

Empirical Properties of Inflation Expectations and the Zero Lower Bound*

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Abstract

Empirical studies analyzing survey data on inflation expectations find that inflation expectations adjust slowly after shocks and that agents have heterogeneous inflation expectations. In models with a zero lower bound on the nominal interest rate currently used for policy analysis, inflation expectations adjust instantly and are homogeneous. Motivated by this tension, the paper solves a New Keynesian model with a zero lower bound and dispersed information on the household side. The information friction changes significantly results regarding shock propagation and policy effectiveness: (1) the deflationary spiral in bad states of the world is less severe, (2) central bank communication about the economy's current state affects consumption (and the sign of this effect reverses when the zero lower bound is binding), (3) forward guidance is less powerful, and (4) the government spending multiplier is smaller. These effects are stronger in states of the world that have a smaller prior probability.

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1 Introduction

Two central questions in Macroeconomics are: how are shocks propagated when the zero lower bound on the nominal interest rate is binding? What policies are effective under these circumstances? The main policy rate of the central bank is currently at zero (or close to zero) in 22 out of 34 OECD countries.¹ Furthermore, unless central banks significantly raise their inflation targets, policy rates are likely to be at zero again in the future.

In New Keynesian models with a binding zero lower bound, movements in household inflation expectations are of great importance for shock propagation and policy effectiveness. Consumption must satisfy the Euler equation for a short-term bond. When monetary policy is constrained by the zero lower bound, the short-term nominal interest rate is constant. For ease of exposition, suppose that the zero lower bound is known to bind for at least T periods and consider the log-linearized Euler equation for the short-term bond. Solving this Euler equation forward implies that current consumption depends on a constant, expected inflation over the next T periods, and expected consumption in period $T+1$. Hence, all the amplification of the shock comes from changes in expected inflation over the next T periods and changes in expected consumption in period $T+1$. Furthermore, government policies like forward guidance or increases in government purchases affect current consumption by changing households' inflation expectations and/or expected consumption in period $T+1$.

It is therefore desirable to model inflation expectations in a way that is consistent with data. Recent papers studying survey data on inflation expectations find that agents' average inflation expectation responds sluggishly to realized shocks to future inflation (Coibion and Gorodnichenko, 2012) and that agents have heterogeneous inflation expectations (see, e.g., Armantier et al., 2011). By contrast, in any model with full information and rational expectations, agents' inflation expectation responds instantly and one-for-one to any realized shock to future inflation, because the shock is in the information set of the agents and agents know how the shock affects future inflation. In addition, all agents have the same expectation of aggregate inflation, because all agents have the same information set and the same perceived law of motion for inflation.

Motivated by the frequent use of New Keynesian models with a zero lower bound in policy analysis, the importance of household inflation expectations in those models, and the tension be-

¹The main policy rate is weakly smaller than 25 basis points in 22 out of 34 OECD countries.

tween model properties and data properties of inflation expectations, this paper studies a New Keynesian model with a zero lower bound and dispersed information on the household side. The assumptions that households have less than perfect information and households have different pieces of information yield the slow adjustment and the dispersion of inflation expectations. It turns out that this model with sluggish and dispersed household inflation expectations has quite different implications for shock propagation and policy effectiveness.

First of all, the deflationary spiral in bad states of the world is less severe than under perfect information. The slow adjustment of household inflation expectations implies that consumption falls slowly. This effect is amplified by the fact that consumption choices of different households are strategic complements when the zero lower bound is binding.²

Second, central bank communication about the state of the economy (without any change in current or future policy) affects aggregate consumption, and the sign of this effect changes when the zero lower bound is binding. The reason for the sign change is that downward movements in household inflation expectations are stabilizing when the zero lower bound is not binding, while they are destabilizing when the zero lower bound is binding. For comparison, in a standard New Keynesian model, central bank communication about the current state has no effect on consumption, because the current state is common knowledge.

Third, a central bank commitment to holding the policy rate at the zero lower bound for longer beyond what is justified by contemporaneous economic conditions (“forward guidance”) has smaller effects on consumption than under perfect information (and can even reduce current consumption). The reason is twofold. Households that do not update their inflation expectations do not change their consumption. Households that do update their inflation expectations experience a positive and a negative effect on their inflation expectations. On the one hand, future monetary policy is expected to be more expansionary (if the announcement is credible). On the other hand, today’s commitment to such a policy may reveal that the economy is *currently* in a bad state of the world, and the sluggishness of household inflation expectations implies that households’ average inflation expectation is above the true conditional mean of future inflation in bad states of the world.³

Fourth, the government spending multiplier is smaller than under perfect information. The

²Consumption choices of different households are strategic substitutes when the zero lower bound is not binding.

³This negative effect on household inflation expectations is probably more pronounced when the central bank justifies the policy with deflationary pressures.

reason is twofold. Households that do not update their inflation expectations do not change their consumption. Households that do update their inflation expectations in response to the policy announcement experience a positive effect and a negative effect on their inflation expectations. State-contingent expansionary fiscal policy raises inflation in a given state and reveals the state.

Fifth, if individual beliefs about inflation cover regions where the zero lower bound is non-binding and regions where the zero lower bound is binding, then individual consumption depends on several moments of the conditional distribution of inflation and aggregate consumption is not simply a function of the average inflation expectation. The reason is the kink in the monetary policy rule due to the lower bound on the nominal interest rate.

Finally, the deviations from the perfect-information equilibrium are larger in states of the world that have a smaller prior probability. I therefore believe the results listed above are particularly relevant for thinking about the Great Recession in the U.S. and the sovereign debt crisis in Europe.

In a calibrated version of the model, the response of consumption on impact of a large, negative shock equals about 1/3 of the value under perfect information (while the cumulative response of consumption equals about 1/2 of the value under perfect information). Here the parameter controlling the speed at which households update inflation expectations over time is chosen so as to match the speed at which the gap between inflation and inflation expectations closes in the data, according to the estimates by Coibion and Gorodnichenko (2012).

Literature review: My work builds on the existing literature on the implications of the zero lower bound for shock propagation and policy effectiveness. In contrast to all the existing literature on the zero lower bound, households' short-term inflation expectations are sluggish and dispersed.⁴ In terms of modelling, I follow closely the seminal contributions on the zero lower bound (e.g., Krugman, 1998, Eggertsson and Woodford, 2003) to isolate the implications of slow adjustment and dispersion of household inflation expectations.

The part of the paper on forward guidance is related to the emerging literature on the for-

⁴Andrade et al. (2015) present a model with heterogeneous *medium-term* inflation expectations. All agents agree that the nominal interest rate will equal zero for exactly N periods, but agents disagree about the reason. Pessimists believe that fundamentals revert back to normal after N periods and the central bank is of the no-commitment type. Optimists believe that fundamentals revert back to normal after $N' < N$ periods and the central bank is of the commitment type. In that model, all agents have the same, correct beliefs about the path of the economy up to period N' . The two groups only have different beliefs about the path of the economy in periods N' to N .

ward guidance puzzle. This literature argues that the effects of forward guidance in benchmark New Keynesian models seem unreasonably large and offers explanations for weaker effects of forward guidance (Carlstrom, Fuerst, and Paustian, 2012, Del Negro, Giannoni, and Patterson, 2012, McKay, Nakamura, and Steinsson, 2015).

The part of the paper on fiscal policy is related to the literature on the government spending multiplier at the zero lower bound (e.g., Christiano, Eichenbaum, and Rebelo, 2011, Woodford, 2011), especially the work arguing that the government spending multiplier at the zero lower bound may not be as large as predicted by the benchmark New Keynesian model with a zero lower bound, e.g., because of distortionary taxation (Uhlig and Drautzburg, 2013) or a non-fundamental liquidity trap (Mertens and Ravn, 2014).

The paper is related to the literature on business cycle models with information frictions on the household side (Mankiw and Reis, 2006, Lorenzoni, 2009, Angeletos and La'O, 2013, Maćkowiak and Wiederholt, 2015). In contrast to the existing work on this topic, I study the implications of the zero lower bound. This difference turns out to be important, e.g., because at the zero lower bound, movements in household inflation expectations are destabilizing (rather than stabilizing), actions of different households are strategic complements (rather than strategic substitutes), and the monetary policy rule has a kink.

My work also builds on the empirical literature on inflation expectations (e.g., Coibion and Gorodnichenko, 2012 and 2015, Armantier et al., 2011), which motivated this paper.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 states the optimality conditions of households and firms. Section 4 solves a special case of the model analytically. The main results can be seen directly from this analytical solution. Section 5 solves a calibrated version of the model numerically and presents quantitative results. Section 6 discusses forward guidance, Section 7 discusses the government spending multiplier, and Section 8 concludes.

2 Model

The economy is populated by households, firms, and a government. The government consists of a monetary authority and a fiscal authority. The model setup is close to a standard New Keynesian model with a zero lower bound (in particular, Eggertsson and Woodford, 2003), but in contrast

to all the existing literature on the zero lower bound households' short-term inflation expectations adjust slowly to shocks and are dispersed.

Households. The economy is populated by a continuum of households of mass one. Households are indexed by $i \in [0, 1]$. The preferences of household i are given by

$$E_0^i \left[\sum_{t=0}^{\infty} \beta^t e^{\xi_{i,t}} \left(\frac{C_{i,t}^{1-\gamma} - 1}{1-\gamma} - N_{i,t} \right) \right],$$

where $C_{i,t}$ is consumption of the household in period t , $N_{i,t}$ is labor supply of the household in period t , and $\xi_{i,t}$ is a preference shock. Here E_0^i is the expectation operator conditioned on the information of the household in period zero. The parameter $\beta \in (0, 1)$ is a discount factor and $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution.

Following Eggertsson and Woodford (2003), I study the response of the economy to a temporary increase in households' desire to save. In period zero, each household is hit by a preference shock $\xi_{i,0} \in \{\xi_L, \xi_H\}$ with $\xi_L < \xi_H < 0$. To obtain a closed-form solution of the model, I initially assume stochastic decay of the preference shock. That is, in every period $t \geq 1$, $\xi_{i,t} = \xi_{i,t-1}$ with probability μ and $\xi_{i,t}$ returns permanently to its normal value of zero with probability $1 - \mu$. The return to the normal value of zero occurs at the same time for all households. Once the main properties of the model have been shown formally, I also solve the model numerically with deterministic decay. That is, in periods $t \geq 1$, $\xi_{i,t} = \rho \xi_{i,t-1}$ with $\rho \in (0, 1)$.

In contrast to the existing literature, there are two possible aggregate exogenous states in period zero and there is heterogeneity across households in the value of the preference shock. Let λ denote the mass of households with $\xi_{i,0} = \xi_H$. Let $1 - \lambda$ denote the mass of households with $\xi_{i,0} = \xi_L$. The two possible aggregate exogenous states in period zero differ in terms of the mass of households who experience the high realization of the preference shock: $\lambda \in \{\lambda_{bad}, \lambda_{good}\}$ with $0 < \lambda_{bad} < \lambda_{good} < 1$. To study the implications of imperfect information about the aggregate state, one has to introduce at least two possible aggregate states. To ensure that the own preference shock does not perfectly reveal the aggregate state, one has to assume that each realization of the preference shock is possible in both aggregate states. In the following, think of the good state as an aggregate shock that would cause a severe recession under perfect information. Think of the bad state as an aggregate shock that would cause the worst recession in a century under perfect information. Let $\theta \in (0, 1)$ denote

the prior probability of the good state.⁵

Households can save or borrow by holding (positive or negative amounts of) nominal government bonds. Let $B_{i,t}$ denote the bond holdings of household i between periods t and $t+1$. The evolution of the bond holdings of household i is given by

$$B_{i,t} = R_{t-1}B_{i,t-1} + W_{i,t}N_{i,t} + D_{i,t} - P_tC_{i,t} + Z_{i,t}.$$

Here R_{t-1} denotes the gross nominal interest rate on bond holdings between periods $t-1$ and t , $W_{i,t}$ is the nominal wage rate for labor supplied by household i in period t , and $D_{i,t}$ denotes the difference between dividends received by the household in period t and nominal lump-sum taxes paid by the household in period t . The term $P_tC_{i,t}$ is the household's consumption expenditure, where P_t denotes the price of the final good in period t . The term $Z_{i,t}$ is a net transfer that is specified below. The household can save or borrow (i.e., bond holdings can be positive or negative), but the household cannot run a Ponzi scheme. All households have the same initial bond holdings in period minus one.

For simplicity, I assume that households can trade state-contingent claims with one another in period minus one (i.e., when all households are still ex-ante identical). Each household is hit by a preference shock in period zero and the preference shocks of all households revert permanently back to zero in a stochastic period, denoted T . The contingent claims are settled in that period $T \geq 1$. A state-contingent claim specifies a payment to the household who purchased the claim that is contingent on the individual history of the household and the aggregate history of the economy (i.e., the claim is contingent on $\xi_{i,0}$, λ , and T). The term $Z_{i,t}$ in the flow budget constraint is the net transfer associated with these state-contingent claims. This term equals zero in all periods apart from period T . The fact that agents can trade these state-contingent claims in period minus one implies that in equilibrium all households will have the same post-transfer wealth in period T . This simplifies the analysis: to solve for consumption of each household one does not have to keep track of the dynamics of the wealth distribution in periods $0 \leq t \leq T-1$. A similar assumption is made in Lucas (1990), Lorenzoni (2010), and Curdia and Woodford (2011).

Finally, let us turn to how households form their expectations. I assume that households have correct prior beliefs about the probability of the good state, the distribution of types in the different

⁵Throughout the paper, the recession is more severe in the state with the lower λ . Furthermore, the efficient allocation will feature no recession. I therefore refer to the state with the lower λ as the bad state.

exogenous states, and the dynamics of inflation in the good state and the bad state. In period zero, each household observes the realization of the own preference shock and updates his or her beliefs about the current aggregate state and future inflation using Bayes' rule. Households that experience different local conditions (here different preference shocks) form different beliefs about inflation. Thereafter, there is slow updating of beliefs, as in Mankiw and Reis (2002, 2006). In every period $0 \leq t < T$, a constant fraction $\omega \in [0, 1]$ of randomly selected households learns the exact size of the shock that hit the economy in period zero (e.g., by processing information contained in the news media or prices) and moves to full-information rational expectations (FIRE) about inflation. The remaining households do not update their beliefs about inflation. In the calibrated version of the model, I set the value of ω so as to match the speed at which the gap between future inflation after a shock and the average expectation of future inflation after the same shock closes in the data (according to the estimates by Coibion and Gorodnichenko (2012)).

The idea is *not* that information is not publicly available. The idea is that processing information is costly and therefore in every period only a fraction of households update their beliefs about the current state of the economy and future inflation. The standard assumption is $\omega = 1$, i.e., all households move *instantly* to full-information rational expectations about inflation, but this assumption is inconsistent with both the slow adjustment and the dispersion of household inflation expectations in the data.

Firms. There are final good firms and intermediate good firms. To illustrate as clearly as possible the effects of household imperfect information, firms are assumed to have perfect information.⁶ The final good is produced by competitive firms using the technology

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{\psi}{\psi-1}},$$

where Y_t denotes output of the final good, $Y_{j,t}$ is input of intermediate good j , and $\psi > 1$ is the elasticity of substitution between intermediate goods. Final good firms have fully flexible prices. Profit maximization of final good firms implies the following demand function for good j

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\psi} Y_t,$$

⁶I use the terms “perfect information” and “full information” interchangeably. An agent has perfect information if in every period t the agent knows the entire history of the economy up to and including period t .

where $P_{j,t}$ denotes the price of intermediate good j and P_t is the price of the final good. Furthermore, the zero profit condition of final good firms implies

$$P_t = \left(\int_0^1 P_{j,t}^{1-\psi} dj \right)^{\frac{1}{1-\psi}}.$$

The intermediate good j is produced by a monopolist using the technology

$$Y_{j,t} = N_{j,t}^\varrho \quad \text{with} \quad N_{j,t} = \left(\int_0^1 N_{i,j,t}^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}.$$

Here $Y_{j,t}$ is output, $N_{j,t}$ is composite labor input, and $N_{i,j,t}$ is type i labor input of monopolist j . Type i labor is labor supplied by household i . The parameter $\varrho \in (0, 1]$ is the elasticity of output with respect to composite labor and $\eta > 1$ is the elasticity of substitution between types of labor. Cost minimization implies that the demand for type i labor of monopolist j is given by

$$N_{i,j,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\eta} N_{j,t},$$

where

$$W_t = \left(\int_0^1 W_{i,t}^{1-\eta} di \right)^{\frac{1}{1-\eta}}.$$

Furthermore, cost minimization implies that the wage bill of monopolist j in period t equals $W_t N_{j,t}$. Monopolists producing intermediate goods are subject to a price-setting friction, as in Calvo (1983). Each monopolist can optimize its price with probability $1 - \alpha$ in any given period. With probability α the monopolist producing good j sets the price

$$P_{j,t} = P_{j,t-1}.$$

How monopolists value profit in different states of the world is determined by the ownership structure. I assume that each monopolist is owned by a single household and takes the household's marginal utility of consumption as given, because the household also owns many other firms.⁷

Monetary policy. The monetary authority sets the gross nominal interest rate according to the rule

$$R_t = \max \left\{ 1, R\Pi_t^\phi \right\},$$

⁷To ensure that households are ex-ante identical in period minus one, ownership is assigned randomly in period zero. The individual history of a household in period T then consists of the realization of the preference shock and the realization of ownership in period zero. Since the state-contingent claims specify a payment that is contingent on the individual history, in equilibrium all households have the same post-transfer wealth in period T .

where $R = (1/\beta)$ denotes the nominal interest rate in the non-stochastic steady state with zero inflation, $\Pi_t = (P_t/P_{t-1})$ is the inflation rate, and $\phi > 1$ is a parameter. According to the last equation, the monetary authority follows a Taylor rule as long as the implied net nominal interest rate is non-negative and the monetary authority sets the net nominal interest rate to zero otherwise.

Fiscal policy. The fiscal authority can purchase units of the final good and can finance these purchases with current lump-sum taxes or future lump-sum taxes. The government flow budget constraint in period t reads

$$T_t + B_t = R_{t-1}B_{t-1} + P_tG_t.$$

The government has to finance maturing nominal government bonds and any purchases of the final good, denoted G_t . The government can collect lump-sum taxes, denoted T_t , or issue new bonds.

Until Section 7, $G_t = 0$ in every period. In Section 7, there are government purchases and I study the size of the government spending multiplier. A change in the path of government purchases is assumed to imply a change in the path of lump-sum taxes so as to maintain intertemporal government solvency.

3 Household and firm optimality

This section states the optimality conditions of households and firms and derives the New Keynesian Phillips curve for this economy. The New Keynesian Phillips curve is derived under two different assumptions about wage setting: households set real wage rates and households set nominal wage rates. In Section 4, I solve the model in closed form under the assumption that households set real wage rates and prove the main results. In Section 5, I solve the model numerically under the assumption that households set nominal wage rates and show that results get amplified.

Households. Let $\tilde{W}_{i,t} = (W_{i,t}/P_t)$ denote the real wage rate for type i labor. If households set real wage rates, the first-order conditions for consumption and the real wage rate read

$$C_{i,t}^{-\gamma} = E_t^i \left[\beta \frac{e^{\xi_{i,t+1}}}{e^{\xi_{i,t}}} \frac{R_t}{\Pi_{t+1}} C_{i,t+1}^{-\gamma} \right],$$

and

$$\tilde{W}_{i,t} = \frac{\eta}{\eta - 1} C_{i,t}^\gamma.$$

If households set nominal wage rates, the first-order condition for consumption remains unchanged and the first-order condition for the nominal wage rate reads

$$E_t^i \left[\frac{W_{i,t}}{P_t} \right] = \frac{\eta}{\eta - 1} C_{i,t}^\gamma.$$

Let small letters denote log-deviations from the non-stochastic steady state with zero inflation. Log-linearizing the consumption Euler equation around the non-stochastic steady state yields

$$c_{i,t} = E_t^i \left[-\frac{1}{\gamma} (\xi_{i,t+1} - \xi_{i,t} + r_t - \pi_{t+1}) + c_{i,t+1} \right]. \quad (1)$$

Log-linearizing the two wage setting equations around the non-stochastic steady state yields

$$\tilde{w}_{i,t} = \gamma c_{i,t}, \quad (2)$$

and

$$w_{i,t} = \gamma c_{i,t} + E_t^i [p_t]. \quad (3)$$

Firms. An intermediate good firm j that can adjust its price in period t and is owned by household i sets the price

$$X_{j,t}^i = \arg \max_{P_{j,t} \in \mathbb{R}_{++}} E_t \left[\sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left(\frac{e^{\xi_{i,s}} C_{i,s}^{-\gamma} P_t}{e^{\xi_{i,t}} C_{i,t}^{-\gamma} P_s} \right) \left(P_{j,t} \left(\frac{P_{j,t}}{P_s} \right)^{-\psi} Y_s - W_s \left(\left(\frac{P_{j,t}}{P_s} \right)^{-\psi} Y_s \right)^{\frac{1}{\varrho}} \right) \right].$$

Log-linearizing the first-order condition for the adjustment price around the non-stochastic steady state with zero inflation yields

$$x_{j,t}^i = (1 - \alpha\beta) E_t \left[\sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left(p_s + \frac{1}{1 + \frac{1-\varrho}{\varrho}\psi} (w_s - p_s) + \frac{\frac{1-\varrho}{\varrho}}{1 + \frac{1-\varrho}{\varrho}\psi} y_s \right) \right].$$

Note that the log-linearized adjustment price is independent of who owns the firm and is the same for all adjusting firms. Therefore, one can drop the superscript i and the subscript j . Furthermore, the last equation can be stated in recursive form as

$$x_t = (1 - \alpha\beta) \left(p_t + \frac{1}{1 + \frac{1-\varrho}{\varrho}\psi} (w_t - p_t) + \frac{\frac{1-\varrho}{\varrho}}{1 + \frac{1-\varrho}{\varrho}\psi} y_t \right) + \alpha\beta E_t [x_{t+1}].$$

New Keynesian Phillips curve: Log-linearizing the equation for the price of the final good given in Section 2 and using the fact that adjusting firms are selected randomly and the log-linearized adjustment price is the same for all firms yields

$$p_t = \int_0^1 p_{j,t} dj = \alpha p_{t-1} + (1 - \alpha) x_t.$$

Using the last equation to substitute for the adjustment prices x_t and x_{t+1} in the previous equation and rearranging yields

$$\pi_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \left(\frac{1}{1+\frac{1-\varrho}{\varrho}\psi} (w_t - p_t) + \frac{\frac{1-\varrho}{\varrho}}{1+\frac{1-\varrho}{\varrho}\psi} y_t \right) + \beta E_t [\pi_{t+1}]. \quad (4)$$

Finally, log-linearizing the equation for the wage index presented in Section 2 yields

$$w_t = \int_0^1 w_{i,t} di.$$

Let c_t denote aggregate consumption of the final good. Substituting the log-linearized wage index and the wage setting equation (2) into equation (4) and using $y_t = c_t$ yields the standard New Keynesian Phillips curve

$$\pi_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{\gamma + \frac{1-\varrho}{\varrho}}{1 + \frac{1-\varrho}{\varrho}\psi} c_t + \beta E_t [\pi_{t+1}]. \quad (5)$$

Using instead the wage setting equation (3) yields a modified version of the New Keynesian Phillips curve

$$\pi_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \left(\frac{\gamma + \frac{1-\varrho}{\varrho}}{1 + \frac{1-\varrho}{\varrho}\psi} c_t + \frac{1}{1 + \frac{1-\varrho}{\varrho}\psi} (\bar{E}_t [p_t] - p_t) \right) + \beta E_t [\pi_{t+1}], \quad (6)$$

where $\bar{E}_t [p_t] = \int_0^1 E_t^i [p_t] di$ denotes households' average expectation of the price level. The new term in the modified New Keynesian Phillips curve reflects the following effect: when households' expectations of the price level are above the price level, households set nominal wage rates that are too high, which raises marginal costs and inflation.

4 Analytical solutions

This section presents analytical solutions of the model. I first solve the model under the assumption of perfect information to obtain a benchmark for comparison and to illustrate that movements in household inflation expectations play a crucial role for the propagation of shocks. I then solve a special case of the model with dispersed information on the household side and show how sluggishness and dispersion of household inflation expectations changes the equilibrium.

4.1 Perfect information

Perfect information is a special case of the model: when $\omega = 1$ all households learn instantly the exact size of the aggregate shock that hit the economy in period zero. Building on the work by Eggertsson and Woodford (2003), Christiano, Eichenbaum, and Rebelo (2011), and many others, I initially assume that preference shocks decay stochastically and I consider equilibria with the following properties: consumption, inflation, and the nominal interest rate are constant from period zero until the preference shocks revert permanently back to zero, and the economy is in the non-stochastic steady state with zero inflation thereafter.

It is an equilibrium that all households have the same consumption level in period T because all households have the same post-transfer wealth in period T due to the trade in state-contingent claims in period minus one. Furthermore, it is straightforward to verify that $c_{i,t} = c_t = \pi_t = r_t = 0$ in every period $t \geq T$ satisfies the monetary policy rule given in Section 2, the consumption Euler equation (1) with $\xi_{i,t} = \xi_{i,t+1} = 0$, and the New Keynesian Phillips curve (5).

Let us turn to consumption, inflation, and the nominal interest rate in periods $0 \leq t \leq T - 1$. Since consumption, inflation, and the nominal interest rate are constant over time but depend on the size of the aggregate shock, I replace the time subscript t by the state subscript $s \in \{good, bad\}$. Furthermore, since households have perfect information and the preference shock, inflation, and consumption remain constant with probability μ and revert to their steady-state values of zero with probability $1 - \mu$, the consumption Euler equation (1) reduces to

$$c_{i,s} = -\frac{1}{\gamma} [(\mu - 1)\xi_{i,0} + r_s - \mu\pi_s] + \mu c_{i,s}.$$

Let $\bar{\xi}_s = \lambda_s \xi_H + (1 - \lambda_s) \xi_L$ denote the cross-sectional mean of the preference shock in state s . Integrating across households yields aggregate consumption in state s

$$c_s = -\frac{1}{\gamma} [(\mu - 1)\bar{\xi}_s + r_s - \mu\pi_s] + \mu c_s. \quad (7)$$

The New Keynesian Phillips curves (5) and (6) reduce to

$$\pi_s = \kappa c_s + \beta \mu \pi_s, \quad (8)$$

with

$$\kappa = \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} \frac{\gamma + \frac{1-\varrho}{\varrho}}{1 + \frac{1-\varrho}{\varrho}\psi}.$$

The monetary policy rule reads

$$r_s = \max \{-\ln(R), \phi\pi_s\}. \quad (9)$$

If the zero lower bound on the nominal interest rate is binding, then $\max \{-\ln(R), \phi\pi_s\} = -\ln(R)$. Substituting the New Keynesian Phillips curve (8) and the monetary policy rule (9) into the aggregated consumption Euler equation (7) and rearranging yields⁸

$$c_s = \frac{\frac{1}{\gamma}\bar{\xi}_s + \frac{1}{1-\mu}\ln(R)}{1 - \frac{1}{1-\mu}\frac{\kappa}{1-\beta\mu}}. \quad (10)$$

If the zero lower bound on the nominal interest rate is not binding, then $\max \{-\ln(R), \phi\pi_s\} = \phi\pi_s$. Substituting the New Keynesian Phillips curve (8) and the monetary policy rule (9) into the aggregated consumption Euler equation (7) and rearranging yields

$$c_s = \frac{\frac{1}{\gamma}\bar{\xi}_s}{1 + \frac{1}{1-\mu}\frac{\kappa}{1-\beta\mu}}. \quad (11)$$

Furthermore, the zero lower bound on the nominal interest rate is binding in state s if the cross-sectional mean of the shock is sufficiently negative: $\bar{\xi}_s < \bar{\xi}_{crit}$ with⁹

$$\bar{\xi}_{crit} = -\frac{1 + \frac{1}{1-\mu}\frac{\kappa}{1-\beta\mu}}{\frac{1}{1-\mu}\frac{\kappa}{1-\beta\mu}} \frac{1}{1-\mu} \ln(R). \quad (12)$$

An important insight in the literature on the zero lower bound is that the fall in consumption can be very large when the zero lower bound is binding. Formally, a positive denominator on the right-hand side of equation (10) in combination with the condition $\bar{\xi}_s < \bar{\xi}_{crit}$ implies a negative numerator on the right-hand side of equation (10). Moreover, the denominator on the right-hand side of equation (10) is a difference between two positive numbers that can be arbitrarily small in absolute value. Hence, even if the zero lower bound is only marginally binding in state s , consumption can fall by a very large amount in state s .

To understand why the fall in consumption can be so large, I propose the following decomposition. The aggregated consumption Euler equation (7) can be written as

$$c_s = \frac{1}{\gamma}\bar{\xi}_s - \frac{1}{1-\mu}r_s + \frac{1}{1-\mu}\mu\pi_s.$$

⁸Following common practice in the literature on the zero lower bound, I assume that parameters are such that the denominator in equation (10) is positive. See, e.g., Woodford (2011), Section IV.A.

⁹Substituting equations (8) and (10) (or equations (8) and (11)) into $-\ln(R) = \phi\pi_s$ and solving for $\bar{\xi}_s$ yields equation (12).

Consumption in state s equals the sum of three terms: the first term is the direct effect of the preference shock on consumption, the second term is the effect of the nominal interest rate on consumption, and the third term is the effect of expected inflation on consumption. Substituting in the equilibrium nominal interest rate when the zero lower bound is binding (i.e., $r_s = -\ln(R)$) and equilibrium inflation when the zero lower bound is binding yields

$$c_s = \frac{1}{\gamma} \bar{\xi}_s + \frac{1}{1-\mu} \ln(R) + \frac{1}{1-\mu} \underbrace{\mu \frac{\kappa}{1-\beta\mu} \frac{\frac{1}{\gamma} \bar{\xi}_s + \frac{1}{1-\mu} \ln(R)}{1 - \frac{1}{\gamma} \mu \frac{\kappa}{1-\beta\mu}}}_{\text{expected inflation}}.$$

The first term is negative. The second term is positive because the monetary authority can lower the nominal rate to some extent before the zero lower bound becomes binding, which actually increases consumption. The third term is negative and reflects the effect of the aggregate shock on aggregate consumption coming from movements in inflation expectations. The reason why the fall in consumption can be arbitrarily large for a given size of the shock is the third term.

The model of this subsection predicts that the fall in consumption can be arbitrarily large for a given size of the shock, because the fall in inflation expectations can be arbitrarily large for a given size of the shock. The amplification of the shock comes from movements in household inflation expectations. If household inflation expectations did not move in response to the shock, consumption would be given by the sum of the first two terms. How we model inflation expectations therefore seems crucial for results concerning dynamics at the zero lower bound.

4.2 Imperfect information

To understand the implications of slow adjustment and heterogeneity of household inflation expectations in an economy with a zero lower bound for the nominal interest rate, let us turn to the model with dispersed information on the household side (i.e., $\omega < 1$). I first solve the model analytically in the following special case: households form beliefs about the aggregate state of the economy based only on their own local conditions (i.e., $\omega = 0$) and households set real wage rates. This special case of the model can be solved analytically. In Section 5, I relax these two assumptions. The main results remain unchanged.

As in Section 4.1, I consider equilibria of the following form: consumption, inflation, and the nominal interest rate are constant over time in periods $0 \leq t \leq T - 1$, and the economy is in the

non-stochastic steady state with zero inflation in periods $t \geq T$. The latter is an equilibrium for the same reasons as in Section 4.1: all households have the same post-transfer wealth in period T due to the trade in state-contingent claims in period minus one and $c_{i,t} = c_t = \pi_t = r_t = 0$ in every period $t \geq T$ satisfies the consumption Euler equation (1) with $\xi_{i,t} = \xi_{i,t+1} = 0$, the New Keynesian Phillips curve (5), and the monetary policy rule (9).

Let us turn to consumption, inflation, and the nominal interest rate in periods $0 \leq t \leq T - 1$. Since consumption, inflation, and the nominal interest rate are constant over time but depend on the size of the aggregate shock, I replace the time subscript t by the state subscript $s \in \{good, bad\}$. Since the random period T arrives with probability μ , the consumption Euler equation (1) can be written as

$$c_i = -\frac{1}{\gamma} [(\mu - 1) \xi_{i,0} + E^i [r_S - \mu\pi_S]] + \mu c_i. \quad (13)$$

Here $E^i [r_S - \mu\pi_S]$ is household i 's expectation of the real interest rate. The household's expectation has no time subscript because it is constant over time due to the assumption that $\omega = 0$. (This assumption is relaxed in the following section.) The household's expectation has no state subscript, because the household's expectation only depends on the household's type ($\xi_{i,0} = \xi_H$ or $\xi_{i,0} = \xi_L$). As a result, consumption depends only on the household's type. Integrating across households yields aggregate consumption in state s

$$c_s = -\frac{1}{\gamma} [(\mu - 1) \bar{\xi}_s + \bar{E}_s [r_S - \mu\pi_S]] + \mu c_s, \quad (14)$$

where $\bar{\xi}_s = \lambda_s \xi_H + (1 - \lambda_s) \xi_L$ denotes the cross-sectional mean of the preference shock in state s and $\bar{E}_s [r_S - \mu\pi_S]$ denotes the average expectation of the real interest rate in state s :

$$\bar{E}_s [r_S - \mu\pi_S] = \bar{p}_s^{good} (r_{good} - \mu\pi_{good}) + \bar{p}_s^{bad} (r_{bad} - \mu\pi_{bad}). \quad (15)$$

Here $\bar{p}_s^{good} = \lambda_s p_H^{good} + (1 - \lambda_s) p_L^{good}$ is the average probability that households assign to the good state when the economy is actually in state s , where p_H^{good} denotes the probability that a high type assigns to the good state and p_L^{good} denotes the probability that a low type assigns to the good state. Furthermore, $\bar{p}_s^{bad} = 1 - \bar{p}_s^{good}$ is the average probability that households assign to the bad state when the economy is actually in state s . Finally, the New Keynesian Phillips curve and the monetary policy rule are again given by equations (8) and (9).

An important difference to the case of perfect information is that aggregate consumption in each state depends on the real interest rate in both aggregate states through equations (14)-(15).

For this reason, one cannot solve the model state by state, and one has to distinguish three cases: the zero lower bound is binding in both states, the zero lower bound is binding in no state, and the zero lower bound is binding only in the bad state.

Zero lower bound binding in both states. When the zero lower bound is binding in both states, imperfect information on the household side increases consumption in the bad state. Formally, consumption in the good state equals

$$c_{good} = \frac{\frac{1}{\gamma} \bar{\xi}_{good} + \frac{1}{1-\mu} \ln(R)}{1 - \frac{1}{1-\mu} \frac{\mu\kappa}{1-\beta\mu}} - \bar{p}_{good} \frac{\frac{1}{1-\mu} \frac{\mu\kappa}{1-\beta\mu}}{1 - \frac{1}{1-\mu} \frac{\mu\kappa}{1-\beta\mu}} (c_{good} - c_{bad}), \quad (16)$$

and consumption in the bad state equals

$$c_{bad} = \frac{\frac{1}{\gamma} \bar{\xi}_{bad} + \frac{1}{1-\mu} \ln(R)}{1 - \frac{1}{1-\mu} \frac{\mu\kappa}{1-\beta\mu}} + \bar{p}_{bad}^{good} \frac{\frac{1}{1-\mu} \frac{\mu\kappa}{1-\beta\mu}}{1 - \frac{1}{1-\mu} \frac{\mu\kappa}{1-\beta\mu}} (c_{good} - c_{bad}), \quad (17)$$

with

$$c_{good} - c_{bad} = \frac{\frac{1}{\gamma} (\bar{\xi}_{good} - \bar{\xi}_{bad})}{1 - \frac{1}{1-\mu} \frac{\mu\kappa}{1-\beta\mu}} \frac{1}{1 + \left(\bar{p}_{bad}^{good} + \bar{p}_{good}^{bad} \right) \frac{\frac{1}{1-\mu} \frac{\mu\kappa}{1-\beta\mu}}{1 - \frac{1}{1-\mu} \frac{\mu\kappa}{1-\beta\mu}}} > 0. \quad (18)$$

Here \bar{p}_{bad}^{good} is the average probability assigned to the good state when the economy is actually in the bad state and \bar{p}_{good}^{bad} is the average probability assigned to the bad state when the economy is actually in the good state. To obtain these equations, state equation (14) for the good state and the bad state. Use equation (15) to substitute for the average expectation of the real interest rate, $r_{good} = r_{bad} = -\ln(R)$ to substitute for the nominal interest rate, and equation (8) to substitute for inflation. One obtains a system of two equations in the two unknowns c_{good} and c_{bad} . Rearranging and using $\bar{p}_{good}^{good} + \bar{p}_{good}^{bad} = \bar{p}_{bad}^{good} + \bar{p}_{bad}^{bad} = 1$ yields equations (16)-(18).

Imperfect information on the household side increases consumption in the bad state, because movements in household inflation expectations are *destabilizing* at the zero lower bound and thus the slow adjustment of household inflation expectations keeps consumption high in the bad state. More formally, in the bad state, households assign positive probability to the good state, which is a state with a higher inflation rate and a lower real interest rate. This attenuates the fall in consumption in the bad state.

To understand the *magnitude* of the effect of dispersed information, I follow Angeletos and La'O (2013) by drawing an analogy between the equilibrium of the economy and the perfect Bayesian equilibrium of a fictitious game. Consider the following abstract game. There is a continuum of agents and each agent i chooses an action $c_i \in \mathbb{R}$. There are two types of agents and the aggregate state is the cross-sectional distribution of types. Let c_s denote the average action in the population. Payoff functions are such that the action of agent i equals a linear combination of an agent-specific fundamental, ϖ_i , and the agent's expectation of the average action, $E^i [c_S]$:

$$c_i = (1 - \varsigma) \varpi_i + \varsigma E^i [c_S]. \quad (19)$$

When $\varsigma > 0$, actions are said to be *strategic complements*. It is well known from the literature on dispersed information that dispersed information has larger effects when the degree of strategic complementarity in actions, $\varsigma \in (0, 1)$, is larger.

Let us return to the economy presented before. Substituting $r_s = -\ln(R)$ and $\pi_s = \frac{\kappa}{1-\beta\mu} c_s$ into equation (13) yields an equation of the form (19) with

$$\varpi_i = \frac{\frac{1}{\gamma} \xi_{i,0} + \frac{1}{1-\mu} \ln(R)}{1 - \frac{1}{1-\mu} \frac{\mu\kappa}{1-\beta\mu}} < 0 \quad \text{and} \quad \varsigma = \frac{\frac{1}{\gamma} \frac{\mu\kappa}{1-\beta\mu}}{1 - \mu \frac{\mu\kappa}{1-\beta\mu}} > 0.$$

Furthermore, returning to equations (16)-(18), note that for a given size of the information friction (i.e., for given values of \bar{p}_{bad}^{good} and \bar{p}_{good}^{bad}) and for a given difference between consumption in the good state under perfect information and consumption in the bad state under perfect information (i.e., for a given value of the numerator in equation (18)), the effect of dispersed information is larger when ς is larger and the effect becomes arbitrarily large as ς converges to one.¹⁰ In sum, at the zero lower bound, dispersed information on the household side can have very large effects, because consumption choices of different households are strategic complements.

Zero lower bound binding in no state. When the zero lower bound is binding in no state, imperfect information on the household side decreases consumption in the bad state. Formally, consumption in the good state equals

$$c_{good} = \frac{\frac{1}{\gamma} \bar{\xi}_{good}}{1 + \frac{1}{1-\mu} \frac{(\phi-\mu)\kappa}{1-\beta\mu}} + \bar{p}_{good}^{bad} \frac{\frac{1}{\gamma} \frac{(\phi-\mu)\kappa}{1-\beta\mu}}{1 + \frac{1}{1-\mu} \frac{(\phi-\mu)\kappa}{1-\beta\mu}} (c_{good} - c_{bad}), \quad (20)$$

¹⁰One can change the value of $\frac{1}{1-\mu} \frac{\mu\kappa}{1-\beta\mu}$ and hold the value of the numerator in equation (18) constant by adjusting the value of $\bar{\xi}_{good} - \bar{\xi}_{bad}$. Finally, recall that the literature assumes that $\frac{1}{1-\mu} \frac{\mu\kappa}{1-\beta\mu}$ is smaller than one (Footnote 6).

and consumption in the bad state equals

$$c_{bad} = \frac{\frac{1}{\gamma}\bar{\xi}_{bad}}{1 + \frac{\frac{1}{\gamma}(\phi-\mu)\kappa}{1-\mu} \frac{1}{1-\beta\mu}} - \bar{p}_{bad}^{good} \frac{\frac{1}{\gamma}(\phi-\mu)\kappa}{1-\mu} \frac{1}{1-\beta\mu} (c_{good} - c_{bad}), \quad (21)$$

with

$$c_{good} - c_{bad} = \frac{\frac{1}{\gamma}(\bar{\xi}_{good} - \bar{\xi}_{bad})}{1 + \frac{\frac{1}{\gamma}(\phi-\mu)\kappa}{1-\mu} \frac{1}{1-\beta\mu}} > 0. \quad (22)$$

$$1 - \left(\bar{p}_{good}^{bad} + \bar{p}_{bad}^{good} \right) \frac{\frac{1}{\gamma}(\phi-\mu)\kappa}{1-\mu} \frac{1}{1-\beta\mu}$$

The derivation is almost identical to the derivation in the previous case. The only difference is that one has to use $r_{good} = \phi\pi_{good}$ and $r_{bad} = \phi\pi_{bad}$ instead of $r_{good} = r_{bad} = -\ln(R)$ to substitute for the nominal interest rate.

Household imperfect information decreases consumption in the bad state, because movements in household inflation expectations are *stabilizing* when the Taylor principle is satisfied (i.e., when the central bank lowers the nominal interest rate more than one-for-one with inflation). More formally, in the bad state, households assign positive probability to the good state, which is the state with the higher inflation rate and the state with the higher real interest rate due to the Taylor principle. In other words, in the bad state, households believe that the central bank may still be fighting high inflation. This amplifies the fall in consumption in the bad state.

Fortunately, the perfect-information fall in consumption in the bad state is not that large to begin with when the zero lower bound is not binding (the first term on the right-hand side of equation (21)). Furthermore, consumption choices of different households are strategic substitutes when the Taylor principle is satisfied, which limits the *magnitude* of the effect of household dispersed information on consumption (the second term on the right-hand side of equation (21)).¹¹

Zero lower bound binding only in the bad state. When the zero lower bound is binding only in the bad state, the sign of the effect of household imperfect information depends on the sign of the difference between the real rate in the good state and the real rate in the bad state. Formally, consumption in the good state equals

$$c_{good} = \frac{\frac{1}{\gamma}\bar{\xi}_{good}}{1 + \frac{\frac{1}{\gamma}(\phi-\mu)\kappa}{1-\mu} \frac{1}{1-\beta\mu}} + \bar{p}_{good}^{bad} \frac{\frac{1}{\gamma}}{1-\mu} [(r_{good} - \mu\pi_{good}) - (r_{bad} - \mu\pi_{bad})], \quad (23)$$

¹¹Substituting $r_s = \phi\pi_s$ and $\pi_s = \frac{\kappa}{1-\beta\mu}c_s$ into equation (13) yields an equation of the form (19) with $\varsigma < 0$.

and consumption in the bad state equals

$$c_{bad} = \frac{\frac{1}{\gamma}\bar{\xi}_{bad} + \frac{1}{1-\mu}\ln(R)}{1 - \frac{\frac{1}{\gamma}\mu\kappa}{1-\mu} \frac{1}{1-\beta\mu}} - \bar{p}_{bad}^{good} \frac{\frac{1}{1-\mu}}{1 - \frac{\frac{1}{\gamma}\mu\kappa}{1-\mu} \frac{1}{1-\beta\mu}} [(r_{good} - \mu\pi_{good}) - (r_{bad} - \mu\pi_{bad})], \quad (24)$$

with

$$(r_{good} - \mu\pi_{good}) - (r_{bad} - \mu\pi_{bad}) = \frac{\frac{(\phi-\mu)\kappa}{1-\beta\mu} \frac{\frac{1}{\gamma}\bar{\xi}_{good}}{1 + \frac{\frac{1}{\gamma}(\phi-\mu)\kappa}{1-\beta\mu}} - \left[-\ln(R) - \frac{\mu\kappa}{1-\beta\mu} \frac{\frac{1}{\gamma}\bar{\xi}_{bad} + \frac{1}{1-\mu}\ln(R)}{1 - \frac{\frac{1}{\gamma}\mu\kappa}{1-\mu} \frac{1}{1-\beta\mu}} \right]}{1 - \bar{p}_{good}^{bad} \frac{\frac{\frac{1}{\gamma}(\phi-\mu)\kappa}{1-\mu} \frac{1}{1-\beta\mu}}{1 + \frac{\frac{1}{\gamma}(\phi-\mu)\kappa}{1-\beta\mu}} - \bar{p}_{bad}^{good} \frac{-\frac{\frac{1}{\gamma}\mu\kappa}{1-\mu} \frac{1}{1-\beta\mu}}{1 - \frac{\frac{1}{\gamma}\mu\kappa}{1-\mu} \frac{1}{1-\beta\mu}}}. \quad (25)$$

Note that $(r_{good} - \mu\pi_{good}) - (r_{bad} - \mu\pi_{bad})$ is the difference between the real interest rate in the good state and the real interest rate in the bad state, and the numerator on the right-hand side of equation (25) is the difference between the real interest rate in the good state *under perfect information* and the real interest rate in the bad state *under perfect information*. The derivation is almost identical to the derivation in the previous two cases. The only difference is that one has to use $r_{good} = \phi\pi_{good}$ and $r_{bad} = -\ln(R)$ to substitute for the nominal interest rate.

When the zero lower bound is binding only in the bad state, imperfect information on the household side increases/decreases/does not affect consumption in the bad state if the real rate in the good state is lower/higher/equal to the real rate in the bad state.¹² All three cases are possible. Aggregate consumption and inflation are monotonic functions of the aggregate shock, $\bar{\xi}_s$, but in an economy with a monetary policy rule (9) the real interest rate $r_s - \mu\pi_s$ is a non-monotonic function of inflation.

Furthermore, note that in the case when the zero lower bound is binding only in the bad state, individual consumption depends on several moments of the conditional distribution of inflation. For comparison, when the zero lower bound is binding in both states, individual consumption equals

$$c_i = \frac{1}{\gamma}\xi_{i,0} + \frac{1}{1-\mu}\ln(R) + \frac{1}{1-\mu}\mu E^i[\pi_S],$$

and aggregate consumption equals

$$c_s = \frac{1}{\gamma}\bar{\xi}_s + \frac{1}{1-\mu}\ln(R) + \frac{1}{1-\mu}\mu \int_0^1 E^i[\pi_S] di.$$

¹²The denominator on the right-hand side of equation (25) is positive.

When the zero lower bound is binding in no state, individual consumption equals

$$c_i = \frac{1}{\gamma} \xi_{i,0} - \frac{\frac{1}{\gamma}}{1-\mu} (\phi - \mu) E^i [\pi_S],$$

and aggregate consumption equals

$$c_s = \frac{1}{\gamma} \bar{\xi}_s - \frac{\frac{1}{\gamma}}{1-\mu} (\phi - \mu) \int_0^1 E^i [\pi_S] di.$$

In both cases, individual consumption depends only on the conditional mean of inflation and aggregate consumption depends only on the average inflation expectation. By contrast, when the zero lower bound binds only in the bad state, individual consumption equals

$$c_i = \frac{1}{\gamma} \xi_{i,0} - \frac{\frac{1}{\gamma}}{1-\mu} (\phi - \mu) E^i [\pi_S] - \frac{\frac{1}{\gamma}}{1-\mu} p_i^{bad} (-\ln(R) - \phi \pi_{bad}),$$

and aggregate consumption equals

$$c_s = \frac{1}{\gamma} \bar{\xi}_s - \frac{\frac{1}{\gamma}}{1-\mu} (\phi - \mu) \int_0^1 E^i [\pi_S] di - \frac{\frac{1}{\gamma}}{1-\mu} \bar{p}_s^{bad} (-\ln(R) - \phi \pi_{bad}).$$

Individual consumption depends on the conditional mean of inflation, the probability that the household assigns to the bad state, and inflation in the bad state. Aggregate consumption depends on the average inflation expectation, the average probability assigned to the bad state, and inflation in the bad state. Nevertheless, one can solve the model analytically. See equations (23)-(25).

Switches. In the interpretation (not the derivation) of equations (16)-(18), (20)-(22), and (23)-(25), I assumed up to now that the set of states with a binding zero lower bound under imperfect information equals the set of states with a binding zero lower bound under perfect information. These sets may differ. In particular, when the zero lower bound is binding in no state under perfect information, the zero lower bound may be binding in the bad state under imperfect information. The reason is that imperfect information decreases consumption in the bad state.¹³ Furthermore, when the zero lower bound is binding only in the bad state under perfect information (and the real rate is higher in the bad state than in the good state), the zero lower bound may be binding in both states under imperfect information. The reason is that imperfect information decreases

¹³Consumption in the bad state is then given by equation (11) under perfect information and by equation (24) under imperfect information.

consumption in the good state.¹⁴ However, these potential switches do not change the statements about how imperfect information changes consumption in the two states.

Conditional probabilities. So far we used no properties of the conditional probabilities p_i^{good} and p_i^{bad} and the average probabilities \bar{p}_s^{good} and \bar{p}_s^{bad} , apart from the fact that probabilities are non-negative and sum to one. The next paragraph gives the expressions for these probabilities and points out that \bar{p}_{bad}^{good} and \bar{p}_{good}^{bad} can be of very different size.

The probability that a high type assigns to the good state by Bayes' law equals

$$p_H^{good} = \frac{\lambda_{good}\theta}{\lambda_{good}\theta + \lambda_{bad}(1-\theta)}, \quad (26)$$

while the probability that a low type assigns to the good state by Bayes' law equals

$$p_L^{good} = \frac{(1-\lambda_{good})\theta}{(1-\lambda_{good})\theta + (1-\lambda_{bad})(1-\theta)}. \quad (27)$$

The average probability assigned to the good state when the economy is in the bad state equals

$$\bar{p}_{bad}^{good} = \lambda_{bad}p_H^{good} + (1-\lambda_{bad})p_L^{good}. \quad (28)$$

The average probability assigned to the bad state when the economy is in the good state equals

$$\bar{p}_{good}^{bad} = \lambda_{good}p_H^{bad} + (1-\lambda_{good})p_L^{bad}. \quad (29)$$

The variable \bar{p}_{bad}^{good} is an increasing function of the prior probability of the good state, θ , while the variable \bar{p}_{good}^{bad} is a decreasing function of the prior probability of the good state, θ . Hence, when the good state has a high prior probability (i.e., the bad state is a rare event) and the zero lower bound is binding in both states, household imperfect information has a small negative effect on consumption in the good state (equation (16)) and a large positive effect on consumption in the bad state (equation (17)).

5 Numerical solutions

This section relaxes the simplifying assumptions of the previous section and presents quantitative results. I first discuss the parameter values that serve as benchmark parameter values throughout the section.

¹⁴Consumption in the good state is then given by equation (11) under perfect information and by equation (16) under imperfect information.

5.1 Calibration

The main ideas underlying the parameter choices are the following. The preference, technology, price stickiness, and policy parameters are set to their most standard values. The information friction parameter ω is chosen so as to match the speed at which the gap between inflation and inflation expectations closes in survey data. Finally, the shock parameters are chosen so that under perfect information $\bar{\xi}_{good}$ would create a severe recession and $\bar{\xi}_{bad}$ would create the worst recession in almost a century.

One period corresponds to one quarter. I assume a long-run annual real interest rate of 4% and set $\beta = 0.99$. The intertemporal elasticity of substitution is $(1/\gamma) = 1$ and the elasticity of output with respect to labor is $\varrho = (2/3)$. These are the most common values in the business cycle literature. The elasticity of substitution between intermediate goods is $\psi = 10$, which implies a long-run markup of 11%, a common target in the New Keynesian literature. The probability that a firm cannot adjust its price in a given quarter is $\alpha = 0.66$, implying that one third of prices change per quarter, a value consistent with micro evidence on prices once sales prices have been removed. See Nakamura and Steinsson (2008). For these parameters, the slope of the New Keynesian Phillips curve is $\kappa = 0.045$. I set $\phi = 1.5$, which is the most standard value for the coefficient on inflation in a Taylor rule.

Coibion and Gorodnichenko (2012) estimate impulse responses of inflation and inflation expectations to shocks. Under the null of full-information rational expectations, inflation expectations should adjust to a realized shock by the same amount as the conditional mean of future inflation. Full information implies that the shock is in the information set of the agents. Rational expectations implies that agents understand how the shock affects future inflation. By contrast, Coibion and Gorodnichenko (2012) find that the responses of inflation expectations to shocks are dampened and delayed relative to the responses of future inflation to the same shocks. After an inflationary (disinflationary) shock, inflation expectations rise (fall) by less than future inflation and this difference becomes smaller over time and eventually converges to zero. This result is obtained for all four types of shocks they consider (technology shocks, news shocks, oil shocks, unidentified shocks), inflationary and disinflationary shocks, and different types of agents.¹⁵ As a next step, Coibion and

¹⁵For professional forecasters, the inflation forecasts are from the Survey of Professional Forecasters and the time sample is 1976-2007. For households, the inflation forecasts are from the Michigan Survey of Consumers and the time

Gorodnichenko (2012) estimate the degree of information rigidity that matches the empirical speed of response of inflation expectations to shocks. In the context of a sticky information model, their estimated degree of information rigidity corresponds to the fraction of agents that do not update their inflation expectations in a given quarter. On page 143 they write: “This procedure yields estimates between 0.86 and 0.89 for technology, news, and oil price shocks as well as for unidentified shocks.” Based on these estimates, I set $\omega = 1 - 0.875$.¹⁶

Finally, let us turn to the shock parameters. I set the persistence of the preference shocks to $\mu = 0.8$, which is a common value in the New Keynesian literature on the zero lower bound. I set $\xi_H = -0.05$ and $\xi_L = -0.075$, which implies that the shock term $(1 - \mu)\xi_{i,0}$ in the consumption Euler equation equals -1% for a high type and -1.5% for a low type. I set the fraction of high types in the good state to $\lambda_{good} = (3/4)$ and the fraction of high types in the bad state to $\lambda_{bad} = (1/4)$. The shock term $(1 - \mu)\bar{\xi}_s$ in the aggregated Euler equation thus equals -1.125% in the good state and -1.375% in the bad state. Under perfect information, the zero lower bound is marginally binding in the good state and clearly binding in the bad state (for comparison, $(1 - \mu)\bar{\xi}_{crit} = -1.09\%$). Furthermore, under perfect information, consumption drops by 4% in the good state and by 13% in the bad state. Hence, I think of the good state as a shock that would create a serious recession under perfect information, while I think of the bad state as a shock that would create the worst recession since World War II under perfect information. Finally, I set $\theta = 0.9$. That is, the prior probability of the bad state equals 10%. This seems a reasonable value given that recessions with a 13 percent fall in consumption are rare.

5.2 Slow updating of inflation expectations

This subsection presents the solution of the model when households update their inflation expectations slowly over time (i.e., $0 < \omega < 1$). The next three paragraphs describe how the model is solved. The following paragraph presents the solution for the benchmark parameter values.

sample is 1976-2007. For each group, Coibion and Gorodnichenko (2012) study the response of the average forecast.

¹⁶These estimates are for professional forecasters. Coibion and Gorodnichenko (2012) obtain slightly lower estimates of the degree of information rigidity for households (see their Table 4). In the benchmark calibration, I use the estimates that Coibion and Gorodnichenko (2012) obtain for inflation expectations data from the Survey of Professional Forecasters. In Section 5.5, I show that results are essentially unchanged when one uses the estimates that Coibion and Gorodnichenko (2012) obtain for inflation expectations data from the Michigan Survey of Consumers.

There are four types of households at each point in time t : households with a high realization of the preference shock in period zero ($\xi_{i,0} = \xi_H$) and full-information rational expectations in period t (“high, informed types”), households with a low realization of the preference shock in period zero ($\xi_{i,0} = \xi_L$) and full-information rational expectations in period t (“low, informed types”), households with a high realization of the preference shock in period zero and imperfect information in period t (“high, uninformed types”), and households with a low realization of the preference shock in period zero and imperfect information in period t (“low, uninformed types”).

The consumption Euler equation of a high, informed type in period t in state $s \in \{good, bad\}$ reads

$$c_{s,t}^{hi} = -\frac{1}{\gamma} ((\mu - 1) \xi_H + r_{s,t} - \mu\pi_{s,t+1}) + \mu c_{s,t+1}^{hi}. \quad (30)$$

The consumption Euler equation of a low, informed type in period t in state $s \in \{good, bad\}$ reads

$$c_{s,t}^{li} = -\frac{1}{\gamma} ((\mu - 1) \xi_L + r_{s,t} - \mu\pi_{s,t+1}) + \mu c_{s,t+1}^{li}. \quad (31)$$

The consumption Euler equation of a high, uninformed type in period t reads

$$\begin{aligned} c_t^{hu} &= -\frac{1}{\gamma} \left((\mu - 1) \xi_H + p_H^{good} (r_{good,t} - \mu\pi_{good,t+1}) + p_H^{bad} (r_{bad,t} - \mu\pi_{bad,t+1}) \right) \\ &+ \omega \left(p_H^{good} \mu c_{good,t+1}^{hi} + p_H^{bad} \mu c_{bad,t+1}^{hi} \right) + (1 - \omega) \mu c_{t+1}^{hu}. \end{aligned} \quad (32)$$

The parameter ω reflects the fact that an uninformed household becomes informed in the next period with probability ω . Furthermore, p_H^{good} and p_H^{bad} are the probabilities that a high type assigns to the good state and the bad state. The consumption Euler equation of a low, uninformed type in period t reads

$$\begin{aligned} c_t^{lu} &= -\frac{1}{\gamma} \left((\mu - 1) \xi_L + p_L^{good} (r_{good,t} - \mu\pi_{good,t+1}) + p_L^{bad} (r_{bad,t} - \mu\pi_{bad,t+1}) \right) \\ &+ \omega \left(p_L^{good} \mu c_{good,t+1}^{li} + p_L^{bad} \mu c_{bad,t+1}^{li} \right) + (1 - \omega) \mu c_{t+1}^{lu}. \end{aligned} \quad (33)$$

Aggregate consumption is a weighted average of the consumption of the four types of households. The weight on consumption of a particular type equals the mass of this type. Hence, aggregate consumption in the good state equals

$$c_{good,t} = \left[1 - (1 - \omega)^{t+1} \right] \left[\lambda_{good} c_{good,t}^{hi} + (1 - \lambda_{good}) c_{good,t}^{li} \right] + (1 - \omega)^{t+1} \left[\lambda_{good} c_t^{hu} + (1 - \lambda_{good}) c_t^{lu} \right], \quad (34)$$

and aggregate consumption in the bad state equals

$$c_{bad,t} = \left[1 - (1 - \omega)^{t+1}\right] \left[\lambda_{bad} c_{bad,t}^{hi} + (1 - \lambda_{bad}) c_{bad,t}^{li}\right] + (1 - \omega)^{t+1} \left[\lambda_{bad} c_t^{hu} + (1 - \lambda_{bad}) c_t^{lu}\right]. \quad (35)$$

The mass of uninformed households equals $(1 - \omega)$ in period zero, $(1 - \omega)^2$ in period one, and so on. The mass of informed households equals one minus the mass of uninformed households. Finally, the New Keynesian Phillips curve in the good state reads

$$\pi_{good,t} = \kappa c_{good,t} + \beta \mu \pi_{good,t+1}, \quad (36)$$

and the New Keynesian Phillips curve in the bad state reads

$$\pi_{bad,t} = \kappa c_{bad,t} + \beta \mu \pi_{bad,t+1}. \quad (37)$$

For the benchmark parameter values, the zero lower bound is binding in each period in both states. The system of equations can then be written as a linear difference equation

$$A_t x_t = b + B x_{t+1}, \quad (38)$$

with

$$x_t = \begin{pmatrix} c_{good,t}^{hi} \\ c_{bad,t}^{hi} \\ c_{good,t}^{li} \\ c_{bad,t}^{li} \\ c_t^{hu} \\ c_t^{lu} \\ c_{good,t} \\ c_{bad,t} \\ \pi_{good,t} \\ \pi_{bad,t} \end{pmatrix}. \quad (39)$$

The matrix A_t is a function of t , because aggregate consumption depends on the mass of households with full-information rational expectations and this mass increases over time. Let $A = \lim_{t \rightarrow \infty} A_t$ denote the limit of the matrix A_t as t goes to infinity. The matrix A also equals the value of the matrix A_t at each point in time in the special case of the model where all households have full-information rational expectations already in period zero (i.e., $\omega = 1$). I first solve for the steady

state of the linear difference equation (38) after replacing A_t by A . The solution $x = (A - B)^{-1} b$ equals the equilibrium under perfect information presented in Section 4.1. Thereafter, I compute the earlier x_t from equation (38) and the endpoint $x_{10,000} = x$. That is, I assume the following. If preference shocks have not yet reverted back, 10,000 periods after the shock the economy has converged to the solution under perfect information.

Figure 1 shows the solution for the benchmark parameter values. Each line depicts consumption in periods $0 \leq t \leq T - 1$. Recall that the preference shocks revert back to zero with probability $1 - \mu = 0.2$ in any given period and the economy is in the non-stochastic steady state with zero inflation thereafter. The thin black line shows consumption in the good state. The thick black line shows consumption in the bad state. For comparison, the dotted upper line shows consumption in the good state under perfect information. The dotted lower line shows consumption in the bad state under perfect information. Each dot corresponds to one quarter. The slow adjustment of household inflation expectations increases consumption in the bad state. The reason is that downward movements in household inflation expectations are destabilizing. The effect of dispersed information is large and persistent. The reason is that Coibion and Gorodnichenko (2012) estimate a speed of updating of inflation expectations that is far away from $\omega = 1$ and consumption choices of different households are strategic complements when the zero lower bound is binding. Finally, dispersed information has only a small negative effect on consumption in the good state and a large positive effect on consumption in the bad state. The reason is the small prior probability of the bad state.

5.3 Deterministic decay

Let us introduce deterministic decay as a next step towards relaxing the simplifying assumptions of Section 4. Formally, $\xi_{i,t} = \rho \xi_{i,t-1}$ in periods $1 \leq t \leq T - 1$. Fundamentals now change deterministically in addition to the stochastic decay. In the consumption Euler equations (30)-(33), the term $(\mu - 1) \xi_{i,0}$ with $\xi_{i,0} \in \{\xi_L, \xi_H\}$ then becomes $(\mu\rho - 1) \rho^t \xi_{i,0}$ and the nominal interest rate is given by $r_{s,t} = \max\{-\ln(R), \phi\pi_{s,t}\}$. To solve the model, I make a guess regarding the number of periods for which the zero lower bound is binding in the good state and in the bad state. If the guess turns out to be incorrect, I update the guess until a fixed point is reached.

Figure 2 shows aggregate consumption (upper panel) and the nominal interest rate (lower panel)

in periods $0 \leq t \leq T - 1$ for the benchmark parameter values, $\rho = 0.99$, $\mu = 0.95$, $\xi_H = -0.05 * 1.4$, and $\xi_L = -0.075 * 1.4$. I assume some stochastic decay to match the observation that in December of 2008 the Federal Reserve expected the zero lower bound episode to be shorter than seven years.¹⁷ Furthermore, I scale the preference shocks in order to keep roughly constant the *perfect-information* response of consumption on impact in the two states across subsections. The good state is again an aggregate shock that would create a serious recession under perfect information. The bad state is again an aggregate shock that would create the worst recession since World War II under perfect information.

The effect of household dispersed information is large and persistent. The thin black lines show consumption and the nominal interest rate in the good state. The thick black lines show consumption and the nominal interest rate in the bad state. For comparison, the dotted upper line in the upper panel shows consumption in the good state under perfect information. The dotted lower line in the upper panel shows consumption in the bad state under perfect information. The first difference to Figure 1 is that consumption in the bad state is almost flat over time for many quarters and then converges slowly back to the non-stochastic steady state. The reason why consumption is almost flat over time in the first couple of years is that there are two forces working in opposite directions. On the one hand, the fundamentals are improving, which drives aggregate consumption up. On the other hand, more and more households have updated their expectations since the period of the shock, which drives aggregate consumption down. In the first couple of years, these two effects roughly cancel. The second difference to Figure 1 is that the economy can now transit through all three regions discussed in Section 4.2. In the first two years, the zero lower bound is binding in both states. In the following 5 years, the zero lower bound is binding only in the bad state. Thereafter, the zero lower bound is binding in no state. Finally, turning to magnitudes, in the bad state the response of consumption on impact of the shock equals about 1/3 of the value under perfect information.

5.4 Households set nominal wage rates

Finally, let us relax the assumption that households set real wage rates. When households set nominal wage rates and household inflation expectations adjust slowly over time, the New Keynesian

¹⁷Another explanation is that even the Fed assigned some probability to the wrong state in December of 2008.

Phillips curve changes. The New Keynesian Phillips curve is given by equation (6). As a result, there is a time- and state-varying intercept in equations (36)-(37) equal to $\frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{1}{1+\frac{1-\epsilon}{\epsilon}\psi} (\bar{E}_t [p_t] - p_t)$. To solve the model, I make a guess regarding the value of this term in every period in each state. I solve the difference equation (38), which now has a time-varying vector b_t , using the same endpoint as before. Afterwards, I compute the actual path of $(\bar{E}_t [p_t] - p_t)$ in each state. If the initial guess turns out to be incorrect, I update the guess until a fixed point is reached.

Figure 3 shows aggregate consumption (upper panel) and the nominal interest rate (lower panel) for the parameter values of the previous subsection when households set nominal wage rates. The line styles in Figure 3 equal the line styles in Figure 2. The main difference to Figure 2 is that consumption falls even less in the bad state. The reason is simple. When households' expectations of the price level are above the price level, households set nominal wage rates that are too high, which raises marginal costs and inflation. This attenuates the deflationary spiral and increases aggregate consumption. Turning to magnitudes, in the bad state the response of consumption on impact of the shock equals about 1/4 of the value under perfect information.

The economy's path in the bad state matches basic features of the U.S. economy during the Great Recession: (i) inflation falls little, (ii) household inflation expectations fall even less, (iii) consumption falls by several percentage points and is almost flat over time for several years (relative to trend), and (iv) the zero lower bound is binding for about seven years.

5.5 Faster updating of inflation expectations

Figure 4 shows the solution for $\omega = 1 - 0.77 = 0.23$ instead of $\omega = 1 - 0.875 = 0.125$. The value 0.77 is the average estimate of the degree of information rigidity that Coibion and Gorodnichenko (2012) obtain when they use inflation forecasts from the Michigan Survey of Consumers (see their Table 4). Otherwise the model setup and parameter values are the same as in the previous subsection. In the bad state, the response of consumption on impact of the shock is somewhat larger than in Figure 3 (about 1/3 instead of 1/4 of the value under perfect information).

5.6 Lower prior probability of the good state

Figure 5 shows the solution for $\theta = 0.5$ instead of $\theta = 0.9$. Otherwise the model setup and parameter values are the same as in Section 5.4. The responses of consumption and inflation on impact of the

shock become larger in both states.

6 Monetary policy

6.1 Central bank communication about the current state

Central bankers frequently make statements about the current state of the economy. For example, the Federal Reserve’s official statement after a regular Federal Open Market Committee (FOMC) meeting typically starts with a paragraph on the current state of the economy.

In most models, central bank communication about the current state is irrelevant, because the current state is common knowledge and central bank communication also does not affect equilibrium selection. In the model studied here, central bank communication about the current state of the economy affects aggregate consumption, because the current state is not common knowledge. Moreover, the sign of this effect switches when the zero lower bound is binding.

To show this result as clearly as possible, let us return to the closed-form solutions of Section 4. Suppose that in period zero the central bank makes a correct statement about the aggregate state of the economy, and this statement reaches a fraction $\zeta \in [0, 1]$ of randomly selected households.¹⁸ Then, the probabilities \bar{p}_{bad}^{good} and \bar{p}_{good}^{bad} given by equations (28)-(29) have to be multiplied by a factor of $1 - \zeta$, because a fraction ζ of households is reached by the central bank communication and these households hold correct beliefs about the aggregate state of the economy.

The effect on consumption can be seen directly from equations (16)-(18), (20)-(22), and (23)-(25). When the zero lower bound is binding in both states (or more generally the real interest rate is lower in the good state than in the bad state), the communication *decreases* consumption in the bad state. In the bad state, households’ inflation expectations are above the true conditional mean of future inflation. Therefore, the central bank communication reduces the inflation expectations of the households that are reached. Furthermore, at the zero lower bound, reductions in households’ inflation expectations are destabilizing. Hence, aggregate consumption falls. By contrast, when the zero lower bound is binding in no state (or more generally the real interest rate is higher in the

¹⁸The individual history of a household in period T then consists of the realization of the preference shock in period zero, the realization of ownership in period zero, and whether the central bank statement reaches the household. Since the state-contingent claims specify a payment that is contingent on the individual history, in equilibrium all households have the same post-transfer wealth in period T .

good state than in the bad state), the same communication *increases* consumption in the bad state. In the bad state, households' inflation expectations are once again above the true conditional mean of future inflation and the central bank communication reduces the inflation expectations of the households that are reached. The Taylor principle implies that downward movements in households' inflation expectations are stabilizing. Hence, aggregate consumption increases.

These results imply that central banks probably face a new trade-off when inflation expectations adjust slowly downwards and the zero lower bound is binding. On the one hand, communicating low inflation risk to the public at the zero lower bound is destabilizing. On the other hand, the central bank probably would like to communicate truthfully the current state to the public in order to maintain a good reputation. The following excerpt from the September 4, 2014, press conference with the President of the European Central Bank (ECB) after that day's meeting of the ECB Governing Council suggests that this trade-off is present in practice:¹⁹

“Question: Mr Draghi, isn't there a risk that with the ECB emphasising so much the risk of low inflation that this itself could trigger a de-anchoring of expectations?”

Draghi: Well, you see, this question is actually a question we also asked ourselves. But the answer to this question is: would the truth be a risk? In other words, do we really think that telling people things other than the truth would affect their behaviour? And the answer is no. We think that we ought to state things as they are. We don't see deflation. We have seen, as a matter of fact, low inflation for a long time. As I've said several times, the longer the period of low inflation the higher the risks of de-anchoring.”

6.2 Central bank communication about future policy

Let us turn to forward guidance. Following Bassetto (2015), I define forward guidance as a direct statement by the central bank about the future path of its policy tools. These statements can take different forms. Campbell et al. (2012) distinguish between Odyssean forward guidance, which publicly commits the central bank to a future action, and Delphic forward guidance, which merely forecasts macroeconomic performance and likely monetary policy actions. Eggertsson and Woodford (2003) have pointed out that in a simple New Keynesian model a commitment by the

¹⁹See www.ecb.europa.eu/press/pressconf/2014/html/is140904.en.html

central bank to create inflation in the future is a powerful way of stimulating the economy when the zero lower bound is binding. I therefore first study the effects of Odyssean forward guidance and then turn to Delphic forward guidance.

Consider again the closed-form solutions of Section 4. Suppose that in period zero the central bank publicly commits itself to a future path of the nominal interest rate and this statement reaches a fraction $\zeta \in [0, 1]$ of randomly selected households. In the good state, the central bank announces that it will maintain a long-run inflation target of zero and set the nominal interest rate in periods $t \geq T$ accordingly. In the bad state, the central bank announces that it will increase the long-run inflation target from zero to $\bar{\pi} > 0$ and set the nominal interest rate in periods $t \geq T$ accordingly in order to raise inflation expectations in periods $t < T$.

Suppose that the zero lower bound is binding in both states with and without forward guidance. Without forward guidance, consumption in the two states is given by equations (16)-(18) and (28)-(29). With forward guidance, consumption in the good state equals

$$c_{good} = \frac{\frac{1}{\gamma}\bar{\xi}_{good} + \frac{1}{1-\mu}\ln(R) + \bar{p}_{good}^{bad}\left(\frac{1}{1-\beta\mu}\frac{1}{\gamma}\bar{\pi} + \bar{c}\right)}{1 - \frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}} - \bar{p}_{good}^{bad}\frac{\frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}{1 - \frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}(c_{good} - c_{bad}), \quad (40)$$

and consumption in the bad state equals

$$c_{bad} = \frac{\frac{1}{\gamma}\bar{\xi}_{bad} + \frac{1}{1-\mu}\ln(R) + \left(1 - \bar{p}_{bad}^{good}\right)\left(\frac{1}{1-\beta\mu}\frac{1}{\gamma}\bar{\pi} + \bar{c}\right)}{1 - \frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}} + \bar{p}_{bad}^{good}\frac{\frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}{1 - \frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}(c_{good} - c_{bad}), \quad (41)$$

with

$$c_{good} - c_{bad} = \frac{\frac{1}{\gamma}(\bar{\xi}_{good} - \bar{\xi}_{bad}) - \left(1 - \bar{p}_{good}^{bad} - \bar{p}_{bad}^{good}\right)\left(\frac{1}{1-\beta\mu}\frac{1}{\gamma}\bar{\pi} + \bar{c}\right)}{1 - \left(1 - \bar{p}_{good}^{bad} - \bar{p}_{bad}^{good}\right)\frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}. \quad (42)$$

Here $\bar{c} \geq 0$ denotes consumption in the non-stochastic steady state with inflation rate $\bar{\pi} > 0$, and the probabilities \bar{p}_{bad}^{good} and \bar{p}_{good}^{bad} are given by the expressions on the right-hand sides of equations (28)-(29) multiplied by the factor $1 - \zeta$, because the central bank's state-contingent statement reveals the aggregate state to the households that are reached by the central bank communication. The derivation of equations (40)-(42) is in Appendix A.

Forward guidance is less powerful than under perfect information and the effect on consumption can even have the opposite sign compared to perfect information. Under perfect information, consumption without forward guidance is given by equation (10). Consumption in the bad state

with forward guidance is given by equation (41) with $\bar{p}_{bad}^{good} = 0$. Forward guidance increases consumption in the bad state by $\left(\frac{1}{1-\beta\mu}\frac{1}{\gamma}\bar{\pi} + \bar{c}\right) / \left(1 - \frac{\frac{1}{\gamma}}{1-\mu} \frac{\mu\kappa}{1-\beta\mu}\right)$. Under imperfect information, this term is multiplied by $1 - \bar{p}_{bad}^{good}$ because not all households are aware of the fact that the commitment to a higher inflation target of $\bar{\pi} > 0$ is currently in place. Moreover, households that are reached by the central bank communication assign probability one to the bad state. Thus, \bar{p}_{bad}^{good} in the second term on the right-hand side of equation (41) differs from \bar{p}_{bad}^{good} in the second term on the right-hand side of equation (17) by a factor of $1 - \zeta$, which reduces consumption in the bad state. In other words, households that are reached by the central bank communication experience a positive and a negative effect on their inflation expectations: they learn that the long-run inflation target has been raised and they learn that the economy is in the bad state.

Due to the second effect, forward guidance can decrease aggregate consumption in the bad state. To see this as clearly as possible, suppose that all households are reached by the communication ($\zeta = 1$). Consumption in the bad state without forward guidance is given by equations (17)-(18) and (28)-(29). Consumption in the bad state with forward guidance is given by equation (41) with $\bar{p}_{bad}^{good} = 0$. Forward guidance increases aggregate consumption in the bad state if and only if

$$\frac{1}{1-\beta\mu}\frac{1}{\gamma}\bar{\pi} + \bar{c} > \bar{p}_{bad}^{good} \frac{\frac{1}{\gamma}(\bar{\xi}_{good} - \bar{\xi}_{bad}) \frac{\frac{1}{\gamma}}{1-\mu} \frac{\mu\kappa}{1-\beta\mu}}{1 - \left(1 - \bar{p}_{good}^{bad} - \bar{p}_{bad}^{good}\right) \frac{\frac{1}{\gamma}}{1-\mu} \frac{\mu\kappa}{1-\beta\mu}},$$

where \bar{p}_{bad}^{good} and \bar{p}_{good}^{bad} are given by equations (28)-(29).

Next, consider Delphic forward guidance, which merely forecasts macroeconomic performance and likely monetary policy actions. For this purpose, let us return to Section 5.4 and Figure 3. Suppose that in period zero the central bank makes a correct statement about the number of periods that the zero lower bound is expected to bind and this statement reaches all households ($\zeta = 1$). In the bad state this number of periods is much larger than in the good state. Therefore, the statement reveals the aggregate state to households, and aggregate consumption in the bad state is now given by the dotted lower line instead of the thick black line in Figure 3. Hence, Delphic forward guidance unambiguously decreases aggregate consumption in the bad state. In practice, the effect of Delphic forward guidance is probably much weaker, because the official statement by the central bank does not reach all households ($\zeta \in [0, 1)$).

The results in this subsection are related to two literatures. First, the literature on signaling

effects of monetary policy (e.g., Melosi, 2015, and Tang, 2015) studies environments where the current policy rate provides information about current fundamentals to the private sector. Note that this channel is muted when the zero lower bound is binding. I instead study a model where forward guidance provides information about the current state to households. Second, the emerging literature on the forward guidance puzzle (Carlstrom, Fuerst, and Paustian, 2012, Del Negro, Giannoni, and Patterson, 2012, McKay, Nakamura, and Steinsson, 2015) argues that the effects of forward guidance in benchmark New Keynesian models seem unreasonably large and offers explanations for weaker effects of forward guidance. I offer complementary explanations for weaker effects of forward guidance (not all households are reached and those that are reached may revise their inflation expectations downwards).

7 Fiscal policy

Shortly after the Federal Reserve lowered the target for the federal funds rate to 0-0.25 percent in December 2008, the U.S. Congress passed a major fiscal stimulus package - the American Recovery and Reinvestment Act of 2009. An interesting question is how expansionary fiscal policy in a bad state affects consumption in a world where household inflation expectations adjust slowly.

To address this question formally, consider the closed-form solutions of Section 4. Suppose that the level of government spending equals G in the long run (i.e., in periods $t \geq T$). Let $g_t = \ln(G_t/G)$ denote the log-deviation of current government spending from long-run government spending. In period zero, the fiscal authority makes an announcement about the level of government spending during the recession (i.e., in periods $0 \leq t < T$) and this announcement reaches a fraction $\zeta \in [0, 1]$ of randomly selected households. In the good state, the government announces $g_t = g_{good} = 0$. In the bad state, the government announces $g_t = g_{bad} = g > 0$.

Government spending changes the New Keynesian Phillips curve (5), but not the consumption Euler equation (1). Substituting the log-linearized wage index, the wage setting equation (2), and the log-linearized resource constraint $y_t = (C/Y)c_t + (G/Y)g_t$ into equation (4) yields the modified New Keynesian Phillips curve

$$\pi_t = \kappa_c c_t + \kappa_g g_t + \beta E_t [\pi_{t+1}],$$

where

$$\kappa_c = \frac{(1-\alpha)(1-\alpha\beta)\gamma + \frac{1-\rho}{\rho}\frac{C}{Y}}{\alpha \left(1 + \frac{1-\rho}{\rho}\psi\right)} \quad \text{and} \quad \kappa_g = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{\frac{1-\rho}{\rho}\frac{G}{Y}}{1 + \frac{1-\rho}{\rho}\psi}.$$

It is then straightforward to derive aggregate consumption in the good state and aggregate consumption in the bad state. When the zero lower bound is binding in both states, consumption in the good state equals

$$c_{good} = \frac{\frac{1}{\gamma}\bar{\xi}_{good} + \frac{1}{1-\mu}\ln(R) + \bar{p}_{good}^{bad} \frac{1}{1-\mu} \frac{\mu\kappa_g}{1-\beta\mu} g}{1 - \frac{1}{1-\mu} \frac{\mu\kappa_c}{1-\beta\mu}} - \bar{p}_{good}^{bad} \frac{\frac{1}{1-\mu} \frac{\mu\kappa_c}{1-\beta\mu}}{1 - \frac{1}{1-\mu} \frac{\mu\kappa_c}{1-\beta\mu}} (c_{good} - c_{bad}), \quad (43)$$

and consumption in the bad state equals

$$c_{bad} = \frac{\frac{1}{\gamma}\bar{\xi}_{bad} + \frac{1}{1-\mu}\ln(R) + \left(1 - \bar{p}_{bad}^{good}\right) \frac{1}{1-\mu} \frac{\mu\kappa_g}{1-\beta\mu} g}{1 - \frac{1}{1-\mu} \frac{\mu\kappa_c}{1-\beta\mu}} + \bar{p}_{bad}^{good} \frac{\frac{1}{1-\mu} \frac{\mu\kappa_c}{1-\beta\mu}}{1 - \frac{1}{1-\mu} \frac{\mu\kappa_c}{1-\beta\mu}} (c_{good} - c_{bad}), \quad (44)$$

with

$$c_{good} - c_{bad} = \frac{\frac{1}{\gamma}(\bar{\xi}_{good} - \bar{\xi}_{bad}) - \left(1 - \bar{p}_{good}^{bad} - \bar{p}_{bad}^{good}\right) \frac{1}{1-\mu} \frac{\mu\kappa_g}{1-\beta\mu} g}{1 - \left(1 - \bar{p}_{good}^{bad} - \bar{p}_{bad}^{good}\right) \frac{1}{1-\mu} \frac{\mu\kappa_c}{1-\beta\mu}}. \quad (45)$$

The probabilities \bar{p}_{bad}^{good} and \bar{p}_{good}^{bad} are again given by the expressions on the right-hand sides of equations (28)-(29) multiplied by the factor $1 - \zeta$, because the government's state-contingent state-ment reveals the aggregate state to the households that are reached by the communication. The derivation of equations (43)-(45) is in Appendix B.

There are two differences to the case of perfect information. First, the expression $\frac{1}{1-\mu} \frac{\mu\kappa_g}{1-\beta\mu} g$ in the first term on the right-hand side of equation (44) is multiplied by a factor of $1 - \bar{p}_{bad}^{good}$, because the government's announcement does not reach all households (or equivalently, not all households update their inflation expectations based on the announcement). Second, the probability \bar{p}_{bad}^{good} in the second term on the right-hand side of equation (44) is multiplied by a factor of $1 - \zeta$, because households that are reached by the communication assign probability one to the bad state.

Due to the second effect, the state-contingent fiscal policy can reduce aggregate consumption in the bad state. To see this as clearly as possible, suppose that the government's announcement reaches all households ($\zeta = 1$). Then, consumption in the bad state with state-contingent fiscal policy is given by equation (44) with $\bar{p}_{bad}^{good} = 0$. Consumption in the bad state without state-contingent fiscal policy is given by equations (17)-(18) and (28)-(29). The policy increases aggregate

consumption in the bad state if and only if

$$\kappa_g g > \bar{p}_{bad}^{good} \kappa (c_{good} - c_{bad}),$$

where $c_{good} - c_{bad}$ is given by equation (18) and \bar{p}_{bad}^{good} is given by equation (28). Substituting in the expressions for κ_g and κ yields

$$\frac{G}{Y} \frac{\frac{1-\rho}{\rho}}{\gamma + \frac{1-\rho}{\rho}} g > \bar{p}_{bad}^{good} (c_{good} - c_{bad}).$$

The results in this section are related to other work arguing that the government spending multiplier at the zero lower bound may not be as large as predicted by the benchmark New Keynesian model with a zero lower bound, e.g., because of distortionary taxes (Uhlig and Drautzburg, 2013) or a non-fundamental liquidity trap (Mertens and Ravn, 2014). I offer complementary arguments (not all households revise their inflation expectations directly after the government’s announcement of the fiscal package and those that do may revise their inflation expectations downwards).

8 Conclusion

In New Keynesian models, movements in household inflation expectations are of great importance for the propagation of shocks and the effectiveness of policy, especially when the nominal interest rate is at zero. It is therefore desirable to model household inflation expectations in a way that is consistent with data. To this end, I assume that households have different pieces of information and update their inflation expectations slowly over time. The resulting model has properties that are quite different from a model with full-information rational expectations on the household side. The deflationary spiral takes off slowly in bad states of the world. Central bank communication about the current state affects aggregate consumption, and the sign of this effect switches when the zero lower bound is binding. Forward guidance is less powerful than under perfect information, and the effect on consumption can even have the opposite sign compared to perfect information. The government spending multiplier is smaller than under perfect information. All these results are more pronounced when the economy ends up in an aggregate state with a small prior probability, such as the financial crisis in the U.S. and the sovereign debt crisis in Europe.

One interesting extension would be to introduce slow updating of inflation expectations on the

firm side. I conjecture that this modification would reinforce the main conclusions.²⁰ Another interesting extension would be to move from a model with an exogenous information structure to a rational inattention model, as in Sims (2003). One could then study how policy-makers' action and communication strategies affect households' attention to fundamentals (ω) and to official statements by policy-makers (ζ). Thereafter, one could study policy-makers' optimal action and communication strategies. I leave these extensions to future research.

²⁰Kiley (2014) studies the effects of various policies in a model with sticky information instead of sticky prices on the firm side. Coibion and Gorodnichenko (2015b) show that a standard Phillips curve can match the absence of a persistent decline in inflation during the Great Recession once model inflation expectations on the firm side are replaced by survey data.

A Derivation of equations (40)-(42)

First, state the consumption Euler equation of the four types of households: high, informed types (i.e., households with $\xi_{i,0} = \xi_H$ that are reached by the communication), low, informed types (i.e., households with $\xi_{i,0} = \xi_L$ that are reached by the communication), high, uninformed types (i.e., households with $\xi_{i,0} = \xi_H$ that are not reached by the communication), and low, uninformed types.

The consumption Euler equation of a high, informed type in the good state reads

$$c_{good}^{hi} = -\frac{1}{\gamma} [(\mu - 1) \xi_H - \ln(R) - \mu\pi_{good}] + \mu c_{good}^{hi}.$$

In the bad state, it reads

$$c_{bad}^{hi} = -\frac{1}{\gamma} [(\mu - 1) \xi_H - \ln(R) - (\mu\pi_{bad} + (1 - \mu) \bar{\pi})] + \mu c_{bad}^{hi} + (1 - \mu) \bar{c}.$$

The consumption Euler equation of a low, informed type in the good state reads

$$c_{good}^{li} = -\frac{1}{\gamma} [(\mu - 1) \xi_L - \ln(R) - \mu\pi_{good}] + \mu c_{good}^{li}.$$

In the bad state, it reads

$$c_{bad}^{li} = -\frac{1}{\gamma} [(\mu - 1) \xi_L - \ln(R) - (\mu\pi_{bad} + (1 - \mu) \bar{\pi})] + \mu c_{bad}^{li} + (1 - \mu) \bar{c}.$$

The consumption Euler equation of a high, uninformed type reads

$$\begin{aligned} c^{hu} &= -\frac{1}{\gamma} \left[(\mu - 1) \xi_H - \ln(R) - \left(p_H^{good} \mu\pi_{good} + p_H^{bad} (\mu\pi_{bad} + (1 - \mu) \bar{\pi}) \right) \right] \\ &\quad + p_H^{good} \mu c^{hu} + p_H^{bad} \left(\mu c^{hu} + (1 - \mu) \bar{c} \right). \end{aligned}$$

The consumption Euler equation of a low, uninformed type reads

$$\begin{aligned} c^{lu} &= -\frac{1}{\gamma} \left[(\mu - 1) \xi_L - \ln(R) - \left(p_L^{good} \mu\pi_{good} + p_L^{bad} (\mu\pi_{bad} + (1 - \mu) \bar{\pi}) \right) \right] \\ &\quad + p_L^{good} \mu c^{lu} + p_L^{bad} \left(\mu c^{lu} + (1 - \mu) \bar{c} \right). \end{aligned}$$

Hence, aggregate consumption in the good state equals

$$\begin{aligned} c_{good} &= \zeta \left[\lambda_{good} c_{good}^{hi} + (1 - \lambda_{good}) c_{good}^{li} \right] + (1 - \zeta) \left[\lambda_{good} c^{hu} + (1 - \lambda_{good}) c^{lu} \right] \\ &= \frac{1}{\gamma} (1 - \mu) \bar{\xi}_{good} + \frac{1}{\gamma} \ln(R) + \frac{1}{\gamma} \mu\pi_{good} + \mu c_{good} \\ &\quad + \bar{p}_{good}^{bad} (1 - \mu) \left(\frac{1}{\gamma} \bar{\pi} + \bar{c} \right) - \bar{p}_{good}^{bad} \frac{1}{\gamma} \mu (\pi_{good} - \pi_{bad}). \end{aligned}$$

Aggregate consumption in the bad state equals

$$\begin{aligned}
c_{bad} &= \zeta \left[\lambda_{bad} c_{bad}^{hi} + (1 - \lambda_{bad}) c_{bad}^{li} \right] + (1 - \zeta) \left[\lambda_{bad} c^{hu} + (1 - \lambda_{bad}) c^{lu} \right] \\
&= \frac{1}{\gamma} (1 - \mu) \bar{\xi}_{bad} + \frac{1}{\gamma} \ln(R) + \frac{1}{\gamma} \mu \pi_{bad} + \mu c_{bad} \\
&\quad + \left(1 - \bar{p}_{bad}^{good} \right) (1 - \mu) \left(\frac{1}{\gamma} \bar{\pi} + \bar{c} \right) + \bar{p}_{bad}^{good} \frac{1}{\gamma} \mu (\pi_{good} - \pi_{bad}).
\end{aligned}$$

The probabilities \bar{p}_{bad}^{good} and \bar{p}_{good}^{bad} are given by the expressions on the right-hand sides of equations (28)-(29) multiplied by the factor $(1 - \zeta)$.

Second, state the New Keynesian Phillips curve for the good state and the bad state. In the good state, the New Keynesian Phillips curve reads

$$\pi_{good} = \kappa c_{good} + \beta \mu \pi_{good}.$$

In the bad state, the New Keynesian Phillips curve reads

$$\pi_{bad} = \kappa c_{bad} + \beta (\mu \pi_{bad} + (1 - \mu) \bar{\pi}).$$

Finally, using the last two equations to substitute for π_{good} and π_{bad} in the previous two equations and rearranging yields equations (40)-(42).

B Derivation of equations (43)-(45)

First, state the consumption Euler equation of the four types of households: high, informed types (i.e., households with $\xi_{i,0} = \xi_H$ that are reached by the announcement), low, informed types (i.e., households with $\xi_{i,0} = \xi_L$ that are reached by the announcement), high, uninformed types (i.e., households with $\xi_{i,0} = \xi_H$ that are not reached by the announcement), and low, uninformed types.

The consumption Euler equation of a high, informed type in state $s \in \{good, bad\}$ reads

$$c_s^{hi} = -\frac{1}{\gamma} [(\mu - 1) \xi_H - \ln(R) - \mu \pi_s] + \mu c_s^{hi}.$$

The consumption Euler equation of a low, informed type in state $s \in \{good, bad\}$ reads

$$c_s^{li} = -\frac{1}{\gamma} [(\mu - 1) \xi_L - \ln(R) - \mu \pi_s] + \mu c_s^{li}.$$

The consumption Euler equation of a high, uninformed type reads

$$c^{hu} = -\frac{1}{\gamma} \left[(\mu - 1) \xi_H - \ln(R) - \left(p_H^{good} \mu \pi_{good} + p_H^{bad} \mu \pi_{bad} \right) \right] + p_H^{good} \mu c^{hu} + p_H^{bad} \mu c^{hu}.$$

The consumption Euler equation of a low, uninformed type reads

$$c^{lu} = -\frac{1}{\gamma} \left[(\mu - 1) \xi_L - \ln(R) - \left(p_L^{good} \mu \pi_{good} + p_L^{bad} \mu \pi_{bad} \right) \right] + p_L^{good} \mu c^{lu} + p_L^{bad} \mu c^{lu}.$$

Hence, aggregate consumption in the good state equals

$$\begin{aligned} c_{good} &= \zeta \left[\lambda_{good} c_{good}^{hi} + (1 - \lambda_{good}) c_{good}^{li} \right] + (1 - \zeta) \left[\lambda_{good} c^{hu} + (1 - \lambda_{good}) c^{lu} \right] \\ &= \frac{1}{\gamma} (1 - \mu) \bar{\xi}_{good} + \frac{1}{\gamma} \ln(R) + \frac{1}{\gamma} \mu \pi_{good} + \mu c_{good} - \bar{p}_{good}^{bad} \frac{1}{\gamma} \mu (\pi_{good} - \pi_{bad}). \end{aligned}$$

Furthermore, aggregate consumption in the bad state equals

$$\begin{aligned} c_{bad} &= \zeta \left[\lambda_{bad} c_{bad}^{hi} + (1 - \lambda_{bad}) c_{bad}^{li} \right] + (1 - \zeta) \left[\lambda_{bad} c^{hu} + (1 - \lambda_{bad}) c^{lu} \right] \\ &= \frac{1}{\gamma} (1 - \mu) \bar{\xi}_{bad} + \frac{1}{\gamma} \ln(R) + \frac{1}{\gamma} \mu \pi_{bad} + \mu c_{bad} + \bar{p}_{bad}^{good} \frac{1}{\gamma} \mu (\pi_{good} - \pi_{bad}). \end{aligned}$$

The probabilities \bar{p}_{bad}^{good} and \bar{p}_{good}^{bad} are given by the expressions on the right-hand sides of equations (28)-(29) multiplied by the factor $1 - \zeta$.

Second, state the New Keynesian Phillips curve for the good state and the bad state. In the good state, the New Keynesian Phillips curve reads

$$\pi_{good} = \kappa_c c_{good} + \beta \mu \pi_{good}.$$

In the bad state, the New Keynesian Phillips curve reads

$$\pi_{bad} = \kappa_c c_{bad} + \kappa_g g + \beta \mu \pi_{bad}.$$

Finally, using the last two equations to substitute for π_{good} and π_{bad} in the previous two equations and rearranging yields equations (43)-(45).

References

- [1] Andrade, Philippe, Gaetano Gaballo, Eric Mengus, and Benoit Mojon (2015). “Forward Guidance and Heterogeneous Beliefs,” manuscript, Banque de France and HEC Paris.
- [2] Angeletos, George-Marios, and Jennifer La’O (2013). “Sentiments,” *Econometrica*, 81(2), 739-780.
- [3] Armantier, Olivier, Wändi Bruine de Bruin, Giorgio Topa, Wilbert van der Klaauw, and Basit Zafar (2011). “Inflation Expectations and Behavior: Do Survey Respondents Act on Their Beliefs?” *Federal Reserve Bank of New York Staff Reports*, no. 509.
- [4] Bassetto, Marco (2015). “Forward Guidance: Communication, Commitment, or Both?” manuscript, Federal Reserve Bank of Chicago and University College London.
- [5] Calvo, Guillermo A. (1983). “Staggered Prices in a Utility-Maximizing Framework,” *Journal of Monetary Economics*, 12(3), 383-398.
- [6] Campbell, Jeffrey, Charles Evans, Jonas Fisher, and Alejandro Justiniano (2012). “Macroeconomic Effects of Federal Reserve Forward Guidance,” *Federal Reserve Bank of Chicago Working Papers*, no. 2012-03.
- [7] Carlstrom, Charles, Timothy Fuerst, and Matthias Paustian (2012). “Inflation and Output in New Keynesian Models with a Transient Interest Rate Peg,” *Bank of England Working Papers*, no. 459.
- [8] Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo (2011). “When is the Government Spending Multiplier Large?” *Journal of Political Economy*, 119(1), 78-121.
- [9] Coibion, Olivier, and Yuriy Gorodnichenko (2012). “What Can Survey Forecasts Tell Us about Information Rigidities?” *Journal of Political Economy*, 120(1), 116-159.
- [10] Coibion, Olivier, and Yuriy Gorodnichenko (2015). “Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts,” *American Economic Review*, 105(8), 2644-2678.

- [11] Coibion, Olivier, and Yuriy Gorodnichenko (2015b). “Is the Phillips Curve Alive and Well After All? Inflation Expectations and the Missing Disinflation,” *American Economic Journal: Macroeconomics*, 7(1), 197-232.
- [12] Curdia, Vasco, and Michael Woodford (2011). “The Central-Bank Balance Sheet as an Instrument of Monetary Policy,” *Journal of Monetary Economics*, 58(1), 54-79.
- [13] Del Negro, Marco, Marc Giannoni, and Christina Patterson (2012). “The Forward Guidance Puzzle,” *Federal Reserve Bank of New York Staff Reports*, no. 574.
- [14] Drautzburg, Thorsten, and Harald Uhlig (2013). “Fiscal Stimulus and Distortionary Taxation,” *Federal Reserve Bank of Philadelphia Working Papers*, no. 13-46.
- [15] Eggertsson, Gauti, and Michael Woodford (2003). “The Zero Bound on Interest Rates and Optimal Monetary Policy,” *Brookings Papers on Economic Activity*, 2003(1), 139-211.
- [16] Kiley, Michael (2014). “Policy Paradoxes in the New Keynesian Model,” Finance and Economics Discussion Series, Federal Reserve Board.
- [17] Krugman, Paul (1998). “It’s Baaack: Japan’s Slump and the Return of the Liquidity Trap,” *Brookings Papers on Economic Activity*, 1998(2), 137-205.
- [18] Lorenzoni, Guido (2009). “A Theory of Demand Shocks,” *American Economic Review*, 99(5), 2050-2084.
- [19] Lorenzoni, Guido (2010). “Optimal Monetary Policy with Uncertain Fundamentals and Dispersed Information,” *Review of Economic Studies*, 77(1), 305-338.
- [20] Lucas, Robert E. Jr. (1990). “Liquidity and Interest Rates,” *Journal of Economic Theory*, 50(2), 237-264.
- [21] Maćkowiak, Bartosz, and Mirko Wiederholt (2015). “Business Cycle Dynamics under Rational Inattention,” manuscript, European Central Bank and Goethe University Frankfurt.
- [22] Mankiw, N. Gregory, and Ricardo Reis (2002). “Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve,” *Quarterly Journal of Economics*, 117(4), 1295-1328.

- [23] Mankiw, N. Gregory, and Ricardo Reis (2006). “Pervasive Stickiness,” *American Economic Review*, 96(2), 164-169.
- [24] McKay, Alisdair, Emi Nakamura, and Jon Steinsson (2015). “The Power of Forward Guidance Revisited,” manuscript, Boston University and Columbia University.
- [25] Mertens, Karel, and Morten Ravn (2014). “Fiscal Policy in an Expectations-Driven Liquidity Trap,” *Review of Economic Studies*, 81(4), 1637-1667.
- [26] Nakamura, Emi, and Jón Steinsson (2008). “Five Facts About Prices: A Reevaluation of Menu Cost Models,” *Quarterly Journal of Economics*, 123(4), 1415-1464.
- [27] Woodford, Michael (2011). “Simple Analytics of the Government Expenditure Multiplier,” *American Economic Journal: Macroeconomics*, 3(1), 1-35.

Figure 1: consumption over time, benchmark

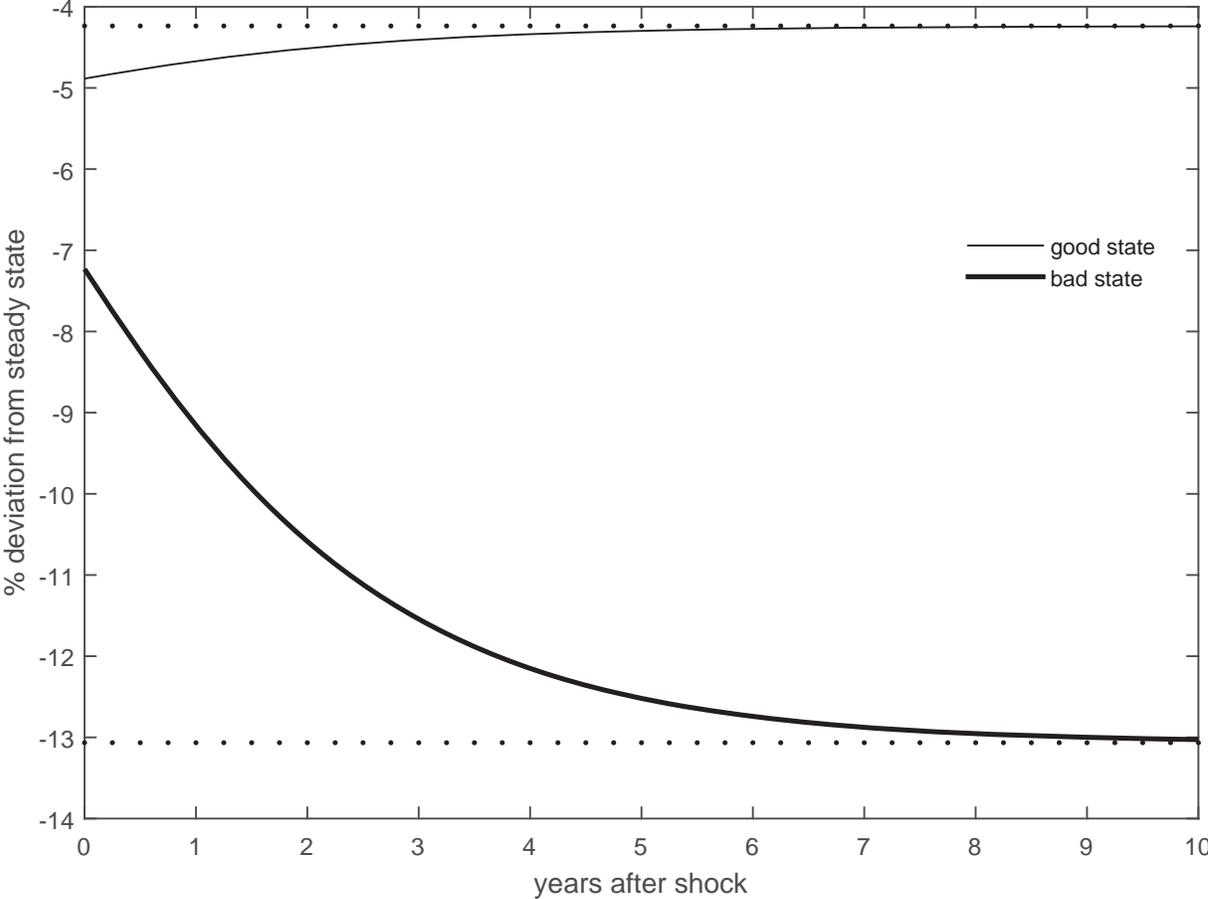


Figure 2: consumption and nominal interest rate, deterministic decay

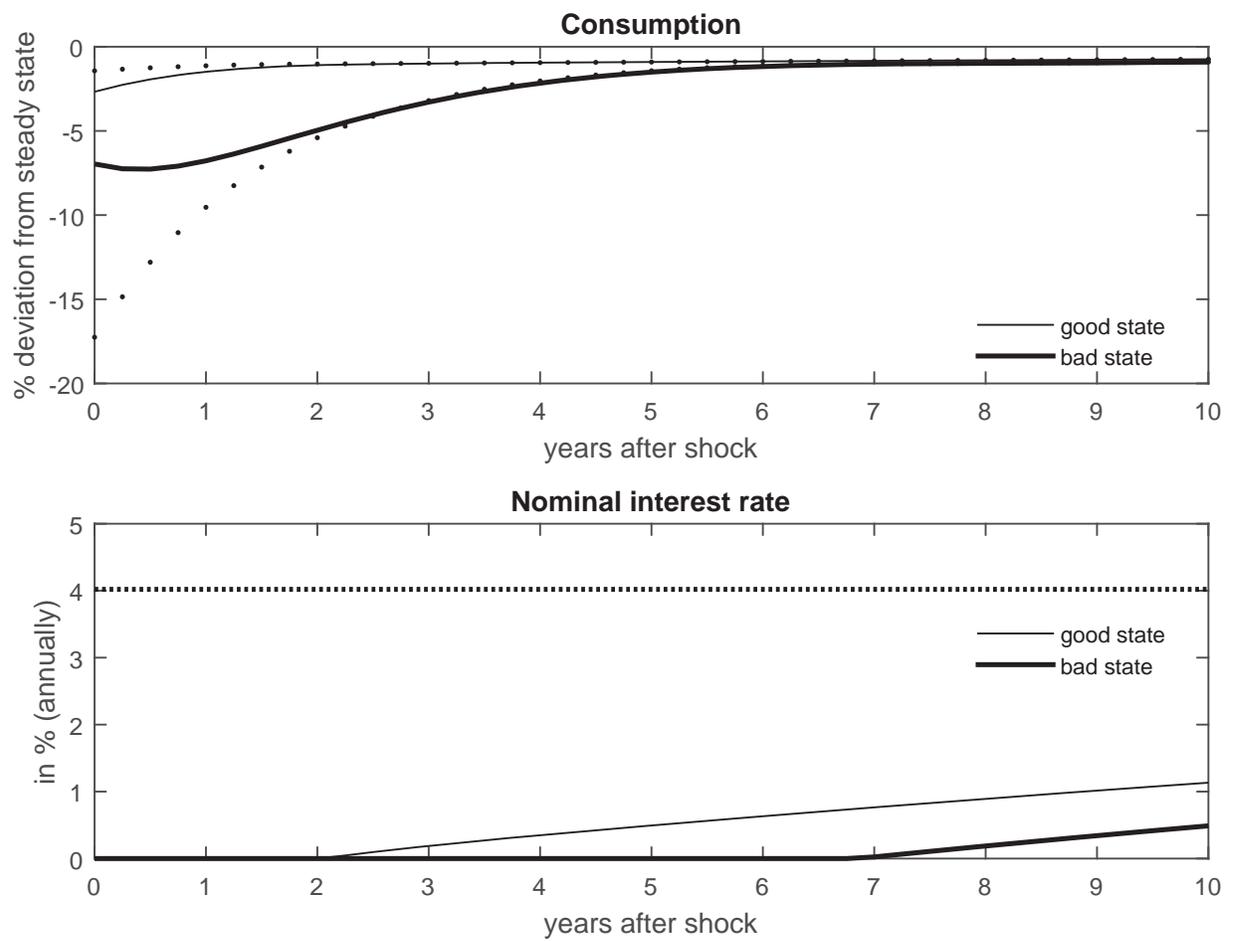


Figure 3: consumption and nominal interest rate, households set nominal wage rate

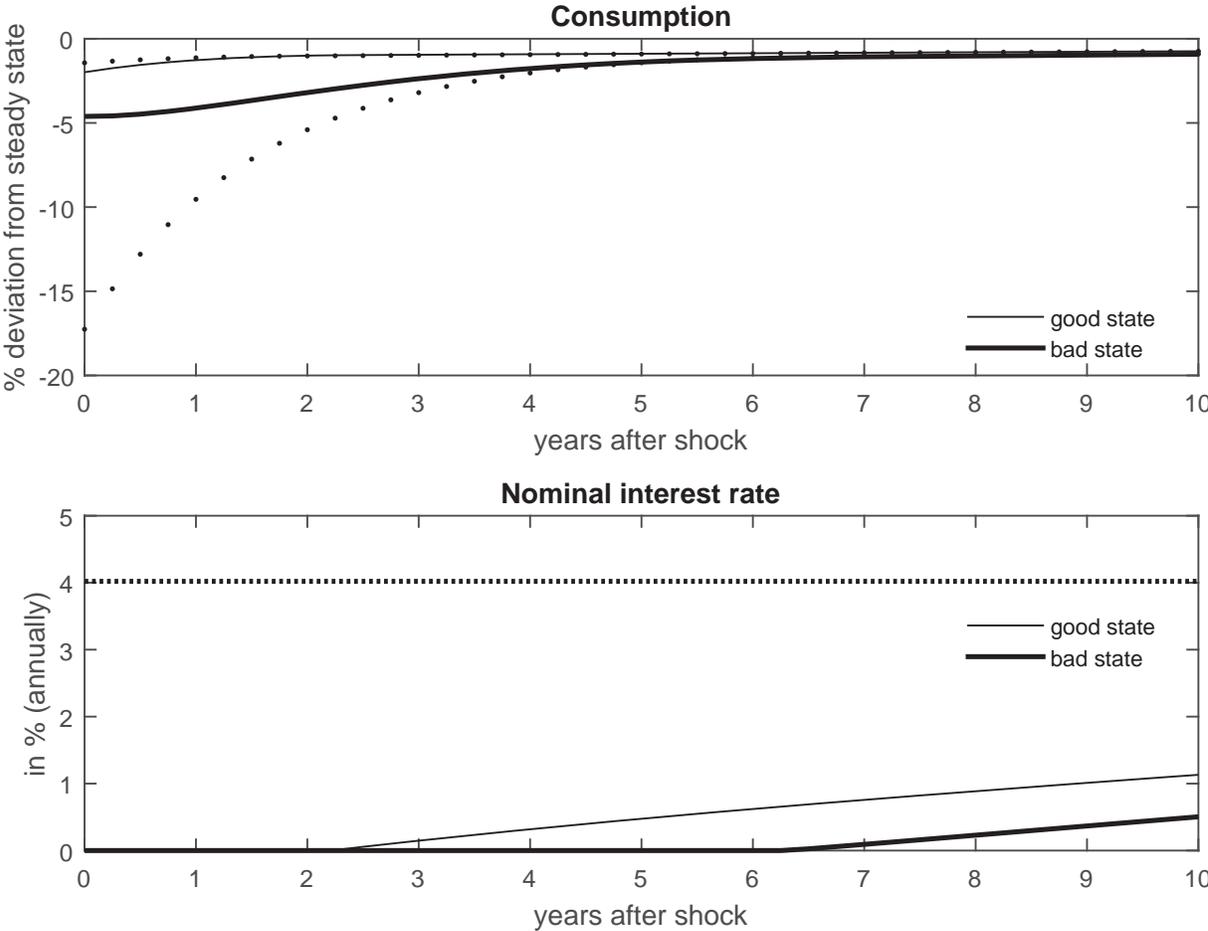


Figure 4: faster updating of inflation expectations

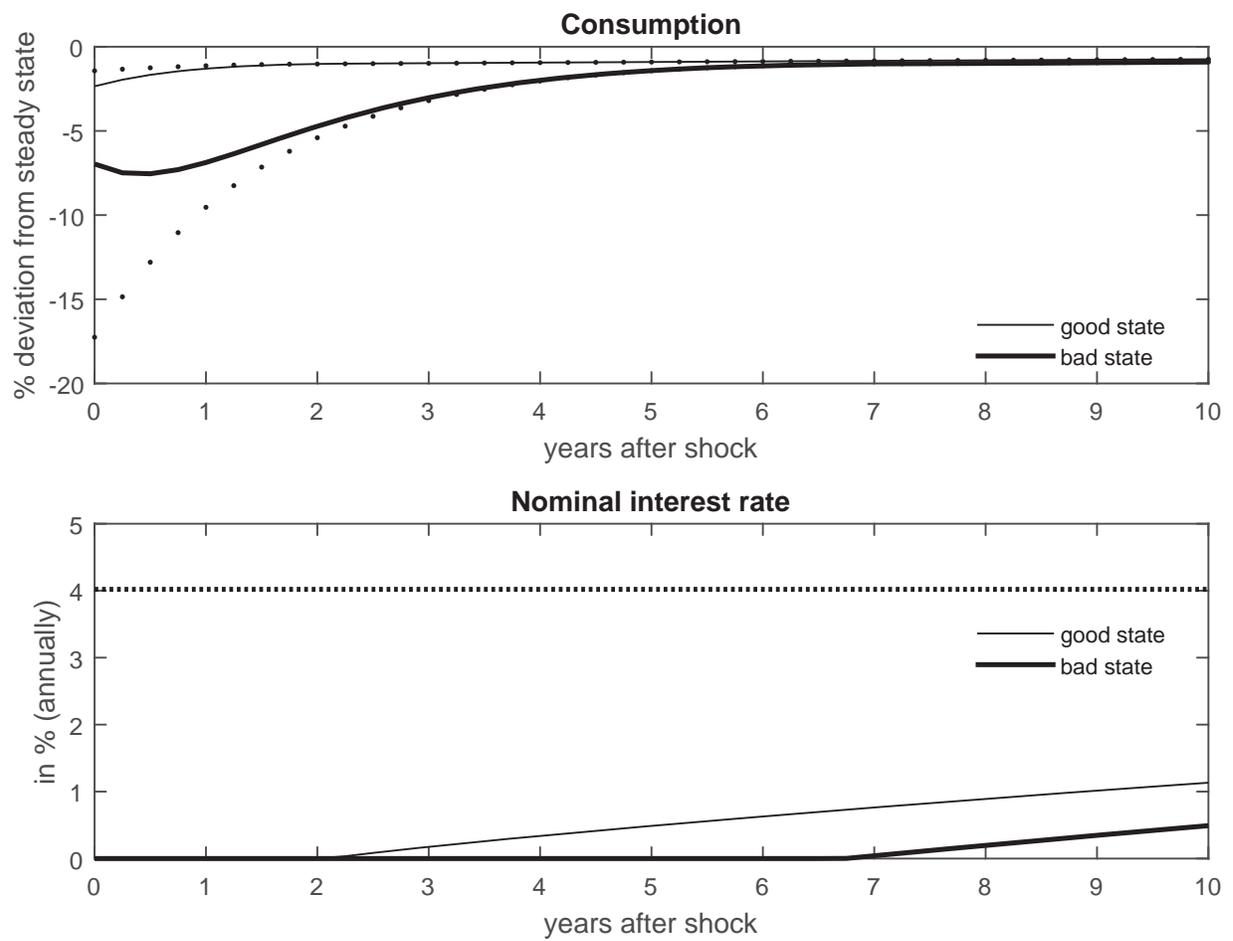


Figure 5: lower prior probability of the good state

