Optimal Domestic (and External) Sovereign Default

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¹The views expressed here do not necessarily reflect those of the FRB Philadelphia or The Federal Reserve System.
Reinhart & Rogoff’s Forgotten History of Domestic Debt

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5. Largely a “forgotten” story in macroeconomics literature
**Eurozone Debt Crisis as a Domestic Default**

- Most Eurozone public debt held within Europe
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- Caveat: Eurozone is not a fiscal union
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▶ Quantitative analysis via time series simulations (long-run, default events, business cycle correlations, sensitivity)

▶ Study model’s mechanism in RME functions and perform sensitivity analysis
Questions

▶ Can distributional incentives and social value of debt support equilibria with public debt?
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▶ If equilibria with debt exist, do they feature dynamics in which default risk and default events are observed?

▶ Can the model account for key facts of debt-crisis dynamics (debt ratios, rising spreads, low default prob., foreign v. domestic debt)?
Main Findings

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- Debt exhibits protracted fluctuations
Overview Model

- Introduce endogenous public debt and default in a model of heterogeneous agents, incomplete markets, and public debt with aggregate risk.

- Agents face idiosyncratic income shocks $y$, aggregate government expenditure shocks $g$, and save in non-contingent, pari-passu government bonds with a no-borrowing constraint.

- Utilitarian government pays for $g$, $B$ and lump sum transfers $\tau$ with income taxes $\tau^y$ and by issuing debt $B'$ at price $q$.

- Public debt sold to both foreign and domestic creditors.

- Study Recursive Markov Equilibrium without commitment.
Environment: Households

- Unit measure of households with preferences

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad u(c_t) = c_t^{1-\sigma} / (1 - \sigma) \]

where \( \beta \in (0, 1) \) and \( c_t \) is individual consumption.

- Agents receive income \( y_t \in \mathcal{Y} = \{ \underline{y}, \ldots, \bar{y} \} \). Income is iid across households, and persistent with transition \( \pi(y_{t+1}, y_t) \).

\[ \log(y_{t+1}) = (1 - \rho_y) \mu_y + \rho_y \log(y_t) + u_t, \quad |\rho_y| < 1, \quad u \sim N(0, \sigma_u) \]
Households (cont.)

- If the government does not default, the budget constraint is

\[ c_t + q_t b_{t+1} = y_t (1 - \tau^y) + b_t + \tau^{d=0}_t \]

- If the government defaults, the market for public debt closes and re-opens next period. The budget constraint is:

\[ c_t = y_t (1 - \tau^y) - \phi(g_t) + \tau^{d=1}_t \]
International Investors

- Pricing of gov. bonds is simplified by introducing risk-neutral competitive investors a’la Eaton-Gersovitz

- Expected profits:
  \[
  \Omega_t = -q_t \hat{B}_{t+1} + \frac{(1 - p_t)}{(1 + \bar{r})} \hat{B}_{t+1}
  \]

- FOC yields arbitrage of expected risky return and international risk free rate $\bar{r}$. 
Government

- Gov. expenditures follow exogenous Markov process
 \[ g_t \in \mathcal{G} \equiv \{g, \ldots, \bar{g}\} \]
  with transition prob. matrix \( F(g_{t+1}, g_t) \),
  independent of income shocks.

  \[
  \log(g_{t+1}) = (1 - \rho_g) \mu_g + \rho_g \log(g_t) + e_t, \quad |\rho_g| < 1, \ e \sim N(0, \sigma_e)
  \]

- If \( d_t = 0 \), the gov. budget constraint is:
  \[
  \tau_t^{d=0} = \tau^y Y - B_t - g_t + q_t B_{t+1}
  \]

- If \( d_t = 1 \), the gov. budget constraint is:
  \[
  \tau_t^{d=1} = \tau^y Y - g_t
  \]
Timing of Actions and Participation

1. Realizations of exogenous shocks $y$ and $g$ are observed.

2. Individual states $\{b, y\}$, wealth distribution $\Gamma_t(b, y)$ and aggregate states $\{B, g\}$ are known.

3. Income taxes are paid. Government chooses to default or not, $d_t \in \{0, 1\}$:
   - If $d_t = 0$, debt is repaid, new debt market opens, government sets supply of debt, lump-sum transfers satisfy GBC
     \[
     (\tau_t = \tau^yY - B_t - g_t + q_tB_{t+1}),
     \]
     agents and foreign investors choose bond holdings with price $q_t$.
   - If $d_t = 1$, debt is not paid to all creditors, output cost $\phi(g)$, debt market does not open, transfers satisfy GBC
     \[
     (\tau_t = \tau^yY - g_t).
     \]

4. Agents consume, period $t$ ends.
Recursive Markov Competitive Eq. (given gov. policies)

Given $\Gamma_0(b, y)$, $d(B, g)$, $B'(B, g)$, and $\tau^d(B', B, g)$, a Recursive Markov Equilibrium (RMCE) is a value function, households’ decision rules, bond price and transition function $H^d(\Gamma, B, g, g')$ such that:

1. Given prices and policies, the value function and saving decision rule solve the households’ problem

2. The foreign investor’s arbitrage condition holds

3. The distribution evolves according to $H^d\in\{0,1\}(\Gamma, B, g, g')$

4. The government budget constraint is satisfied period by period

5. The asset market clears: $\hat{B}' = B'^d - B' $

6. The aggregate resource constraint is satisfied
Government’s Default Decision

$$\max_{d \in \{0, 1\}} \left\{ W^{d=0}(B, g), W^{d=1}(g) \right\}$$
Government’s Default Decision

\[ \max_{d \in \{0,1\}} \left\{ W^{d=0}(B, g), W^{d=1}(g) \right\} \]

- Social Welfare Functions:

\[ W^{d=0}(B, g) = \int_{y \times B} V^{d=0}(b, y, B, g) \, d\omega(b, y), \]

\[ W^{d=1}(g) = \int_{y \times B} V^{d=1}(y, g) \, d\omega(b, y). \]

- Welfare weights are given by joint cdf.:

\[ \omega(b, y) = \sum_{y_i \leq y} \pi^*(y_i) \left( 1 - e^{-\frac{b}{\bar{\omega}}} \right) \]
**Government’s Debt Decision**

- The value for each household of an alternative debt level $\tilde{B}'$
  
  $$\tilde{V}(b, y, B, g, \tilde{B}') = \max_{\{c \geq 0, b' \geq 0\}} u(c) + \beta E_{(y', g')}|(y, g)[V(b', y', \tilde{B}', g')]$$

  s.t. $c + q(\tilde{B}', g)b' = b + y(1 - \tau y) + \tau(\tilde{B}', B, g)$

- The optimal government policy is the solution to:
  
  $$\max_{\tilde{B}'} \int_{Y \times B} \tilde{V}(b, y, B, g, \tilde{B}')d\omega(b, y).$$
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- A Recursive Markov Equilibrium with Endogenous Policies is an RMCE for which $B'(B, g)$ and $d(B, g)$ are the optimal debt and default decision rules.
Eq. Implications I: Demand for Bonds

- Assuming differentiability, FOC with respect to $b'$:

\[ u'(c) \leq \beta E(y', g')|(y, g) \left[ (1 - d(B', g')) \frac{u'(c')}{q(B', g)} \right] \]

with equality if $b' > 0$

- Larger default set reduces the expected marginal benefit of $b'$

- Higher default prob. lowers $b'$, except for high enough $(b, y)$, who demand more bonds at higher risk premia

- Even if $d' = 0$, marginal benefit affected by future default risk (reduces bond demand for most $(b, y)$)
Eq. Implications II: Public Debt for Liquidity

- Using $\tilde{b} = (b - B)$, agent’s and gov. budget constraint imply:

\[
c = y + \tilde{b} - q(B', g)\tilde{b}' - \tau^y(y - Y) - g \\
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- Debt redistributes resources
  - Repaying $B$ favors the wealthy (agents with $\tilde{b} > 0$)
  - Issuing $B'$ favors the poor (agents with $\tilde{b}' < 0$)
  - Default risk erodes the effect of $B'$: $q$ falls as $B'$ rises, which affects wealth distribution and default choice (feedback mechanism)
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- Income tax insures against idiosyncratic shocks
Eq. Implications III: Default Incentives

- Consumption differences in repayment v. default states:

\[ \Delta c \equiv c^{d=0} - c^{d=1} = \tilde{b} - q(B', g)\tilde{b}' + \phi(g) \]

- The two first terms in RHS reflect distributional effects of \( B \) and \( B' \)
**Eq. Implications III: Default Incentives**

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- The two first terms in RHS reflect distributional effects of $B$ and $B'$

- Larger mass with $\tilde{b} < 0$ and low $q$ (high default risk), imply more agents with $\Delta c < 0$ and higher default incentives

- Larger mass with $\tilde{b}' < 0$ reduces fraction of agents with $\Delta c < 0$: "static" default incentives decrease as fraction of future net borrowers increases
Two Simple Examples

1. **Distributional Incentives**
   - One period model with given fraction of rich and poor
   - Gov. always default as second best policy to attain efficient consumption dispersion unless rich weight more in the SWF than their actual share of wealth
   - Extended to two period model with uncertainty and optimal choice of debt/default (D’Erasmo and Mendoza (2015))

2. **Social Value of Debt**
   - What is the welfare cost of “surprise” default in an economy with full commitment
   - Welfare costs: 1.35% for $B/Y$ up to 5%
   - Social value of debt and agents in favor in repayment decrease monotonically with $B/Y$
Quantitative Analysis
Calibration - Spain

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-Free Rate (%)</td>
<td>$\bar{r}$</td>
<td>2.07</td>
</tr>
<tr>
<td>Risk Aversion</td>
<td>$\sigma$</td>
<td>1.00</td>
</tr>
<tr>
<td>Autocorrel. Income</td>
<td>$\rho_y$</td>
<td>0.85</td>
</tr>
<tr>
<td>Std Dev Error</td>
<td>$\sigma_u$</td>
<td>0.25</td>
</tr>
<tr>
<td>Avg. Income</td>
<td>$\mu_y$</td>
<td>0.75</td>
</tr>
<tr>
<td>Autocorrel. G</td>
<td>$\rho_g$</td>
<td>0.88</td>
</tr>
<tr>
<td>Std Dev Error</td>
<td>$\sigma_e$</td>
<td>0.02</td>
</tr>
<tr>
<td>Avg. Gov. Consumption</td>
<td>$\mu_g$</td>
<td>0.18</td>
</tr>
<tr>
<td>Proportional Income Tax</td>
<td>$\tau_y$</td>
<td>0.35</td>
</tr>
<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
<td>0.885</td>
</tr>
<tr>
<td>Welfare Weights</td>
<td>$\omega$</td>
<td>0.051</td>
</tr>
<tr>
<td>Default Cost</td>
<td>$\phi_1$</td>
<td>0.603</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Moments (%)</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Ratio Domestic Debt</td>
<td>74.31</td>
<td>74.43</td>
</tr>
<tr>
<td>Avg. Spread Spain</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>Avg. Debt to GDP Spain (maturity adjusted)</td>
<td>5.88</td>
<td>5.56</td>
</tr>
</tbody>
</table>
Time-Series Dynamics: Long Run and Pre-Crisis

Table: Long-run and Pre-Crisis Moments: Data v. Model

<table>
<thead>
<tr>
<th>Moment (%)</th>
<th>Data Avg.</th>
<th>Peak Crisis</th>
<th>Model Average</th>
<th>Prior Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gov. Debt $B$</td>
<td>5.43*</td>
<td>7.43</td>
<td>5.88</td>
<td>7.95</td>
</tr>
<tr>
<td>Domestic Debt $B^d$</td>
<td>4.04</td>
<td>4.85</td>
<td>4.29</td>
<td>4.84</td>
</tr>
<tr>
<td>Foreign Debt $\hat{B}$</td>
<td>1.39</td>
<td>2.58</td>
<td>1.59</td>
<td>3.11</td>
</tr>
<tr>
<td>Ratio $B^d/B$</td>
<td>74.34*</td>
<td>65.28</td>
<td>74.31</td>
<td>60.94</td>
</tr>
<tr>
<td>Tax Revenues $\tau^yY$</td>
<td>25.24</td>
<td>24.85</td>
<td>26.60</td>
<td>26.60</td>
</tr>
<tr>
<td>Gov. Expenditure $g$</td>
<td>18.12*</td>
<td>20.50</td>
<td>18.13</td>
<td>18.18</td>
</tr>
<tr>
<td>Transfers $\tau$</td>
<td>7.04</td>
<td>7.06</td>
<td>8.35</td>
<td>8.73</td>
</tr>
<tr>
<td>Spread</td>
<td>0.94*</td>
<td>4.35</td>
<td>0.94</td>
<td>7.22</td>
</tr>
</tbody>
</table>

Note: * identifies moments used as calibration targets.
Time-Series Dynamics: Cyclical Properties

**Table:** Cyclical Moments: Data v. Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standard Deviation</th>
<th>Correl($x, hhdi$)</th>
<th>Correl($x, g/GDP$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.85</td>
<td>0.84</td>
<td>0.43</td>
</tr>
<tr>
<td>Trade Balance/GDP</td>
<td>0.63</td>
<td>0.55</td>
<td>-0.31</td>
</tr>
<tr>
<td>Spreads</td>
<td>1.04</td>
<td>2.46</td>
<td>-0.44</td>
</tr>
<tr>
<td>Gov. Debt / GDP</td>
<td>1.58</td>
<td>1.23</td>
<td>-0.18</td>
</tr>
<tr>
<td>Dom. Debt / GDP</td>
<td>1.68</td>
<td>0.32</td>
<td>-0.32</td>
</tr>
</tbody>
</table>

Note: $hhdi$ denotes household disposable income. In the model, $hhdi = Y + \tau + \tau y Y$ and $TB = Y - C - g$. 
Time-Series Dynamics: Event Analysis

Panel (i): Debt and Default

Panel (ii): Gov. Exp. and Gov Transfers

Panel (iii): Spreads

Panel (iv): $\bar{\alpha}(B, g)(\%)$

Highlights

Optimal Domestic (and External) Sovereign Default

Pablo D’Erasmo and Enrique Mendoza
Individual Gains from Default as a Function of $B$

Panel (i): $\alpha(b = 0, y, B, g_L) \%$

Panel (ii): $\alpha(b = 0.2, y, B, g_L) \%$

Panel (iii): $\alpha(b = 0, y, B, g_H) \%$

Panel (iv): $\alpha(b = 0.2, y, B, g_H) \%$
Social Distribution of $\alpha$ (for different $B$ and $g$)

Panel (i): CDF of $\alpha$ at $g = g_L$ (cdf)

Panel (ii): CDF of $\alpha$ at $g = g_H$ (cdf)
Bond Prices & Debt Laffer Curves

Panel (i): Eq. Price Function $q(B', g)$

Panel (ii): Laffer Curve $q(B', g)B'$
Conclusions

- Tradeoff between distributional incentives to default and social value of debt for self-insurance, liquidity provision and risk-sharing supports RME with debt exposed to default risk.

- A rich feedback mechanism links debt issuance and default choices, government bond prices, the agent’s optimal plans and the dynamics of the distribution of bonds across agents.

- Results largely consistent with the data:
  - Rapidly rising spreads at high debt ratios in periods leading to a default (rising dist. incentives, falling social value).
  - Long-run and pre-default averages are consistent with data counterparts, at low default frequency and with spreads of up to 700 basis points.
  - Model also consistent with key cyclical moments observed in the data (e.g. correlation of $g/GDP$ and spreads).
The Forgotten History of Domestic Defaults

Figure 5. The Runup in Domestic and External Debt on the Eve of Default, Average Default Episode: 1800-2006

Sources: See Data Appendices I and II in Reinhart and Rogoff (2008).
## Euro Area Fiscal and Debt Situation 2011

<table>
<thead>
<tr>
<th></th>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>62.73</td>
<td>43.34</td>
<td>24.48</td>
<td>50.60</td>
<td>-2.51</td>
<td>0.71</td>
</tr>
<tr>
<td>Germany</td>
<td>51.49</td>
<td>45.04</td>
<td>19.27</td>
<td>44.50</td>
<td>1.69</td>
<td>0.00</td>
</tr>
<tr>
<td>Greece</td>
<td>133.10</td>
<td>26.73</td>
<td>17.38</td>
<td>42.40</td>
<td>-2.43</td>
<td>13.14</td>
</tr>
<tr>
<td>Ireland</td>
<td>64.97</td>
<td>14.43</td>
<td>18.38</td>
<td>34.90</td>
<td>-9.85</td>
<td>6.99</td>
</tr>
<tr>
<td>Italy</td>
<td>100.22</td>
<td>61.72</td>
<td>20.42</td>
<td>46.20</td>
<td>1.22</td>
<td>2.81</td>
</tr>
<tr>
<td>Portugal</td>
<td>75.84</td>
<td>33.64</td>
<td>20.05</td>
<td>45.00</td>
<td>-0.29</td>
<td>7.63</td>
</tr>
<tr>
<td>Spain</td>
<td>45.60</td>
<td>64.19</td>
<td>20.95</td>
<td>35.70</td>
<td>-7.04</td>
<td>2.83</td>
</tr>
<tr>
<td>Avg.</td>
<td>76.28</td>
<td>41.30</td>
<td>20.13</td>
<td>42.76</td>
<td>-2.74</td>
<td>4.87</td>
</tr>
<tr>
<td>Median</td>
<td>64.97</td>
<td>43.34</td>
<td>20.05</td>
<td>44.50</td>
<td>-2.43</td>
<td>2.83</td>
</tr>
<tr>
<td>GDP (w. avg)</td>
<td>66.49</td>
<td>49.18</td>
<td>21.02</td>
<td>44.99</td>
<td>-1.06</td>
<td>1.80</td>
</tr>
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</table>

Optimal Domestic (and External) Sovereign Default

Pablo D’Erasmo and Enrique Mendoza
Euro Area Evolution Debt and Spreads

Panel (i): Ireland

Panel (ii): Spain

Panel (iii): Greece

Panel (iv): Italy

Panel (v): France

Panel (vi): Portugal

Introduction

Environment

Markov Competitive Equil.

Examples

Results

Optimal Domestic (and External) Sovereign Default
Pablo D’Erasmo and Enrique Mendoza
Banks’ Exposure to Sov. Risk (2011.q2)
Banks’ Exposure to Agg. Credit Risk (2011.q2)
Definitions

- **Reinhart and Rogoff (2008):**
  - Domestic Public debt is issued under home legal jurisdiction.
  - In most countries, it has been denominated in local currency and held mainly by residents.

- **Kumhof and Tamer (2005):**
  - BIS aggregates comprehensive data on individual securities from market sources. The definition is very conservative.
  - Classifies as domestic security: issues by residents, target at resident investors in domestic currency.
Related Literature

1. Incomplete Markets - Role of Debt:
   - Het. Agents: Aiyagari & McGrattan (98); Azzimonti, de Francisco and Quadrini (14); Heathcote (05); Floden (01); Bhandari, Evans Golosov and Sargent (16);
   - Rep. Agent: Aiyagari et al. (02); Presno and Pouzo (14);

2. External Default: Arellano (08); Aguiar and Gopinath (06); Cuadra, Sanchez & Sapriza (08); Dias, Richmond & Wright (12); Sosa Padilla (14); Du and Schreger (16)

3. Interaction with Domestic Agentes: Guembel & Sussman (09); Broner, Martin & Ventura (10); Gennaioli, Martin & Rossi (14); Aguiar and Amador (14); Mengus (14)

4. Het. Agents - Default: Dovis, Golosov and Shourideh (16); Aguiar, Amador, Farhi and Gopinath (15)
Recursive Individual Agent’s Problem

▶ Beginning-of-period value, before $d$ is chosen:

$$V(b, y, B, g) = (1 - d(B, g))V^{d=0}(b, y, B, g) + d(B, g)V^{d=1}(y, g)$$
Recursive Individual Agent’s Problem

- Beginning-of-period value, before \( d \) is chosen:

\[
V(b, y, B, g) = (1 - d(B, g))V^{d=0}(b, y, B, g) + d(B, g)V^{d=1}(y, g)
\]

- If \( d = 0 \), the agent’s payoff is:

\[
V^{d=0}(b, y, B, g) = \max_{\{c \geq 0, b' \geq 0\}} u(c) + \beta E_{y', g'|y, g}[V(b', y', B', g')]
\]

s.t. \( c + q(B'(B, g), g)b' = b + y(1 - \tau^y) + \tau^{d=0}(B'(B, g), B, g) \)
Recursive Individual Agent’s Problem

- Beginning-of-period value, before $d$ is chosen:

$$V(b, y, B, g) = (1 - d(B, g))V^{d=0}(b, y, B, g) + d(B, g)V^{d=1}(y, g)$$

- If $d = 0$, the agent’s payoff is:

$$V^{d=0}(b, y, B, g) = \max_{\{c \geq 0, b' \geq 0\}} u(c) + \beta E_{y', g'|y, g}[V(b', y', B', g')]$$

subject to:

$$c + q(B'(B, g), g)b' = b + y(1 - \tau^y) + \tau^{d=0}(B'(B, g), B, g)$$

- If $d = 1$, the agents’s payoff is:

$$V^{d=1}(y, g) = u(y(1 - \tau^y) - g + \tau^y Y - \phi(g)) + \beta E_{y', g'|y, g}[V^{d=0}(0, y', 0, g')]$$
Recursive Problem of International Investors

- Arbitrage condition for bond prices:

\[ q(B', g) = \frac{1 - p(B', g))}{(1 + \bar{r})}, \]

where \( p(B', g) \) is the default probability given by

\[ p(B', g) = \sum_{g'} d(B', g') F(g', g). \]

- If supply of debt is short of domestic demand, agents buy bonds abroad at risk-free price
Definition (RME): Aggregates

- Aggregate Consumption is
  \[ C = \int_{Y \times B} c \ d\Gamma(b, y), \]

- Aggregate income is
  \[ Y = \int_{Y \times B} y \ d\Gamma(b, y), \]

- The domestic asset demand is
  \[ B^{d'} = \int_{Y \times B} b' \ d\Gamma(b, y). \]

- The aggregate resource constraint in the no default periods is
  \[ C + g = Y + \hat{B} - q(B', g)\hat{B}', \]

  and in the default period is
  \[ C + g = Y - \phi(g). \]
Simple Example I: Distributional Incentives

- One-period economy where gov. has issued $B$.

- Same $y$ for all agents, default can cost a fraction $\phi$ of $y$

- Exogenous wealth distribution:
  - Fraction $\gamma$ holds $b^L = B - \epsilon$
  - Fraction $(1 - \gamma)$ holds $b^H = \frac{B - \gamma b^L}{1 - \gamma} = B + \frac{\gamma}{1 - \gamma} \epsilon$
  - $\epsilon \in [0, B]$ is exogenous demand for gov. bonds

- Government solves: $\max_{d \in \{0, 1\}} \left\{ W^{d=0}(B, g), W^{d=1}(g) \right\}$,

\[
W^{d=0}(B, g) = \omega u(y - g + b^L - B) + (1 - \omega) u(y - g + b^H - B)
\]

\[
W^{d=1}(g) = u(y(1 - \phi) - g)
\]
Distributional Incentives to Default

Efficient consumption dispersion chosen by planner satisfies:

\[
\frac{u'(y - g + \frac{\gamma}{1-\gamma} \epsilon^{SP})}{u'(y - g - \epsilon^{SP})} = \left(\frac{\omega}{\gamma}\right) \left(\frac{1 - \gamma}{1 - \omega}\right).
\]

If \( \phi = 0 \):

- \( \omega \geq \gamma \Rightarrow \text{default is always optimal for any } \epsilon > 0 \)
- \( \omega < \gamma \Rightarrow \exists \hat{\epsilon} > 0 : \text{if } \epsilon < \hat{\epsilon} \text{ repayment is optimal} \)

If \( \phi > 0 \):

- For any \( \{\omega, \gamma\} \Rightarrow \exists \hat{\epsilon} > 0 : \text{if } \epsilon < \hat{\epsilon} \text{ repayment is optimal} \)
- Repayment range widens as \( \gamma - \omega \) or \( \phi \) increase (i.e. tolerance for dispersion is akin to default costs)
**Distributional Mechanism** *(given B)*

\[ \epsilon^{SP} \text{ and default decision } (\phi = 0) \]
Distributional Mechanism (given $B$)
Simple Ex. II: Social Value of Debt

- Compare an economy with government committed to repay with one experiencing a once-and-for-all unanticipated default.

- In both cases $\bar{q} = 1/(1 + \bar{r})$ (gov. committed/default is a surprise).

- Compensating variation in consumption for each agent:

$$\alpha(b, y, B, g) = \left[ \frac{V_{d=1}(y, g)}{V_c(b, y, B, g)} \right]^{\frac{1}{1-\sigma}} - 1$$

- Social value of public debt:

$$\bar{\alpha}(B, g) = \int \alpha(b, y, B, g)d\omega(b, y)$$
Social Value of Debt (cont.)

<table>
<thead>
<tr>
<th>$B/GDP$</th>
<th>$B^d/GDP$</th>
<th>$\tau/GDP$</th>
<th>$\bar{\alpha}(B, \mu_g)$</th>
<th>$\bar{\alpha}(B, g)$</th>
<th>$\bar{\alpha}(B, \bar{g})$</th>
<th>hh’s $\alpha &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>4.5</td>
<td>32.4</td>
<td>-1.35</td>
<td>-2.49</td>
<td>-0.94</td>
<td>12.4</td>
</tr>
<tr>
<td>10.0</td>
<td>4.5</td>
<td>30.8</td>
<td>-0.66</td>
<td>-1.82</td>
<td>-0.23</td>
<td>49.3</td>
</tr>
<tr>
<td>15.0</td>
<td>4.5</td>
<td>29.0</td>
<td>0.05</td>
<td>-1.14</td>
<td>0.51</td>
<td>79.5</td>
</tr>
<tr>
<td>20.0</td>
<td>4.5</td>
<td>26.6</td>
<td>0.77</td>
<td>-0.44</td>
<td>1.26</td>
<td>94.2</td>
</tr>
</tbody>
</table>

Note: All moments are in percentage.

- Social value of debt (i.e. cost of a surprise default) is large and monotonically decreasing in $B/GDP$
Social Value of Debt (cont.)

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<tr>
<th>$B/GDP$</th>
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- Social value of debt (i.e. cost of a surprise default) is large and monotonically decreasing in $B/GDP$

- Estimates are significantly larger than those in Aiyagari & McGrattan (98) (which find a max. value of 0.1 percent)
**Social Value of Debt (cont.)**

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<thead>
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<th>$B/GDP$</th>
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<th>$\tau/GDP$</th>
<th>$\bar{\alpha}(B, \mu_g)$</th>
<th>$\bar{\alpha}(B, g)$</th>
<th>$\bar{\alpha}(B, \bar{g})$</th>
<th>hh’s $\alpha &gt; 0$</th>
</tr>
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<tbody>
<tr>
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<td>4.5</td>
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<td>-0.66</td>
<td>-1.82</td>
<td>-0.23</td>
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</tr>
<tr>
<td>15.0</td>
<td>4.5</td>
<td>29.0</td>
<td>0.05</td>
<td>-1.14</td>
<td>0.51</td>
<td>79.5</td>
</tr>
<tr>
<td>20.0</td>
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<td>0.77</td>
<td>-0.44</td>
<td>1.26</td>
<td>94.2</td>
</tr>
</tbody>
</table>

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- Social value of debt (i.e. cost of a surprise default) is large and monotonically decreasing in $B/GDP$

- Estimates are significantly larger than those in Aiyagari & McGrattan (98) (which find a max. value of 0.1 percent)

- Higher debt ratios reduce transfers and the extent to which the government can redistribute
Default Cost and Maturity Adjustment

▶ Default Cost Function:

\[ \phi(g) = \phi_1 \max\{0, (\mu_g - g)^{1/2}\} \]

▶ Maturity Adjustment:

▶ Bonds issued in year \( t \) promise to pay one unit in year \( t + 1 \) and \((1 - \delta)^{s-1}\) units in year \( t + s \) for \( s > 1 \)

▶ Duration can be written as: \( D = \frac{1+r^*}{r^*+\delta} \)

▶ If we let \( \overline{B} \) denote the value of total outstanding debt and \( B \) represents the maturity adjusted (one period) stock of debt, \( B \) can be written as

\[ B = \frac{\overline{B}}{D} \]
Time Series Dynamics: Event Analysis

- Debt accelerates just before default with foreign and domestic holdings rising but the former rising faster.

- A lower value of $g$ weakens the incentives to default and allows the government to increase $B$ and $\tau$ (resulting in a reduction in $\bar{\alpha}$).

- Higher debt results in higher spreads that spike when $g$ rises.

- The increase in $g$ strengthens default incentives resulting in a sharp increase in $\bar{\alpha}$ causing a “sudden” default.

- The sudden default and the surge in spreads (both occurring with unchanged debt) may look as if equilibrium multiplicity is the culprit but this is not the case.
Gains of default across \((B, y)\)

- Gains from default differ sharply for the non-debt holders (low \(b\)) and debt holders (high \(b\))
  - Non-debt holders receive the same lump-sum transfers and pay same taxes that debt-holders but do not suffer wealth losses from a default
  - Gains are non-monotonic in income
    - Low wealth high income agents value repayment because they would like to start building a buffer
    - High wealth, low income agents value repayment more because they would like to use their buffer stock
  - Default gains are convex in government debt: non-debt holders value increasingly more redistribution of resources in their favor when a larger \(B\) is defaulted on
Individual Gains from Default as a Function of $g$

Panel (i): $\alpha(b = 0, y, B_L, g)\%$

Panel (ii): $\alpha(b = 0.2, y, B_L, g)\%$

Panel (iii): $\alpha(b = 0, y, B_H, g)\%$

Panel (iv): $\alpha(b = 0.2, y, B_H, g)\%$
Gains of default across $(g, y)$

- Individual default gains are increasing and convex in $g$ for $g < \mu_g$
  - Default risk increases with $g$
  - Exogenous default cost falls as $g$ rises
- Response of default gains to increases in $g$ is weaker for high-income agents
Welfare Gain of Default and Tax Differential

- The social value of default rises with $B$ with the same convex pattern identified in the individual gains of default.

- Social gains from default rise much faster at at $g \leq \mu_g$

- Social gains yield smaller numbers than individual gains because they reflect government’s aggregation.
Social Gain of Default

Panel (i): Social Value of Default $\bar{\alpha}$

$$\bar{\alpha}(B, g) = \int_{B \times Y} \alpha(b, y, B, g) d\omega(b, y)$$
Social Distribution of $\alpha$ (for different $B$ and $g$)

- Welfare weights $\omega(b, y)$ are exogenous but the social distribution of gains from default across agents varies endogenously with the aggregate states $(B, g)$.

- The non-linear, non-monotonic responses of the individual $\alpha$'s to changes in $B$ and $g$ imply that the $\alpha$'s move in different directions across $(b, y)$ pairs when $(B, g)$ changes.

- The social distribution of default gains shifts to the right as $B$ rises, and a larger fraction of agents are assessed as benefiting from a default.
Bond Prices & Debt Laffer Curves

- Price function has similar shape that those observed in EG models

- For debt that carries default risk, prices are lower at higher $g$ because the probability of default is increasing in $g$

- For low $g$ (and long-run $B$), debt is sold at the risk free price and below the maximum of the Laffer curve.

- For average or high $g$ the government chooses $B'$ to maximize resources.

- On the equilibrium path, we also observe $B'$ choices that are interior and carry default risk ($g = g_9$)
Sensitivity I: Government Welfare Weights

<table>
<thead>
<tr>
<th>Moment (%)</th>
<th>benchmark</th>
<th>$\tilde{\omega} = 0.051$</th>
<th>$\tilde{\omega} = 0.0435$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$z = 0.025$</td>
<td>$z = 0$</td>
<td></td>
</tr>
<tr>
<td><strong>Long Run Averages</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gov. Debt $B$</td>
<td>5.88</td>
<td>4.22</td>
<td>4.56</td>
</tr>
<tr>
<td>Dom. Debt $B^d$</td>
<td>4.29</td>
<td>3.84</td>
<td>4.16</td>
</tr>
<tr>
<td>Default Frequency</td>
<td>0.93</td>
<td>1.00</td>
<td>0.53</td>
</tr>
<tr>
<td>Spreads</td>
<td>0.94</td>
<td>1.01</td>
<td>0.54</td>
</tr>
<tr>
<td>Transf $\tau$</td>
<td>8.35</td>
<td>8.39</td>
<td>8.38</td>
</tr>
<tr>
<td>Frac. Hh’s $b = 0$</td>
<td>68.74</td>
<td>69.15</td>
<td>67.41</td>
</tr>
<tr>
<td>$\bar{\alpha}(B, g)$</td>
<td>-0.341</td>
<td>-0.306</td>
<td>-0.483</td>
</tr>
<tr>
<td><strong>Averages Prior Default</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gov. Debt $B$</td>
<td>7.95</td>
<td>6.00</td>
<td>6.12</td>
</tr>
<tr>
<td>Dom. Debt $B^d$</td>
<td>4.84</td>
<td>4.76</td>
<td>4.66</td>
</tr>
<tr>
<td>Spreads</td>
<td>7.22</td>
<td>6.84</td>
<td>4.56</td>
</tr>
<tr>
<td>Def. Th. $\tilde{b}(\mu_y)$</td>
<td>0.073</td>
<td>0.051</td>
<td>0.051</td>
</tr>
<tr>
<td>%. Favor Repay $(1-\omega(\tilde{b}(\mu_y), \mu_y))$</td>
<td>23.45</td>
<td>21.99</td>
<td>29.98</td>
</tr>
<tr>
<td>%. Favor Repay $(1-\gamma(\tilde{b}(\mu_y), \mu_y))$</td>
<td>3.68</td>
<td>4.16</td>
<td>4.07</td>
</tr>
</tbody>
</table>

Note: Benchmark model parameters are $\tilde{\omega} = 0.051$, $z = 0$.

$$\omega(b, y) = \sum_{y_i \leq y} \pi^*(y_i) \left(1 - e^{-\frac{(b+z)}{\tilde{\omega}}}\right)$$
# Sensitivity II: Preferences and Income Process

<table>
<thead>
<tr>
<th>Moment (%)</th>
<th>bench.</th>
<th>0.85</th>
<th>0.90</th>
<th>0.5</th>
<th>2</th>
<th>0.200</th>
<th>0.300</th>
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<tbody>
<tr>
<td><strong>Long Run Averages</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gov. Debt $B$</td>
<td>5.88</td>
<td>5.96</td>
<td>6.32</td>
<td>5.06</td>
<td>6.80</td>
<td>6.28</td>
<td>6.40</td>
</tr>
<tr>
<td>Dom. Debt $B^d$</td>
<td>4.29</td>
<td>1.16</td>
<td>6.24</td>
<td>0.02</td>
<td>6.82</td>
<td>1.22</td>
<td>6.39</td>
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<tr>
<td>Foreign Debt $\hat{B}$</td>
<td>1.59</td>
<td>4.80</td>
<td>0.08</td>
<td>5.04</td>
<td>-0.02</td>
<td>5.06</td>
<td>0.01</td>
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<tr>
<td>Def. Freq.</td>
<td>0.93</td>
<td>1.02</td>
<td>0.27</td>
<td>19.58</td>
<td>0.25</td>
<td>0.29</td>
<td>0.49</td>
</tr>
<tr>
<td>Spreads</td>
<td>0.94</td>
<td>1.027</td>
<td>0.266</td>
<td>24.340</td>
<td>0.249</td>
<td>0.296</td>
<td>0.490</td>
</tr>
<tr>
<td>Transf $\tau$</td>
<td>8.35</td>
<td>8.35</td>
<td>8.35</td>
<td>9.20</td>
<td>8.34</td>
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<tr>
<td>Frac. Hh’s $b = 0$</td>
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<td>91.66</td>
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<td>98.96</td>
<td>22.25</td>
<td>93.27</td>
<td>61.19</td>
</tr>
<tr>
<td>$\bar{\alpha}(B, g)$</td>
<td>-0.341</td>
<td>-0.506</td>
<td>-0.305</td>
<td>-0.646</td>
<td>-0.448</td>
<td>-0.320</td>
<td>-0.323</td>
</tr>
<tr>
<td><strong>Averages Prior Default</strong></td>
<td></td>
<td></td>
<td></td>
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<td>6.31</td>
<td>8.72</td>
<td>8.17</td>
<td>8.46</td>
</tr>
<tr>
<td>Dom. Debt $B^d$</td>
<td>4.84</td>
<td>1.27</td>
<td>8.34</td>
<td>0.03</td>
<td>8.72</td>
<td>1.32</td>
<td>8.42</td>
</tr>
<tr>
<td>Foreign Debt $\hat{B}$</td>
<td>3.11</td>
<td>6.72</td>
<td>0.13</td>
<td>6.28</td>
<td>0.00</td>
<td>6.85</td>
<td>0.04</td>
</tr>
<tr>
<td>Spreads</td>
<td>7.22</td>
<td>7.03</td>
<td>3.76</td>
<td>43.49</td>
<td>3.72</td>
<td>3.59</td>
<td>4.69</td>
</tr>
</tbody>
</table>

Note: Benchmark model parameters are $\beta = 0.885$, $\sigma = 1$ and $\sigma_u = 0.25$.  

Optimal Domestic (and External) Sovereign Default  
Pablo D’Erasmo and Enrique Mendoza
**Sensitivity III: Default Cost**

<table>
<thead>
<tr>
<th>Moment (%)</th>
<th>( \phi_1 )</th>
<th>( \psi )</th>
<th>( \hat{\psi} )</th>
<th>( \hat{g} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>bench.</td>
<td>0.35</td>
<td>0.75</td>
<td>0.35</td>
<td>0.75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Long Run Avg</th>
<th>( \hat{\phi} )</th>
<th>( \hat{\psi} )</th>
<th>( \hat{\beta} )</th>
<th>( \hat{\varphi} )</th>
<th>( \hat{\gamma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gov. Debt ( B )</td>
<td>5.88</td>
<td>5.59</td>
<td>6.04</td>
<td>7.23</td>
<td>5.37</td>
</tr>
<tr>
<td>Dom. Debt ( B^d )</td>
<td>4.29</td>
<td>4.30</td>
<td>4.31</td>
<td>4.35</td>
<td>4.29</td>
</tr>
<tr>
<td>Foreign Debt ( \hat{B} )</td>
<td>1.59</td>
<td>1.29</td>
<td>1.73</td>
<td>2.88</td>
<td>1.08</td>
</tr>
<tr>
<td>Def. Freq.</td>
<td>0.93</td>
<td>0.49</td>
<td>0.95</td>
<td>2.89</td>
<td>0.13</td>
</tr>
<tr>
<td>Spreads</td>
<td>0.94</td>
<td>0.494</td>
<td>0.955</td>
<td>2.976</td>
<td>0.135</td>
</tr>
<tr>
<td>Transf ( \tau )</td>
<td>8.34</td>
<td>8.36</td>
<td>8.35</td>
<td>8.33</td>
<td>8.36</td>
</tr>
<tr>
<td>Frac. Hh’s ( b = 0 )</td>
<td>68.74</td>
<td>68.78</td>
<td>68.71</td>
<td>65.51</td>
<td>68.87</td>
</tr>
<tr>
<td>( \tilde{\alpha}(B, g) )</td>
<td>-0.341</td>
<td>-0.230</td>
<td>-0.449</td>
<td>-0.668</td>
<td>-0.136</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Avg Prior Default</th>
<th>( \hat{\phi} )</th>
<th>( \hat{\psi} )</th>
<th>( \hat{\beta} )</th>
<th>( \hat{\varphi} )</th>
<th>( \hat{\gamma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gov. Debt ( B )</td>
<td>7.95</td>
<td>6.92</td>
<td>8.48</td>
<td>11.76</td>
<td>5.96</td>
</tr>
<tr>
<td>Dom. Debt ( B^d )</td>
<td>4.84</td>
<td>4.66</td>
<td>4.90</td>
<td>5.48</td>
<td>4.42</td>
</tr>
<tr>
<td>Foreign Debt ( \hat{B} )</td>
<td>3.11</td>
<td>2.26</td>
<td>3.57</td>
<td>6.28</td>
<td>1.54</td>
</tr>
<tr>
<td>Spreads</td>
<td>7.22</td>
<td>4.64</td>
<td>7.19</td>
<td>15.42</td>
<td>1.59</td>
</tr>
</tbody>
</table>

Note: Benchmark model parameters are \( \phi_1 = 0.572 \), \( \hat{g} = u_g = 0.182 \) and \( \psi = 1/2 \).

\[
\phi(g) = \phi_1 \max\{0, (\hat{g} - g)^\psi\}
\]
### Sensitivity IV: Proportional Income Taxes

<table>
<thead>
<tr>
<th>Moment (%)</th>
<th>benchmark</th>
<th>$\tau^y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.20</td>
<td>0.45</td>
</tr>
</tbody>
</table>

**Long Run Averages**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Gov. Debt $B$</td>
<td>5.88</td>
<td>6.40</td>
<td>6.34</td>
</tr>
<tr>
<td>Dom. Debt $B^d_d$</td>
<td>4.29</td>
<td>6.42</td>
<td>2.36</td>
</tr>
<tr>
<td>Foreign Debt $\hat{B}$</td>
<td>1.59</td>
<td>-0.02</td>
<td>3.98</td>
</tr>
<tr>
<td>Def. Freq.</td>
<td>0.93</td>
<td>0.49</td>
<td>0.52</td>
</tr>
<tr>
<td>Spreads</td>
<td>0.94</td>
<td>0.49</td>
<td>0.52</td>
</tr>
<tr>
<td>Transf $\tau$</td>
<td>8.35</td>
<td>8.34</td>
<td>8.34</td>
</tr>
<tr>
<td>Frac. Hh’s $b = 0$</td>
<td>68.74</td>
<td>59.81</td>
<td>85.87</td>
</tr>
</tbody>
</table>

### Averages Prior Default

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Gov. Debt $B$</td>
<td>7.95</td>
<td>8.45</td>
<td>8.06</td>
</tr>
<tr>
<td>Dom. Debt $B^d_d$</td>
<td>4.84</td>
<td>8.43</td>
<td>2.60</td>
</tr>
<tr>
<td>Foreign Debt $\hat{B}$</td>
<td>3.11</td>
<td>0.01</td>
<td>5.47</td>
</tr>
<tr>
<td>Spreads</td>
<td>7.22</td>
<td>4.71</td>
<td>4.56</td>
</tr>
</tbody>
</table>

Note: Benchmark model parameters are $\tau^y = 0.35$. 

Optimal Domestic (and External) Sovereign Default

Pablo D'Erasmo and Enrique Mendoza
Introduction

Environment

Markov Competitive Equil.

Examples

Results

Sensitivity I: Government Welfare Weights

► Increasing $z$ for given $\bar{\omega}$:
  ▶ Weights of agents at $b = 0$ increases considerably (0 vs 38.62 percent)
  ▶ The default threshold and the fraction that benefit from repayment drop
  ▶ These changes reflect stronger incentives to default and less desire to issue debt

► Decreasing $\bar{\omega}$ for given $z$:
  ▶ Stronger incentives to default put an additional constraint on government borrowing
  ▶ Incentives to use debt for redistribution decrease: lower average debt and spreads
Sensitivity II: Preferences and Income Process

- Observed changes in $B^d$ are standard: increasing incentives for self-insurance by rising $\beta$, $\sigma$ or $\sigma_u$ increases domestic holdings.
- Higher $\beta$, $\sigma$ or $\sigma_u$ also allows the government to issue higher levels of debt: default incentives decrease (lower spreads).
- The benefit of defaulting as a mechanism for redistribution that cannot happen via self-insurance decreases.
- The scenario with lower $\beta$ results in higher debt levels and spreads: similar mechanism to external debt literature.
Sensitivity III: Income Taxes and Default Cost

- As the cost of default increases (higher $\phi_1$, lower $\psi$ or higher $\hat{g}$) the government is able to borrow more.
- Everything else equal the default probability decreases; however, the higher level of debt results in higher spreads.
- Higher spreads induce a higher domestic demand for government bonds.
- The average welfare cost of default increases.
Default Decision

\[ d(B, g) = 1 \]

\[ d(B, g) = 0 \]
**Default Event Analysis: Additional Cases**

Panel (i): Government Debt ($B'$)

Panel (ii): Government Expenditures (g)

Panel (iii): Government Transfers ($\tau$)

Panel (iv): Spreads (%)
Preferences over Repayment

Panel (i): Fraction Favor Repayment (all)

Panel (ii): Fraction Favor Repayment (cond. on y)
**Time-Series Dynamics between Default Events**

Panel (i): Debt and Default

Panel (ii): Gov. Exp. and Transfers

Panel (iii): Spreads

Panel (iv): $\bar{\alpha}(B, g)$

Optimal Domestic (and External) Sovereign Default

Pablo D’Erasmo and Enrique Mendoza
\[ \alpha(b, y, B, g) \text{ (for different } B \text{ at } g = \mu_g) \]

Panel (i): \( \alpha(b, y, B, \mu_g) \) at \( B_L \)

Panel (ii): \( \alpha(b, y, B, \mu_g) \) at \( B_H \)
**Optimal Debt** $B'(B, g)$

**Optimal Government Debt** $B'(B, g)$

The graph illustrates the optimal government debt $B'(B, g)$ as a function of government debt $B$. The lines $g_L$, $g_M$, $g_H$, and the 45° line represent different scenarios or constraints on the debt. The graph shows how the optimal debt adjusts based on the level of government debt.

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Optimal Domestic (and External) Sovereign Default

Pablo D'Erasmo and Enrique Mendoza
“Average” Wealth Distribution $\bar{\Gamma}(b, y)$ and Welfare Weights $\omega(b, y)$

Panel (i): Cond. Wealth and Welfare Weights Dist. ($y = y_L$)

Panel (ii): Cond. Wealth and Welfare Weights Dist. ($y = y_M$)

Panel (iii): Cond. Wealth and Welfare Weights Dist. ($y = y_H$)