Optimal Trend Inflation

Klaus Adam
University of Mannheim

Henning Weber
Deutsche Bundesbank

September 2017
Add firm heterogeneity (productivity) to otherwise standard sticky price economy
Add firm heterogeneity (productivity) to otherwise standard sticky price economy

Productivity at firm level displays *systematic* trends:
- life cycle: firms start small/unproductive, become productive, exit
- product life cycle: new products, higher quality, initially higher price
Add firm heterogeneity (productivity) to otherwise standard sticky price economy

Productivity at firm level displays *systematic* trends:
- life cycle: firms start small/unproductive, become productive, exit
- product life cycle: new products, higher quality, initially higher price

Productivity trends at the firm level
⟹ strongly affect optimal inflation dynamics
    & rationalizes positive steady state inflation
Large part of existing sticky price literature:

abstracts from firm level heterogeneity, except for price heterogeneity
Introduction

- Large part of existing sticky price literature:
  - abstracts from firm level heterogeneity, except for price heterogeneity
- Technically motivated: aggregating 2-dim. heterogeneity a challenge

*Strong economic implications*: zero inflation optimal
Large part of existing sticky price literature:
abstracts from firm level heterogeneity, except for price heterogeneity

Technically motivated: aggregating 2-dim. heterogeneity a challenge

*Strong economic implications*: zero inflation optimal

Productivity of price adjusting firms equal to productivity of non-adjusting firms
Introduction

- Large part of existing sticky price literature:
  abstracts from firm level heterogeneity, except for price heterogeneity
- Technically motivated: aggregating 2-dim. heterogeneity a challenge
  
  *Strong economic implications*: zero inflation optimal

- Productivity of price adjusting firms equal to productivity of non-adjusting firms

- Adjusting firms’ price = price of non-adjusting firms
  
  \[
  \implies \text{strong force towards zero inflation}
  \]


Idiosyncratic firm level productivity $\Leftrightarrow$ without systematic trend

Do not look at optimal inflation

Results suggest zero inflation optimal:

$\text{av. prod. of adjusting firm} \approx \text{av. prod. of non-adjusting firm}$
Enrich basic homogeneous firm setup by adding:

- Firm entry & exit

- Measure $\delta$ of randomly selected firms: very negative productivity shock & exit

- Exiting firms replaced by same measure of newly entering firms

- Alternative interpretations of setup possible (product substitution, quality improvements)
Firm-level productivity trends driven by 3 underlying trends:

- **aggregate trend**: productivity gains experienced by all firms
- **experience trend**: firms become more productive over time
- **cohort trend**: productivity level for new cohort of firms
Production function of firm $j \in [0, 1]$:

$$Y_{jt} = A_t Q_{t-s_{jt}} G_{jt} \left( K_{jt}^{1-1/\phi} L_{jt}^{1/\phi} - F_t \right),$$

where $s_{jt}$ is time since last $\delta$-shock

$$A_t = a_t A_{t-1},$$

$$Q_t = q_t Q_{t-1},$$

$$G_{jt} = \begin{cases} 1 & \text{if } s_{jt} = 0, \\ g_t G_{jt-1} & \text{otherwise.} \end{cases}$$

$(a_t, q_t, g_t)$ arbitrary stationary process w mean $a, q, g$
Production function of firm $j \in [0, 1]$: 

$$Y_{jt} = A_t Q_{t-s_j} G_{jt} \left( K_{jt}^{\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t \right),$$ 

where $s_j$ is time since last $\delta$-shock

$$A_t = a_t A_{t-1},$$
$$Q_t = q_t Q_{t-1},$$
$$G_{jt} = \begin{cases} 
1 & \text{if } s_j = 0, \\
 g_t G_{jt-1} & \text{otherwise}.
\end{cases}$$

$(a_t, q_t, g_t)$ arbitrary stationary process w mean $a, q, g$

Three productivity trends: $a, q$ and $g$
Introduction

- Production function of firm $j \in [0, 1]$:

$$Y_{jt} = A_t Q_{t-sjt} G_{jt} \left( K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t \right),$$

where $s_{jt}$ is time since last $\delta$-shock

$$A_t = a_t A_{t-1},$$
$$Q_t = q_t Q_{t-1},$$
$$G_{jt} = \begin{cases} 
1 & \text{if } s_{jt} = 0, \\
g_{jt} G_{jt-1} & \text{otherwise.}
\end{cases}$$

$(a_t, q_t, g_t)$ arbitrary stationary process w mean $a, q, g$

- Three productivity trends: $a, q$ and $g$

- Measure $\delta$ of firms: productivity drops to zero & exit
Production function of firm $j \in [0, 1]$: 

$$Y_{jt} = A_t Q_{t-s_{jt}} G_{jt} \left( K_{jt}^{\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t \right),$$

where $s_{jt}$ is time since last $\delta$-shock

$$A_t = a_t A_{t-1},$$

$$Q_t = q_t Q_{t-1},$$

$$G_{jt} = \begin{cases} 
1 & \text{if } s_{jt} = 0, \\
 g_t G_{jt-1} & \text{otherwise.} 
\end{cases}$$

$(a_t, q_t, g_t)$ arbitrary stationary process w mean $a, q, g$

Three productivity trends: $a, q$ and $g$

Measure $\delta$ of firms: productivity drops to zero & exit

Special cases w/o firm level trends: $\delta = 0$ or if $q_t \equiv g_t$
Figure: Productivity dynamics in a setting with firm entry and exit
Setup naturally generates positive steady state inflation, if young firms initially less productive than non-exiting incumbents.
Setup naturally generates positive steady state inflation, if young firms initially less productive than non-exiting incumbents

In line with young firms being small

\[ \text{av. prod. adjusting firm} < \text{av. prod. non-adjusting firm} \]

\[ \text{relative price of adj. to non-adj. firm larger than one} \]

Strength of effect independent of turnover rate

\[ \delta > 0 \]

Discontinous jump of optimal inflation:

\[ \delta = 0 \rightarrow \delta > 0 \]
Introduction

- Setup naturally generates positive steady state inflation, if young firms initially less productive than non-exiting incumbents.

- In line with young firms being small:
  - $\text{av. prod. adjusting firm} < \text{av. prod. non-adjusting firm}$
  - $\Rightarrow$ relative price of adj. to non-adj. firm larger than one

- Inefficient that existing firms adjust: price dispersion/adjustment costs
  - $\Rightarrow$ positive rates of inflation optimal in steady state
Introduction

- Setup naturally generates positive steady state inflation, if young firms initially less productive than non-exiting incumbents.

- In line with young firms being small:
  \[ \text{av. prod. adjusting firm} < \text{av. prod. non-adjusting firm} \]
  \[ \implies \text{relative price of adj. to non-adj. firm larger than one} \]

- Inefficient that existing firms adjust: price dispersion/adjustment costs:
  \[ \implies \text{positive rates of inflation optimal in steady state} \]

- **Strength of effect independent of turnover rate** \( \delta > 0 \)
  Discontinous jump of optimal inflation: \( \delta = 0 \rightarrow \delta > 0 \)
 Aggregate NL model in closed form & determine opt. inflation
• Aggregate NL model in closed form & determine opt. inflation

• Optimal gross steady state inflation rate

\[ \Pi^* = \frac{g}{q}, \]

independent of TFP trend \( a \).
Introduction

- Aggregate NL model in closed form & determine opt. inflation

- Optimal gross steady state inflation rate

\[ \Pi^* = \frac{g}{q}, \]

independent of TFP trend \( a \).

- Optimal inflation

  - cannot be inferred from aggregate productivity trends
  - has to know firm level trends & shocks to these trends
Introduction

- Aggregate NL model in closed form & determine opt. inflation

- Optimal gross steady state inflation rate

\[
\Pi^* = \frac{g}{q},
\]

independent of TFP trend \(a\).

- Optimal inflation
  - cannot be inferred from aggregate productivity trends
  - has to know firm level trends & shocks to these trends

- Optimal inflation \(\Pi^* = 1\) if \(\delta = 0\).
Introduction

- What is the optimal inflation rate of the US economy?
What is the optimal inflation rate of the US economy?

Extend model to multi-sector economy: sector-specific price stickiness & sector-specific trends in TFP ($a_{zt}$), experience ($q_{zt}$) and cohort ($g_{zt}$)
Introduction

- What is the optimal inflation rate of the US economy?
- Extend model to multi-sector economy: sector-specific price stickiness & sector-specific trends in TFP ($a_{zt}$), experience ($q_{zt}$) and cohort ($g_{zt}$)
- Derive in closed form an analytical expression for the optimal SS inflation rate
What is the optimal inflation rate of the US economy?

Extend model to multi-sector economy: sector-specific price stickiness & sector-specific trends in TFP ($a_{zt}$), experience ($q_{zt}$) and cohort ($g_{zt}$)

Derive in closed form an analytical expression for the optimal SS inflation rate

Model-consistent approach for estimating SS inflation rate from firm level trends: 147million firm observations from the LBD database (US Census)
What is the optimal inflation rate of the US economy?

Extend model to multi-sector economy: sector-specific price stickiness & sector-specific trends in TFP \((a_{zt})\), experience \((q_{zt})\) and cohort \((g_{zt})\)

Derive in closed form an analytical expression for the optimal SS inflation rate

Model-consistent approach for estimating SS inflation rate from firm level trends: 147 million firm observations from the LBD database (US Census)

Estimated optimal infl. rate steadily declined:

1986: \(\approx 2\%\) \(\quad \Rightarrow \quad\) 2013: \(\approx 1\%\)
Related Literature

- Few papers: inflation $\Leftrightarrow$ productivity dynamics

- All of them find negative inflation rates optimal:
  - Wolman (JMCB, 2011): two sector economy with different sectorial productivity trends, homogeneous firms in each sector, neg. inflation optimal despite monetary frictions being absent
  - Amano, Murchison & Rennison (JME, 2009): homogeneous firm model with sticky prices and wages & aggregate growth; wages more sticky than prices; to depress wage-markups deflation turns out optimal.

• Zero lower bound cannot justify positive average rates of inflation: Adam & Billi (2006), Coibion, Gorodnichenko & Wieland (2012)

• Brunnermeier and Sannikov (2016): idiosyncratic risk → positive inflation increasingly optimal


• Positive inflation possibly optimal in models with endogenous entry: Corsetti & Bergin (2008), Bilbiie, Ghironi & Melitz (2008), Bilbiie, Fujiwara & Ghironi (2014)
Outline of Remaining Talk

1. **Sticky price model with δ-shocks**
2. Aggregation & optimality of flex price equilibrium
3. Optimal inflation: main result
4. Multi-sector extension & empirical strategy
Consider a Calvo sticky price setup: price stickiness parameter $\alpha$ (main results extend to menu cost setting)

- Continuum of sticky price firms, Dixit-Stiglitz aggregate $Y_t$
- Random sample $\delta$ receives $\delta$-shocks
- Firm productivity dynamics as described before
- Competitive labor and capital markets
Household problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left( \frac{[C_t V(L_t)]^{1-\sigma} - 1}{1-\sigma} \right)$$

s.t.

$$C_t + K_{t+1} + \frac{B_t}{P_t} =$$

$$(r_t + 1 - d)K_t + \frac{W_t}{P_t}L_t + \int_0^1 \frac{\Theta_{jt}}{P_t} \, dj + \frac{B_{t-1}}{P_t}(1 + i_{t-1}) - T_t$$

Existence of balanced growth path:

$$\beta < (aq)^{\phi\sigma} \text{ and } (1 - \delta)(g/q)^{\theta-1} < 1$$
Outline of Remaining Talk

1. Sticky price model with $\delta$-shocks

2. Aggregation & optimality of flex price equilibrium

3. Optimal inflation: main result

4. Multi-sector extension & empirical strategy
• Highlight the differences relative to a model with homogeneous firms

• Will spare you the derivation behind the results...
Aggregate output $Y_t$:

\[ Y_t = \frac{A_t Q_t}{\Delta_t} \left( K_t^{1 - \frac{1}{\phi}} L_t^{\frac{1}{\phi}} - F_t \right), \]

with $K_t, L_t$ aggregate capital, labor and $F_t \geq 0$ fixed costs

$\Delta_t$: captures joint distribution of prices & productivities:

\[ \Delta_t = \int_0^1 \left( \frac{Q_t}{G_{jt} Q_t s_{jt}} \right) \left( \frac{P_{jt}}{P_t} \right)^{-\theta} dj \] (1)
Price Level

- Price level: exp.-weighted average of product prices

\[ P_t = \left( \int_{0}^{1} (P_{jt})^{1-\theta} \,dj \right)^{\frac{1}{1-\theta}} \]

\[ = \int_{0}^{1} \left( \frac{Y_{jt}}{Y_t} \right) P_{jt} \,dj \]

Price level accounts for product substitution (as statistical agencies do)
Price Level

- Price level: exp.-weighted average of product prices

\[ P_t = \left( \int_0^1 (P_{jt})^{1-\theta} \, dj \right)^{\frac{1}{1-\theta}} \]

\[ = \int_0^1 \left( \frac{Y_{jt}}{Y_t} \right) P_{jt} \, dj \]

Price level accounts for product substitution (as statistical agencies do)

- Inflation:

\[ \Pi_t = \frac{P_t}{P_{t-1}}. \]
Aggregation Price Level Dynamics

Evolution of the aggregate price under opt. price setting:

\[ P_{t}^{1-\theta} = \left( \delta_{\text{new firms}} + (1 - \alpha)(1 - \delta) \left( \frac{p_{t}^{n}(\theta-1) - \delta}{1 - \delta} \right) \right) P_{t,t}^{1-\theta} + \alpha(1 - \delta)P_{t-1}^{1-\theta} \]

\[ (p_{t}^{n})^{\theta-1} = \delta + (1 - \delta) \left( \frac{g_{t}}{q_{t}} \right)^{\theta-1}. \]
Aggregate Price Level Dynamics

\[ g_t \equiv q_t \implies \text{no firm level trends and} \ (p^n_t)^{\theta-1} \rightarrow 1 \ \text{and} \]

\[ P_t^{1-\theta} = (\delta + (1-\alpha)(1-\delta))(P^*_t)^{1-\theta} + \alpha(1-\delta)(P_{t-1})^{1-\theta} \]

If - in addition - \( \delta = 0 \):

\[ P_t^{1-\theta} = (1-\alpha)(P^*_t)^{1-\theta} + \alpha(P_{t-1})^{1-\theta} \]

Standard price evolution equation in homogeneous firm models.
Attaining efficiency requires
- eliminating firm’s monopoly power by an output subsidy
- choosing $\Delta_t$ in the production function

\[ Y_t = \frac{A_t Q_t}{\Delta_t} \left( K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} - F_t \right), \]

equal to

\[ \Delta_t = \Delta_t^e = \left( \int_0^1 \left( \frac{Q_t}{G_{jt} Q_{t-s_j t}} \right)^{1-\theta} dj \right)^{\frac{1}{1-\theta}} \]
Attaining efficiency requires

- eliminating firm’s monopoly power by an output subsidy
- choosing \( \Delta_t \) in the production function

\[
Y_t = \frac{A_t Q_t}{\Delta_t} \left( K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} - F_t \right),
\]

equal to

\[
\Delta_t = \Delta_t^e = \left( \int_0^1 \left( \frac{Q_t}{G_{jt} Q_{t-s_{jt}}} \right)^{1-\theta} \right)^{\frac{1}{1-\theta}}.
\]

\( \Delta_t = \Delta_t^e \) decentralized by prices satisfying

\[
\frac{P_{jt}}{P_t} = \frac{1}{\Delta_t^e} \frac{Q_t}{G_{jt} Q_{t-s_{jt}}}.
\]
Proposition: With flexible prices ($\alpha = 0$) & appropriate output subsidy, the equilibrium allocation is efficient.

The optimal inflation rate is indeterminate....
1. Sticky price model with $\delta$-shocks

2. Aggregation & optimality of flex price equilibrium

3. Optimal inflation: main result

4. Multi-sector extension & empirical strategy
Efficiency under Sticky Prices

- Empirically relevant case $E[g_t] > E[q_t] \Leftrightarrow$ new firms small/new products expensive
Empirically relevant case \( E[g_t] > E[q_t] \iff \text{new firms small/new products expensive} \)

From

\[
(p^n_t)_{\theta-1} = \delta + (1 - \delta) \left( \frac{p^n_{t-1} g_t}{q_t} \right)^{\theta-1}.
\]

we tend to get that \((p^n_t)^{\theta-1} > 1\).
Empirically relevant case $E[g_t] > E[q_t] \iff$ new firms small/new products expensive

From

$$(p^n_t)^{\theta-1} = \delta + (1-\delta) \left( p^n_{t-1} \frac{g_t}{q_t} \right)^{\theta-1}.$$  

we tend to get that $(p^n_t)^{\theta-1} > 1$.

Price level equation

$$P_{t-\theta} = (\delta + (1-\alpha)(1-\delta) \frac{(p^n_t)^{\theta-1} - \delta}{1-\delta})(P^*_t)^{1-\theta} + \alpha(1-\delta)(P_{t-1})^{1-\theta},$$

$\implies$ old firms choose higher $(P_{tj})^{1-\theta}$ than new firms

$\implies$ since $1 - \theta < 0$: old firms to set lower prices than new firms
Efficiency under Sticky Prices

- Empirically relevant case $E[g_t] > E[q_t] \Leftrightarrow$ new firms small/new products expensive

- From
  
  $$(p^n_t)^{\theta-1} = \delta + (1 - \delta) \left(p^n_{t-1} \frac{g_t}{q_t}\right)^{\theta-1}. $$

  we tend to get that $(p^n_t)^{\theta-1} > 1$.

- Price level equation

  $$P^{1-\theta}_t = (\delta + (1 - \alpha)(1 - \delta) \left(p^n_t\right)^{\theta-1} - \delta \left(P^*_t\right)^{1-\theta} + \alpha(1 - \delta)(P^*_{t-1})^{1-\theta},$$

  $\implies$ old firms choose higher $(P^*_{tj})^{1-\theta}$ than new firms

  $\implies$ since $1 - \theta < 0$: old firms to set lower prices than new firms

- Efficiency: old firms that adjust must choose same price as old firms that do not adjust
Efficiency under Sticky Prices

- Empirically relevant case $E[g_t] > E[q_t] \iff$ new firms small/new products expensive

- From

$$\left(p^n_t\right)^{\theta-1} = \delta + (1 - \delta) \left(p^n_{t-1} \frac{g_t}{q_t}\right)^{\theta-1}.$$  

we tend to get that $\left(p^n_t\right)^{\theta-1} > 1$.

- Price level equation

$$P^{1-\theta}_t = (\delta + (1 - \alpha)(1 - \delta) \frac{\left(p^n_t\right)^{\theta-1} - \delta}{1 - \delta})(P^*_{t,t})^{1-\theta} + \alpha(1 - \delta)(P_{t-1})^{1-\theta},$$

$\implies$ old firms choose higher $(P_{tj})^{1-\theta}$ than new firms  

$\implies$ since $1 - \theta < 0$: old firms to set lower prices than new firms

- Efficiency: old firms that adjust must choose same price as old firms that do not adjust

- Need to allow for inflation to achieve efficiency!
**Proposition:** Suppose (1) there is an appropriate output subsidy and (2) initial prices in $t = -1$ reflect firms’ relative productivities, i.e., $P_{j,-1} \propto 1/(Q_{-1-sj,-1}G_{j,-1})$ for all $j \in [0, 1]$. The eq. allocation is efficient under sticky price if

$$\Pi_t^* = \left( \frac{1 - \delta / (\Delta_t^{e,1-\theta})}{1 - \delta} \right)^{\frac{1}{\theta-1}}$$

for all $t \geq 0$, where $(\Delta_t^{e,1-\theta}) = \delta + (1 - \delta) (\Delta_{t-1}^{e} q_t / g_t)^{1-\theta}$. 

---

Adam & Weber (University of Mannheim & CEPR Deutsche Bundesbank)
Proposition: Suppose (1) there is an appropriate output subsidy and (2) initial prices in $t = -1$ reflect firms’ relative productivities, i.e.,

$$P_{j,-1} \propto 1/(Q_{-1-s_{j,-1}} G_{j,-1})$$

for all $j \in [0, 1]$. The eq. allocation is efficient under sticky price if

$$\Pi^*_t = \left( \frac{1 - \delta/(\Delta_t^{e})^{1-\theta}}{1 - \delta} \right)^{\frac{1}{\theta-1}}$$

for all $t \geq 0$, where

$$(\Delta_t^{e})^{1-\theta} = \delta + (1 - \delta) (\Delta_{t-1}^{e} q_t / g_t)^{1-\theta}.$$ 

Prop holds for arbitrary initial prod. distributions & arbitrary shock processes (consistent with balanced growth)
Proposition: Suppose (1) there is an appropriate output subsidy and (2) initial prices in $t = -1$ reflect firms’ relative productivities, i.e., $P_{j,-1} \propto 1/(Q_{j,-1} G_{j,-1})$ for all $j \in [0, 1]$. The eq. allocation is efficient under sticky price if

$$
\Pi_t^* = \left( \frac{1 - \delta / (\Delta_t^e)^{1-\theta}}{1 - \delta} \right)^{\frac{1}{\theta-1}}
$$

for all $t \geq 0$, where $(\Delta_t^e)^{1-\theta} = \delta + (1 - \delta) (\Delta_{t-1}^e q_t / g_t)^{1-\theta}$.

Prop holds for arbitrary initial prod. distributions & arbitrary shock processes (consistent with balanced growth)

Proof works as follows: under the inflation rate (2)
1. new firms choose relative price as in the flex price economy
2. existing firms do not want to adjust their price.
3. with initial prices ’right’ & output subsidy $\implies$ flex price alloc.
Efficiency under Sticky Prices

\[ \Pi_t^* = \left( \frac{1 - \delta / (\Delta^e_t)^{1-\theta}}{1 - \delta} \right)^{\frac{1}{\theta-1}} \]

- In the absence \( \delta \)-shocks/firm level trends (\( \delta = 0 \) and/or \( g_t \equiv q_t \)) get familiar result:

\[ \Pi_t^* \equiv 1 \]
\[
\Pi^*_t = \left( \frac{1 - \delta}{(\Delta^e_t)^{1-\theta}} \right)^{\frac{1}{\theta-1}}
\]

- **In the absence** \(\delta\)-shocks/firm level trends \((\delta = 0 \text{ and/or } g_t \equiv q_t)\) get familiar result:
  \[
  \Pi^*_t \equiv 1
  \]

- Price stability optimal, independently of realized productivity shocks.
Efficiency under Sticky Prices

\[ \Pi_t^* = \left( \frac{1 - \delta / (\Delta_t^e)^{1-\theta}}{1 - \delta} \right)^{\frac{1}{\theta-1}} \]

where \((\Delta_t^e)^{1-\theta} = \delta + (1 - \delta) (\Delta_{t-1}^e q_t / g_t)^{1-\theta} \).

- With firm level trends \((\delta > 0)\), steady state inflation is

\[ \lim \Pi_t^* = \frac{g}{q} \]
Efficiency under Sticky Prices

\[
\Pi_t^* = \left( \frac{1 - \delta}{1 - \delta} \right)^{\frac{1}{\theta-1}} \left( \frac{1 - \delta}{\Delta_t^e \theta} \right)^{\frac{1}{\theta-1}}
\]

where \((\Delta_t^e \theta)^{1-\theta} = \delta + (1 - \delta) (\Delta_{t-1}^e q_t / g_t)^{1-\theta}\).

- **With firm level trends** \((\delta > 0)\), steady state inflation is

  \[\lim \Pi_t^* = \frac{g}{q}\]

- SS inflation positive when \(g > q\)
Efficiency under Sticky Prices

\[ \Pi_t^* = \left( \frac{1 - \delta / (\Delta^e_t)^{1-\theta}}{1 - \delta} \right)^{\frac{1}{\theta-1}} \]

where \( (\Delta^e_t)^{1-\theta} = \delta + (1 - \delta) (\Delta^e_{t-1} q_t / g_t)^{1-\theta} \).

- **With firm level trends** \((\delta > 0)\), steady state inflation is

\[ \lim \Pi_t^* = \frac{g}{q} \]

- SS inflation positive when \(g > q\)
- SS independent of \(\delta\):
  - fewer unproductive firms enter \(\rightarrow\) lower inflation
  - existing firms accumulated more experience \(\rightarrow\) higher inflation
\[ \Pi_t^* = \left( \frac{1 - \delta / (\Delta_t^e)^{1-\theta}}{1 - \delta} \right)^{\frac{1}{\theta-1}} \]  

where \( (\Delta_t^e)^{1-\theta} = \delta + (1 - \delta) (\Delta_{t-1}^e q_t / g_t)^{1-\theta} \).

**Linearization:**

\[ \pi_t^* = (1 - \delta) \pi_{t-1}^* + \delta \left( \frac{g_t}{q_t} - 1 \right) + O(2) \]

- Positive experience shock \((g_t)\): persistent rise in opt. inflation
Efficiency under Sticky Prices

\[ \Pi_t^* = \left( \frac{1 - \delta / (\Delta_t^e)^{1-\theta}}{1 - \delta} \right)^{\frac{1}{\theta - 1}} \]  

where \((\Delta_t^e)^{1-\theta} = \delta + (1 - \delta) (\Delta_{t-1}^e q_t / g_t)^{1-\theta}\).

**Linearization:**

\[ \pi_t^* = (1 - \delta) \pi_{t-1}^* + \delta \left( \frac{g_t}{q_t} - 1 \right) + O(2) \]

- Positive experience shock \((g_t)\): persistent rise in opt. inflation
- Positive cohort shock \((q_t)\): persistent drop in opt inflation
Efficiency under Sticky Prices

\[ \Pi_t^* = \left( \frac{1 - \delta / (\Delta_t^e)^{1-\theta}}{1 - \delta} \right)^{\frac{1}{\theta-1}} \]  

(4)

where \( (\Delta_t^e)^{1-\theta} = \delta + (1 - \delta) (\Delta_{t-1}^e q_t / g_t)^{1-\theta} \).

Linearization:

\[ \pi_t^* = (1 - \delta) \pi_{t-1}^* + \delta \left( \frac{g_t}{q_t} - 1 \right) + O(2) \]

- Positive experience shock \((g_t)\): persistent rise in opt. inflation
- Positive cohort shock \((q_t)\): persistent drop in opt inflation
- \( \lim_{\delta \to 0} : \pi_t^* \) random walk, but \( \text{Var}(\pi_t^*) \to 0. \)
Suppose MP implements $\Pi = 1$ in an economy where $\Pi^* \neq 1$

- Analytical result: strictly positive welfare costs even in the limit $\delta \to 0$
- Numerical illustration highlighting the source of welfare distortions
The Welfare Costs of Strict Price Stability

Assumptions for the analytical result:

- there is an optimal output subsidy and initial prices reflect initial productivities
- there are no aggregate productivity disturbances and $\delta > 0$
- fixed costs of production are zero ($f = 0$)
- disutility of work is given by
  \[ V(L) = 1 - \psi L^\nu, \text{ with } \nu > 1, \psi > 0. \]

- $g/q > \alpha (1 - \delta)$, so that a well-defined steady state with strict price stability exists
- consider the limit $\beta (\gamma^e)^{1-\sigma} \rightarrow 1$
Proposition: Consider a policy implementing the optimal inflation rate \( \Pi^*_t \), which satisfies \( \lim_{t \to \infty} \Pi^*_t = \Pi^* = g/q \). Let \( c(\Pi^*) \) and \( L(\Pi^*) \) denote the limit outcomes for \( t \to \infty \) for consumption and hours under this policy. Similarly, let \( c(1) \) and \( L(1) \) denote the limit outcomes under the alternative policy of implementing strict price stability. Then,

\[
L(1) = L(\Pi^*)
\]

and

\[
\frac{c(1)}{c(\Pi^*)} = \left( \frac{1 - \alpha(1 - \delta)(g/q)^{\theta - 1}}{1 - \alpha(1 - \delta)} \right)^{\frac{\phi \theta}{\theta - 1}} \left( \frac{1 - \alpha(1 - \delta) (g/q)^{-1}}{1 - \alpha(1 - \delta)(g/q)^{\theta - 1}} \right)^{\phi} \leq 1.
\]

For \( g \neq q \) the previous inequality is strict and

\[
\lim_{\delta \to 0} \frac{c(1)}{c(\Pi^*)} < 1
\]
The Welfare Costs of Strict Price Stability

A. Relative cohort price: cohort mean, different inflation rates

B. Relative cohort price: mean and 2 std.dev. bands

Figure: Relative prices and inflation
Figure: Aggregate productivity as a function of gross steady state inflation (optimal inflation rate is 1.02)
Outline of Remaining Talk

1. Sticky price model with $\delta$-shocks
2. Aggregation & optimality of flex price equilibrium
3. Optimal inflation: main result
4. Multi-sector extension & empirical strategy
Goal: quantify inflation rates arising from firm trends

Take into account of sector-specific productivity trends: manufacturing vs services

Present a multi-sector extension of our analytical results & model-consistent empirical strategy
Multi-Sector Extension / Empirical Strategy

- $z = 1, \ldots, Z$ sectors, Dixit-Stiglitz competition, sector output $Y_{zt}$
Multi-Sector Extension / Empirical Strategy

- $z = 1, \ldots, Z$ sectors, Dixit-Stiglitz competition, sector output $Y_{zt}$
- Aggregate output

\[ Y_t = \prod_{z=1}^{Z} (Y_{zt})^{\psi_z} \text{ with } \sum_{z=1}^{Z} \psi_z = 1 \]
Multi-Sector Extension / Empirical Strategy

- $z = 1, \ldots, Z$ sectors, Dixit-Stiglitz competition, sector output $Y_{zt}$
- Aggregate output

$$Y_t = \prod_{z=1}^{Z} (Y_{zt})^{\psi_z} \text{ with } \sum_{z=1}^{Z} \psi_z = 1$$

- Sector-specific TFP, cohort and experience trends

$$a_{zt} = a_z \varepsilon_{zt}^a \text{ and } q_{zt} = q_z \varepsilon_{zt}^q \text{ and } g_{zt} = g_z \varepsilon_{zt}^g,$$
• $z = 1, \ldots, Z$ sectors, Dixit-Stiglitz competition, sector output $Y_{zt}$

• Aggregate output

$$Y_t = \prod_{z=1}^{Z} (Y_{zt})^{\psi_z} \text{ with } \sum_{z=1}^{Z} \psi_z = 1$$

• Sector-specific TFP, cohort and experience trends

$$a_{zt} = a_z \varepsilon^a_{zt} \text{ and } q_{zt} = q_z \varepsilon^q_{zt} \text{ and } g_{zt} = g_z \varepsilon^g_{zt},$$

• Sector-specific price stickiness $\alpha_z \in (0, 1)$
$z = 1, \ldots, Z$ sectors, Dixit-Stiglitz competition, sector output $Y_{zt}$

Aggregate output

$$Y_t = \prod_{z=1}^{Z} (Y_{zt})^{\psi_z} \quad \text{with} \quad \sum_{z=1}^{Z} \psi_z = 1$$

Sector-specific TFP, cohort and experience trends

$$a_{zt} = a_z \varepsilon_{zt}^a \quad \text{and} \quad q_{zt} = q_z \varepsilon_{zt}^q \quad \text{and} \quad g_{zt} = g_z \varepsilon_{zt}^g,$$

Sector-specific price stickiness $\alpha_z \in (0, 1)$

Sector-specific entry/exit rates $\delta_z \in (0, 1)$.
- $z = 1, \ldots, Z$ sectors, Dixit-Stiglitz competition, sector output $Y_{zt}$

- Aggregate output

$$Y_t = \prod_{z=1}^{Z} (Y_{zt})^{\psi_z} \text{ with } \sum_{z=1}^{Z} \psi_z = 1$$

- Sector-specific TFP, cohort and experience trends

$$a_{zt} = a_z \varepsilon_{zt}^a \text{ and } q_{zt} = q_z \varepsilon_{zt}^q \text{ and } g_{zt} = g_z \varepsilon_{zt}^g,$$

- Sector-specific price stickiness $\alpha_z \in (0, 1)$

- Sector-specific entry/exit rates $\delta_z \in (0, 1)$.

- Aggregate price level: $P_t = \prod_{z=1}^{Z} \left( \frac{P_{zt}}{\psi_z} \right)^{\psi_z}$
**Proposition:** Suppose initial prices reflect initial productivity, no economic disturbances, and an optimal output subsidy. Consider the limit $\beta(\gamma^e)^{1-\sigma} \to 1$ and suppose monetary policy implements $\Pi_t = \Pi$ for all $t$. The inflation rate $\Pi$ that maximizes the resulting steady state utility is

$$\Pi^* = \sum_{z=1}^{Z} \omega_z \left( \frac{g_z \gamma^e_z}{q_z \gamma^e} \right),$$

where

$$\frac{\gamma^e_z}{\gamma^e} = \frac{a_z q_z}{\prod_{z=1}^{Z} (a_z q_z) \psi_z}$$

is the growth trend of sector $z$ relative to the growth trend of the aggregate economy in the efficient allocation.

The sector weights $\omega_z \geq 0$ sum to one and are given by

$$\tilde{\omega}_z = \frac{\psi_z \theta \alpha_z (1 - \delta_z) (\Pi \gamma^e / \gamma^e_z)^{\theta} (q_z / g_z)}{[1 - \alpha_z (1 - \delta_z) (\Pi \gamma^e / \gamma^e_z)^{\theta} (q_z / g_z)] [1 - \alpha_z (1 - \delta_z) (\Pi \gamma^e / \gamma^e_z)^{\theta-1}]}.$$
The optimal steady state inflation rate

\[ \Pi^* = \sum_{z=1}^{Z} \psi_z \left( \frac{g_z \gamma_z^e}{q_z \gamma^e} \right) + O(2), \]

\( O(2) \): second order approximation error.
The optimal steady state inflation rate

\[ \Pi^* = \sum_{z=1}^{Z} \psi_z \left( \frac{g_z \gamma^e_z}{q_z \gamma^e} \right) + O(2), \]

\( O(2) \): second order approximation error.

Since \( \psi_z \) and \( \gamma^e_z / \gamma^e \) can be inferred from sectoral data, one only has to estimate \( g_z / q_z \) from firm level data.
The optimal steady state inflation rate

\[ \Pi^* = \sum_{z=1}^{Z} \psi_z \left( \frac{g_z \gamma_z^e}{q_z \gamma^e} \right) + O(2), \]

\( O(2) \) : second order approximation error.

Since \( \psi_z \) and \( \gamma_z^e / \gamma^e \) can be inferred from sectoral data, one only has to estimate \( g_z / q_z \) from firm level data.

How to estimate sector specific productivity trends \( g_z / q_z \)?

- firm level productivity: not observed.....
- firm level prices: not observed....
- firm level employment: productivity->prices->demand/employment
Model implies that $g_z / q_z$ can be estimated from firm level employment trends:

$$\ln(L_{jzt}) = d_{zt} + \eta_z \cdot s_{jzt} + \epsilon_{jzt},$$  

$$d_{zt}$$: sector dummy, $s_{jzt}$ the age of the firm $j$, and $\epsilon_{jzt}$ a stationary residual term, and

$$\eta_z = (\theta - 1) \ln(g_z / q_z).$$
Multi-Sector Extension / Empirical Strategy

- Estimate the firm level trends:
  - 65 BEA private industries
  - use LBD database for US Census Data: 147 million firm employment observations
  - use repeated cross-sections from 1986-2013 to estimate $g_z/q_z$
Estimate the firm level trends:
- 65 BEA private industries
- use LBD database for US Census Data: 147 million firm employment observations
- use repeated cross-sections from 1986-2013 to estimate $g_z / q_z$

Report $(\theta - 1) \Pi^* = (\theta - 1) \sum_{z=1}^{Z} w_z \left( \frac{g_z}{q_z} \frac{\gamma^e_z}{\gamma^e} \right)$
Optimal US Inflation times ($\theta - 1$)
Optimal US Inflation times ($\theta - 1$)
Table 1: Optimal Inflation Rate (Net)

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>TV Weights</th>
<th>LQ Specification</th>
<th>Baseline</th>
<th>TV Weights</th>
<th>LQ Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ = 3.8</td>
<td></td>
<td></td>
<td></td>
<td>θ = 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Pi^*_1996 )</td>
<td>2.34%</td>
<td>2.24%</td>
<td>2.70%</td>
<td>1.64%</td>
<td>1.57%</td>
<td>1.89%</td>
</tr>
<tr>
<td>( \Pi^*_2013 )</td>
<td>1.02%</td>
<td>1.02%</td>
<td>1.45%</td>
<td>0.71%</td>
<td>0.71%</td>
<td>1.01%</td>
</tr>
</tbody>
</table>

Notes: "Baseline" refers to the baseline estimate of \( \Phi_\delta \) with fixed GDP weights and age as single regressor. "TV Weights" refers to the estimate of \( \Phi_\delta \) that is based on time-varying GDP weights. "LQ Specification" refers to the estimate of \( \Phi_\delta \) that is based on a specification with both age and age squared as regressors. The parameter \( \theta \) denotes the product demand elasticity.
Conclusions

- Aggregate in closed form a sticky price model with firm level productivity trends

\[ \Pi = \frac{g}{q} > 1 \]

Productivity disturbances have persistent effects on optimal inflation

Optimal US inflation: dropped from approx 2% in 1986 to 1% in 2013
Conclusions

- Aggregate in closed form a sticky price model with firm level productivity trends
- Trends capture: product substitution, product quality improvements, or firm turnover

Steady state inflation $\Pi = g > 1$

Productivity disturbances have persistent effects on optimal inflation

Optimal US inflation: dropped from approx 2% in 1986 to 1% in 2013
Conclusions

- Aggregate in closed form a sticky price model with firm level productivity trends
- Trends capture: product substitution, product quality improvements, or firm turnover
- Firm level productivity trends key for optimal inflation rate in sticky price models

\[
\pi = g > 1
\]

Productivity disturbances have persistent effects on optimal inflation.

Optimal US inflation: dropped from approx 2% in 1986 to 1% in 2013
Conclusions

- Aggregate in closed form a sticky price model with firm level productivity trends
- Trends capture: product substitution, product quality improvements, or firm turnover
- Firm level productivity trends key for optimal inflation rate in sticky price models
- Steady state inflation $\Pi^* = \frac{g}{q} > 1$
Conclusions

- Aggregate in closed form a sticky price model with firm level productivity trends
- Trends capture: product substitution, product quality improvements, or firm turnover
- Firm level productivity trends key for optimal inflation rate in sticky price models
- Steady state inflation $\Pi^* = \frac{g}{q} > 1$
- Productivity disturbances have persistent effects on optimal inflation
Conclusions

- Aggregate in closed form a sticky price model with firm level productivity trends
- Trends capture: product substitution, product quality improvements, or firm turnover
- Firm level productivity trends key for optimal inflation rate in sticky price models
- Steady state inflation $\Pi^* = \frac{g}{q} > 1$
- Productivity disturbances have persistent effects on optimal inflation
- Optimal US inflation: dropped from approx 2% in 1986 to 1% in 2013