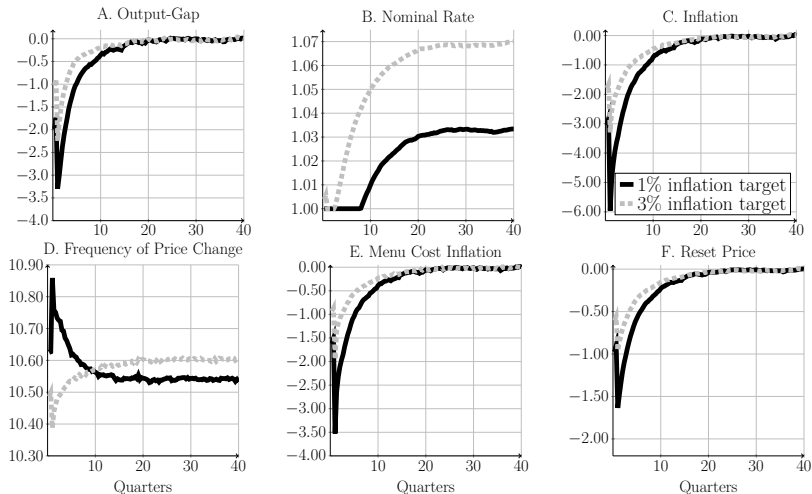
More

- ▶ Firms are exposed to symmetric productivity shocks
  - Positive prod. shock: inflation cancel prod. shock  
⇒ decrease price dispersion owing to idio. shocks
  - Negative prod. shock: inflation cancel prod. shock  
⇒ increase price dispersion owing to idio. shocks
- ▶ At zero inflation: these two forces cancel
- ▶ At low levels of inflation: quantitatively valid
  - Width of the Ss are almost constant (for large idio. shocks)
  - Symmetry of dist. of relative prices (for large idio. shocks)

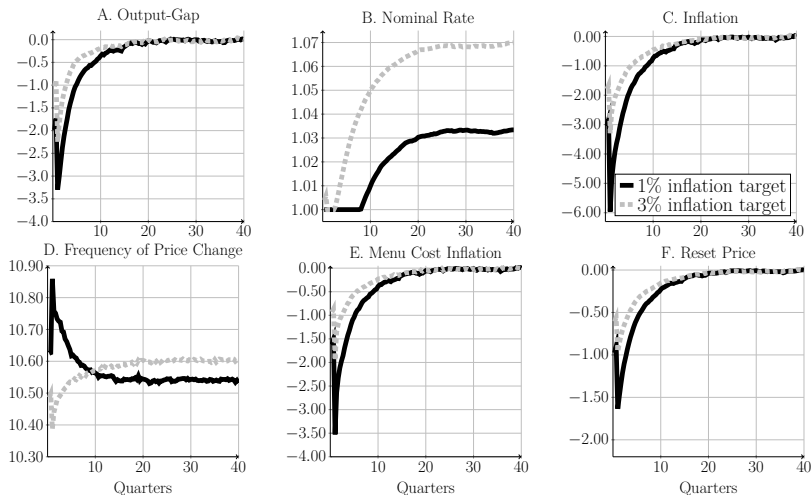
# Zero Lower Bound Dynamics

- ▶ Pricing model also affect business cycle dynamics
- ▶ Inflation target affects the magnitude of a recession at the ZLB:
  - At low inflation, large selection effect *at ZLB*  $\Rightarrow$  large recession
  - At high inflation, low selection effect *at ZLB*  $\Rightarrow$  small recession
- ▶ Methodology: non-linear impulse-response
  - Shock the economy with a risk premium shock ( $2\sigma_q$ )
  - Conditional of low interest rates (percentile 25)
  - Plot
    - Median impulse-response in the menu cost model
    - At 1% and 3% inflation

# Zero Lower Bound Dynamics: 1% IT vs 3% IT

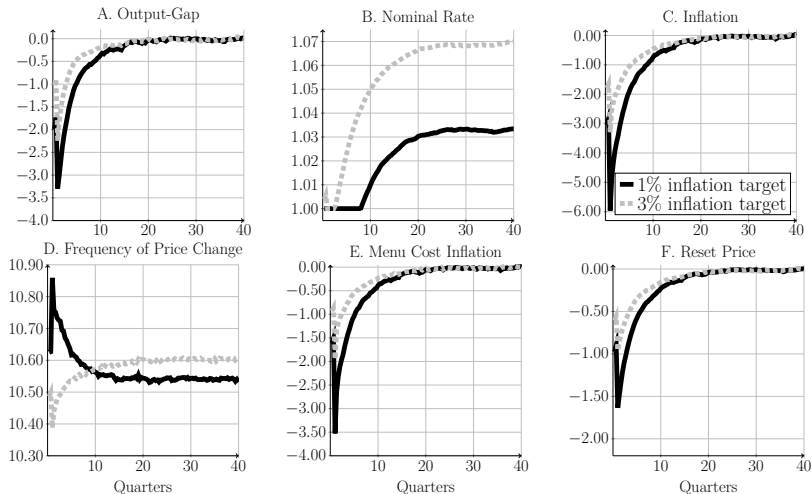


# Zero Lower Bound Dynamics: 1% IT vs 3% IT



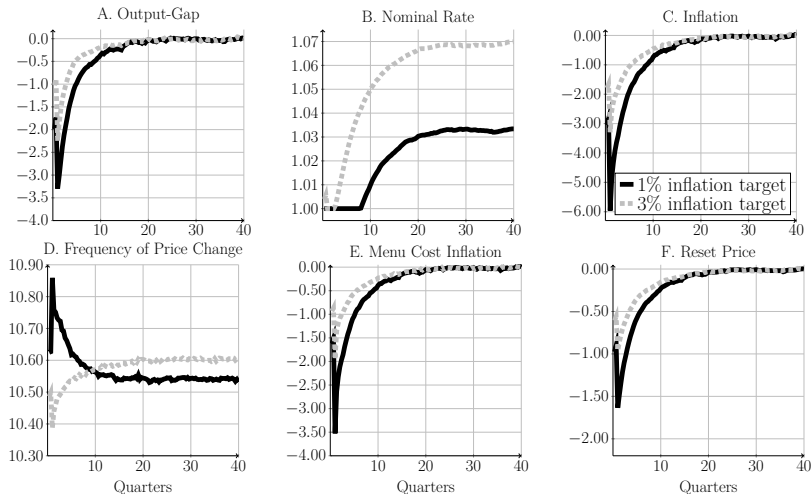
**Economics of Deflationary Spiral:**

# Zero Lower Bound Dynamics: 1% IT vs 3% IT



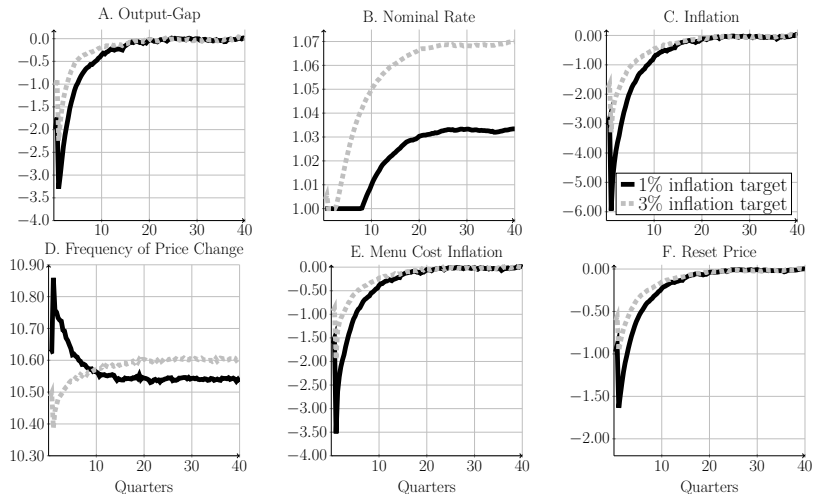
Real interest rate is too high, output gap is depressed

# Zero Lower Bound Dynamics: 1% IT vs 3% IT



A risk premium shocks decreases output gap and inflation

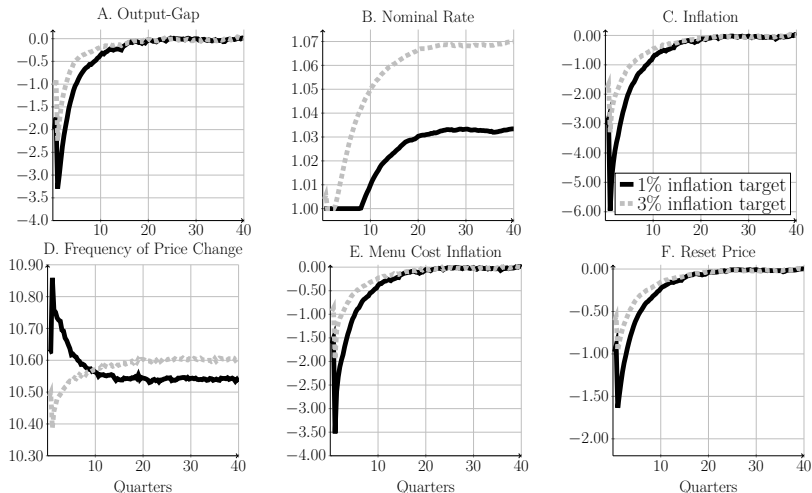
# Zero Lower Bound Dynamics: 1% IT vs 3% IT



Nominal rate does not react, inflation affects 1-1 to real rate

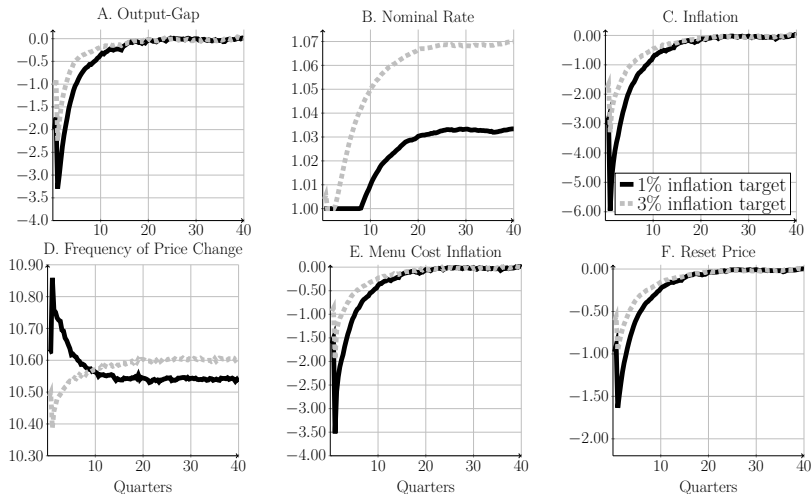


# Zero Lower Bound Dynamics: 1% IT vs 3% IT



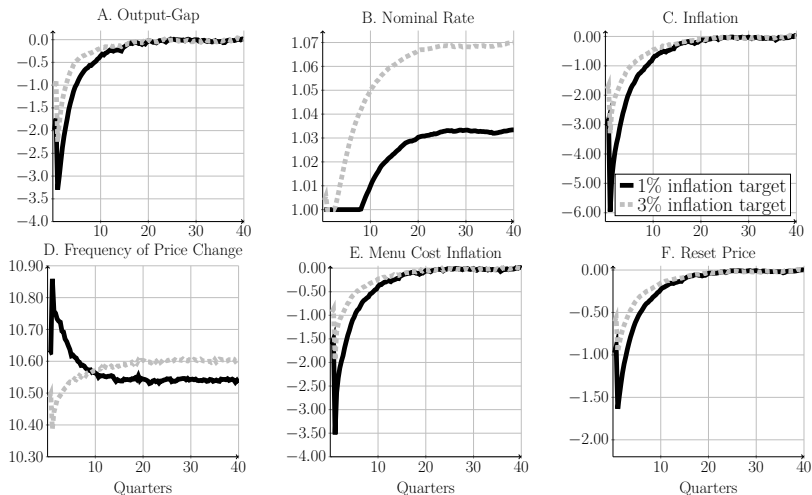
Depressing even more output-gap and inflation!!!!

# Zero Lower Bound Dynamics: 1% IT vs 3% IT



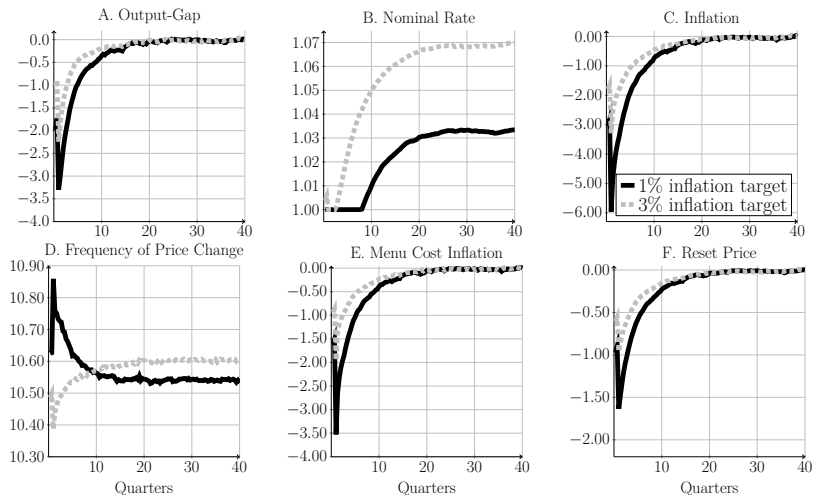
**Economics of Deflationary Spiral in Menu cost model:**

# Zero Lower Bound Dynamics: 1% IT vs 3% IT



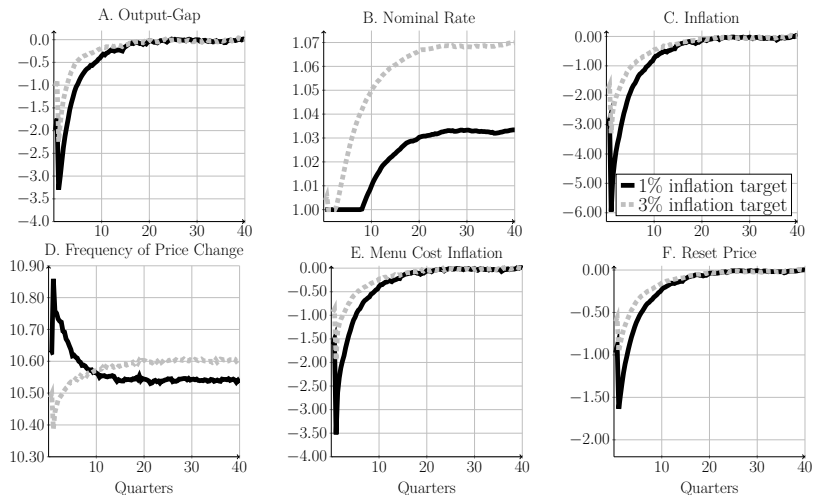
At 1% IT, during the ZLB there is deflation

# Zero Lower Bound Dynamics: 1% IT vs 3% IT



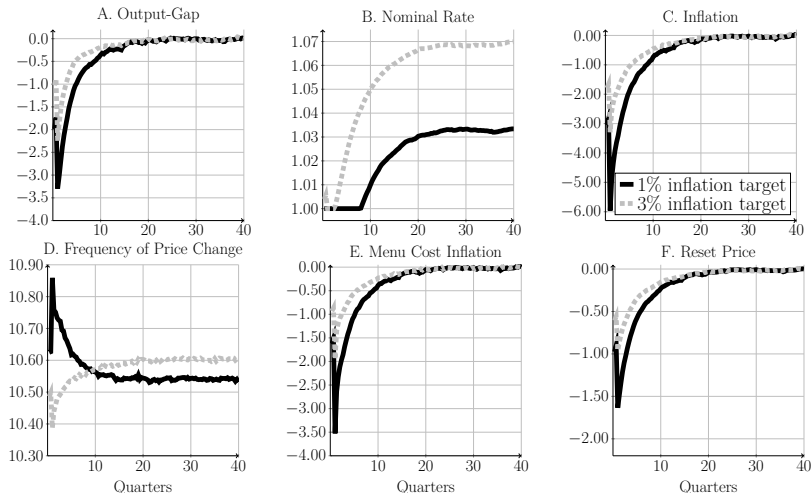
Persistence increase frequency of price change

# Zero Lower Bound Dynamics: 1% IT vs 3% IT



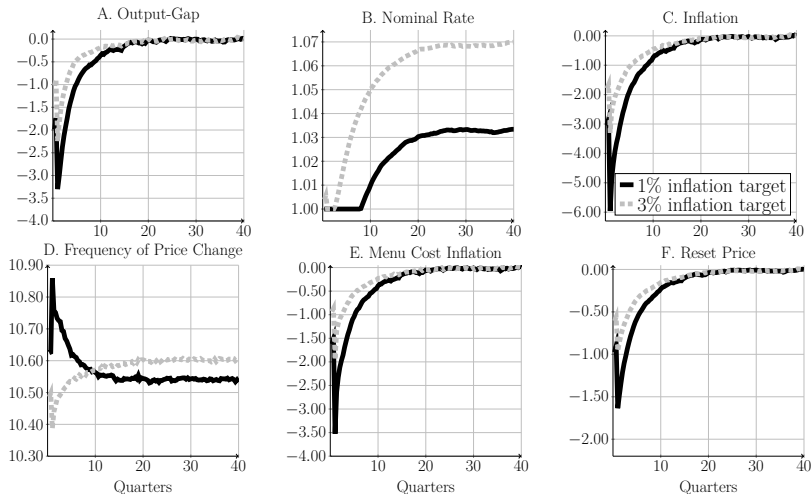
Firms hit the downward adjustment trigger

# Zero Lower Bound Dynamics: 1% IT vs 3% IT



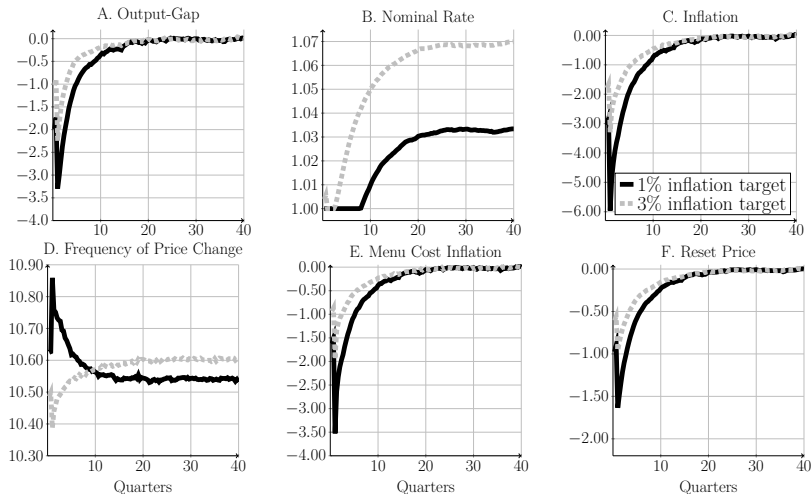
This small measure of firms have a large size of price adjustment

# Zero Lower Bound Dynamics: 1% IT vs 3% IT



1/2 of drop inflation is due to these firms (selection effect)

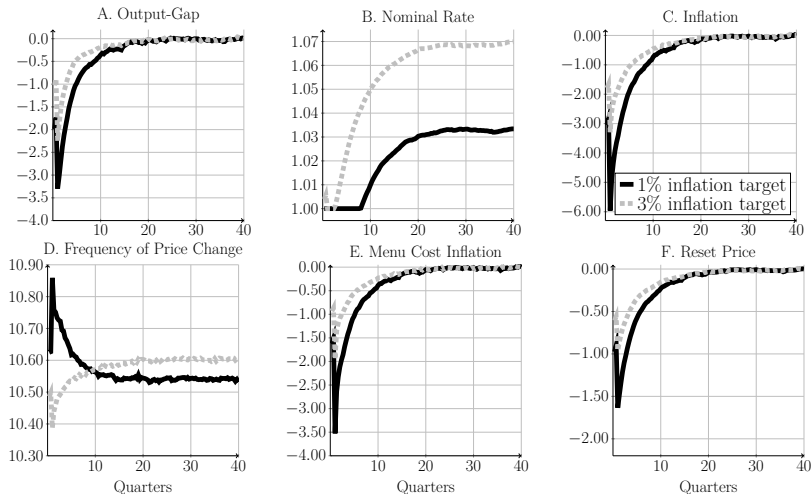
# Zero Lower Bound Dynamics: 1% IT vs 3% IT



At 3% IT, during the ZLB there is positive or zero inflation

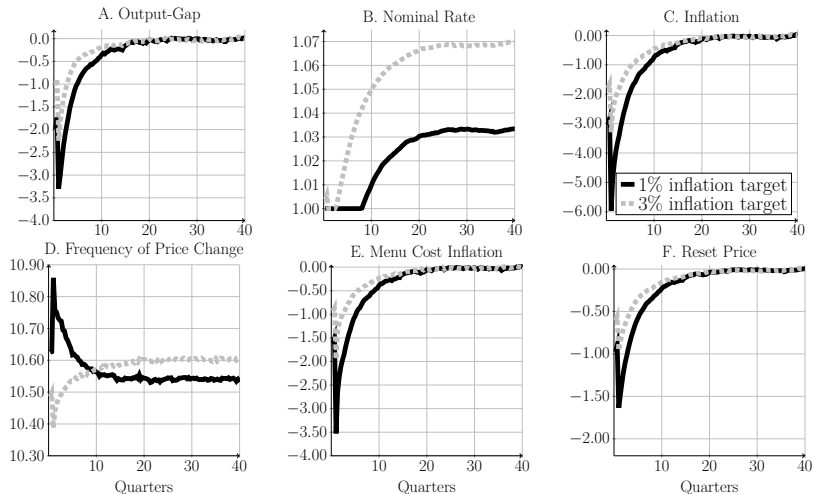


# Zero Lower Bound Dynamics: 1% IT vs 3% IT



Persistence decrease in the frequency of price change

# Zero Lower Bound Dynamics: 1% IT vs 3% IT



No downward price adjustment

# Interaction between ZLB Dynamics and IT

## ZLB dynamics

At low inflation in the ZLB, there is a persistent increase in the frequency of price changes that are large and negative. Higher inflation target eliminates this mechanism.

# Robustness for Optimal IT

- ▶ Increase demand elasticity to 10: IT 3%
- ▶ Reduce freq. ZLB to 8%: IT 2%
- ▶ Expected utility: IT 2.5% with 1/3 reduction of consumption equiv.
- ▶ CRRA preferences: IT 5%
- ▶ Decrease in the growth rate: IT 3.5%

# Conclusion

- ▶ Low real rates are becoming a problem for policy stabilization
- ▶ This paper analyzes optimal IT in
  - A model consist with micro-pricing behavior
  - With the potential to match macroeconomic data
- ▶ Optimal inflation target of 3%
  - Same environment but with Calvo pricing, 1% optimal IT

# Appendix

*The Committee reaffirms its judgment that inflation at the rate of 2 percent, as measured by the annual change in the price index for personal consumption expenditures, is most consistent over the longer run with the Federal Reserve's statutory mandate.*

Statement on Longer-Run Goals and Monetary Policy Strategy  
As amended effective January 28, 2014

- ▶ Taylor rule for interest rate:  $R_t = \max\{1, R_t^*\}$

$$R_t^* = \left(\frac{1 + \bar{\pi}}{\beta}\right)^{1 - \phi_r} (R_{t-1}^*)^{\phi_\pi} \left[ \left(\frac{1 + \pi_t}{1 + \bar{\pi}}\right)^{\phi_{\bar{\pi}}} \left(\frac{X_t}{X_{ss}}\right)^{\phi_{\bar{y}}} \right]^{1 - \phi_\pi} \left(\frac{X_t}{X_{t-1}}\right)^{\phi_{d\bar{y}}} \eta_{rt}$$

- $R_t^*$ : desired i-rate (i-rate Fed would choose absent ZLB)
  - $R_t$ : actual i-rate
  - $\pi_t$ : inflation,  $\bar{\pi}$ : target inflation
  - $X_t$ : output gap
  - $\eta_{rt}$ : monetary shock
- ▶ Stochastic Government Expenditure ( $\eta_{tg} \sim AR(1)$ )

$$C_t + \eta_{tg} = GDP_t$$



- ▶ Quarterly panel data of countries
  - Policy rates/call rates and consumer price index
  - Keep year with constant inflation target
    - Years after 1988
    - Mean inflation less than 4%
- ▶ Frequency of ZLB:  $Pr(i_t < 0.51)$
- ▶ Inflation target:  $\mathbb{E}[\Delta \log(P_t)]$

Country	Historical		After 1988		in/out
	Freq. ZLB	Mean Inf.	ZLB Freq.	Mean Inf.	
Argentina	0	16.21	0	16.21	out
Australia	0	2.53	0	2.53	in
Austria	.1	3.27	.2	2.18	in
Belgium	.34	1.95	.34	1.95	out
Canada	.02	3.64	.05	2.17	in
Chile	.05	3.49	.05	3.49	out
Czech Republic	.26	4.59	.26	4.59	out
Denmark	.05	4.61	.08	2.13	in
Finland	.03	4.68	.06	2.13	in
France	.09	4.33	.2	1.74	in
Germany	.1	2.67	.2	1.91	in
Iceland	0	4.98	0	4.98	out
Ireland	.34	2.21	.34	2.21	out
Israel	.04	3.88	.04	3.88	in
Japan	.3	2.96	.66	.54	in
Luxembourg	.34	2.17	.34	2.17	out
Mexico	0	21.06	0	11.65	out
Netherlands	.34	1.93	.34	1.93	out
New Zealand	0	3.37	0	2.4	in
Norway	0	3.14	0	2.31	in
Peru	0	3.56	0	3.56	in
Poland	0	15.41	0	15.41	out
Portugal	.34	2.1	.34	2.1	out
Singapore	.28	2.01	.29	2.02	in
South Africa	0	7.52	0	7.2	out
Spain	.13	6.65	.2	3.2	in
Sweden	.03	4.37	.07	2.22	in
Switzerland	.29	2.27	.36	1.31	in
United Kingdom	.1	4.98	.22	2.65	in
United States	.11	3.62	.24	2.61	in

- ▶ Consumer Price Index of UK's Office of National Statistics
  - Monthly price quotes goods and services (1100 per month)
  - Time period: 1996m1-1016m3
  - Public available
  - Similar price statistics than other low inflation countries
- ▶ Micro-price statistics for model
  - Filter sales
  - Filter heterogeneity

**S** 2 filters for sales

- 1 Drop price changes with sales flags
- 2 Additional filter: fix  $T_s$  period of sales and  $\epsilon$

$$\mathcal{D}_{T_s}^{i,\epsilon} = \left\{ t : \left| \sum_{j=0}^{T_s} (p_{t+j} - p_{t-1+j}) \right| < \epsilon \right\}$$

Drop price changes between  $t^*$  and  $t^*$  with  $t^* \in \mathcal{D}_{T_s}^{i,\epsilon}$

**H** Filter product level heterogeneity: for each price change

$$\Delta \tilde{p}_{ti} = \frac{\Delta p_{ti} - \mathbb{E}[\Delta p_{ti} | i \in \text{item } j]}{\text{Std}[\Delta p_{ti} | i \in \text{item } j]} \text{Std}[\Delta p_{ti}] + \mathbb{E}[\Delta p_{ti}]$$

- Compute micro-price statistics over  $\Delta \tilde{p}_{ti}$

- ▶ Taylor rule for interest rate:  $R_t = \max\{1, R_t^*\}$

$$R_t^* = \left(\frac{1 + \bar{\pi}}{\beta}\right)^{1 - \phi_r} (R_{t-1}^*)^{\phi_\pi} \left[ \left(\frac{1 + \pi_t}{1 + \bar{\pi}}\right)^{\phi_{\bar{\pi}}} \left(\frac{X_t}{X_{ss}}\right)^{\phi_{\bar{y}}} \right]^{1 - \phi_\pi} \left(\frac{X_t}{X_{t-1}}\right)^{\phi_{d\bar{y}}} \eta_{rt}$$

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$$C_t + \eta_{tg} = GDP_t$$

# Final Good Producer

Return

$$\max_{\{Y_t, \{y_{t,i}\}_i\}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} Q_t \left( P_t Y_t - \int_0^1 p_{ti} y_{ti} di \right) \right] \quad s.t.$$
$$Y_t = \left( \int_0^1 \left( \frac{y_{ti}}{A_{ti}} \right)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

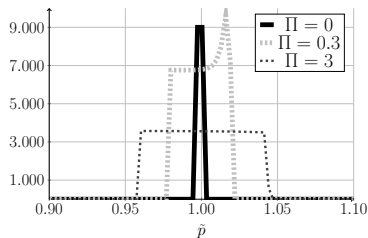
- $Y_t, y_{ti}$ : final output and intermediate inputs
- $Q_t$ : nominal discount factor
- $p_{ti}$ : firm  $i$  nominal price
- $A_{ti}$ : *quality* idiosyncratic shock

$$P_t = \left( \int_0^1 (p_{ti} A_{ti})^{1-\gamma} di \right)^{1/(1-\gamma)} \quad y_t(A_{ti}, p_{ti}) = A_{ti} \left( \frac{A_{ti} p_{ti}}{P_t} \right)^{-\gamma} Y_t$$

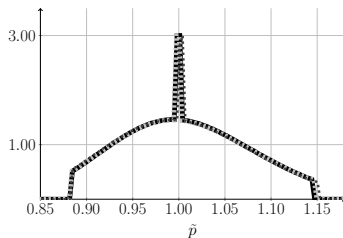
# Menu Cost With and Without Idiosyncratic Shocks

Return

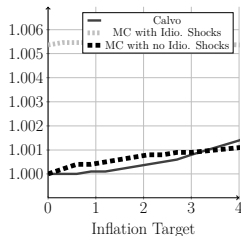
A. Distribution  $f(\tilde{p})$  with no Idiosyncratic Shocks



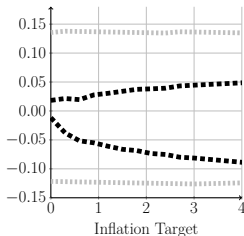
B. Distribution  $f(\tilde{p})$  with Idiosyncratic Shocks



C. Price Dispersion



D. Ss bands



E. Frequency of Price Change

