Highlights

- We introduce a new model for multivariate covariance dynamics based on long-memory behavior of daily returns and daily realized covariance kernels.
- In addition, the model takes into account fat-tailedness in both returns and realized kernels by assuming a Multivariate Student-t distribution for returns and a matrix-F distribution for realized kernels.
- We apply our model on a panel of 15 equities listed at the S&P 500 index from 2001-2012.
- The results show the new fractionally integrated model both statistically and economically outperforms recent alternatives such as the Multivariate HEAVY model (Noureldin et al. 2012) and the Riskmetrics 2006 methodology.

The Multivariate FIGAS model

Our contribution: we connect long memory behavior of both returns and realized measures with their fat-tailedness property by means of the FIGAS tF model. Denote \( y_t \) as a vector of \( k \) returns, and \( R_t \) as a \( k \times k \) realized covariance kernel, specified as:

\[
(1 - L)^d V_t = \Omega + B (1 - L)^d Y_t + A Y_t
\]

where the time-varying conditional covariance matrix is modeled as a FIGAS process:

\[
(1 - L)^d V_t = \Omega + B (1 - L)^d Y_t + A Y_t
\]

with \( L \) the lag operator and \( (1 - L)^d \) the fractional difference operator, defined as

\[
(1 - L)^d = 1 - d \frac{L^1 + \cdots + L^d}{d!} - \cdots
\]

for \( d > -1 \). Further, \( A \) and \( B \) are scalars, and \( x_t \) denotes the scaled score:

\[
x_t = V_t (V_t + 2 \Omega)^{-1} V_t
\]

which depends on the partial derivative of the logarithm of the fat-tailed Multivariate Student-t \( (\nu_0) \) and Matrix-F \( (\nu_1, \nu_2) \) distribution with respect to \( V_t \):

\[
\nabla x_t = \left( \nu_0 + k \nu_0 - 2 + y_t y_t' V_t^{-1} y_t \right) V_t^{-1}
\]

with \( w_t = \frac{\nu_0 + k}{\nu_0} \).

Interpretation of the score:

- Impact of “large values” of \( y_t y_t' \) on \( V_t \) is downweighted by \( w_t \) if density for \( y_t \) is fat-tailed (i.e. \( \nu_0 / \nu_0 > 0 \)).
- Likewise, the inverse term in \( \nabla R_t \), measured by \( V_t^{-1} R_t V_t \), do not automatically lead to substantial changes in the covariance matrix \( V_t \).

Estimation

We estimate the FIGAS tF model by Maximum Likelihood and compare our model against the GAS tF (Janus et al. 2014) M-HEAVY (Noureldin et al. 2012) and the Riskmetrics 2006 models.

Data: 15 assets from S&P 500, from January 2, 2001 until December 30, 2012 (3,017 observations).

Motivation/Literature

Volatility is persistent. Baillie et al. (1996) introduce the Fractionally Integrated GARCH model (FIGARCH) using returns.

Realized measures are highly persistent (Andersen et al. 2001) → HAR model (Corsi, 2009), ARFIMA models (Univariate: Koopman et al. 2005, Multivariate: Chiriac and Voev, 2011).

Important aspect of returns and realized measures: they are fat-tailed and may contain outliers. This has not been taken into account yet by the literature on long-memory volatility models.

In-sample results

- We forecast a 15 x 15 covariance matrix 1, 5, 10, and 22 steps ahead, based on a MW-approach with \( T_w = 1500 \).
- Statistical application: test on predictive ability between models based on the QLIK loss function and the log-score (i.e. density forecasts).
- Economic application: Global Minimum Variance (GMV) weights and test on the difference of the ex-post conditional portfolio standard deviation.

Out-of-sample analysis

- We forecast a 15 x 15 covariance matrix 1, 5, 10, and 22 steps ahead, based on a MW-approach with \( T_w = 1500 \).
- Statistical application: test on predictive ability between models based on the QLIK loss function and the log-score (i.e. density forecasts).
- Economic application: Global Minimum Variance (GMV) weights and test on the difference of the ex-post conditional portfolio standard deviation.