Challenges for Central Banks’ Macro Models*

Jesper Lindé  
Sveriges Riksbank, Stockholm School of Economics and CEPR  
Frank Smets  
ECB, KU Leuven and CEPR  
Rafael Wouters  
National Bank of Belgium and CEPR  
July 2016

Abstract

In this paper we discuss a number of challenges for structural macroeconomic models in the light of the Great Recession and its aftermath. It shows that a benchmark DSGE model that shares many features with models currently used by central banks and large international institutions has difficulty explaining both the depth and the slow recovery of the Great Recession. In order to better account for these observations, the paper analyses three extensions of the benchmark model. First, we estimate the model allowing explicitly for the zero lower bound constraint on nominal interest rates. Second, we introduce time-variation in the volatility of the exogenous disturbances to account for the non-Gaussian nature of some of the shocks. Third and finally, we extend the model with a financial accelerator and allow for time-variation in the endogenous propagation of financial shocks. All three extensions require that we go beyond the linear Gaussian assumptions that are standard in most policy models. We conclude that these extensions go some way in accounting for features of the Great Recession and its aftermath, but they do not suffice to address some of the major policy challenges associated with the use of non-standard monetary policy and macroprudential policies.

JEL Classification: E52, E58  
Keywords: Monetary policy, DSGE, and VAR models, Regime-Switching, Zero Lower Bound, Financial Frictions, Great Recession, Macroprudential policy, Open economy.

*We thank the Editors John B. Taylor and Harald Uhlig, as well as our discussant Jonas Fisher, for very helpful suggestions that significantly improved the chapter. Comments by participants in the Handbook conference at the Becker-Friedman institute in Chicago, the 18th Central Bank Macro Modeling Workshop (hosted by the Reserve Bank of New Zealand in Wellington), and seminars at the ECB and Sveriges Riksbank were also useful. We are heavily indebted to Junior Maih for developing code in RISE which allowed us to estimate models subject to the zero lower bound. A special thank you is also due to Mattias Villani, who allowed us to use his BVAR-code in which we could set steady state priors. Leonard Voltaire and Jessica Radeschnig provided excellent research assistance, but all remaining errors are ours. Frank Smets and Raf Wouters acknowledge financial support from the EU FP7-Project MACFINROBODS, grant 612796. The views, analysis, and conclusions in this paper are solely the responsibility of the authors and do not necessarily agree with ECB, National Bank of Belgium or the Sveriges Riksbank, or those of any other person associated with these institutions.
# Contents

1 Introduction ................................................. 1
2 Common features of central bank models .......................... 4
3 A benchmark model ............................................. 6
   3.1 Firms and Price Setting .................................. 6
   3.2 Households and Wage Setting .............................. 8
   3.3 Market Clearing Conditions and Monetary Policy ........... 11
   3.4 Estimation on Pre-Crisis Data ............................. 12
      3.4.1 Solving the model .................................... 12
      3.4.2 Data .................................................. 12
      3.4.3 Estimation Methodology ............................... 15
      3.4.4 Posterior distributions of the estimated parameters .. 16
4 Empirical performance of benchmark models during the Great Recession .... 17
   4.1 Forecasting performance of benchmark models during the recession ................ 18
   4.2 Economic interpretation of the recession ................... 22
   4.3 Statistical properties of the innovations and their relation to financial indicators 28
5 Augmenting the benchmark model ................................ 32
   5.1 Assessing the impact of the zero lower bound ............... 32
   5.2 Allowing for Time-Varying Volatility ...................... 44
   5.3 Augmenting the Model with Financial Frictions and a Cost-Channel ............ 47
      5.3.1 A regime switching model with occasionally more severe financial frictions 52
      5.3.2 The cost channel of financial spreads and inflation dynamics .......... 56
6 State of Macroeconomic Modeling: Critical Assessment and Outlook ............ 58
Appendix ......................................................... 64
   A Linearized model representation ........................... 64
   B The ZLB algorithm and the likelihood function .............. 66
      B.1 The ZLB algorithm ..................................... 66
      B.2 Computation of the Likelihood Function ................. 71
   C Data ......................................................... 71
      C.1 Benchmark Model ....................................... 71
      C.2 Model with Financial Frictions .......................... 72
1. Introduction

In this paper, we discuss new challenges for structural macroeconomic models used at central banks in light of the Great Recession in United States and other advanced economies. This recession has had widespread implications for economic policy and economic performance, with historically low nominal interest rates and elevated unemployment levels in its aftermath. The fact that the intensification of the crisis in the fall of 2008 was largely unexpected and much deeper than central banks predicted and that the subsequent recovery was much slower, has raised many questions about the design of macroeconomic models at use in these institutions. Specifically, the models have been criticized for omitting key financial mechanisms and shocks stemming from the financial sector.

We start by analyzing the performance of a benchmark macroeconomic model during the Great Recession. The model we use – the well-known Smets and Wouters \cite{126} model – shares many features with the models currently used by central banks. When we analyze this model estimated over the pre-crisis period we find, confirming previous results in Del Negro and Schorfheide \cite{55}, that actual GDP growth was outside the predictive density of the model during the most acute phase of the recession. To account for the depth of the recession, the model needs a cocktail of extremely unlikely shocks that mainly affect the intertemporal decision of households and firms to consume or invest such as risk-premium and investment-specific technology shocks. We then proceed to document that these shocks are non-Gaussian, and strongly related to observable financial variables such as the Baa-Aaa and term spread, suggesting the importance of including financial shocks and frictions to account for large recessions. Moreover, in order to account for the slow recovery, restrictive monetary policy shocks reflecting a binding lower bound on the nominal interest rate, negative investment shocks, and positive price mark-up shocks are needed. This configuration of shocks explains the slow recovery and the missing disinflation following the great recession.

To try to better account for these observations, we proceed to amend the benchmark model along three dimensions. First, we take the zero lower bound (ZLB henceforth) explicitly into account when estimating the model over the full sample. We do this using two alternative approaches. First, we implement the ZLB as a binding constraint on the policy rule with an expected duration that is determined endogenously by the model in each period. Second, we impose the expected duration of the ZLB spells during the recession to be consistent with external information derived from overnight index swap rates. Importantly, we find that the variants of the model estimated
subject to the ZLB constraint typically feature a substantially higher degree of nominal stickiness in both prices and wages which helps to understand the inflation dynamics during the recession period and the subsequent slow recovery. In addition, an important characteristic of these variants of the model is a substantially higher response coefficient on the output gap in the policy rule. Incorporating the ZLB in the estimation and simulation of the model does not materially affect the median forecast of output and inflation in 2008Q3 as the probability of hitting the lower bound is estimated to be low before the crisis. It does however tilt the balance of risks towards the downside in the subsequent periods as the likelihood of monetary policy being constrained increases.

Second, in order to account for the non-Gaussian nature of the shocks driving most recessions, we allow for time-varying volatility in some of the shocks. In line with the previous literature, we find that the empirical performance of the model improves a lot when two regime change processes are allowed in the variance of the shocks. One of those regime switches captures the great moderation period from the mid-1980s to the mid-2000s, when overall macroeconomic volatility was much lower than both before and after this period. The other regime switching process captures the higher volatility of the risk-premium, the monetary policy and the investment-specific technology shocks in recession periods. This regime switching process can account for the non-Gaussian nature of those shocks and also helps widening the predictive density of output growth at the end of 2008 as the probability of a financial recession increases.

Finally, we proceed to examine how the performance and properties of the basic model can be improved by introducing a financial accelerator mechanism and explicit shocks stemming from the financial sector. This exercise is initiated by embedding a variant of the Bernanke, Gertler, and Gilchrist [20] financial accelerator into the workhorse model and estimating it under the standard assumption that the financial sector excerpts a time-invariant influence on business cycles: that is, we follow e.g. Christiano, Motto, and Rostagno [41], and De Graeve [51] and Virginia Queljo von Heideken [116], and assume that the parameters characterizing the financial frictions are constant and that shocks stemming from the financial bloc are Gaussian. In this specification, we do not find that the financial accelerator adds much propagation of other macroeconomic shocks, and that movements in the Baa-Aaa spread we add as observable is mostly explained by the exogenous shock stemming from the financial sector. Driven by this result, and because of the non-Gaussian features of the smoothed shocks in the benchmark model, we examine if the performance of this augmented model can be improved by allowing for regime-switching in the sensitivity of the external finance premium to the leverage ratio, which one may think of as risk-on/risk-off behavior in the financial
sector. We find that allowing for regime-switching in the sensitivity of external finance premium to the leverage ratio introduces a high degree of skewness in the predictive density of the spread and makes the model put non-zero probability in the predictive density on the observed 2008Q4 output growth outcome. Moreover, when we follow Del Negro and Schorfheide [55] and condition on the actual spread outcome during the fourth quarter of 2008 – which is reasonable since the spread reached its quarterly mean in the beginning of October – the model’s ability to account for the severe growth outcome further improves. This result indicates that if we appropriately could integrate the non-linear accelerator dynamics from financial frictions in our models, we may obtain a more realistic predictive density in line with reduced form time-varying volatility models.

The three extensions discussed in this paper go some way to address some of the challenges faced by the benchmark DSGE model in accounting for the Great Recession and its aftermath. They all involve going beyond the linear Gaussian modeling framework. However, they do not suffice to fully address some of the major empirical policy challenges. These new challenges stem from the fact that, following the crisis and hitting the zero lower bound, central banks have implemented a panoply of non-standard monetary policy measures such a Large-Scale-Asset-Purchases and other credit easing policies. Basic extensions of the benchmark model with financial frictions (such as a financial accelerator) are not sufficient to be able to fully analyze the effectiveness of those policies and their interaction with the standard interest rate policy. Similarly, the financial crisis has given rise to the new macroprudential policy domain that aims at containing systemic risk and preserving financial stability. Current extensions of the benchmark model are often not rich enough to analyze the interaction between monetary and macroprudential policy. Being able to do so will require incorporating of a richer description of both solvency (default) and liquidity (bank runs) dynamics with greater complexity in terms of both non-linearities and heterogeneity.

The rest of the paper is structured as follows. Section 2 provides an incomplete survey of the macroeconomic models used by central banks and other international organizations. Following this survey, Section 3 presents the prototype model – the estimated model of Smets and Wouters [125]. This model shares many features of models in use by central banks. The section also discusses the data and the estimation of this model on pre-crisis data. In Section 4, we use this model estimated on pre-crisis data to analyze the crisis episode, which gives us valuable insights into the workings of the model. We also compare the performance of our structural model to a reduced-form benchmark VAR, which is estimated with Bayesian priors. As this analysis points to some important shortcomings of the benchmark model, we augment the baseline model in Section 5 along the three
dimensions discussed above.

Finally, section 6 sums up by discussing some other new and old challenges for structural macro models used in policy analysis and presents some conclusions. An appendix contains some technical details on the model, methods and the data used in the analysis.

2. Common features of central bank models

In this section, we provide an incomplete survey of the key policy models currently in use at central banks and other key policy institutions like the IMF, European Commission, and the OECD. We aim at determining the similarity between models, and assess if – and how – they have been changed in response to the recession and developments since then.

A good starting point for the discussion is the paper by Coenen et al. [49]. Wieland et al. [134] provides a complementary and very useful overview of policy models in use at central banks. An additional advantage with the paper by Wieland et al. is that they have pulled together an archive with well-know estimated macroeconomic models (both policy and academic) that can conveniently be used to run and compare various diagnostic shocks using a Matlab graphical user interface.\(^1\) We nevertheless base our discussion on Coenen et al., as they focus exclusively on models in use at policy institutions. Coenen et al. studies the effects of monetary and fiscal shocks in the key policy models in use at the Bank of Canada (BoC-GEM), the Board of Governors of the Federal Reserve System (with two models, FRB-US and SIGMA), the European Central Bank (NAWM), the European Commission (QUEST), the International Monetary Fund (GIMF), and the OECD (OECD Fiscal). Out of the seven models, six are dynamic stochastic general equilibrium (DSGE) models, while one – the FRB-US – is based on the polynomial adjustment cost (PAC) framework. Hence, an overwhelming majority of key policy institutions today use DSGE models as the core policy tool.\(^2\) The switch from traditional backward-looking macro-econometric models (see e.g. Rudebusch and Svensson [120]) to DSGEs occurred amid the forceful critique by Lucas [112] and Sims [122] of such models, and was made feasible due to the progress in the solution and estimation of such models (see e.g. Blanchard and Kahn [21] and Fair and Taylor [69]) as well as the contribution of Christiano, Eichenbaum and Evans [36] who showed that such models, carefully specified, could feature a realistic monetary policy transmission mechanism. As pointed out by

---

\(^1\) Taylor and Wieland [129] use the database to compare the responses to monetary policy shocks. Wieland and Wolters [135] study the forecasting behavior for a large set of models in the database.

\(^2\) Other prominent institutions that have adopted estimated DSGE model as their core policy tool include Bank of England (COMPASS, see [28]), Norges Bank (NEMO, see [26]), Sveriges Riksbank (RAMSES, see [11]), Federal Reserve Bank of New York [58], and the Federal Reserve Bank of Chicago [25].
Clarida, Galí and Gertler [47], Woodford [137] and Galí [76], these models assign an important role to expectations for macroeconomic stabilization, and this view were embraced by policy makers at central banks. However, although macroeconomic models have been used in scenario analysis and affected policy making more generally, it is probably fair to say that the models impact on the short- and medium-term economic projections have been limited, see e.g. Iversen et al. [95].

As outlined in detail in Tables 1 and 2 by Coenen et al. [49], the DSGE models share many similarities to the seminal models of Christiano, Eichenbaum and Evans [36] (CEE henceforth) and Smets and Wouters [125], [126]. They typically feature imperfect competition in product and labor markets as vehicles to introduce sticky prices and wages. They also include important real rigidities like habit formation, costs of adjusting investment and variable capital utilization. Monetary policy is generally determined by a simple Taylor-type policy rule which allows for interest rate smoothing, but although they share many similarities with the academic benchmark models of CEE and Smets and Wouters [126] (SW07 henceforth), policy models often embed some additional features. One such important feature is that they have a significant share of financially constrained households, ranging between 20 and 50 percent. In some models these are hand-to-mouth households, who take their labor income as given and determine consumption residually from a period-by-period budget constraint. In other models these are liquidity-constrained households, who face the same period-by-period budget constraint but solve an intertemporal decision problem between consumption and work effort. An additional difference between the policy models and the academic style ones, is that the former generally has a much more detailed fiscal sector with many distortionary taxes, types of government spending and various transfers from the government to the households.³

Another interesting observation is that neither CEE nor SW07 include frictions in financial markets or a detailed banking sector in their models.⁴ Four of the seven policy models included financial frictions prior to the crisis. By asking the policy institutions that were part of this study about their development efforts since then, it is clear that efforts have been made towards better integration of financial markets, with a focus on the interaction between banks and the firms in the economy. For instance, following the crisis, financial frictions following the approach of Bernanke, Gertler and Cilchrist [20] have been introduced in (at least) Two of the three models that did not feature them before.⁵

³ These results are broadly in line with the findings of Wieland et al. [134].
⁴ The CEE, but not the SW07-model, includes a working capital – or cost channel – of monetary policy whereby firms have to borrow at the policy rate to finance the wage bill. This channel allows the CEE model to account for the “Price-puzzle” (i.e. that inflation rises on impact following a hike in the policy rate) that often emerges for monetary policy shocks in identified VAR models.
⁵ We are grateful to Günter Coenen (ECB) and John Roberts (Federal Reserve Board) for providing very helpful
The key lesson we draw from this is that while the crisis has had some impact on improving the modeling of the financial sector in DSGE models, it has not so far had a material impact on the type of models used at key policy institutions, which still share many features of the basic model developed by CEE.

3. A benchmark model

In this section, we show the benchmark model environment, which is the model of Smets and Wouters [126], SW-model henceforth. The SW-model builds on the workhorse model by CEE, but allows for a richer set of stochastic shocks. In Section 3.4, we describe how we estimate it using aggregate times series for the United States.

3.1. Firms and Price Setting

**Final Goods Production:** The single final output good \( Y_t \) is produced using a continuum of differentiated intermediate goods \( Y_t(f) \). Following Kimball [101], the technology for transforming these intermediate goods into the final output good is

\[
\int_0^1 G_Y \left( \frac{Y_t(f)}{Y_t} \right) df = 1. \tag{3.1}
\]

As in Dotsey and King [61], we assume that \( G_Y(\cdot) \) is given by a strictly concave and increasing function:

\[
G_Y \left( \frac{Y_t(f)}{Y_t} \right) = \frac{\phi^p_t}{1-(\phi^p_t-1)\epsilon_p} \left[ \left( \frac{\phi^p_t+(1-\phi^p_t)\epsilon_p}{\phi^p_t} \right) \frac{Y_t(f)}{Y_t} + \frac{(\phi^p_t-1)\epsilon_p}{\phi^p_t} \right]^{1-(\phi^p_t-1)\epsilon_p} \frac{1}{\phi^p_t-(\phi^p_t-1)\epsilon_p} + \left[ 1 - \frac{\phi^p_t}{1-(\phi^p_t-1)\epsilon_p} \right], \tag{3.2}
\]

where \( \phi^p_t \geq 1 \) denotes the gross markup of the intermediate firms. The parameter \( \epsilon_p \) governs the degree of curvature of the intermediate firm’s demand curve. When \( \epsilon_p = 0 \), the demand curve exhibits constant elasticity as with the standard Dixit-Stiglitz aggregator. When \( \epsilon_p \) is positive the firms instead face a quasi-kinked demand curve, implying that a drop in the good’s relative price only stimulates a small increase in demand. On the other hand, a rise in its relative price generates a large fall in demand. Relative to the standard Dixit-Stiglitz aggregator, this introduces more strategic complementary in price setting which causes intermediate firms to adjust prices less to a given change in marginal cost. Finally, notice that \( G_Y(1) = 1 \), implying constant returns to scale when all intermediate firms produce the same amount of the good.
Firms that produce the final output good are perfectly competitive in both product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of the output index $Y_t$, taking the price $P_t(f)$ of each intermediate good $Y_t(f)$ as given. Moreover, final goods producers sell the final output good at a price $P_t$, and hence solve the following problem:

$$\max_{\{Y_t, Y_t(f)\}} P_t Y_t - \int_0^1 P_t(f) Y_t(f) \, df,$$

subject to the constraint in (3.1). The first order conditions (FOCs) for this problem can be written

$$\frac{Y_t(f)}{Y_t} = \frac{\phi_t^p}{\phi_t^p - (\phi_t^p - 1)\epsilon_p} \left( \frac{P_t(f)}{P_t} \frac{1}{\Lambda_t^p} \right)^{-\frac{\phi_t^p - (\phi_t^p - 1)\epsilon_p}{\phi_t^p - 1} + \frac{(1-\phi_t^p)\epsilon_p}{\phi_t^p}}$$

$$P_t \Lambda_t^p = \left[ \int P_t(f) \frac{1}{\phi_t^p - (\phi_t^p - 1)\epsilon_p} \, df \right] - \frac{\phi_t^p - 1}{\phi_t^p - (\phi_t^p - 1)\epsilon_p}$$

$$\Lambda_t^p = 1 + \frac{(1-\phi_t^p)\epsilon_p}{\phi_t^p} - \frac{(1-\phi_t^p)\epsilon_p}{\phi_t^p} \int \frac{P_t(f)}{P_t} \, df,$$

where $\Lambda_t^p$ denotes the Lagrange multiplier on the aggregator constraint in (3.1). Note that when $\epsilon_p = 0$, it follows from the last of these conditions that $\Lambda_t^p = 1$ in each period $t$, and the demand and pricing equations collapse to the usual Dixit-Stiglitz expressions, i.e.

$$\frac{Y_t(f)}{Y_t} = \left[ \frac{P_t(f)}{P_t} \right]^{-\frac{\phi_t^p}{\phi_t^p - 1}}, P_t = \left[ \int P_t(f)^{\frac{1}{1-\phi_t^p}} \, df \right]^{1-\phi_t^p}.$$

**Intermediate Goods Production:** A continuum of intermediate goods $Y_t(f)$ for $f \in [0, 1]$ is produced by monopolistic competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces the demand schedule in Equation (3.4) from the final goods firms through the solution to the problem in (3.3), which varies inversely with its output price $P_t(f)$ and directly with aggregate demand $Y_t$.

Each intermediate goods producer utilizes capital services $K_t(f)$ and a labor index $L_t(f)$ (defined below) to produce its respective output good. The form of the production function is Cobb-Douglas:

$$Y_t(f) = \varepsilon_t^a K_t(f)^\alpha \left[ \gamma^t L_t(f) \right]^{1-\alpha} - \gamma^t \Phi,$$

where $\gamma^t$ represents the labor-augmenting deterministic growth rate in the economy, $\Phi$ denotes the fixed cost (which is related to the gross markup $\phi_t^p$ so that profits are zero in the steady state), and $\varepsilon_t^a$ is a total productivity factor which follows a Kydland-Prescott [104] style process:

$$\ln \varepsilon_t^a = \rho_a \ln \varepsilon_{t-1}^a + \eta_t^a, \eta_t^a \sim N(0, \sigma_a).$$
Firms face perfectly competitive factor markets for renting capital and hiring labor. Thus, each firm chooses $K_t(f)$ and $L_t(f)$, taking as given both the rental price of capital $R_{Kt}$ and the aggregate wage index $W_t$ (defined below). Firms can without costs adjust either factor of production, thus, the standard static first-order conditions for cost minimization implies that all firms have identical marginal costs per unit of output.

The prices of the intermediate goods are determined by nominal contracts in Calvo [29] and Yun [139] staggered style nominal contracts. In each period, each firm $f$ faces a constant probability, $1 - \xi_p$, of being able to re-optimize the price $P_t(f)$ of the good. The probability that any firm receives a signal to re-optimize the price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, this is adjusted by a weighted combination of the lagged and steady-state rate of inflation, i.e., $P_t(f) = (1 + \pi_{t-1})^{\epsilon_p} (1 + \pi)^{1-\epsilon_p} P_{t-1}(f)$ where $0 \leq \epsilon_p \leq 1$ and $\pi_{t-1}$ denotes net inflation in period $t - 1$, and $\pi$ the steady-state net inflation rate. A positive value of the indexation parameter $\epsilon_p$ introduces structural inertia into the inflation process. All told, this leads to the following optimization problem for the intermediate firms

$$\max_{P_t(f)} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta \xi_p)^j \frac{\xi_{t+j} P_t}{\xi_t P_{t+j}} \left[ \bar{P}_t(f) \left( \Pi_{s=1}^{j} (1 + \pi_{t+s-1})^{\epsilon_p} (1 + \pi)^{1-\epsilon_p} \right) - MC_{t+j} \right] Y_{t+j}(f) ,$$

where $\bar{P}_t(f)$ is the newly set price and $\beta^j \frac{\xi_{t+j} P_t}{\xi_t P_{t+j}}$ the stochastic discount factor. Notice that given our assumptions, all firms that re-optimize their prices actually set the same price.

As noted previously, we assume that the gross price-markup is time-varying and given by $\phi_t^p = \phi p \eta_t^p$, for which the exogenous component $\eta_t^p$ is given by an exogenous ARMA(1,1) process:

$$\ln \eta_t^p = \rho_p \ln \eta_{t-1}^p + \eta_t - \theta_p \eta_{t-1}^p, \quad \eta_t^p \sim N(0, \sigma_p).$$

3.2. Households and Wage Setting

Following Erceg, Henderson and Levin [66], we assume a continuum of monopolistic competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the production sector; that is, goods-producing firms regard each household’s labor services $L_t(h)$, $h \in [0,1]$, as imperfect substitutes for the labor services of other households. It is convenient to assume that a representative labor aggregator combines households’ labor hours in the same proportions as firms would choose. Thus, the aggregator’s demand for each household’s labor is equal to the sum of firms’ demands. The aggregated labor index $L_t$ has the Kimball [101] form:

$$L_t = \int_0^1 G_L \left( \frac{L_t(h)}{L_t} \right) dh = 1,$$
where the function $G_L(\cdot)$ has the same functional form as does (3.2), but is characterized by the corresponding parameters $\epsilon_w$ (governing convexity of labor demand by the aggregator) and a time-varying gross wage markup $\phi^w_t$. The aggregator minimizes the cost of producing a given amount of the aggregate labor index $L_t$, taking each household’s wage rate $W_t(h)$ as given, and then sells units of the labor index to the intermediate goods sector at unit cost $W_t$, which can naturally be interpreted as the aggregate wage rate. From the FOCs, the aggregator’s demand for the labor hours of household $h$ – or equivalently, the total demand for this household’s labor by all goods-producing firms – is given by

$$
\frac{L_t(h)}{L_t} = G_t^{-1} \left[ \frac{W_t(h)}{W_t} \int_0^1 G_t' \left( \frac{L_t(h)}{L_t} \right) \frac{L_t(h)}{L_t} \, dh \right],
$$

where $G_t' (\cdot)$ denotes the derivative of the $G_t (\cdot)$ function in Equation (3.7).

The utility function of a typical member of household $h$ is

$$
E_t \sum_{j=0}^{\infty} \beta^j \left[ \frac{1}{1 - \sigma_c} (C_{t+j}(h) - \alpha C_{t+j-1}) \right]^{1-\sigma_c} \exp \left( \frac{\sigma_c - 1}{1 + \sigma} L_{t+j}(h)^{1+\sigma} \right),
$$

where the discount factor $\beta$ satisfies $0 < \beta < 1$. The period utility function depends on household $h$’s current consumption $C_t(h)$, as well as lagged aggregate consumption per capita, to allow for external habit persistence (captured by the parameter $\alpha$). The period utility function also depends inversely on hours worked $L_t(h)$.

Household $h$’s budget constraint in period $t$ states that expenditure on goods and net purchases of financial assets must equal to the disposable income:

$$
P_t C_t(h) + P_t I_t(h) + \frac{B_{t+1} (h)}{\varepsilon_t^b R_t} + \int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h)
$$

$$
= B_t(h) + W_t(h) L_t(h) + R_t^k Z_t(h) K_t^p(h) - a(Z_t(h)) K_t^p(h) + \Gamma_t(h) - T_t(h).
$$

Thus, the household purchases part of the final output good (at a price of $P_t$), which is chosen to be consumed $C_t(h)$ or invest $I_t(h)$ in physical capital. Following Christiano, Eichenbaum, and Evans [36], investment augments the household’s (end-of-period) physical capital stock $K_{t+1}^p(h)$ according to

$$
K_{t+1}^p(h) = (1 - \delta) K_t^p(h) + \varepsilon_t^i \left[ 1 - S \left( \frac{I_t(h)}{I_{t-1}(h)} \right) \right] I_t(h).
$$

The extent to which investment by each household turns into physical capital is assumed to depend on an exogenous shock $\varepsilon_t^i$ and how rapidly the household changes its rate of investment according to the function $S \left( \frac{I_t(h)}{I_{t-1}(h)} \right)$, which we assume satisfies $S(\gamma) = 0$, $S'(\gamma) = 0$ and $S''(\gamma) = \phi$ where
\( \gamma \) is the steady state gross growth rate of the economy. The stationary investment-specific shock \( \varepsilon^i_t \) follows the process:

\[
\ln \varepsilon^i_t = \rho_i \ln \varepsilon^i_{t-1} + \eta^i_t, \eta^i_t \sim N(0, \sigma_i).
\]

In addition to accumulating physical capital, households may augment their financial assets through increasing their government nominal bond holdings \( \Delta B_{t} \), from which they earn an interest rate of \( R_t \). The return on these bonds is also subject to a risk-shock, \( \varepsilon^b_t \), which follows:

\[
\ln \varepsilon^b_t = \rho_b \ln \varepsilon^b_{t-1} + \eta^b_t, \eta^b_t \sim N(0, \sigma_b).
\] (3.12)

Fisher [74] shows that this shock can be given a structural interpretation.

We assume that agents can engage in friction-less trading of a complete set of contingent claims to diversify away idiosyncratic risk. The term \( \int \xi_{t+1}B_{D,t+1}(h) - B_{D,t}(h) \) represents net purchases of these state-contingent domestic bonds, with \( \xi_{t+1} \) denoting the state-dependent price, and \( B_{D,t+1}(h) \) the quantity of such claims purchased at time \( t \).

On the income side, each member of household \( h \) earns labor income \( W_t(h) \), capital rental income of \( R^k_t Z_t(h) K^p_t(h) \), and pays a utilization cost of the physical capital equal to \( a(Z_t(h)) K^p_t(h) \) where \( Z_t(h) \) is the capital utilization rate. The capital services provided by household \( h \), \( K_t(h) \) thereby equals \( Z_t(h) K^p_t(h) \). The capital utilization adjustment function \( a(Z_t(h)) \) is assumed to satisfy \( a(1) = 0, a'(1) = r^k \), and \( a''(1) = \psi/(1-\psi) > 0 \), where \( \psi \in [0,1) \) and a higher value of \( \psi \) implies a higher cost of changing the utilization rate. Finally, each member also receives an aliquot share \( \Gamma_t(h) \) of the profits of all firms, and pays a lump-sum tax of \( T_t(h) \) (regarded as taxes net of any transfers).

In every period \( t \), each member of household \( h \) maximizes the utility function in (3.9) with respect to consumption, investment, (end-of-period) physical capital stock, capital utilization rate, bond holdings, and holdings of contingent claims, subject to the labor demand function (3.8), budget constraint (3.10), and transition equation for capital (3.11).

Households also set nominal wages in Calvo-style staggered contracts that are generally similar to the price contracts described previously. Thus, the probability that a household receives a signal to re-optimize its wage contract in a given period is denoted by \( 1 - \xi_w \). In addition, SW07 specify the following dynamic indexation scheme for the adjustment of wages for those households that do not get a signal to re-optimize: \( W_t(h) = \gamma (1 + \pi_{t-1})^{1-w}(1 + \pi)^{1-tw} W_{t-1}(h) \). All told, this leads to
the following optimization problem for the households

$$\max_{W_t(h)} \sum_{j=0}^{\infty} (\beta \xi_w)^j \frac{\tilde{Z}_{t+j} P_l(\cdot)}{\tilde{Z}_{t+j} P_l(\cdot)} \left[ \tilde{W}_t(h) \left( \Pi_{s=1}^j (1 + \pi_{t+s-1})^{\epsilon_w} (1 + \pi)^{1-\epsilon_w} \right) - W_{t+j} \right] L_{t+j}(h),$$

where $\tilde{W}_t(h)$ is the newly set wage and $L_{t+j}(h)$ is determined by Equation (3.7). Notice that with our assumptions all households that re-optimize their wages will actually set the same wage.

Following the same approach as with the intermediate-goods firms, we introduce a shock $\epsilon_t^w$ to the time-varying gross markup, $\phi_t^w = \phi^w \epsilon_t^w$, where $\epsilon_t^w$ is assumed being given by an exogenous ARMA(1,1) process:

$$\ln \epsilon_t^w = \rho_w \ln \epsilon_{t-1}^w + \eta_t^w - \phi_w \eta_{t-1}^w, \eta_t^w \sim N(0, \sigma_w). \tag{3.13}$$

### 3.3. Market Clearing Conditions and Monetary Policy

Government purchases $G_t$ are exogenous, and the process for government spending relative to trend output in natural logs, i.e. $g_t = G_t / (\gamma Y)$, is given by the following exogenous AR(1) process:

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \left( \ln g_{t-1} - \rho_g \ln \epsilon_{t-1}^w \right) + \epsilon_t^g, \epsilon_t^g \sim N(0, \sigma_g).$$

Government purchases neither have any effects on the marginal utility of private consumption, nor do they serve as an input into goods production. The consolidated government sector budget constraint is

$$\frac{B_{t+1}}{R_t} = G_t - T_t + B_t,$$

where $T_t$ are lump-sum taxes. By comparing the debt terms in the household budget constraint in Equation (3.10) with the equation above, one can see that receipts from the risk shock are subject to iceberg costs, and hence do not add any income to the government.\footnote{But even if they did, it would not matter as the government is assumed to balance its expenditures each period through lump-sum taxes, $T_t = G_t + B_t - B_{t+1}/R_t$, so that government debt $B_t = 0$ in equilibrium. Furthermore, as Ricardian equivalence (see Barro, [17]) holds in the model, it does not matter for equilibrium allocations whether the government balances its debt or not in each period.}\footnote{See e.g., Leeper and Leith [106], and Leeper, Traum and Walker [107].}

We acknowledge that this is an extremely simplistic modeling of the fiscal behavior of the government relative to typical policy models, and there might be important feedback effects between fiscal and monetary policies that our model does not allow for.\footnote{As discussed by Benigno and Nisticó [18] and Del Negro and Sims [57], the fiscal links between governments and central banks may be especially important today when central banks have employed unconventional tools in monetary policy. Nevertheless, we maintain our simplistic modeling of fiscal policy throughout the paper, as it allows us to examine the partial...}
implications of amending the benchmark model with more elaborate financial markets modeling and the zero lower bound constraint more directly.

The conduct of monetary policy is assumed to be approximated by a Taylor-type policy rule (here stated in non-linearized form)

$$R_t = \max \left( 0, R_{t-1}^{1-\rho_R} \right) \left( \frac{\Pi_t}{\Pi_t} \right)^r \eta^{1-\rho_R} \left( \frac{Y_t}{Y_t^{pot}} \right)^{r\Delta y} \left( \frac{Y_{t-1}}{Y_{t-1}^{pot}} \right)^{r\Delta y} \xi^r_t, \right), \quad (3.14)$$

where $\Pi_t$ denotes the is gross inflation rate, $Y_t^{pot}$ is the level of output that would prevail if prices and wages were flexible, and variables without subscripts denote steady state values. The policy shock $\xi^r_t$ is supposed to follow an AR(1) process in natural logs:

$$\ln \xi^r_t = \rho \ln \xi^r_{t-1} + \eta^r_t, \eta^r_t \sim N(0, \sigma^r). \quad (3.15)$$

Total output of the final goods sector is used as follows:

$$Y_t = C_t + I_t + G_t + a(Z_t) \bar{K}_t,$$

where $a(Z_t) \bar{K}_t$ is the capital utilization adjustment cost.

### 3.4. Estimation on Pre-Crisis Data

We now proceed to discuss how the model is estimated. To begin with, we limit the sample to the period 1965Q1–2007Q4 to see how a model estimated on pre-crisis data fares during the recession. Subsequently, we will estimate the model on data spanning the crisis.

#### 3.4.1. Solving the model

Before estimating the model, we log-linearize all the equations of the model. The log-linearized representation is provided in Appendix A. To solve the system of log-linearized equations, we use the code packages Dynare (see Adjemian et al. [2]) and RISE (see Maih [114]) which both provide an efficient and reliable implementation of the method proposed by Blanchard and Kahn [21].

#### 3.4.2. Data

We use seven key macro-economic quarterly US time series as observable variables: the log difference of real GDP, real consumption, real investment and the real wage, log hours worked, the log difference of the GDP deflator and the federal funds rate. A full description of the data used is
given in Appendix C. The solid blue line in Figure 3.1 shows the data for the full sample, which spans 1965Q1–2014Q2.\(^8\) From the figure, we see the extraordinary large fall in private consumption, which exceeded the fall during the recession in the early 1980s. The strains in the labor market are also evident, with hours worked per capita falling to a post-war bottom low in early 2010. Finally, we see that the Federal reserve cut the federal funds rate to near zero in 2009Q1 (the FFR is measured as an average of daily observations in each quarter). Evidently, the zero bound was perceived as an effective lower bound by the FOMC committee, and they kept it as this level during the crisis and adopted alternative tools to make monetary policy more accommodating (see e.g. Bernanke [19]). Meanwhile, inflation fell to record lows and into deflationary territory by late 2009. Since then, inflation has rebounded close to the new target of 2 percent announced by the Federal Reserve in January 2012.

The measurement equation, relating the variables in the model to the various variables we match in the data, is given by:

\[
Y_{t, obs} = \begin{bmatrix}
\Delta \ln GDP_t \\
\Delta \ln CONS_t \\
\Delta \ln INVE_t \\
\Delta \ln W^\text{real}_t \\
\ln HOURS_t \\
\Delta \ln PGDP_t \\
FFR_{t}
\end{bmatrix}
= \begin{bmatrix}
\Delta \ln Y_t - \ln Y_{t-1} \\
\Delta \ln C_t - \ln C_{t-1} \\
\Delta \ln I_t - \ln I_{t-1} \\
\Delta \ln (W/P)_t - \ln (W/P)_{t-1} \\
\Delta \ln L_t \\
\Delta \ln \Pi_t \\
\Delta \ln R_t
\end{bmatrix}
\approx \begin{bmatrix}
\tilde{\gamma} \\
\tilde{\gamma} \\
\tilde{\gamma} \\
\tilde{\gamma} \\
\tilde{\gamma} \\
\tilde{\gamma} \\
\tilde{\gamma}
\end{bmatrix}
+ \begin{bmatrix}
\hat{\gamma}_t - \hat{\gamma}_{t-1} \\
\hat{c}_t - \hat{c}_{t-1} \\
\hat{i}_t - \hat{i}_{t-1} \\
\hat{\pi}_t - \hat{\pi}_{t-1} \\
\hat{\Pi}_t \\
\hat{\Pi}_t \\
\hat{\Pi}_t
\end{bmatrix}
\] (3.16)

where \(\Delta \ln\) and \(\Delta \ln\) stand for log and log-difference respectively, \(\tilde{\gamma} = 100 (\gamma - 1)\) is the common quarterly trend growth rate to real GDP, consumption, investment and wages, \(\tilde{\pi} = 100 \pi\) is the quarterly steady-state inflation rate and \(r = 100 (\beta^{-1} \gamma \sigma_e (1 + \pi) - 1)\) is the steady-state nominal interest rate. Given the estimates of the trend growth rate and the steady-state inflation rate, the latter will be determined by the estimated discount rate. Finally, \(\bar{\Pi}\) is steady-state hours-worked, which is normalized to be equal to zero.

Structural models impose important restrictions on the dynamic cross-correlation between the variables but also on the long run ratios between the macro-aggregates. Our transformations in (3.16) impose a common deterministic growth component for all quantities and the real wage, whereas hours worked per capita, the real interest rate and the inflation rate are assumed to have a constant mean. These assumptions are not necessarily in line with the properties of the data and may have important implications for the estimation results. Some prominent papers in the

\(^{8}\) The figure also includes a red-dashed line, whose interpretation will be discussed in further detail within Section 4.
literature assume real quantities to follow a stochastic trend, see e.g. Altig et al. [12]. Fisher [73] argues that there is a stochastic trend in the relative price of investment and examines to what extent shocks that can explain this trend matter for business cycles. There is also an ongoing debate on whether hours worked per capita should be treated as stationary or not, see e.g. Christiano, Eichenbaum and Vigfusson [39], Galí and Rabanal [77], and Boppart and Krusell [24]. Within the context of policy models, it is probably fair to say that less attention and resources have been spent to mitigate possible gaps in the low frequency properties of models and data, presumably partly because the jury is still out on the deficiencies of the benchmark specification, but also partly because the focus is on the near-term behavior of the models (i.e. monetary transmission mechanism, forecasting performance, and historical decomposition) and these shortcomings do not
seriously impair the model’s behavior in this dimension.

3.4.3. Estimation Methodology

Following SW07, Bayesian techniques are adopted to estimate the parameters using the seven U.S. macroeconomic variables in Equation (3.16) during the period 1965Q1–2007Q4. Bayesian inference starts out from a prior distribution that describes the available information prior to observing the data used in the estimation. The observed data is subsequently used to update the prior, via Bayes’ theorem, to the posterior distribution of the model’s parameters which can be summarized in the usual measures of location (e.g. mode or mean) and spread (e.g. standard deviation and probability intervals).⁹

Some of the parameters in the model are kept fixed throughout the estimation procedure (i.e., having infinitely strict priors). We choose to calibrate the parameters we think are weakly identified by the variables included in $\tilde{Y}_t$ in Equation (3.16). In Table 3.1 we report the parameters we have chosen to calibrate. These parameters are calibrated to the same values as had SW07.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibrated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\phi_w$</td>
<td>Gross wage markup</td>
<td>1.50</td>
</tr>
<tr>
<td>$g_y$</td>
<td>Government $G/Y$ ss-ratio</td>
<td>0.18</td>
</tr>
<tr>
<td>$\epsilon_p$</td>
<td>Kimball Elast. GM</td>
<td>10</td>
</tr>
<tr>
<td>$\epsilon_w$</td>
<td>Kimball Elast. LM</td>
<td>10</td>
</tr>
</tbody>
</table>

*Note: The calibrated parameters are adapted from SW07.*

The remaining 36 parameters, which mostly pertain to the nominal and real frictions in the model as well as the exogenous shock processes, are estimated. The first three columns in Table 3.2 shows the assumptions for the prior distribution of the estimated parameters. The location of the prior distribution is identical to that of SW07. We use the beta distribution for all parameters bounded between 0 and 1. For parameters assumed to be positive, we use the inverse gamma distribution, and for the unbounded parameters, we use the normal distribution. The exact location and uncertainty of the prior can be seen in Table 3.2, but for a more comprehensive discussion of our choices regarding the prior distributions we refer the reader to SW07.

⁹ We refer the reader to Smets and Wouters [125] for a more detailed description of the estimation procedure.
Table 3.2: Prior and posterior distributions: 1966Q1–2007Q4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Optimization</th>
<th>Posterior distribution</th>
<th>SW07 results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>type mean std.dev.</td>
<td>mode std.dev.</td>
<td>mean 5% 95% Hess. mode</td>
<td></td>
</tr>
<tr>
<td>Calvo prob. wages $\xi_w$</td>
<td>beta 0.50 0.10</td>
<td>0.79 0.055</td>
<td>0.75 0.61 0.82</td>
<td>0.73</td>
</tr>
<tr>
<td>Calvo prob. prices $\xi_p$</td>
<td>beta 0.50 0.10</td>
<td>0.69 0.051</td>
<td>0.69 0.60 0.76</td>
<td>0.65</td>
</tr>
<tr>
<td>Indexation wages $\omega_w$</td>
<td>beta 0.50 0.15</td>
<td>0.63 0.136</td>
<td>0.58 0.36 0.79</td>
<td>0.59</td>
</tr>
<tr>
<td>Indexation prices $\omega_p$</td>
<td>beta 0.50 0.15</td>
<td>0.23 0.093</td>
<td>0.26 0.13 0.41</td>
<td>0.22</td>
</tr>
<tr>
<td>Gross price markup $\phi_p$</td>
<td>normal 1.25 0.12</td>
<td>1.64 0.76</td>
<td>1.64 1.52 1.71</td>
<td>0.66</td>
</tr>
<tr>
<td>Capital production share $\sigma$</td>
<td>normal 0.30 0.05</td>
<td>0.21 0.018</td>
<td>0.20 0.18 0.24</td>
<td>0.19</td>
</tr>
<tr>
<td>Capital utilization cost $\psi$</td>
<td>beta 0.50 0.15</td>
<td>0.60 0.100</td>
<td>0.59 0.43 0.75</td>
<td>0.54</td>
</tr>
<tr>
<td>Investment adj. cost $\varphi$</td>
<td>normal</td>
<td>4.00 1.50</td>
<td>5.50 1.019</td>
<td>5.48</td>
</tr>
<tr>
<td>Habit formation $\zeta$</td>
<td>beta 0.70 0.10</td>
<td>0.67 0.042</td>
<td>0.69 0.62 0.76</td>
<td>0.71</td>
</tr>
<tr>
<td>Inv subs. elast. of cons. $\sigma_c$</td>
<td>normal 1.50 0.37</td>
<td>1.53 0.138</td>
<td>1.44 1.23 1.69</td>
<td>1.59</td>
</tr>
<tr>
<td>Labor supply elast. $\sigma_l$</td>
<td>normal 2.00 0.75</td>
<td>2.15 0.584</td>
<td>2.03 1.13 2.99</td>
<td>1.92</td>
</tr>
<tr>
<td>Log hours worked in S.S. $l$</td>
<td>normal 0.00 2.00</td>
<td>1.56 0.985</td>
<td>1.15 −0.56 2.72</td>
<td>−0.10</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$100(\beta^{-1} − 1)$</td>
<td>gamma 0.25 0.10</td>
<td>0.13 0.052</td>
<td>0.16</td>
</tr>
<tr>
<td>Quarterly Growth in S.S. $\bar{\gamma}$</td>
<td>normal 0.40 0.10</td>
<td>0.43 0.014</td>
<td>0.43 0.41 0.45</td>
<td>0.43</td>
</tr>
<tr>
<td>Stationary tech. shock $\rho_a$</td>
<td>beta 0.50 0.20</td>
<td>0.96 0.008</td>
<td>0.96 0.93 0.97</td>
<td>0.95</td>
</tr>
<tr>
<td>Risk premium shock $\rho_b$</td>
<td>beta 0.50 0.20</td>
<td>0.18 0.081</td>
<td>0.22 0.10 0.38</td>
<td>0.18</td>
</tr>
<tr>
<td>Invest. spec. tech. shock $\rho_i$</td>
<td>beta 0.50 0.20</td>
<td>0.71 0.053</td>
<td>0.71 0.61 0.80</td>
<td>0.71</td>
</tr>
<tr>
<td>Gov’t cons. shock $\rho_g$</td>
<td>beta 0.50 0.20</td>
<td>0.97 0.008</td>
<td>0.97 0.96 0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>Price markup shock $\rho_p$</td>
<td>beta 0.50 0.20</td>
<td>0.90 0.038</td>
<td>0.89 0.80 0.95</td>
<td>0.90</td>
</tr>
<tr>
<td>Wage markup shock $\rho_w$</td>
<td>beta 0.50 0.20</td>
<td>0.98 0.010</td>
<td>0.97 0.94 0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>Response of $\eta$ to $\sigma^p_w$ $\rho_{\eta w}$</td>
<td>beta 0.50 0.20</td>
<td>0.52 0.086</td>
<td>0.49 0.38 0.67</td>
<td>0.52</td>
</tr>
<tr>
<td>Stationary tech. shock $\sigma_a$</td>
<td>invgamma 0.10 2.00</td>
<td>0.44 0.026</td>
<td>0.45 0.40 0.49</td>
<td>0.45</td>
</tr>
<tr>
<td>Risk premium shock $\sigma_b$</td>
<td>invgamma 0.10 2.00</td>
<td>0.24 0.022</td>
<td>0.24 0.19 0.27</td>
<td>0.24</td>
</tr>
<tr>
<td>Invest. spec. tech. shock $\sigma_i$</td>
<td>invgamma 0.10 2.00</td>
<td>0.41 0.041</td>
<td>0.41 0.34 0.48</td>
<td>0.45</td>
</tr>
<tr>
<td>Gov’t cons. shock $\sigma_g$</td>
<td>invgamma 0.10 2.00</td>
<td>0.50 0.028</td>
<td>0.51 0.46 0.57</td>
<td>0.52</td>
</tr>
<tr>
<td>Price markup shock $\sigma_p$</td>
<td>invgamma 0.10 2.00</td>
<td>0.12 0.015</td>
<td>0.13 0.10 0.15</td>
<td>0.14</td>
</tr>
<tr>
<td>MA(1) price markup shock $\sigma_p^p$</td>
<td>invgamma 0.10 2.00</td>
<td>0.74 0.080</td>
<td>0.72 0.46 0.83</td>
<td>0.74</td>
</tr>
<tr>
<td>Wage markup shock $\sigma_w$</td>
<td>invgamma 0.10 2.00</td>
<td>0.31 0.025</td>
<td>0.30 0.25 0.34</td>
<td>0.24</td>
</tr>
<tr>
<td>MA(1) wage markup shock $\sigma_{\eta w}$</td>
<td>beta 0.50 0.20</td>
<td>0.95 0.030</td>
<td>0.92 0.77 0.95</td>
<td>0.88</td>
</tr>
<tr>
<td>Quarterly infl. rate. in S.S. $\pi$</td>
<td>gamma 0.62 0.10</td>
<td>0.79 0.114</td>
<td>0.82 0.65 1.01</td>
<td>0.81</td>
</tr>
<tr>
<td>Inflation response $r_\pi$</td>
<td>normal 1.50 0.25</td>
<td>2.01 0.174</td>
<td>2.07 1.75 2.33</td>
<td>2.03</td>
</tr>
<tr>
<td>Output gap response $r_y$</td>
<td>normal 0.12 0.05</td>
<td>0.10 0.023</td>
<td>0.10 0.05 0.13</td>
<td>0.08</td>
</tr>
<tr>
<td>Diff. output gap response $r_{\Delta y}$</td>
<td>normal 0.12 0.05</td>
<td>0.23 0.026</td>
<td>0.23 0.18 0.27</td>
<td>0.22</td>
</tr>
<tr>
<td>Mon. pol. shock std $\sigma_r$</td>
<td>invgamma 0.10 2.00</td>
<td>0.23 0.014</td>
<td>0.24 0.21 0.26</td>
<td>0.24</td>
</tr>
<tr>
<td>Mon. pol. shock pers. $\rho_r$</td>
<td>beta 0.50 0.20</td>
<td>0.12 0.002</td>
<td>0.15</td>
<td>0.12</td>
</tr>
<tr>
<td>Interest rate smoothing $\rho_B$</td>
<td>beta 0.75 0.10</td>
<td>0.82 0.022</td>
<td>0.82</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Log marginal likelihood | Laplace −961.81 | MCMC −960.72 |

Note: Data for 1965Q1–1965Q4 are used as pre-sample to form a prior for 1966Q1, and the log-likelihood is evaluated for the period 1966Q1–2007Q4. A posterior sample of 250,000 post burn-in draws was generated in the Metropolis-Hastings chain. Convergence was checked using standard diagnostics such as CUSUM plots and the potential scale reduction factor on parallel simulation sequences. The MCMC marginal likelihood was numerically computed from the posterior draws using the modified harmonic estimator of Geweke [84].

3.4.4. Posterior distributions of the estimated parameters

Given these calibrated parameters in Table 3.1, we obtain the joint posterior distribution mode for the estimated parameters in Table 3.2 on pre-crisis data in two steps. First, the posterior mode and an approximate covariance matrix, based on the inverse Hessian matrix evaluated at the mode, is obtained by numerical optimization on the log posterior density. Second, the posterior distribution is subsequently explored by generating draws using the Metropolis-Hastings algorithm. The proposal distribution is taken to be the multivariate normal density centered at the previous
draw with a covariance matrix proportional to the inverse Hessian at the posterior mode; see Schorfheide [121] and Smets and Wouters [125] for further details. The results in 3.2 shows the posterior mode of all the parameters along with the approximate posterior standard deviation obtained from the inverse Hessian at the posterior mode. In addition, it shows the mean along with the 5th and 95th percentiles of the posterior distribution, and finally, the last column reports the posterior mode in the SW07 paper.

There two important features to notice with regards to the posterior parameters in Table 3.2. First, the policy- and deep-parameters are generally very similar to those estimated by SW07, reflecting a largely overlapping estimation sample (SW07 used data for the 1965Q1–2004Q4 period to estimate the model). The only noticeable difference relative to SW07 is that the estimated degree of wage and price stickiness is somewhat more pronounced (posterior mode for $\xi_w$ is 0.79 instead of 0.73 in SW07, and the mode for $\xi_p$ has increased from 0.65 (SW07) to 0.69). The tendency of an increased degree of price and wage stickiness in the extended sample is supported by Del Negro, Giannoni and Schorfheide [53], who argue that a New Keynesian model similar to ours augmented with financial frictions points towards a high degree of price and wage stickiness to fit the behavior of inflation during the Great Recession. Second, the estimated variances of the shocks are somewhat lower (apart from the wage markup shock). Given that SW07 ended their estimation in 2004, and the so-called “Great Moderation” was still in effect from 2005 into the first half of 2007, the finding of reduced shock variances is not surprising.

4. Empirical performance of benchmark models during the Great Recession

We will now assess the performance of our benchmark DSGE model during the great recession in a number of dimensions. First and foremost, we study the forecasting performance of the model during the most intense phase of the recession, i.e. the third and fourth quarters of 2008. In addition, we look into what the model has to say about the speed of recovery in the economy during the post-crisis period. In this exercise, we benchmark the performance of the DSGE model against a standard Bayesian VAR, which includes the same set of variables.

Second, we examine how the model interprets the “Great Recession”, and assess the plausibility of the shocks the model needs to explain it. We do this from both a statistical and economic viewpoint.
4.1. Forecasting performance of benchmark models during the recession

We now use the DSGE model estimated on data up to 2007Q4 to forecast for the out-of-sample data. We start to make forecasts for 1, 2, ..., 12 quarters ahead in the third and fourth quarter of 2008, conditional on observing data up to and including 2008Q3 and 2008Q4 respectively. Forecasts starting in these quarters are of particular interest as output plummeted in 2008Q4 (about $-9.75$ percent at an annualized quarterly rate) and in 2009Q1 (roughly $-5.75$ percent at an annualized rate). To provide a benchmark for the DSGE forecasts, we also report the forecasts of a Bayesian vector auto-regressive (BVAR) model estimated on the same sample. While both models have been estimated for the same time series stated in (3.16), we only show results for a subset of variables; the federal funds rate, output growth and price inflation (where inflation and output growth have been transformed into yearly rates by taking 4-quarter averages). Warne, Coenen, and Christoffel [133] study how the predictive likelihood can be estimated, by means of marginalization, for any subset of the observables in linear Gaussian state-space models. Our exposition below is less formal and focuses on the univariate densities.\textsuperscript{10}

The BVAR uses the standard Doan-Litterman-Sims [60] prior on the dynamics and an informative prior on the steady state following the procedure outlined in Villani [132]. We select the priors on the steady state in the BVAR to be consistent with those used in the DSGE model, which facilitates comparison between the two models. In both the DSGE and the BVAR, the median projections and 50-, 90- and 95-percent uncertainty bands are based on 10,000 simulations of respective model in which we allow for both shock and parameter uncertainty.\textsuperscript{11}

In Figure 4.1, the left column shows the forecasts in the DSGE conditional on observing data up to 2008Q3. As can be seen in the upper left panel, the endogenous DSGE model forecast predicted yearly GDP growth (four quarter change of log-output) to be about unchanged, whereas actual economic activity fell dramatically in the fourth quarter. Moreover, the 95-percent uncertainty band suggests that the large drop in output was completely unexpected from the point of view of the DSGE model. Thus, in line with Del Negro and Schorfheide [55], our estimated model carries the implication that the “Great Recession” as late as of observing the outcome in 2008Q3 was a highly unlikely tail event. Turning to yearly inflation and the federal funds rate in the middle and bottom left panels, we also see that they fell considerably more than predicted by the model, but

\textsuperscript{10} We perform these forecasts on \textit{ex post} data, collected on September 25th 2014 (see Appendix C).

\textsuperscript{11} For an extensive comparison of the forecasting performance of the Smets and Wouters model along with a comparison to a BVAR and Greenbook forecasts on real-time data, see Edge and Gürkaynak [69] and Wieland and Wolters [135]. Adolfson et al. [7] and Andersson et al. [9] examine the forecasting properties of an open economy DSGE model on Swedish data.
their decline are within or close to the 95-percent uncertainty bands of the linearized DSGE model and hence, cannot be considered as tail events to the same extent as the Great Recession.

Turning to the results for the BVAR, which are reported in the right column in Figure 4.1, we see that the forecast distribution in the BVAR for yearly GDP growth is both quantitatively and qualitatively very similar to that in the DSGE model. Hence, the Great Recession was also a highly unlikely tail event according to the BVAR model. Given that the BVAR and the DSGE are both linearized models, the relatively high degree of similarity of the two model forecasts is not completely surprising. We also see that the uncertainty bands for the output roughly are equally-sized in the DSGE as those in the BVAR model. This finding is neither obvious nor trivial as the DSGE model does not have a short-lag BVAR representation. The BVAR, on the other hand, does not impose nearly as many cross-restrictions on the parameter space as the DSGE model. Hence,
allowing for parameter uncertainty will tend to increase the uncertainty bands considerably more in the BVAR relative to the DSGE model (the BVAR has around 190 free parameters, while the DSGE has 36). On net, these two forces appear to cancel each other out.

Moreover, as is clear from Figure 4.2, the high degree of coherence between the DSGE and BVAR output growth forecasts also holds up when conditioning on the state in 2008Q4 and using the estimated models to make predictions for 2009Q1, 2009Q2,...,2011Q4. For yearly inflation and the federal funds rate, the forecasts conditional on the state in 2008Q3 are very similar, as can be seen in the middle rows in Figure 4.1. However, for the forecast made conditional on the state in 2008Q4 (Figure 4.2), the DSGE and BVAR forecasts differ substantially, at least qualitatively. In this period, the BVAR predicts a pro-longed period with near-zero inflation and a federal funds rate well below zero for two years, whereas the modal outlook in the DSGE model is that inflation would quickly return to near 2 percent and that the federal funds rate should therefore be increased steadily throughout the forecast horizon. The zero lower bound is not much of a concern in the DSGE model, while the BVAR suggests that it should be a binding constraint longer than two years.

Apart from failing to predict the crisis in the first place, both the BVAR and the DSGE model also have a clear tendency to forecast a quick recovery. For the benchmark DSGE model, this feature is evident already from Figure 3.1. In this figure, the red-dotted line shows the one-sided filtered Kalman projections of the observed variables; that is, the projection for period $t$ given all available information in period $t-1$. By comparing the one-sided filtered Kalman projections against the outcome (the blue-solid line) it is evident that the benchmark DSGE model predicts that growth in output, consumption and investment would pick up much quicker than they did following the recession. Hence, consistent with the findings in Chung, Laforte, and Reifschneider [46], the benchmark DSGE model consistently suggests a V-shaped recovery and that better times were just around the corner, whereas the outcome is consistent with a much more slower recovery out of the recession as is evident from Figures 4.1 and 4.2. Figure 4.3 shows sequential BVAR forecasts 1,2,...,12 quarters ahead for the period 2008Q3–2014Q1 conditional on observing the state up to the date in which the forecasts start. In line with the results for the DSGE model, the results in this figure indicate that the BVAR also tends to predict a quick recovery of economic activity. Consistent with this reasoning, the forecasts for the level of output (as deviation from the deterministic trend), shown in the bottom row in Figures 4.1 and 4.2, display that both the DSGE
and the BVAR models overestimate the speed of recovery out of the recession.\textsuperscript{12}

The slow recovery following the recession is consistent with the work by Reinhart and Rogoff [119] and Jordà, Schularick and Taylor [96], who suggest that recoveries from financial crises are slower than recoveries from other recessions. The empirical observation by Reinhart and Rogoff has also been corroborated in subsequent theoretical work by Queralto [117] and Anzoategui et al. [15].\textsuperscript{13} As our benchmark equilibrium model does not include the mechanisms of Queralto, it has a hard time accounting for the slow recovery following the recession, both in terms of the level and the growth rate of GDP. Our benchmark models – both the DSGE and the BVAR – rely

\textsuperscript{12} For both the BVAR and the DSGE model, the series for detrended output is the smoothed estimate from the DSGE model. When we construct the forecast of detrended output in the BVAR, we accumulate the projected quarterly growth rate of output after subtracting the estimated steady state growth rate in each period.

\textsuperscript{13} Notwithstanding these results, Howard, Martin and Wilson [92] argue out that the finding pertains to the level of economic activity, and not the growth rate (which is what we focused on in Figure 3.1).
on significant influence of adverse exogenous shocks which weighs on economic activity during the recovery. While this might be deemed to be a significant weakness of these models, it should be noted that some major negative events may have contributed to hold back the recovery; e.g. the European debt crisis which intensified in May 2010, and the showdown between the Republicans and democrats in the congress which created significant uncertainty in the U.S. economy according to estimates by Fernandez-Villaverde et al. [72]. With these events in mind, it is not entirely implausible that the models need some adverse shocks to account for the slow recovery.

4.2. Economic interpretation of the recession

As indicated in the previous section, both the DSGE and BVAR models are dependent on major adverse shocks to account for the recession. In this section, we examine what shocks are filtered out as the drivers of the recession and its aftermath. We will focus entirely on the benchmark DSGE, as it would be hard to identify all the shocks in the BVAR model. We extract the smoothed shocks
through the Kalman filter by using the model estimated on the pre-crisis period for the full sample (without re-estimating the parameters).

In Figure 4.4, the left column shows the two-sided smoothed Kalman filtered innovations (e.g. $\eta_t^0$ for the technology shock in Equation 3.5) for the seven shock processes in the model using the posterior mode parameters. In the right column, we show the two-sided smoothed shock processes in levels (e.g. $\epsilon_t^0$ for the technology shock in Equation 3.5). The blue solid-line indicates the in-sample period, and the blue-dotted line the out-of-sample period. The Grey bars are NBER dated recessions.

Figure 4.4: Smoothed Innovations and Shocks in Model Estimated on Pre-Crisis Data.

Before analyzing the role various shocks played during the crisis and its aftermath, it is insightful to discuss if there are any signs in the pre-crisis shocks about what events might have been causal for the crisis itself. As is clear from the left column in the figure, there is nothing that stands
out in the innovations between 2000 and the burst of the crisis. There were a string of positive innovations to technology during 2003–2005, which led to a run-up in technology (right upper panel) during this period. To the extent that households and firms expected this positive development to continue and were taken off-guard by the adverse outcomes 2006 and onward, this could have been a contributing factor to the crisis. Christiano et al. [40] argue that over-optimistic expectations of future technology have been associated with credit cycles that have contributed to boom-bust cycles in the real U.S. economy in a model with a more elaborate financial sector.  

Our benchmark model does not include a financial sector and thus, cannot be used to assess this possibility explicitly. Loose monetary policy have also been argued as a possible driver for the crisis, see e.g. Taylor [128]. Our estimated model lend some, but limited, support to this view; although the estimated policy rule suggest that monetary policy was on average expansionary between 2002–2006, the magnitude of the deviations are not very large though, as seen from the lower panels in Figure 4.4. Based on the shock decomposition, it is therefore hard to argue that the Fed’s conduct of monetary policy was causal for the crisis.  

With this discussion in mind, we now turn to the crisis and its aftermath. As is clear from Figure 4.4, the key innovations happened to technology, investment specific technology (the Tobin’s Q-shock), and the risk-premium shock during the most intense phase of the recession. More specifically, the model filters out a very large positive shock to technology (about 1.5 percent as shown in the upper left panel, which corresponds to a 3.4 standard error shock) in 2009Q1. In 2008Q4 and 2009Q1, the model also filters out two negative investment specific technology shocks (about −1 and −1.5 percent – or 2.0 and 3.7 standard errors – respectively). The model moreover filters out a large positive risk shocks in 2008Q3–Q4, and in 2009Q1 (0.5, 1.5, and 0.5 percent respectively, equivalent to 1.9, 6.0 and 2.8 standard errors). These smoothed shocks account for the bulk of the sharp decline in output, consumption and investment during the acute phase of the crisis at the end of 2008 and the beginning of 2009. Our finding of a large positive technology shock in the first quarter of 2009 may at first glance be puzzling, but can be understood from Figures 3.1 and 4.2. In these, we see that output (as deviation from trend) fell less during the recession than did hours worked per capita. Hence, labor productivity rose sharply during the most acute

\[14\] The focus of Christiano et al. is what monetary policy should do to mitigate the inefficient boom-bust cycle. They do not consider the role macro-prudential regulation could play to mitigate the cycle.

\[15\] The main reason why our policy shocks are much smaller in magnitude than those computed by Taylor is that we consider a more elaborate policy rule with considerable interest rate smoothing (\(\rho_R = 0.82\), see Table 3.2). One could argue about whether one should allow for interest rate smoothing or whether this persistence should be attributed to the exogenous monetary policy shock (i.e. a higher \(\rho_*\) in the process for \(\varepsilon_{\ast t}\) in Equation 3.15). In our estimated model, however, the log marginal likelihood strongly favors a high degree of interest rate smoothing and low persistence of the exogenous policy shocks (i.e. a combination of high \(\rho_R\) and low \(\rho_*\)).
phase of the recession. The model replicates this feature of the data by filtering out a sequence of positive technology shocks. These technology shocks will stimulate for output, consumption and investment. The model thus needs some really adverse shocks that depresses these quantities even more and causes hours worked per capita to fall, and this is where the positive risk premium and investment specific technology shocks come into play. These shocks cause consumption (risk premium) and investment (investment specific) – and thereby GDP – to fall. Lower consumption and investment also causes firms to hire less labor, resulting in hours worked per capita to fall.

Another shock that helps account for the collapse in activity at the end of 2008 is the smoothed monetary policy shock shown in the bottom left panel (expressed at a quarterly rate). This shock becomes quite positive in 2008Q4 and 2009Q1; in annualized terms it equals roughly 150 (1.6 standard errors) and 250 (2.8 standard errors) basis points in each of these quarters respectively. As the actual observations for the annualized federal funds rate is about 50 and 20 basis points, these sizable policy shocks suggests that the zero lower bound is likely to have been a binding constraint, at least in these quarters. This finding is somewhat different from those of Del Negro and Schorfheide [55] and Del Negro, Giannoni, and Schorfheide [53], who argued that the zero lower bound was not a binding constraint in their estimated models.

The large smoothed innovations translate into very persistent movements in some of the smoothed shock processes, reported in the right column in Figure 4.4. For the simple AR(1) shock processes, the degree of persistence is governed by the posterior for $\rho$. As can be seen from Table 3.2, the posterior for $\rho_a$ ($\rho_b$) is very high (low), whereas the posterior for $\rho_i$ is somewhere in between. It is therefore not surprising that the technology process is almost permanently higher following the crisis, whereas the risk shock process quickly recedes towards steady state. Our finding of a very persistent rise in the exogenous component of total factor productivity (TFP) is seemingly at odds with Christiano, Eichenbaum and Trabandt [37], who reports that TFP fell in the aftermath of the recession. Christiano et al., Gust, Lopez-Salido and Smith [89] also report negative innovations to technology in 2008 (see Figure 5 in their paper). While a closer examination behind the differences in the results would take us too far, we note that our findings aligns very well with Fernald [70]. Specifically, our smoothed innovations to technology are highly correlated with the two TFP measures computed by Fernald [70], as can be seen from Table 4.1. The table shows the correlations between our technology innovations $\eta^t$, shown in the left column in Figure 4.4, and the period-by-period change in the raw and utilization-corrected measure of TFP by Fernald. From the first column in the Table, we learn that the correlation between our innovations and his raw
measure is almost 0.5 for the estimation sample period. As we are studying first differences and innovations, this correlation must be considered quite high. Even more reassuring for our model is that the correlation between our smoothed innovation series and Fernald’s utilization adjusted series is as high as 0.6. When extending the sample to include the crisis and post-crisis period, we see that these correlations remain high; if anything, they become slightly higher. We believe this lends support for our basic result that weak TFP growth was not a key contributing factor to the crisis.

Table 4.1: Correlations between Smoothed and Actual TFP shocks.

<table>
<thead>
<tr>
<th>TFP measure</th>
<th>Pre-Crisis: 66Q1–07Q4</th>
<th>Full: 66Q1–14Q2</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr(ΔRaw, η_t^T)</td>
<td>0.483</td>
<td>0.522</td>
</tr>
<tr>
<td>corr(ΔCorrected, η_t^T)</td>
<td>0.602</td>
<td>0.608</td>
</tr>
</tbody>
</table>

Note: “ΔRaw” denotes the first difference of the quarterly unadjusted measure in Fernald [70], while “ΔCorrected” is the first difference Fernald’s capacity utilization adjusted TFP measure. In the model, the smoothed estimates of the innovations η_t^T (see Equation 3.5) are used. This series is depicted in the upper left column of Figure 4.4.

For the two markup shocks, we notice that they are not nearly as highly correlated as the technology shock although the estimated AR(1) coefficients for these processes are quite high (0.89 for the price markup shock, and 0.97 for the wage markup shock, see Table 3.2). The reason why their correlation is so low is the estimated MA(1) coefficients, θ_p and θ_w in Equations (3.6) and (3.13) are rather high, i.e. 0.72 and 0.92 respectively. Despite the generally low correlation of the price shock process during the pre-crisis period, we see that its outcome is driven by a sequence of positive innovations during the crisis period. This finding is in line with Fratto and Uhlig [75], who found that price markup shocks played an important role to avoid an even larger fall in inflation during the crisis, and contributed to the slow decline in employment during the post-crisis recovery.\footnote{16} The wage markup shock process does not display any clear pattern after the pre-crisis period, but it is clear that its variance has increased since the end of the 1990s suggesting that the model provides a less accurate description of wage-setting behavior in the U.S. labor market since then. However, it should be kept in mind that this finding may not necessarily remain if alternative wage series are used.\footnote{17}

\footnote{16} The prominent role of the price and wage markup for explaining inflation and behavior of real wages in the SW-model have been criticized by Chari, Kehoe and McGrattan [34] as implausibly large. Galí, Smets and Wouters [80], however, shows that the size of the markup shocks can be reduced substantially by allowing for preference shocks to household preferences.

\footnote{17} Because of potential measurement problems pertaining to wages, Galí, Smets, and Wouters [80] and Justiniano, Primiceri and Tambalotti [98] use two series for real wage growth when estimating their DSGE model.
The historical decompositions in Figure 4.5 summarizes the impact of the various shocks on the output growth, inflation, federal funds rate and output as deviation from a trend during 2007Q1–2014Q2 in the benchmark model estimated on data up to 2007Q4 (see Table 3.2). Notice that the scale on the left- and right-axes are not the same (except for the 2-sided smoothed output as deviation from trend): the left axis shows the contributions of the various shocks to fluctuations around the steady state, whereas the right axis shows evolution of each variable in levels. Thus, for each period the sum of the bars on the left axis plus the steady state value for each variable (not shown) equals the actual outcome (thin line). For output as deviation from trend, the steady state value is nil, why the sum of the bars directly equals the smoothed values.

Figure 4.5: Historical Decompositions of Yearly Output Growth (4-quarter change), Yearly Inflation (4-quarter change), Fed Funds Rate and Output (deviation from trend). Left axis shows the contributions of the shocks (bars) to fluctuations around the steady state; right axis shows actual outcomes (in levels).
As seen from the figure, the risk premium, the investment specific technology and the monetary policy shocks are the key drivers behind the decline in output during the recession period, whereas TFP as discussed above had some offsetting impact on output. However, all four shocks contributed to the gradual decline in inflation. The nominal interest rate would clearly have dropped below zero in absence of the zero bound constraint. The slow recovery is attributed to the persistence of the shocks that were responsible for the recession, but also captures new unexpected headwinds along with positive innovations to markups in prices and wages. Interestingly, the negative impact of the risk premium shock is relatively short-lived. To a large extent this of course reflects that the model is not rich enough to propagate financial shocks sufficiently, but it is also conceivable that this partly captures the stimulus coming from the non-conventional monetary policy actions.

The continuously low interest rate is consistent with the weak state of the economy during this period; output (as deviation from trend) is well below its pre-crisis trend and inflation persistently below its targeted rate, and sustained sub-par growth (slow or non-existent recovery in output as deviation from trend). As the pre-crisis model features a moderate degree of price and wage stickiness, inflation would have fallen persistently into negative territory in the absence of other shocks. This is counter-factual relative to the data, and the missing deflation in the model estimated on pre-crisis data is accounted for by inflationary markup shocks.

While the smoothed shocks – that the model needs to explain the crisis period – are not too surprising given the model’s specification, it is nevertheless clear that the benchmark model needs a highly unlikely combination of adverse shocks in 2008Q4 and 2009Q1 to account for the most intense phase of the recession. Therefore, we now discuss the statistical properties of the shocks and examine if they correlate with some key observable financial variables not included in our set of observables.

4.3. Statistical properties of the innovations and their relation to financial indicators

Table 4.2 provides an overview of the statistical properties of the estimated structural shocks and of the forecast errors for the seven observed macro variables. Most of the forecast errors display a significant amount of kurtosis, a feature that they inherit from the underlying macro variables. For the structural shocks, the problems are mostly concentrated in two shocks – the monetary policy and the risk premium shock – that display highly significant deviations from the underlying Gaussian assumption. The structural innovations in the policy rate and the risk premium are
characterized by a highly skewed and fat-tailed distribution.\textsuperscript{18} We identified the large disturbances in these shocks already in the previous section as crucial drivers of the recent recession, but Table 4.2 illustrates that both processes were already affected by non-Gaussian innovations in the pre-crisis model as well. As observed in Figure 4.4, these negative outliers occur mostly during the recession periods.

<table>
<thead>
<tr>
<th>Innovations in</th>
<th>Sample Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-Crisis: 66Q1–07Q4</td>
</tr>
<tr>
<td>Technology</td>
<td>mean 0.04     std 0.44     skew 0.43*     kurt 4.09*</td>
</tr>
<tr>
<td>Risk premium</td>
<td>mean 0.00     std 0.24     skew 0.74**     kurt 5.12**</td>
</tr>
<tr>
<td>Inv.spec.techn.</td>
<td>mean 0.02     std 0.42     skew 0.09     kurt 3.95*</td>
</tr>
<tr>
<td>Exog.spending</td>
<td>mean −0.07    std 0.50     skew 0.30     kurt 3.66</td>
</tr>
<tr>
<td>Price markup</td>
<td>mean 0.00     std 0.12     skew −0.14    kurt 3.49</td>
</tr>
<tr>
<td>Wage markup</td>
<td>mean 0.01     std 0.31     skew 0.10     kurt 3.89</td>
</tr>
<tr>
<td>Monetary policy</td>
<td>mean −0.03    std 0.23     skew 0.76**     kurt 8.09**</td>
</tr>
</tbody>
</table>

This feature implies that the predictive density of linear Gaussian DSGE models underestimates systematically the probability of these large recession events. This observation is important because it means that the model considers the strong economic downturns that we typically observe during recession periods as extremely unlikely tail events.\textsuperscript{19} Linear Gaussian models may therefore be

\textsuperscript{18} The innovations in the structural shocks are also characterized by a significant ARCH effect illustrating the systematic time-varying volatility structures.

\textsuperscript{19} This observation is consistent with the findings presented by Chung, Laforte, and Reifsneider [46].
inappropriate instruments for analyzing policy questions related to risk scenario’s or stress test exercises.

Table 4.3: Correlation between Innovations and Financial Indicators.

<table>
<thead>
<tr>
<th>Innovations in</th>
<th>Innovations</th>
<th>Innovations</th>
<th>Innovations</th>
<th>Innovations</th>
<th>Innovations</th>
<th>Innovations</th>
<th>Innovations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sigma_a )</td>
<td>( \sigma_b )</td>
<td>( \sigma_i )</td>
<td>( \sigma_g )</td>
<td>( \sigma_p )</td>
<td>( \sigma_w )</td>
<td>( \sigma_r )</td>
</tr>
<tr>
<td>Technology</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk premium</td>
<td>(-0.11)</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inv.spec.techn.</td>
<td>(-0.19)</td>
<td>(-0.08)</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exog.spending</td>
<td>0.01</td>
<td>0.27</td>
<td>(-0.06)</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price markup</td>
<td>(-0.03)</td>
<td>0.18</td>
<td>0.05</td>
<td>0.13</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage markup</td>
<td>0.00</td>
<td>(-0.01)</td>
<td>(-0.07)</td>
<td>(-0.21)</td>
<td>(-0.09)</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Monetary policy</td>
<td>0.09</td>
<td>(-0.17)</td>
<td>(-0.05)</td>
<td>0.17</td>
<td>(-0.05)</td>
<td>(-0.04)</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Innovations</th>
<th>Innovations</th>
<th>Innovations</th>
<th>Innovations</th>
<th>Innovations</th>
<th>Innovations</th>
<th>Innovations</th>
<th>Innovations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baa-Aaa</td>
<td>(-0.10)</td>
<td>0.39</td>
<td>(-0.21)</td>
<td>0.28</td>
<td>0.04</td>
<td>(-0.02)</td>
<td>0.04</td>
</tr>
<tr>
<td>Term spread</td>
<td>0.11</td>
<td>0.33</td>
<td>(-0.11)</td>
<td>(-0.04)</td>
<td>(-0.07)</td>
<td>0.10</td>
<td>(-0.46)</td>
</tr>
<tr>
<td>Ted Spread</td>
<td>(-0.20)</td>
<td>(-0.13)</td>
<td>0.13</td>
<td>0.18</td>
<td>0.14</td>
<td>(-0.02)</td>
<td>0.34</td>
</tr>
<tr>
<td>return S&amp;P</td>
<td>0.14</td>
<td>(-0.24)</td>
<td>0.18</td>
<td>(-0.20)</td>
<td>(-0.13)</td>
<td>0.02</td>
<td>(-0.13)</td>
</tr>
<tr>
<td>return Fin</td>
<td>0.02</td>
<td>0.03</td>
<td>0.01</td>
<td>(-0.05)</td>
<td>(-0.14)</td>
<td>0.02</td>
<td>(-0.10)</td>
</tr>
<tr>
<td>return HP</td>
<td>(-0.07)</td>
<td>(-0.07)</td>
<td>0.25</td>
<td>(-0.06)</td>
<td>0.00</td>
<td>0.02</td>
<td>(-0.14)</td>
</tr>
<tr>
<td>VOX</td>
<td>(-0.12)</td>
<td>0.10</td>
<td>0.03</td>
<td>0.13</td>
<td>0.09</td>
<td>0.01</td>
<td>(-0.05)</td>
</tr>
</tbody>
</table>

Note: The data sources are provided in Appendix C.

It is also interesting to note that the two structural shocks that generate most of the extreme events are directly related to the intertemporal decisions and to the developments in the monetary and the financial sector of the economy. The non-Gaussian nature of financial returns, spreads and risk premiums is widely documented in the financial literature. Therefore, it appears like a natural hypothesis to assume that the non-Gaussian shocks that are identified in our macro model reflect the influence – or the feedback – from financial disruptions to the rest of the economy. To support this argument, we calculate the correlations between our estimated structural innovations and a set of popular financial returns and spreads. We selected seven measures related to the different segments of the financial sector and for which long time series are available: the Baa-Aaa spread, the term spread, the Ted spread, the return on the S&P index, the return on the Fama-French
financial sector portfolio, the change in the Shiller house price index and the VOX index. Table 4.3 summarizes the correlation between these seven financial indicators and our seven structural innovations. The strongest correlations in this table – exceeding 0.3 in absolute terms – are observed between our identified risk premium innovation and the Baa-Aaa and Term spreads, and between the monetary policy innovation and the Term and Ted spreads.

Table 4.4: Regression Analysis of Innovations and Financial Indicators.

<table>
<thead>
<tr>
<th>Innovations in</th>
<th>Pre-crisis sample</th>
<th>Full sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-crisis sample</td>
<td>Full sample</td>
</tr>
<tr>
<td></td>
<td>$\sigma_b$</td>
<td>$\sigma_i$</td>
</tr>
<tr>
<td>Contemporaneous impact from financial indicator on innovations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BAA-AAA</td>
<td>0.29*</td>
<td>-0.57*</td>
</tr>
<tr>
<td>Term spread</td>
<td>0.10*</td>
<td>-0.05</td>
</tr>
<tr>
<td>Ted Spread</td>
<td>-0.09*</td>
<td>0.16*</td>
</tr>
<tr>
<td>return S&amp;P</td>
<td>-0.64</td>
<td>1.51*</td>
</tr>
<tr>
<td>return Fin</td>
<td>0.46*</td>
<td>-0.24</td>
</tr>
<tr>
<td>return HP</td>
<td>1.35</td>
<td>5.41*</td>
</tr>
<tr>
<td>VOX</td>
<td>0.38</td>
<td>0.00</td>
</tr>
</tbody>
</table>

F / p-value

<table>
<thead>
<tr>
<th>F / p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.00/0.00</td>
</tr>
<tr>
<td>4.45/0.00</td>
</tr>
<tr>
<td>15.80/0.00</td>
</tr>
<tr>
<td>11.79/0.00</td>
</tr>
<tr>
<td>4.86/0.00</td>
</tr>
<tr>
<td>14.31/0.00</td>
</tr>
<tr>
<td>0.04/2.97</td>
</tr>
<tr>
<td>0.17/3.22</td>
</tr>
<tr>
<td>0.60/4.39</td>
</tr>
<tr>
<td>0.15/3.11</td>
</tr>
<tr>
<td>0.14/3.15</td>
</tr>
<tr>
<td>0.57/4.08</td>
</tr>
</tbody>
</table>

Skew/kurt resid

<table>
<thead>
<tr>
<th>F / p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.73/0.06</td>
</tr>
<tr>
<td>1.53/0.11</td>
</tr>
<tr>
<td>2.11/0.01</td>
</tr>
<tr>
<td>1.67/0.06</td>
</tr>
<tr>
<td>2.05/0.02</td>
</tr>
<tr>
<td>1.62/0.07</td>
</tr>
</tbody>
</table>

Granger Causality regressions

To see the strong linkages between some of the smoothed shocks and the financial variables in an alternative way, we regress the structural innovations on this set of financial observables. The results of these multivariate regressions are shown in Table 4.4. In contemporaneous regressions, the significant coefficients are again only apparent in the risk premium, monetary policy and – at a slightly weaker significance level – for the investment specific technology innovation. The most interesting feature of the regression results is that the remaining unexplained variation (i.e. the regression residuals) are basically normally distributed. Thus, shock outliers seem to coincide with periods of clear financial stress as measured by our observed financial indicators. Also noteworthy is that in Granger causality regression tests, none of the financial indicators carry significant predictive power for the structural innovations. Because financial variables can essentially be observed in real time however, they can still provide timely indications of big structural innovations. Including
these variables in our list of observables can therefore be very useful to improve the model now-
cast and the conditional forecast performance.\textsuperscript{20} Even so, this strategy will probably not improve
the out-of-sample prediction performance of our linearized models \textit{ex ante} to the observation of
financial stress signals. It might also require non-Gaussian and non-linear models to exploit this
information from financial variables more efficiently in our macro models.

5. Augmenting the benchmark model

As the analysis in Section 4 suggested that the benchmark model suffers from some important
shortcomings, we study in this section to which extent its performance can be improved by allowing
for zero lower bound on policy rates, time-varying volatility of the shocks, and by introducing
financial frictions and a cost-channel into the model. The modeling of financial frictions follows
the basic approach in the seminal work of Bernanke, Gertler and Gilchrist [20]. In contrast to the
analysis in Section 3.4, we estimate the different perturbations of the model on data including the
crisis period in this section.

5.1. Assessing the impact of the zero lower bound

We assess the impact of imposing the zero lower bound (ZLB) in the estimation in two alternative
ways. These procedures differ in the way the duration of the ZLB spells is determined. In our
first approach, the incidence and duration of the ZLB spells are endogenous and consistent with
the model expectations. In the second approach, we model them as “exogenous” and require
the model to match information from the market-based overnight index swap rates following Del
Negro, Giannoni, and Schorfheide [53]. In both approaches, we make use of the same linearized
model equations (stated in Appendix A), except that we impose the non-negativity constraint on
the federal funds rate. To do this, we adopt the following policy rule for the federal funds rate

\begin{equation}
\hat{R}_t^* = \rho_R \hat{R}_{t-1} + (1 - \rho_R)(r_{\pi} \hat{\pi}_t + r_y (\hat{ygap}_t) + r_{\Delta y} \Delta (\hat{ygap}_t)) , \\
\hat{R}_t = \max \left(-\bar{\tau}, \hat{R}_t^* + \hat{\varepsilon}_t^* \right) .
\end{equation}

The policy rule in (5.1) assumes that the interest rate set by the bank, \( \hat{R}_t \), equals \( \hat{R}_t^* + \hat{\varepsilon}_t^* \) if
unconstrained by the ZLB. \( \hat{R}_t \), in turn, is a shadow interest rate that is not subject to the policy
shock \( \hat{\varepsilon}_t^* \). Note that \( \hat{R}_t \) in the policy rule (5.1) is measured as percentage point deviation of the

\textsuperscript{20}See Del Negro and Schorfheide [55] for strong evidence in this direction.
federal funds rate from its quarterly steady state level \((\bar{r})\), so restricting \(\hat{R}_t\) not to fall below \(-\bar{r}\) is equivalent to imposing the ZLB on the nominal policy rate.\(^{21}\) In its setting of the shadow- or notional-rate we assume that the Fed is smoothing over the actual interest rate, as opposed to the notional rate \(\hat{R}_t^*\). We made this assumption to preserve the property that \(\varepsilon_t\) is close to white noise. Smoothing over the notional rate in (5.1) would cause the policy shock to become highly persistent, with an AR(1) coefficient roughly equal to \(\rho_R\).\(^{22}\)

To impose the policy rule (5.1) when we estimate the model, we use the method outlined in Hebden, Lindé and Svensson [91]. This method is convenient because it is quick even when the model contains many state variables, and we provide further details about the algorithm in Appendix A.\(^{23}\) In a nutshell, the algorithm imposes the non-linear policy rule in Equation (5.1) through current and anticipated shocks (add-factors) to the policy rule. More specifically, if the projection of \(\hat{R}_{t+h}\) in (5.1) given the filtered state in period \(t\) in any of the periods \(h = 0, 1, ..., T\) for some sufficiently large non-negative integer \(T\) is below \(-\bar{r}\), the algorithm adds a sequence of anticipated policy shocks \(\varepsilon_{t+h|t}\) such that \(E_t \hat{R}_{t+h} \geq 0\) for all \(h = \tau_1, \tau_1 + 1, ..., \tau_2\). If the added policy shocks put enough downward pressure on the economic activity and inflation, the duration of the ZLB spell will be extended both backwards (\(\tau_1\) shrinks) and forwards (\(\tau_2\) increases) in time. Moreover, as we think about the ZLB as a constraint on monetary policy, we further require all current and anticipated policy shocks to be positive whenever \(\hat{R}_t^* < -\bar{r}\). Imposing that all policy shocks are strictly positive whenever the ZLB binds, amounts to think about these shocks as Lagrangian multipliers on the non-negativity constraint on the interest rate, and implies that we should not necessarily be bothered by the fact that these shocks may not be normally distributed even when the ZLB binds for several consecutive periods \(t, t+1, ..., t+T\) with long expected spells each period (\(h\) large).

We will subsequently refer to this method as “Endogenous ZLB duration”, as it implies that both the incidence and the duration of the ZLB is endogenous determined by the model subject to the criterion to maximize the log marginal likelihood. In this context, it is important to understand that the non-negativity requirement on the current and anticipated policy shocks for each possible state and draw from the posterior, forces the posterior itself to move into a part of the parameter

\(^{21}\) See (3.16) for the definition of \(\hat{r}\). If writing the policy rule in levels, the first part of (5.1) bee replaced by (3.14) (omitting the policy shock), and the ZLB part would bee \(R_t = \max(1, R_t^* \varepsilon_t^*)\).

\(^{22}\) To see this, replace \(\hat{R}_{t-1}\) with \(\hat{R}_{t-1}^*\) in the first equation in (5.1) and then substitute \(\hat{R}_t = \hat{R}_t^* + \varepsilon_t^*\) from the second equation to write the unconstrained policy rule with the actual policy rate \(\hat{R}_t\). Then, the residual will be \(u_t^* = \varepsilon_t^* - \rho_R \varepsilon_{t-1}^*\). Hence, the residual \(u_t^*\) will be roughly white noise in this case when \(\varepsilon_t^*\) has an AR(1)-root \(\rho_R\).

\(^{23}\) Iacoviello and Guerrieri [94] have subsequently shown how this method can be applied to solve DSGE models with other types of asymmetry constraints.
space where the model can account for long ZLB spells which are contractionary to the economy. Without this requirement, DSGE models with endogenous lagged state variables may experience sign switches for the policy shocks, so that the ZLB has a stimulative rather than contractionary impact on the economy even for fairly short ZLB spells as documented by Carlstrom, Fuerst and Paustian [32]. As discussed in further detail in Hebden et al., the non-negativity assumption for all states and draws from the posterior also mitigates the possibility of multiple equilibria (indeterminacy). Finally, it is important to point our that when the ZLB is not a binding constraint, we assume the contemporaneous policy shock $\varepsilon_t^r$ in Equation (5.1) can be either negative or positive; in this case we do not use any anticipated policy shocks as monetary policy is unconstrained.

However, a potentially serious shortcoming of the method we adapt to assess the implications of the ZLB is that it relies on perfect foresight and hence does not explicitly account for the role of future shock uncertainty as in the work of Adam and Billi [1] and Gust, Lopez-Salido and Smith [89]. Even so, we implicitly allow for parameter and shock uncertainty by requiring that the filtered current and anticipated policy shocks in each time point are positive for all parameter and shock draws from the posterior whenever the ZLB binds. More specifically, when we evaluate the likelihood function and find that $E_t R_{t+h} < 0$ in the modal outlook for some period $t$ and horizon $h$ conditional on the parameter draw and associated filtered state, we draw a large number of sequences of fundamental shocks for $h = 0, 1, ..., 12$ and verify that the policy rule (5.1) can be implemented for all possible shock realizations through positive shocks only. For those parameter draws this is not feasible, we add a smooth penalty to the likelihood which is set large enough to ensure that the posterior will satisfy the constraint.

To provide a reference point for the ZLB estimations we start out by estimating the model for 2008Q4 implies that the ZLB would be a binding constraint in 2009Q1 through 2009Q3 in the modal outlook. For this period we generated 1,000 shock realizations for 2009Q1, 2009Q2, ..., 2011Q4 and verified that we could implement the policy rule (5.1) for all forecast simulations of the model through non-negative current and anticipated policy shocks. For the draws with adverse shocks, the duration of the ZLB was prolonged substantially during the forecast horizon, with expected ZLB spells close to 4 years occurring. We provide further details in Appendix B how the likelihood function is constructed when we impose the ZLB in the estimations.

Alternatively, we could implement this type of restriction by using a stochastic filter in which the prediction is calculated by integrating over a simulated forecast distribution. Parameter values that generate explosive paths and positive outliers typical for sign reversal realizations would be punished automatically in the likelihood evaluation.
the full sample period, but disregarding the existence of the ZLB. The posterior mode and standard deviation in this case are shown in the first two columns in Table 5.1, and labeled “No ZLB model”. The only difference between these results and those reported in Table 3.2 is that the sample period has been extended from 2007Q4 to 2014Q2. By comparing the results, a noteworthy difference is that the estimated degree of wage and price stickiness has increased even further relative to the pre-crisis sample. The posterior mode for the sticky wage parameter ($\xi_w$) has increased from 0.79 to 0.83, and the sticky price parameter ($\xi_p$) from 0.69 to 0.75. Relative to the SW07 posterior mode, $\xi_w$ has increased from 0.73 to 0.83 and $\xi_p$ from 0.65 to 0.75. These increases are substantial, considering that the sample has been expanded with less than 10 years and that these parameters affect the slope of the wage and price pricing curves in a non-linear fashion, implying an even sharper reduction in the slope coefficients for the forcing variables (wage markup and marginal costs, respectively) in the linearized price and wage equations. Evidently, the much higher degree of price and wage stickiness is only partly driven by the fact that prices and real wages fell modestly relative to output during the Great Recession (as can be seen in Figure 3.1); even before the recession materialized there was already a strong trend in the data towards higher stickiness parameters, consistent with the findings by Del Negro, Giannoni and Schorfheide [53].

Even so, we note that our estimated full sample model without the ZLB still features a much lower degree of price and wage stickiness than the policy model recently estimated by Brave et al. [25].

In Figure 5.1, we plot conditional forecast distributions for selected variables for the “No ZLB model” posterior in Table 5.1. In the left column, the forecast is conditional on the state in 2008Q3, whereas in the other two columns it is conditional on the filtered state in 2008Q4. Similarly to the results for the pre-crisis models in Figure 4.1, the results in the left column shows that the severe drop in economic activity in 2008Q4 was outside the 95th percent uncertainty bands, even though the model is estimated on the full sample. This thus should be considered as an in-sample exercise. However, the median forecast conditional on the state in 2008Q4 is very accurate for yearly output growth and output (as deviation from trend) and the actual outcome is well within the uncertain bands for these variables, even disregarding the ZLB. For the federal funds rate, we see that the median forecast for the federal funds rate falls only slightly below nil for three quarters

---

27 This finding implies that the lower slope does not seem to be related to aggregate volatility, consistent with the findings by Vavra [130].

28 As different models make alternative assumptions about strategic complements in price and wage setting, we have the reduced form coefficient for the wage and price markups in mind when comparing the degree of price and wage stickiness. In our benchmark model this coefficient equals 0.012 at the posterior mode for the New Keynesian Phillips curve which is similar to the estimate of Del Negro et al. [58] (0.016). The estimate of Brave et al. [25], however, the mode is as low as 0.002.
This seemingly suggests that the ZLB was not much of a binding constraint during the Great Recession, consistent with the finding and interpretation in Del Negro, Giannoni and Schorfheide [53]. This interpretation however ignores the fact that the forecast distribution for the federal funds rate has considerable mass below nil. Shifting this part of the distribution to 0 and above may therefore change the median outlook considerably.

Table 5.1: Posterior distributions in SW-Model: 1966Q1–2014Q2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No ZLB model</th>
<th>Endogenous ZLB duration</th>
<th>OIS-based ZLB duration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Posterior</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>mode</td>
<td>std.dev.</td>
<td>mode</td>
</tr>
<tr>
<td>Calvo prob. wages</td>
<td>0.83</td>
<td>0.040</td>
<td>0.85</td>
</tr>
<tr>
<td>Calvo prob. prices</td>
<td>0.75</td>
<td>0.039</td>
<td>0.83</td>
</tr>
<tr>
<td>Indexation wages</td>
<td>0.69</td>
<td>0.122</td>
<td>0.57</td>
</tr>
<tr>
<td>Indexation prices</td>
<td>0.22</td>
<td>0.081</td>
<td>0.25</td>
</tr>
<tr>
<td>Gross price markup</td>
<td>1.60</td>
<td>0.073</td>
<td>1.46</td>
</tr>
<tr>
<td>Capital production share</td>
<td>0.19</td>
<td>0.015</td>
<td>0.16</td>
</tr>
<tr>
<td>Capital utilization cost</td>
<td>0.80</td>
<td>0.075</td>
<td>0.73</td>
</tr>
<tr>
<td>Investment adj. cost</td>
<td>4.58</td>
<td>0.941</td>
<td>4.61</td>
</tr>
<tr>
<td>Habit formation</td>
<td>0.62</td>
<td>0.054</td>
<td>0.62</td>
</tr>
<tr>
<td>Inv subs. elast. of cons.</td>
<td>1.49</td>
<td>0.138</td>
<td>1.02</td>
</tr>
<tr>
<td>Labor supply elast.</td>
<td>1.81</td>
<td>0.555</td>
<td>2.03</td>
</tr>
<tr>
<td>Hours worked in S.S.</td>
<td>–0.40</td>
<td>1.178</td>
<td>–0.18</td>
</tr>
<tr>
<td>Discount factor</td>
<td>0.10</td>
<td>0.042</td>
<td>0.13</td>
</tr>
<tr>
<td>Quarterly Growth in S.S.</td>
<td>0.41</td>
<td>0.014</td>
<td>0.42</td>
</tr>
<tr>
<td>Stationary tech. shock</td>
<td>0.96</td>
<td>0.008</td>
<td>0.97</td>
</tr>
<tr>
<td>Risk premium shock</td>
<td>0.40</td>
<td>0.104</td>
<td>0.85</td>
</tr>
<tr>
<td>Invest. spec. tech. shock</td>
<td>0.84</td>
<td>0.039</td>
<td>0.85</td>
</tr>
<tr>
<td>Gov’t cons. shock</td>
<td>0.97</td>
<td>0.007</td>
<td>0.98</td>
</tr>
<tr>
<td>Price markup shock</td>
<td>0.92</td>
<td>0.030</td>
<td>0.88</td>
</tr>
<tr>
<td>Wage markup shock</td>
<td>0.97</td>
<td>0.010</td>
<td>0.98</td>
</tr>
<tr>
<td>Response of g_t to ε_t</td>
<td>0.51</td>
<td>0.077</td>
<td>0.52</td>
</tr>
<tr>
<td>Stationary tech. shock</td>
<td>0.46</td>
<td>0.025</td>
<td>0.48</td>
</tr>
<tr>
<td>Risk premium shock</td>
<td>0.19</td>
<td>0.026</td>
<td>0.10</td>
</tr>
<tr>
<td>Invest. spec. tech. shock</td>
<td>0.36</td>
<td>0.032</td>
<td>0.31</td>
</tr>
<tr>
<td>Gov’t cons. shock</td>
<td>0.49</td>
<td>0.025</td>
<td>0.48</td>
</tr>
<tr>
<td>Price markup shock</td>
<td>0.12</td>
<td>0.013</td>
<td>0.13</td>
</tr>
<tr>
<td>MA(1) price markup shock</td>
<td>0.80</td>
<td>0.058</td>
<td>0.79</td>
</tr>
<tr>
<td>Wage markup shock</td>
<td>0.37</td>
<td>0.022</td>
<td>0.36</td>
</tr>
<tr>
<td>MA(1) wage markup shock</td>
<td>0.96</td>
<td>0.013</td>
<td>0.96</td>
</tr>
<tr>
<td>Quarterly infl. rate. in S.S.</td>
<td>0.81</td>
<td>0.102</td>
<td>0.76</td>
</tr>
<tr>
<td>Inflation response</td>
<td>1.69</td>
<td>0.153</td>
<td>1.86</td>
</tr>
<tr>
<td>Output gap response</td>
<td>0.05</td>
<td>0.016</td>
<td>0.10</td>
</tr>
<tr>
<td>Diff. output gap response</td>
<td>0.24</td>
<td>0.027</td>
<td>0.24</td>
</tr>
<tr>
<td>Mon. pol. shock std</td>
<td>0.23</td>
<td>0.013</td>
<td>0.22</td>
</tr>
<tr>
<td>Mon. pol. shock pers.</td>
<td>0.21</td>
<td>0.070</td>
<td>0.10</td>
</tr>
<tr>
<td>Interest rate smoothing</td>
<td>0.80</td>
<td>0.028</td>
<td>0.83</td>
</tr>
<tr>
<td>Log marginal likelihood</td>
<td>–1146.69</td>
<td></td>
<td>–1151.99</td>
</tr>
</tbody>
</table>

Note: See notes to Table 3.2. The “No ZLB model” neglects the presence of the zero lower bound in the estimations, whereas the “Endogenous ZLB duration” allows the duration of the ZLB to be endogenous as described in the main text. Finally, the “OIS-based ZLB duration” imposes the duration of the ZLB in each point in time according to OIS rates for the federal funds rate between 2008Q4 and 2011Q2.

To examine this possibility, the third column in Figure 5.1 reports the forecast distribution when sampling parameters and shocks from the posterior distribution for the “No ZLB model” in Table 5.1, but with the unconstrained policy rule replaced by the policy rule in (5.1). This means
that the actual and expected federal funds rate will respect the ZLB during the forecast horizon. Importantly, the 1,000 different shock realizations used to construct the forecast distribution in the ZLB case are identical to those used to construct the unconstrained forecast distribution. Given the state in 2008Q4 the only difference between the results in the second and third column is that the federal funds rate is constrained from falling below zero. As can be seen from the panels for output growth and output as deviation from trend, imposing the ZLB on the federal funds rate widens their uncertainty bands downwards quite notably. For output as deviation from trend, the lower 95th percentile shifts down from roughly −10 percent to nearly −20 percent in 2010. Hence, in the absence of unconventional monetary policies and coordination between monetary and fiscal policy (i.e. fiscal stimulus when the economy enters a long-lived liquidity trap), the baseline model suggests that the ZLB may be associated with large economic costs.

On the other hand, the upper-95th percent bands for these variables are also much higher when the federal funds rate is constrained to fall below zero conditional on the given state in 2008Q4. For detrended output, the upper 95th percentile is above 10 percent in 2009. For yearly inflation, the upper 95th percentile is above 6 percent. Despite these elevated upper uncertainty bands for output growth, detrended output and inflation, the upper 95th percentile for the federal funds rate is lower than the corresponding percentile in the unconstrained policy rate distribution. This seemingly goes against the specification of the policy rule in (5.1) as the systematic part of the policy rule governing \( \hat{R}_t \) calls for a high policy rate whenever inflation, output growth and the output gap is high. The reason why this does not happen in the conditional ZLB distribution in Figure 5.1 is that the model estimated without the imposed ZLB constraint needs large negative current and anticipated policy shocks \( \hat{\varepsilon}_{t+h|t} \) to satisfy \( E_t \hat{R}_{t+h} \geq 0 \). In essence, when the economy is hit by some really adverse shocks in these simulations and the policy rate is constrained to respond to these shocks for a sufficiently long period, inflation expectations and economic activity fall to such a large extent that a sequence of negative instead of positive policy shocks \( \hat{\varepsilon}_{t+h|t} \) for \( h = 0, 1, \ldots, \tau_2 \) are needed to prevent the federal funds rate to fall below nil. As discussed in Hebden et al. [91] and Carlstrom et al. [32], the switch in signs of the policy shocks only happens in the relatively few draws for which the policy rate is expected to be constrained by the lower bound for a very prolonged period of time (i.e. \( \tau_2 \) is large). This also explains why the upper 95th percentiles for inflation and output shifts up so much while the 90th percentile is roughly unchanged relative to the unconstrained distribution. The 90th percentile is associated with simulations of favorable fundamental shocks and parameter draws for which no large negative policy shocks are needed to
prevent the policy rate to fall below nil.

We believe this result – that the ZLB can trigger adverse shocks to have sharply expansionary effects on the economy – is an unpalatable feature of the model. Therefore when we re-estimate the model subject to the ZLB constraint on the federal funds rate, we believe it is crucial to impose the additional constraint – discussed in the beginning of this section – that the parameters of the economy have to be such that all current and expected policy shocks used to impose the policy rule in (5.1) are positive whenever the ZLB binds. By imposing this constraint, we ensure that the re-estimated model does not feature any sign reversals of the policy shocks even for the most long-lived liquidity traps in our forecast distributions.

The estimation results for this variant of the model are reported in Table 5.1 and labeled
“Endogenous ZLB duration”. We use this label because both the incidence and duration of the ZLB spells are endogenous estimation outcomes in the model, and do not necessarily conform with other commonly used measures of the expected future path of the federal funds rate such as overnight index swap (OIS, henceforth) rates. By comparing the results with the “No ZLB model”, we see that imposing the ZLB in the estimations have quite important implications for the posterior distribution. First of all the degree of price and wage stickiness is elevated even further, and the estimated parameters imply a slope of the New Keynesian Phillips curve of 0.006. This is somewhat lower than the median estimates of literature which cluster in the range of about 0.009–0.014, but well within standard confidence intervals provided by empirical studies (see e.g. Adolfson et al. [5], Altig et al. [12], Galí and Gertler [78], Galí, Gertler, and López-Salido [79] and Lindé [109]). In addition, the higher degree of nominal wage stickiness makes marginal costs even more sticky in the ZLB model. Together these features makes inflation and inflation expectations more slow to react to various shocks and therefore allow the model to cope with long spells at the ZLB without triggering indeterminacy problems (i.e. switches in signs for the policy shocks). This finding is consistent with Erceg and Lindé [67], who argue that a low slope of the Phillips curve is consistent with the development during the recent crisis where inflation and inflation expectations have fallen very moderately despite large contractions in output. It is also consistent with many recent papers which have estimated similar DSGE models, see e.g. Brave et al. [25] and Del Negro, Giannoni and Schorfheide [53].

In addition to the higher degree of wage and price stickiness, there are two other important differences. Firstly, the coefficient on the output gap the in policy rule (eq. 5.1), \( r_y \), is about twice as high as in the “No ZLB model”. To the extent the output gap becomes significantly negative during the Great Recession, this will tend to push down the path of the federal funds rate and extend the duration of the ZLB. Secondly, the persistence coefficient in the risk premium shock process, \( \rho_b \), increases sharply from 0.40 to 0.85. However, since the posterior mode for \( \sigma_b \) is reduced from 0.19 to 0.10, the unconditional variance for the risk-premium shock nevertheless falls slightly (from 0.044 to 0.039) in the ZLB model. Therefore the higher persistence does not imply a significantly larger role for the risk-premium shocks (apart from expectational effects). Even so, the likelihood prefers naturally more persistence in the shock process of the risk premium above a repeated set of positive innovations to explain the duration of the crisis and the slow pace of the recovery, but this shift in the posterior distribution of the parameters goes with a cost during the tranquil periods. This time variation in the role of the financial wedge over periods with more or
less financial stress will be further discussed in section 5.3.

Figure 5.2 shows the forecast distribution (given the state in 2008Q4) in the “Endogenous ZLB duration” variant of the model. The left column gives the results when the ZLB is counterfactually neglected, whereas the right column shows the results when the ZLB is imposed. As expected, we see that the forecast distribution in the variant of the model which counterfactually neglects the ZLB features symmetric uncertainty bands around the modal outlook, and is a little bit too optimistic about the outlook for output relative to the model which imposes the ZLB (right column). More surprisingly is that the modal outlook for 2008Q4 in the model estimated and imposing ZLB constraint (right column in Figure 5.2) differs very little to the modal outlook in the “No ZLB model” which completely neglects the ZLB (the middle column in Figure 5.1). Obviously, a key
difference is that the median path of the federal funds rate is constrained by the lower bound in 2009, but below nil in the unconstrained version of the model. Still, the quantitative difference for the median projection is small. The most noticeable difference between the No ZLB model and the model estimated under the ZLB is the uncertainty bands: they are wider and downward skewed in the model that imposes the ZLB constraint (the right column of Figure 5.2) compared to the No ZLB model that neglects the presence of the ZLB constraint.

However, the forecast distributions in the “No ZLB model” (the right column in Figure 5.1) – which enforces the ZLB *ex post* – differs dramatically to the forecast distributions in the model estimated under the ZLB constraint (the right column in Figure 5.2). The higher degree of wage and price stickiness in the model estimated under the ZLB constraint insulate the economy from the disaster scenarios and the indeterminate equilibria, and therefore shrink the uncertainty bands considerably. Overall this suggests that taking the ZLB into account in the estimation stage may be of key importance in assessing its economic consequences, and that it is not evident that models estimated on pre-crisis data can be useful for policy analysis when the economy enters into a long-lived liquidity trap. In such situations, the pre-crisis policy models may feature too much flexibility in price and wage setting, and e.g. yield implausibly large fiscal multipliers as noted by e.g. Erceg and Lindé [67].

Another interesting feature of the model which neglects the ZLB and the variant of the model which is constrained to impose (5.1) through positive current and anticipated policy shocks is that the former has a higher log-marginal likelihood (−1146.7 vs. −1152). This implies that imposing the ZLB on the model is somewhat costly in terms of data coherence. However, as suggested by the small differences in the conditional forecast distributions in Figures 5.1 (middle column) and 5.2 (right column), it is not evident if this difference in log marginal likelihood is important from an economic viewpoint, although it is large enough to be sizable in terms of a Bayesian posterior odds ratio.

As the model is endogenously determining the incidence and duration of the ZLB spell, it is interesting to note that according to the model, the ZLB is expected in 2008Q4 to be a binding constraint from 2009Q1 through 2009Q3 in the modal outlook. The expected positive policy shocks we use to impose the ZLB partially substitute for the exceptionally huge risk premium shocks that drive the economy to the ZLB in the first place.\(^{29}\) The constraint is then expected to be binding

\(^{29}\) This high substitutability between anticipated monetary shocks that capture the effect of the ZLB on the one hand and the risk premium shock on the other hand, implies that it is very difficult to quantify accurately the precise impact of the ZLB on growth during the crisis. For instance, when a lagged shadow rate is used in the monetary policy rule instead of the lagged actual rate, the anticipated monetary policy shocks needed to impose the ZLB becomes
during 2009 with a maximum duration of five quarters given the state in 2009Q1, and from 2010Q2 and onward, the model expects the interest rate to lift off already in the next quarter. The short duration of the ZLB spells is consistent with the findings of Chung, Laforte, and Reifschneider [46]. The fact that the federal funds rate has remained at the ZLB since then is by the model explained either as a result of expansionary monetary policy actions – forward guidance – or as standard policy reactions to unexpected headwinds. The filtered shocks suggest a dominant role for the second interpretation.

As noted previously, an alternative to letting the DSGE model determine the expected duration of the ZLB in each time period is to use OIS data for the federal funds rate as observables when estimating the model. By doing so, we follow Del Negro, Giannoni, and Schorfheide [53] and require that the expected federal funds rate in the model matches the OIS data in each point in time when the ZLB is binding, i.e. from 2008Q4 and onward. We use OIS data (acquired from the Federal Reserve Board) for 1, 2, ..., 12 quarters’ expected federal funds rates, and require the model to match those rates exactly through anticipated policy shocks following the general idea outlined by Maih [113]. The appealing feature of Maih’s algorithm is that it does not require us to include standard deviations for each of the anticipated policy shocks we use to fit the OIS data, and that the log-marginal likelihood can be compared to the models which does not condition on OIS data.

Before we turn to the results in Table 5.1, there are two additional important pieces of information. First, as we interpret the OIS data as expected means of future federal funds rates, we set them equal to nil in each point in time whenever they are lower than 50 basis points. We do this as our OIS estimation procedure does not explicitly account for future shock uncertainty, and the projected path of the interest rate from the model should therefore be viewed as a modal outlook (which will be lower than the mean of the forecast distribution when the ZLB binds). Second, because the Federal Reserve did not use explicit time-dependent forward guidance until August 2011, we restrict all anticipated policy shocks to be positive prior to this date. After this date we do not impose any signs on the anticipated policy shocks, because credible forward guidance – or a “lower for longer policy” – in the spirit of Williams and Reifschneider [118] and Eggertsson and Woodford [65], which extends the duration of the ZLB, is better viewed as expansionary than contractionary policy. Specifically, we allow the model to explain the sharp flattening of the OIS curve between the second and third quarter in 2011 with negative policy shocks, and do not impose this flattening to be associated with a noticeable deterioration in the economic outlook. According...
to the data, however, the magnitude of these expansionary “forward guidance” shocks are modest: interpreting the long ZLB spells as a deliberate “lower for longer” decision by the policy makers would further boost the predicted recovery by the model which goes against the observed slow and disappointing recovery in growth following the crisis.\(^{30}\)

The results when imposing the incidence and duration of the ZLB to adhere with OIS rates are shown in the left panel in Table 5.1, labeled “OIS-based ZLB duration”. Relative to the posterior “Endogenous ZLB duration”, or which the incidence and duration of the ZLB is determined endogenously in the model, we see that the degree of price stickiness is elevated further (from 0.83 to 0.89), and now implies a slope of the Phillips curve (i.e. direct sensitivity of current inflation to marginal cost) of 0.003. This is substantially lower than e.g. the estimate in Altig et al. [12], but still higher than Brave et al. [25]. To square this estimate with the micro-literature is a challenge, and probably requires a combination of firm-specific capital (as in Altig et al.), firm-specific labor (as in Woodford [137]), and a higher sensitivity of demand to relative prices (i.e. higher Kimball parameter \(\varepsilon_p\)). Apart from the higher stickiness we also see an elevated role for risk–premium shock in this model (\(\rho_0\) rises sharply from 0.85 to 0.97, whereas the std of the innovations only falls moderately from 0.10 to 0.08), and that the degree of habit formation consumption (\(\varkappa\)) and investment adjustment costs (\(\varphi\)) rises somewhat. Finally, the response coefficient for the output gap in the policy rule is increased further, and is now three times higher than in the model which neglects the presence of the ZLB.

The reason why these parameters are further changed relative to the “No ZLB model” is that the OIS data generally imposes longer-lived ZLB episodes than the model endogenously produces. In order to be able to explain those episodes with positive anticipated policy shocks through 2011Q2 the model needs to make dynamics more sluggish and explain the rebound in inflation during 2010 with temporary shocks. However, enforcing this sluggish dynamics on the model is rather costly in terms of log-marginal likelihood, which falls from \(-1152\) in the model with endogenous ZLB duration to \(-1175.2\) for the OIS-based ZLB duration. This is a sizable drop and a possible interpretation is that the SW-model, despite imposing the ZLB constraint, was more optimistic about the recovery than market participants during this episode.

\(^{30}\) There is a growing literature on the effectiveness of forward guidance. While Andrade et al. [3] argue mainly on theoretical grounds that forward guidance may not be effective when agents have heterogeneous beliefs, Campbell et al. [30], Williams [136] and Del Negro, Giannoni and Patterson [52] argue on empirical grounds that forward guidance have had some positive impact. Even so, Del Negro, Giannoni and Patterson recognize that forward guidance may be too potent in a standard New Keynesian model relative to what the empirical evidence supports, and therefore integrate perpetual youth structure into the model to reduce its effectiveness. By and large, our estimated model produces results that are in line with their findings and suggests that forward guidance have had some, but limited, impact on the economy.
There are of course other possibilities as to why the ZLB episodes in the model are short-lived relative to what OIS data suggest. They include that; (i) the model miss-measures the size and persistence of the relevant output gap, (ii) the model-consistent or rational expectation hypothesis fails to capture the stickiness and persistence in expectations that might be caused by learning dynamics or information filtering issues, (iii) the steady state natural real rate has fallen (e.g. due to lower trend growth) and this has caused the (gross) steady state nominal interest rate $R$ in Equation (3.14) to fall; \textit{ceteris paribus} this calls for an extended ZLB duration, and (iv), the Federal Reserve decided to respond more vigorously to the negative output gap (i.e. $r_y$ in Equation (3.14) increased) from the outset of the Great Recession and thereafter.\footnote{To the extent that these mechanisms are at work, they should be picked up in our estimated model as expansionary monetary policy shocks due to the presumption in our analysis that the Fed before and after the crisis (i.e. upon exit from the ZLB) adheres to the same Taylor-type policy rule (Equation 5.1), and that agents form their expectations accordingly.} Yet other possibilities is that our model above misses out on time-varying volatility of the shocks and omits financial frictions and the cost-channel of monetary policy. We explore these latter possibilities below.

5.2. Allowing for Time-Varying Volatility

As documented above, the prototype linear Gaussian model with constant volatility does not provide a realistic predictive density for the forecast, in particular around severe recession periods or periods of high financial and monetary stress. A large share of the research effort on DSGE models since the financial crisis and the Great recession has tried to overcome these weaknesses of the basic DSGE setup. By now, most models used in academia and in policy institutions contain financial frictions and financial shocks in an effort to introduce stronger amplification mechanisms in the model. As we will discuss in the next section however, to the extent that even the modified models adopt a Gaussian linear framework, they still depend on extremely large shocks to predict important recessions. The explicit modeling of the non-linear macro-finance interactions is complex and ambitious and the research in that direction has not yet been integrated in empirical macro models. A technically feasible avenue to improve the predictive densities of the linear DSGE model is to allow for a more complicated stochastic structure. Here we illustrate this approach by considering a Markov Switching (MS) stochastic structure following Liu, Waggoner and Zha [111].\footnote{We use the RISE toolbox to implement this exercise, see Maih [114].} By allowing for such a shock structure, the hope is that the estimated model can capture the phenomena that the economic outlook is sometimes very uncertain (i.e. the economy is filtered to be in the high volatility regime), without necessarily destroying its ability to provide reasonably narrow forecast
uncertainty bands in normal times (i.e. in the low volatility regime).

Low frequency changes in the shock variances have been analyzed by Fernandez-Villaverde and Rubio-Ramirez [71] and Justiniano and Primiceri [97] via stochastic volatility processes. Chib and Ramamurthy [35] and Curdia, Del Negro and Greenwald [50] show that a Student’s t-distribution for the innovations is also strongly favored by the data as it allows for rare large shocks. The latter authors makes the point that the time-variation in shock variances should contain both a low and a high frequency component.

To capture these insights, we consider a version of the benchmark model in which we allow for two independent Markov Switching processes in the shock variances. Each Markov process can switch between a low and a high volatility regime. One process affects the volatility of all the structural innovations with exception of the wage markup shock, based on the observation that the wage markup and the observed real wage variable has a completely different volatility profile compared to the other shocks and variables as shown in Figures 3.1 and 4.4. The second Markov process is restricted to the non-Gaussian structural shocks as identified in Table 4.4 in Section 4.3: this process affects the volatility in the monetary policy, the risk premium and the investment specific innovations. The volatility in these three shocks is scaled by both the common ($\sigma_c$) and the monetary/financial volatility factor ($\sigma_{mf}$). The typical process for these three shocks is now written as follows:

$$\tilde{\epsilon}_t = \rho \tilde{\epsilon}_{t-1} + \sigma_{mf}(s_{mf}) \cdot \sigma_c(s_c) \cdot \sigma \cdot \eta_t, \eta_t \sim N(0, 1).$$

The estimated transition probabilities are summarized by the following matrices:

$$Q_c \begin{pmatrix} \text{low} \\ \text{high} \end{pmatrix} = \begin{bmatrix} 0.95 & 0.07 \\ 0.05 & 0.93 \end{bmatrix} \quad Q_{mf} \begin{pmatrix} \text{low} \\ \text{high} \end{pmatrix} = \begin{bmatrix} 0.92 & 0.46 \\ 0.08 & 0.54 \end{bmatrix}.$$

The relative volatility of the two regimes are estimated as:

$$\sigma_c \begin{pmatrix} \text{low} \\ \text{high} \end{pmatrix} = \begin{bmatrix} 1 \\ 1.74 \end{bmatrix} \quad \text{and} \quad \sigma_{mf} \begin{pmatrix} \text{low} \\ \text{high} \end{pmatrix} = \begin{bmatrix} 1 \\ 2.33 \end{bmatrix}.$$

In figure 5.3, we plot the smoothed regime probabilities for the model estimated over the complete sample. A filtered probability near unity (zero) implies that the economy is filtered to be in the high (low) volatility regime.

The common volatility process captures the great moderation phenomena. The high volatility regime is typically preferred during most of the seventies and the first half of the eighties, while the low volatility regime is active during the great moderation and is interrupted by the financial
crisis and the resulting Great Recession. Both regimes are estimated to be persistent and the relative volatility during the high volatility regime is almost twice as high as in the low volatility regime. The monetary/financial volatility process captures the increase in the volatility during most of the recession periods and in the late seventies-early eighties-episode of increased monetary policy uncertainty. The expected duration of this high volatility/financial stress regime is relatively short-lived with a quarterly transition probability of 0.46 percent. The estimated parameters that describe the regimes and the regime probabilities are very stable when estimating the model for the pre-crisis period or for the complete sample.

Table 5.2: Log marginal likelihood of alternative regime switching specifications.

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>Pre-Crisis: 66Q1–07Q4</th>
<th>Full sample: 66Q1–14Q2</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Regime Switching (RS)</td>
<td>−961.8</td>
<td>−1146.7</td>
</tr>
<tr>
<td>RS in common process</td>
<td>−894.6</td>
<td>−1060.9</td>
</tr>
<tr>
<td>RS in mon/fin process</td>
<td>−911.8</td>
<td>−1082.1</td>
</tr>
<tr>
<td>RS in common and mon/fin process</td>
<td>−881.7</td>
<td>−1046.0</td>
</tr>
</tbody>
</table>

Note: None of the models in this table are estimated subject to the ZLB on policy rates.

Table 5.2 shows that the estimated log marginal likelihood of our model with switching volatility outperforms the log marginal likelihood of the homoscedastic Gaussian models by far. In this sense, our results confirm the results in the literature based stochastic volatility or t-distributed shocks. In contrast with Liu et al. [111] and in support of the results of Curdia et al. [50], we find strong
evidence in favor of a setup that allows for multiple sources of volatility changes. The time-varying volatility structure requires sufficient flexibility to account for a common low frequency trend on the one hand, and a more cyclical high frequency process that controls mainly the monetary and financial shocks on the other hand.\textsuperscript{33}

Accounting for the non-Gaussian stochastic structure drastically improves the log marginal likelihood of our models, but leaves the estimated parameters, i.e. the central forecasts and identified innovations, relatively unaffected. Most of the gains are realized because the predictive densities attribute appropriate probabilities to the extreme tail events: the large downturns in recessions and the corresponding sharp responses in policy rates. To illustrate this property, we consider the predictive forecast distribution with the pre-crisis model conditional on data up to 2008Q3, and we calculated the percentile interval that contains the 2008Q4 realized output growth observation (see Figure 5.4). For our baseline pre-crisis model, the realized 2008Q4 growth rate falls completely outside of the simulated predictive densities based on 10,000 draws with parameter and shock volatility, as is clear from the left panel in the figure (see also Figure 4.1). In contrast, in the model with Markov Switching volatility, almost 1 percent of the simulated forecasts fall below the 2008Q4 realization, as shown in the figure’s right panel.\textsuperscript{34} The Markov Switching volatility structure, by allowing for a mixture of normal distributions, gives more probability to the tails in general. In addition, the probability of the high volatility regimes in both the high and the low frequency Markov processes increased already by 2008Q3 because the magnitude of the realized shocks preceding the fourth quarter observation were relatively large.

5.3. Augmenting the Model with Financial Frictions and a Cost-Channel

We incorporate a financial accelerator mechanism into the benchmark model in Section 3 following the basic approach of Bernanke, Gertler and Gilchrist \cite{20}. Thus, the intermediate goods producers rent capital services from entrepreneurs rather than directly from households. Entrepreneurs purchase physical capital from competitive capital goods producers (at price $\hat{Q}_t^k$, and resell it back

\textsuperscript{33} Our restrictive setup of two processes improve the log marginal likelihood by 115 for the complete sample. More flexible structures could easily improve this result but this goes with a cost because these setups are less robust, are computational much more intensive and lack an intuitive interpretation of the regimes. Curdia et al. report a gain of 154 in the log marginal likelihood for a setup that contains a combination of shock specific stochastic volatility and t-distributed innovations.

\textsuperscript{34} We want to emphasize that it is not the case that we are more content with this model just because it gives a positive probability that the great recession could indeed happen. As pointed out earlier, our rationale for going in this direction is that the models with regime-switching in shock variances and the propagation of financial frictions (see analysis in the next section) improves the statistical properties of the model (as suggested by the strong improvement in log marginal likelihood) and makes sense from an economic viewpoint (supporting the widely held belief that financial frictions are key to understand the crisis).
at the end of each period), with the latter employing the same technology to transform investment goods into finished capital goods as described by equation 3.11). To finance the acquisition of physical capital ($\hat{k}_t$), each entrepreneur combines his net worth ($\hat{NW}_t^c$) with a loan from a bank, for which the entrepreneur must pay an external finance premium due to an agency problem. We follow Christiano, Motto and Rostagno [42] by assuming that the debt contract between entrepreneurs and banks is written in nominal terms (rather than real terms as in Bernanke, Gertler and Gilchrist [20]). Banks, in turn, obtain funds to lend to the entrepreneurs by receiving deposits from households, with households bearing no credit risk (reflecting assumptions about free competition in banking and the ability of banks to diversify their portfolios). In equilibrium, shocks that affect entrepreneurial net worth – i.e., the leverage of the corporate sector – induce fluctuations in the corporate finance premium.\footnote{For further details about the setup, see Bernanke, Gertler and Gilchrist, and Christiano, Motto and Rostagno. Excellent expositions are also provided by Christiano, Trabandt and Walentin [44] and Gilchrist, Ortiz and Zakrajsek [85].}

When estimating the model with the financial friction mechanism embedded, we add one more observable variable, the widely-used Baa-Aaa corporate credit spread (see Appendix C for exact definition and data sources). This spread plays a key role in the Bernanke-Gertler-Gilchrist framework. Since we also want to learn about the importance of shocks originating in the financial sector, and because we need as many shocks as observables to avoid stochastic singularity, we also add a “net worth” shock to the set of estimated shocks. We derive this shock by allowing the survival probability of the entrepreneurs to vary over time. Hence, this shock will enter in the accumulation equation for the entrepreneurs net worth. An alternative would have been to allow for a shock
directly in the equation which relates the spread (or equivalently, the external finance premium) to the entrepreneurs leverage ratio following e.g. Del Negro and Schorfheide [55] or Christiano, Motto and Rostagno [42]. We preferred, however, not to add a shock directly in the spread equation in an attempt to elevate the endogenous propagation of the financial accelerator mechanism.\footnote{Christiano, Motto and Rostagno [42] embed a complete banking sector into their model and estimate it using 17 time-series and an equal number of shocks. A benefit, however, of our more modest perturbation of the model size and number of observables matched is that it allows for a straightforward comparison with the findings in the benchmark SW-model.} Even so, the equation for the external finance premium,

\[ E_t \dot{R}_{t+1}^e - \dot{R}_t^b = \chi \left( \ddot{Q}_t^k + \ddot{k}_t - \ddot{N}W_t^e \right), \tag{5.2} \]

still contains a shock because we assume that the financing rate of the banks, \( \dot{R}_t^b \), is not the risk-free rate set by the central bank, but rather the sum of the policy rate \( \dot{R}_t \) and the risk-premium shock \( \varepsilon_t^b \).

As recent research by Christiano, Eichenbaum and Trabandt [37] and Gilchrist et al. [87] emphasize the importance of firms financing conditions for their price setting behavior, we also embed a cost channel into the model. Specifically, we assume that firms have to borrow short to finance their wage bill following CEE [36]. As shown in the CEE paper, the working capital channel can cause inflation to rise following a tightening of monetary policy if firms financing costs rise sufficiently. To allow for sharp increases in firms financing costs, we assume that the relevant financing rate is the expected nominal return on capital for the entrepreneurs as opposed to the risk-free policy rate. However, instead of imposing that all firms borrow to finance their entire wage bills as in CEE, we estimate a parameter, \( \nu \), which determines the share of firms that are subject to working capital, so that the expression for log-linearized marginal costs becomes

\[ \tilde{m}c_t = (1 - \alpha) \left( \tilde{w}_t + \tilde{R}_t^f \right) + \alpha \tilde{r}_t^k - \tilde{\varepsilon}_t^a, \]

where \( \tilde{R}_t^f \) is the effective working capital interest rate given by

\[ \tilde{R}_t^f = \frac{\nu R}{\nu R + 1 - \nu} E_t \dot{R}_{t+1}^e, \tag{5.3} \]

in which \( E_t \dot{R}_{t+1}^e \) is the nominal expected return on capital for the entrepreneurs. From Equation (5.3), we notice that \( \tilde{R}_t^f = E_t \dot{R}_{t+1}^e \) when \( \nu = 1 \).

The SW-model embedded with the financial friction mechanism and the cost-channel thus include five additional estimated parameters; \( \nu \), the two parameters for the AR(1) process for net worth (\( \rho_{nw} \) and \( \sigma_{nw} \)), the monitoring cost parameter \( \mu \) which indirectly determines the sensitivity of
the external finance premium to the entrepreneurs leverage ratio ($\chi$ in Equation 5.2), and a constant ($\bar{c}_{sp}$) which captures the mean of the credit spread. Estimation results for three specifications of the model are provided in Table 5.3; first we have the “Pre-crisis sample” (sample 1966Q1–2007Q4 without the ZLB), second, the full sample (66Q1–14Q2) when imposing the ZLB constraint with endogenous duration, and third we study a variant of the model with the ZLB which allows the key parameter $\mu$ to switch stochastically between a high and low value. The adopted priors for the five new parameters are provided in the notes to the table. The priors for the other parameters are the same as before (and already stated in Table 3.2).

In the pre-crisis model, the external finance premium delivers only a very modest amplification of the standard shocks. The estimated elasticity of the spread to the net worth ratio is small (with $\mu = 0.033$ and $\chi$ in Equation 5.2 equals 0.012, implying an annualized spread sensitivity of 0.048), a result that is in line with the estimates reported in Gilchrist, Ortiz, and Zakrajsek [85]. The exogenous risk-premium shock and – to a lower degree – the monetary policy shock are most impacted by the introduction of the FA mechanism because they have the biggest impact on the price of capital and net worth. The net worth channel tends to support the persistence in the response of investment to these shocks. The low sensitivity of the spread to the traditional shocks also implies that most of the fluctuations in the external finance premium are generated by the new exogenous shock that is assumed to hit directly the net worth of the entrepreneurs. This highly volatile shock explains up to 70 percent of the variance in the spread and one-third of the variance in investment. As such, the net worth shock substitutes for the exogenous risk premium and for the investment-specific technology shock. The latter also captures financial frictions as suggested by Justiniano, Primiceri and Tambalotti [99]. Overall, the impact of the net worth shock on the macro-dynamics remains modest and one important reason for this is that the net worth shock typically crowds out private consumption and this clashes with the observed strong co-movement between consumption, and investment over the business cycle.\textsuperscript{37}

The direct comparison of the marginal likelihood with the baseline model is complicated because the financial frictions model (FF model henceforth) has an additional observable in the form of the Baa-Aaa spread. When we estimate the FF-model without this additional observable, the log marginal likelihood improves by a factor of 10 when no additional shock is considered and by a factor of 20 when the net worth shock is retained. With a posterior mode for $\mu = 0.2$ in this variant

\textsuperscript{37}This crowding out problem is not present for our reduced-form risk-premium shock $\varepsilon_{t}^{b}$ in Equation (3.12), see Fisher [74] for a structural interpretation of this risk-premium shock.
of the model, the estimated sensitivity of the spread to the net worth ratio in this model is much higher, i.e. 0.08, or 0.32 in annualized terms. This result is more supportive for an important tendogenous amplification effect of the standard shocks through the net worth channel (see also

Table 5.3: Posterior distributions in SW Model with Financial Frictions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pre-crisis sample mean</th>
<th>Endogenous ZLB duration mean</th>
<th>Endog. ZLB dur. with regime switch mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calvo prob. wages</td>
<td>$\xi_w$</td>
<td>0.72</td>
<td>0.83</td>
</tr>
<tr>
<td>Calvo prob. prices</td>
<td>$\xi_p$</td>
<td>0.68</td>
<td>0.84</td>
</tr>
<tr>
<td>Indexation wages</td>
<td>$\tau_w$</td>
<td>0.67</td>
<td>0.63</td>
</tr>
<tr>
<td>Indexation prices</td>
<td>$\tau_p$</td>
<td>0.21</td>
<td>0.23</td>
</tr>
<tr>
<td>Gross price markup</td>
<td>$\phi_p$</td>
<td>1.61</td>
<td>1.45</td>
</tr>
<tr>
<td>Capital production share</td>
<td>$\alpha$</td>
<td>0.21</td>
<td>0.17</td>
</tr>
<tr>
<td>Capital utilization cost</td>
<td>$\psi$</td>
<td>0.44</td>
<td>0.50</td>
</tr>
<tr>
<td>Investment adj. cost</td>
<td>$\varphi$</td>
<td>4.71</td>
<td>4.61</td>
</tr>
<tr>
<td>Habit formation</td>
<td>$\pi$</td>
<td>0.77</td>
<td>0.67</td>
</tr>
<tr>
<td>Inv subs. elast. of cons.</td>
<td>$\sigma_c$</td>
<td>1.27</td>
<td>0.97</td>
</tr>
<tr>
<td>Labor supply elast.</td>
<td>$\sigma_l$</td>
<td>1.50</td>
<td>1.58</td>
</tr>
<tr>
<td>Hours worked in S.S.</td>
<td>$l$</td>
<td>0.85</td>
<td>1.08</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$100(\beta^{-1} - 1)$</td>
<td>0.13</td>
<td>0.12</td>
</tr>
<tr>
<td>Stationary tech. shock</td>
<td>$\rho_0$</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>Risk premium shock</td>
<td>$\rho_b$</td>
<td>0.26</td>
<td>0.83</td>
</tr>
<tr>
<td>Invest. spec. tech. shock</td>
<td>$\rho_1$</td>
<td>0.80</td>
<td>0.84</td>
</tr>
<tr>
<td>Gov’t cons. shock</td>
<td>$\rho_g$</td>
<td>0.96</td>
<td>0.97</td>
</tr>
<tr>
<td>Price markup shock</td>
<td>$\rho_p$</td>
<td>0.92</td>
<td>0.89</td>
</tr>
<tr>
<td>Wage markup shock</td>
<td>$\rho_w$</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>Response of $g_t$ to $\varepsilon^2$</td>
<td>$\rho_{g\rho}$</td>
<td>0.49</td>
<td>0.53</td>
</tr>
<tr>
<td>Stationary tech. shock</td>
<td>$\sigma_a$</td>
<td>0.47</td>
<td>0.49</td>
</tr>
<tr>
<td>Risk premium shock</td>
<td>$\sigma_b$</td>
<td>0.21</td>
<td>0.11</td>
</tr>
<tr>
<td>Invest. spec. tech. shock</td>
<td>$\sigma_{i}$</td>
<td>0.35</td>
<td>0.31</td>
</tr>
<tr>
<td>Gov’t cons. shock</td>
<td>$\sigma_g$</td>
<td>0.47</td>
<td>0.47</td>
</tr>
<tr>
<td>Price markup shock</td>
<td>$\sigma_p$</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>MA(1) price markup shock</td>
<td>$\sigma_p$</td>
<td>0.75</td>
<td>0.79</td>
</tr>
<tr>
<td>Wage markup shock</td>
<td>$\sigma_w$</td>
<td>0.31</td>
<td>0.37</td>
</tr>
<tr>
<td>MA(1) wage markup shock</td>
<td>$\sigma_w$</td>
<td>0.92</td>
<td>0.96</td>
</tr>
<tr>
<td>Quarterly infl. rate. in S.S.</td>
<td>$\pi$</td>
<td>0.78</td>
<td>0.73</td>
</tr>
<tr>
<td>Inflation response</td>
<td>$r_\pi$</td>
<td>1.91</td>
<td>1.78</td>
</tr>
<tr>
<td>Output gap response</td>
<td>$r_g$</td>
<td>0.07</td>
<td>0.10</td>
</tr>
<tr>
<td>Diff. output gap response</td>
<td>$r_{\Delta g}$</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>Mon. pol. shock std</td>
<td>$\sigma_r$</td>
<td>0.23</td>
<td>0.22</td>
</tr>
<tr>
<td>Mon. pol. shock pers.</td>
<td>$\rho_r$</td>
<td>0.14</td>
<td>0.10</td>
</tr>
<tr>
<td>Interest rate smoothing</td>
<td>$\rho_R$</td>
<td>0.81</td>
<td>0.84</td>
</tr>
<tr>
<td>Net worth shock pers.</td>
<td>$\rho_{nw}$</td>
<td>0.25</td>
<td>0.30</td>
</tr>
<tr>
<td>Net worth shock std</td>
<td>$\rho_{nw}$</td>
<td>0.27</td>
<td>0.19</td>
</tr>
<tr>
<td>Working capital share</td>
<td>$\nu$</td>
<td>0.34</td>
<td>0.64</td>
</tr>
<tr>
<td>Credit spread in S.S.</td>
<td>$\delta_{sp}$</td>
<td>1.51</td>
<td>1.28</td>
</tr>
<tr>
<td>Monitoring cost</td>
<td>$\mu_1$</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>Mon. monitoring cost - Regime 1</td>
<td>$\mu_1$</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>Mon. monitoring cost - Regime 2</td>
<td>$\mu_2$</td>
<td>0.08</td>
<td>0.01</td>
</tr>
<tr>
<td>Trans. Prob. – R1 to R2</td>
<td>$p_{12}$</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>Trans. Prob. – R2 to R1</td>
<td>$p_{21}$</td>
<td>0.16</td>
<td>0.05</td>
</tr>
<tr>
<td>Log marginal likelihood</td>
<td></td>
<td>Laplace $-987.80$</td>
<td>Laplace $-1112.00$</td>
</tr>
</tbody>
</table>

Note: For the financial friction parameters, we use the same prior as for the other exogenous shocks (stated in Table 3.2). For $\mu$ and $\varepsilon_{sp}$, we use a normal distribution with means 0.25 and 1.00 and standard deviations 0.10 and 0.50 respectively. Finally, for $\nu$ we use a beta distribution with mean 0.50 and standard deviation 0.20. The “Pre-crisis sample” neglects the presence of the ZLB and is estimated on data up to 2007Q4, whereas the “Endogenous ZLB duration” imposes the ZLB as described in Section 5.1, and is estimated up to 2014Q2. “Endog. ZLB dur. with regime switch” also imposes the ZLB, but allows $\mu$ to vary stochastically between a low ($\mu_1$) and high ($\mu_2$) value. For $\mu_1$ and $\mu_2$, we use a normal distribution with means 0.025 and 0.25, and standard deviations 0.01 and 0.10 respectively. For the transition probabilities $p_{12}$ and $p_{21}$, we use a beta distribution with means 0.10 and 0.30 and standard deviations 0.05 and 0.10 respectively.
De Graeve [51] for a similar result). This observation suggests that the use of the Baa-Aaa spread as an observable for the external finance premium in the model can be too restrictive. Baa-Aaa spread is only one specific measure for default risk, and the cost of credit for firms is determined by various risks and constraints in the financial sector.\footnote{Gilchrist et al. [85] and Gilchrist and Zakrajsek [86] present alternative indicators of the default spread that have a stronger predictive power for economic activity than the Baa-Aaa spread.}

Not surprisingly, when we evaluate the performance of the FF-model for the complete sample including the 2008Q4–2009Q1 crisis period, the monitoring cost parameter $\mu$ and the implied elasticity of the spread to the net-worth ratio doubles. Perhaps surprisingly, the standard error of the exogenous net-worth shock is substantially lower, 0.19 versus 0.27 in the model estimated on pre-crisis data. We interpret this finding to imply that the endogenous amplification becomes more important when including the crisis period in the estimation sample. As we also impose the ZLB constraint in the estimation of this model, the estimated nominal wage and price stickiness is again very high (0.83 and 0.84 respectively) so that all the expected policy shocks that are required for the model to respect the ZLB constraint are positive. It is also striking that in this full-sample model, the estimated fraction of the wage bill that requires external financing is substantially higher than in the pre-crisis version, supporting the argument in Christiano, Eichenbaum, and Trabandt [37] that this channel was important during crisis. The magnitude of this cost channel increases from 0.33 to 0.64, but in both models the uncertainty in the posterior distribution for this parameter is very high. These two observations, the time-variation in the role of financial frictions and the potential role of the cost channel for the inflation dynamics, are discussed in more detail below.

### 5.3.1. A regime switching model with occasionally more severe financial frictions

Pre-crisis DSGE models typically neglected the role of financial frictions. This additional transmission mechanism was considered non-vital for forecasting output and inflation during the great moderation period, and by Occam’s razor arguments this mechanism was typically left out. However, as our discussion of the in-sample innovations illustrated, there was already strong evidence in our estimated pre-crisis model for occasionally big disturbances that seemed to be highly correlated with financial spreads and return indicators. When looking at these results from a broader perspective that also gives appropriate attention to the potential risks around the central banks forecast, these outliers should not be disregarded. A linear Gaussian approach is not the most efficient framework for handling these issues. The instability in the estimated parameters of our
FF-model depending on the estimation sample clearly illustrates these limitations. To more efficiently capture the time-varying relevance of the financial frictions in our model, we therefore consider here a Markov switching setup in which the constraints from the financial frictions can become much more binding occasionally.

In our Regime Switching Financial Friction model (RS-FF), we allow for two possible regimes: one regime (high-FF) with a high monitoring costs – implying a high sensitivity of the spread to the net worth position – and another regime (low-FF) with a low monitoring costs and low sensitivity of spread to leverage.\textsuperscript{39} The estimation results for this model is reported last in Table 5.3, and the data prefer this RS-FF-setting compared to the linear FF-model as shown by the gain in the log marginal likelihood of more than 30 in the pre-crisis context (not shown) and around 50 in the sample with the recent crisis.\textsuperscript{40} The transition probabilities and the regime-specific $\mu$ parameter are given by:

\[
Q_{FF} \begin{pmatrix} \text{low} \\ \text{high} \end{pmatrix} = \begin{bmatrix} 0.96 & 0.16 \\ 0.04 & 0.84 \end{bmatrix}, \quad \mu_{FF} \begin{pmatrix} \text{low} \\ \text{high} \end{pmatrix} = \begin{bmatrix} 0.029 \\ 0.084 \end{bmatrix}.
\]

The estimation results indicate that the elasticity of the spread to the leverage ratio varies between the two regimes by a factor of 2.7. As shown in Figure 5.5, the high-FF regime is active mainly around the two recession periods in the seventies, and its probability increases slightly

\textsuperscript{39} Christiano, Motto, and Rostagno \textsuperscript{[43]} focus instead on the distribution of the idiosyncratic productivity risk as the source for time-varying financial frictions. Levin, Natalucci and Zakrasjeck \textsuperscript{[108]} identify the time variation in the bankruptcy cost parameter, the equivalent of our monitoring cost, as the source for the counter-cyclical external premium behavior.

\textsuperscript{40} Suh and Walker (2016) also finds support for time-variation in parameters governing financial frictions.
Figure 5.6: Distributions for Output Growth (4-quarter change) in 2008Q4 given state in 2008Q3 in financial friction models with constant parameters (left panel) vs. regime-switching (right panel).

During all recessions. When evaluated over the more recent period the probability of the high-FF regime starts to rise early in 2008 and remains active during the financial crisis in 2009, but quickly returns to the low-FF regime after 2009. The higher marginal likelihood is due to the time-varying volatility in the spread: in the high-FF regime, the financial friction is strongly binding and the spread reacts more than twice as strong to the leverage ratio. The impact of shocks on investment is also higher but the magnitude of the amplification is moderate up to a factor of 1.5 maximum. The expected period-by-period persistence of the high-FF regime is limited (0.84) and this reduces the impact of spread increases on the discounted value of future expected returns on investment.

As evidenced in Figure 5.6, the central forecast of the single-regime pre-crisis FF-model, conditional on data up to 2008Q3 is completely missing the magnitude of the 2008Q4 downturn just as the benchmark SW-model without financial frictions. By comparing the no-financial friction model – left panel in Figure 5.4 – with the constant parameter FF-model – left panel in Figure 5.6, we see that the distribution around the FF-forecast is more disperse due to the extra volatility that is generated by the spread and the additional net worth shock. As a result, the extreme negative output growth realization of 2008Q4 now falls within the 0.25 percent interval of the predictive density, which is some improvement relative to the baseline model. The pre-crisis RS-FF model, shown in the right panel in Figure 5.6, further improves on this result because the probability of being in the high friction regime increased in 2008Q3 (56 percent against an unconditional probability of 20 percent) and this introduces a high degree of skewness in the predictive density of the spread. While the pre-crisis FF-model predicts a 1 percent upper tail for the expected spread
above 2.3 percentage points in 2008Q4, this becomes as high as 3 percentage points in the RS-FF model. The probability of the observed 2008Q4 output growth outcome now lies around the 0.5 percent tail interval, which is still small but at least the *ex post* realized event obtains some non-zero probability in the predictive density. This result indicates that if we appropriately could integrate the non-linear accelerator dynamics from financial frictions in our DSGE models we may obtain a more realistic predictive density that resembles these from the reduced form time-varying volatility models such as our RS-volatility example in Section 5.2.

Given the important role of the spread in the short run forecast, it is also informative to show how a conditional forecast, conditional on the timely observation of the spread, performs in the crisis period. Therefore, we make a forecast conditional on the 2008Q3 state of the economy as filtered by the pre-crisis FF-model but now we also provide the model with the information that the spread increased to the exceptionally high observed level of 3.02 percentage points in 2008Q4 (from 1.55 percentage points in 2008Q3). This conditioning is plausible in real time as the spread already in the beginning of the fourth quarter in 2008 (mid-October) had reached 3 percentage points. Figure 5.7 shows the unconditional (left panel) and conditional (right panel) forecast distributions for GDP growth in 2008Q4. As seen from the figure, the forecast conditional on the timely information from the spread display a median prediction for annual GDP growth of −2.11 percent in 2008Q4 and −1.92 percent in 2009Q1 (not shown), which should to be compared to the observed −3.61 and −4.42 percent in the actual data and unconditional forecast of −1.05 (left panel in the figure) and 0.06 (not shown) percent.

In the RS-FF model, the result depends very much on the regime in which the economy is finding itself in 2008Q3: the impact of conditioning on the spread is most disturbing when the economy is in the low friction regime. Extreme high spreads are very difficult to reconcile with the low friction regime, with its low elasticity of spread to leverage, and therefore the spreads are translated in huge negative shocks in net worth and/or risk premiums which then also result in worse output growth predictions of −2.53 and −3.01 percent in 2008Q4 and 2009Q1. The real-time information on the spread and the presence of the additional transmission mechanism allow the FF-model to considerably improve the accuracy of the central forecast in the crisis period. Our results confirm the findings of Del Negro and Schorfheide [55], who also compare the predictive performance of a standard SW setup with an augmented SW-FF model. They observe that the

---

41 This somewhat counter-intuitive result of the RS-FF model is related to the nature of the conditional forecast exercise: conditioning on a given spread observation has larger effects when that observation deviates more from the baseline unconditional forecast. The gain from the RS-FF model is precisely that the unconditional forecast will show larger dispersion in the high-FF regime and lower dispersion in the low-FF regime.
relative performance of the two models changes over time. On average the model without financial frictions generates more accurate forecasts, but during the recent financial crisis a SW-FF model – that also exploits the timely information on spread and interest rate – produces better forecasts for output and inflation. Del Negro, Hasegawa, and Schorfheide [54] built on these results and develop a new method for combining predictive densities from recursively estimated models using time-varying weights. As in our RS-approach, this dynamic linear prediction pooling relies on weights that follow an exogenous process. The next step in this research agenda would be to endogenize the occurrence of financial stress periods during which constraints are reinforced and additional feedback mechanisms are activated.\footnote{Various approaches have been developed in this context: Iacoviello and Guerrieri [88] with occasionally binding constraints, Dewachter and Wouters [59] with third order non-linear approximations and Bocola [22] with a combination of occasionally binding constraints and non-linear risk premiums.}

5.3.2. The cost channel of financial spreads and inflation dynamics

In section 4.2, when we discussed the economic interpretation of the great recession through the lense of the baseline SW-model, we observed that the model requires a series of positive mark up shocks to explain the maintained inflation rate during the period of slow recovery and persistent negative output gap. These positive mark up shocks are necessary despite the high estimate of nominal stickiness in price and wage setting. This trend towards more nominal stickiness was already present in the sub-sample estimates presented by SW07. The high nominal stickiness also plays a crucial role in the explanation of the recent inflation dynamics by Del Negro, Giannoni, and
Schorfheide [53] and Fratto and Uhlig [75]. These positive markup shocks disappear completely in our version of the SW-model, in which we implement the ZLB, and that feature an even higher degree of nominal stickiness. The question arises whether this estimated stickiness parameter should be interpreted effectively as a sign of pure nominal stickiness in the price setting practice or whether it reflects some other mechanism that lowered the responsiveness of inflation to the slack in production capacity.

As noted by Christiano, Eichenbaum, and Trabandt [37], one mechanism that might contribute to this inflation resilience, in particular during periods of increased financial constraints and high financing costs, is the cost channel. Firms that are financially constrained and that must finance their operations with expensive external capital can experience an increase in their marginal production costs if these financing costs dominate the influence of the other cost components. Related to this cost channel, firms can have other arguments to keep their prices high during periods of financial constraints: high markups can be necessary for firms to generate sufficient cash flow or firms might be forced by their financing constraints to give up on market share (see Gilchrist et al. [87]). Note that this cost channel also plays a crucial role in the explanation of the inflation inertia following a monetary policy shock in CEE [36].

Our FF-model contains a parameter that controls the strength of the cost channel. This parameter reflects the fraction of the wage bill that firms have to finance with credit. In this setup, we assume that the external finance premium is also affecting the cost for these intertemporal loans of the firms. In the pre-crisis model this fraction of the wage bill on which the financial cost applies, is estimated to be quite low (0.33) and the posterior distribution has a large uncertainty margin around this mode. This parameter increases to 0.63 in the complete sample estimation, still with a large uncertainty, but at least there is some indication that the cost channel was more relevant during the recent crisis. To examine the potency of this channel in our model, Figure 5.8 plots the impulse response functions of the three shocks that directly affect the external financing costs – the monetary policy shock, the exogenous risk premium shock, and the wealth shock – on the marginal cost and inflation for the two extreme values (zero and one) of the cost channel parameter. Given the large estimation uncertainty around the magnitude of the cost channel parameter, these two extreme values are not completely unlikely and their relevance can probably change depending on the nature of the financial shocks and the constraints. We plot the results for both the pre-crisis model, with a moderate degree of nominal stickiness, and the full sample ZLB model with a high degree of stickiness.
In both model versions and for all three shocks, it is obvious that marginal cost behaves quite different if the cost channel is fully active compared to a situation in which the cost channel is completely absent. The presence of the cost channel implies that the marginal cost increases at least during the first quarters following each of these shocks. The persistence of this positive effect depends on the type of shock and tends to be shorter for the risk-premium shock and most persistent for the net-worth shock.

The impact on inflation can differ substantially depending on the volatility of the cost shock and on the persistence of the shock relative to the degree of nominal stickiness which determines the degree of forward-lookingness in price setting. In the pre-crisis model, the exogenous risk-premium shock is highly volatile, but short lived. Combined with the moderate degree of stickiness the cost channel drastically changes the response of inflation to this shock. Inflation rises on impact due to the high risk-premium component in the financing costs, but the effect is very short lived. In the model with ZLB constraint – with more stickiness – the price setting is more forward looking and the persistence of the shock is crucial. In such a context, the smooth inflation process is dependent on the long-run expected marginal cost. In this case, only the net worth shock has a sufficiently persistent effect on the financing cost to exert a positive impact on inflation; the temporarily high risk free rate and risk premium shock are missing sufficient persistence to have a substantial impact on the inflation dynamics.

From this impulse response analysis, it follows that the cost channel can contribute to the slow response of inflation in a financial crisis context. When the external finance shock for firms are sufficiently high and/or sufficiently persistent, as it is the case for a net worth shock that is expected to have long lasting effects on the financing costs, this inflationary pressure from the cost channel can be quantitatively important. These results illustrate that the financial crisis should not necessarily be viewed as a purely negative aggregate demand shock without an impact on the supply side of the economy. With both aggregate demand and aggregate supply shifting inward by the financial shock, inflation should not necessarily be expected to react that much in a financial crisis situation.

6. State of Macroeconomic Modeling: Critical Assessment and Outlook

In this section, we conclude by discussing both “new” and “old” challenges for macroeconomic models. As evidenced above, the financial crisis has generated new challenges for macroeconomic models used at central banks. When the Great Recession and the financial crisis are included in the
Figure 5.8: The transmission of financial shocks: monetary policy (left column); risk premium (middle column) and net-worth (right column) shock. Panel A: Pre-crisis model; Panel B: Endogenous ZLB model.
estimation sample, we must adjust the specification and empirical estimation strategy of our policy models. Our chapter provides some avenues for moving in that direction, and suggests that the gains of doing so may be considerable. Our suggested modifications have in common of moving away from the standard linear Gaussian setup by including time variation in exogenous and endogenous disturbances. An important short-cut, however, in our adopted Markow Switching framework is that the regime changes are modeled as exogenous events and hence, unrelated to the conduct of policy. At this stage we therefore consider our extensions as a shortcut for truly endogenous non-linear and state dependent propagation mechanisms. Further progress on the specification of non-linear methods, solution and filtering techniques, as well as computational techniques, are ongoing for analyzing non-linear integrated macro-finance models. Together with a broader set of observable variables, these models should allow us to more efficiently identify the nature of shocks, their transmission, and their implications for policy. At this stage, it is important that different theoretical frameworks should be exploited to formulate and validate alternative model specifications.

There were also well-known challenges for central bank models prior to the financial crisis, and they have not been mitigated by the evidence brought forward by the crisis. The balanced growth and the stationarity assumptions provide discipline to the model forecasts, but these long term restrictions often conflict with the observed stochastic trends in many important macro-ratios. This mismatch between the theoretical assumptions and the empirical properties can result in overestimation of the persistence in the endogenous frictions and exogenous shocks. It may also be necessary to re-evaluate the forecast implications of full information and rational expectations in the models with alternative assumptions about information and expectations formation building on the seminal work of Evans and Honkapohja [68], Sims ([123],[124]) and Woodford [138].

Macro-models necessarily abstract of many sector-details. Recently, a lot of effort have been devoted to model the financial sector. In the standard Smets-Wouters [126] model analyzed in this chapter, the risk premium shock combines the impact of credit supply conditions, risk aversion, anticipations about future policy actions and the effect of quantitative easing (QE) policies targeting yield curve or risk spreads. Integrating the analysis of financial markets explicitly into general equilibrium is hence of first-order importance, both for firms (the focus of our chapter) and households, e.g. along the lines suggested by Iacoviello [93], and Liu, Wang and Zha [111].

43 For instance, the influential work of Del Negro et al. [56] suggested that workhorse closed economy DSGE models suffered from misspecification problems. Adolfson et al. [10] confirmed this finding for a standard open economy model.
Other models incorporate an active role for financial institutions in the credit supply process or the asset pricing functions: Christiano, Motto and Rostagno ([41], [42], [43]), Gerali et al. [81] and Gertler and Kiyotaki [82] are inspiring examples. Innovative new macro-finance models, as in e.g. Brunnermeier and Sannikov [27], He and Krishnamurthy [90] and Mendoza [115] suggest that strong endogenous risk and feedback channels between the real and the financial sectors can go a long way in explaining the change in volatility and correlations between tranquil and stress periods. A more explicit recognition of default in both the financial and non-financial private sectors as in Clerc et al. [48] is also an important avenue.

However, other sectors of the economy also have very similar problems in that the exogenous shocks represent a large range of influences that might call for different policy responses depending on the specific underlying distortion or inefficiency. One obvious example is the labor market with very diverging underlying trends in labor participation at intensive and extensive margins, and with shocks and distortions affecting both the labor supply and demand conditions. More work is needed to examine in which dimensions the labor market implications of the standard New Keynesian sticky wage model analyzed by Galí, Smets and Wouters [80] fall short relative to the data, and if recent work with a more elaborate labor market modeling (see e.g. Gertler, Sala and Trigari [83], Christiano, Trabandt and Walentin [45], and Christiano, Eichenbaum and Trabandt [38]) can remedy those shortcomings. Some prominent economists, like Kocherlakota [103], have recently reiterated that incomplete insurance and heterogeneity in labor and product markets is key for understanding the propagation and welfare costs of business cycles. Thus, the representative agents framework preserved by Gertler et al., and Christiano et al., may not be sufficient in the end, although it represents a clear step forward relative to current generation of policy models.

In a world increasingly integrated through trade of goods and services and more globalized financial markets, policy models also need to be able to account for the impact of foreign shocks. Two old challenges for open economy models is to account for the high degree of observed co-movement between real quantities (see e.g. Backus, Kehoe, and Kydland [16], and Justiniano and Preston [100]), and the relationship between interest rate differentials and exchange rate movements (i.e. the uncovered interest rate parity condition, see e.g. Eichenbaum and Evans [64], and Chaboud and Wright [33]). A voluminous literature deals with these issues, but there is yet no consensus on the “solutions” to these challenges.\footnote{The estimated open economy DSGE model developed by Adolfson et al. ([6], [10], [4]) which early on was integrated into operational use at the Riksbank (see [8]) attempted to account for this by modifying the UIP condition following the insights in Duarte and Stockman [62] and allowing for a common unitroot technology shock.}
Another key challenge posed for macro models at use in central banks following the crisis is that they have to provide a framework where topical questions can be addressed. First, they have to provide a framework where the central bank can use both conventional monetary policy (manipulating short rates) and unconventional policies (large scale asset purchases (LSAPs) and QE) to affect the economy. A serious treatment of unconventional monetary policy in policy models seems to imply that we have to tackle one old key-challenge in macro modeling, namely the failure of the expectations hypothesis (see e.g. Campbell and Shiller [31]), in favor of environments where the expectations hypothesis does not necessarily hold. One theoretical framework consistent with the idea that large scale asset purchases can reduce term premiums for different maturities and put downward pressure on long-term yields is the theory of preferred habit, see e.g. Andrés, López-Salido, and Nelson [13] and Vayanos and Vila [131]. Extensions in this direction appear crucial for evaluating the unconventional monetary policy measures during the crisis. Second, apart from analyzing unconventional policies during the crisis, the aftermath of the crisis have brought a renewed focus on financial stability issues, which implies that we need to be able to integrate financial stability considerations into macro models traditionally used for monetary policy analysis only. This involves stress testing exercises and the creation of an environment with an effective role for various macro-prudential tools. This requires a more realistic modeling of the interbank market as the one by Boissy, Collard, and Smets [23]. The “3D model” developed by Clerc et al. [48] and IMF’s GIMF model with banks (see Andrle et al. [14]) represent important steps in this direction. Unconventional monetary policy and macro-prudential instruments have important distributional effects and this calls for sufficient heterogeneity among agents that are affected by these measures. As mentioned before, the actual and potential budgetary implications of these measures require an explicit modeling of the systematic fiscal reaction function.

We believe the benchmark model analyzed in this chapter can serve as the starting point to analyze various extensions for topical questions and policy purposes. Specific model extensions combined with broader set of observed data should help us to better identify the various blocks. This applies equally for the financial, fiscal, labor market and the open economy blocks of the models. Bayesian methodology provides the tools to evaluate and combine these model predictions. In this endeavor, a challenge will be to keep the model size manageable by finding the most parsimonious ways to capture the necessary frictions and shocks, and to understand its implications as the models become increasingly complicated. To keep the models tractable, a critical decision point will be which frictions and shocks that are really needed in the core model, and which features that
can be abstracted from in the core model and instead meaningfully analyzed in satellite models. Developing and maintaining empirically validated models with strong theoretical foundations is a daunting task ahead for policy making institutions, even the ones with the most resources.
Appendix

A. Linearized model representation

In this appendix, we summarize the log-linear equations of the basic SW-model stated in Section 3. The complete model also includes the seven exogenous shocks \( \varepsilon_a, \varepsilon_i, \varepsilon_p, \varepsilon_w, \varepsilon_r \) and \( g_t \), but their processes are not stated here as they were already shown in the main text. Consistent with the notation of the log-linearized endogenous variables \( \hat{x}_t = dx_t/x \), the exogenous shocks are denoted with a ‘hat’, i.e. \( \hat{\varepsilon}_t = \ln \varepsilon_t \).

First, we have the consumption Euler equation:

\[
\hat{b}_c = 1 + (1 + \gamma) \hat{E}_t b_c - \frac{(\sigma_c - 1) \omega^L}{\sigma_c (1 + \gamma)} (E_t \hat{L}_t + \hat{L}_t) + \hat{\varepsilon}_b, \tag{A.1}
\]

where \( \gamma \) is the external habit parameter, \( \sigma_c \) the reciprocal of the intertemporal substitution elasticity, \( \omega^L \) the steady state nominal labor earnings to consumption ratio, and the exogenous risk premium shock \( \hat{\varepsilon}_b \) is re-scaled so that it enters additive with a unit coefficient.

Next, we have the investment Euler equation:

\[
\hat{i}_t = \frac{1}{(1 + \beta \gamma)} (\hat{i}_{t-1} + \beta \gamma \hat{E}_t \hat{i}_t + \frac{1}{\gamma^2 \varphi} \hat{Q}_t^k) + \hat{\varepsilon}_i, \tag{A.2}
\]

where \( \beta = \beta \gamma^{-\sigma_c} \), \( \varphi \) is the investment adjustment cost, and the investment specific technology shock \( \hat{\varepsilon}_i \) has been re-scaled so that it enters linearly with a unit coefficient. Additionally \( i_1 = 1/(1 + \beta) \) and \( i_2 = i_1/\psi \), where \( \beta \) is the discount factor and \( \psi \) is the elasticity of the capital adjustment cost function.

The price of capital is determined by:

\[
\hat{Q}_t^k = -(\hat{R}_t - \hat{E}_t \hat{\pi}_{t+1}) + q_1 \hat{E}_t r_{t+1}^k + (1 - q_1) \hat{E}_t Q_{t+1}^k + \frac{\sigma_c (1 + \gamma)}{1 - \gamma} \hat{\varepsilon}_b. \tag{A.3}
\]

where \( q_1 \equiv r^k / (r_* + (1 - \delta)) \) in which \( r^k \) is the steady state rental rate to capital, \( \delta \) the depreciation rate, and \( \hat{\varepsilon}_b \) is multiplied by \( \frac{\sigma_c (1 + \gamma)}{1 - \gamma} \) reflecting the re-scaling of this shock in the consumption Euler equation (A.1).

Fourth, we have the optimal condition for the capital utilization rate \( \hat{u}_t \):

\[
\hat{u}_t = (1 - \psi)/\psi \hat{r}_t^k, \tag{A.4}
\]

where \( \psi \) is the elasticity of the capital utilization cost function and capital services used in production \( \hat{k}_t \) is defined as:

\[
\hat{k}_t = \hat{u}_t + \hat{u}_{t-1}, \tag{A.5}
\]

64
where \( \hat{k}_{t-1} \) is the physical capital stock which evolves according to the capital accumulation equation:

\[
\hat{k}_t = \kappa_1 \hat{k}_{t-1} + (1 - \kappa_1) \hat{j}_t + \kappa_2 \hat{\varepsilon}_t
\]  
(A.6)

with \( \kappa_1 = (1 - (i_s/K_s)) \) and \( \kappa_2 = (i_s/K_s)\gamma^2\varphi \).

The following optimal capital/labor input condition also holds:

\[
\hat{k}_t = \hat{w}_t - \hat{r}_t + \hat{L}_t,
\]  
(A.7)

where \( \hat{w}_t \) is the real wage.

The log-linearized production function is given by:

\[
\hat{y}_t = \phi_p \left( (1 - \alpha) \hat{L}_t + \hat{\varepsilon}_t \right),
\]  
(A.8)

in which \( \phi_p \) is the fixed costs of production corresponding to the gross price markup in the steady state, and \( \hat{\varepsilon}_t \) is the exogenous TFP process.

Aggregate demand must equal aggregate supply:

\[
\hat{y}_t = \frac{c_s}{y_s} \hat{c}_{t} + \frac{i_s}{y_s} \hat{i}_{t} + \frac{n^k}{y_s} \hat{n}_{t},
\]  
(A.9)

where \( g_t \) represents the exogenous demand component.

Next, we have the following log-linearized price-setting equation with dynamic indexation \( t_p \):

\[
\hat{\pi}_t - \tau_p \hat{\pi}_{t-1} = \pi_1 (E_t \hat{\pi}_{t+1} - \tau_p \hat{\pi}_t) - \pi_2 \hat{\mu}_t + \hat{\varepsilon}_t^p,
\]  
(A.10)

where \( \pi_1 = \beta, \pi_2 = (1 - \xi_p\beta)(1 - \xi_p)/(\xi_p(1 + (\phi_p - 1)\epsilon_p)) \), \( 1 - \xi_p \) is the probability of each firm being able to re-optimize the price each period, \( \epsilon_p \) is the curvature of the aggregator function (eq. (3.2)), and the markup shock \( \hat{\varepsilon}_t^p \) has been re-scaled to enter with a unit coefficient. The price markup \( \hat{\mu}_t^p \) equals the inverse of the real marginal cost, \( \hat{\mu}_t^p = - \hat{\alpha} \hat{c}_t \), which in turn is given by:

\[
\hat{\alpha} \hat{c}_t = (1 - \alpha) \hat{w}_t^{real} + \alpha \hat{r}_t - \hat{z}_t.
\]  
(A.11)

We also have the following wage-setting equation allowing for dynamic indexation of wages for non-optimizing households:

\[
(1 + \bar{\beta}\gamma)\hat{w}_t^{real} - \hat{w}_{t-1}^{real} - \bar{\beta}\gamma E_t \hat{w}_{t+1}^{real} =
\]  
(A.12)
where \( \phi_w \) the gross wage markup, \( 1 - \xi_p \) is the probability of each household being able to re-optimize its wage each period, \( \epsilon_w \) is the curvature of the aggregator function (eq. 3.7), and \( \sigma_l \) determines the elasticity of labor supply given \( \sigma_c \) (see Equation (3.9)). The exogenous wage markup shock \( \bar{\epsilon}_t^w \) has been re-scaled to enter linearly with a unit coefficient.

Finally, we have the monetary policy rule:

\[
\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) (r_N \hat{\eta}_t + r_y \hat{y}^{gap}_t + r_\Delta \Delta \hat{y}^{gap}_t) + \bar{\epsilon}_t^r, \tag{A.13}
\]

where \( \hat{y}^{gap}_t = \hat{y}_t - \hat{y}^{pot}_t \), or in words: the difference between actual output and the output prevailing in the flexible price and wage economy in absence of the inefficient price and wage markup shocks. We solve for \( \hat{y}^{pot}_t \) by setting \( \xi_p = \xi_w = 0 \) (or arbitrary close to nil) and removing \( \bar{\epsilon}_t^w \) and \( \bar{\epsilon}_t^p \) from the system of equations given by (A.1) – (A.13). Note that when we impose the ZLB on the model, Equation (A.13) is replaced by Equation (5.1).

**B. The ZLB algorithm and the likelihood function**

This appendix provides some details on the ZLB algorithm we use and how the likelihood function takes the ZLB into account. For more details on the ZLB algorithm we refer to Hebden, Lindé, and Svensson [91], whereas more details on the computation of the likelihood is provided by Lindé, Maih, and Wouters [110].

**B.1. The ZLB algorithm**

The DSGE model can be written in the following practical state-space form,

\[
\begin{bmatrix}
X_{t+1} \\
H x_{t+1|t}
\end{bmatrix} = A \begin{bmatrix}
X_t \\
x_t
\end{bmatrix} + B i_t + \begin{bmatrix}
C \\
0
\end{bmatrix} \varepsilon_{t+1}. \tag{B.1}
\]

Here, \( X_t \) is an \( n_X \)-vector of predetermined variables in period \( t \) (where the period is a quarter) and \( x_t \) is a \( n_x \)-vector of forward-looking variables. The \( i_t \) is generally a \( n_i \)-vector of (policy) instruments but in the cases examined here it is a scalar – the central bank’s policy rate – giving \( n_i = 1 \). The \( \varepsilon_t \) is an \( n_\varepsilon \)-vector of independent and identically distributed shocks with mean zero and covariance matrix \( I_{n_\varepsilon} \), while \( A, B, C, \) and \( H \) are matrices of the appropriate dimension. Lastly \( x_{t+\tau|t} \) denotes \( E_t x_{t+\tau} \), i.e. the rational expectation of \( x_{t+\tau} \) conditional on information available in period \( t \). The forward-looking variables and the instruments are the non-predicted variables.\(^{45}\)

\(^{45}\) A variable is predetermined if its one-period-ahead prediction error is an exogenous stochastic process (Klein [102]). For (B.1), the one-period-ahead prediction error of the predetermined variables is the stochastic vector \( C \varepsilon_{t+1} \).
The variables are measured as differences from steady-state values, in which case their unconditional means are zero. In addition, the elements of the matrices $A$, $B$, $C$, and $H$ are considered fixed and known.

We let $i_t^*$ denote the policy rate when we disregard the ZLB. We call it the *unrestricted* policy rate. We let $i_t$ denote the actual or *restricted* policy rate that satisfies the ZLB,

$$i_t + \bar{i} \geq 0,$$

where $\bar{i} > 0$ denotes the steady-state level of the policy rate and we use the convention that $i_t$ and $i_t^*$ are expressed as deviations from the steady-state level. The ZLB can therefore be written as

$$i_t + \bar{i} = \max \{ i_t^* + \bar{i}, 0 \}. \quad (B.2)$$

We assume the unrestricted policy rate follows the (possibly reduced-form) unrestricted linear policy rule,

$$i_t^* = f_X X_t + f_x x_t, \quad (B.3)$$

where $f_X$ and $f_x$ are row vectors of dimension $n_X$ and $n_x$ respectively. From (B.2) it then follows that the restricted policy rate is given by:

$$i_t + \bar{i} = \max \{ f_X X_t + f_x x_t + \bar{i}, 0 \}. \quad (B.4)$$

Consider now a situation in period $t \geq 0$ where the ZLB may be binding in the current or the next finite number $T$ periods but not beyond period $t + T$. That is, the ZLB constraint

$$i_{t+\tau} + \bar{i} \geq 0, \quad \tau = 0, 1, ..., T \quad (B.5)$$

may be binding for some $\tau \leq T$, but we assume that it is not binding for $\tau > T$,

$$i_{t+\tau} + \bar{i} > 0, \quad \tau > T.$$

We will implement the ZLB with anticipated shocks to the unrestricted policy rule, using the techniques of Laséen and Svensson [105]. Thus, we let the restricted and unrestricted policy rate in each period $t$ satisfy

$$i_{t+\tau,t} = i_{t+\tau,t}^* + z_{t+\tau,t}, \quad (B.6)$$

for $\tau \geq 0$. The ZLB policy rule in (B.4) – as we explain in further detail below – implies that all current and future anticipated shocks $z_{t+\tau,t}$ in (B.6) must be non-negative, and that $z_{t,t}$ is strictly positive in periods when the ZLB is binding.
Disregarding for the moment when \( z_t \) are non-negative, we follow Laséen and Svensson [105] and call the stochastic variable \( z_t \) the deviation and let the \((T+1)\)-vector \( z_t^{T} \equiv (z_{t,t}, z_{t+1,t}, \ldots, z_{t+T,t})' \) denote a projection in period \( t \) of future realizations \( z_{t+\tau}, \tau = 0, 1, \ldots, T \), of the deviation. Furthermore, we assume that the deviation satisfies

\[
z_t = \eta_{t,t} + \sum_{s=1}^{T} \eta_{t,t-s}
\]

for \( T \geq 0 \), where \( \eta_t \equiv (\eta_{t,t}, \eta_{t+1,t}, \ldots, \eta_{t+T,t})' \) is a \((T+1)\)-vector realized in the beginning of period \( t \). For \( T = 0 \), the deviation is given by \( z_t = \eta_t \). For \( T > 0 \), the deviation is given by the moving-average process

\[
z_{t+\tau,t+1} = z_{t+\tau,t} + \eta_{t+\tau,t+1}
\]

where \( \tau = 1, \ldots, T \). It follows that the dynamics of the projection of the deviation can be written more compactly as

\[
z_{t+1} = A_2 z_t + \eta_{t+1}^{T+1},
\]

where the \((T+1) \times (T+1)\) matrix \( A_2 \) is defined as

\[
A_2 \equiv \begin{bmatrix}
0_{T \times 1} & I_T \\
0 & 0_{1 \times T}
\end{bmatrix}.
\]

Hence, \( z_t \) is the projection in period \( t \) of current and future deviations, and the innovation \( \eta_t \) can be interpreted as the new information received in the beginning of period \( t \) about those deviations.

Let us now combine the model, (B.1), the dynamics of the deviation, (B.7), the unrestricted policy rule, (B.3), and the relation (B.6). Taking the starting period to be \( t = 0 \), we can then write the combined model as

\[
\begin{bmatrix}
\tilde{X}_{t+1} \\
\tilde{H} \tilde{x}_{t+1|t}
\end{bmatrix} = \tilde{A} \begin{bmatrix}
\tilde{X}_t \\
\tilde{x}_t
\end{bmatrix} + \begin{bmatrix}
C_{0_{n_x \times (T+1)}} \\
0_{(T+1) \times n_e}
\end{bmatrix} \begin{bmatrix}
0_{n_x \times (T+1)} \\
I_{T+1}
\end{bmatrix} \begin{bmatrix}
\tilde{\varepsilon}_{t+1} \\
\eta^{T+1}_t
\end{bmatrix}
\]

for \( t \geq 0 \), where

\[
\tilde{X}_t \equiv \begin{bmatrix}
X_t \\
z_t
\end{bmatrix}, \quad \tilde{x}_t \equiv \begin{bmatrix}
x_t \\
\mathbf{i}_t^*
\end{bmatrix}, \quad \tilde{H} \equiv \begin{bmatrix}
H_{0_{n_x \times 1}, 0_{n_x \times 1}} \\
0_{1 \times n_x} & 0 & 0
\end{bmatrix}.
\]
Under the standard assumption of the saddle-point property (that the number of eigenvalues of \( \hat{A} \) with modulus larger than unity equals the number of non-predetermined variables, here \( n_x + 2 \)), the system of difference equations (B.8) has a unique solution and there exist unique matrices \( M \) and \( F \) returned by the Klein [102] algorithm such that the solution can be written:

\[
\tilde{x}_t = F \tilde{X}_t \equiv \begin{bmatrix} F_x \\ F_i \end{bmatrix} \tilde{X}_t \tilde{X}_{t+1} = M \tilde{X}_t + \begin{bmatrix} C \varepsilon_{t+1} \\ \eta^{t+1} \end{bmatrix} = \begin{bmatrix} M_{XX} & M_{Xz} \\ 0_{(T+1) \times n_x} & A_z \end{bmatrix} \begin{bmatrix} X_t \\ z^t \end{bmatrix} + \begin{bmatrix} C \varepsilon_{t+1} \\ \eta^{t+1} \end{bmatrix},
\]

for \( t \geq 0 \), and where \( X_0 \) in \( \tilde{X}_0 \equiv (X_0', z^0)' \) is given but the projections of the deviation \( z^0 \) and the innovations \( \eta^t \) for \( t \geq 1 \) (and thereby \( z^t \) for \( t \geq 1 \)) remain to be determined. They will be determined such that the ZLB is satisfied, i.e. Equation (B.4) holds. Thus, the \textit{policy-rate projection} is given by

\[
i_{t+\tau,t} = F_i M^\tau \begin{bmatrix} X_t \\ z^t \end{bmatrix}
\]

for \( \tau \geq 0 \) and for given \( X_t \) and \( z^t \).

We will now show how to determine the \((T+1)\)-vector \( z^t \equiv (z_t, z_{t+1}, ..., z_{t+T})' \), i.e. the projection of the deviation, such that policy-rate projection satisfies the ZLB restriction (B.5) and the policy rule (B.4).

When the ZLB restriction (B.5) is disregarded or not binding, the policy-rate projection in period \( t \) is given by

\[
i_{t+\tau,t} = F_i M^\tau \begin{bmatrix} X_t \\ 0_{(T+1) \times 1} \end{bmatrix}, \quad \tau \geq 0.
\]

The policy-rate projection disregarding the ZLB hence depends on the initial state of the economy in period \( t \), represented by the vector of predetermined variables \( X_t \). If the ZLB is disregarded, or not binding for any \( \tau \geq 0 \), the projections of the restricted and unrestricted policy rates will be the same,

\[
i_{t+\tau,t} = i_{t+\tau,t}^* = f_X X_{t+\tau,t} + f_x \tilde{x}_{t+\tau,t}, \quad \tau \geq 0.
\]

Assume now that the policy-rate projection according to (B.10) violates the ZLB for one or several periods, that is,

\[
i_{t+\tau,t} + \bar{i} < 0, \quad \text{for some } \tau \text{ in the interval } 0 \leq \tau \leq T.
\]

In order to satisfy the ZLB, we then want to find a projection of the deviation \( z^t \) such that the policy-rate projection satisfies (B.5) and

\[
i_{t+\tau,t} + \bar{i} = \max\{i_{t+\tau,t}^*, \bar{i}\} = \max\{f_X X_{t+\tau,t} + f_x \tilde{x}_{t+\tau,t} + \bar{i}, 0\}
\]

(B.12)
for \( \tau \geq 0 \). This requires that the projection of the deviation satisfies a non-negativity constraint

\[
z_{t+\tau,t} \geq 0, \quad \tau \geq 0, \tag{B.13}
\]

and that the policy-rate projection and the projection of the deviation satisfies the complementary-slackness condition

\[
(i_{t+\tau,t} + \bar{\iota}) z_{t+\tau,t} = 0, \quad \tau \geq 0. \tag{B.14}
\]

Notice that the complementary-slackness condition implies that \( z_{t+\tau,t} = 0 \) if \( i_{t+\tau,t} + \bar{\iota} > 0 \).

For given \( X_t \), we now proceed under the presumption that there exists a unique projection of the deviation \( z^t \) that satisfies (B.9) and (B.12)–(B.14).\(^{46}\) We call this projection of the deviation and the corresponding policy-rate projection the *equilibrium* projection. This projection of the deviation either has all elements equal to zero (in which case the ZLB is not binding for any period) or has some elements positive and other elements zero. Let

\[
\mathcal{T}_t = \{0 \leq \tau \leq T \mid z_{t+\tau,t} > 0\}
\]

denote the set of periods for which the projection of the deviation are positive in equilibrium.

For each \( \tau \in \mathcal{T}_t \), the solution will satisfy

\[
i_{t+\tau,t} + \bar{\iota} = F_i M^\tau \begin{bmatrix} X_t \\ z^t \end{bmatrix} + \bar{\iota} = 0 \quad \text{for} \quad \tau \in \mathcal{T}_t. \tag{B.15}
\]

Let \( n_{\mathcal{T}_t} \) denote the number of elements of \( \mathcal{T}_t \), that is, the number of periods that the ZLB binds. The equation system (B.15) then has \( n_{\mathcal{T}} \) equations to determine the \( n_{\mathcal{T}} \) elements of \( z^t \) that are positive. From the system (B.15), it is clear that the solution for \( z^t \) and the set \( \mathcal{T}_t \) will depend on \( X_t \) as well as the initial situation, and thereby also on the initial innovation \( \varepsilon_t \). For other periods (that is \( \tau \not\in \mathcal{T}_t \)), the ZLB will not be binding and the elements in \( z^t \) will be zero. The equation system (B.15) and the periods in the set \( \mathcal{T}_t \) hence refer to the periods where the ZLB is *strictly* binding, that is, when \( z_{t+\tau,t} \) is positive. Furthermore, it is important to notice that the set of periods \( \tau \) in (B.11), for which the policy-rate projection (B.10) violates the ZLB, is not necessarily the same as the set of periods \( \mathcal{T}_t \) for which the ZLB is strictly binding in equilibrium. That is because the projections of the predetermined and forward-looking variables \( X_{t+\tau,t} \) and \( x_{t+\tau,t} \), that determine the unrestricted policy rate differ, depending on whether \( z^t \) is zero or not. This means that the whole policy-rate path is affected when the ZLB is imposed.

\(^{46}\) This assumption is discussed in further detail in Hebden, Lindé, and Svensson [91].
The difficulty in imposing the ZLB is to find the set $T_t$ for which the ZLB is strictly binding in equilibrium, that is, to find the periods for which the equation system (B.15) applies. Once this is done, solving the equation system (B.15) is trivial. Hebden et al. [91] outline a simple shooting algorithm to find the set $T_t$.

**B.2. Computation of the Likelihood Function**

To compute the likelihood function, we follow the general idea outlined by Maih [113]. Maih’s algorithm allows us to add anticipated policy shocks (using the algorithm outlined above) to the state space formulation of the model and filter those shocks with the Kalman filter to impose the zero lower bound on policy rates in the estimation. The appealing feature of Maih’s algorithm is that it does not require us to include standard deviations for each of the anticipated policy shocks. Thus, the log-marginal likelihood can be directly compared to the models which does not impose the ZLB. For further details on the computation of the likelihood function in the face of the ZLB constraint, we refer to Lindè, Maih, and Wouters [110].

**C. Data**

In this appendix, we provide the sources on the data we use in the analysis.

**C.1. Benchmark Model**

The benchmark model is estimated using seven key macro-economic time series: real GDP, consumption, investment, hours worked, real wages, prices, and a short-term interest rate. The Bayesian estimation methodology is extensively discussed by Smets and Wouters [125]. GDP, consumption and investment were taken from the U.S. Department of Commerce – Bureau of Economic Analysis data-bank – on September 25, 2014. Real gross domestic product is expressed in billions of chained 2009 dollars. Nominal personal consumption expenditures and fixed private domestic investment are deflated with the GDP-deflator. Inflation is the first difference of the log of the implicit price deflator of GDP. Hours and wages come from the BLS (hours and hourly compensation for the non-farm business, NFB, sector for all persons). Hourly compensation is divided by the GDP price deflator in order to get the real wage variable. Hours are adjusted to take into account the limited coverage of the NFB sector compared to GDP (the index of average hours for the NFB sector is multiplied with the Civilian Employment (16 years and over) . The aggregate real
variables are expressed per capita by dividing with the population size aged 16 or older. All series are seasonally adjusted. The interest rate is the Federal Funds Rate. Consumption, investment, GDP, wages, and hours are expressed in 100 times log. The interest rate and inflation rate are expressed on a quarterly basis during the estimation (corresponding with their appearance in the model), but in the figures the series are reported on an annualized (400 times first log difference) or yearly (100 times the four-quarter log difference) basis.

C.2. Model with Financial Frictions

The first seven variables are exactly those used to estimate the benchmark model, which are described in Appendix C.1. In addition to those series, this model features an interest rate spread. Following Bernanke, Gertler, and Gilchrist [20], this spread is measured as the difference between the BAA corporate interest rate and the U.S. 10-year government yield.
References


84


