Inflation Variability and the Level of Inflation

Paul Ho and Mark Watson
Princeton University

– Work-in-Progress –

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Does the variability of inflation increase when the level of inflation increases?

Old Question:


Issues:

(i) What variability? (Anticipated/Unanticipated, Long-run/Short-run, etc.)

(ii) What variation in level? (Cross-section, time-series, etc.)
Ball and Cecchetti (BPEA 1990) "Inflation and Uncertainty at Short and Long Horizons" …

(1) \[ \pi_t = \hat{\pi}_t + \eta_t, \]
(2) \[ \hat{\pi}_t = \hat{\pi}_{t-1} + \epsilon_t, \]

(10) \[ \sigma^2_\epsilon(t) = \beta_0 + \beta_1 \hat{\pi}_{t-1}; \]
(11) \[ \sigma^2_\eta(t) = \delta_0 + \delta_1 \hat{\pi}_{t-1}. \]

Their findings:

- US: \( \beta_1 > 0, \delta_1 \approx 0 \)
- Different for some other countries

Our contribution: Revisit, augmenting their analysis with (a) more data and (b) improved filtering tools.
Framework: Univariate UCSV model (with outlier adjustment: SW(2015))

\[ \pi_t = \tau_t + \varepsilon_t \]

\[ \tau_t = \tau_{t-1} + \sigma_{\Delta \tau} \times \eta_{\tau,t} \]

\[ \varepsilon_t = \sigma_{\varepsilon} \times s_t \times \eta_{\varepsilon,t} \]

\[ \Delta \ln(\sigma_{\varepsilon,t}^2) = \gamma_{\varepsilon} \nu_{\varepsilon,t} \]

\[ \Delta \ln(\sigma_{\Delta \tau,t}^2) = \gamma_{\Delta \tau} \nu_{\Delta \tau,t} \]

(\( \eta_{\varepsilon}, \eta_{\tau}, \nu_{\varepsilon}, \nu_{\Delta \tau} \)) are iidN(0, I_4)

\( s_t = \text{i.i.d. multinomial with values 1, 5, 10} \)

and probability 0.975, 1/60, and 1/120
US Inflation

(a) Inflation and $\tau_i$

(b) $\sigma_{\Delta\tau, t}$

(c) $\sigma_{\epsilon, t}$
UCSV model with regressors in volatility process

\[ \pi_t = \tau_t + \varepsilon_t \]

\[ \tau_t = \tau_{t-1} + \sigma_{\Delta\tau,t} \times \eta_{\tau,t} \]

\[ \varepsilon_t = \sigma_{\varepsilon,t} \times s_t \times \eta_{\varepsilon,t} \]

\[ \ln(\sigma_{\varepsilon,t}^2) = x_t' \delta + \xi_{\varepsilon,t} \quad \text{with} \quad \Delta \xi_{\varepsilon,t} = \gamma_{\varepsilon} \nu_{\varepsilon,t} \]

\[ \ln(\sigma_{\Delta\tau,t}^2) = x_t' \beta + \xi_{\Delta\tau,t} \quad \text{with} \quad \Delta \xi_{\Delta\tau,t} = \gamma_{\Delta\tau} \nu_{\Delta\tau,t} \]

\((\eta_{\varepsilon}, \eta_{\tau}, \nu_{\varepsilon}, \nu_{\Delta\tau})\) are iidN(0, I_4)

\[ x_t = \tau_{t-1} \quad \text{(Ball-Cecchetti:} \quad \beta > 0, \quad \delta \approx 0 \ ?) \]
Implications for forecasting:

\[ \pi_{t+h} = (\tau_{t+h} - \tau_t) + \tau_{t|t} + (\tau_t - \tau_{t|t}) + \varepsilon_{t+h} \]

\[ \tau_{t+h} - \tau_t = \sum_{i=1}^{h} \sigma_{\Delta \tau, t+i} \eta_{\tau, t+i} \]

\[ \ln(\sigma_{\Delta \tau, t}^2) = x_t' \beta + \xi_{\Delta \tau, t} \quad \text{with} \quad \Delta \xi_{\Delta \tau, t} = \gamma_{\Delta \tau} \nu_{\Delta \tau, t} \]

\[ (\eta_{\tau, t}, \nu_{\Delta \tau, t}) \sim \text{iidN}(0, I_2) \]

Three cases

(i) \( \gamma_{\Delta \tau} = \beta = 0 \)

(ii) \( \gamma_{\Delta \tau} > 0, \beta = 0 \)

(iii) \( \gamma_{\Delta \tau}, \beta > 0 \)
Illustrative Predictive Densities

2-year ahead predictive densities

\[ \gamma_{\Delta \tau} = 0, \beta = 0 \]
Illustrative Predictive Densities

2-year ahead predictive densities

- $\gamma_{\Delta \tau} = 0, \beta = 0$
- $\gamma_{\Delta \tau} = 1, \beta = 0$
Illustrative Predictive Densities
Some empirical results

\[ \ln(\sigma_{\epsilon,t}^2) = x_t' \delta + \xi_{\epsilon,t} \quad \text{with} \quad \Delta \xi_{\epsilon,t} = \gamma_{\epsilon} \nu_{\epsilon,t} \]

\[ \ln(\sigma_{\Delta \tau,t}^2) = x_t' \beta + \xi_{\Delta \tau,t} \quad \text{with} \quad \Delta \xi_{\Delta \tau,t} = \gamma_{\Delta \tau} \nu_{\Delta \tau,t} \]

(We'll set \( \delta = 0 \) for the preliminary results presented here)
\[
\ln(\sigma^2_{\Delta \tau, t}) = x_t' \beta + \xi_{\Delta \tau, t} \quad \text{with} \quad \Delta \xi_{\Delta \tau, t} = \gamma_{\Delta \tau} v_{\Delta \tau, t}
\]

\[x_t = \tau_{t-1}\]
UCSV and $\beta \neq 0$
UCSV and $\beta \neq 0$
Alternative Regressors

\[
\ln(\sigma_{\Delta \tau, t}^2) = x_t'\beta + \xi_{\Delta \tau, t} \quad \text{with} \quad \Delta \xi_{\Delta \tau, t} = \gamma_{\Delta \tau} \nu_{\Delta \tau, t}
\]

\[
x_t = (\tau_{t-1} + \tau_{t-2} + \ldots + \tau_{t-h})/h
\]

so \(
\Delta \ln(\sigma_{\Delta \tau, t}^2) = \beta (\tau_{t-1} - \tau_{t-h-1})/h + \gamma_{\Delta \tau} \nu_{\Delta \tau, t}
\)
Posteriors for $\beta$ with different values of $h$
Model Selection

Consider $n$ models:

**Common features:**

- $\pi_t = \tau_t + \varepsilon_t$
- $\tau_t = \tau_{t-1} + \sigma_{\Delta \tau,t} \times \eta_{\tau,t}$
- $\varepsilon_t = \sigma_{\varepsilon,t} \times s_t \times \eta_{\varepsilon,t}$
- $\Delta \ln(\sigma^2_{\varepsilon,t}) = \gamma_{\varepsilon} \nu_{\varepsilon,t}$

**Differences:**

Model $i$ ($M_i$): \[ \ln(\sigma^2_{\Delta \tau,t}) = x_{i,t}' \beta_i + \xi_{\Delta \tau,t} \] with $\Delta \xi_{\Delta \tau,t} = \gamma_{\Delta \tau} \nu_{\Delta \tau,t}$

Let $Y = \{\pi_t\}_{t=1}^T$.
We want to calculate $P(M_i|Y)$ for $i = 1, \ldots, n$. 
Let \( Z = \left\{ \tau_t, \ln(\sigma_{\Delta \tau, t}^2) \right\}_t^T \) and \( Y = \left\{ \pi_t \right\}_t^T \)

Note:

(1) \( \Pr(M_i | Y) = \int \Pr(M_i | Y, Z) f(Z | Y) \, dZ \)

(2) \( \Pr(M_i | Y, Z) = \Pr(M_i | Z) \)

(3) \( f(Z | Y) = \sum_{j=1}^n f(Z | Y, M_j) \Pr(M_j | Y) \)

\[
\Pr(M_i | Y) = \sum_{j=1}^n \left[ \int \Pr(M_i | Z) f(Z | Y, M_j) \, dZ \right] \Pr(M_j | Y)
\]

Thus:

\[
= \sum_{j=1}^n a_{ij} \Pr(M_j | Y), \text{ where } a_{ij} = \mathbb{E} \left[ \Pr(M_i | Z) | Y, M_j \right]
\]
Stacking: \( P(M \mid Y) = A \; P(M \mid Y) \)

so \( P(M \mid Y) \) is an eigenvector of \( A \) corresponding to a unit eigenvalue.

Estimation:

\[
\hat{a}_{ij} = \frac{1}{n_j} \sum_{k=1}^{n_j} P(M_i \mid Z_k) \quad \text{with} \quad Z_k \sim f(Z \mid Y, M_j)
\]
Model probabilities: \( x_t = \frac{x_t}{h} = (\tau_{t-1} + \tau_{t-2} + \ldots + \tau_{t-h})/h \)

\[
P(M_h | Y) \]

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Compare model \( M_h \) (with \( h = 4 \)) to UCSV model

\[
P(M_4 | Y) \quad P(UCSV | Y) \]

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<td>0.74</td>
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Implications for forecasting:

\[
\pi_{t+h} = (\tau_{t+h} - \tau_t) + \tau_{t|t} + (\tau_t - \tau_{t|t}) + \epsilon_{t+h}
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\[
\tau_{t+h} - \tau_t = \sum_{i=1}^{h} \sigma_{\Delta\tau, t+i} \eta_{\tau, t+i}
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\[
\ln(\sigma^2_{\Delta\tau, t}) = x_t'\beta + \xi_{\Delta\tau, t} \quad \text{with} \quad \Delta\xi_{\Delta\tau, t} = \gamma_{\Delta\tau} \nu_{\Delta\tau, t}
\]

\[
(\eta_{\tau, t}, \nu_{\Delta\tau, t}) \sim \text{iidN}(0, I_2)
\]

Three Five cases

(i) \( \gamma_{\Delta\tau} = \beta = 0 \)

(ii) \( \gamma_{\Delta\tau} > 0, \beta = 0 \) (Posterior Median from UVSCO)

(iii) \( \gamma_{\Delta\tau}, \beta > 0 \) (Posterior Median from \( h = 4 \) model)

(iv)-(v) Same as (ii) and (iii) but with parameter uncertainty, 

\[ [\sigma_{\epsilon, T}, \sigma_{\Delta\tau, T}, \gamma_{\epsilon}, \gamma_{\Delta\tau}, \beta]. \]
(ii) $\gamma_{\Delta \tau} > 0, \; \beta = 0$  (Posterior Median from UVSCO)

(iii) $\gamma_{\Delta \tau}, \beta > 0$  (Posterior Median from $h = 4$ model)
(iv)-(v) Same as (ii) and (iii) but with parameter uncertainty, 
\[\sigma_{\epsilon T}, \sigma_{\Delta \tau T}, \gamma_{\epsilon}, \gamma_{\Delta \tau}, \beta\].
A quick tour of Germany, Italy, and Sweden

(Data 1960 – 2014)
Germany

(a) Inflation and $\tau_t$

(b) $\sigma_{\Delta \tau, t}$

(c) $\sigma_{\epsilon, t}$
Germany

(a) $\tau_i$

(b) $\sigma_{\Delta \tau, i}$
Germany
Italy

(a) Inflation and $\tau_i$

(b) $\sigma_{\Delta \tau_i}$

(c) $\sigma_{\epsilon_i}$
Italy

(a) $\tau_i$

(b) $\sigma_{\Delta\tau, t}$
Italy
Sweden

(a) Inflation and $\tau_I$

(b) $\sigma_{\Delta \tau, t}$

(c) $\sigma_{\epsilon, t}$

1960 1980 2000 2020

1960 1980 2000 2020

33
Sweden

(a) $\tau_i$

(b) $\sigma_{\Delta \tau, i}$
Sweden

\[ \beta \]

\[ \gamma_{\Delta \tau} \]

- Blue: UCSVO (\( \beta = 0 \))
- Red: \( \beta \neq 0 \)
Ball and Cecchetti (BPEA 1990) "Inflation and Uncertainty at Short and Long Horizons" …

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