The Joint Dynamics of the U.S. and Euro-area Inflation: Expectations and Time-varying Uncertainty

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Central Banks and Inflation Anchoring

- The Federal Reserve System and the European Central Bank have adopted a mandate of price stability.

- Price stability is devised to foster economic activity and employment.

- Both central banks monitor closely various measures of inflation expectations.

- Inflation expectations come in two forms:
  - market-based (inflation swaps or TIPS breakeven inflation rates)
  - survey-based.
Measuring Inflation Anchoring

Traditionally, central banks use different tools to gauge the anchoring of inflation expectations:

- Assess the stability of medium- to long-run inflation expectations (Beechey, Johannsen and Levin, 2011).
- Evaluate the extent of the pass-through of short-run inflation expectations, or of news, on medium- and long-run inflation expectations (e.g. Gürkaynak, Levin and Swanson, 2010).

These measures reflect the stability of the conditional mean of inflation.

Problem: conditional means (1st-order moments) can be stable even if uncertainty (2nd-order moment) is relatively high.
Measuring Inflation Anchoring

Country 1. Conditional expectation $E_t(\pi_{t+h}^{(1)})$

Country 2. Conditional expectation $E_t(\pi_{t+h}^{(2)})$
Measuring Inflation Anchoring

Country 1. Conditional expectation $E_t(\pi_{t+h}^{(1)})$

Country 2. Conditional expectation $E_t(\pi_{t+h}^{(2)})$

Country 1. $P_t(1.5 < \pi_{t+h}^{(1)} < 2.5)$
Conditional variance of inflation: 2

Country 2. $P_t(1.5 < \pi_{t+h}^{(2)} < 2.5)$
Conditional variance of inflation: 0.2
Measuring Inflation Anchoring

⇒ We propose to think of the anchoring of inflation expectations in terms of conditional distributions. Specifically:

\[
\text{Measure of anchoring} \equiv \mathbb{P}_t(\pi_{t+h} \in [a, b]),
\]

where \([a, b]\) is an interval deemed consistent with price stability.
Evaluating Conditional Distribution of Inflations

- Inflation derivatives data (inflation caps and floors) could be used to derive such probabilities. But these data are affected by liquidity and risk premia.

- Such probabilities can be derived from surveys where respondents (professional forecasters) are asked to provide probabilities of future inflation outcomes falling within given ranges.

- SPF’s limitations:
  - only certain horizons are available,
  - the targeted measure of inflation is survey-specific,
    - (y-o-y in the euro area, yearly averages of y-o-y inflation rates in the US)
  - infrequent (quarterly) or irregular (FOMC frequency) releases.

- More frequent –monthly– surveys are available (Blue Chip and Consensus Forecasts), but these provide only first-order moments.
Evaluating Conditional Distribution of Inflations

- We propose a methodology able to "digest" various types of inflation-based information so as to give, as an output, the distribution of inflation at any future horizon.

- These outputs can further be used to compute distribution-based anchoring measures.

- Our approach is based on a flexible dynamic factor model of inflation.

- We jointly account for the US and EA inflation, allowing us to study the probability of future joint inflation outcomes.
Results Overview

- The model fits the survey-implied first and second moments reasonably well.
- Larger inflation uncertainty in the US than in the EA.
- Conditional correlations between future US and EA inflation rates significantly trended up since 2010.
- The increase in correlations reflects increasing interconnectedness between the economies.
- Substantial movements in our measures of inflation expectations’ anchoring during the crisis.
- Second-order moments appear to be related to the US and EA Economic Policy Uncertainty indices (Baker et al., 2015)
## Data

<table>
<thead>
<tr>
<th>Survey</th>
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<td><strong>US surveys</strong></td>
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<td>NY Fed’s PDS</td>
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<td>Blue Chip</td>
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<td>Consensus Forecasts</td>
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<td><strong>EA surveys</strong></td>
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<td>ECB SPF</td>
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<tr>
<td>Consensus Forecasts</td>
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Our Approach

Estimation of the dynamic factor model

Inputs
(Inflation and survey data)
- Inflation rates
- Blue Chip forecasts
- Consensus Forecasts
- Philly Fed SPF$s^*$
- US PDS$^*$
- ECB SPF$s^*$

Outputs
- Conditional distributions of future inflation rates.
- "Synthetic" surveys (any horizon or inflation def.)
- Inflation expectations comparable across areas.
- Expectations of future inflation comovements (US and EA)

*: Surveys in blue provide second-order moments.
The Model: Inflation and Its Driving Factors

- $\pi^{(i)}_{t,t+h}$: annualized inflation rate in economy $i$ between $t$ and dates $t + h$

\[
\pi^{(i)}_{t,t+h} = \frac{12}{h} \log \left( \frac{P^{(i)}_{t+h}}{P^{(i)}_t} \right), \text{ where } P^{(i)}_t \text{ is a price index.}
\]

- $\pi^{(i)}_{t-12,t}$ is a linear combination of factors gathered in the $n \times 1$ vector $Y_t$:

\[
\pi^{(i)}_{t-12,t} = \bar{\pi}^{(i)} + \delta^{(i)}' Y_t.
\]

- $Y_t$ follows:

\[
Y_t = \Phi Y_{t-1} + \Theta (z_t - \bar{z}) + \Sigma(z_t) \varepsilon_{Y,t}, \quad \varepsilon_{Y,t} \sim \mathcal{N}(0, I),
\]

where $z_t$ is an exogenous factor driving $Y_t$'s conditional variance.

- $Y_t$ feature stochastic volatility $\Rightarrow$ inflation uncertainty.
The Model: Transition Equations

- $z_t$ follows a multivariate auto-regressive gamma process (time-discretized Cox-Ingersoll-Ross process). VAR representation:

$$z_t = \mu_z + \Phi_z z_{t-1} + \Omega(z_{t-1}) \varepsilon_{z,t},$$

where $\varepsilon_{z,t}$ has a conditional zero mean and an $ld$ covariance matrix.

- $X_t = (Y_t', z_t')'$ follows a VAR process:

$$X_t = \begin{bmatrix} Y_t \\ z_t \end{bmatrix} = \mu_X + \Phi_X \begin{bmatrix} Y_{t-1} \\ z_{t-1} \end{bmatrix} + \Sigma_X(z_{t-1}) \varepsilon_X,t,$$

where $\varepsilon_X,t$ is a unit-variance martingale difference sequence.
The Model: Measurement Equations

There are three sets of measurement equations:

- **Realized inflation:**
  \[ \pi_{t-12,t} = \bar{\pi}(i) + \delta(i)' Y_t \]

- **Survey-based expectations of future inflation rates:**
  \[ SPF_t = \bar{\pi} + a + b' X_t + \text{diag}(\sigma_{avg}) \eta_{t}^{avg} \]

- **Survey-based variances:**
  \[ VSPF_t = \alpha + \beta' X_t + \text{diag}(\sigma_{var}) \eta_{t}^{var} \]
The Model: Key Property

- Key property: $X_t$ is an "affine" process

  $\Rightarrow$ Conditional first- and second-order moments of any linear combination of future $X_t$ values are available in closed form.

- Notably, closed-form formula to compute:

  - $\mathbb{E}_t(\pi_t^{(i)}_{t,t+h})$
    (as in Consensus Forecasts for maturities up to 5 years)

  - $\mathbb{E}_t(\pi_t^{(i)}_{t+h-12,t+h})$ and $\text{Var}_t(\pi_t^{(i)}_{t+h-12,t+h})$
    (as in EA SPFs)

  - $\mathbb{E}_t(\pi_t^{(i)}_{t+h-21,t+h-9} + \pi_t^{(i)}_{t+h-18,t+h-6} + \pi_t^{(i)}_{t+h-15,t+h-3} + \pi_t^{(i)}_{t+h-12,t+h})$
    (as in Philly Fed SPFs for horizons up to 2 years)

  - $\mathbb{E}_t(\pi_t^{(i)}_{t+60,t+120})$ and $\text{Var}_t(\pi_t^{(i)}_{t+60,t+120})$
    (as in the U.S. Primary Dealer Survey)

- non-affine stochastic volatility models
The Model: Estimation

- Model-implied equivalent of survey-based point estimates ($E_t$) and of survey-based uncertainty ($Var_t$) are affine in $X_t$.

$\Rightarrow$ The model has a linear state-space representation.

- The model is estimated by quasi maximum likelihood, using the Kalman filter which makes it possible
  - to simultaneously estimate
    - the model parameters and
    - the latent factors $X_t$
  - to handle missing observations (all surveys are not available every month).
Model Fit of the 1-year Inflation

Euro area

\[ \pi_{t-12,t} \]

\[ E_t(\pi_{t+12}) \]

\[ \text{Var}(\pi_{t+12}) \]

U.S.

\[ \pi_{t-12,t} \]

\[ E_t(\pi_{t+12}) \]

\[ \text{Var}(\pi_{t+12}) \]
Model Fit of the Longer-term Inflation

\[ \pi_{t-12}, t \]

Euro area

\[ \hat{\pi}_{t+48, t+60} \]

U.S.

\[ \sigma_t(\pi_{t+24}) \]

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Survey-Implied 1-year Inflation Distribution

Euro area (1-year horizon)

- 11-2004, Fitted (model)
- 11-2004, Survey (observed)
- 11-2014, Fitted (model)
- 11-2014, Survey (observed)

U.S. (1-year horizon)

- 01-2005, Fitted (model)
- 01-2005, Survey (observed)
- 01-2014, Fitted (model)
- 01-2014, Survey (observed)
Inflation Uncertainty (conditional std dev.)

1-year 4 years ahead

1-year 9 years ahead

5-year 5 years ahead

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Model-Implied Low Inflation Probabilities

Euro area – Proba. of an inflation lower than 0%

U.S. – Proba. of an inflation lower than 0%

Euro area – Proba. of an inflation lower than 1%

U.S. – Proba. of an inflation lower than 1%

option-implied probabilities

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Joint Dynamics of US and EA Inflation
Inflation Comovements in the US and the EA

Standard deviations of EA and US inflation

Covariances of EA and US inflation

Correlation between EA and US inflation

Probability of joint deflation

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Measuring the Anchoring of Inflation Expectations

Euro area – Proba. that inflation in [1.5%,2.5%]

U.S. – Proba. that inflation in [1.5%,2.5%]

Euro area – Proba. that inflation in [1%,3%]

U.S. – Proba. that inflation in [1%,3%]
Conclusion

- **Our model:**
  - Dynamic factor model estimated using various US and EA surveys.
  - Derive various model outputs that are consistent with survey-based inflation expectations.
  - Aggregate survey-based information and inter- and extrapolate it.
  - Compute survey-consistent probabilities that future inflation — for any horizon — falls within a given range.

- **Our findings:**
  - Future inflation correlations increased since the Great Recession.
  - Joint deflation probabilities in the US and EA are currently negligible.
  - Probabilities of US 5y5f inflation ∈ [1.5%, 2.5%] increased since the crisis and are currently > than 0.6.
  - Probabilities of EA 5y5f inflation ∈ [1.5%, 2.5%] declined since the crisis and are currently ≈ 0.8.
Thank you for your attention!
Appendix

Model-Implied Conditional Distributions of Inflation
Factor Loadings

Euro area, Y loadings of conditional expectations

U.S., Y loadings of conditional expectations

Euro area, z loadings of conditional expectations

U.S., z loadings of conditional expectations

Euro area, z loadings of conditional variances

U.S., z loadings of conditional variances

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Term Structure of Inflation Expectations

Euro area

U.S.

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Joint Dynamics of US and EA Inflation
Appendix

Joint Conditional Distribution of Inflation

(a) 1–year horizon

(b) 5–year horizon
Literature


- **Augmented models**: Kozicki and Tinsley (2006), Ghysels and Wright (2009), Kim and Orphanides (2012)
**Surveys**

**Euro area, 1-year horizon**
(ECB SPF)

**U.S., 1-year horizon**
(Philly Fed SPF)

**U.S., 5-year 5 years ahead**
(Primary Dealer Survey)

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Joint Dynamics of US and EA Inflation
Standard Inflation Model with Stochastic Volatility

- Model building on Stock and Watson (2007):
  \[ \pi_t = \pi_{t-1} + \sigma_t \eta_t, \]
  \[ \ln(\sigma_t^2) = \rho \ln(\sigma_{t-1}^2) + \gamma \nu_t, \text{ where } [\eta_t, \nu_t]' \sim i.i.d. \mathcal{N}(0, I_d). \]

- In this model:
  \[ \nabla \text{ar}_t(\pi_{t+h}) = \nabla \text{ar}_t(\sigma_{t+1} \eta_{t+1} + \cdots + \sigma_{t+h} \eta_{t+h}) = \mathbb{E}_t(\sigma_{t+1}^2 + \cdots + \sigma_{t+h}^2) \]
  \[ = \mathbb{E}_t(\sigma_{t+1}^2) + \cdots + \mathbb{E}_t(\sigma_{t+h}^2), \]
  \[ \sigma_{t+j}^2 | \sigma_t \sim \exp \mathcal{N} \left( \rho^j \ln \sigma_t^2, \gamma^2 \frac{1-\rho^j}{1-\rho} \right) \Rightarrow \mathbb{E}_t(\sigma_{t+j}^2) = (\sigma_t^2)^{\rho^j} \exp \left( j \gamma^2 \frac{1-\rho^j}{2(1-\rho)} \right). \]
  \[ \text{Then: } \nabla \text{ar}_t(\pi_{t+h}) = \sum_{j=1}^{h} (\sigma_t^2)^{\rho^j} \exp \left( j \gamma^2 \frac{1-\rho^j}{2(1-\rho)} \right). \]

\[ \Rightarrow \nabla \text{ar}_t(\pi_{t+h}) \text{ (even log-transformed) is not a linear function of } \ln(\sigma_t^2). \text{ The estimation is less straightforward than in our affine case.} \]
Low Inflation Probabilities

Euro area – Proba. of an inflation lower than 0%

U.S. – Proba. of an inflation lower than 0%

Euro area – Proba. of an inflation lower than 1%

U.S. – Proba. of an inflation lower than 1%