Motivation

- Policy relevant forecasts commonly required at quarterly horizons from 1 - 12
- Policy analysis requires structural models - often DSGE models
- DSGE Models estimated by ML or Bayesian methods - in both cases using one-step prediction errors to build likelihood
- Might h > 1-step forecast errors → produce sensible estimates?
- Improve forecast performance?

Method

- State space representation of the linearised DSGE model
  \[ y_t = HE_t, \quad t = 1, \ldots, T \]
  \[ \xi_t = C \xi_{t-1} + v_t. \]
- \( y_t \) observed variables, \( x_t \) unobserved vector of states that may be estimated using the Kalman filter to provide \( \hat{\xi}_{t-1} = E(\xi_t|Y_{1:t-1}) \) and \( \hat{\xi}_t = E(\xi_t|Y_t) \) where \( Y_t = (y_t, \ldots, y_1) \)
- As parameter matrices \( H \) and \( C \) are unknown, can be conveniently estimated using ML based on the prediction error decomposition
- Normally uses a 1-step error to do so, but can equally use \( h \)-step where \( h > 1 \), thus using a vector of errors
- Might be interpreted as a MOM approach akin to cross-validation
- Precursor: Frank Schorfheide JoE 2005 VAR forecasting under misspecification, who specifies a loss function in terms of prediction errors

Model

- Use Smets and Wouters 2007
- Seven macro series - output, consumption and investment growth, inflation, wage growth, hours, interest rate
- Estimate by maximising likelihood (as in eg Ireland JEEA 2013)
- S&W 1966Q1-2004Q4: we use 1954Q3-1997Q4 to use earlier data while still retaining observations for forecast evaluation
- Despite shorter and overlapping sample, results similar to S&W

Global optimum?

- Core results outcome of maxima from 100 starting points
- To check robustness, compared to results from only 10 starting points
- Most estimates similar
- Where there appear to be large differences, in fact not economically significant (as in insensitive parts of the parameter space)

Forecasts

- Sample 1954Q3-1997Q4 evaluated 1998Q1-2007Q4 to exclude crisis
- Sample 1954Q3-2000Q4Q4 evaluated 2001Q1-2010Q2 to include crisis
- Recursive estimates starting 1954Q3-1997Q4 recursively evaluated 1998Q1-2010Q2

RMSFE 1998Q1-2007Q4

- Multi-step outperforms benchmark in most cases, some cases by large margins

DM stats 1998Q1-2007Q4

- Might hypothesise that forecast performance at \( h \) optimised using \( h \)-step errors, but not so

RMSFE 2001Q1-2010Q2

- Less good post-crisis but multi-step still better

DM stats 2001Q1-2010Q2

- Again multi-step best performer

DM stats rolling 1998Q1-2010Q2

- Rather less significant cases

Why does this work?

- In general, introducing additional moment conditions
- As parameter estimates largely unchanged, we hypothesise improvement due partly to improved estimate of the state

Monte Carlo

- 200 data points are simulated from true DGP
- Add noise to the observed variables (noise to signal ratios of 0.5, 1.0 and 2.0)
- Estimate the state vector for \( h = 1, 4 \) and 8
- Calculate MSFE (between the actual state observations and the smoothed estimates) for last 50 periods and repeat 1000 times

\[
\begin{array}{lcccc}
\text{Noise to Signal Ratio} & \text{Relative Mean Square Error} & \text{MSFE}(h=1) & \text{MSFE}(h=4) & \text{MSFE}(h=8) \\
0.5 & 0.991 & 0.975 & 0.968 & 0.968 \\
1.0 & 0.987 & 0.968 & 0.968 & 0.968 \\
2.0 & 0.932 & 0.894 & 0.894 & 0.894 \\
\end{array}
\]

- As \( h \) rises estimates improve due to cross-equation restrictions
- As noise to signal ratio increases these restrictions become more important
- Expanding evaluation period from 50 to 200 performance increases dramatically - evaluation over entire sample even more so
- The additional cross-equation restrictions particularly useful for estimation of initial values of state vector

Paper and code

- Published in Economic Letters as A New Approach to Multi-Step Forecasting using Dynamic Stochastic General Equilibrium Models - longer version in BoE WP series
- Code available from Kostas’ site:
  [https://sites.google.com/site/konstantinostheodoridis/publications](https://sites.google.com/site/konstantinostheodoridis/publications)

Conclusions

- If concerned with forecasts as well as structure, may help to optimise over predictive power
- Turns out to improve forecasts with minimal changes to structure
- In most cases RRMSFE improved, in many cases significantly