

# Systemic Loops and Liquidity Regulation\*

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## Abstract

Banks are typically exposed to spirals between liquidity scarcity and solvency risk. We build a network model of optimizing banks featuring contagion on both sides of balance sheets: runs on short term liabilities and banks' liquidity hoarding induce liquidity freezes; fire sale externalities and interconnected debt defaults produce asset risk. We use the model, which is calibrated to European data via simulated method of moments, to study the effects of phase-in increases of liquidity coverage ratios. Interestingly we find that the systemic risk profile of the system is not improved and might even deteriorate. Based on those insights we propose an alternative approach: differential (across banks) application of coverage ratios based on a systemic importance ranking help to mitigate the externalities and deliver a much more stable system.

**Keywords:** *bank runs, liquidity scarcity, interconnections, contagion, phase-in.*

**JEL:** *D85, G21, G28, C63, L14.*

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# 1 Introduction

Systemic risk is usually associated with *contagion*. Indeed, contagion is a key ingredient in explaining how a small shock can lead to large system-wide losses (i.e. how systemic risk emerges). Contagion itself is a multifaceted phenomenon as it can occur on both the liability and asset sides of banks' balance sheets. But in order to properly account for how systemic risk can arise, contagion is not enough: *amplification* is also needed. Amplification mechanisms are critical in deepening contagion effects and in particular in generating self-reinforcing dynamics. Understanding how contagion arises through its various channels and how it gets endogenously amplified is paramount for crisis prevention. At the current juncture prudential regulation is undertaking two main avenues. Equity requirements are meant to control and prevent the spread of losses on banks' asset side. Liquidity requirements, newly introduced in Basel III and subsequent regulations (CRD IV and CRR), aim at mitigating the impact of liquidity freezes. A unified theory of contagion and its interaction with amplification mechanisms is not yet available, although many recent and prominent contributions have examined in depth various individual channels of contagion. We move a step forward in this direction by providing a unified model that captures the interplay between these channels, in the context of a micro-founded framework with a meaningful role for regulation. We focus on the newly adopted liquidity regulation, which has been motivated by the widespread observation that banks' solvency crises are often the result of liquidity freezes (namely distress on the banks' liability side).

To build a theory of contagion it is essential to endeavour toward a model with interlinkages. We do so by building a banking network model which features interlinkages on both the asset and the liability side of banks' balance sheets. In our model banks optimally solve portfolio decisions (choosing both interbank lending and borrowing, liquid and non-liquid assets, and short term liabilities) subject to equity and liquidity requirements. Banks trade in interbank and non-liquid asset markets. They enter the first to insure against liquidity shortages<sup>1</sup>, but once inside they are also exposed to risks of debt default. In both markets prices are determined endogenously and fire sale externalities materialize in the non-liquid asset market. Those endogenous clearing processes together with banks' optimizing decisions contribute to determine the contagion channels in our model as described below.

Banks' short term funding comes from interbank borrowing and short term liabilities<sup>2</sup>. Liquidity is *scarce* in our model for two reasons. First, banks are risk averse and therefore tend to hoard liquidity in the face of shocks. Second, short term funding is obtained by resorting to external investors who assess the quality of their asset investment based on information about banks' returns. When news of non-performing banks' assets arrive, an information coordination problem among

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<sup>1</sup>Trading partners in the interbank market are matched based on an entropy algorithm, which spreads trading relationships as evenly as possible. See [Upper \(2011\)](#) for a methodological overview and [Upper & Worms \(2004\)](#) for an early application to the interbank contagion literature.

<sup>2</sup>We will occasionally use the term deposits for simplicity, although those are meant to be non-insurable short term liabilities, as is the vast majority of banks' outside short-term funding.

depositors of the bank takes place. Specifically, through an underlying global game mechanism (along the lines of [Morris & Shin \(2003\)](#) or [Carlsson & van Damme \(1993\)](#)), if returns fall below a certain threshold investors run the bank. Because of interbank freezes or investors' runs, banks might experience liquidity shortages. The latter typically lead to banks' solvency crises: as postulated in [Diamond & Rajan \(2005\)](#) (among others) illiquid banks quickly turn into insolvent banks as liquidity shortage forces project liquidations. The ensuing asset losses render illiquid banks also insolvent. In turn insolvency of some banks puts further strains on other banks. It is those links and the feedback loops between liquidity and solvency that motivated policy makers to consider liquidity requirements so central in the design of the most recent regulatory architecture. Notice that interbank markets in our framework play a dual role: on the one side banks, experiencing liquidity shortage, enter the interbank market for insurance motives and to mitigate the impact of runs on short term liabilities; on the other side, interbank lending exposes banks to default risk. The impacts of liquidity shortage on systemic risk in our model will always result from the balance between those two effects.

Our model also features a rich structure for contagion on the asset side. Both interbank lending and investment in non-liquid assets carry some risk on returns. The interbank market features direct network linkages thereby creating a direct channel for loss propagation. Defaulting banks impose losses on their creditors, who might in turn be unable to honour their debts thereby amplifying the network externalities. On the other side returns on non-liquid assets are heterogenous across banks<sup>3</sup> and are subject to shocks. When an adverse shock materializes banks engage into fire sales of non-liquid assets in order to fulfill regulatory requirements. Market prices fall endogenously due to the readjustment triggered by the tâtonnement mechanism. The ensuing fall in asset prices produces accounting losses on all exposed banks (pecuniary externalities).

Notice that the model features systemic feedback loops arising from the endogenous interaction of contagion on both sides of the balance sheet. Feedback loops in turn induce amplification effects. On the one side, liquidity shortage (due to interbank debt defaults or to investors' bank runs) force banks to liquidate assets and to engage into fire sales. Hence liquidity shortage triggers contagion on the asset side. On the other side, when banks' asset returns fall due to accounting losses, news of the bad performance reach investors, who might then run the bank. In this case asset risk feedbacks onto liquidity risk. Past literature on banking networks (see [Caccioli, Farmer, Foti & Rockmore \(2014\)](#) or [Glasserman & Young \(2014\)](#)) pointed out that a single contagion channel can hardly explain systemic bank crises. The two side contagion channels coupled with the feedback loops just described allows our model to produce realistic banking panics: this also makes the model suitable for the study of crises prevention policies, such as liquidity requirements.

The model is calibrated to the network of large European banks presented in [Alves, Ferrari,](#)

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<sup>3</sup>This captures the fact that banks have different performing investment opportunities, either because of luck or because of their monitoring abilities.

Franchini, Heam, Jurca, Langfield, Laviola, Liedorp, Sánchez, Tavoraro & Vuillemeys (2013). Calibration of the policy parameters is done based on regulatory requirements. The rest is instead obtained through a method of simulated moments: parameters are chosen so as to match some empirical targets. This strategy contributes to the realism and the empirical validity of the model.

We use quantitative simulations to conduct policy analysis. Prior to that we verify whether our model matches a number of banking network statistics: and indeed it does it remarkably well. Also based on this we judge it well suited for the analysis of prudential regulation. Specifically we simulate the model in response to shocks and to a gradual introduction (phase-in) of the liquidity coverage ratio (LCR hereafter). We find that a phased-in increase of the LCR produces undesired negative consequences in the dynamic of systemic risk<sup>4</sup>. In the initial steps of the phase-in arrangement systemic risk presents a mild reduction, but in the last step this is reversed, providing no net gain overall. The reason for this is twofold. First, under high LCR the insurance benefits of interbank trading fade away and leave space only to contagion channels. Second, an LCR requirement which is equal for all banks has distortive effects when applied to institutions which are otherwise very diverse in their exposures and balance sheet structures. Liquidity ratios have beneficial effects by limiting interbank leverage and the exposure to non-liquid assets of large banks (those with high returns on assets). This limits the scope for loss propagation through network and fire sale externalities. However their introduction has detrimental effects by creating unnecessary liquidity shortages also on banks which were only mildly exposed to contagion risk. The detrimental effects tend to out-weigh the beneficial ones in the process of phase-in.

Motivated by this finding we conduct a second policy experiment which focuses on the cross-section dimension of liquidity regulation by incorporating a macro-prudential element into an otherwise flat micro-prudential requirement. We increase liquidity requirements to systemically important banks<sup>5</sup>, while at the same time reducing them for the others, in a “liquidity-neutral” way (i.e. required liquidity stays the same as in the benchmark model with no macro-prudential requirements). This alternative approach is actually effective in reducing systemic risk monotonically. The differential regulation helps in maximizing the beneficial effects and minimizing the detrimental ones. Systemically important banks are in fact forced to raise internal liquidity buffers and to reduce their exposure to interbank and non-liquid asset markets, thereby reducing the likelihood of contagion. This mitigates the propagation of contagion. The other banks are instead able to free up liquidity thereby compensating for the shortage induced by the introduction of the LCR on systemically important banks. Overall this manoeuvre helps to restore the function of liquidity insurance in the interbank market.

The rest of the paper is structured as follows. The next section provides a literature review. Section 3 describes the model. Section 4 discusses the shock propagation mechanism, presents

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<sup>4</sup>Systemic risk is computed as the percentage of assets lost over total assets of the system.

<sup>5</sup>These banks are identified based on the methodology proposed by the Basel Committee on Banking Supervision (BCBS) to identify systemically important financial institutions (SIFIs).

the measure of systemic risk used and outlines the methodology to rank banks according to their systemic importance. Section 5 presents the calibration and results for the benchmark version of the model, while Section 6 presents the policy experiments and examines the role of prudential regulation, with particular focus on the role of liquidity coverage ratios. Finally, Section 7 concludes.

## 2 Related Literature

The empirical literature on contagion is vast. We do not review it here since ours is an applied theory contribution. We therefore focus on theoretical contributions which explore some of the channels embedded in our model.<sup>6</sup>

The classic contribution by [Allen & Gale \(2000\)](#) is often cited as being among the first to assess the propagation of risk in an interbank network. The authors examine a circular network of banks interlinked through cross-deposits which can be run in the face of shocks. They show the existence of a monotonically decreasing relation between systemic risk and the degree of connectivity. A trade off between insurance/liquidity and risk contagion is at the heart of their result. As banks' connectivity increases, liquidity supply and insurance provision increases (the beneficial effect), but risk propagation increases too. Our model features a similar trade-off between insurance motives, which operate primarily in the interbank market, and contagion propagation channels, which are multifaceted in our model.

Contagion and risk transmission have been extensively studied in models of cross-holdings.<sup>7</sup> A key contribution in this branch of the literature is the now classic paper by [Eisenberg & Noe \(2001\)](#), which presents a clearing algorithm to solve for the equilibrium payment vector in an interbank system characterized by interlocking exposures among institutions. The mechanism proposed by these authors is in fact used in the shock transmission process featured in this paper. More recently, [Elliott, Golub & Jackson \(2014\)](#) have used cross-holdings to analyze the structure and resilience of the European interbank system. Contrary to this family of papers, which analyze networks with exogenous links and heuristic decision rules, the model presented here features fully fledged micro-founded decisional processes.

Contagion through fire sale externalities is considered among others in [Cifuentes, Ferrucci & Shin \(2005\)](#) and [Allen & Carletti \(2008\)](#). The model by [Bluhm, Faia & Krahnen \(2014\)](#) builds on the former by considering portfolio maximizing banks and trading in inter-connected interbank markets. [Aldasoro, Delli Gatti & Faia \(2015\)](#) assess the extent to which liquidity hoarding, generated by risk averse banks, can amplify systemic risk associated with interbank inter-connections and fire sale externalities. None of these models consider runs on the liability side, the feedback loops between

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<sup>6</sup>For recent overviews of the literature on systemic risk and interbank exposure networks the reader is referred to the review articles by [Benoit, Colliard, Hurlin & Pérignon \(2015\)](#) and [Hüser \(2015\)](#) respectively.

<sup>7</sup>See for instance the contribution by [Gai & Kapadia \(2010\)](#).

contagion risk on the two sides of banks' balance sheet and the impact of liquidity regulation. Also contrary to most of the past literature we bring our model closer to the data by calibrating based on method of moments.<sup>8</sup>

Runs on the liability side are modeled in our model through a global game à la [Morris & Shin \(2003\)](#) or [Carlsson & van Damme \(1993\)](#). Other authors have modeled alternative forms of bank runs through global games. For instance [Goldstein & Piazner \(2005\)](#) extend the [Diamond & Dybvig \(1983\)](#) model using global game techniques with the goal of deriving an expression for the probability of bank runs. An early contribution can be found in [Dasgupta \(2004\)](#). More recently, [Anand, Gai & Marsili \(2013\)](#) use the global game theoretical apparatus to model a run in the interbank market in order to assess the relevance of roll-over risk.<sup>9</sup> Contrary to those papers, we use global game techniques to model a fundamental bank run on banks' short term liabilities held by uninformed investors. Furthermore, this feature is added to the amplification and contagion channels already present on the asset side. We focus on fundamental bank runs which are triggered by news on asset returns rather than by liquidity shocks on investors. Our goal is to show how news on banks' asset returns can activate risk contagion on both sides of the balance sheet: on the liability side through investors' run and on the asset side through fire sales.

Interbank markets in our model play a crucial role since they can function both as liquidity risk insurer and as risk propagation devices. For this reason our paper is also related to a recent literature assessing the role of interbank markets (see for instance [Afonso & Lagos \(2015\)](#)). In some cases interbank trading is modeled based on bilateral relationships and prices are formed through bargaining arrangements. While this assumption captures well the functioning of most Anglo-Saxon interbank markets, our focus on European markets calls for considering centralized pricing mechanisms, like the tâtonnement.<sup>10</sup>

Finally, the paper is also related to the literature on macro-prudential policies, with particular focus on interbank networks. [Haldane & May \(2011\)](#) note that liquidity requirements, which are the key regulatory tool studied here, can be seen as a way of short-circuiting the negative spillovers arising from fire sales and liquidity hoarding. The model presented here helps to substantiate their arguments. The policy experiments studied in section 6.2 also relate to the seminal contribution by [Gai, Haldane & Kapadia \(2011\)](#). Contrary to them, the model in this paper features endogenous decision-making, is calibrated to match certain elements of a real world network and follows closely the state of the art of the regulatory rule-book.

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<sup>8</sup>Calibrations to real datasets can be found for instance in [Gofman \(2014\)](#) and [Blasques, Bräuning & van Lelyveld \(2015\)](#).

<sup>9</sup>[Ahnert \(2014\)](#) also provides a model of rollover risk in a context of a global game in order to rationalize the need for liquidity regulation. Our model takes liquidity regulation as a fact of life and asks instead how it can be made better by incorporating macro-prudential elements.

<sup>10</sup>As noted in [European Central Bank \(2012\)](#), around 60% of interbank repo transactions in the Euro area take place via CCP-based electronic trading.

### 3 The Model

In our model risk averse banks solve optimal portfolio problems by choosing short term liabilities, liquid and non-liquid asset investment and interbank borrowing and lending. Since banks are risk averse, the concavity of the maximization problem guarantees that banks are exposed on both sides of the interbank market as they can be both borrowers and lenders. Our model features two markets: an interbank market with direct network links for borrowing and lending and a non-liquid asset market with indirect links through buying and selling. Finally there is supply of short term funding to banks from investors who can run the bank by coordinating their expectations on banks' asset returns.

Banks in the model are heterogenous with respect to the returns on non-liquid assets and with respect to their initial equity holding. Differential returns on assets are a realistic feature which depends upon a combination of luck and project monitoring ability. Banks' heterogeneity also results in different optimal balance sheets positions: it is this balance sheets heterogeneity that provides the scope for trading in our model. A general feature which is recurrent in the model is that banks experiencing high returns have higher incentives to invest in non-liquid assets, an activity which is accomplished through higher leveraging in the interbank market and through higher investors' short term liabilities.

Different channels of contagion are present in the model both on the asset and liability side of banks and they tend to interact with each other. First, direct linkages exist in the interbank market so that debt defaults endogenously propagate losses. Second, indirect linkages emerge through asset commonality and fire sales: when asset price swings materialize due to fire sales, the balance sheets of all banks exposed to non-liquid assets are affected. Third, price swings further affect also the banks' liability side and *systemic feedback loops* between the asset and liability side emerge. On the one side, when asset losses materialize, they turn into liquidity shortages. This is for two reasons: first, since banks are risk averse they tend to hoard more liquidity; second, the spread of news triggers runs on short term liabilities. On the other side, the liquidity shortage following a run forces banks to early liquidation and to further fire sales. At any moment in this process banks might find themselves unable to fulfill their interbank obligations, thereby going into default and further transmitting distress. The fire sale and distress propagation on the asset side can feed back into additional liability side distress and viceversa. Those feedback loops between the asset and the liability side are the most salient novelty of our paper. This feature is compounded by the fact that banks in the model are risk averse, and therefore tend to hoard liquidity in the face of shocks. Banks in our model enter the interbank market to insure against asset and liquidity risk: trading partners in the interbank market are indeed matched based on an entropy algorithm, which spreads trading relationships as evenly as possible. However the insurance benefits have to be balanced against the contagion channels described above to determine overall systemic risk.

The balance sheet structure of our model banking system results endogenously from the interaction between risk averse banks' optimizing decisions, a price tâtonnement process and the matching algorithm (the latter capturing the insurance motives). Bank runs also contribute to the endogenous formation of the banks' balance sheet structure as they affect liquidity.

The markets in the model are designed so that prices are determined through a tâtonnement process managed by a central counterparty. In the interbank market a Walrasian auctioneer collects all demand and supply and adjust prices upward/downward according to the existence of excess demand/supply. Once equilibrium in the interbank market is achieved, actual partners are assigned by a maximum entropy method. Once the model is set up, exogenous shocks to the non-liquid asset portion of banks' balance sheet materialize. In the aftermath of these shocks the second tâtonnement process goes into action in the market for non-liquid assets: this process guarantees convergence of the fire sale process such that after a finite number of rounds a new equilibrium price is obtained.

The financial system is made up of  $N$  banks. Each bank represents a node in the interbank market network, whereas lending and borrowing relationships represent the links connecting the nodes. A link between banks  $i$  and  $j$  is indicated by the element  $x_{ij} \in \mathbb{R}_{\geq 0}$  and it stands for the amount lent by  $i$  to  $j$ , implying that the network is *weighted*. Additionally, since lending from bank  $i$  to  $j$  does not imply the existence of a reciprocal relationship, the network is *directed* (i.e.  $x_{ij} \neq x_{ji}$ ,  $i \neq j$ ).

In the following subsections we go into the details of the different aspects of the model, starting from the objective function and going through the different constraints until specifying the full problem of the bank.

### 3.1 Banks' Objective Function

Table 1 presents the balance sheet of a generic bank  $i$ , highlighting in bold fonts the choice variables of the bank. On the asset side banks can hold high-quality liquid assets (or just *cash* for short,  $c_i$ ), they can lend to other banks in the interbank market (total interbank lending  $l_i$ ) and they can invest in non-liquid assets  $a_i$  which are marked-to-market at price  $p$ . These uses of funds can be funded by short term funding (or deposits for short,  $d_i$ ), borrowing from all other banks in the interbank market ( $b_i$ ) and equity ( $e_i$ ). The latter element is the only balance sheet item which is calibrated (for more details on the calibration see section 5 below).

Assets	Liabilities
<b><math>c_i</math></b>	<b><math>d_i</math></b>
<b><math>l_i</math></b>	<b><math>b_i</math></b>
$pa_i$	$e_i$

**Table 1:** The balance sheet of bank  $i$ .



Banks choose their desired balance sheet, for given prices, by solving a concave optimization problem. Hence the first step is to characterize their objective function.

Banks' profits are given by the returns on lending in the interbank market (lending  $l_i$  at the interest rate  $r^l$ ) plus returns from investments in non-liquid assets (investment of  $a_i$  with rate of return  $r_i^a$  and price  $p$ ), minus the expected costs from interbank borrowing ( $b_i$  with associated interest rate  $r_i^b$ ) and the cost of servicing deposits ( $d_i$  with associated interest rate  $r_i^d$ ). In order to keep the exposition compact, we relegate the derivation of the expression for profits (and their variance) to [Appendix A](#). The choice variables of bank  $i$  are high quality liquid assets ( $c_i$ , referred to as *cash* for short), non-liquid assets ( $a_i$ ) and interbank lending ( $l_i$ ) on the asset side, and interbank borrowing ( $b_i$ ) and short term funding ( $d_i$ , referred to as *deposits* for short) on the liability side. The difference between assets and liabilities is covered by bank equities ( $e_i$ ).

We assume that banks have risk-averse preferences also featuring precautionary saving motives (utility third derivative being positive). This specification captures the idea that bankers or bank managers tend to become cautious in times of distress and that their precautionary motives increase when uncertainty increases (we will return to this point later). See [Afonso & Shin \(2011\)](#) or [He & Krishnamurthy \(2013\)](#) for similar characterizations.

The bank's preferences are represented through a CRRA utility function over profits:

$$U(\pi_i) = \frac{(\pi_i)^{1-\sigma}}{1-\sigma} \quad (1)$$

where  $\sigma$  represents the bank's risk aversion.

As mentioned above, the convex maximization problem serves both to allow for interior solutions for borrowing and lending, and to put precautionary behavior into the model.<sup>11</sup> Furthermore, in this context, the variance of assets' returns will factor in the banks' decision: an increase in the variance of profits has a negative impact on the expected utility of banks. This naturally reduces the extent of their investment in non-liquid assets and their involvement in the interbank market, producing both liquidity hoarding and a credit crunch.

Another important aspect of the concave optimization problem is that in non-linear set-ups, the variance in assets' returns affects the bank's decision. Higher variance in assets' returns reduces expected banks' utility, thereby reducing the extent of their involvement both in lending as well non-liquid assets investment. The impact of volatility of banks' choices will have an amplifying effect on shocks. Consider an adverse shock which forces banks to fire sale assets and to reduce their exposure on interbank lending: under the non-linear set-up the ensuing volatility in asset returns and asset prices will amplify balance sheet repositioning. In this context it is both convenient and standard to take a second order Taylor approximation of the expected utility of profits around the

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<sup>11</sup>The CRRA function is characterized by convex marginal utility (i.e. a positive third derivative). The consequence of this is that higher uncertainty induces higher expected marginal utility at the optimum. This increase in expected marginal utility induces cautiousness in banks and makes them hoard more liquidity.

expected value of profits.

The second order approximation of Equation 1 in the neighborhood of the expected value of profits  $E[\pi_i]$  is given by<sup>12</sup>:

$$U(\pi_i) \approx U(E[\pi_i]) + U_\pi(\pi_i - E[\pi_i]) + \frac{1}{2}U_{\pi\pi}(\pi_i - E[\pi_i])^2 \quad (2)$$

Taking expectations on both sides of Equation 2 and using the law of iterated expectations to eliminate terms yields:

$$E[U(\pi_i)] \approx U(E[\pi_i]) + \frac{1}{2}U_{\pi\pi}\sigma_\pi^2$$

where  $\sigma_\pi^2 = E[(\pi_i - E[\pi_i])^2]$  stands for the variance of profits (with the bank subindex omitted for simplicity).

From Equation 1, it is straightforward to see that  $U_{\pi\pi} = -\sigma E[\pi_i]^{-(1+\sigma)}$ .<sup>13</sup> The expected utility of profits can therefore be written as:

$$E[U(\pi_i)] \approx \frac{E[\pi_i]^{1-\sigma}}{1-\sigma} - \frac{\sigma}{2}E[\pi_i]^{-(1+\sigma)}\sigma_\pi^2 \quad (3)$$

Upon a duality argument the banks' maximization problem can in fact be written as a convex optimization where the objective function is given by the approximation in Equation 3. Details on the derivation and precise form of the profit function and the approximation of variance of profits are provided in Appendix A.

### 3.2 Banks' Regulatory Constraints

Table 1 above presented the balance sheet of bank  $i$ . This is of course a constraint that banks must satisfy at all times with equality. In equation form the balance sheet constraint of bank  $i$  looks as follows:

$$c_i + pa_i + \underbrace{l_{i1} + l_{i2} + \dots + l_{ik}}_{\equiv l_i} = d_i + \underbrace{b_{i1} + b_{i2} + \dots + b_{ik'}}_{\equiv b_i} + e_i \quad (4)$$

where  $c_i$  represents cash holdings,  $a_i$  denotes the volume and  $p$  the price of non-liquid assets (such that  $pa_i$  is the market value of the non liquid portion of the bank's portfolio),  $d_i$  stands for deposits and  $e_i$  for equity.  $l_{ij}$  is the amount lent to bank  $j$  where  $j = 1, 2, \dots, k$  and  $k \leq N-1$  is the cardinality of the set of borrowers of bank  $i$ ;  $b_{ij}$  is the amount borrowed from bank  $j$ , where  $j = 1, 2, \dots, k'$

<sup>12</sup>Note that all partial derivatives are also evaluated at  $E[\pi]$ .

<sup>13</sup>Under certainty equivalence (namely when the third derivative equals zero) the equality  $E[U(\pi_i)] = U(E[\pi_i])$  holds in all states. With CRRA utility the third derivative is positive, implying that expected marginal utility increases with the variability of profits. Additionally, the *expected utility of profits* is no longer equal to the *utility of expected profits* since one must subtract a term which depends both on the volatility of banks' profits and the risk aversion parameter. This is a direct consequence of Jensen's inequality and provides the standard rationale for precautionary saving.

and  $k' \leq N - 1$  is the cardinality of the set of lenders to bank  $i$ . Hence  $l_i = \sum_{j=1}^k l_{ij}$  stands for total interbank lending and  $b_i = \sum_{j=1}^{k'} b_{ij}$  stands for total interbank borrowing.<sup>14</sup> Equation 4 is a balance sheet constraint that banks must fulfill with equality. Note that regarding the interbank market, the choice variables of banks are *total* interbank borrowing and lending.

The bank's optimization decision is subject to three additional constraints, two of a regulatory nature and one of a behavioral type. The behavioral constraint is captured in our model by a run constraint, which introduces the global game into the framework and will be explained in more detail in the next subsection.

The two regulatory requirements are based upon Basel III prescriptions and take the following form:

$$\frac{c_i + pa_i + l_i - d_i - b_i}{\omega_a pa_i + \omega_l l_i} \geq \gamma \quad (5)$$

$$\frac{c_i}{\omega_d d_i + \omega_b b_i - \min\{\tilde{\omega}_l l_i, 0.75(\omega_d d_i + \omega_b b_i)\}} \geq \alpha \quad (6)$$

Equation 5 represents an equity requirement, prescribing that equity at market prices as a share of risk-weighted assets must not fall below a given threshold. Cash (i.e. high quality liquid assets) is risk-less in the model and therefore does not show up in the denominator of Equation 5.  $\omega_a$  and  $\omega_l$  represent the risk weights on non-liquid assets and interbank lending respectively. The parameter  $\gamma$  is set by the regulator and defines the threshold that banks need to comply with at all times.

Equation 6 is the model representation of the *Liquidity Coverage Ratio* (LCR) again following the guidelines of the Basel III accord. The numerator consists of the stock of high quality liquid assets, which in our model is referred to as *cash*. The denominator is given by a measure of expected cash outflows, minus the minimum of expected cash inflows and 75% of expected cash outflows.<sup>15</sup> The risk weights in the denominator,  $\omega_d$ ,  $\omega_b$  and  $\tilde{\omega}_l$ , apply respectively to deposits, interbank borrowing and interbank lending. Finally,  $\alpha$  stands for the actual liquidity coverage ratio. The economic rationale behind this regulatory requirement implies that banks' liquid assets should suffice to cover expected net outflows.<sup>16</sup>

The analysis of the effects of liquidity ratios in a model characterized by equilibrium liquidity freezes through bank runs is a main novelty of our paper<sup>17</sup>. The liquidity regulatory requirements

<sup>14</sup>Since banks cannot lend to nor borrow from themselves, we have  $l_{ii} = b_{ii} = 0 \forall i = 1, \dots, N$ .

<sup>15</sup>In the Basel III regulation expected outflows/inflows refer to the following 30 days. Our framework is static so we make no reference to a time horizon.

<sup>16</sup>Additionally, one could include the interest rate associated to each of the outflows. For the sake of simplicity we leave this element out of the model. Based upon the regulation,  $\alpha$  should be 100% after full implementation. This is the benchmark value that we assign to this parameter. In our policy experiments below we also consider the fact that prior to the full implementation of the Basel regime there is a "phase-in" period which is designed as follows. Starting from January 1, 2015 the parameter  $\alpha$  is set to 60%, while afterwards there will be annual increments of 10% until the parameter reaches the regulatory value of 100%.

<sup>17</sup>Besides the LCR, another important element of the Basel III proposal on liquidity regulation is the Net Stable

are a main pillar of the new regulatory architecture and certainly a frontier of the policy regime design. Given its experimental nature there is at the same time great interest but also scant or no academic literature on the assessment of its effects.

### 3.3 Bank Run on Short Term Liabilities

Banks rely heavily on short term funding for the smooth course of their operational activity. In fact one of the main concerns which the bankers are confronted with is the possibility that bank runs or market freezes might drain liquidity in their balance sheet. As other authors have also shown (see [Diamond & Rajan \(2005\)](#)) banks' liquidity crises usually forego solvency problems. For this reason and given our focus on the analysis of contagion on the liability side as well as on the ability of liquidity requirements containing such contagion, we introduce micro-foundations for fundamental banks runs on uninsured short term liabilities. We do so by modeling expectations through a global game perspective. Investors of uninsured banks' short term liabilities (uninsured deposits, covered and uncovered bonds, etc.) coordinate their expectation formation processes so that a run is triggered in equilibrium upon observing a certain threshold for the banks asset returns. Banks are aware of this possibility and in fact they do take into account this run region when solving their optimization problem. All investors observe the fundamental value of the asset, as summarized by the return  $r_i^a$ , with some noise. Hence there is incomplete information about the fundamental value, albeit each depositor receives a private signal. The bank instead knows the return on its asset portfolio.

We start by presenting the reduced form of the equation describing the run region: micro-foundations through a global game follow in this section. The reduced form, which also represents the additional constraint (beyond the regulatory constraints) faced by banks, reads as follows:

$$\exp(-\varepsilon_i) r_i^a \frac{a_i}{p} + r^l l_i - r_i^b b_i \geq r_i^d d_i \quad (7)$$

where  $\varepsilon_i$  stands for the shock realization on the non-liquid asset portfolio,  $r_i^b$  is the cost of interbank borrowing and  $r_i^d$  is the interest paid on deposits. In their optimization, banks take into account the restriction that in expectation their return on assets (minus interbank outflows) should be high enough to satisfy depositors. This aspect is captured by equation 7.

Equation 7 can be manipulated to isolate a threshold for the idiosyncratic component  $\varepsilon_i$  below which a run occurs. Let us assume that  $\varepsilon$  follows distribution  $\Gamma$ , with density function  $\theta$  and cumulative distribution function  $\Theta$ . The share of deposits being withdrawn will be then given by

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Funding Ratio (NFSR), which requires banks to maintain a stable funding profile relative to their asset composition and their off-balance sheet activities. The incorporation of the NFSR is left for future research.

$\rho_i = \int_{-\infty}^{\tilde{\varepsilon}_i} \theta(\varepsilon) d\varepsilon = \Theta(\tilde{\varepsilon}_i)$ , where:

$$\tilde{\varepsilon}_i = \log \left( \frac{r_i^a a_i / p}{r_i^d d_i + r_i^b b_i - r_i^l l_i} \right) \quad (8)$$

The shock  $\varepsilon_i$  can therefore be interpreted as a news shock, which prompts depositors of the bank to withdraw a portion of their funds.

As explained above the reduced form for the run region identified by 8 can be rationalized through a switching strategy in a simultaneous incomplete information game among many depositors. We follow [Morris & Shin \(2003\)](#) and [Carlsson & van Damme \(1993\)](#) and obtain the threshold strategy above as the unique equilibrium of a global game among depositors who have a binary decision with two actions: “run” and “no run”. Let us assume that there are  $m = \{1, \dots, M\}$  depositors and define  $\eta$  as the fraction of depositors who run the bank. Each depositor,  $m$ , receives a private signal regarding the realization of banks’ non-liquid asset returns which takes the following form:

$$\vartheta_m = \varepsilon_i + \mu_m \quad (9)$$

where  $\mu_m$  are small errors which are independently distributed with a cumulative distribution  $\mathbb{F}$  given by the normal distribution,  $N(0, \sigma_\mu^2)$ . The signal can be thought of as the depositor private information or opinion regarding bank  $i$ ’s health. Notice that while agents have different information, none has an informational advantage vis-à-vis the others. Each depositor decides whether to run or not depending on the signal. The latter has a dual function. On the one side it suggests, albeit with some errors, if bank  $i$  is healthy or not. On the other side a signal provides an agent information about the other depositors’ signals, thereby allowing the agent some inference about the other depositors’ actions. Seeing a bad signal about bank  $i$  assets’ returns ultimately provides information about the probability that a run occurs. Guessing other depositors’ actions is of fundamental importance in informational games with complementarities. Indeed each individual depositor payoff will depend upon the amount of bank  $i$  funds which are left after other depositors have run. Each depositor will decide whether to run or not when his expected payoff, conditional on his signal and on the run probability by other depositors, equals zero. Following [Morris & Shin \(2003\)](#) and [Carlsson & van Damme \(1993\)](#) we can show that the decision to run follows a unique switching strategy (equal for all depositors) which is given by equation 8. To prove this result in what follows we assume that  $b_i = 0, l_i = 0$  and  $p = 1$ : the result can be easily generalized.

**Lemma 1.** *The unique equilibrium for the run game amounts to all depositors of bank  $i$  choosing the threshold strategy:*

$$\tilde{\vartheta} \leq \frac{r_i^d d_i}{r_i^a a_i} \quad (10)$$

**Proof.** We start by computing the expected payoff of each depositor,  $m$ , of bank  $i$  from running conditional on signal  $\vartheta_m$  and conditional on depositors  $j \neq m$  running when  $\vartheta_j \leq h$ . The value is as follows:

$$\Upsilon(\vartheta_m, h) = \left[ \mathbb{E}[\varepsilon \mid \vartheta_m] r_i^a a_i - \eta r_i^d d_i \mathbb{F}\left(\frac{h - \vartheta_m}{\sqrt{2}\sigma_\mu}\right) \right] \frac{1}{1 - \eta} \quad (11)$$

The depositor who decides to run gets the expected value of bank  $i$  assets,  $\mathbb{E}[\varepsilon \mid \vartheta_m] r_i^a a_i$ , minus the funds withdrawn by the fraction  $\eta$  of depositors who have run previously and in proportion of the depositors who have not yet run the bank. The funds withdrawn by depositors who run shall be weighted by the probability that their signal is below the threshold  $h$ . Notice that the probability mass  $\mathbb{F}\left(\frac{h - \vartheta_m}{\sqrt{2}\sigma_\mu}\right)$  has been obtained using the fact that  $\vartheta_j \mid \vartheta_m \sim N(\vartheta_m, 2\sigma_\mu^2)$ . Each depositor  $m$  will run when the expected payoff in Equation 11 equals zero. In other words the depositor terminates the contract with the bank when on the margin he or she expects to receive zero. We can re-write the payoff function as follows:

$$\Upsilon(\vartheta_m, h) = \left[ \vartheta_m r_i^a a_i - \eta r_i^d d_i \mathbb{F}\left(\frac{h - \vartheta_m}{\sqrt{2}\sigma_\mu}\right) \right] \frac{1}{1 - \eta} \quad (12)$$

The function in Equation 11 is monotonically increasing with respect to  $\vartheta_m$  and with respect to  $h$ . Therefore there must be a unique value of  $\vartheta_m$  for which the function is zero. This value is the unique threshold defining the switching strategy common to all depositors (hence when  $\eta = 1$ ), which can be formalized as follows:  $s_m(\vartheta) = \text{run}$  if  $\tilde{\vartheta} \leq \frac{r_i^d d_i}{r_i^a a_i}$  and  $s_m(\vartheta) = \text{no run}$  otherwise.<sup>18</sup> Through an iterative argument one can show that this switching strategy survives even after many iterations of the game.

Intuitively when the depositor receives a signal that the ratio between banks' asset returns and the returns that banks must pay to uninformed investors is too low, he expects a high probability of bank runs. Indeed the signal also suggests that other depositors might have received a similar signal. If all investors follow the same switching strategy the likelihood of a bank run rises. This in turn raises the incentives of each individual depositor to run, thereby making the run a self-fulfilling prophecy.

### 3.4 Bank's Optimization

Having detailed the banks' objective function as well as all constraints, we are now in the position to outline the banks' optimization problem which reads as follows:

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<sup>18</sup>Note that the threshold of our bank run global game corresponds to the threshold defined in Equation 8 to the extent that  $\tilde{\vartheta}$  is given by the following transformation of  $\tilde{\varepsilon}$ :  $\tilde{\vartheta} = \exp(-\tilde{\varepsilon})$ .

$$\begin{aligned}
& \text{Max}_{\{c_i, n_i, l_i, b_i, d_i\}} E[U(\pi_i)] \\
& \text{s.t. } (4), (6), (5), (7) \\
& c_i, n_i, l_i, b_i, d_i \geq 0
\end{aligned} \tag{P}$$

### 3.5 Price Tâtonnement: Interbank and Non-Liquid Asset Markets

Given a calibration of the relevant parameters, a solution to problem P for all banks in the system will yield notional quantities for the banks' choice variables. Given individual optimal demands the equilibrium prices in each market (interbank as well as non-liquid asset markets) are reached through a price tâtonnement process. This process captures the economic rationale and the agents' behavior of centralized markets such as those characterized by Clearing Houses or Central Counterparties. Centralized counterparty clearing is well spread in European financial markets<sup>19</sup> and it plays an important role also in some U.S. markets (see for instance FedWire). From a theoretical point of view the importance of central clearing is analyzed for instance in [Duffie & Zhu \(2011\)](#).

We describe the functioning of the tâtonnement starting from the interbank market and subsequently detailing the process for the non-liquid asset market.

Individual banks submit their demand and supply of interbank funding to a walrasian auctioneer. The latter aggregates individual requests to obtain aggregate demand and supply of interbank funds, respectively  $B = \sum_{i=1}^N b_i$  and  $L = \sum_{i=1}^N l_i$ . Upon discrepancies between aggregate demand and supply the auctioneer adjusts the interest rate. The new rate that is obtained by the auctioneer is subsequently used for another round of optimization by banks. The auctioneer again collects all individual demands and supplies as before, and adjusts the interest rate accordingly. Banks re-optimize again given this new interbank rate, and the process goes on until an equilibrium is achieved, i.e. a rate for which  $L = B$ .

In the aftermath of a shock, and given an equilibrium price in the interbank market, a price tâtonnement process starts in the market for non-liquid assets. We model this process along the lines of [Cifuentes et al. \(2005\)](#). The market price is set to 1 at the initial equilibrium. Upon an adverse shock to asset returns banks engage into fire sales. Fire sales generate an excess supply of assets which, for given excess market demand, results into a new equilibrium price. The adjustment process can be described analytically as follows. Denote the individual bank's optimal supply of non-liquid assets with  $s_i$ . Since  $s_i$  is decreasing in  $p$ ,<sup>20</sup> the aggregate sales function,  $S(p) = \sum_i s_i(p)$ , will

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<sup>19</sup>See footnote 9.

<sup>20</sup>To see this, in the denominator of [Equation 5](#) replace  $a_i$  by  $a_i - s_i$  (where  $s_i$  represent the sales of non-liquid

also be decreasing in  $p$ . We define an aggregate market demand function as  $\Delta(p)$  ( $\Delta : [p, 1] \rightarrow [p, 1]$ ). An equilibrium price will solve the following fixed point problem:  $\Delta(p) = d^{-1}(S(p))$ .

Prior to any shock the above equilibrium condition is satisfied for  $p = 1$ . In response to a shock raising aggregate excess supply the price adjusts through a step-wise process until it reaches a second stable equilibrium, the existence of which is guaranteed if and only if the supply curve lies above the demand curve, at least for a range of values. The existence of the second stable equilibrium can be guaranteed by assuming the following inverse demand function<sup>21</sup> :

$$p = \exp(-\beta \sum_i s_i), \quad (13)$$

where  $\beta$  is a (positive) constant that scales the price responsiveness with respect to non-liquid assets sold.

Numerically the step-wise adjustment functions as follows. The initial shock to non-liquid assets is reflected in an initial reduction in price to, say,  $p_0$ . At such a price banks will offer  $S(p_0)$ , but this in turn pushes the price further down to  $p_1 = d^{-1}(S(p_0))$ , prompting further aggregate sales of  $S(p_1)$ . The adjustment goes on until demand and supply meet again at the new (fire-sale) equilibrium price  $p^* < p = 1$ .

### 3.6 Matching Trading Partners

Once equilibrium prices in the interbank market have been determined through the process described above, actual trading allocations are assigned based upon a matching algorithm. Given the equilibrium vectors,  $\mathbf{l} = [l_1 \ l_2 \ \dots \ l_N]$  and  $\mathbf{b} = [b_1 \ b_2 \ \dots \ b_N]$ , resulting from the tâtonnement process, we need to match pairs of banks for the actual trading to take place. The matching results in an interbank matrix  $\mathbf{X}$ , whose element  $x_{ij}$  indicates the exposure (through lending) of bank  $i$  to bank  $j$ . Matrix  $\mathbf{X}$  effectively summarizes all bilateral lending/borrowing relationships in the banking network<sup>22</sup>.

The method employed to obtain the interbank matrix is the maximum entropy approach, by means of the RAS algorithm<sup>23</sup>, a procedure based on the assumption that banks distribute their lending and borrowing as evenly as possible. The economic rationale behind it based upon a risk sharing argument. By engaging in many trading relations and by atomizing the demands for funds across many intermediaries banks effectively diversify the contagion risk stemming from interbank debt default. Beyond an economic rationale this algorithm has also the advantage of incorporating

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assets in order to fulfill the equity requirement) and solve for  $s_i$ , as in Cifuentes et al. (2005).

<sup>21</sup>Such a function can be rationalized by assuming the existence of noise traders in the market.

<sup>22</sup>Vectors  $\mathbf{l}$  and  $\mathbf{b}$  correspond, respectively, to the row and column sums of matrix  $\mathbf{X}$ .

<sup>23</sup>The method is based on a biproportional matrix balancing technique originally developed in the context of classic input-output analysis for the purposes of matrix updating. See Miller & Blair (2009) for a complete treatment of the topic. See also Upper (2011) and references therein.



additional information besides the marginals of the target matrix. To this purpose we use the matrix of exposures between large European banks as a prior. This additional twist allows us to provide also empirical validity to the resulting trading network (more details on the procedure are provided in Section 5.2 below).

### 3.7 Equilibrium Definition

After having specified the maximization process, the clearing mechanism and the expectation formation process we are now in the position to define the competitive equilibrium.

**Definition.** A competitive equilibrium in our model is defined as follows:

- (i) A quintuple  $(l_i, b_i, n_i, c_i, d_i)$  for each bank  $i$  that solves the optimization problem P.
- (ii) Depositors and banks expectations compatible with equation 11.
- (iii) A clearing price in the interbank market,  $r^l$ , which satisfies  $B = L$ , with  $B = \sum_{i=1}^N b_i$  and  $L = \sum_{i=1}^N l_i$ .
- (iv) A trading-matching algorithm for the interbank market.
- (v) A clearing price for the market of non-liquid assets,  $p$ , that solves the fixed point:  $\Theta(p) = d^{-1}(s(p))$ .

## 4 Shocks, Systemic Risk and Systemic Importance

Our goal is to assess the impact of different regulatory requirements on contagion and systemic risk. To this purpose we need to define the shocks that hit the system, the measure to evaluate the damage caused by the unfolding of the contagion cascade, and the methodology to rank banks in order to tailor the prudential requirements to their ranking of systemic importance.

Banks in our model are subject to shocks to non-liquid assets which are modeled according to an exogenous multivariate distribution (details on the calibration are provided below). Once banks are shocked, all the contagion and amplification channels in the model start to interact in a truly systemic loop. The initial fall in the price of non-liquid assets forces banks which are unable to fulfill the regulatory requirements into fire sales. Some banks might not be able to honor their interbank commitments, thereby transmitting further losses to lender banks. The news of falls in asset prices will induce investors to run the banks' short term liabilities. The ensuing shortage of liquidity might in turn trigger further fire sales that feed back as accounting losses for all banks in the system. The contagion mechanism receives further amplification by the banks' precautionary behavior.

The shock transmission process, which is solved through clearing mechanisms akin to those featured in payment systems (see Eisenberg & Noe (2001)), takes into account the endogenous loss propagation just described. To guarantee convergence of the fire sale process we use the

mechanism put forward in [Cifuentes et al. \(2005\)](#). This mechanism guarantees that a new (fire sale) equilibrium price of non-liquid assets will be achieved in the aftermath of a shock, thereby allowing for the computation of the damage done by the default cascade.

Once the shock transmission process has run its course, we rely on a simple metric to compute systemic risk. In particular, we use the ratio between assets of defaulting banks to total assets of the system, which can be interpreted as the default probability of the system. Our metric for systemic risk then reads as follows:

$$\Phi = \frac{\sum_{\Omega} assets_{\Omega}}{\sum_i assets_i} \quad (14)$$

where  $\Omega \in i$  identifies the set of defaulting banks. Defaulting banks are those that cannot fulfill regulatory requirements even after selling all non-liquid assets and/or suffer massive liquidity shortage that prevents from fulfilling the liquidity requirement and/or cannot honour their interbank commitments.

In order to rank banks according to their systemic importance we use the so-called G-SIB methodology developed by the Basel Committee on Banking Supervision (BCBS hereafter)<sup>24</sup>. We focus in particular on the adaptation of the methodology devised for the European banking system (see [EBA \(2014\)](#) for the so-called O-SII methodology). The methodology constructs an index by identifying a set of categories that capture different aspects of the systemic importance of financial institutions. Each category is in turn composed by a set of core indicators which are the observables upon which the measurement is based. For our model we focus on three categories (each accounting for 1/3 of the final index): size, interconnectedness (both on the liability and asset sides) and complexity on the asset side.<sup>25</sup> The first is simply given by total assets. Intuitively this indicator captures the contribution to aggregate risk of too-big-to-fail institutions. The second category encompasses interbank assets and interbank liabilities, each accounting for 1/2 of the score corresponding to the category. This second category captures importance based on the direct linkages of banks to the rest of the system. Finally, complexity is a feature of non-liquid assets. This indicator gives the extent of risk contribution stemming from fire sale externalities. To assess the likelihood that a bank is conducive to fire sale externalities one shall examine the uncertainty attached to asset returns. Higher complexity (uncertainty in pricing) in asset returns renders banks' assets more illiquid, hence more prone to fire sales. Indeed in the BCBS definition complex assets include OTC derivatives as well as level 3 assets (assets whose price is not determined by the market nor by reference models: examples include complex bilateral contracts). Importantly all of the indicators used are measurable and do not require knowledge of the precise structure of the

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<sup>24</sup>See [BCBS \(2013, 2014\)](#).

<sup>25</sup>By experimenting with additional criteria (including deposits and different measures of network centrality, among others) we found rankings of systemic importance characterized by very high rank correlation relative to the ranking produced by the criteria we chose (correlation always above 98%).

interbank network.<sup>26</sup>

Following BCBS (2014), the score corresponding to each indicator is computed as follows: the number for each bank is divided by the corresponding sample total and multiplied by 10000 to express the final result in basis points. For example, the indicator for total assets for bank  $i$  in basis points will be given by:  $\frac{\text{assets}_i}{\sum_{j=1}^N \text{assets}_j} \times 10000$ . If a category has more than one indicator then these indicators are averaged to produce the score of the category. Then the score of the different categories are averaged to produce the final systemic importance score. The EBA guidelines provide some leeway in setting up the cut-off score to determine the group of systemically important banks (see EBA (2014), pp. 20-21). In particular, an admissible range for the cut-off is established between 275 and 425 basis points. We choose 400 basis points as this cut-off clearly identifies a group of 10 systemic banks.

## 5 Calibration, Matching and Baseline Results

Prior to simulating the model we must choose values for all exogenous parameters. The ultimate aim of the paper is to evaluate policy options, therefore the realism of the model-based banking system is an essential intermediate goal. The target system that we aim to mimic is the network of large European banks.

The next subsection shows the calibration of all model parameters and presents the baseline configuration of the banking system, in order to compare what is obtained from the model with the data. As discussed in the section above, in order to obtain the full model results after the optimization the interbank matrix needs to be estimated. In the second subsection we turn our attention to this issue and present the method by which we obtain this matrix in a way that preserves the structural features of the target real interbank matrix.

### 5.1 Calibration and Moment Matching

The calibration of the model is composed of two blocks: the first concerns the calibration of all parameters that depend on policy or that can otherwise be pinned down, the second relates to the calibration of other parameters which we calibrate using a simulated method of moments approach in order to help the final configuration of the model get closer to the data.

**Policy Parameters and Calibrated Quantities** We limit ourselves to set ex ante only the parameters for which there is a clear policy prescription that allows us quantification. All policy

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<sup>26</sup>Given the nature of our model we cannot capture the category *substitutability* which is present in the EBA methodology. Appendix C presents the mapping between the methodology presented in EBA (2014) and our adaptation.

parameters, with the exception of the risk weights  $\tilde{\omega}_l$  and  $\omega_b$ ,<sup>27</sup> are consistent with the Basel III requirements for Europe.<sup>28</sup> The (fully phased-in) liquidity requirement parameter,  $\alpha$ , is set to 100%, while the risk weight on deposits,  $\omega_d$ , is set to 10%. The equity ratio requirement is set to 8%, while the risk weights on non-liquid assets and interbank lending are set to  $\omega_a = 100\%$  and  $\omega_l = 20\%$  respectively.

Recall that in our model banks are heterogenous ex ante with respect to the distribution of equities. The initial distribution of equities is therefore calibrated using data for the set of large European banks presented in [Alves et al. \(2013\)](#). The latter present a network of 54 banks, whereas we set the number of banks in our analysis to  $n = 49$ .<sup>29</sup>

The interest rate on deposits ( $r_i^d$ ) is randomly drawn from a uniform distribution within a range between the interbank interest rate and a mark-up of up to 100 basis points.<sup>30</sup> Notice that the interest rates on deposits contains an exogenous part, the distribution of which we set ex ante, but also depends upon the endogenous determination of the interbank rate. The price elasticity for non-liquid asset,  $\beta$ , is set so as to induce a 10% price decline in the event in which the entire stock of non-liquid assets is sold.<sup>31</sup> Following [Memmel & Sachs \(2013\)](#) and [Georg \(2013\)](#), the loss-given-default parameter and banks' risk aversion are set to  $\xi = 0.5$  and  $\sigma = 2$  respectively. For precautionary saving to emerge the latter parameter must be larger than 1. Finally, the vector of shocks to non-liquid assets, which is the starting point of the shock transmission process, is drawn from a multivariate normal distribution with a mean of 1, a variance of 5 and zero covariance. To evaluate the shock transmission process in the model and assess the systemic risk profile of the system, 1000 shock realizations are drawn.

**Matching Moments** For the remaining parameters in our model it is not possible to infer values from data. At the same time we want to make sure that our model properties are consistent with empirical counterparts. To this end some parameters of the model are obtained through a simulated method of moments approach.<sup>32</sup>

The logic behind the approach is to consider several combinations of the parameters of interest and evaluate the model to generate model-based moments. The latter are compared with the

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<sup>27</sup>The regulation specifies that the denominator of the LCR shall be composed of short term expected outflows/inflows under a high stress scenario. In the context of the setting presented here, this leaves some room for interpretation on what would constitute a reasonable share of potential outflows/inflows for interbank market activity in a high stress scenario. For this reason these parameters are actually obtained through the simulated method of moments, explained in detail below.

<sup>28</sup>See the [Regulation No 575/2013 of the European Parliament and of the Council](#) of 26 June 2013.

<sup>29</sup>We had to exclude some banks for which either equity was negative or leverage was abnormal.

<sup>30</sup>The variance of returns is computed accordingly as  $\sigma_{r_i^d}^2 = \frac{1}{12} \left( \max(r_i^d) - \min(r_i^d) \right)^2$ .

<sup>31</sup>See [Greenwood, Landier & Thesmar \(2015\)](#) for price responsiveness in fire sales processes.

<sup>32</sup>This strategy has been used in the context of interbank networks by [Gofman \(2014\)](#) and [Blasques et al. \(2015\)](#). The spirit of our approach draws from these contributions, in particular the former. This specific application of the method, as well as the adaptation of the G-SIB methodology and the algorithm to obtain the interbank matrix, were initially used in chapter 6 of Aldasoro's Ph.D. thesis.

counterpart data-based target moments and the optimal combination of parameters is chosen as that which minimizes the distance between model-based and target moments.<sup>33</sup> Formally, the objective function of the optimization problem is the following:

$$\min_{\theta} \frac{1}{k_2} \hat{\mathbf{m}}(\theta)' \mathbf{W} \hat{\mathbf{m}}(\theta) \quad (15)$$

where  $\theta$  stands for the  $k_1 \times 1$  vector of parameters to be chosen,  $\hat{\mathbf{m}}(\theta) = \frac{\mathbf{m}(\theta) - \bar{\mathbf{m}}}{\bar{\mathbf{m}}}$  represents a  $k_2 \times 1$  vector with percentage deviations of the model-based moments ( $\mathbf{m}(\theta)$ ) relative to target data moments ( $\bar{\mathbf{m}}$ ) and  $\mathbf{W}$  is a  $k_2 \times k_2$  weighting matrix (with  $k_1 < k_2$ ). As in Gofman (2014), we consider percentage deviations from target since the moments we wish to match are measured in different units.

The method of moments is used to estimate the following parameters: the upper limit for the interval of returns on non-liquid assets ( $r_{max}^a$ ), which we draw from a uniform distribution between the range  $U(0.01, r_{max}^a)$ ; the upper limit on the probability of default ( $\delta_{max}$ ), which is randomly drawn from a uniform distribution on the range  $U(0, 0.02 + \delta_{max})$ ;<sup>34</sup> the values for  $\omega_b$  and  $\tilde{\omega}_l$ , which are ex ante assumed to be identical. The grid of values considered are as follows:  $r_{max} \in \{0, 0.01, \dots, 0.2\}$ ,  $\delta_{max} \in \{0, 0.005, \dots, 0.02\}$  and  $\omega_b = \tilde{\omega}_l \in \{0, 0.1, \dots, 1\}$ .

The moments to be matched are the maximum level of assets in the system, the skewness of the distribution of assets, average leverage (assets over equity) and average interbank assets. In the data the banking system is characterized by a positively skewed distribution of assets, a fact we can match well. The estimation based on the above statistics is typically characterized by a trade-off between matching the distribution of total assets and the distribution of leverage. A similar, albeit milder, trade-off exists between matching the distribution of assets and that of interbank assets. We try to strike an optimal balance among all those statistics.<sup>35</sup>

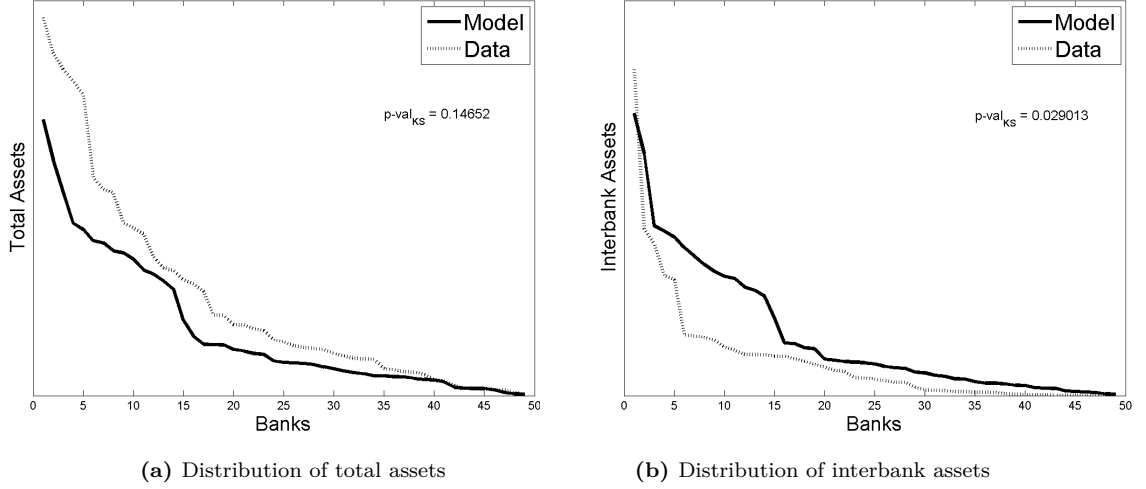
The simulated method of moments delivers values for the three parameters of interest of 0.03 for  $r_{max}^a$ , 0 for  $\delta_{max}$  and 0.2 for  $\omega_b$  ( $= \tilde{\omega}_l$ ). The deviations of moments with respect to the data counterpart are -26% for the maximum level of assets, -5% for the skewness of the distribution of assets, -32% for the average leverage and 67% for the average level of interbank assets.

Figure 1 presents the distribution of total assets and interbank assets for both the model and the data. Additionally it presents p-values for two-sided Kolmogorov-Smirnov tests to assess closeness between the model-based distributions and the empirical equivalent. We cannot reject the null-hypothesis that the series for total assets come from the same distribution. For interbank

<sup>33</sup>Note that the goal of this exercise is not to identify some deep structural parameters but rather to achieve a realistic banking system to be subsequently used for policy analysis.

<sup>34</sup>The variance of returns on non-liquid assets and default probability are computed accordingly. The default probability  $\delta$  is defined in A

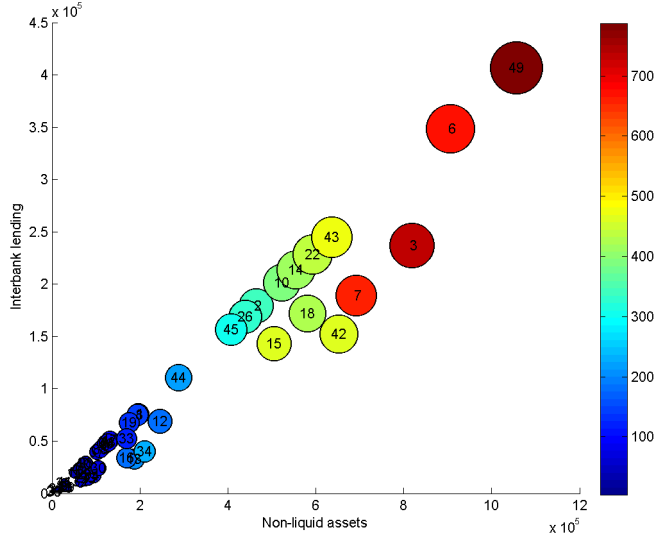
<sup>35</sup>For the weighting matrix  $\mathbf{W}$  we have a diagonal matrix, but we do not choose an identity matrix as we want to put more weight on matching some specific elements, as in Blasques et al. (2015). In particular, the weight on the maximum level of assets ( $w_{11}$ ) is set to 50, while the one on average leverage ( $w_{33}$ ) is set to 10.



**Figure 1:** Distribution of total and interbank assets for model and data.  $p\text{-val}_{KS}$  denotes the p-value of a two-sided Kolmogorov-Smirnov test.

assets we cannot reject at the 1% confidence level. We conclude that the model, albeit a stylized representation of a complex reality, provides a good fit of the data.

Prior to the policy experiments and before obtaining the interbank matrix, Figure 2 presents a summary snapshot of the balance sheet of the model banking system. The charts puts the emphasis on non-liquid asset investment and interbank lending, two of the main channels of contagion in the model. Each bubble in the snapshot represents a bank. The axes measure the amount of interbank lending and non-liquid assets for each bank and the color of the bubble indicates the extent of systemic importance as measured by the indicator outlined in section 4. Large banks, those who start with a high level of equity, are the ones who invest more (have the largest amount of non-liquid assets) and also leverage more (external funding is needed to cover the intense investment activity). Those banks are the ones that contribute the most to systemic risk. In fact the more leveraged they are on the interbank market the higher is the loss that they would transmit to the system in case of debt default. Second, as they invest heavily in non-liquid assets they are obliged to hold a high level of bank capital: upon an adverse shock those banks are forced into massive fire sales thereby prompting severe asset price declines and transmitting large accounting losses to other banks. Hence, not surprisingly those banks are assigned a high index of systemic importance (red colored ball in the snapshot).



**Figure 2:** Baseline configuration. Nodes size indicates total assets, while node color denotes the systemic importance ranking.

## 5.2 Interbank Matrix and Network Properties

The second and last part of the process of matching the network of large European banks consists in obtaining the interbank matrix from the optimal balance sheet structure obtained above. The optimization problem of banks delivers the row sum and column sum of the interbank matrix (i.e. total lending and total borrowing by bank respectively). Given the optimal balance sheet quantities chosen by banks, we reconstruct the interbank matrix using the RAS algorithm, as noted in [subsection 3.6](#). This algorithm efficiently delivers the maximum entropy solution and can incorporate any additional information besides the marginals of the target matrix. In particular, we use the matrix of exposures between large European banks as a prior. This allows us to generate an interbank matrix which respects the optimal quantities chosen by banks in our model while at the same time preserving as much as possible the structural features of the network we want to mimic.

Given our baseline network representation we can compute a number of traditional network metrics and compare them to the data equivalent. [Table 2](#) presents this comparison for a number of network metrics.<sup>36</sup> The table immediately shows that the matching is almost perfect.

[Alves et al. \(2013\)](#) note that the European banking system features a network with some “national champions” which are heavily connected between themselves, therefore exhibiting a relatively

<sup>36</sup>Note that the two networks present the same number of nodes, namely 49. An [online appendix](#) presents a more formal treatment of the network measures shown in [Table 2](#), which are standard in the literature.

	<b>Model</b>	<b>Data</b>
Density (%)	63.14	63.14
Average Degree	30.31	30.31
Average Path Length	1.17	1.17
Betweenness Centrality (Av.)	0.04	0.05
Eigenvector Centrality (Av.)	0.08	0.10
Clustering Coefficient (Av.)	0.41	0.40
Assortativity		
<i>out-in degree</i>	-0.25	-0.25
<i>in-out degree</i>	-0.04	-0.04
<i>out-out degree</i>	-0.06	-0.06
<i>in-in degree</i>	-0.11	-0.11
Modularity (Maximum)	0.22	0.22
Reciprocity	0.72	0.72
Reciprocity (normalized)	0.25	0.25

**Table 2:** Network indicators of model and data

high density of 63%<sup>37</sup>. Our model replicates the density metric very well. The average number of connections per bank is 30 in the data<sup>38</sup>, implying an average path length close to 1. Both numbers are matched well by the model. We compute centrality metrics and clustering coefficients as averages for all nodes in the network. For the former we consider eigenvector and betweenness centrality and in both cases the model is very close to the data. For the latter, which represents the tendency of neighbors of a given bank to be connected between themselves, the model is also notably close to the data. Interbank networks typically present clustering coefficients which are larger than in random networks with the same degree distribution, but smaller than other economic networks such as input-output or trade networks. The network of large European banks presents a particularly large average clustering coefficient.

The assortativity coefficient measures the tendency of high (low) degree nodes to be connected to other high (low) degree nodes. The empirical evidence suggests that interbank networks are dis-assortative (i.e. present negative assortativity), a feature closely associated to the existence of a core-periphery structure and which implies that high degree nodes tend to be connected with low degree nodes. The network of large European banks is no exception and our simulated network mimics this feature very well. The modularity of a network measures the extent to which the network presents communities or modules within which the connections are maximized. Both the data and the model show positive modularity, indicating that there are more connections between nodes of the same type (i.e. nodes belonging to the same community) than one can expect by chance.

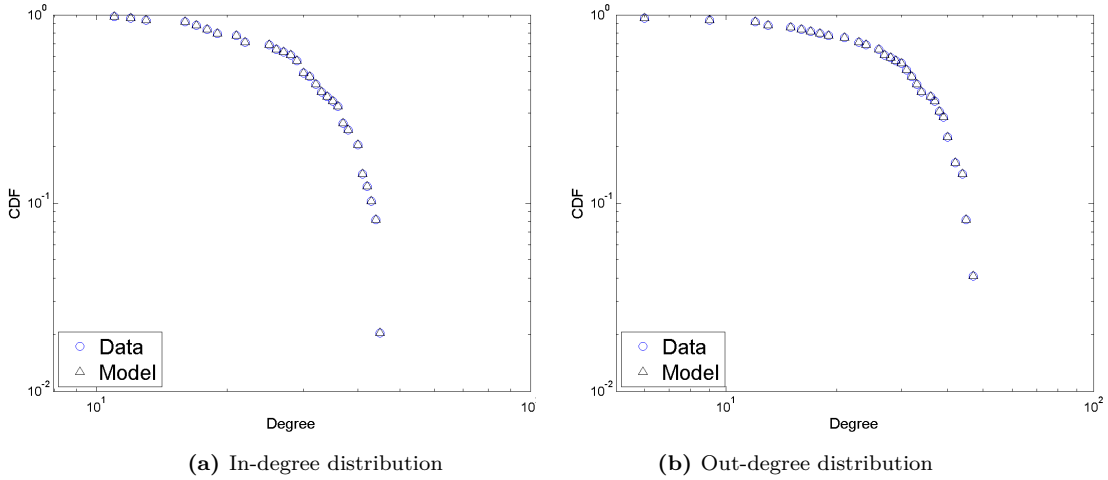
<sup>37</sup>This is somehow in contrast with the observations of other banking systems that are characterized by low density with graphs featuring a small core of highly connected banks and a large, loosely connected periphery.

<sup>38</sup>This is actually a large number relative to country-specific studies.



Finally, the reciprocity index quantifies how many connections in one direction are reciprocated by another connection going in the opposite direction. Again, the indicator for both model and data coincide: if bank A lends to bank B there is a 72% probability that bank B also lends to A.<sup>39</sup>

The last statistic that we consider is the degree distribution (the distribution of the number of connections). This informs about the underlying microstructure of the market. Degree distributions which resemble Poisson processes are typical of random networks with atomistic/competitive agents. Skewed distributions (like power laws) are indicative of networks with few “hubs” with high degrees and a large majority of nodes with low degrees. For the sake of completeness, Figure 3 presents the distribution of in- and out-degrees in both the model and the data in log-log scale as is standard in the literature. The model-based and the data-based distributions are very close to each other and they are both very much skewed.



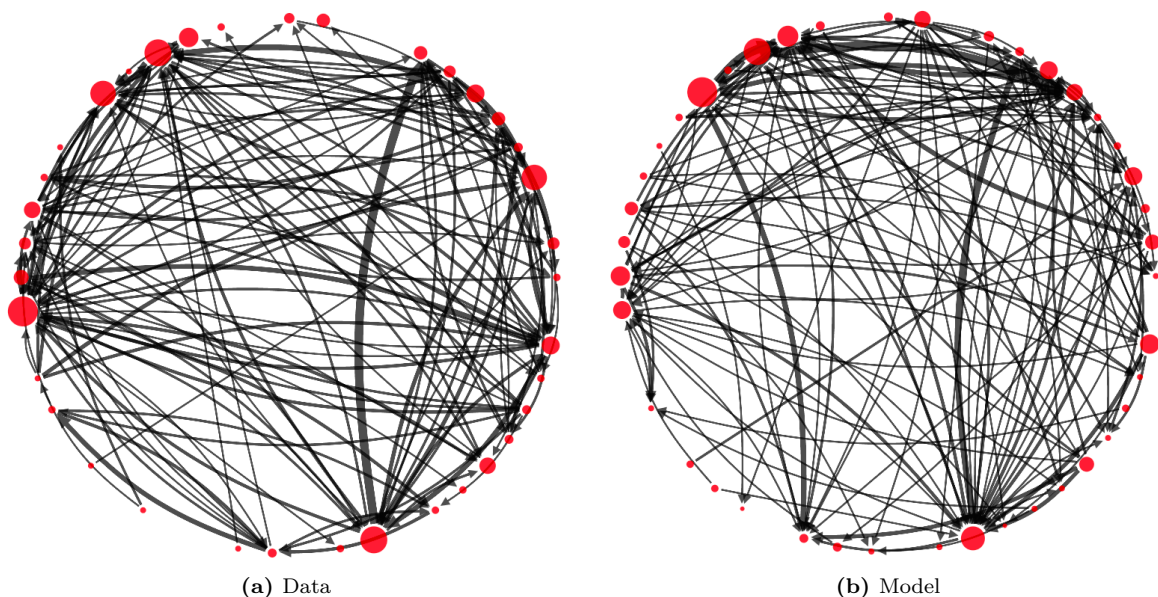
**Figure 3:** Degree distribution for model and data in log-log scale.

Finally, Figure 4 presents the network configuration (graphs) for both model and data. In order to have better visibility, only the largest 150 links in value are shown for each chart.<sup>40</sup> For both networks this represents close to 10% of all non-zero links (in number), whereas in terms of exposures, the top 150 links account for roughly half of all exposures in the data and about 60% in

<sup>39</sup>The unnormalized reciprocity indicator ranges from 0 to 1. We also present a normalized version of reciprocity which allows for better comparability between different networks and also provides a measure of the reciprocity present in the network relative to a random network with the same number of nodes and links. The value of normalized reciprocity of 0.25 indicates that more links are reciprocated than could be expected based on a random network that preserves nodes’ degrees. For more details on network methods and the relevant references see the [online appendix](#).

<sup>40</sup>Both networks are quite dense (see Table 2), so if one plots all the links present in the system it is hard to visually appreciate where the bulk of the action is. Appendix B presents both network charts without a cap on the number of links shown.

the model. In both data and model the big players in terms of size account for a big share of the market and transact the largest amounts, in particular between themselves. The last observation is another manifestation of the non-random nature, but rather the hub-based characterization of our banking network.



**Figure 4:** Network charts. Node size indicates total assets. Arrows go from lender to borrower and their width indicates size of exposures. Only the top 150 links in terms of value are shown.

Three elements are responsible for the model’s ability in matching data. The first is that our model is fairly rich as it includes several realistic channels of contagion. Second, relevant model parameters have been calibrated by a method of simulated moments: this allows the model-based distribution of variables to stay close to the true data generating process. Finally, the interbank matrix obtained from the model mimics very well the structural features of the real world counterpart.

## 6 Policy Experiments

We have constructed a fairly rich banking system featuring several contagion channels. We have even ensured that the model is realistic and delivers properties which are very close to those observed in the data. Equipped with this model we are now in the position to conduct policy experiments. Our main goal is to assess to which extent the new liquidity regulations are able to contain contagion and systemic risk. There is pretty much agreement in the academic literature that liquidity crises

tend to precede and lead to widespread bank insolvency (see [Diamond & Rajan \(2005\)](#) among others). Banks operate largely by relying on short term liabilities: when those become scarce (either because of investor runs or because of interbank market freezes) banks are forced into liquidation of productive projects and/or fire sales. The latter quickly turn a liquidity crisis into an insolvency one. As explained earlier our model captures this link and also the feedback loops between illiquidity and insolvency. The mechanism just described also provided the main motivation for the Basel regulators to convincingly introduce new liquidity requirements.

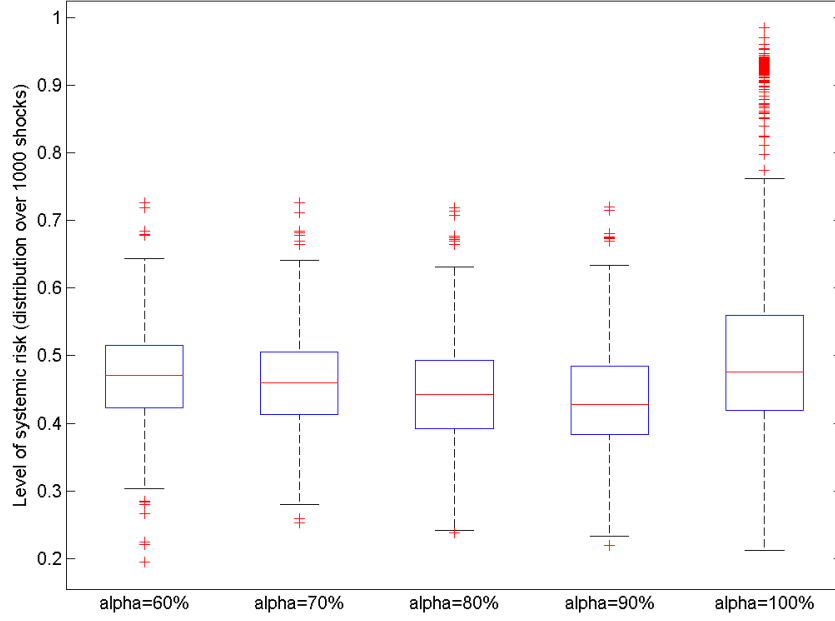
We therefore simulate our model to assess the impact of liquidity coverage ratios on the network in general and on systemic risk. In reality new regulations are introduced gradually (through a phase-in): we take this aspect into account in our simulations below.

## 6.1 Phase-in of the *Liquidity Coverage Ratio*

In the first policy experiment we evaluate the model and its response to shocks as the liquidity coverage ratio (LCR) is taken from its initial state to its full implementation as devised in the law. This translates into evaluating the model and submitting it to shocks for  $\alpha \in \{60\%, 70\%, 80\%, 90\%, 100\%\}$ . For each value of the parameter  $\alpha$  the model is simulated from scratch and shocked 1000 times in order to evaluate the distribution of systemic risk across all realizations of the shock vector. Note that for every model (i.e. for every value of  $\alpha$ ), the  $1000 \times 1$  shock vector remains the same, allowing for better comparability.

[Figure 5](#) presents the path of systemic risk for each phase of the LCR implementation. One would expect that a continuous increase in the coverage ratio brings about a monotonic decrease in the systemic risk profile. The figure shows that this is not the case. There is a mild reduction in the first steps, but the final move to 100% undoes the initial risk reduction. Furthermore, as the fully phased-in level of  $\alpha$  is reached, there is a substantial increase in the number of high systemic risk outliers (i.e. the number of shock realizations in which a high share of the system is wiped out by the initial shocks).

The rationale for our results is as follows. An increase in LCR has both beneficial as well as detrimental effects. The increase in LCR has obviously several beneficial effects. Generally speaking a higher LCR limits the degree of interbank leveraging, thereby limiting the likelihood of risk cascades through debt defaults. Also it requires banks to maintain liquidity buffers in anticipation of possible bank runs. This mechanism reduces the extent of project liquidation and fire sales in the face of investors' runs. An increase in LCR however can also produce severe liquidity shortages and have two types of detrimental effects. First, high LCR can severely impair the function that interbank markets have as insurance devices. LCR reduce the overall supply of liquidity in interbank markets, thereby reducing the pool of liquidity available for insurance. Second, in this particular policy experiment LCR are applied equally to all banks, which are however very different in terms of



**Figure 5:** Systemic risk for different stages of the phase-in of LCR.

their liquidity needs and asset exposure. Policy regimes in which instruments are applied equally to heterogeneous agents (banks in our case) typically induce relative distortions. For banks which are only mildly exposed to interbank leverage and non-liquid asset investment the high LCR produces an unnecessary liquidity shortage. Those banks are typically the ones which (by experiencing low returns on non-liquid assets) tend to be liquidity suppliers in the interbank market. By forcing those banks to retain more liquidity internally, the regulator implicitly reduces interbank supply. Liquidity scarcity increases also for highly leveraged banks which are in greater need of it. Overall the insurance function of the interbank market is impaired and contagion propagation is amplified.

Another interesting observation emerges from the simulations. While systemic risk increases on average for the initial part of the phase in, the risk of a few banks in fact decreases. This effect is primarily due to asset substitution. While the price of non-liquid assets collapses as described above, the returns on interbank lending increase due to liquidity scarcity. Banks which are exposed to non-liquid assets experience large accounting losses, while banks whose asset structure is more reliant on interbank market activity experience an increase in returns and a fall in risk. At the aggregate level, however, accounting losses imposed on all banks by fire sales tend to outweigh the gains, driving overall systemic risk upwards.

The above policy experiment shows that one problematic aspect of the phasing in of policies

stems from applying the same policy requirements to banks featuring differential portfolios. Some institutions start with less stable funding sources and/or more exposure to asset risk. For those institutions, an increase in the coverage ratio induces early asset liquidation, with losses which also impair the balance sheet of other banks thereby producing negative externalities. Those effects are unintended consequences of the prudential regimes. With those insights we then propose a new and more creative policy experiment which might help to limit the above-mentioned unintended consequences.

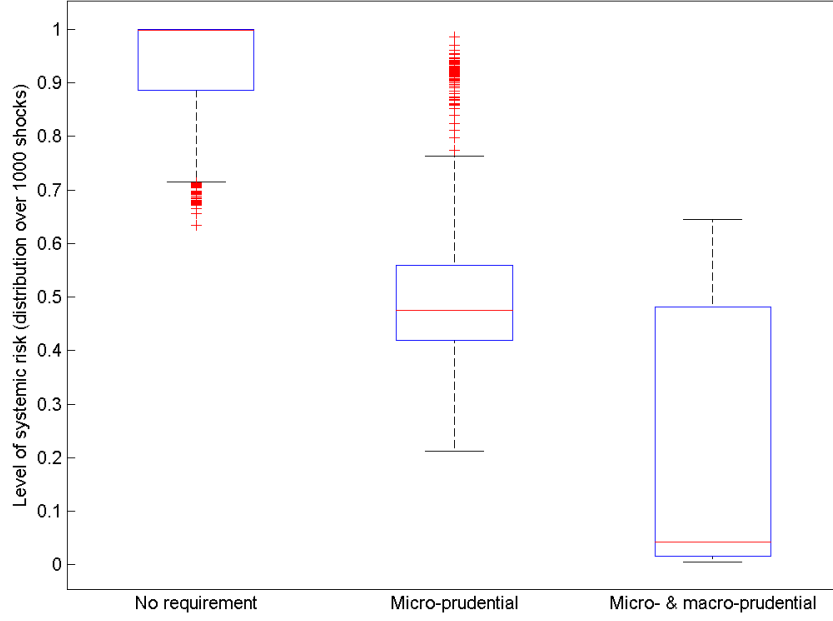
## 6.2 Mixing Micro- and Macro-Prudential Liquidity Requirements

Given the above findings we now propose and assess a policy experiment in which the LCR is applied differentially across banks, based upon their degree of systemic importance. In particular, systemically important banks are subject to stricter requirements than the others. In this respect one can think of this policy experiment as blending elements of micro and macro prudential regulation: while LCR are applied at the bank level (micro prudential regulation), their application is conditional on the index of systemic importance, as per [section 4](#) (the macro-prudential aspect). We start from a situation in which LCR has been already fully phased in (i.e.  $\alpha = 100\%$ ) and the calibration remains the same as in the benchmark simulation. The alternative policy regime can be described as follows. After computing the systemic importance index score for each bank in the benchmark scenario, we select the 10 banks with the highest index:<sup>41</sup> those banks are required to maintain a value  $\omega_d = 12.5\%$ . For the remaining “non-systemic” banks, the weight  $\omega_d$  is reduced so as to keep invariant the system-wide LCR requirement. [Figure 6](#) shows the impact of this policy on systemic risk. As before, each of the boxes summarizes the distribution of 1000 shock realizations. The right box in [Figure 6](#), labeled “Micro- and macro-prudential” represents this new policy experiment. Results are compared to the extreme case in which the LCR weight on deposits is set to zero (box labelled “No requirement”) and to the case of a  $\omega_d = 10\%$  equal for all banks (box labelled “Micro-prudential requirement”, corresponding to the right-most box in [5](#)).

[Figure 6](#) shows that this alternative differential approach performs considerably better in terms of risk profile than the prudential regime in which all banks are taxed equally. Systemic risk decreases monotonically. Note that this result is achieved for a constant level of required liquidity in the system. The rationale for this result is as follows. Relative to the scenario with no regulation, the introduction of the LCR has beneficial effects in that it reduces the exposure of banks to interbank debt and to short term debt subject to runs, fostering instead the build up of internal liquidity buffers. In the scenario in which the LCR is applied equally to all banks however there are also some detrimental effects (as explained in the previous section) due to the spillovers from the systemically important banks to the others. Instead, in the new scenario in which LCR are

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<sup>41</sup>These are the banks which present a systemic importance score above 400 basis points.



**Figure 6:** Systemic risk for different prudential regimes.

applied differentially, there are effectively only the beneficial effects of LCR regulations. Systemic important banks have to raise their internal liquidity buffers, hence they reduce their exposure to interbank and non-liquid asset markets thereby reducing the likelihood of contagion. As for the other banks, they are not liquidity constrained as in the scenario with common-to-all regulation. On the contrary those banks can free up liquidity and help to mitigate the interbank liquidity shortage. Overall the insurance function of interbank markets is preserved, while asset and debt contagion are mitigated.

## 7 Concluding Remarks

Understanding the unfolding of cascades and the interaction between contagion and amplification mechanisms is key for effective regulation and crisis prevention. We build a banking network model which provides a unified framework to study these systemic loops. The model features distress and contagion stemming from both the asset and liability sides of banks' balance sheets. Contagion can arise due to network externalities, pecuniary (fire sale) externalities, liquidity hoarding, and bank runs on short term liabilities. The latter represents a novel liability-side channel in our model. Banks can enter interbank markets for insurance motives. However, the beneficial effects of

insurance have to be balanced with the above-mentioned contagion channels. Taken together, all those channels explain the emergence and fluctuations in systemic risk.

The model is calibrated to match certain aspects of the network of large European banks using a method of moments procedure. Given the empirical validity of our model we use it to assess prudential regulation. We focus in particular on the recently adopted liquidity regulation. Given that in our model liquidity freezes, due to bank runs or interbank defaults, ignite solvency crises, the environment we analyze is particularly well suited to answer those questions. We study the effects of a phase-in of the liquidity coverage ratio (LCR). We find surprisingly that LCR, while reducing systemic risk in the initial phase, might produce an increase of it in the final phase. First, high LCR reduce the insurance function of interbank markets. Second, when applied equally to all banks LCR impose unnecessary liquidity shortages on banks which are mildly leveraged and which would otherwise act as interbank liquidity providers. Motivated by this result we investigate if imposing LCR differentially across banks and conditional on an index of systemic importance might deliver a better result. Our results show that this simple skewing of requirements toward more systemic banks is highly effective in delivering a more stable system.

We assessed prudential requirements solely based upon minimizing a systemic risk criteria. However prudential regulators typically face trade/offs between fostering investment and smoothing systemic risk. It would then be of interest to study the design of optimal policy upon a criterion that could strike a balance between the two opposing forces. We leave this for future research.

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## A Banks' Profit Function and Variance of Profits

**Profits** The bank's profits are given by the returns on lending in the interbank market (at the interest rate  $r^l$ ) plus returns from investments in non-liquid assets (with rate of return  $r_i^a$ ) minus the expected costs from interbank borrowing and the cost of servicing depositors. When lending an amount  $l_{ij}$  to bank  $j$ , bank  $i$  expects to earn the following amount:

$$\underbrace{(1 - \delta_j) (r^l + r_j^p) l_{ij}}_{\text{with no default}} + \underbrace{\delta_j (r^l + r_j^p) (1 - \xi) l_{ij}}_{\text{with default}} \quad (16)$$

where  $\xi$  stands for a loss-given-default parameter and  $\delta_j$  for the default probability of  $j$ . If bank  $j$  cannot default, then bank  $i$  simply gets the risk-free rate:

$$l_{ij} r^l \quad (17)$$

Equating 16 and 17, one can solve for the fair risk premium charged to counterparty  $j$ :

$$r_j^p = \frac{\xi \delta_j}{1 - \xi \delta_j} r^l \quad (18)$$

The premium is calculated such that, by lending to  $j$ , bank  $i$  expects to get  $r^l l_{ij}$  (to see this, plug the premium back into 16). Condition 16 can also be interpreted as a participation constraint: bank  $i$  will lend to bank  $j$  only if it gets an expected return from lending equal to the risk free rate, i.e. the opportunity cost of lending. Aggregating over all counterparties (and recalling that  $l_i = \sum_{j=1}^k l_{ij}$ ), we obtain the overall gain that bank  $i$  expects to achieve through interbank lending, namely  $r^l l_i$ . On the other hand, as a borrower, bank  $i$  must always pay the premium associated to its own default probability.<sup>42</sup> Therefore the cost of borrowing is given by:  $r_i^b b_i = (r^l + r_i^p) b_i = \frac{1}{1 - \xi \delta_i} r^l b_i$ . That part of profits derived from interbank market activity is therefore given by:  $r^l l_i - (r^l + r_i^p) b_i$ .

We assume that banks have different investment abilities, and/or access to investment opportunities with varying degrees of profitability. This is reflected in heterogenous returns on non-liquid assets, which are also exogenous to the model. The gains from investment in non-liquid assets are given by  $r_i^a \frac{a_i}{p}$ , where  $r_i^a$ ,  $a_i$  and  $p$  stand respectively for the heterogenous return on non-liquid assets, the stock and the price of such assets.

Finally, banks must pay depositors an amount equal to  $r_i^d d_i$ , where  $r_i^d$  is the interest on deposits.<sup>43</sup> Given all these elements, profits of bank  $i$  take the following form:

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<sup>42</sup>Since banks charge a fair risk premium, the returns from non-defaulting borrowers offset losses stemming from defaulting borrowers. On the other hand, borrowing banks must always pay the premium.

<sup>43</sup>Details on the calibration of the bank-specific interest on deposits are provided in [section 5](#).

$$\pi_i = r_i^a \frac{a_i}{p} + r^l l_i - (r^l + r_i^p) b_i - r_i^d d_i = r_i^a \frac{a_i}{p} + r^l l_i - \frac{1}{1 - \xi \delta_i} r^l b_i - r_i^d d_i \quad (19)$$

**Variance of Profits** We now derive an expression for the variance of profits. Notice that volatility derives from risk in non-liquid asset returns, in borrowing default premia and in returns on deposits<sup>44</sup>. Hence profits' volatility reads as follows:

$$\begin{aligned} \sigma_\pi^2 &= \text{Var} \left( r_i^a \frac{a_i}{p} + r^l l_i - \frac{1}{1 - \xi p d_i} r^l b_i - r_i^d d_i \right) \\ &= \left( \frac{a_i}{p} \right)^2 \sigma_{r_i^a}^2 - (b_i r^l)^2 \text{Var} \left( \frac{1}{1 - \xi p d_i} \right) + 2 a_i r^l b_i \text{cov} \left( r_i^a, \frac{1}{1 - \xi p d_i} \right) \\ &\quad - d_i^2 \text{Var}(r_i^d) + 2 a_i d_i \text{cov}(r_i^a, r_i^d) + 2 r^l b_i d_i \text{cov} \left( \frac{1}{1 - \xi p d_i}, r_i^d \right) \end{aligned} \quad (20)$$

We know that  $\delta_i \in [0, 1]$ . Furthermore, even when  $f(\delta_i) = \frac{1}{1 - \xi \delta_i}$  is a convex function, over a realistic range of  $\delta_i$  it is essentially linear and it is therefore sensible to obtain the variance of  $f(\delta_i)$  through a first order Taylor approximation around the expected value of  $\delta_i$ , which yields:

$$\text{Var} \left( \frac{1}{1 - \xi \delta_i} \right) = \xi^2 (1 - \xi E[\delta_i])^{-4} \sigma_{\delta_i}^2 \quad (21)$$

We assume that the ex ante correlation between return on non-liquid assets and costs of borrowing is zero, hence we can set the first covariance term in Equation 20 to zero. Additionally, given the assumptions on the interest on deposits, we can also set the two remaining covariance terms to zero.

This leaves us with the following expression for the variance of profits, which can then be plugged in the expression for the objective function in the main text (Equation 3):

$$\sigma_\pi^2 = \left( \left( \frac{a_i}{p} \right)^2 \sigma_{r_i^a}^2 - (b_i r^l)^2 \xi^2 (1 - \xi E[\delta_i])^{-4} \sigma_{\delta_i}^2 - d_i^2 \text{Var}(r_i^d) \right)$$

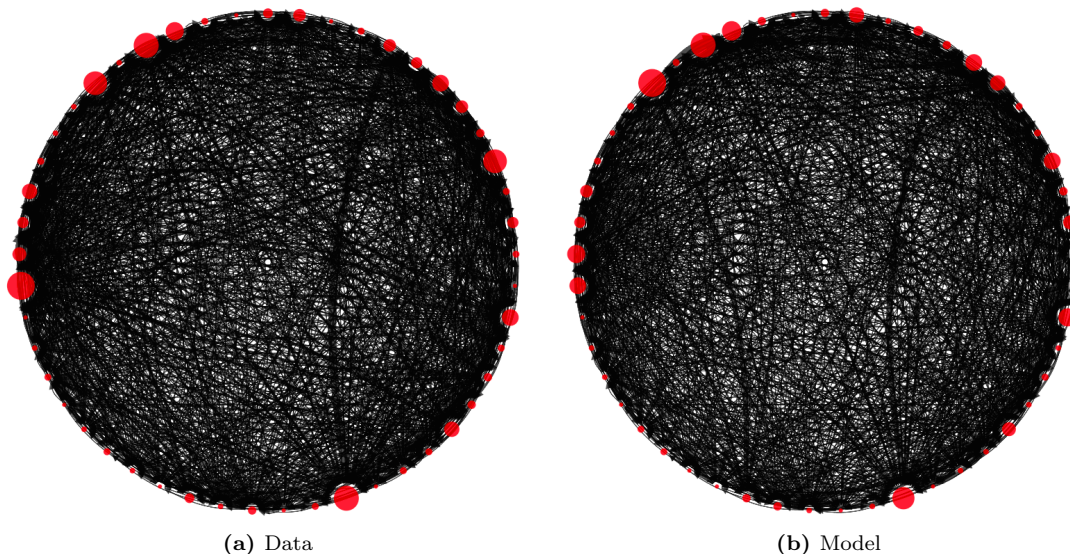
## B Additional Figures

## C Systemic Importance Methodology

The indicator-based measurement approach that we use to rank banks in this paper is based upon a set of categories that capture different aspects of systemic importance. Each category is in turn composed by a set of core indicators which are the observables upon which the measurement is

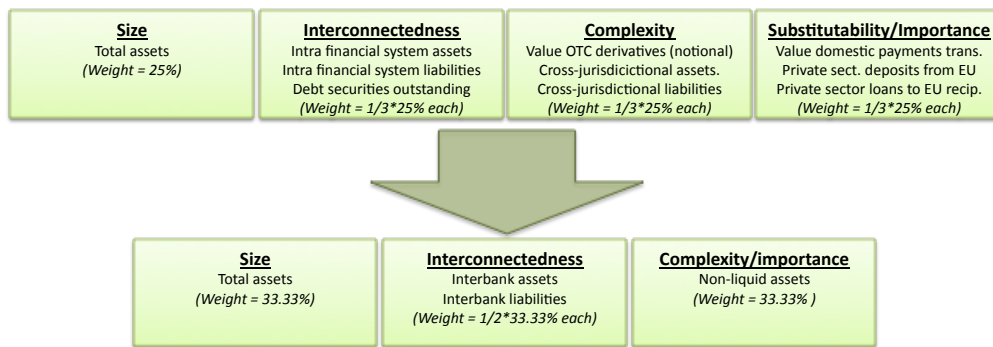
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<sup>44</sup>In setting up the system there is no uncertainty on the price of non-liquid assets, which is set to 1.



**Figure 7:** Network charts. Node size indicates total assets. Arrows go from lender to borrower and their width indicates size of exposures. All links are shown.

based. Figure 8 presents the categories and indicators of the O-SII (other systemically important institutions) methodology proposed in EBA (2014) as the European adaptation of the G-SIB methodology developed by the BCBS and the Financial Stability Board. We try to stay closer to the European version of the methodology since the model is calibrated to a sub-system of European banks. The criteria and indicators outlined in EBA (2014) are in fact very similar to the methodology presented by the BCBS to assess the importance of global systemically important banks. The main difference between the criteria presented by the BCBS and the European Banking Authority (EBA) is that the former has 5 categories whereas the latter has 4. The fifth category in the BCBS proposal relates to the global nature of the exercise by focusing on *cross-jurisdictional activities* of banks. In the EBA proposal, cross-jurisdictional assets and liabilities are also present but they are subsumed into the *complexity* category.



**Figure 8:** Indicators for scoring process to rank systemically important institutions, based on BCBS (2013, 2014) and EBA (2014). Upper level indicates the O-SII criteria and lower level denotes the adaptation to our framework